TRANSACTIONS

OF THE

AMERICAN INSTITUTE

OF

ELECTRICAL ENGINEERS

JULY TO DECEMBER, 1920.

VOL. XXXIX, PART II

PUBLISHED BY THE
American Institute of Electrical Engineers
33 West Thirty-ninth Street
New York, N. Y., U. S. A.
1920

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Press of McIlroy & Emmet, 22 Thames Street, New York

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C. J. Fechheimer **Vol. xxxix—1920, pp. 291-299**

The method proposed makes use of idle operation of the machine as a motor, the voltage being varied, and speed kept constant. After deducting the armature $I^2 R$ losses from the watts input, the remaining watts are plotted against the voltage. A formula is derived based upon the assumption that the watts are equal to constant windage and friction loss plus core loss which latter varies as a constant power of the voltage. An example is given of the close agreement with the test curve in the case of an induction motor; and other examples are cited.

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P. M. Lincoln **Vol. xxxix—1920, pp. 1477-1479**

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C. L. Fortescue **Vol. xxxix—1920, pp. 1481-1484**

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A general discussion.

3. UNITS, MEASUREMENTS AND INSTRUMENTS

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THE MEASUREMENT OF PROJECTILE VELOCITIES

P. E. Klopsteg and A. L. Loomis **Vol. xxxix—1920, pp. 327-353**

The paper discusses the requirements imposed by proving ground practise upon a chronograph which is intended for general ammunition testing. Instruments of the standard pre-war pattern were entirely inadequate in number for testing the immense quantities of ammunition contracted for by the Government during the war. The instrument which was developed, and adopted as a standard ordnance chronograph, is designated "Aberdeen chronograph." An account of its development is given. The Aberdeen chronograph, and the procedure in determining velocities by this means, are described in detail. For the sake of comparison, the Boulenge chronograph is briefly described. Comparative results as to speed and accuracy of measurement are given.

Discussion, pages 354-358, by Messrs. E. E. F. Creighton, C. H. Sharp, R. Kegerreis and P. E. Klopsteg.

A general discussion and description of certain special problems to be solved.

A NEW FORM OF VIBRATION GALVANOMETER

P. G. Agnew **Vol. xxxix—1920, pp. 359-367**

The instrument described, has a sensitivity higher than other forms of the moving-iron type, but less than that of the most sensitive forms of the moving-coil type, has the advantages of sturdiness, quick responsiveness, and freedom from the effects of external vibration. It consists essentially of a fine steel wire, mounted on one pole of a permanent magnet and so arranged that the free end of the wire may vibrate between the poles of an electromagnet through which the current to be detected passes.

Discussion, pages 368-370, by Messrs. J. B. Whitehead, C. H. Sharp, W. H. Pratt, P. E. Klopsteg, and P. G. Agnew.

A PRECISION GALVANOMETRIC INSTRUMENT FOR MEASURING THERMOELECTRIC E. M. Fs.

T. R. Harrison and P. D. Foote **Vol. xxxix—1920, pp. 371-402**

A new principle has been developed whereby an ordinary millivoltmeter may be converted into an instrument in which the usual errors arising from a variable line resistance are entirely eliminated. The instrument measures true e. m. f. in a simple circuit or if connected across a resistance or network through which a current flows it indicates the potential drop which would have existed had the instrument not been connected. Various wiring diagrams are shown and methods are discussed for constructing instruments of zero temperature coefficient and properly damped. A new deflection potentiometer is described which offers considerable advantage over the ordinary type for small e. m. fs. in a circuit of variable resistance.

Discussion, pages 403-406, by Messrs. J. B. Whitehead, A. E. Kennelly, W. D. A. Peaslee, E. M. Hewlett, C. H. Sharp and H. B. Brooks.

NOTES ON THE SYNCHRONOUS COMMUTATOR

J. B. Whitehead and T. Isshiki **Vol. xxxix—1920, pp. 407-438**

In the use of the synchronous commutator in series connection as a suppressor, serious errors may arise due to relatively small amounts of capacity in the commutator and galvanometer circuits. The magnitude of these errors is studied for a number of different connections, and methods for eliminating them are pointed out. A number of wave forms are given, indicating the nature of the errors.

An appendix gives a theoretical analysis of two cases investigated and shows a close agreement with the experimental observations.

Discussion, pages 439-441, by Messrs. P. G. Agnew, E. D. Doyle, A. E. Kennelly, J. H. Morecroft and J. B. Whitehead.

OSCILLOGRAPHS AND THEIR TESTS

A. E. Kennelly, R. N. Hunter and A. A. Prior **Vol. xxxix—1920, pp. 443-487**

A method and technique for the testing and calibration of oscillographs is described, using an auxiliary vibrator or "oscillographmeter" for the production of Lissajous optical figures whereby the resonant frequency f_0 of the tested oscillograph may be readily ascertained. From this and one other test, which is preferably a comparative calibration at 60~ and at the resonant frequency, the bluntness of resonance B of the oscillograph is determined.

From the two essential constants f_0 and B of an oscillograph, its indication at any assigned frequency can be corrected for the inertia of its vibratory system. At high frequencies, the correction may be relatively large.

A number of oscillographs have been tested for their f_0 and B. The principal results obtained are reported in the paper.

Discussion, pages 488-494, by Messrs. F. S. Dellenbaugh, Jr., B. W. St. Clair, N. E. Bonn, M. A. Rusher and A. E. Kennelly.

THE ACCURACY OF COMMERCIAL ELECTRICAL MEASUREMENTS

H. B. Brooks **Vol. xxxix—1920, pp. 495-589**

The paper discusses the accuracy required in commercial electrical measurements, and the means of obtaining it. Conditions of use, external disturbing influences, and features of design and construction affecting the accuracy are considered in detail. The best types of instrument for measuring voltage and current are mentioned.

The sources of error in electrodynamic wattmeters are discussed, including the effect of instrument transformer errors. The principal factors affecting the accuracy of watthour meters are given. In conclusion, some improvements which should be soon forthcoming are mentioned.

Discussion, pages 590-615, by Messrs. W. H. Pratt, J. R. Craighead, E. P. Peck, F. V. Magalhaes, F. H. Bowman, A. Maxwell, H. H. Sticht, B. W. St. Clair, A. L. Ellis, H. P. Sleeper and H. B. Brooks.

A general discussion referring particularly to field practise.

SOME PRACTICAL EXPERIENCE WITH EMBEDDED TEMPERATURE DETECTORS IN LARGE GENERATORS

F. D. Newbury and C. J. Fechheimer **Vol. xxxix—1920, pp. 971-991**

A description of the various factors affecting the location of temperature detectors and the results obtained.

Discussion, pages 992-995, by C. J. Fechheimer.

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Discussion, pages 1111-1114, by Messrs. C. L. Fortescue, A. E. Kennelly, J. Slepian, L. W. Chubb, F. W. Peek, Jr. and J. B. Whitehead.

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A general discussion.

BRIDGE METHODS FOR ALTERNATING-CURRENT MEASUREMENTS

D. I. Cone **Vol. xxxix—1920, pp. 1743-1762**

This paper presents a resume of simple methods of utilizing "bridge" networks in alternating-current measurements of impedances and their components of effective resistance, self and mutual inductance and capacitance, and in frequency measurement.

Discussion, pages 1763-1765, by Messrs. W. A. Hillebrand, H. J. Ryan, W. D. Scott, H. V. Carpenter and D. I. Cone.

THE MEASUREMENT OF MAXIMUM DEMAND AND THE DETERMINATION OF LOAD FACTOR

P. A. Borden **Vol. xxxix—1920, pp. 1847-1883**

This paper gives a bird's-eye view of the situation as it exists today and, in an endeavor to reconcile some of the different opinions on the subject to show an actual comparison of the performances of a number of demand-measuring devices. From this comparison have been deduced some interesting facts which would seem to have important bearing upon the status of industrial load measurement.

Discussion, pages 1884-1894, by Messrs. P. M. Lincoln, C. I. Hall, W. H. Pratt and P. A. Borden.

4. INSULATION AND DIELECTRIC PHENOMENA

VENTILATION AND TEMPERATURE IN LARGE TURBO GENERATORS

B. G. Lamme **Vol. xxxix—1920, pp. 915-947**

A description of the various methods used for obtaining required ventilation with their advantages and disadvantages.

Discussion, pages 948-949, by Messrs. R. B. Williamson and W. J. Foster.

TEMPERATURES IN LARGE ALTERNATING-CURRENT GENERATORS

W. J. Foster **Vol. xxxix—1920, pp. 951-964**

A description of design practise in large alternators.

Discussion, pages 965-969, by Messrs. F. D. Newbury, W. I. Slichter, P. Torchio, A. S. Loizeaux, S. Haar, S. R. Bergman and W. J. Foster.

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HIGH-TENSION INSULATOR PORCELAIN

W. D. A. Peaslee **Vol. xxxix—1920, pp. 1179-1187**

Porcelain high-tension insulator requirements as to mechanical strength, ability to resist, sudden changes of temperature, porosity, homogeneity and temperature, coefficient of resistivity. Suggestion as to the influences of the Piezo electric effect and the deterioration is presented with a brief discussion of the degree of progress in art.

Discussion, pages 1188-1194, by Messrs. H. B. Vincent, C. L. Fortescue, R. M. Spurck, G. I. Gilchrest and W. D. A. Peaslee.

A general discussion with plea for the keeping of records by operators.

FACTORS CONTROLLING THE DESIGN AND SELECTION OF SUSPENSION INSULATORS

W. D. A. Peaslee **Vol. xxxix—1920, pp. 1645-1667**

A discussion of the factors entering into the design and operating behavior of suspension insulators and problems to be solved in designing a suspension insulator to overcome the objectionable features shown by experience to affect seriously the operation of the insulators in service.

Factors to be taken into consideration in the selection of suspension insulators for a given condition are given and a brief discussion of the general trend of future improvements is presented.

Discussion incorporated with that of paper by F. W. Peek, Jr. on "Electrical Characteristics of the Suspension Insulator—II."

UNIT VOLTAGE DUTIES IN LONG SUSPENSION INSULATORS

H. J. Ryan and H. H. Henline **Vol. xxxix—1920, pp. 1669-1684**

This paper deals chiefly with the quantitative relations that exist between the maximum and average voltage unit-duties in line suspension insulators made up of units in common use.

Discussion incorporated with that of paper by F. W. Peek, Jr., on "Electrical Characteristics of the Suspension Insulator—II."

ELECTRICAL CHARACTERISTICS OF THE SUSPENSION INSULATOR—II

The Line Insulators at the Higher Voltages

F. W. Peek, Jr. **Vol. xxxix—1920, pp. 1685-1705**

This paper reviews the duties of the line insulator at voltages above 100 kv. and compares them with the duties imposed by the lower voltages. The discussion is based in general upon data and operating experiences of many investigations made particularly during the last few years.

Discussion, pages 1706-1741, by Messrs. J. B. Fisken, M. T. Crawford, E. R. Stauffacher, L. C. Williams, G. E. Quinan, W. A. Hillebrand, K. A. Hawley, S. C. Lindsay, L. Lauridsen, A. C. Pratt, C. P. Osborne, D. W. Proebstel, J. C. Clark, H. H. Schoolfield, W. D. A. Peaslee, F. W. Peek, Jr. and H. J. Ryan.

STUDIES IN LIGHTNING PROTECTION ON 4000-VOLT CIRCUITS—II

D. W. Roper **Vol. xxxix—1920, pp. 1895-1940**

The paper continues the investigations described by the author in another paper presented before the Institute in June, 1916.

In the paper an endeavor is made to list the several factors which affect lightning arrester performance and to describe the methods of eliminating these several variables so as to permit the presentation of curves which show the relative merits of the arresters under investigation. The final curves show the performance of the several types of arresters as affected by their density. These results show that four of the types of arresters are practically identical in their protective value and about 60 per cent as efficient as a fifth type, while the results for the sixth type, being limited to a much smaller number of arresters, and a shorter period of time, do not appear to be conclusive.

Discussion, pages 1941-1966, by Messrs. C. P. Steinmetz, J. L. R. Hayden, E. E. F. Creighton, V. E. Goodwin, H. B. Gear, E. Bennett, H. R. Woodrow, H. R. Summerhayes and D. W. Roper.

LIGHTNING ARRESTER SPARK GAPS—II

Chester T. Allcutt **Vol. xxxix—1920, pp. 1967-1980**

This paper presents data giving the discharge characteristics of a commercial type of impulse gap under different conditions. Test data are also presented giving the characteristics of certain experimental gap structures designed to minimize the effects of adverse weather conditions, such as rain, fog, etc. A brief discussion of some of the factors that determine the degree of protection afforded by a lightning arrester spark gap is included in the paper. The term "protection factor" is defined and curves giving the protection factor of certain types of gap are presented.

No discussion.

LIFE AND PERFORMANCE TESTS OF O F LIGHTNING ARRESTERS

N. A. Lougee **Vol. xxxix—1920, pp. 1981-1994**

No discussion.

5. ELECTRIC CONDUCTORS

EDDY CURRENT LOSSES IN ARMATURE CONDUCTORS

R. E. Gilman **Vol. xxxix—1920, pp. 997-1048**

A mathematical determination of eddy current losses with formulas derived to meet various conditions.

Discussion, pages 1049-1056, by Messrs. S. L. Henderson and V. Karapetoff.

A general discussion including a difficult mathematical method of obtaining the same results.

6. MAGNETIC PROPERTIES AND TESTING OF IRON

MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-NICKEL ALLOYS

T. D. Yensen **Vol. xxxix—1920, pp. 791-815**

Part 1. This investigation was undertaken to determine whether any iron-nickel alloys could be found having a higher saturation value than pure iron. Alloys were prepared containing 0-100 per cent Ni.

Part II. Previous investigations on commercial iron-nickel alloys have shown that 25 to 35 per cent alloys have irreversible magnetic and electrical transformation points occurring below ordinary temperatures. The present investigation confirms these results for pure alloys.

Discussion, pages 816-822, by Messrs. S. L. Gokhale and L. D. Yensen.

A general discussion.

9. ELECTRICAL MACHINERY AND APPARATUS

A METHOD FOR SEPARATING NO-LOAD LOSSES IN ELECTRICAL MACHINERY

C. J. Fechheimer **Vol. xxxix—1920, pp. 291-299**

The method proposed makes use of idle operation of the machine as a motor, the voltage being varied, and speed kept constant. After deducting the armature $I^2 R$ losses from the watts input, the remaining watts are plotted against the voltage. A formula is derived based upon the assumption that the watts are equal to constant windage and friction loss plus core loss which latter varies as a constant power of the voltage. An example is given of the close agreement with the test curve in the case of an induction motor; and other examples are cited.

Discussion, pages 300-308, by Messrs. V. Karapetoff, W. F. Dawson, W. I. Slichter, P. L. Alge and C. J. Fechheimer.

A general theoretical discussion.

FLASHING OF 60-CYCLE SYNCHRONOUS CONVERTERS AND SOME SUGGESTED REMEDIES

M. W. Smith **Vol. xxxix—1920, pp. 631-658**

A general discussion of the causes of flashing, action of machines under various conditions and some remedies which have been put to practical test.

Discussion, incorporated with that of paper by R. J. Wensley on "Automatic Substations for Heavy City Service."

AUTOMATIC RAILWAY SUBSTATIONS

F. W. Peters **Vol. xxxix—1920, pp. 659-676**

This paper reviews the broad range of conditions to which railway automatic substations have been applied and also discusses the economies and operating advantages effected by their use. A description is given of the modern equipment with details of its operation. Special reference is made to improvements in design of control apparatus, to the positive sequence of starting the machines and the protection afforded the apparatus against overloads or other irregularities either outside or internal to the stations.

Discussion, incorporated with that of paper by R. J. Wensley on "Automatic Substations for Heavy City Service."

AUTOMATIC SUBSTATIONS FOR HEAVY CITY SERVICE

R. J. Wensley **Vol. xxxix—1920, pp. 677-689**

A description of the advantages to be obtained in many cases through the installation of automatic substations both in improved service and in reduction in installed feeder copper etc.

Discussion, (including that of papers by J. J. Linebaugh, M. W. Smith and F. W. Peters), pages 690-710, by Messrs. F. D. Newbury, S. Q. Hayes, C. H. Jones, D. C. Hershberger, C. A. Butcher, D. Bowman, J. F. Tritle, L. D. Bale, E. W. Cook, J. L. Burnham, M. W. Smith, F. W. Peters and R. J. Wensley.

A description of practical experiences with flashing and with automatic substations on various large installations; C. M. & St. P. R. R., City of Cleveland, etc.

THE BALDWIN-WESTINGHOUSE CHICAGO, MILWAUKEE AND ST. PAUL ELECTRIC LOCOMOTIVES

N. W. Storer **Vol. xxxix—1920, pp. 711-740**

A description in detail of the mechanical and electrical features of the locomotives and their action in operation.

Discussion, incorporated with that of paper by A. F. Batchelder and S. T. Dodd on "Passenger Locomotives for Chicago. Milwaukee and St. Paul Railway."

PASSENGER LOCOMOTIVES FOR CHICAGO, MILWAUKEE AND ST. PAUL RAILWAY

A. F. Batchelder and S. T. Dodd **Vol. xxxix—1920, pp. 741-752**

A detailed description of the G. E. designed locomotives both mechanical and electrical features and their operation.

Discussion, (including that of paper by N. W. Storer), pages 753-760, by Messrs. S. Q. Hayes, R. L. Wilson, E. H. Martindale, R. J. Wensley, F. D. Hall, R. E. Ferris, A. M. Candy, L. J. Hibbard, C. Townley, C. M. Davis, S. T. Dodd and N. W. Storer.

THE FIXATION OF ATMOSPHERIC NITROGEN BY THE SILENT ELECTRIC DISCHARGE PROCESS—I

C. F. Harding and K. B. McEachron **Vol. xxxix—1920, pp. 761-788**

A detailed description of the process of obtaining nitric acid from the air by corona discharge from the original apparatus down to the most up-to-date equipment.

Discussion, pages 789-790, by Messrs. Finch, Benjamin, K. B. McEachron and Tucker.

CLASSIFICATION OF LARGE TURBO GENERATOR FAILURES

Philip Torchio **Vol. xxxix—1920, pp. 903-905**

General analysis of causes of failures.

Discussion, pages 906-913, by Messrs. W. J. Foster, F. D. Newbury, R. B. Williamson. W. F. Dawson, R. Treat, J. Lyman and Philip Torchio. A general discussion.

VENTILATION AND TEMPERATURE IN LARGE TURBO GENERATORS

B. G. Lamme **Vol. xxxix—1920, pp. 915-947**

A description of the various methods used for obtaining required ventilation with their advantages and disadvantages.

Discussion, pages 948-949, by Messrs. R. B. Williamson and W. J. Foster.

TEMPERATURES IN LARGE ALTERNATING-CURRENT GENERATORS

W. J. Foster **Vol. xxxix—1920, pp. 951-964**

A description of design practise in large alternators.

Discussion, pages 965-969, by Messrs. F. D. Newbury, W. I. Slichter, P. Torchio, A. S. Loizeaux, S. Haar, S. R. Bergman and W. J. Foster.

A general discussion.

SOME PRACTICAL EXPERIENCE WITH EMBEDDED TEMPERATURE DETECTORS IN LARGE GENERATORS

F. D. Newbury and C. J. Fechheimer **Vol. xxxix—1920, pp. 971-991**

A description of the various factors affecting the location of temperature detectors and the results obtained.

Discussion, pages 992-995, by C. J. Fechheimer.

EDDY CURRENT LOSSES IN ARMATURE CONDUCTORS

R. E. Gilman **Vol. xxxix—1920, pp. 997-1048**

A mathematical determination of eddy current losses with formulas derived to meet various conditions.

Discussion, pages 1049-1056, by Messrs. S. L. Henderson and V. Karapetoff.

A general discussion including a difficult mathematical method of obtaining the same results.

THEORY OF SPEED AND POWER FACTOR CONTROL OF LARGE INDUCTION MOTORS BY NEUTRALIZED POLYPHASE ALTERNATING-CURRENT COMMUTATOR MACHINES

John I. Hull **Vol. xxxix—1920, pp. 1135-1169**

Theory of induction motor control, discussing single-range (below synchronism only) speed and power factor control by means of a constant-speed series commutator motor, by means of a constant-speed shunt commutator motor, by means of a constant-speed compound excited commutator motor; double range (all speeds above or below synchronism) speed and power factor control by means of a constant-speed shunt commutator motor; and double-range (either above or below synchronism) operation remote from synchronism.

Discussion, pages 1170-1177, by Messrs. R. E. Hellmund, C. W. Kincaid, and J. I. Hull.

Discussion of relative advantages of the "Kraemer" and "Scherbius" systems.

THE USE OF REACTORS ON LARGE CENTRAL STATION SYSTEMS

R. F. Schuchardt **Vol. xxxix—1920, pp. 1195-1208**

An introductory paper to "The Stability of Operation of High-Power Generating Systems" by C. P. Steinmetz. A description of reactor experience of the Commonwealth Edison Company of Chicago.

Discussion, pages 1204-1214, by Messrs. C. P. Steinmetz, A. E. Bauhan, R. A. Hentz, D. C. Jackson, P. Torchio, J. Lyman, C. J. Holslag and B. G. Jamieson.

An approximation of short-circuit current developed in Chicago system during trouble and outline of reactor practise by other companies.

POWER CONTROL AND STABILITY OF ELECTRIC GENERATING STATIONS

C. P. Steinmetz **Vol. xxxix—1920, pp. 1215-1270**

A full description of the theoretical and practical application of reactors.

Discussion, pages 1271-1287, by Messrs. R. E. Doherty, V. Karapetoff, H. R. Woodrow, E. G. Merrick, M. Brooks, J. A. Johnson, D. C. Jackson, D. W. Roper, P. Torchio and H. R. Summerhayes.

A general discussion.

DESIGN OF CONSTANT-CURRENT GENERATORS FOR ARC WELDING

K. L. Hansen **Vol. xxxix—1920, pp. 1387-1402**

A description of the various factors encountered in arc welding and the generator characteristics necessary to overcome them. Various types of field windings are discussed.

Discussion, page 1403, by Messrs. R. W. Owens and S. R. Bergman.

AUTOMATIC ARC WELDING APPARATUS

S. R. Bergman and R. L. Unland **Vol. xxxix—1920, pp. 1405-1407**

An abstract of a paper descriptive of generator with four-pole field and two-pole armature possessing inherent electrical characteristics desirable for single operator arc welding. No external resistors or regulating devices are required.

Discussion, page 1408, by Messrs. K. L. Hansen and S. R. Bergman.

ARC WELDING MACHINES OF THE WILSON WELDER AND METALS COMPANY

Alexander Churchward **Vol. xxxix—1920, pp. 1409-1410**

An abstract of a paper descriptive of apparatus furnishing constant current to the arc and in which length of the arc is limited. The use of the carbon pile is described. The system produces constant heat per unit area in the weld, not in the arc.

No discussion.

CHARACTERISTICS AND PERFORMANCE OF ARC WELDING APPARATUS

A. M. Candy **Vol. xxxix—1920, pp. 1411-1416**

An abstract of a paper descriptive of the increased efficiency of the constant-potential welding circuit and the decreased size and cost of apparatus by changing the generated circuit potential from 75 to 60 volts. Recent developments are described of both a-c. and d-c. types particularly an interconnected constant-current variable-voltage d-c. generator and exciter.

Discussion, pages 1417-1421, by Messrs. C. J. Holslag, W. O. Noble, S. R. Bergman, A. M. Candy and J. C. Lincoln.

A general discussion.

ARC WELDING MACHINERY OF THE U. S. LIGHT AND HEAT CORPORATION

W. A. Turbayne **Vol. xxxix—1920, pp. 1423-1426**

An abstract of a paper descriptive of an arc welding machine wherein control of the current and voltage and length of arc is accomplished by inherent action of the machine windings.

No discussion.

RECENT DEVELOPMENTS IN ELECTRO-PERCUSSIVE WELDING

Douglas F. Miner **Vol. xxxix—1920, pp. 1427-1431**

An abstract of a paper descriptive of machines utilizing the discharge of an electrolytic condenser for fusion substantially simultaneous with a percussive engagement.

Discussion, page 1432, by Messrs. J. C. Lincoln and D. F. Miner.

ELECTRIC ARC WELDING APPARATUS

Robert E. Kinkead **Vol. xxxix—1920, pp. 1433-1434**

An abstract of a paper on electric arc welding apparatus.

No discussion.

A-C. TRANSFORMERS FOR ARC WELDING

C. J. Holslag **Vol. xxxix—1920, pp. 1435-1442**

Abstract of a paper descriptive of the design and advantages of a-c. welding apparatus.

Discussion, pages 1443-1447, by Messrs. W. O. Noble, J. C. Lincoln, C. J. Holslag and B. W. David.

A general discussion.

THE APPLICATION OF D-C. GENERATORS TO EXCITER SERVICE

C. A. Boddie and F. L. Moon **Vol. xxxix—1920, pp. 1595-1616**

It is the purpose of this paper chiefly to discuss the relation of exciter design to problems, such as voltage range, responsiveness, stability, and type of drive, with especial reference to the requirements of automatic voltage regulators.

Discussion, pages 1617-1623, by Messrs R. E. Doherty and C. A. Boddie.

THE MEASUREMENT OF MAXIMUM DEMAND AND THE DETERMINATION OF LOAD FACTOR

P. A. Borden **Vol. xxxix—1920, pp. 1847-1883**

This paper gives a bird's-eye view of the situation as it exists today; and, in an endeavor to reconcile some of the different opinions on the subject to show an actual comparison of the performances of a number of demand-measuring devices. From this comparison have been deduced some interesting facts which would seem to have important bearing upon the status of industrial load measurement.

Discussion, pages 1884-1894, by Messrs. P. M. Lincoln, C. I. Hall, W. H. Pratt and P. A. Borden.

LIFE AND PERFORMANCE TESTS OF O F LIGHTNING ARRESTERS

N. A. Lougee **Vol. xxxix—1920, pp. 1961-1994**

No discussion.

ELECTROSTATIC CONDENSERS

V. E. Goodwin **Vol. xxxix—1920, pp. 1995-2015**

The first part is confined to fundamental characteristics of condensers and their relations to electric circuits containing inductance and resistance, and includes a discussion of the effects of switching condensers on and off such circuits. The second part of the paper describes some of the more important applications of condensers and illustrates the great demand there is in the electrical industry for this class of apparatus.

Discussion, pages 2016-2019, by Messrs. E. E. F. Creighton, C. T. Allcutt, V. E. Goodwin and H. R. Summerhayes.

GASEOUS CONDUCTION LIGHT FROM LOW-VOLTAGE CIRCUITS

D. McFarlan Moore **Vol. xxxix—1920, pp. 2021-2047**

A description of the development of light production with gaseous conductors, dwelling particularly on the latest and most efficient lamps of this type.

No discussion.

11. POWER PLANTS AND CENTRAL STATIONS

ESSENTIAL STATISTICS FOR GENERAL COMPARISON OF STEAM POWER PLANT PERFORMANCE

W. S. Gorsuch **Vol. xxxix—1920, pp. 91-98**

This paper briefly outlines a method for preparing statistical reports relating to power generation in steam power plants, whereby fairly close comparisons can be made of the efficiencies between different plants, without going into a detailed study of the thermal characteristics of the plants, or the intricate subject of power costs.

The essential items of steam power plant performance that should be recorded and a uniform method of expressing them are given in tabular form, followed by an illustration demonstrating the advantage of the proposed method.

Discussion, pages 99-100, by Messrs. E. J. Cheney and W. S. Gorsuch.

ECONOMICAL SUPPLY OF ELECTRIC POWER FOR THE INDUSTRIES AND THE RAILROADS OF THE NORTHEAST ATLANTIC SEABOARD

W. S. Murray **Vol. xxxix—1920, pp. 101-186**

A description of the super-power plan of generation and distribution of electric power whereby a saving will be made annually of over $300,000,000.

Contributions by W. L. Emmet, J. F. Johnson, H. G. Reist, F. D. Newbury, W. B. Potter, Philip Torchio, Percy H. Thomas, W. D. A. Peaslee and A. O. Austin.

No discussion.

FACTORS IN EXCITATION SYSTEMS OF LARGE CENTRAL STATION STEAM PLANTS

J. W. Parker and A. A. Meyer **Vol. xxxix—1920, pp. 1563-1573**

This paper points out the most essential requirements of excitation schemes; outlines two general methods followed in the design of such systems and from which a variety of schemes are built up; and discusses briefly the merits as well as demerits of factors determining the success of various schemes.

No discussion.

EXCITER PRACTISE IN THE NORTHWEST

J. D. Ross **Vol. xxxix—1920, pp. 1625-1633**

The author gives in table form the generator installation and the exciter sets, with size, voltage, drive, regulators and system of exciter connections for the principal hydroelectric plants in operation in the Northwest. He analyzes their essential characteristics and the difficulties encountered with automatic voltage regulators in plants having generators of different sizes and makes. The author advocates for an entirely new large hydroelectric or steam plant a system of excitation in which each generator is supplied with its own shunt-wound exciter driven by a prime mover and furnished with its own regulator. In an existing plant, not designed on the unit system, the use of one large exciter to operate the entire plant through field rheostats is the best compromise, the old exciter system being used as the duplicate for emergency.

Discussion incorporated with that of paper by Cox and Michener on "Generator Excitation Practise in the Hydroelectric Plants of the Southern California Edison Company."

GENERATOR EXCITATION PRACTISE IN THE HYDROELECTRIC PLANTS OF THE SOUTHERN CALIFORNIA EDISON COMPANY

H. H. Cox and H. Michener **Vol. xxxix—1920, pp. 1633-1636**

A description of apparatus installed, method of operating and changes being made and proposed.

Discussion, pages 1637-1644, by Messrs. C. J. Fechheimer and J. Lyman.

Discussion deals chiefly with amount of current required to charge a transmission line, etc.

12. PARALLEL OPERATION

THE USE OF REACTORS ON LARGE CENTRAL STATION SYSTEMS

R. F. Schuchardt **Vol. xxxix—1920, pp. 1195-1203**

An introductory paper to "The Stability of Operation of High-Power Generating Systems" by C. P. Steinmetz. A description of reactor experience of the Commonwealth Edison Company of Chicago.

Discussion, pages 1204-1214, by Messrs. C. P. Steinmetz, A. E. Bauhan, R. A. Hentz, D. C. Jackson, P. Torchio, J. Lyman, C. J. Holslag and B. G. Jamieson.

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A general discussion.

13. TRANSMISSION LINES

ECONOMICAL SUPPLY OF ELECTRIC POWER FOR THE INDUSTRIES AND THE RAILROADS OF THE NORTHEAST ATLANTIC SEABOARD

W. S. Murray **Vol. xxxix—1920, pp. 101-166**

A description of the super-power plan of generation and distribution of electric power whereby a saving will be made annually of over $300,000,000.

Contributions by W. L. Emmet, J. F. Johnson, H. G. Reist, F. D. Newbury, W. B. Potter, Philip Torchio, Percy H. Thomas, W. D. A. Peaslee and A. O. Austin.

No discussion.

INHERENT REGULATION OF CONTINUOUS CURRENT CIRCUITS

A. L. Ellis and B. W. St. Clair **Vol. xxxix—1920, pp. 309-330**

This paper discusses the voltage changes, inherent in d-c. circuits, upon change of load. These variations are independent of *i r* drops or speed of prime movers. A simple means of mitigating their effect is given.

Discussion, pages 331-336, by Messrs. V. Karapetoff, H. R. Summerhayes, L. W. Thompson, B. W. St. Clair and Philip Torchio.

A general discussion.

14. ELECTRIC SERVICE DISTURBANCES AND PROTECTION

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A general discussion.

SHORT-CIRCUIT PROTECTION FOR DIRECT-CURRENT SUBSTATIONS

J. J. Linebaugh **Vol. xxxix—1920, pp. 617-629**

The author includes an outline of the progress made in protection of direct-current machinery from short circuits since the publication of a paper at Atlantic City on this subject.

The improvements mentioned include the refinement and perfection in details of the flash barriers; a new design of high-speed circuit breaker for both direct-current substations and electric locomotives; a new high reluctance commutating pole 60-cycle synchronous converter and a new design of protected brush holder. An instructive analysis of conditions during direct-current short circuits is shown by several photographs, oscillographs and diagrams. Special reference is made to operating results on the electric zone of the Chicago, Milwaukee and St Paul Railroad.

Discussion incorporated with that of paper by R. J. Wensley on "Automatic Substations for Heavy City Service."

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CLASSIFICATION OF LARGE TURBO GENERATOR FAILURES

Philip Torchio **Vol. xxxix—1920, pp. 902-905**

General analysis of causes of failures.

Discussion, pages 906-913, by Messrs. W. J. Foster, F. D. Newbury, R. B. Williamson, W. F. Dawson, R. Treat, J. Lyman and Philip Torchio.

A general discussion.

THE CORONA VOLTMETER AND THE ELECTRIC STRENGTH OF AIR

A Natural Secondary Standard of Voltage

J. B. Whitehead and T. Isshiki **Vol. xxxix—1920, pp. 1067-1110**

An improved form of the corona voltmeter is described. Precision measurements of crest values of high alternating voltage taken in the high-tension circuit are compared with the indications of the corona voltmeter.

The law of corona has been determined to a higher degree of accuracy, and a modification in the form of the law as heretofore accepted is revealed.

As based on the precision voltage measurements the corona voltmeter is proposed as a natural secondary standard of high voltages. Its advantages as a standard, and its practical operation are described.

Discussion, pages 1111-1114, by Messrs. C. L. Fortescue, A. E. Kennelly, J. Slepian, L. W. Chubb, F. W. Peek, Jr. and J. B. Whitehead.

A discussion of the relative advantages of various methods of measurement of high voltages.

THE USE OF REACTORS ON LARGE CENTRAL STATION SYSTEMS

R. F. Schuchardt **Vol. xxxix—1920, pp. 1195-1203**

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A general discussion.

VOLTAGE STRESSES IN REACTORS IN SERVICE

F. H. Kierstead and R. Meeker **Vol. xxxix—1920, pp. 1289-1328**

In this paper the authors present experimental data which indicate the voltage stresses existing in current-limiting reactors under different conditions. Physical explanations of the phenomena are offered.

Discussion, pages 1329-1335, by Messrs. J. F. Peters, P. Torchio, H. R. Woodrow, C. L. Fortescue, H. B. Dwight, A. Nyman, J. Slepian and F. H. Kierstead.

A general discussion.

FACTORS CONTROLLING THE DESIGN AND SELECTION OF SUSPENSION INSULATORS

W. D. A. Peaslee **Vol. xxxix—1920, pp. 1645-1667**

A discussion of the factors entering into the design and operating behavior of suspension insulators and problems to be solved in designing a suspension insulator to overcome the objectionable features shown by experience to affect seriously the operation of the insulators in service.

Factors to be taken into consideration in the selection of suspension insulators for a given condition are given and a brief discussion of the general trend of future improvements is presented.

Discussion incorporated with that of paper by F. W. Peek, Jr. on "Electrical Characteristics of the Suspension Insulator—II."

UNIT VOLTAGE DUTIES IN LONG SUSPENSION INSULATORS

H. J. Ryan and H. H. Henline **Vol. xxxix—1920, pp. 1669-1684**

This paper deals chiefly with the quantitative relations that exist between the maximum and average voltage unit-duties in line suspension insulators made up of units in common use.

Discussion incorporated with that of paper by F. W. Peek, Jr., on "Electrical Characteristics of the Suspension Insulator—II."

ELECTRICAL CHARACTERISTICS OF THE SUSPENSION INSULATOR—II

The Line Insulators at the Higher Voltages

F. W. Peek, Jr. **Vol. xxxix—1920, pp. 1685-1705**

This paper reviews the duties of the line insulator at voltages above 100 kv. and compares them with the duties imposed by the lower voltages. The discussion is based in general upon data and operating experiences of many investigations made particularly during the last few years.

Discussion, pages 1706-1741, by Messrs. J. B. Fisken, M. T. Crawford, E. R. Stauffacher, L. C. Williams, G. E. Quinan, W. A. Hillebrand, K. A. Hawley, S. C. Lindsay, L. Lauridsen, A. C. Pratt, C. P. Osborne, D. W. Proebstel, J. C. Clark, H. H. Schoolfield, W. D. A. Peaslee, F. W. Peek, Jr. and H. J. Ryan.

STUDIES IN LIGHTNING PROTECTION ON 4000-VOLT CIRCUITS—II

D. W. Roper **Vol. xxxix—1920, pp. 1895-1940**

The paper continues the investigations described by the author in another paper presented before the Institute in June, 1916.

In the paper an endeavor is made to list the several factors which affect lightning arrester performance and to describe the methods of eliminating these several variables so as to permit the presentation of curves which show the relative merits of the arresters under investigation. The final curves show the performance of the several types of arresters as affected by their density. These results show that four of the types of arresters are practically identical in their protective value and about 60 per cent as efficient as a fifth type, while the results for the sixth type, being limited to a much smaller number of arresters, and a shorter period of time, do not appear to be conclusive.

Discussion, pages 1941-1966, by Messrs. C. P. Steinmetz, J. L. R. Hayden, E. E. F. Creighton, V. E. Goodwin, H. B. Gear, E. Bennett, H. R. Woodrow, H. R. Summerhayes and D. W. Roper.

LIGHTNING ARRESTER SPARK GAPS—II

Chester T. Allcutt **Vol. xxxix—1920, pp. 1967-1980**

This paper presents data giving the discharge characteristics of a commercial type of impulse gap under different conditions. Test data are also presented giving the characteristics of certain experimental gap structures designed to minimize the effects of adverse weather conditions, such as rain, fog, etc. A brief discussion of some of the factors that determine the degree of protection afforded by a lightning arrester spark gap is included in the paper. The term "protection factor" is defined and curves giving the protection factor of certain types of gap are presented.

No discussion.

LIFE AND PERFORMANCE TESTS OF O F LIGHTNING ARRESTERS

N. A. Lougee **Vol. xxxix—1920, pp. 1981-1994**

No discussion.

15. DISTRIBUTION SYSTEMS

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No discussion.

POWER FACTOR CORRECTION ON DISTRIBUTION SYSTEMS

D. M. Jones **Vol. xxxix—1920, pp. 1767-1793**

A general description of power factor, its relation to capital invested, effect on losses and regulation. Sources of bad power factor, remedies, etc.

Discussion, pages 1794-1806, by Messrs. J. E. Woodbridge, W. J. Davis, Jr., L. T. Mervin, H. J. Ryan, R. F. Hayward, H. T. Plumb, H. V. Carpenter, C. A. Whipple and D. M. Jones.

ECONOMIC STUDY OF SECONDARY DISTRIBUTION

P. O. Reyneau and H. P. Seelyee **Vol. xxxix—1920, pp. 1807-1843**

The purpose of this paper is to study from an economical viewpoint the conditions generally met with in secondary distribution and to furnish as far as possible guidance for the designer. Means are furnished for readily discovering the limitations of any problem.

Discussion, pages 1844-1846, by Messrs. W. I. Slichter, D. W. Roper and H. P. Seelyee.

16. CONTROL, REGULATION AND SWITCHING

INHERENT REGULATION OF CONTINUOUS CURRENT CIRCUITS

A. L. Ellis and B. W. St. Clair **Vol. xxxix—1920, pp. 309-330**

This paper discusses the voltage changes, inherent in d-c. circuits, upon change of load. These variations are independent of $i\,r$ drops or speed of prime movers. A simple means of mitigating their effect is given.

Discussion, pages 331-336, by Messrs. V. Karapetoff, H. R. Summerhayes, L. W. Thompson, B. W. St. Clair and Philip Torchio.

A general discussion.

AUTOMATIC RAILWAY SUBSTATIONS

F. W. Peters **Vol. xxxix—1920, pp. 659-676**

This paper reviews the broad range of conditions to which railway automatic substations have been applied and also discusses the economies and operating advantages effected by their use. A description is given of the modern equipment with details of its operation. Special reference is made to improvements in design of control apparatus, to the positive sequence of starting the machines and the protection afforded the apparatus against overloads or other irregularities either outside or internal to the stations.

Discussion, incorporated with that of paper by R. J. Wensley on "Automatic Substations for Heavy City Service."

AUTOMATIC SUBSTATIONS FOR HEAVY CITY SERVICE

R. J. Wensley **Vol. xxxix—1920, pp. 677-689**

A description of the advantages to be obtained in many cases through the installation of automatic substations both in improved service and in reduction in installed feeder copper etc.

Discussion, (including that of papers by J. J. Linebaugh, M. W. Smith and F. W. Peters), pages 690-710, by Messrs. F. D. Newbury, S. Q. Hayes, C. H. Jones, D. C. Hershberger, C. A. Butcher, D. Bowman, J. F. Tritle, L. D. Bale, E. W. Cook, J. L. Burnham, M. W. Smith, F. W. Peters and R. J. Wensley.

A description of practical experiences with flashing and with automatic substations on various large installations; C. M. & St. P. R. R., City of Cleveland, etc.

REACTIVE POWER AND MAGNETIC ENERGY

Joseph Slepian **Vol. xxxix—1920, pp. 1115-1132**

The relation between reactive power and magnetic and electrostatic energies is stated and proved and its utility is illustrated by deriving the connection between reactive power and the size of machines, between the magnetic energy of an a-c. system and the field excitations of the synchronous machines therein, and by giving the physical significance of power factor under unbalanced conditions.

Discussion, page 1133, by Mr. C. L. Fortescue.

THEORY OF SPEED AND POWER FACTOR CONTROL OF LARGE INDUCTION MOTORS BY NEUTRALIZED POLYPHASE ALTERNATING-CURRENT COMMUTATOR MACHINES

John I. Hull **Vol. xxxix—1920, pp. 1135-1169**

Theory of induction motor control, discussing single-range (below synchronism only) speed and power factor control by means of a constant-speed series commutator motor, by means of a constant-speed shunt commutator motor, by means of a constant-speed compound excited commutator motor; double range (all speeds above or below synchronism) speed and power factor control by means of a constant-speed shunt commutator motor; and double-range (either above or below synchronism) operation remote from synchronism.

Discussion, pages 1170-1177, by Messrs. R. E. Hellmund, C. W. Kincaid, and J. I. Hull.

Discussion of relative advantages of the "Kraemer" and "Scherbius" systems.

THE USE OF REACTORS OF LARGE CENTRAL STATION SYSTEMS

R. F. Schuchardt **Vol. xxxix—1920, pp. 1195-1203**

An introductory paper to "The Stability of Operation of High-Power Generating Systems" by C. P. Steinmetz. A description of reactor experience of the Commonwealth Edison Company of Chicago.

Discussion, pages 1204-1214, by Messrs. C. P. Steinmetz, A. E. Bauhan, R. A. Hentz, D. C. Jackson, P. Torchio, J. Lyman, C. J. Holslag and B. G. Jamieson.

An approximation of short-circuit current developed in Chicago system during trouble and outline of reactor practise by other companies.

POWER CONTROL AND STABILITY OF ELECTRIC GENERATING STATIONS

C. P. Steinmetz **Vol. xxxix—1920, pp. 1215-1270**

A full description of the theoretical and practical application of reactors.

Discussion, pages 1271-1287, by Messrs. R. E. Doherty, V. Karapetoff, H. R. Woodrow, E. G. Merrick, M. Brooks, J. A. Johnson, D. C. Jackson, D. W. Roper, P. Torchio and H. R. Summerhayes.

A general discussion.

CALCULATION OF MAGNETIC FORCE ON DISCONNECTING SWITCHES

Herbert Bristol Dwight **Vol. xxxix—1920, pp. 1337-1350**

The practical problem of finding the magnetic force tending to open a disconnecting switch is expressed in concise form in formulas (20) and (21), and their derivation and the assumptions on which they are based are given.

Curves are also presented in Figs. No. 3 and No. 4 from which the force may be found without using the formulas.

Discussion, pages 1351-1355, by Messrs. V. Karapetoff, A. Nyman, E. G. Merrick and H. B. Dwight.

A discussion of other mathematical methods of obtaining similar results.

POWER FACTOR IN POLYPHASE CIRCUITS

Vol. xxxix—1920, pp. 1449-1450

Preliminary report of Special Joint Committee.

POLYPHASE POWER FACTOR

F. C. Holtz **Vol. xxxix—1920, pp. 1451-1456**

An abstract of a paper descriptive of the determination of power factor by various methods.

Discussion, combined with that of other papers in the symposium.

POWER FACTOR IMPROVEMENT DEPENDENT UPON ADEQUATE METERING

Wm. L. Brown **Vol. xxxix—1920, pp. 1457-1464**

An abstract of a paper descriptive of the practical questions involved in metering of loads of low power factor and what central stations are actually doing.

Discussion combined with that of the other papers in the symposium.

POWER FACTOR IN POLYPHASE SYSTEMS

Francis B. Silsbee **Vol. xxxix—1920, pp. 1465-1467**

An abstract of a paper descriptive of the mutual relations of the various definitions of power factor and their respective merits.

Discussion combined with that of the other papers in the symposium.

POWER FACTOR AND UNBALANCE ON A POLYPHASE SYSTEM

Carl J. Fechheimer **Vol. xxxix—1920, pp. 1469-1473**

An abstract of a paper descriptive of balanced and unbalanced polyphase systems without neutral current.

Discussion combined with that of the other papers in the symposium.

POLYPHASE POWER FACTOR

H. L. Wallau **Vol. xxxix—1920, pp. 1475-1476**

An abstract of a paper descriptive of the most applicable definition of power factor.

Discussion, combined with that of the other papers in the symposium.

POLYPHASE POWER FACTOR

P. M. Lincoln **Vol. xxxix—1920, pp. 1477-1479**

An abstract of a paper descriptive of the applicable definition.

Discussion, combined with that of the other papers in the symposium.

POLYPHASE POWER REPRESENTATION BY MEANS OF SYMMETRICAL COORDINATES

C. L. Fortescue **Vol. xxxix—1920, pp. 1481-1484**

An abstract of a paper descriptive of power representation by means of symmetrical coordinates.

Discussion, combined with that of the other papers in the symposium.

MEASUREMENT OF POWER FACTOR ON UNBALANCED POLYPHASE CIRCUITS

R. D. Evans **Vol. xxxix—1920, pp. 1485-1487**

A scientific definition of power factor and description of devices for measuring power factor and unbalance.

Discussion combined with that of the other papers in the symposium.

POLYPHASE POWER FACTOR AND UNBALANCED LOADS

Philip Torchio **Vol. xxxix—1920, pp. 1489-1490**

Data giving comparative results of the amount of capacity of electrical equipment required for generating and transmitting a unit amount of power under different conditions of unbalancing, all giving same average value of power factor.

Discussion combined with that of the other papers in the symposium.

POWER FACTOR IN POLYPHASE CIRCUITS

W. H. Pratt **Vol. xxxix—1920, pp. 1491-1495**

A comparison of the two proposed definitions of power factor from various aspects.

Discussion, (including that of papers by Holtz, Brown, Silsbee, Fechheimer, Wallau, Lincoln, Fortescue, Evans and Torchio), pages 1496-1520 by Messrs. F. C. Holtz, V. Karapetoff, J. C. Lincoln, C. L. Fortescue, R. D. Evans, A. Nyman, R. K. Honaman, A. E. Kennelly, Philip Torchio, H. B. Dwight, W. H. Pratt, G. A. Sawin, J. R. Craighead and W. V. Lyon.

A general discussion.

CONSIDERATIONS WHICH DETERMINE THE SELECTION AND GENERAL DESIGN OF AN EXCITER SYSTEM

J. T. Barron and A. E. Bauhan **Vol. xxxix—1920, pp. 1521-1551**

Excitation systems may be divided into (a) Central Systems with separately driven exciters, (b) Central Systems with direct-connected exciters, (c) Individual Systems with direct-connected exciters and (d) Individual Systems with separately driven exciters.

The paper discusses the factors which must be considered in determining which of the above systems is the best to use in any particular case.

The paper also considers subjects related to the completion of an excitation design layout.

Discussion, pages 1552-1561, by Messrs. R. C. Muir, W. F. Sims, E. G. Merrick, V. Karapetoff, W. J. Foster and A. E. Bauhan.

A description of experience with various systems and a tabular comparison of the various exciter systems.

FACTORS IN EXCITATION SYSTEMS OF LARGE CENTRAL STATION STEAM PLANTS

J. W. Parker and A. A. Meyer **Vol. xxxix—1920, pp. 1563-1573**

This paper points out the most essential requirements of excitation schemes; outlines two general methods followed in the design of such systems and from which a variety of schemes are built up; and discusses briefly the merits as well as demerits of factors determining the success of various schemes.

No discussion.

EXCITERS AND SYSTEMS OF EXCITATION

H. R. Summerhayes **Vol. xxxix—1920, pp. 1575-1594**

A description of the various factors to be considered in the selection and design of an exciter system.

No discussion.

THE APPLICATION OF D-C. GENERATORS TO EXCITER SERVICE

C. A. Boddie and F. L. Moon **Vol. xxxix—1920, pp. 1595-1616**

It is the purpose of this paper chiefly to discuss the relation of exciter design to problems, such as voltage range, responsiveness, stability, and type of drive, with especial reference to the requirements of automatic voltage regulators.

Discussion, pages 1617-1623, by Messrs. R. E. Doherty and C. A. Boddie.

EXCITER PRACTISE IN THE NORTHWEST

J. D. Ross **Vol. xxxix—1920, pp. 1625-1632**

The author gives in table form the generator installation and the exciter sets, with size, voltage, drive, regulators and system of exciter connections for the principal hydroelectric plants in operation in the Northwest. He analyzes their essential characteristics and the difficulties encountered with automatic voltage regulators in plants having generators of different sizes and makes. The author advocates for an entirely new large hydroelectric or steam plant a system of excitation in which each generator is supplied with its own shunt-wound exciter driven by a prime mover and furnished with its own regulator. In an existing plant, not designed on the unit system, the use of one large exciter to operate the entire plant through field rheostats is the best compromise, the old exciter system being used as the duplicate for emergency.

Discussion incorporated with that of paper by Cox and Michener on "Generator Excitation Practise in the Hydroelectric Plants of the Southern California Edison Company."

GENERATOR EXCITATION PRACTISE IN THE HYDROELECTRIC PLANTS OF THE SOUTHERN CALIFORNIA EDISON COMPANY

H. H. Cox and H. Michener **Vol. xxxix—1920, pp. 1633-1636**

A description of apparatus installed, method of operating and changes being made and proposed.

Discussion, pages 1637-1644, by Messrs. C. J. Fechheimer and J. Lyman.

Discussion deals chiefly with amount of current required to charge a transmission line, etc.

POWER FACTOR CORRECTION ON DISTRIBUTION SYSTEMS

D. M. Jones **Vol. xxxix—1920, pp. 1767-1793**

A general description of power factor, its relation to capital invested, effect on losses and regulation. Sources of bad power factor, remedies, etc.

Discussion, pages 1794-1806, by Messrs. J. E. Woodbridge, W. J. Davis, Jr., L. T. Mervin, H. J. Ryan, R. F. Hayward, H. T. Plumb, H. V. Carpenter, C. A. Whipple and D. M. Jones.

ELECTROSTATIC CONDENSERS

V. E. Goodwin **Vol. xxxix—1920, pp. 1995-2015**

The first part is confined to fundamental characteristics of condensers and their relations to electric circuits containing inductance and resistance, and includes a discussion of the effects of switching condensers on and off such circuits. The second part of the paper describes some of the more important applications of condensers and illustrates the great demand there is in the electrical industry for this class of apparatus.

Discussion, pages 2016-2019, by Messrs. E. E. F. Creighton, C. T. Allcutt, V. E. Goodwin and H. R. Summerhayes.

18. LIGHTING AND LAMPS

THE SERIES SYSTEM OF STREET LIGHTING DISTRIBUTION

W. P. Hurley **Vol. xxxix—1920, pp. 1-15**

The series system of distribution has been used almost universally for street lighting since the first use of electric lamps.

Lamps, both arc and incandescent are very simple and more efficient when designed for series operation than are the multiple type. The maintenance of constant power at the lamp terminals where lamps are thinly scattered over a wide area is much easier with a series system of distribution than with any other system. The burning of all street lamps in the city for certain specified hours makes it desirable to turn the whole circuit on and off from a certain point so that such a system whether series or multiple cannot be used to distribute power for other purposes. As therefore, a special system is necessary for the street lamps, it has usually been made of the series type for the above reasons.

The special apparatus required to operate a series system from constant potential is very simple and inexpensive.

Discussion, pages 16-32, by Messrs. A. E. Betts, E. Sweitzer, F. F. Fowle, G. N. Chamberlin, F. W. Parker, E. N. Lake, F. A. Vaughn, Mr. Cameron, C. H. Shepherd, W. A. Del Mar, H. Nixon, Mr. Snyder, C. E. Skinner, J. E. Royer and W. P. Hurley.

A general discussion of series distribution practise and apparatus.

MULTIPLE SYSTEMS OF DISTRIBUTION FOR STREET LIGHTING

Ward Harrison **Vol. xxxix—1920, pp. 33-48**

The advantages in simplicity and flexibility of multiple connected lamps are discussed with particular reference to the frequent changes and extensions of street lighting service required in growing cities. The more general adoption of multiple street lighting is stated to be contingent upon fuller standardization of suitable methods of control applicable generally to existing electrical power distribution systems. Different devices in use or proposed for control of multiple street lamps are briefly described and the characteristics desirable in such apparatus are outlined. Attention is directed to the small differences in efficiency of present multiple and series incandescent lamps.

Discussion, pages 49-56, by Messrs. N. B. Hinson, W. Harrison, C. E. Skinner, J. M. Humiston, C. H. Shepherd, F. A. Vaughn, H. Goodwin, Jr., F. H. Bernhard, Mr. Peaslee, Mr. Cameron and G. Nixon.

A discussion of practise in various cities, Southern California Edison etc.

CONSTANT POTENTIAL SERIES LIGHTING

C. P. Steinmetz **Vol. xxxix—1920, pp. 57-72**

A general description of the advantages and disadvantages of constant potential series systems, with appendix descriptive of Chicago's system.

Discussion, pages 73-89, by Messrs. J. R. Cravath, F. A. Vaughn, D. W. Roper, W. Harrison, and F. H. Murphy.

A general discussion including a description of the Milwaukee system with outdoor substations, etc.

GASEOUS CONDUCTION LIGHT FROM LOW-VOLTAGE CIRCUITS

D. McFarlan Moore **Vol. xxxix—1920, pp. 2021-2047**

A description of the development of light production with gaseous conductors, dwelling particularly on the latest and most efficient lamps of this type.

No discussion.

20. MISCELLANEOUS APPLICATIONS OF ELECTRICITY

THE FIXATION OF ATMOSPHERIC NITROGEN BY THE SILENT ELECTRIC DISCHARGE PROCESS—I

C. F. Harding and K. B. McEachron **Vol. xxxix—1920, pp. 761-788**

A detailed description of the process of obtaining nitric acid from the air by corona discharge from the original apparatus down to the most up-to-date equipment.

Discussion, pages 789-790, by Messrs. Finch, Benjamin, K. B. McEachron and Tucker.

DESIGN OF CONSTANT-CURRENT GENERATORS FOR ARC WELDING

K. L. Hansen **Vol. xxxix—1920, pp. 1357-1402**

A description of the various factors encountered in arc welding and the generator characteristics necessary to overcome them. Various types of field windings are discussed.

Discussion, page 1403, by Messrs. R. W. Owens and S. R. Bergman.

AUTOMATIC ARC WELDING APPARATUS

S. R. Bergman and R. L. Unland **Vol. xxxix—1920, pp. 1405-1407**

An abstract of a paper descriptive of generator with four-pole field and two-pole armature possessing inherent electrical characteristics desirable for single operator arc welding. No external resistors or regulating devices are required.

Discussion, page 1408, by Messrs. K. L. Hansen and S. R. Bergman.

ARC WELDING MACHINES OF THE WILSON WELDER AND METALS COMPANY

Alexander Churchward **Vol. xxxix—1920, pp. 1409-1410**

An abstract of a paper descriptive of apparatus furnishing constant current to the arc and in which length of the arc is limited. The use of the carbon pile is described. The system produces constant heat per unit area in the weld, not in the arc.

No discussion.

CHARACTERISTICS AND PERFORMANCE OF ARC WELDING APPARATUS

A. M. Candy **Vol. xxxix—1920, pp. 1411-1416**

An abstract of a paper descriptive of the increased efficiency of the constant-potential welding circuit and the decreased size and cost of apparatus by changing the generated circuit potential from 75 to 60 volts. Recent developments are described of both a-c. and d-c. types particularly an interconnected constant-current variable-voltage d-c. generator and exciter.

Discussion, pages 1417-1421, by Messrs. C. J. Holslag, W. O. Noble, S. R. Bergman, A. M. Candy and J. C. Lincoln.

A general discussion.

ARC WELDING MACHINERY OF THE U. S. LIGHT AND HEAT CORPORATION

W. A. Turbayne **Vol. xxxix—1920, pp. 1423-1426**

An abstract of a paper descriptive of an arc welding machine wherein control of the current and voltage and length of arc is accomplished by inherent action of the machine windings.

No discussion.

RECENT DEVELOPMENTS IN ELECTRO-PERCUSSIVE WELDING

Douglas F. Miner **Vol. xxxix—1920, pp. 1427-1431**

An abstract of a paper descriptive of machines utilizing the discharge of an electrolytic condenser for fusion substantially simultaneous with a percussive engagement.

Discussion, page 1432, by Messrs. J. C. Lincoln and D. F. Miner.

ELECTRIC ARC WELDING APPARATUS

Robert E. Kinkead **Vol. xxxix—1920, pp. 1433-1434**

An abstract of a paper on electric arc welding apparatus.

No discussion.

A-C. TRANSFORMERS FOR ARC WELDING

C. J. Holslag **Vol. xxxix—1920, pp. 1435-1442**

Abstract of a paper descriptive of the design and advantages of a-c. welding apparatus.

Discussion, pages 1443-1447, by Messrs. W. O. Noble, J. C. Lincoln, C. J. Holslag and B. W. David.

A general discussion.

21. TELEPHONY AND TELEGRAPHY

PRINTING TELEGRAPH SYSTEMS

John H. Bell **Vol. xxxix—1920, pp. 167-230**

This paper describes the Creed, Murray Automatic, Siemans & Halske Baudot and American Multiplex Printing Telegraph Systems, and their methods of operation. A discussion of the operating features of the systems described is dealt with under the following headings:

Accuracy; Speed of Service; Operator Output; Maintenance; Line Economies; and Flexibility.

Discussion is combined with that of paper by Campbell and Foster on "Maximum Output Networks For Telephone Substation and Repeater Circuits."

MAXIMUM OUTPUT NETWORKS FOR TELEPHONE SUBSTATION AND REPEATER CIRCUITS

G. A. Campbell and R. M. Foster **Vol. xxxix—1920, pp. 231-280**

Ideal telephone substation and repeater circuits are shown to present output and input requirements which can be met by a type of circuit containing four resistances each of which has maximum output; maximum output is found to involve biconjugacy. The necessary formulas for proportioning these circuits and also circuits having superfluous elements are derived.

Discussion (including that of paper by J. H. Bell), pages 281-290, by Messrs. G. O. Squier, E. Russel, C. E. Davies, H. A. Emmons, P. M. Rainey, J. P. Edwards, E. R. Shute, R. E. Chetwood, G. R. Benjamin, L. Espenscheid, O. B. Blackwell, J. H. Cuntz, J. H. Bell and G. A. Campbell.

Discussion chiefly on printing telegraph including description of its accomplishments in France with the A. E. F.

22. MISCELLANEOUS TOPICS AND INSTITUTE AFFAIRS

ESSENTIAL STATISTICS FOR GENERAL COMPARISON OF STEAM POWER PLANT PERFORMANCE

W. S. Gorsuch **Vol. xxxix—1920, pp. 91-98**

This paper briefly outlines a method for preparing statistical reports relating to power generation in steam power plants, whereby fairly close comparisons can be made of the efficiencies between different plants, without going into a detailed study of the thermal characteristics of the plants, or the intricate subject of power costs.

The essential items of steam power plant performance that should be recorded and a uniform method of expressing them are given in tabular form, followed by an illustration demonstrating the advantage of the proposed method.

Discussion, pages 99-100, by Messrs. E. J. Cheney and W. S. Gorsuch.

ECONOMICAL SUPPLY OF ELECTRIC POWER FOR THE INDUSTRIES AND THE RAILROADS OF THE NORTHEAST ATLANTIC SEABOARD

W. S. Murray **Vol. xxxix—1920, pp. 101-166**

A description of the super-power plan of generation and distribution of electric power whereby a saving will be made annually of over $300,000,000.

Contributions by W. L. Emmet, J. F. Johnson, H. G. Reist, F. D. Newbury, W. B. Potter, Philip Torchio, Percy H. Thomas, W. D. A Peaslee and A. O. Austin.

No discussion.

A METHOD FOR SEPARATING NO-LOAD LOSSES IN ELECTRICAL MACHINERY

C. J. Fechheimer **Vol. xxxix—1920, pp. 291-299**

The method proposed makes use of idle operation of the machine as a motor, the voltage being varied, and speed kept constant. After deducting the armature $I^2 R$ losses from the watts input, the remaining watts are plotted against the voltage. A formula is derived based upon the assumption that the watts are equal to constant windage and friction loss plus core loss which latter varies as a constant power of the voltage. An example is given of the close agreement with the test curve in the case of an induction motor; and other examples are cited.

Discussion, pages 300-308, by Messrs. V. Karapetoff, W. F. Dawson, W. I. Slichter, P. L. Alge and C. J. Fechheimer.

A general theoretical discussion.

THE MEASUREMENT OF PROJECTILE VELOCITIES

P. E. Klopsteg and A. L. Loomis **Vol. xxxix—1920, pp. 337-353**

The paper discusses the requirements imposed by proving ground practise upon a chronograph which is intended for general ammunition testing. Instruments of the standard pre-war pattern were entirely inadequate in number for testing the immense quantities of ammunition contracted for by the Government during the war. The instrument which was developed and adopted as a standard ordnance chronograph, is designated "Aberdeen chronograph." An account of its development is given. The Aberdeen chronograph, and the procedure in determining velocities by this means, are described in detail. For the sake of comparison, the Boulenge chronograph is briefly described Comparative results as to speed and accuracy of measurement are given.

Discussion, pages 354-358, by Messrs. E. E. F. Creighton, C. H. Sharp, R. Kegerreis and P. E. Klopsteg.

A general discussion and description of certain special problems to be solved

NOTES ON THE SYNCHRONOUS COMMUTATOR

J. B. Whitehead and T. Isshiki **Vol. xxxix—1920, pp. 407-438**

In the use of the synchronous commutator in series connection as a suppressor, serious errors may arise due to relatively small amountt of capacity in the commutator and galvanometer circuits. The magnitude of these errors is studied for a number of different connections, and methods for eliminating them are pointed out. A number of wave forms are given, indicating the nature of the errors.

An appendix gives a theoretical analysis of two cases investigated and shows a close agreement with the experimental observations.

Discussion, pages 439-441, by Messrs. P. G. Agnew, E. D. Doyle, A. E. Kennelly, J. H. Morecroft and J. B. Whitehead.

AUTOMATIC RAILWAY SUBSTATIONS

F. W. Peters **Vol. xxxix—1920, pp. 659-676**

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A description of practical experiences with flashing and with automatic substations on various large installations; C. M. & St. P. R. R., City of Cleveland, etc.

THE BALDWIN-WESTINGHOUSE CHICAGO, MILWAUKEE AND ST. PAUL ELECTRIC LOCOMOTIVES

N. W. Storer **Vol. xxxix—1920, pp. 711-740**

A description in detail of the mechanical and electrical features of the locomotives and their action in operation.

Discussion, incorporated with that of paper by A. F. Batchelder and S. T. Dodd on "Passenger Locomotives for Chicago, Milwaukee and St. Paul Railway."

PASSENGER LOCOMOTIVES FOR CHICAGO, MILWAUKEE AND ST. PAUL RAILWAY

A. F. Batchelder and S. T. Dodd **Vol. xxxix—1920, pp. 741-752**

A detailed description of the G. E. designed locomotives both mechanical and electrical features and their operation.

Discussion, (including that of paper by N. W. Storer), pages 753-760, by Messrs. S. Q. Hayes, R. L. Wilson, E. H. Martindale, R. J. Wensley, F. D. Hall, R. E. Ferris, A. M. Candy, L. J. Hibbard, C. Townley, C. M. Davis, S. T. Dodd and N. W. Storer.

AN ENGINEERING ANALYSIS OF THE LABOR PROBLEM

Calvert Townley **Vol. xxxix—1920, pp. 823-831**

Presidents address.

TECHNICAL COMMITTEE REPORTS

Vol. xxxix—1920, pp. 833-901

VENTILATION AND TEMPERATURE IN LARGE TURBO GENERATORS

B. G. Lamme **Vol. xxxix—1920, pp. 915-947**

A description of the various methods used for obtaining required ventilation with their advantages and disadvantages.

Discussion, pages 948-949, by Messrs. R. B. Williamson and W. J. Foster.

TEMPERATURES IN LARGE ALTERNATING-CURRENT GENERATORS

W. J. Foster **Vol. xxxix—1920, pp. 951-964**

A description of design practise in large alternators.

Discussion, pages 965-969, by Messrs. F. D. Newbury, W. I. Slichter, P. Torchio, A. S. Loizeaux, S. Haar, S. R. Bergman and W. J. Foster.

A general discussion.

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Joseph Slepian **Vol. xxxix—1920, pp. 1115-1132**

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Discussion, page 1133, by Mr. C. L. Fortescue.

HIGH-TENSION INSULATOR PORCELAIN

W. D. A. Peaslee **Vol. xxxix—1920, pp. 1179-1187**

Porcelain high-tension insulator requirements as to mechanical strength, ability to resist, sudden changes of temperature, porosity, homogeneity and temperature, coefficient of resistivity. Suggestion as to the influences of the Piezo electric effect and the deterioration is presented with a brief discussion of the degree of progress in art.

Discussion, pages 1188-1194, by Messrs. H. B. Vincent, C. L. Fortescue, R. M. Spurck, G. I. Gilchrest and W. D. A. Peaslee.

A general discussion with plea for the keeping of records by operators.

VOLTAGE STRESSES IN REACTORS IN SERVICE

F. H. Kierstead and R. Meeker **Vol. xxxix—1920, pp. 1289-1328**

In this paper the authors present experimental data which indicate the voltage stresses existing in current-limiting reactors under different conditions. Physical explanations of the phenomena are offered.

Discussion, pages 1329-1335, by Messrs. J. F. Peters, P. Torchio, H. R. Woodrow, C. L. Fortescue, H. B. Dwight, A. Nyman, J. Slepian and F. H. Kierstead.

A general discussion.

CALCULATION OF MAGNETIC FORCE ON DISCONNECTING SWITCHES

Herbert Bristol Dwight **Vol. xxxix—1920, pp. 1337-1350**

The practical problem of finding the magnetic force tending to open a disconnecting switch is expressed in concise form in formulas (20) and (21), and their derivation and the assumptions on which they are based are given.

Curves are also presented in Figs. No. 3 and No. 4 from which the force may be found without using the formulas.

Discussion, pages 1351-1355, by Messrs. V. Karapetoff, A. Nyman, E. G. Merrick and H. B. Dwight.

A discussion of other mathematical methods of obtaining similar results

CONSIDERATIONS WHICH DETERMINE THE SELECTION AND GENERAL DESIGN OF AN EXCITER SYSTEM

J. T. Barron and A. E. Bauhan **Vol. xxxix—1920, pp. 1521-1551**

Excitation systems may be divided into (a) Central Systems with separately driven exciters, (b) Central Systems with direct-connected exciters, (c) Individual Systems with direct-connected exciters and (d) Individual Systems with separately driven exciters.

The paper discusses the factors which must be considered in determining which of the above systems is the best to use in any particular case.

The paper also considers subjects related to the completion of an excitation design layout.

Discussion, pages 1552-1561, by Messrs. R. C. Muir, W. F. Sims, E. G. Merrick, V. Karapetoff, W. J. Foster and A. E. Bauhan.

A description of experience with various systems and a tabular comparison of the various exciter systems.

FACTORS IN EXCITATION SYSTEMS OF LARGE CENTRAL STATION STEAM PLANTS

J. W. Parker and A. A. Meyer **Vol. xxxix—1920, pp. 1563-1573**

This paper points out the most essential requirements of excitation schemes; outlines two general methods followed in the design of such systems and from which a variety of schemes are built up; and discusses briefly the merits as well as demerits of factors determining the success of various schemes.

No discussion.

EXCITERS AND SYSTEMS OF EXCITATION

H. R. Summerhayes **Vol. xxxix—1920, pp. 1575-1594**

A description of the various factors to be considered in the selection and design of an exciter system.

No discussion.

THE APPLICATION OF D-C. GENERATORS TO EXCITER SERVICE

C. A. Boddie and F. L. Moon **Vol. xxxix—1920, pp. 1595-1616**

It is the purpose of this paper chiefly to discuss the relation of exciter design to problems, such as voltage range, responsiveness, stability, and type of drive, with especial reference to the requirements of automatic voltage regulators.

Discussion, pages 1617-1623, by Messrs. R. E. Doherty and C. A. Boddie.

EXCITER PRACTISE IN THE NORTHWEST

J. D. Ross **Vol. xxxix—1920, pp. 1625-1633**

The author gives in table form the generator installation and the exciter sets, with size, voltage, drive, regulators and system of exciter connections for the principal hydroelectric plants in operation in the Northwest. He analyzes their essential characteristics and the difficulties encountered with automatic voltage regulators in plants having generators of different sizes and makes. The author advocates for an entirely new large hydroelectric or steam plant a system of excitation in which each generator is supplied with its own shunt-wound exciter driven by a prime mover and furnished with its own regulator. In an existing plant, not designed on the unit system, the use of one large exciter to operate the entire plant through field rheostats is the best compromise, the old exciter system being used as the duplicate for emergency.

Discussion incorporated with that of paper by Cox and Michener on "Generator Excitation Practise in the Hydroelectric Plants of the Southern California Edison Company."

GENERATOR EXCITATION PRACTISE IN THE HYDROELECTRIC PLANTS OF THE SOUTHERN CALIFORNIA EDISON COMPANY

H. H. Cox and H. Michener **Vol. xxxix—1920, pp. 1633-1636**

A description of apparatus installed, method of operating and changes being made and proposed.

Discussion, pages 1637-1644, by Messrs. C. J. Fechheimer and J. Lyman. Discussion deals chiefly with amount of current required to charge a transmission line, etc.

FACTORS CONTROLLING THE DESIGN AND SELECTION OF SUSPENSION INSULATORS

W. D. A. Peaslee **Vol. xxxix—1920, pp. 1645-1667**

A discussion of the factors entering into the design and operating behavior of suspension insulators and problems to be solved in designing a suspension insulator to overcome the objectionable features shown by experience to affect seriously the operation of the insulators in service.

Factors to be taken into consideration in the selection of suspension insulators for a given condition are given and a brief discussion of the general trend of future improvements is presented.

Discussion incorporated with that of paper by F. W. Peek, Jr. on "Electrical Characteristics of the Suspension Insulator—II."

UNIT VOLTAGE DUTIES IN LONG SUSPENSION INSULATORS

H. J. Ryan and H. H. Henline **Vol. xxxix—1920, pp. 1669-1684**

This paper deals chiefly with the quantitative relations that exist between the maximum and average voltage unit-duties in line suspension insulators made up of units in common use.

Discussion incorporated with that of paper by F. W. Peek, Jr., on "Electrical Characteristics of the Suspension Insulator—II."

ELECTRICAL CHARACTERISTICS OF THE SUSPENSION INSULATOR—II

The Line Insulators at the Higher Voltages

F. W. Peek, Jr. **Vol. xxxix—1920, pp. 1685-1705**

This paper reviews the duties of the line insulator at voltages above 100 kv. and compares them with the duties imposed by the lower voltages. The discussion is based in general upon data and operating experiences of many investigations made particularly during the last few years.

Discussion, pages 1706-1741, by Messrs. J. B. Fisken, M. T. Crawford, E. R. Stauffacher, L. C. Williams, G. E. Quinan, W. A. Hillebrand, K. A. Hawley, S. C. Lindsay, L. Lauridsen, A. C. Pratt, C. P. Osborne, D. W. Proebstel, J. C. Clark, H. H. Schoolfield, W. D. A. Peaslee, F. W. Peek, Jr. and H. J. Ryan.

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Presented at the 357th meeting of the American Institute of Electrical Engineers, Chicago, Ill., January 9, 1920.

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THE SERIES SYSTEM OF STREET LIGHTING DISTRIBUTION

BY W. P. HURLEY

Sales Engineer, Westinghouse Electric & Mfg. Co.

The series system of distribution has been used almost universally for street lighting since the first use of electric lamps.

Lamps, both arc and incandescent are very simple and more efficient when designed for series operation than are the multiple type. The maintenance of constant power at the lamp terminals where lamps are thinly scattered over a wide area is much easier with a series system of distribution than with any other system. The burning of all street lamps in the city for certain specified hours makes it desirable to turn the whole circuit on and off from a certain point so that such a system whether series or multiple cannot be used to distribute power for other purposes. As therefore, a special system is necessary for the street lamps, it has usually been made of the series type for the above reasons.

The special apparatus required to operate a series system from constant potential is very simple and inexpensive. The constant-current moving-coil transformer is the main factor in the maintenance of the necessary constant current. The film cut-out socket for the lamp enables the continuity of the circuit to be maintained when the incandescent lamp breaks or is removed. Both are very simple and reliable even with unskilled operators.

Any system of street lighting of sufficient merit to supersede the series system must depend primarily on the development of simple, reliable and inexpensive control apparatus for the individual lamps to enable them to be operated on the existing multiple distribution circuits.

A SERIES system of lighting distribution is one in which all the current in the circuit passes through each individual lamp in turn. The power as measured in watts taken by each individual lamp is determined by the resistance drop across that lamp, the current in all the lamps being the same. By properly regulating this current at the station, the wattage and illumination is maintained at a constant value, regardless of the pressure drop in the conducting wires of the system. Special short-circuiting devices are employed to maintain the continuity of the circuit when the lamps burn out and to maintain a constant current in the circuit with the various conditions of load.

Such series systems are employed on a large scale to operate various types of lamps for street lighting. Conspicuous examples are the Mazda lamps in Chicago, the flame carbon arc lamps in Chicago and Indianapolis, the metallic flame or luminous arc lamps in Detroit, Pittsburgh and St. Louis as well as the old carbon arc lamps, both open and enclosed, which these lamps replaced. Where a suitable current and frequency are used the same circuit may operate both arc and incandescent lamps without difficulty.

As distinguished from the series system the multiple system of distribution requires lamps of the same voltage as the circuit rating, each of which has a resistance permitting the desired current to be taken. A simple switch at each lamp disconnects it individually from the circuit. As practically all factory and residence lighting is so operated approximately 95 per cent of the incandescent lamps used in this country are on multiple systems.

Other systems combine the principles of the series system with that of the multiple system by connecting a number of lamps in series and operating this series of lamps on a multiple circuit of higher voltage than is suitable for single lamps. Street cars thus use five of the substantial 110-volt lamps in series on their 550-volt circuits. However when one lamp breaks a filament, the other four will not operate as the circuit is then open. In some street lighting systems, notably at Milwaukee, series street lamps are connected in series and operated on constant potential in a similar manner. In such cases alternating current is employed and special devices are provided to maintain the continuity of the circuit when individual lamps go out.

EXTENT OF USE

Approximately 1,000,000, street lamps in the United States are supplied by series distribution circuits. This constitutes practically the entire street lighting distribution system of a large majority of the operating companies. Although requiring but a small proportion of the total kv-a. generating capacity, the system is of great importance to operating companies, because of the conspicuous position and essential duty

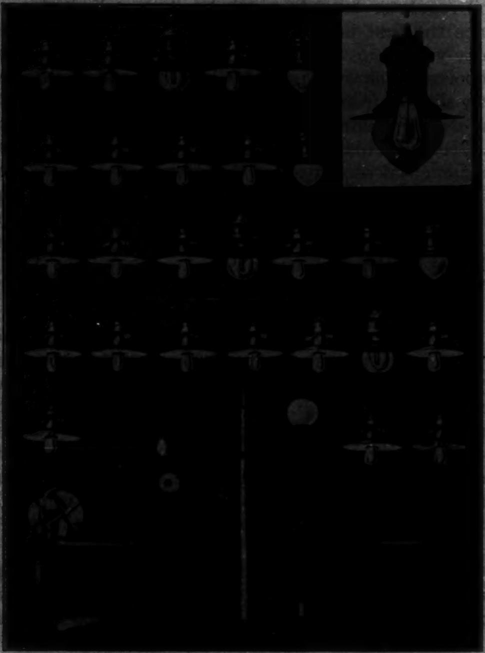

FIG. 1—STREET LIGHTING APPARATUS AS CONNECTED TO A CONSTANT-CURRENT MOVING COIL REGULATOR

of each small unit. It is especially notable that in their street lighting work, many operating companies render service to every one of the inhabitants whereas only 20 per cent or less, of the people may be making use of electricity in their homes. Yet, its total actual cost may be only 1 or 2 per cent of the total city budget and seldom over one dollar ($1.00) per year, per inhabitant.

DEVELOPMENT

Although a great many other systems involving substitutes for the constant-current generator or moving-coil regulator have been tried out on a limited scale since the first introduction of the electric lamp some forty years ago, the series system is of as great relative importance as ever in street lighting, and is changed only slightly from the original concept brought out with the development of the first arc lamp by Mr. Charles F. Brush. This is an indication of the simplicity and fundamental soundness of the scheme.

The change from direct current to alternating current involved practically no change other than the lamps themselves, and the station equipment. The lines were seldom involved.

ADVANTAGE OF A SERIES SYSTEM

Street lamps constitute a comparatively small load in kv-a. which must be operated at various spacings over a wide area. The line cost per kv-a. of installation is relatively high. This is offset to a certain extent by a load factor of approximately 45 per cent where a 4000-hr. per year schedule is maintained. Furthermore, special precautions must be taken to maintain constant power at the lamps, regardless of the wide distribution. The series system is particularly adaptable to this service.

Consider that power is supplied to a 500-watt lamp every 500 feet along a street. A conductor smaller than No. 8 gage, is seldom used, as it would be too weak to be dependable, from a mechanical standpoint. From current-carrying consideration, a No. 14 wire would be ample although the watts lost in transmission, would be approximately four times as high as in a typical street lighting system.

If 110-volt multiple distribution is attempted, even with No. 8 conductor, not over two such lamps, the most distant being 1000 feet away, can be successfully operated, because of the poor regulation resulting from the voltage drop in the conductor. On a similar series circuit, 100 or more of such lamps, or a much larger number of smaller lamps, with as much as 40 miles of line wire, are readily operated.

Under such conditions there would not be over 50 kw. in load per square mile of territory served, and where the lamps are small, or spaced even further apart due to the presence of gas lamps in many locations or due to a scattered population, this load may be of the order of 5 kw. per square mile and 1 kw. per mile of single wire.

The low current in the series circuit prevents excessive drop in the line even with No. 8 conductor. The regulator compensates perfectly for this drop. The voltage of a single circuit is thus the sum of the voltages of the lamps on this circuit, and the line drop. It is evident that the long high-voltage circuits have the great advantage of less apparatus per kilowatt of load because fewer circuits will be required. Also a lower percentage of the line is used in simply running back to the station for purposes of control.

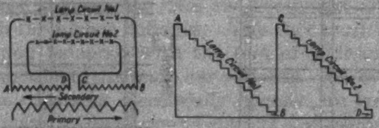

Fig. 2—Regulator with Two Interconnected Secondary Circuits and Development of the Diagram Showing the Voltage to Ground in the Various Parts of the Circuit.

The size of such a circuit may be limited by any one of several factors.

1. Voltage of the circuit to ground.
2. Number of lamps to operate on one circuit.
3. Insulation of lines and equipment.

Series lighting circuits on poles are subject to many grounds due to trees, lines breaking, crosses with other circuits and other causes. If an excessive voltage is supplied to any circuit an accidental ground at any point on the circuit causes a heavy strain on the insulation at other points because the drop across the large number of lamps is so great. This drop can be greatly reduced by cutting the circuit in parts and connecting a portion only of the generator or regulator winding between these sections. The whole combination consisting of from two to four sections is thus connected in series as shown in Fig. 2 and takes the same current throughout the circuits. Such a system has been used in the d-c. arc generators for many years and also in the constant-current regulating transformers.

Grounds must be promptly removed from such a system. Where double grounds occur, one on each of two circuits, they may, if of low resistance, both be placed near enough the station (Fig. 2 at *B* and *C*) to practically short-circuit one section of the regulator winding and the adjacent lamps, thus causing the current in that part of the circuit to increase to nearly 100 per cent of normal, while the remaining lamps would get practically no current. This double ground rarely occurs with good line construction, maintenance and methodical testing of the circuits. Equality of current in the two circuits may be insured by a relay so connected as to disconnect the regulator or circuits if current in the two circuits becomes unbalanced. In general the merits of such a system of interconnection are so great that it is in very general use, particularly where the voltage of a circuit would otherwise exceed 5000.

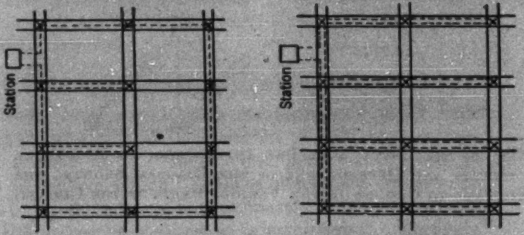

FIG. 3—DIAGRAM OF A SERIES CIRCUIT ON STREETS SHOWING THE SAVING OF WIRE THAT MAY BE MADE BY USING A SINGLE-WIRE CIRCUIT

In many cases one circuit is used for the all night lamps of a white way system and the others for the half night lamps. The high light load efficiency of the regulating transformer makes this arrangement very suitable.

The number of lamps on a circuit is influenced largely by the density of the lighting units and their size. In extreme cases 500 are found on a single circuit where there is a wide area of suburban territory to be lighted only by low candle power lamps. The numerous lamp loops of such circuits greatly increase the chances of open-circuiting the whole system. To reduce the consequences of this the wiring should be arranged in loops so that cut-outs can be installed which automatically short-circuit and disconnect a broken loop (Fig. 3 shows typical loops).

The cost of insulating lines and equipment rises rapidly with the higher voltages. This fact must be considered and balanced with the cost of additional circuits necessary to maintain low voltages. On overhead lines some systems successfully operate arc lamps up to 8000 volts. Underground circuits are seldom operated at more than 5000 volts as any breakdown in their insulation is much more serious from the standpoint of repair.

It is of great advantage to be able, in a series circuit, to operate the lamps on one street from one wire of the circuit alone without it being necessary to have the return circuit on the same street (Fig. 3). This may save as much as 50 per cent of the total wire required, and as many street lighting circuits are on pole lines already necessary for residence lighting such a use of a single wire system means the saving of a very considerable portion of the total expense. Little or no additional wiring in such single-wire circuits is necessary to create loops by which large sections can be isolated either automatically as previously described or by means of a jumper. The automatic current regulation of the constant-current moving-coil transformer makes this perfectly feasible even though 90 per cent or more of the lamps be so cut out of circuit.

Line reactance with such small currents involved may usually be ignored as its only effect is to slightly reduce the load carrying capacity of the regulator. Variations of the lamps themselves from their rated voltage have a much greater effect on the total regulator capacity. For overhead lines this reactance drop at 60 cycles in single-wire circuits is approximately 50 per cent and in return-wire circuits, 25 per cent respectively of the resistance drop.[1]

CONTROL

The operation of the street lighting lamps from dark until dawn or over some other fixed period makes a special condition of control not required for any other type of service. It is of great advantage from the standpoint of time and labor to be able to control all these lamps from a single point. Such has generally been the practise, thus making it impracticable to use the street lighting circuits and lines for any other service. Therefore, where a separate circuit is run for street lighting the series system has many advantages from the standpoint of installation costs and regulation over any other system.

LAMP CHARACTERISTICS

In general it has been well worth while to make special lamps for series operation on account of the higher electrical efficiency or lumens per watt input.

In direct-current series arc lamps the wasteful sustaining resistance necessary to the multiple arc is omitted and the constant-current regulating device itself sustains the arc without material loss of power. In consequence of this difference, the direct-current arc lamp electrical efficiency is approximately 70 per cent for the multiple and 95 per cent for the series lamp. Similar figures for an alternating-current system of enclosed carbon arc lamps so widely used at one time, were 90 per cent and 95 per cent respectively. The smaller apparent difference is, of course, due to the use of a reactance coil, instead of a resistance in the multiple arc lamp for sustaining effect.

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[1] Formulas in Pender's handbook or in Bureau of Standards, Vol. 4, No. 2, page 317.

The series system has been retained for use with Mazda lamps, as the efficiency of these lamps when made for a series system is much better than that of multiple lamps, being, in the case of the 100-c.p., 6.6-amperes series lamp, some 20 per cent higher than a 110-volt lamp of such a candle power.

Direct-current arc lamps were used on the early series circuits because of the greater steadiness and efficiency of the arc on direct current than on alternating current. The alternating-current enclosed arc although much less efficient became very popular when alternating-current systems of generation and distribution became common because a series alternating-current circuit could be derived, from the usual 2300-volt constant-potential bus by a simple constant-current regulating transformer. This transformer had only a fraction of the first cost, maintainence cost or floor space of the motor generator or rectifier set necessary to operate a direct-current series circuit from the alternating-current constant potential bus. Yet the improvement, from the illumination standpoint, over all predecessors of the metallic flame, or luminous arc lamp introduced about 1907, was such that many of the direct-current series circuits necessary for their operation have since been installed.

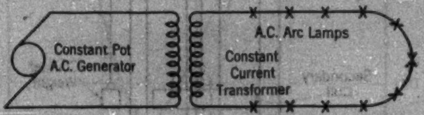

FIG. 4—CONNECTIONS OF A SERIES CIRCUIT TO A CONSTANT-POTENTIAL SOURCE BY MEANS OF A CONSTANT-CURRENT TRANSFORMER

Where incandescent lamps only are used there is obviously no need for direct current and it is well worth the slight expense to change the regulating equipment from direct to alternating current when direct-current series arc lamps are replaced by incandescent lamps. Such a change usually only involves the replacement of the station apparatus by a constant-current regulating transformer. An alternating-current circuit enables the use of the 20-ampere lamps from auto-transformers where high candle power lamps are desired. Safety coils can also be used on alternating current enabling the use of series lamps on fire alarm brackets, traffic posts and bridges in a way practically impossible on a high-voltage direct-current circuit with any degree of safety.

Frequency has had but slight effect in the development or use of series circuits. Apparatus has been available for all commercial frequencies except that arc lamps were never satisfactory on 25-cycle circuits.

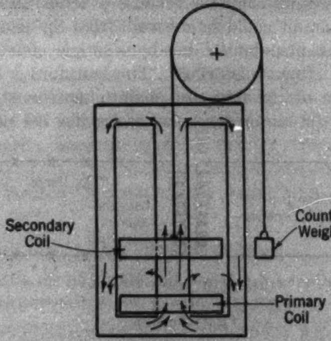

FIG. 5—FLUX RELATIONS IN A MOVING-COIL REGULATING TRANSFORMER SHOWING THE LEAKAGE OF FLUX BETWEEN THE STATIONARY AND THE COUNTERBALANCED MOVING-COIL

APPARATUS

In the simplest and most commonly used series systems, a special constant direct-current generator or a moving-coil transformer for alternating current is required at the station to control the circuit and to maintain the correct current at the lamp.

In the old direct-current arc lighting machine, special designs utilized the reactance of the armature to prevent excess current in the lamp circuits, and used devices for shifting the brushes to obtain the necessary voltage up to the limits of the machine. Some of these regulators were extremely accurate and were sensitive enough to operate the high efficiency incandescent lamps now commonly used. The constant-current regulator commonly used for alternating current depends upon the electrical repulsion existing between the primary and secondary coils of a transformer under load to produce and maintain a constant current in the secondary or lamp circuit. Such a regulator is of great value because it can automatically maintain a fixed secondary current through any number of lamps from one up to its maximum capacity or with a wide fluctuation

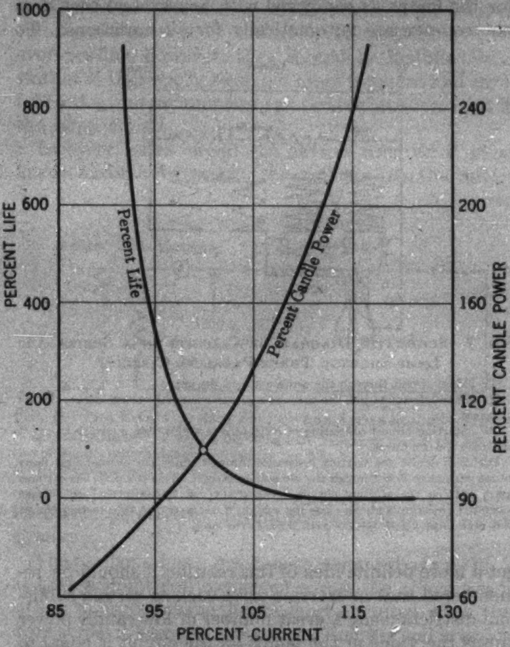

Fig. 6—Curves Showing the Change in Candle Power and in Lamp Life with a Change of Current in a Tungsten Filament

of the voltage on its supply circuit. When it is realized that an increase of 3 per cent in the current of the lamp circuit cuts the life of an incandescent lamp in half and thus doubles the renewal cost, it can readily be appreciated that such a regulator is very cheap protection for the lamps as compared with any system that does not compensate automatically for all variations. To get a more definite idea of this relation it should be remembered that in extreme cases such as certain suburban districts using a great number of low candle power lamps the value of the lamps on the circuit is equal to that of the regulator controlling them. Under normal conditions these lamps are renewed about three times per year. Accurate and reliable regulation is therefore necessary to keep these lamps up to rated candle power without getting at times a current sufficient to cause premature burnouts and increased expense in lamp renewals as well as in deductions for lamp outage. Renewals of four times per year would in this case require excess lamps equal the value of the regulator. The

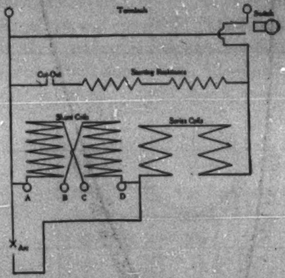

FIG. 7—SCHEMATIC DIAGRAM OF CIRCUITS IN A SERIES ARC LAMP SHOWING THREE PARALLEL CIRCUITS:

(1) Main circuit through the series coil and the arc.

(2) Shunt circuit to pull the carbons together when the voltage across the arc increases.

(3) Starting circuit consisting of resistance in series with a switch to cut it out when the arc burns.

The drop across the starting resistance through which the current first flows causes current to flow through the arc and series coil This coil pulls the carbons apart opening the starting circuit The shunt coil prevents the arc voltage becoming excessive and can close the switch, if necessary, in the starting circuit thus maintaing a safe current path through the lamp.

moving coil regulator is the only device which automatically compensates accurately for short circuits and double grounds on the series line or for voltage variation of the constant potential supply.

Such regulators are so substantial and reliable that small sizes have been successfully mounted on poles in transformer cases and operated by a time switch. Wherever possible however, they should be installed in a station so that the line can get more frequent and careful testing and an ammeter can be continuously kept in the lamp circuit.

In every series circuit the devices used on it must have a means of by-passing the current when the device itself is inoperative to prevent the opening of the whole circuit.

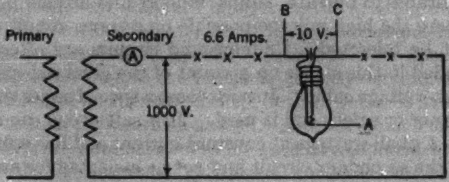

FIG. 8—THE FILM CUT-OUT SOCKET FOR INCANDESCENT LAMPS

When lamp filament *A* breaks, the voltage *BC* rises from that of the lamp to that of an open-circuited regulator causing the film to break down and short-circuit the lamp.

The arc lamp has a mechanism arranged as in Fig. 7 so that a shunt coil connected across the arc causes a switch to be closed and the arc short-circuited when the drop across it becomes excessive. Another coil in series with the arc and line acts to give the correct spacing of the carbons or electrodes.

For the incandescent lamp a simple form of switch is incorporated in the socket. An insulating film is placed between the two sides of this switch. This film has normally only to withstand the drop across the lamp filament, *A* (Fig. 8) of 10 volts. However, if the lamp burns out the filament itself gives the effect of an infinite resistance, and the full generated voltage of the regulator or generator is impressed upon the insulating film, causing it to puncture and short-circuit the lamp filament. In a moving coil regulator this maximum voltage is approximately 25 per cent more than the full-load voltage. A switch in the socket operated by the insertion of the lamp in this socket short-circuits this film when the lamp is removed.

Such film cut-out sockets are very simple and inexpensive, and enable men with little training easily to renew the lamps, even under trying conditions of lamp location and bad weather.

In general, devices other than lamps have seldom been used to any great extent on street series circuits. On account of the dark until daylight schedule it is often desirable to operate lamps indoors and around places where the high voltage possible on a series circuit cannot be tolerated. Underground circuits are often included in this status on account of the great expense of high-voltage cable. In such cases a special series transformer or safety coil is used. This safety coil can supply a small wattage at constant current and low voltage on the secondary circuit and yet be insulated for an extremely high voltage on the main series circuit.

Instruments or meters of a special nature are seldom required for series circuits. A high grade ammeter should be in series with the lamps for upon it depends the accuracy of the adjustment of the regulator and the life and candle power of all the lamps. The possible accuracy and permanence of the adjustment of the moving-coil regulator is well comparable to that of the ammeter, and little difficulty is experienced in its operation. Municipalities sometimes require a chart of the current in the series circuit and maintain curve drawing ammeters for this purpose. In other cities a jack on a pole allowing the insertion in the circuit of a plug with ammeter fulfills all requirements.

MAINTENANCE

Maintenance and testing of series circuits is special only in that checks through the day are advisable in order to repair before dark any damage that may have occurred during the hours of non use. Troubles usually show in the form of grounds and open circuits. Simple electrical tests locate troubles, of this nature. They must be promptly removed to avoid damage to the lamps or interruption of service during the dark hours.

SUMMARY

The advantage of the series systems of street lighting distribution consist of (1) effectiveness; (2) simplicity of control; and (3) high electrical efficiency of lines and lamps.

The disadvantages are (1) high cost of line per dollar of income; (2) high voltage; (3) special lamps.

The ideal system ultimately should be one to operate a highly efficient standard lamp from the existing multiple distribution lines necessary for residences and factories. Complicated special apparatus and special wiring, other than to the lamps is thereby eliminated. Reliable and inexpensive control devices must be available to turn the lamps on and off. Although much time and expense have been spent on such development ever since the lighting industry started, none has ever been sufficiently perfected to be worthy of generally replacing the simple series system.

DISCUSSION ON "THE SERIES SYSTEM OF STREET LIGHTING DISTRIBUTION" (HURLEY), CHICAGO, ILL., JANUARY 9, 1920.

A. E. Betts: I would like to find out what success different people have met with in the use of the refractors on such lamps, and also what effect it has had on illumination, after they have been cleaned.

E. Sweitzer: I am of the opinion that a refractor does materially benefit the distribution of illumination. It is found with the ordinary diffusing globe the natural angle of distribution is around 45 deg. below the horizontal. In the use of refractors the maximum angle is raised to about 10 or 15 deg., and consequently you get a very much wider distribution. I think fundamentally that is the basic reason for using refractors. I venture to say that there are very few installations having high candle-power where the refractors are not being used. Where large units are used I think it is of decided advantage. As far as maintenance cost is concerned, it is not excessively high. They are made quite heavily and with ordinary service, breakage should be very light. The operating records of central station companies who are using street lighting units with refractors will well bear out my contention.

A. E. Betts: The idea is that after a refractor has been in use for a short period of time it becomes very dirty and unless you institute a system of periodic cleaning, the lighting will be very materially reduced and the refractor of no benefit.

E. Sweitzer: That is true. From a purely commercial point of view I want to say that both central station companies and municipalities have gotten to the point of seriously abusing street lighting systems such as are now in operation since Mazda Lamps have become as prevalent as they are. Heretofore when they used arc lamps they had to get around and trim them at certain periods. A lamp had to be trimmed every hundred hours or 150 hours, and at that time the lamps were naturally cleaned, but when you come to Mazda Lamps the operating company seems to think "Well, here is something that will last a thousand hours" and while the contracting company calls for the lamps to be maintained in proper form, in a great many instances they are not given near the attention that the arc lamps were. If the refractor is permitted to get dirty naturally it absorbs some light, but if they are given reasonable attention, possibly, I would say six to eight times a year, there would be a very slight loss of light. Does that answer your question?

A. E. Betts: Yes, it does, only our experience is a little bit to the contrary. We find that if they are allowed to go even six weeks the amount of dirt accumulation on the refractor very materially reduces the light.

E. Sweitzer: Isn't it true that it is due very largely to the atmospheric conditions? Taking it as a whole I don't believe they should require such frequent attention.

A. E. Betts: We have not made any tests on the lamps, although we intend to do so, but it is evident that the light is very materially reduced.

F. F. Fowle: Inductive interference from series distribution systems is due fundamentally to two causes. One is the character of the current in the circuit and the other is the nature of the layout of the line circuit itself. In the early days with the use of old T. H. and Brush arc machines inductive interferences with telephone systems from series distribution, using generators of this type, were very severe, more severe than is encountered ordinarily today. That was due in large part to the fact that the wave form produced by those machines was very irregular and in the case of both machines there was a very peaked wave of electromotive force which arose instantaneously to very high and abrupt values and of course produced a very serious disturbance. The current was a constant in the sense that it had a constant average or effective value, but the instantaneous value even though it was so-called direct current, was anything but constant. With the passing out of direct-current series lighting and the introduction of the alternating-current system, of course, we come to the modern conditions. Series circuits are, generally speaking, of three types, the so-called open-loop series circuit, in which the line wire is led from lamp to lamp by the most available direct route without regard to anything else, and in that type of circuit the configuration of it is generally an irregular open loop. The second type is the closed loop circuit, in which the line wires are kept parallel so that while it is a series circuit it is a parallel two-wire series circuit in which the return wire is always kept on the adjacent pin, or at least on the same pole line with its mate.

The third type, so-called, is a combination of the two or a mixed loop circuit. The open loop circuit gives trouble of course from inductive interference if it is in the same neighborhood with telephone circuits, particularly circuits of open wire. The trouble with telephone circuits in cable is considerably diminished. The electrostatic induction is entirely removed by placing the telephone circuits in a cable, within a grounded metal sheet, but magnetic induction is affected but slightly. A practically sure and efficient remedy for inductive interference in series distribution can be obtained by using the closed loop. In that case the inductive interference is usually less than it would be from the ordinary multiple distribution system with the same spacing of conductors. Of course, its current is limited in value; the current is very frequently less than would be flowing in a multiple distribution system,—at least when the load is of fair proportions. There are no very effective means, from the telephone standpoint, to combat induction from the open loop construction in ordinary city distribution. It is usually impractical to transpose against it, because the points of change or discontinuity in the lighting circuit occur with such frequency that any attempt to put transpositions in the telephone circuit is impractical for the reason that the poles are not close enough together or they don't come close enough to the theoretical transposition point, and it would in any case make a badly cut up and disarranged telephone distribution system, that is, according to the open wire.

It is also of some advantage to place the lamps alternately on each side of the series circuit. Where the two wires of the series circuit leave the station, let us say the first lamp is in wire No. 1 and the next in No. 2, the third in No. 1, the fourth in No. 2, and so on. That means the point of zero potential or the neutral point in the series circuit will come at the far end and the drop of potential will be uniform in the two wires, so that if taking maximum values of one wire at the station at some particular instant as 1000 volts positive, the opposite end is 1000 volts negative, and at the far end of the loop at that same instant the potential is zero.

I would like to say a few words about the use of line conductors for series distribution. Some years ago I had an opportunity to make an experimental installation of copper plated steel lined wire in a small series distribution system. The lamps were closed carbon lamps operating on about 5 amperes, and there were 19 or 20 lamps in the circuit. It was operated from a 2300-volt bus through a regulating reactance. This circuit was carefully tested in normal operation for current, voltage, energy input, power factor, and then in the day time the lamps were all strapped out with a jumper at the terminals of each lamp. We then measured the true continuous current resistance of the lamp circuit and we also put a normal operating current around the line circuit, with the lamps cut out, and measured the impedance, effective resistance, and effective reactance of the line circuit and power factor. After these tests were completed, we took down the line wire, which was No. 6 medium hard drawn triple weather-proof copper, and on the same pin positions and precisely the same positions throughout the entire circuit we strung up some No. 8 hard drawn triple weather-proof copper cased steel. Then we repeated the whole series of tests and I can give you a few of the results that we obtained. We found that the No. 6 annealed copper had a breaking load of an average of 750 lb., a potential strength of approximately 38,000 lb. per sq. in.; No. 8 hard drawn, an average breaking load of 1200 lb. approximately, and about 97,000 lb. per sq. in. A test on the No. 6 copper circuit with the lamps cut out showed that it had a true resistance of about 9.05 ohms and the effective a-c. resistance, about 9.25 ohms. The ratio of the effective a-c. resistance to the true d-c. resistance was about 1.02 per cent. That is only about 2 per cent skin effect in that copper wire at sixty cycles. The true resistance of the copper clad steel circuit was practically four times as much, and the skin effect factor was practically the same as it was with the copper wire, that is, between two and three per cent. Taking into account the actual diameter of the wire and the temperature, we found that the conductivity of the copper wire was 97 per cent.

Another interesting result was the calculations on the measurements of the effective reactance of the copper circuit. This was a mixed loop circuit but more of the open loop type than of the closed loop. It was found that the total reactance of the mixed loop circuit, as calculated from the measurements, was the same as though it had been a parallel loop circuit from end to end, with a spacing of 11.5 ft. between the two wires. The measurements with the lamps in circuit showed that the power factor at the bus, which would include, of course, the power factor of the line and the power factor of the reactance combined, was, for the No. 6 copper circuit about 62 per cent, and for the No. 8 copper clad steel circuit about 66 per cent. The power factor of the line circuit with the lamps in it was only about 92 per cent for the No. 6 copper, and the same figure for the copper clad steel. That is to say, that shows that there was no particular additional reactive effect because of the steel core in the copper clad steel wire. The main effect of it was increased line resistance. The average watts lost in the line in this particular case was about 230 watts for the copper circuit and about a thousand watts for the copper clad steel circuit, that is, roughly, in the ratio of four to one.

In the matter of structural advantages and economies I gave you the figures on the breaking load of the two wires. Of course the No. 8 copper clad steel is much the stronger of the two and naturally would be the more reliable conductor, from a structural standpoint. However, its disadvantage is having about four times the resistance of the copper, and the increased energy loss.

Without going into details, taking the cost of copper wire as a basis, which at that time was in the neighborhood of sixteen cents, and assuming a schedule of 3000 hr. per annum of operation, it worked out that the turning point in economy, as between the two wires, was when the energy cost about 0.55 of a cent per kw-hr. If the cost of energy was more than that there was economy in using copper clad steel, but if the cost of energy was less than that, there was no economy in doing that. There was, of course, considerable increase for reliability, and the inference would be there was somewhat less maintenance cost. That circuit has been in service for some years and as far as I know it has never given any trouble.

G. N. Chamberlin (read by C. E. Skinner): In setting forth the advantages of the series system of distribution, I think Mr. Hurley has been more than fair in giving what he considers the disadvantages.

HIGH VOLTAGE

Modern arc lamps cannot be used to advantage on multiple circuits and series incandescent circuits are as a rule I believe not above 5000 volts. It is doubtful if this voltage, with the circuit wires widely separated and the flow of current restricted to practically normal, is not safer than 2200-volt lines carried, as they usually are, closely spaced and backed by unlimited current, in case of short circuit or double grounds.

SPECIAL LAMPS

It is well known that losses from theft where multiple lamps are used for street lighting are very large and even series lamps are at times brought in for free renewal.

Several central stations are now using an auto transformer, so that the low-voltage, high-current, large base lamps can be used on multiple circuits for no other purpose than to eliminate the loss from theft.

Series lamps have greater intrinsic brilliancy or a more compact filament. This allows of the more effective use of scientifically designed auxiliaries (reflectors, globes and refractors.)

It is characteristic of the incandescent lamp filament that its diameter decreases during life, due to evaporation. The decreased diameter results in increased resistance. With a constant-current supply the wattage is increased and the candle power maintained, while on any form of multiple or mixed circuit there must be a continual changing of the line potential in order to maintain contant candle-power and candle-power is what the central station sells and wishes to maintain as nearly constant as possible.

The speaker fully realizes the advantage to the manufacturer, distributor and central station of having the fewest possible types of lamps, but he believes that the series street lighting lamp is fully justified.

There is soon to be installed at Saratoga, N. Y., an ornamental duoflux unit consisting of 1000 candle-power and 250 candle-power lamp in one globe. A small evacuated bulb containing mercury and suitable terminals, tilts one way and then the other by successive throwing on of the circuit. The 1000 candle-power lamp will be used until midnight and the 250 candle-power from midnight until daylight. Simple switching apparatus can be made use of where a separate circuit is controllable from the station.

F. W. Parker (read by C. E. Skinner): In Mr. Hurley's paper under "apparatus" the statement is made "The moving coil regulator is the only device which automatically compensates accurately for short circuits and double grounds on the series line or for voltage variation of the constant potential supply." He further states that "The constant-current regulator depends upon the electrical repulsion existing between the primary and secondary coils under load to produce and maintain a constant current in the secondary or lamp circuit. Such a regulator is of great value because it can automatically maintain a fixed secondary current through any number of lamps from one up to its maximum capacity."

Is it not a fact that the repulsion action depends upon the strength of the magnetic field set up by the current in the primary and secondary coils of the regulator? The magnetic fields being proportional to the value of the current. Under short-circuit conditions coming on a part of the series circuit, it must necessarily result in a greater current, hence the repulsion action lags behind the current thereby introducing a time element during which the lamps remaining in the circuit are subjected to abnormal current.

What would be the value of this current with a short of fifty, seventy-five or eighty per cent, and under said conditions would the remaining lamps in the series circuit be injured or destroyed?

Regulators with stationary coils built with high inherent reaction, and operating upon the magnetic leakage principle are used for the control of series incandescent lamp circuits. With this type of regulator the time element mentioned above is entirely removed and it is possible to get any range of regulation desired by adjusting the inherent reaction. Is such equipment now available, and when built as described above, will it give full protection to series burning lamps when operating from a constant potential supply?

F. F. Fowle: The example is shown at the right of Fig. 3 in Mr. Hurley's paper. The example of the open loop circuit is shown at the left in Fig. 3.

E. N. Lake: I don't understand that the previous speaker intended to recommend a special copper clad wire for distribution, but if that was his intention I want to say that the use of copper clad wire may, under certain climatic conditions, prove to be very unsatisfactory. I refer particularly to locations where the lines may be close to salt water.

F. F. Fowle: I would like to ask Mr. Lake if he has reference to the bare wire or to the weather-proofed wire?

E. N. Lake: The bare wire.

F. F. Fowle: I will say, in general, that the corrosion of copper clad steel wire with regard to its life, would be the same as that of bare steel. It sometimes happens as you say, near salt water, that it corrodes rapidly, but I know that so far as recommending it is concerned there are a great many thousands of miles of that wire that are used both bare and insulated, and it is giving on the whole, very satisfactory service. I can see no reason why you shouldn't recommend it where the economics of the situation warrant.

E. N. Lake: To take an example, would you recommend it for use across salt marshes?

F. F. Fowle: Yes.

E. N. Lake: In the bare condition?

F. F. Fowle: Yes.

F. A. Vaughn: I would like to ask Mr. Lake where the corrosion occurs. I would get from inference, that it occurred on the steel, and with reference to your own statement it would seem to me it would make no difference, if the copper coating was perfect, whether there were steel wire inside or not. I should like to ask Mr. Lake if the salt water, or the salt air, did get in to the steel through the copper coating.

E. N. Lake: The particular installation that I refer to was along the border of salt water and it became necessary to replace the wire with solid copper wire. It was not possible, from the brief examination that was made, to determine whether or not it was defective wire. The assumption would be, of course, that it was defective in manufacture, but the fact is that, due to the processes of manufacture, you cannot always depend upon a uniform coating of copper over the steel core.

F. F. Fowle: Perhaps I can explain a little about the manufacture which will make it clear why corrosion sometimes occurs, and why the thin places sometimes occur. The copper clad steel wire has been made in this country by three different processes. It was first made by a company that has since gone out of business. It is now made by the Griffith and Roth processes. At one time the company which manufactured it under the Monarch process started by using steel rounds and coated them with copper. Then for some reason or other they abandoned that process and began using square steel bars and coated them with copper. That undoubtedly had a bad effect. When steel rods were used and coated with copper the steel was well centered in the copper coating, and it drew down, that is, it rolled down very uniformly in the hot rolling, and then drew down very uniformly in the cold drawing, except in one or two cases, but with the square sections there was obviously a thinner coating of copper at the four tips of the squares than there would be at the middle of the four sides, or between the tips of the squares, and therefore bare steel was more likely to occur at these points, and it did occur more frequently. While it was made at that time, with the square steel I think it gave some trouble. As far as I know at this time the two companies that are making it are both using round steel.

As to the character of the copper on the outside of the steel, all the researches that I have made on it have never indicated that the copper was anything but good metallic copper, unless the copper was oxidized. I have analyzed cases where we found that the wire had low conductivity and where it was determined upon getting the relative area of the copper and steel that the conductivity should have been normal. Upon an analysis being made of the copper it was found that the low conductivity was caused from within the copper itself. Copper is exceedingly sensitive to conductivity. A few tenths of a per cent of impurity in copper will effect the conductivity immensely, so that it is necessary to be careful to get the purest grade of copper on the copper clad steel, as it is to get it in the ordinary copper wire. I have known of many cases of premature corrosion of copper clad steel that didn't occur as a result of the steel showing through the copper, which really accounts for corrosion in many cases.

E. N. Lake: It would be very interesting to know whether there is any considerable quantity of it over salt marshes. I think it would be well to make a further investigation.

F. F. Fowle: I can't say specifically as to whether it is installed over salt marshes, although a great deal of it is in use in this country for railroad signal circuits. I think that, however, is made of weather proof wire.

Mr. Cameron: It is true that when a short circuit involving an appreciable amount of the street lighting circuit occurs, there is a time lag before the transformer accommodates itself to the new conditions. That interval isn't a matter of hours or minutes; it is of seconds or fractions of seconds, and one feature which helps even to reduce that interval is the fact that the action of the tungsten filament to increase this current is not instantaneous, either. The lamp comes up to heat after the current has increased. There is a time lag there also, which, for all practical purposes, eliminates the "leisureness" of the transformer in meeting the new conditions.

As to the second question, if it is so important that even intervals of that instantaneous nature are worthy of comment, how much more necessary is it to have a device which meets that condition automatically and instantaneously, rather than with a device where it is necessary to manually change taps after you have become acquainted with the conditions on the circuit, which is not a matter of seconds or fractions of seconds, but perhaps minutes or hours, in a division where the current may change ten per cent, which is one quarter the life of the lamp, before the next tap brings you a better condition. That is, your transformer, without regard to steps, automatically and instantaneously does it.

C. H. Shepherd: The Commissioners of Lincoln Park operate a street and park lighting system of some 1700 street series lamps which are supplied with power from 18—7½—ampere series circuits with voltages ranging from 2000 to 7000. Each circuit has its individual transformer, regulator, oil switch, control panel and instruments, the power for the entire system being derived from a 12,000-volt 3-phase power line from The Sanitary District of Chicago.

From the standpoint of the lay-out engineer these circuits divide themselves into three main classes, namely, open loop, closed loop and combination.

In order to properly segregate defective portions of circuits, pot heads are installed at various points to facilitate not only this operation, but the locating of trouble and maintenance of continuity of service as well. At the feeding points of all loops, disconnecting pot heads are installed with triple braided jumpers so that any loop may be disconnected, and shorted out of circuit in case of trouble. At the ends of all loops, pot heads are installed with ground connections arranged so as to be made immediately available in case of necessity, and where two circuits run parallel on the same boulevard it is thereby made an easy matter to select for service the two good sides of these circuits or loops in case of the development of a ground or open on one side of each. The pot heads installed at the ends of the circuits are also available for rotation testing and for opening any circuit for capacity tests. Pot heads are also installed at the ends of feeder lines on the energy side of the first lamp on each circuit and the last lamp respectively, thereby allowing the feed line to be disconnected and closed across for the purpose of making a bridge test.

In cases where it is necessary to interrupt the continuity of the cable sheath in such places where pot heads, triple braided lamp legs, or similar devices are installed, the cable sheath is bonded across the break and pot head sleeves, belled at the triple braided end and wiped into a sealed joint, 18 in. back of the bell, are installed, thus effectually taking care of the accumulation of static at these points. In cases where by polarity tests the cable is shown to be positive to ground ground rods are installed and bonded to the cable sheath but in places where the cable shows negative to ground the cable is either insulated, or no action is taken at all where the process of insulating would be too difficult.

In checking the rotation of lamps on circuits, a ground is placed at the center of the loop in question, and another ground placed on the right or left respectively of the circuit at the switchboard. This connection cuts out the lamps on the grounded leg allowing the lamps on the ungrounded leg to burn, with the feeder transformer at half-tap making it a very easy matter to check up a circuit diagram by merely running along the circuit in a machine and checking off the lamps on the right or left according to whether they burn or not.

On long closed loops sometimes involving an entire circuit, the lamps are connected on the step by step system, or electrically staggered on the line cables. This method cutting down the greatest length of cable between lamps, to two-lamp spans. Obviously this construction facilitates the locating of trouble and reduces the necessary cuts to a minimum. The rotation of lamps is checked after each change by the method outlined above.

Series circuits on this system are always treated as condensers, the conductor forming one plate and the grounded cable sheath, the other. In order to determine whether or not these circuits should be operated grounded or ungrounded, potential gradient tests were made covering each circuit in question. On ungrounded circuits the potential gradient is almost uniform due to the fact that there is no ground on the circuit, allowing the zero potential point to shift position to compensate for the changes in reactance on the circuit due to lamp burn-outs, regulator movements, etc. and the prevailing electrostatic conditions throughout the entire length of the circuit, including the feeder cable. With the apparent electrical center of the circuit grounded the potential gradient is unbalanced and shows a lack of uniformity due to the fact that the zero potential point is fixed by being grounded and the former flexibility and freedom for adjustment of the ungrounded circuit in response to changes of electrostatic and electromagnetic conditions is thereby seriously hampered. Such unbalance due to action of the regulator and to changes in electrostatic capacity caused by variations in the specific inductive capacity of the various dielectrics used in the cable insulation along the circuits are very apparent in making such a test.

After investigation of the potential gradient situation obtaining on this system, it was apparent that the advantages on an ungrounded system outweighed those inherent on a grounded system and it was therefore decided to operate the system ungrounded as it is a very easy matter to place grounds at the salient points of the circuit whenever necessary.

In the early days of this system the series circuits carried 7½-ampere enclosed-carbon arc lamps with an efficiency no better than the average for these units. Some five or six years ago the arc lamps were eliminated and a compensator construction substituted on all existing standards, each compensator supplying power to a 15-ampere 400-candle power type C lamp. As extensions to the system were being rapidly made, it was decided after certain experiments to feed lamps on all new construction from the secondary side of suitable series-multiple transformers and after a series of experiments and tests had been conducted with this end in view, a standard transformer was adopted and installed from that time on. These transformers have a partly open core—the primary and secondary windings being insulated by a dielectric having a strength of 20,000 volts, 60 cycle. The line and lamp connections are made with tinned brass wiping sleeves which are fitted with suitable lead sleeves soldered on, wiped to the cable sheath, properly insulated, filled, refilled and sealed, thus establishing a bell at each terminal and a 100 per cent cable sheath bond. This construction operates as well submerged as dry, no static trouble being experienced, as the secondaries are not grounded, but are carried in 600-volt duplex rubber and lead cable direct to the lamp.

The safety island light system is fed by transformers of similar construction, but of a different suitable characteristic and capacity, each transformer having as a secondary load five standard 56-watt railway type B lamps.

The capacity of the type C transformers is approximately 225 watts with a regulation within one-half of 1 per cent of normal full-load current and a rise in pressure from full load to open secondary of not more than 200 per cent of normal, the power consumption expressed as a figure including line loss, averaging about 272 watts per unit. The island light transformers on the other hand have a capacity of approximately 220 watts with a very low power factor, and a rise in pressure from full load to open secondary of not more than 50 per cent normal full-load pressure.

By the elimination of the arc lamps and substitution of the devices mentioned above, each circuit has been made metallically solid, allowing a number of advantages which did not exist previous to the time mentioned. At the present time three continuity devices are in service, *viz.*: film cut-outs, compensators, and series-multiple transformers.

Some two years ago a testing ground bus with suitable quick connecting devices was installed on the main series circuit switchboard allowing certain and rapid trouble shooting by means of using the lamps themselves as indicators. This process is extremely simple, it being only necessary in the case of a ground on the exterior circuit, to connect to the ground bus the side of the circuit nearest to the trouble as determined by a current test. The circuit is then cut in at half tap, the lamps on the grounded side naturally being cut out and a quick trip made to the last lamp out and the first lamp burning. This point being reached it is quickly checked by a reversal of the ground at the switchboard, the defective cable being then narrowed down to two lamp spans, after which the trouble is quickly located at the exact point by a visual test, magneto or bridge. In the winter of 1918 we had 161 cases of trouble and the average time of location was 17 minutes. The cost of this ground bus was about $150 and it paid for itself many times over from the first month of use.

As stated above, any feeder lines may be disconnected and shorted out at the feeder point leaving this line available in case of trouble for a Murray loop, or Varley loop test, or both. Open circuits on similar lines may be easily located by disconnecting sections, by the use of a capacity bridge, or by breaking down the insulation at the open point by means of high tension to ground.

As an adjunct to the above, a progressive total of cable lengths from right to left and from left to right on each series circuit is being prepared which we anticipate will greatly facilitate the locating of points at any given distance from the source of supply of any circuit, when such distance is indicated by means of a bridge or loop test.

W. A. Del Mar: Some open-circuit series-arc lighting systems have been installed. A pessimistic theorist might say that they would be objectionable from the point of view of inductive losses, excessive impedance, etc. I would like to know whether any engineers, who are familiar with such systems, are encountering any difficulties in operation.

Due to the small current used in series arc lighting, usually less than ten amperes, this objection is probably not important. Nevertheless, it would be very interesting to hear of some actual results in practical operation.

Henry Nixon: It is a fact that we have noted some differences in the current value at the phase end and the ground end of a long circuit. It hasn't been particularly serious, except that in the longer lines it has been necessary to install reactance in series at the point where the return circuit starts. That difference in the amperage between the phase and the grounded ends of the circuit I believe has never been more than about 0.4 of an ampere, but on some of our circuits we operate at a higher amperage than the rated amperage for the lamp. Mr. Synder, our operating engineer, was talking to me about it the other day and in the course of our discussion he drew my attention to the fact that we were now getting a longer life out of the four ampere lamps used on such circuits than we were out of similarly designed lamps.

It was our idea, from the few figures we had at hand, that the average amperage at which the lamps were operating—I don't know whether I make that clear or not—the average at which the average lamp was operating would be slightly less than the rated amperage of the lamp, giving a longer life to that lamp. So far as cable trouble is concerned, the mechanical troubles and grounds, etc., we consider them negligible. The only trouble we have is due to citizens driving parkway stakes through the cable and cutting it in two. This cable has been in service since 1913 and very few cases of trouble have come up except from that cause. I don't know whether Mr. Snyder has any figures here or not.

Mr. Snyder: I have not.

F. A. Vaughn: May I add that the Milwaukee system, I think, is the type of system that Mr. Del Mar speaks of, and while the system is not entirely installed, there are hundreds of thousands of feet of cable already in and operating, and this system is largely of the open-loop circuit type. The cable is broken up into isolated sections very often, and so far there has been no noticeable or appreciable effect on the system as it is operating at the present time. I don't know of any cases of trouble that have arisen from this cause, and while I think there are some of the engineers of the Bureau of Illumination Service here, and they have not called my attention to troubles of that kind, if I am incorrect, I hope they will correct me at this time, but it is a fact that there has been no noticeable effect from this theoretical thing that Mr. Del Mar speaks of.

C. E. Skinner: On the question of steel tape cable for series distribution, when it first came up there were quite a number of us that thought it was doubtful whether the inductive effect of the steel tape would permit its use, but a few tests on that point dispelled our fears and it is used extensively now. About 1912 there was quite a lot of it put in under my direction at Champaign, Ill. As far as I know they have had no mechanical troubles due to the breaking through of the steel tape sheet, and no electrical troubles to speak of outside of defective joints which were defective when they were laid.

F. F. Fowle: Talking about the inductiveness of the steel tape cable, which is quite comparable with the case of steel cables for ordinary power conductors, research has proved that it is a very interesting fact that whereas some of the grades of steel, that is, mild steel, are highly inductive, that a steel that contains as much carbon as the Sells-Martin is practically non-inductive. That is, measured by the skin effect, it is almost negligible. I made a series of tests on some steel cables of all grades up to the high strength strand. The high strength strand was merely a trade name, but it is really a steel which has something like sixty or seventy points of carbon, and the skin effect, even with a 3/8-in., seven-wire strand made of that grade of steel is on the order of only one or two per cent, whereas the strength of the same size of strand made of ordinary wire strands, the skin effect is very marked indeed, so that, by analogy, the magnetic circuit that would be created around a steel tape cable if the proper grade of steel is used, would be practically negligible, but if made of mild soft steel there might be a noticeable effect.

C. E. Skinner: There are tests on this that are a matter of record, and can be found with a little research. One test in particular on a certain open loop, a street lighting circuit, a No. 6 steel tape lead covered cable, the voltage drop over that was only a little over double the ohmic drop.

Has Mr. Vaughn had any experience with the use of straight lead cables without the use of tape. It seems that the tape is put on merely as a protection against accidents, as I understand it.

F. A. Vaughn: In the older Milwaukee system practically all is lead sheet cable without the steel tape. Of course that doesn't bring into account, however, the magnetic effect that Mr. Del Mar speaks of, if that is what you have reference to, because the steel or iron tape is missing. Of course, the effect from the lead sheet is present on the high-voltage a-c. circuits.

C. E. Skinner: There are some lead-covered cables being used without the steel tape, buried directly in the ground.

Member: I would like to ask Mr. Shepherd if he has had any experience with the steel line transformers. That is one point that has not been brought up, and I believe it would be interesting to hear something about those transformers, the "pull" type regulating transformers with the moving coil.

J. E. Royer (by letter): I will give you some idea of what is used in Spokane.

About a year ago all the overhead lights were changed from the 7.5-ampere series arc lamps to the 6.6-ampere Westinghouse Luxolite fixture with the incandescent lamp. The maintenance with the new system is about the same as under the old series arcs, but as the incandescent lamps take less voltage, it permitted putting more lamps per circuit on the same station equipment and in so doing spare transformer capacity was gained. Out of 1500 lamps in circuit at the beginning, 1558 replacements have been made during the year giving about 2000 hours as the average life per lamp, although in a few cases lamps have gone as high as 4500 hours burning. The causes for renewals are many, the greater per cent being breakage, high voltage (crosses) and faulty manufacture. The voltage of the primary circuit ranges from 5000 to 7000 volts while with the old series arcs in some circuits it was as high as 10,000 volts.

On sidewalk lighting the 6.6-ampere luminous d-c. arcs are used on constant current transformers with rectifier tubes, in most cases four tubes being used, two tubes in parallel on each side but only one tube per side taking the load. We have had quite a little trouble with tubes during the past year in the matter of hours life. The later tubes do not seem to have the good qualities the earlier ones had. I hope something in the way of care and treatment of tubes will come up in the discussion as we have tried drying, washing and rest, but as I said before, the life of the later tubes does not come up to what we used to get.

In the matter of testing circuits in daylight hours, we test with 220 d-c. every three hours for open or ground.

W. P. Hurley: The moving coil regulator has an inherent reactance of approximately 40 per cent when the coils are in the full-load position. This value increases to 100 per cent depending on the load which in turn determines the position of the coils. Tests with an oscillograph have shown that such a regulator has under full-load and short-circuit conditions, enough reactance to amply protect the series lamps remaining in the circuit. The current returns to approximately normal value within less than one second. These regulators are, in many installations, started by time switches without separating the coils or injuring the lamps. The maximum current in the lamp is approximately three times the normal current when the regulators are so started.

Presented at the 357th meeting of the American Institute of Electrical Engineers, Chicago, Ill., January 9, 1920.

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MULTIPLE SYSTEMS OF DISTRIBUTION FOR STREET LIGHTING

BY WARD HARRISON
Engineering Department, National Lamp Works

The wider use of multiple lamps supplied from standard secondary distribution systems is considered as a step in simplifying operating problems and reducing the disporportionate cost which goes to cover fixed charges on special equipment and additional lines for street lighting only. The advantages in simplicity and flexibility of multiple connected lamps are discussed with particular reference to the frequent changes and extensions of street lighting service required in growing cities. The more general adoption of multiple street lighting is stated to be contingent upon fuller standardization of suitable methods of control applicable generally to existing electrical power distribution systems. Different devices in use or proposed for control of multiple street lamps are briefly described and the characteristics desirable in such apparatus are outlined. Attention is directed to the small differences in efficiency of present multiple and series incandescent lamps.

THE problem of securing street lighting most economically from multiple connections is one whose possibilities are far from exhausted, for multiple street lighting circuits and suitable control devices have not as yet progressed to the same degree of standardization as those of the series type. In the past, each individual case has been worked out, as far as possible, by adapting existing apparatus to meet the requirements of the particular situation. The purpose of the present paper is to review the systems now in use with their methods of control, and to indicate some of the more promising avenues of development.

EXTENT OF USE

In European cities, multiple street lighting systems are practically as common for both arc and incandescent lamps as are series systems in this country. Even here, where series circuits predominate, there are, notwithstanding, a large number of straight multiple systems in operation. It is true that they are largely confined to cities where low-tension current only can be used. Of these cities, New York, with many thousands of such lamps, is the most prominent example.

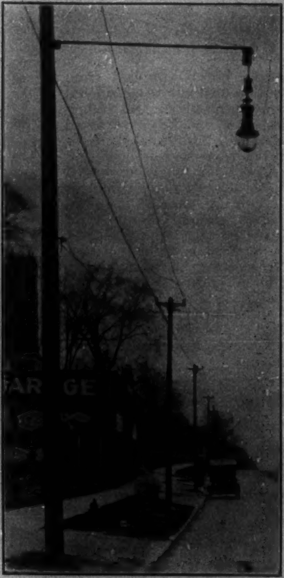

FIG. 1—TYPICAL OVERHEAD SERIES ARC LAMP POST SHOWING INSULATING HANGER

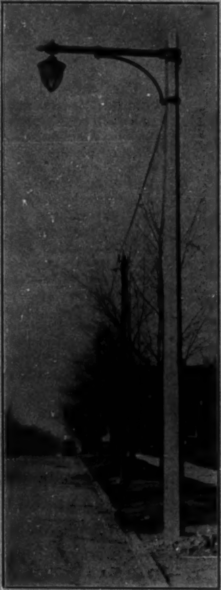

FIG. 2—SIMPLIFIED TYPE OF FIXTURE DESIGNED FOR MULTIPLE INCANDESCENT STREET LAMP

It is worthy of note, however, that in cities nominally lighted from series circuits, considerable numbers of street lamps will be found operated from the multiple distribution system.

ADVANTAGES

The following advantages which contributed to the standardization of multiple distribution for all other types of electric service apply also to street lighting.

1. Widest choice in number and sizes of lamps to be connected to circuit.

2. Ease of adding more lamps or lamps of higher wattage.

3. Simplicity and low cost of fixtures and accessories.

4. Safety.

The matter of flexibility is of immediate importance in the problem of designing street lighting for American cities and towns. There is not only steady growth in population and area covered, but there are also unexpected demands from time to time for increased street lighting service in given areas. The result, with series circuits of units of 50 to 100 lamps included on a single line, is that there must be an almost continual process of rearrangement and redivision of lamps among different circuits. In many cases service can only be provided after considerable delay and at relatively high cost, because existing circuits are so completely loaded that the addition of a few lamps makes necessary an extended revision of the system. No such limitations attend when the street lamps are operated from the general multiple distribution system.

An additional advantage of multiple street lighting is that, as mentioned above, the multiple circuit is now used exclusively by central stations for every other service. Were it used generally in street lighting as well, the reduction in types of apparatus required and the resulting simplicity of operation would benefit both the electrical manufacturer and the central station. At the present time, when a new real estate sub-division or allotment is opened, it is common practise for the lighting company to run a set of 2300-volt feeders to cover the plot, and to furnish service to houses as they are built. At the same time, a series transmission line is extended to furnish the necessary street lighting for the sub-division, and, in anticipation of increased requirements for service, both supply circuits are installed with a large excess capacity. The principal reasons for this duplication of service have been:

1. The arc lamp is inherently a constant current device and gives its best operation and greatest efficiency on series rather than on multiple circuits.

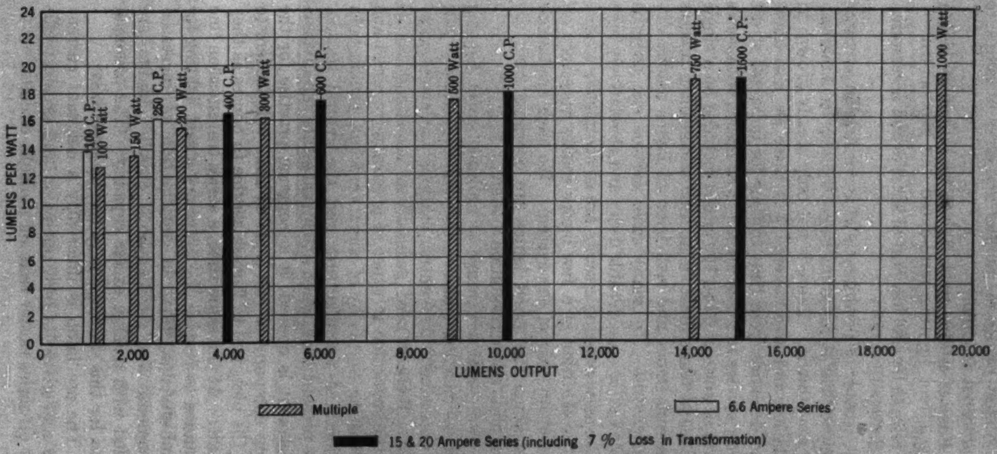

Fig. 3—Comparative Efficiency of Series and Multiple Incandescent Lamps

2. The series circuit is the simplest and most efficient method of supplying energy to comparatively small units scattered over wide areas; in many cases electric street lighting ante-dated the general use of electricity in residences by a considerable period.

3. A separate system of distribution has furnished a convenient means of automatically lighting and extinguishing street lamps from the central station.

There is an insistent demand for the extension of electric street lighting service in territory not now served by series lines, as for example, in Ohio cities, where in the past a considerable portion of the lighting has been furnished by natural gas, the supply of which is being rapidly depleted. In such territory the residences are, for the most part, furnished with current from overhead circuits along the rear lot lines, and the introduction of a pole along the street, to carry the series circuit has often met with a decided protest from the property holders. On the other hand, the cost of underground construction, with ducts running parallel to the curb, is almost prohibitive, especially in view of the increasing frequency with which concrete driveways now cross the parking in residence districts. High-tension underground lines passing through private property from the rear lot lines to the street, also involves expensive construction if safety is to be assured. Taken altogether, the conditions point toward the use of low-tension multiple circuits from the pole lines in the rear to the posts on the street, brought out either overhead or underground, as occasion may require. Under these circumstances the underground construction may be in the form of a relatively inexpensive lightly-insulated sheathed conductor protected simply by fuses at the transformer. Such a conductor is very much less expensive than that designed for high-tension work, and it can be installed without opening a wide trench across the lawn. If such a conductor is accidentally severed no serious harm can result, and the loss in itself is small.

CONTROL

Today if incandescent street lamps were to be operated throughout the twenty-four hours instead of at night only, there would be no discussion as to the desirability of connecting them to the existing multiple distribution system, but the obvious difficulty in this practise lies in obtaining a thoroughly satisfactory method for turning the lamps on and off. Of course, wherever the load is sufficiently concentrated to justify a separate circuit from the substation, as in the case of many so-called White Way installations in the business streets of cities, the lamps may be switched at the sub-station in the same way as in the case of series circuits. Ususally, however, to bring the circuits back to the sub-station entails a prohibitive expense in copper and it is necessary to supply the lamps from several different transformers or feeders located nearer the lamps. For example, in the case of residence districts, it would be particularly desirable if the street lamps could be supplied from the same circuits located in each block, which now provide service for the residential consumers, as in Fig. 8.

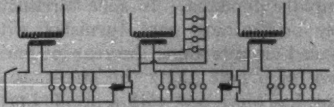

FIG. 4—CASCADE CONTROL OF MULTIPLE STREET LAMPS

The simplest plan is of course to have the lamps turned on and off by hand through the agency of a patrol messenger. Where labor is cheap, as in some of the European and Asiatic countries, this has been found a satisfactory and economical solution. In this country there are many cities where the White Way lighting is turned on and off by the policemen on duty. However, in view of the higher cost of labor in the United States and the extended distances between lamps, hand control is less desirable. Furthermore, the fact that gas companies are forced to rely upon manual control, and the difficulties which they have experienced, have served to deter electric lighting companies from resorting to it, excepting where there was no practical alternative in the way of a mechanical or electrical control.

Most of the efforts toward the development of remote control of multiple street lamps on 110-220-volt circuits can be grouped under three general types of systems,—cascading, relay switches operated from a pilot wire, and automatic clock or impulse-operated relay switches without separate control circuit. Diagrams illustrating these three methods of remote control are shown in Figs. 4, 5 and 6.

In the cascade method the section of lamps nearest the sub-station or control point are switched on and off by hand or a clock control switch. Energizing of this section serves to close a relay operating the switch for a second section. At the end of the second section is a similar relay which, being energized, closes the circuit for the third section of lamps, and so on. Opening the switch on the first section of lamps serves to de-energize the other sections of lamps in the same order. It is obvious that a method of this kind can be used for either alternating- or direct-current systems, or even a combination, and the load may be carried from as many separate transformers or sets of feeders as desired.

In the cascade system a failure of current supply in any section will turn out the lamps in succeeding sections of that cascade. However, this difficulty can be minimized by provision for hand operation of the switches in case of failure of one of the units.

The so-called pilot wire control utilizes a somewhat different principle for switching the lamps on and off. In this method, as will be seen from Fig. 5, lamps of the system are grouped in sections of suitable size to be controlled by a single switch and the separate switches are operated from a pilot wire which is energized from the sub-station or control point. The pilot wire can obviously be made of low current carrying capacity since it does not carry the lamp current but simply serves to energize the switch relays. Switch relays of several different models have been developed in connection with this pilot wire control. The simplest type consists of a solenoid-operated switch, which is closed or opened, depending on whether the current is on or off the control wire. Preferably, switches of this type should be closed when the pilot wire is de-energized, so that if the pilot circuit is accidentally opened the lamps will all be lighted, even though the break should occur during the daytime and therefore result in some waste of current.

Another type includes a mechanism with on and off positions. With this, the first time the control wire is energized the lamps are lighted. The next time the lamps are turned off, and so on. Continuous application of current to the control wire is not necessary with this form, but there exists a possibility of lamps being left turned off when it was desired to have them on, or vice versa, due to the switches getting out of step. A pilot or signal switch at the control station has been suggested to obviate this uncertainty. Also, one manufacturer has provided for convenient hand setting of individual switches which might drop out of step.

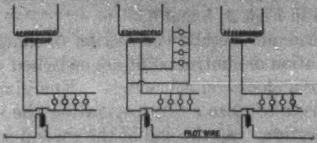

FIG. 5—PILOT WIRE RELAY SWITCH CONTROL

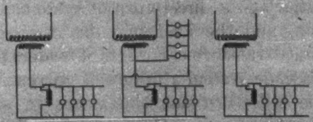

FIG. 6—CONTROL BY AUTOMATIC CLOCK OR IMPULSE OPERATED RELAY SWITCHES WITHOUT SEPARATE CONTROL CIRCUIT

In an endeavor to avoid the objections raised against the impulse-operated switches of the type just described other forms have been developed which operate from impulses of different strength. One of this type is illustrated in Fig. 7. The solenoid has two plungers so adjusted that a certain current will lift one and close the circuit. The impression of double this voltage on the control circuit lifts the second plunger which releases the catch and opens the lamp circuit. Such an arrangement using different impulses for the on and off position, has the advantage of avoiding the possibility of switches falling out of step. For instance, when turning on the lamps the control circuit can be given several impulses if desired without fear of leaving the switches in the wrong position.

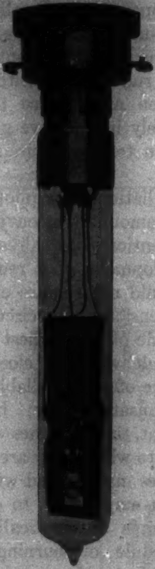

FIG. 7—SMALL RELAY SWITCH OPERATED IN INERT GAS

In order to reduce the wattage required to operate the switch to the lowest possible value and to be able to carry a large number on a single control circuit, the parts are made light in weight. Corrosion and the resultant failure of switches is avoided by sealing the entire mechanism in a glass bulb filled with an inert gas.

The capacity of these small switches is limited, but where a heavy current is to be handled they can be used as relays to operate a circuit breaker.

The pilot wires for operating the types of switches described may connect the switches in series. A greater flexibility in making extensions results if the switches can be operated in parallel, and in this case the neutral or grounded side of the distribution system may be used as one side of the control circuit and only one wire need be run to each switch. With a switch which operates on an impulse transmitted at low voltage, a relatively inexpensive galvanized iron wire such as is used in telegraph circuits, will frequently prove satisfactory.

Since the installation of a pilot or control circuit involves a certain amount of inconvenience and expense, considerable attention has already been directed toward the development of a remote-control switch system which would not require extra wiring beyond the necessary extension of the circuit from the feeders to the lamps. One of the earliest devices for accomplishing this result was the clock or time switch. Early designs were often unreliable in operation, and therefore quite unsatisfactory; however, there are available at present, time switches which withstand the severe conditions to which they are subjected. Again, time clock switches may be had which are electrically wound and which may be set to change the time of switching the lamps automatically, according to a prearranged schedule of burning. Experience has indicated that clocks require a considerable amount of attention and regulation to keep them operating and in close agreement with each other. This factor, together with the question of initial cost, has hindered the more extended use of these devices.

A tiny synchronous motor-operated clock switch requiring an almost negligible wattage, is also available for remote control switching on alternating-current circuits. This switch would find its best application in systems whose average frequency is well maintained and where there are but few interruptions of service.

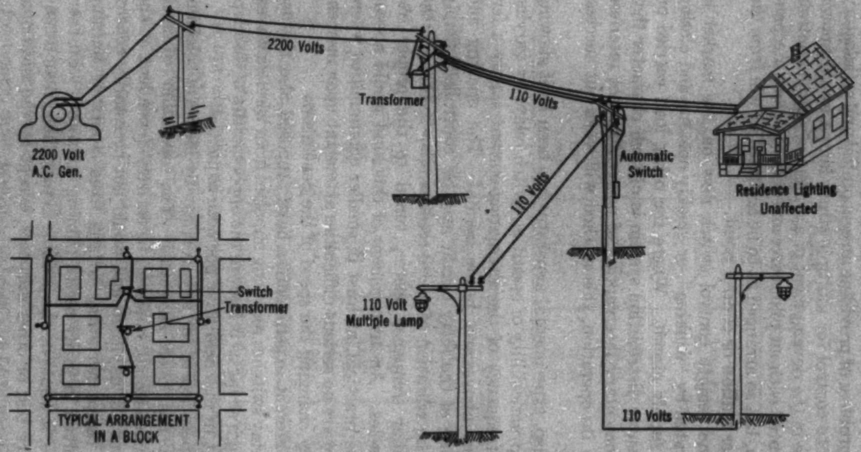
2200 Volt
A.C. Gen.
2200 Volts
Transformer
110 Volts
110 Volts
Automatic
Switch
Residence Lighting
Unaffected
110 Volt
Multiple Lamp
110 Volts
Switch
Transformer
TYPICAL ARRANGEMENT
IN A BLOCK

FIG. 8—PLAN OF OPERATING STREET LAMPS FROM SECONDARIES OF A HOUSE LIGHTING SYSTEM

Otherwise the switches are liable to require a considerable amount of resetting.

Quite a different and a more satisfying solution of the problem of providing an economical and satisfactory remote control for multiple street lamps would appear to lie in the development of a system of switch controls operated without pilot wires and without clock mechanisms, but so arranged as to be actuated at will from the power station or control point.

One remote control switch now on the market is operated by "winking", or quickly opening and closing the supply circuit. Where the other load carried on the same distribution system is of such character that a momentary interruption of power does not interfere with its operation, this method will work out successfully.

Also, experimental apparatus has been built showing the practicability of operating switch relays by varying the frequency of the alternating-current supply a few cycles above and below normal. The opening and closing of the relay circuit may be controlled simply by the vibration of metal reeds similar to those in the familiar frequency meter, tuned to the proper frequencies. An arrangement of this kind would appear to be practical, at least in smaller communities where there are no serious difficulties attendant upon momentarily changing the speed of the generators.

Different experimenters have proposed also the use of relay switches receiving impulses from wireless waves, though so far as is known, the opportunities of development in this field have not been exploited.

Operation of the switch relays by means of selenium cells has been suggested. The operation of such switches would, however, be entirely dependent on the amount of daylight, and would not necessarily control the lamps according to the fixed hours of burning involved in many street lighting contracts. Furthermore, it would not be possible to turn the system on and off for test or other purposes, except by the additional provision of hand-operated switches.

The development of a phantom-circuit-operated control switch has been described elsewhere[1], and with further development this plan would appear to have promising possibilities. At present, however, the expense of the apparatus is too great to permit of an individual control for each lamp.

TABLE I.

COMPARATIVE EFFICIENCIES OF GAS-FILLED INCANDESCENT UNITS.

MULTIPLE LAMPS

Watts	Lumens	Lumens per watt
100	1,260	12.57
150	2,040	13.66
200	3,100	15.51
300	4,840	16.11
500	8,750	17.45
750	13,900	18.48
1,000	19,300	19.33

SERIES LAMPS

Amperes	Nominal candle power	Approx. watts	Approx. lumens.	Lumens per watt.
6.6	100	72	1,000	13.96
6.6	250	155	2,500	16.11
15.	400	240*	4,000	16.65*
20.	600	344*	6,000	17.45*
20.	1,000	556*	10,000	18.00*
20.	1,500	809*	15,000	18.55*

*Includes 7 per cent allowance for loss in transformation.

It seems reasonable to believe, however, that apparatus operated without a pilot wire should soon be developed by means of which street lamps can be lighted and extinguished at will, in a manner which is not open to any of the foregoing objections. At the same time it is evident that such a system, to be successful, must not involve a high cost per unit installed; also, it must not require the addition of any considerable amount of equipment on the 2200 lines; finally, its mode of operation, particularly if it involves the use of high frequency, must be such that there is no possibility of injury to transformer insulation or to any apparatus forming a part of the present standard distribution system.

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[1] *G. E. Review, Vol. XX, No. 2, Feb. 1917.*

LAMP CHARACTERISTICS

The serious loss in light output when incandescent lamps are operated at less than rated voltage, is shown graphically in Fig. 9. Because of this characteristic it is essential with multiple systems that lamps be used whose ratings correspond closely to the voltage measured at the socket. In most cases with series systems, the operation of all lamps at proper efficiency is assured if the current is maintained at the correct value at the station. In some multiple installations a large number of lamps may be carried on comparatively long branches, so that there exists a considerable difference in voltage between sockets near the feeders and at the far end. This difference in voltage may become so great that the expedient of using lamps of two or even three different voltages is sometimes followed. Under these circumstances every lamp post must be marked with a symbol indicating the proper

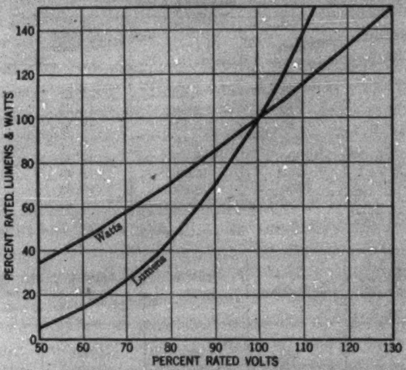

FIG. 9—VOLTAGE, CANDLE POWER AND WATTAGE CHARACTERISTICS OF MULTIPLE TUNGSTEN FILAMENT LAMPS

voltage of lamp to be used in the replacement of burnouts. In spite of this precaution, however, confusion of lamp voltages is very likely to arise unless the maintenance of the system is supervised with unusual care. A plan which is usually preferable is to provide for a very small drop between the street lamp and the supply circuit by running short branches and operating from one to three lamps on a branch. Table I shows the efficiency of the various sizes of multiple and series lamps computed in lumens per watt. For the 15-and 20-ampere units an allowance of 7 per cent has been made to cover loss in transformation. In Fig. 3 these data have been plotted graphically so that the comparative efficiency of series and multiple lamps of equivalent output may be readily seen. In the case of those series lamps which are available both in 6.6 and higher amperages, the more efficient type has been shown. The very slight inferiority of multiple lamps from the standpoint of efficiency is worthy of note.

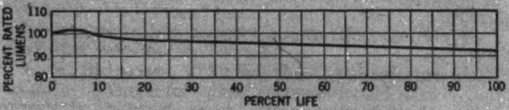

FIG. 10—TYPICAL LIFE-CANDLE POWER PERFORMANCE OF MULTIPLE TUNGSTEN FILAMENT LAMPS

The depreciation in candle power or falling off in light output during life, is comparatively slight for the modern types of gas-filled incandescent lamps. The curve of Fig. 10, based on life tests of a large number of lamps of the sizes customary in street lighting, illustrates the performance which may be expected.

CONCLUSION

A fundamental reason for the increasing interest in multiple systems for street lighting is the growing realization that with present systems but a small part of the total cost of operation per lamp per year (usually not more than one-third) is to cover the actual cost of light; *i. e.*, cost for current and lamp renewals, or electrodes; the major portion consists of fixed charges on special equipment, floor space rental in substations, and proportionate charges for the use of pole lines and underground ducts. Furthermore, these latter charges are abnormally high per kv-a. owing to the small load carried on the average series circuit. Twenty-three hundred and 11,000-volt multiple circuits may be loaded at 400 to 4000 kv-a., while a 6.6-ampere circuit occupying an adjoining duct, is usually considered fully loaded at 20-30 kv-a.

A DISCUSSION ON "MULTIPLE SYSTEMS OF DISTRIBUTION FOR STREET LIGHTING" (HARRISON), CHICAGO, ILL., JANUARY 9, 1920.

N. B. Hinson: I would like to ask Mr. Harrison why it is the switch is put on the secondary circuit rather than on the primary side of the transformer.

Ward Harrison: The reason we have shown the control in each case on the secondary side is that we feel in any system of multiple distribution it is desirable to be able to utilize the transformer already installed for house lighting circuits, as shown in Fig. 8 of the paper. Again, any system of switching has the possibility of getting out of order, and also the possibility of being grounded. If you can keep these switches off the 2200-volt line and install them on the 110, a ground or short circuit has no more serious effect on the system than if it were in your own house.

C. E. Skinner: The fact that the transformer would usually be there for other purposes would answer that question in most cases. The street lighting would simply be a part of the whole general scheme of multiple distribution. Where Mr. Harrison says that the question of control is one of the governing features he hit the nail squarely on the head, were it not for the limited distances that multiple circuits can be operated from the source of supply, the transformer or whatever it may be, the question of control would not be difficult, but on account of the fact that if you are going to use street lighting on a multiple plan you can only economically supply a very limited number of lamps from one center, means that you have to have a lot of control points.

N. B. Hinson: A list of the different types of street lighting systems of the Southern California Edison Company, Southern Division, will give some idea of the kinds of circuits used and tendency to adopt series circuits as standard.

There are seven cities with d-c. series arcs, three with a-c. series arcs, and two with both a-c. and d-c. series arcs. Six cities have multiple incandescent over head systems, only one of these being a very large city. There the lamps are controlled by a special primary circuit which feeds the street lighting transformers. Seven cities have series incandescent overhead systems, most of which are fed from constant-current transformers in the substations. A few are controlled by outdoor type constant-current transformers mounted on poles and operated by time switches. Eight cities have multiple lamps in ornamental posts only, and six series lamps in ornamental posts only. Some of the above cities which have overhead circuits also have ornamental. Twenty-seven of these systems are multiple lights, and twenty-two series. All of the multiple being old installations, the series are newer systems.

All of the series ornamental systems except one, are cut up into smaller systems so as to limit the voltage on each individual circuit to a value of less than 600 volts; only one system being operated at high voltage. This has been done as it is not considered safe to carry the high voltage of the series circuit direct to the post. Most of the multiple ornamental systems are operated from a special primary system which feeds the street lighting transformers.

The reason for the almost universal use of series circuits is due to the lower first cost of the system, either overhead or underground for ornamental posts, the reduced maintenance due to the longer life of lamps, and the increased efficiency of the series lamp, particularly on the smaller size units, which size predominates.

A brief description of a city which is lighted by ornamental posts only and which were installed quite recently, may be of interest.

Due to the high cost of material and the special requirements of a particular installation, a new type of system has been developed, which we have christened "Conduit Return". It has proven very satisfactory for other installations than the one for which it was designed. A brief description of the original conduit return system may be of interest.

The city in question has three distinct sections, long blocks and very irregular streets. It was desired to operate the lamps on a double schedule, that is, all of the lamps were to burn until 12 o'clock and then about half were to be extinguished and the remainder to burn all night. This means two circuits, and it was agreed that one side of the street should be the midnight and the other the all night. This simplifies the wiring, but the sections were so long in some cases a complete loop would run the voltage higher than that at which the rubber covered wire could be operated, and in other sections no complete loops could be made because of the streets, and in all of the sections there was an odd number of streets; also all the pole lines are in alleys and in one section the whole territory is underground so that the possible feed points were limited.

With the conduit return system these features were not difficulties at all. Each side of the street was a circuit to itself, the current flowing out on the wire through all the lamps and returning on the conduit to the transformer. The conduit was bonded together in the base of each post and special care taken to see that the joints were made so as to offer as little resistance as possible. The system has practically no reactance, the voltage drop and energy loss in the circuit are less than with a copper return circuit. Extensions are very easily made, trouble easy to locate and the first cost is very much less. The conduit was ½ in. galvanized with No. 8 D. B. R. C. wire.

At the feed points S. L. transfomers are located with a protective device to limit the voltage in case of an open circuit. These are fed from the substation by four overhead high-voltage series circuits which are regulated by four constant current transformers in the substation, two of which control the midnight circuit, and two the all-night. In this particular case it was more satisfactory to run out high-voltage circuits with S. L. transformers than to use R. O. transformers and a control primary.

J. M. Humiston: I want to give a little history of a municipally owned plant that I am familiar with, more for the sake of throwing some light on the discussion and pointing out some absurdities rather than giving any information to this meeting. This municipal plant originally served an area of about two-thirds of a square mile and was a Nernst lamp system operating at 220 volts. As the Nernst lamp went out of use the lamps were replaced with carbon filament lamps burning at 220 volts. Later these lamps were replaced, first by a 100-watt tungsten 220-volt lamp, and now they are gradually being replaced by 150-watt tungsten lamps, none of these lamps being of the nitrogen filled type. No effort has been made by means of refractor globes to improve the illumination. I imagine that the 220-volt pressure lamp has been one reason why none of these lamps have been stolen, because I have heard no complaints of thefts. About 1907, some five years after this Nernst lamp was put in operation, another part of the town was supplied with about 50 arc lights on a series circuit. About five years ago these were changed to the series incandescent type. At the present time there is one circuit consisting of 32 tungsten 350-watt series lamps burning on a circuit with a regulator, the circuit being supplied by 4000 volts at the bus bar, and another similar circuit with 83 of these same lamps. The 83 lamps, of course, are burning at approximately half the candle-power that they would have if the circuits were rearranged in such a fashion as to put the proper number of lamps on a circuit. These two systems are supplemented in part of the town by a post lighting system, the posts being about 100 feet apart. Some trouble has been experienced through these post lamps being damaged by autos. Four or five posts have broken off in this way. The original multiple system, using the 150-watt lamps has lights spaced at an average of 200-ft. centers, and that part of the town is the best lighted of any district that I know of around Chicago, with the exception of the boulevards and places where special effort has been made.

About 3 years ago I suggested changing this multiple system to a series system, but I found that there was opposition from the force that was maintaining the system because they didn't want to take the trouble to trim the trees, and they had had almost no trouble from tree contacts against these 220-volt wires, and where the series system was in force they had had a great deal of trouble. In spite of the fact that the series system was in a district with comparatively few trees, and the multiple system was in a district with a great many trees. After surveying this I finally concluded to drop the matter of changing this to a series system, and now I am rather favorable toward maintaining the multiple system as it stands. The secondaries of this multiple system are entirely separated from any commercial distribution. There are five or six transformers located at proper intervals which serve the secondaries of this multiple system, the lamps being turned on, by the operator at the station charging the primaries that feed these transformers.

C. H. Shepherd: I would like to ask Mr. Harrison as to how the economies of multiple street lighting would be affected by the question of metering; whether the current would be metered or not, and if not, why not?

Ward Harrison: I think in general that it would be uneconomical to meter the current to each lamp, but rather to measure at reasonable intervals, if necessary, the wattage taken by each lamp, or by a predetermined group of lamps; although I don't see why this is any more necessary than to measure the current taken in the series circuits at the present time. In general, these lamps should be regarded as flat rate customers such as electric signs. There would be no objection, however, to treating the lamps as regular residence consumers with a wattmeter back on the pole, which could be read by the same man who reads the meters in the houses.

F. A. Vaughn: In connection with that point—just incidentally—it might be of interest to record that in Milwaukee during the transitory period from the old series system to the new series system an arrangement was made with the public utility to supply lamps on these short individual multiple circuits at a given rate per lamp, based upon the wattage rating of the lamp, and those lamps were temporarily burned for a number of months, many of them on that rate schedule. This was a flat rate schedule based on the rating of the multiple lamp. I wish that Mr. Harrison had mentioned in connection with the manual turning on and off of the lamps, that it is not so much the cheapness of the labor as the undependability of the labor that is a factor acting against adopting that as a good system. It seems to me that under the present-day circumstances undependability of the labor which you can get for that manual extinguishing and turning-on of the lamps is absolutely against its feasibility.

H. Goodwin, Jr.: Mr. Harrison has given us a summary of many methods of turning off and on the lamps, and he has mentioned the selenium cell. It might be interesting to add to this collection of methods a method which is in practical use for turning off and on gas buoys at sea and on lonely rocks or headlands. This daylight switch has been in satisfactory operation for many years. It is a Swedish invention but is in use in quite a number of lights in this country, particularly in Alaska and remote sections of the coast. This device consists of a steel cylinder covered with lampblack. There is a frame on top of this cylinder to which are attached steel posts of just the same material as the steel cylinder. There are about six or eight of those posts. They are covered with gold leaf. They reflect the light and the cylinder absorbs the light. Therefore, during daylight one expands and the others remain at the temperature of the air. There is a yoke around the bottom. Now if you pivot a lever you can get a considerable motion on the end of the lever, due to the light falling on the cylinder, and that is actually made to open and close a gas valve, which turns the gas buoys off during the day. This apparatus can be made so sensitive, and the adjustment so fine, that it could be set in this room with the lights turned on and yet if you hold a match within one foot of it, that would give enough additional light to turn the lights off. A satisfactory daylight switch has been developed. It is slightly expensive at the present time, but if sufficient demand is created, they could undoubtedly be produced in quantities that would allow them to be sold at a reasonable price.

Mr. Harrison also mentioned the pulling in of cable from rear lots and along parkways, etc. I don't know if you are all familiar with the method that has been used in Philadelphia to a considerable extent. The apparatus consists of a cable claw with two channel irons set parallel, there being a roller on each end, a straight plow share and a bulb on the lower end. You dig a hole in the ground, set this down into it, shackle the cable to the rear end and by means of an electric truck or a gasoline winch set up sixty or one hundred, or one thousand feet ahead, just pull that along 18 inches below the surface. The rear roller coming along closes the slot so you can hardly find it after the plow has passed. In that way they can pull in many thousand feet of cable in parkways, etc. in a single day.

F. A. Vaughn: I wonder if I may add, that I think the difficulty of laying the parkway cable under the driveways would be largely minimized if the driving-method was used. The ratio that Mr. Harrison gave as to the cost of crossing the street and crossing driveways, as compared with a block of cable, I think would be materially changed if a "jacking" system which we use in Milwaukee were utilized. I believe this is used here in Chicago—some modification of it, at least—for driving a pipe under the pavement or under the driveway. That is particularly easy on all the narrower driveways and is perfectly feasible even on asphalt covered streets.

Apropos of the control of the circuits I should like to record that in Milwaukee on the multiple side of the system, that is, on the primary transformers that feed the series circuits, we use time-switches and are having very good success with them. In order to show the magnitude of their present use, I might say that the Milwaukee system is about 40 per cent complete and ultimately will have between nine and ten thousand units, so we have had a pretty fair chance to try out the control by the time-switch method.

F. H. Bernhard: I wonder if Mr. Harrison could throw any light on the subject as to why this system is used so extensively in Europe and also some of the results we have compared with our system here.

Ward Harrsion: I am not familiar with all the causes underlying the greater use of the multiple system in Europe, but one reason is the fact that lamps are spaced much closer in a great many of the foreign cities than we have been in the habit of spacing them in this country. The line drop in the multiple system of distribution in these cities would consequently be less objectionable than here where there is a wider spacing of units; also, the fact that labor is cheaper there, or at least that it has been, affords an inexpensive method of turning the lights on and off.

Mr. Peaslee: The use in Europe is very common of what they call a "Pensioner", or a man that draws a pension from the government. I know this is the case in the lines in Paris and any time that any of the companies over there are incorporated they have some sort of a clause that gives them a government privilege of using men who are drawing war pensions, and men who are paid a pension in a great many of those cases are used, for a very slight sum, to go around and handle the switches, both in the gas lighting and electric lighting. It costs the company probably three or four cents a day extra to get a man to handle three or four blocks of that switching. And the reliability of that labor is really remarkable to anyone who has had experience with American labor, and that has been a very important feature of the continuance of it, and also of the continuance of the multiple systems in that part of the country.

Mr. Cameron: Isn't it a fundamental fact also that in most European cities, particularly of the smaller class, the streets are so narrow that the street in most cases provides also a sidewalk. The wires are usually fastened to the buildings, and must necessarily, from the standpoint of safety, be of low voltage.

George Nixon: As stated in Mr. Harrison's paper there yet remains much to be done in perfecting the multiple system of street lighting. In time, however, there may be developed specially for this purpose reliable remote control switches and automatic time switches and manufactured in quantities sufficiently large to be sold at a low enough price to allow their practical use to become quite common.

Among the practical applications of the multiple lights for street lighting not particularly mentioned by Mr. Harrison, is the lighting of small villages and towns from service off of a transmission line.

Often the amount of business to be obtained in a small village does not warrant the installation of a transformer unless all the business can be served from one transformer. On the same pole with the transformer is installed the time switch which automatically turns the street lights on each evening and off each morning. One side of the street lights would be connected to a special secondary wire and the other side to one of the secondary wires serving the residences.

The time switch is wound up once a week by the patrolman in that section of the line.

Since the swift automobile and auto trucks have come into such extensive use it has become more necessary than ever that these small villages be well lighted.

Presented at the Chicago Meeting of the American Institute of Electrical Engineers, January 9, 1920.

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CONSTANT POTENTIAL SERIES LIGHTING

BY CHAS. P. STEINMETZ

Chief Consulting Engineer, General Electric Co.; Prof. Electrophysics, Union College, Schenectady, N. Y.

UNTIL recent years, street lighting in the United States has been almost exclusively by the high-voltage constant-current series system, either direct current or alternating current, and low-voltage constant-potential multiple systems have been used to a very limited extent only. This was for two reasons, one social-political, the other electrical.

To the democratic character of the country, only such form of lighting appeared suitable, as was applicable alike to the scattered suburbs and to the more densely populated center of the city. Thus high voltage had to be used, either a series system, or alternating current with individual transformers. The latter was excluded at first by the absence of a satisfactory alternating-current arc lamp and by the high cost and low efficiency of the small individual transformers. Thus American street lighting developed on the high-voltage series system, and much later only, with the gradual growth of civic pride of the city population, special lighting features were developed such as "White Way Lighting." The alternating-current arc lamp largely replaced the direct-current arc lamp; the small transformers became more efficient and less expensive; but still more efficient and economical was the constant-current transformers operating a whole circuit of alternating arc lamps, so that the conditions which led to the use of the high-voltage series system did not materially change.

The other reason was the electrical characteristic of the arc, which was the only illuminant of sufficient intensity and efficiency for street lighting. The arc is unsteady on constant potential, and requires a constant current or at least a circuit in which the voltage drops greatly with the increase of current. Thus, to operate an arc lamp on constant-voltage supply, a large series resistance or reactance is necessary, which makes it inefficient and, with alternating current, greatly lowers the power factor, and even then, the light is not as steady as when operating on a constant-current circuit.

As seen, of these two conditions, the first one requires high voltage, whether constant potential or constant current; the second one required constant current, which economically means series connection and high voltage. The first one still applies with practically the same force as before, but the second one has entirely changed, and thus makes a reconsideration and revision of our views on street lighting desirable.

With the exception of the luminous arc, which by its high efficiency and superior quality of light—perfect whiteness—holds its own, especially in high intensity and decorative lighting, such as "white way lighting," exposition lighting, etc., the arc lamp has practically disappeared from street lighting, and the gas-filled mazda lamp has taken its place. With the disappearance of the arc lamp however, disappeared the necessity of the constant-current circuit. As an incandescent lamp, the gas-filled mazda lamp is a dead resistance, equally stable on constant potential as on constant current, and while the necessity of using high voltage still remains—especially in American cities, which as a rule cover a far larger territory with their street-lighting districts, than European cities of the same population—the need of converting from constant potential to constant current has ceased, and the high-voltage constant-potential street-lighting systems with gas-filled mazda lamps, thus have become possible and are increasing in frequency, either as high-voltage constant-potential series systems, or as high-voltage alternating-current multiple systems with individual transformers.

Unlike the carbon-filament incandescent lamp of old, which has a negative temperature coefficient of resistance, the metal-filament mazda lamp has a positive temperature coefficient, that is with a change of circuit conditions, in the mazda lamp the voltage varies more than the current, while in the carbon-filament lamp the current varied more than the voltage. Therefore, in a constant-potential circuit, the mazda lamp,—other things being equal—will have a better life than in a constant-current circuit having the same percentage fluctuation of current, as the constant-potential circuit has fluctuations of voltage, while the reverse was the case with the old carbon-filament lamp. Constant potential thus is somewhat preferable for the mazda lamp, that is to give the same life of the mazda lamp, the regulation of the constant-current system must be closer than that of the constant-potential system.

The regulation of the available constant-current systems, such as given by the moving-coil constant-current transformer, reactor or similar devices, is amply close for the successful operation of mazda lamps. It is true that the incandescent lamp is much more sensitive to current variations than the arc lamp was, and while arc lamps are not affected by momentary current variations of 50 to 100 per cent, such as may occur in case of a surging of the circuit due to instability of the arc lamps, swinging grounds, etc., such current variations may be disastrous to the incandescent lamp.

However, in the constant-current mazda lamp circuit if reasonable care is taken of the insulation, the danger of surging hardly exists, and it is only where mazda lamps are operated in series with arc lamps, and the circuit is overloaded and the arc lamps allowed to get out of proper adjustment, that an impairment of the life of the mazda lamp, due to current surges, may occur.

As regards the regulation of constant-potential street-lighting systems, whether high-voltage series, or multiple systems with individual transformers, the problem of regulation of the voltage supply is eliminated by the fact that domestic lighting constitutes a large part of the load of the electric supply system, and the voltage regulation satisfactory for domestic lighting is equally satisfactory for constant-potential—series or multiple—street lighting. That is, constant-potential street lighting is operated from the same supply circuits and in the same manner as domestic lighting is.

Necessary in all series systems is a shunt protective device, that is, a device keeping the circuit closed in case one of the lamps open-circuits. Otherwise either all the lamps would go out or, if the circuit voltage is high enough, arcing occurs and causes more or less damage. The simplest form of shunt protective device is the film cut-out, used very largely in constant-current incandescent lamp circuits. Usually however, one of two other functions are combined with the shunt protective device; to automatically restore operation and to maintain the regulation of the circuit. In incandescent lamp circuits, the former is not necessary, as an open circuiting of the lamp means a burn-out, the lamp then has to be replaced, and when doing so, the film cut-out is renewed. In arc lamps however, the arc not infrequently breaks momentarily, and if then the film cut-out short-circuited the lamp, it would be put out of service. Therefore, in arc circuits, as a part of the arc lamp, a shunt magnet is used closing the circuit through a starting resistance, the latter giving sufficient voltage drop to allow the arc to start again when the electrodes come together.

In constant-potential series circuits, the operation of the film cut-out would reduce the circuit resistance, thereby increase the circuit current and impair the regulation and the life of the lamps. Thus the shunt protective device when operating, must insert a resistance or reactance into the circuit, such as maintains the total circuit impedance unchanged. For this purpose, the same device may be used as in arc lamps, that is, shunt magnet and starting resistance, making the latter equal to the lamp resistance. Such is used in the series operation of a number of arc lamps in direct-current circuits. Or a reactance may be used instead of the resistance, as in the series operation of two flame carbon arc lamps on constant-potential 110-volt circuits.

The objection to this device is the use of moving parts and contacts, and as series incandescent circuits are almost invariably alternating, as shunt protective device maintaining the circuit regulation, a shunted reactance is used, which remains continuously in circuit, and when the lamp is in normal operation, by-paths only a negligible current, operating below saturation; but when the lamp burns out, the shunt reactance magnetically saturates and thereby drops in reactance to the value required to maintain the regulation. A constant shunt reactance cannot give regulation, since with the lamp burned out, but all other parts of the circuit the same, the total circuit impedance necessarily is higher, thus the circuit current lower, and the reactance in this case thus must decrease, to maintain the same circuit impedance. However, with the lamp burned out, the voltage drop across the shunt reactance is much higher than with the lamp in operation, since in the former case, the reactance voltage combines in approximate quadrature with the remaining circuit voltage.

The requirements of such a regulating shunt reactance thus are: (1) with the lamp in operation, to consume as little current and power as possible. (2) When the lamp opens, to drop by saturation to such a value of reactance as to give the same total circuit impedance as before, and thus maintain the circuit regulation. (3) To maintain the circuit regulation over as wide a range as possible, up to at least 10 to 20 per cent of burned-out lamps, so as to give ample time for renewal. These requirements can be fulfilled by proper design in a very perfect manner.[1]

Such shunt reactance may be, and usually is combined with other features. It may be used as an autotransformer, so as to operate lamps of different current consumption in the same circuit or to operate a larger number of lamps in one circuit, without excessive voltage, by having the circuit current larger than the lamp current. Or it may be used as a transformer, with sufficient insulation between primary and secondary so as to make the secondary, that is, the lamp, a safe low-voltage circuit, especially when locating the transformer away from the lamp, and grounding it. However, in this case care must be taken in the design of the transformer to guard against dangerous peak voltage. Assuming a 6.6-ampere 300-watt lamp. The normal lamp voltage, and thus transformer secondary voltage, then is 45.5 volts. At open circuit, that is, with the lamp burned out, the secondary voltage then would be of the magnitude of 100 volts effective. However, the voltage wave may be very greatly distorted by saturation, having high voltage peaks—at the moment of magnetic reversal—reaching into dangerous values.

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[1] Regarding the mathematics hereof, see "Theory and Calculations of Electric Circuits." Chapter XV.

The foremost disadvantage of the constant-potential series system is its lesser flexibility. From a constant-current transformer any number of lamps can be operated, with equally good regulation, from full load down to a fraction of it—though it is economically undesirable to operate a constant-current transformer much below full load, as with decreasing load, the power factor rapidly decreases. In a constant-potential series system, however, the number of lamps is given by the circuit voltage. Thus from a 2300-volt constant-potential supply could be operated $\frac{2300}{45}$ = 51 lamps of 300-watts 45-volts, neither more nor less. This limitation is overcome by the use of transformer taps and idle regulating reactors. Thus considering a 100-lamp circuit, operated from an ordinary constant-potential transformer with 4500-volt secondary; giving the transformer secondary one 4 per cent and two 8 per cent taps, and using up to 3 idle reactors in series to the circuit, would give a 20 per cent range of load, which probably would be sufficient for most purposes.

A further disadvantage of the constant-potential system is its sensitivity to grounds. If two dead grounds occur in the same circuit, and thereby short-circuit a part of the circuit, the current in the remaining part of the circuit is increased, and if the short-circuited part of the circuit is considerable, all the lamps in the remaining part of the circuit may be burned out. How serious this is, depends on the quality of the circuit. However, with any reasonably well built and operated circuit, the probability of two dead grounds in the same circuit should be so remote as hardly to require much consideration.

On the other hand the great advantage of the constant-potential series system is its high power factor, of 95 to 99 per cent, the better efficiency, and especially, that it requires no station apparatus such as constant-current transformers, but can, and would be operated from a transformer on a pole in the distribution system, just like domestic lighting, and the mass of lines or cables running back to the stations, which are characteristic of the usual constant-current systems, due to the limited power per circuit, thus is eliminated.

In such constant-potential series lighting system, all the street-lighting circuits may be operated from one single feeder or a few feeders, which are connected or disconnected from the station at the proper time. While an advantage over the use of a separate feeder from the station for each street-lighting circuit, this is not as simple and convenient as connecting the constant-potential transformers on the poles in the street-lighting districts, which operate the street lighting, to the domestic-lighting feeders in their respective territory. This latter however requires some means of connecting and disconnecting the street-lighting transformers at the proper time of starting or stopping the street lights, since the domestic-lighting feeders are continuously alive.

Such may be done by manual operation of the switches, though such is rarely satisfactory.

Or it may be done by cascade operation, that is, having the street-lighting circuits overlap, and each start the next one, the first being started from the station. This may introduce some complications and limitations of the location of the transformers, and has the disadvantage that, if one switch fails, all the circuits beyond it are out of service.

	I Constant-current series system	II Constant-potential series system	III Constant-potential multiple system
(1) Type of lamp.	Arc lamp or incandescent lamp.	Incandescent lamps only.	Incandescent lamps only.
(2) Station apparatus.	Constant-current transformer or reactor, etc.	No station apparatus.	No station apparatus.
(3) Number of station circuits.	A separate circuit for every lighting circuit.	No special circuits from station.	No special circuits from station.
(4) Number of wires per circuit.	Single-wire circuit.	Single-wire circuit.	Two-wire circuit.
(5) Accessories at lamp.	Film cut-out, or auto-transformer or series transformer.	Auto - transformer or series transformer.	Constant-potential transformer.
(6) Size of circuits.	Limited number of lamps per circuit.	Limited number of lamps per circuit.	Unlimited number of lamps per circuit.
(7) Flexibility.	Great flexibility in number of lamps.	Lesser flexibility.	Great flexibility in number of lamps.
(8) Power factor.	80 to 85 per cent with incandescent lamps. 70 to 75 per cent with arc lamps.	95 to 99 per cent.	Practically unity.
(9) Safety.	High voltage at lamp except when using transformer.	High voltage at lamp except when using transformer.	Low voltage at lamp, due to transformer

Or time switches may be used, that is, switches operated by a clock located on the pole at the transformer. Such clocks however, exposed to weather, are not always as reliable as desirable.

Or a pilot circuit may be used, that is, a low-voltage direct-current circuit carried to all the switches. This means an additional circuit.

Another method, which in many cases is very promising, is the use of the phantom switch, that is, a switch operated with direct current by a polarized relay, so that one direction of the direct current closes it, the opposite direction opens it, with the direct current sent to the switch over a phantom circuit, such as are extensively used in telephony. That is, a direct current is sent through a balanced reactance in the station, into the neutral of the alternating-current distribution system, and at the switch passes from the alternating neutral through another balanced reactance to the relay which operates the switch. Such arrangement permits electrical long-distance operation without any additional circuit. It is not applicable in circuits with grounded neutral. However, in primary distribution circuits a grounded neutral is hardly ever used.

Comparing then the three types of high-voltage street-lighting systems; the constant-current series system, the constant-potential series system, and the constant-potential multiple system: (See page 64)

NOTES

(2) For I, constant-current pole transformers have been designed, without moving parts, and are in successful use to some extent, especially for smaller circuits; but usually are either lower in power factor or inferior in regulation.

Theoretically, II may be operated directly from the primary distribution feeders, but usually it will be preferable to interpose constant-potential transformers on a pole in the lighting district, so as to reduce the trouble from grounds.

(3) In II or III, provision must however be made for starting and stopping the circuit, as discussed in the preceding.

(5) The film cut-out, as the simplest shunt protective device, is applicable only in I.

To secure low voltages at the lamp, so as to make it safe even when the circuit is alive, requires transformers in I as well as II.

However, these transformers are series transformers, that is, with a moderate number of primary turns, while in III, the transformer has to step down from the total circuit voltage to the lamp voltage, requiring many turns of fine wire in the primary, and thus is more expensive to build and insulate.

In III, several lamps may be operated from the same transformer, if near together, but this then means a two-wire secondary circuit in addition to the primary circuit. The same could also be done in I or II.

(6) Both series circuits are limited in the number of lamps per circuit, but this limitation, while serious in a constant-current circuit which has to go back to the station, is of no moment in the constant-potential circuit, as the latter does not need to run back to the station but is fed from the transformer on the pole in the lighting district, and such transformer is of sufficient size to be efficient and economical.

In addition to paper by Dr. Steinmetz a description of Chicago's lighting system was presented by Mr. Henry Nixon, as follows:

I have been asked to give a description of Chicago's new group lighting system. In order that you may know the general type of distribution system to which these group circuits have been added, I think it might be well to cover briefly our transmission and station equipment.

The energy used is three-phase, sixty-cycle, transmitted from the main substation to the thirteen municipal street lighting substations by three-conductor, 12,000-volt cables installed in municipal duct lines. The apparatus in all of these substations is similar, differing only in manufacture and in location in the station, except for three of our stations in which synchronous motors are operated for the benefit of the transmission system. The substation transformation is from 12,000 to 5050 volts, the secondary bus being of the three-wire, grounded neutral type. The path of the circuit is from the phase side of this bus, through an oil switch, ammeter and reactance type of regulator, out to the line, the return coming into the station and terminating on the grounded neutral bus.

The circuits are carried out of the station in sets of four, in 8-conductor, 5000-volt, lead-covered cables to a point where the first distribution to circuits is desired. At that point it branches usually into a 4-conductor and four single-conductor cables, the 4-conductor cable being carried on to a second point of distribution where it branches into four single conductors. From these branch points, the circuit construction varies according to the types of lamps and the local conditions to be met.

Chicago's street lighting system consists of some 50,000 incandescent lamps operated on four general classes of circuit construction. About one-half of the system is of the 600 candle power type of lamp mounted at a height of twenty-two feet. About eighty per cent of these 600 candle power lamps are operated on aerial circuits, using the No. 6 hard-drawn, weatherproof copper wire, and are fed out of the multi-conductor cables by means of single-conductor cables and potheads in the usual way. The remaining twenty per cent of these 600 candle power lamps are operated on underground circuits of the usual type, using 5000-volt, single-conductor, lead-covered cable in the street in ducts, and in some cases utilizing the area ways under the sidewalks.

In about 1913 the street lighting in the better residential districts was changed from gas to electric by the installation of 100 candle power, 4-ampere lamps on the existing gas posts, utilizing an adapter to fit the lamp socket to the post. Five thousand volt, steel-armored cable was installed in the parkway about eighteen inches below the surface. The circuits were quite long, averaging about 250 lamps to the circuit, and the result was that a large number of lamps were put out of service when the circuit was opened, due to the breaking of a post, the cutting of the cable, or other accidents. These circuits were also fed from the multi-conductor cables as explained before. This method of lighting residential streets proved very satisfactory in every regard except that it was open to the serious objection of being dangerous to both the public and the department employes. The posts were only 8 ft., 6 in. high, with a 14-in. ball globe surrounding the lamp and socket. When one of these globes was broken or removed, the binding post of the socket was exposed, and the potential of that binding post being very high, in some instances 5000 volts to ground, it was very hazardous for our men to work upon it, and also dangerous to the public. There are not many instances on record where citizens have been hurt in that way, but of course it only takes a few such accidents to cause you to look about for a safer system. Especially is this true, because when posts are knocked down, quite often the circuit would not be broken. We had one instance of this kind where a policeman attempting to remove such a pole was killed. Boys trying to steal the lamps from the sockets have also been injured, and some deaths have been caused in this manner.

In looking around for a method which would prove safer than the straight series system, two methods of circuit construction presented themselves, the individual transformer method and the group lighting system.

The group lighting system consists of several small series circuits, each operated from a series transformer, the primaries of which are energized by a 5000-volt main supply circuit, which in turn, in our system, is fed from the multi-conductor cable. This system is somewhat cheaper to install than the individual transformer system, it gives the same safety to the public, and is slightly more efficient in operation than the transformer method, and hence it was decided upon for installation in Chicago.

A 6.6-ampere, 72-watt, 100 candle power incandescent lamp is used, mounted upon a cast iron post 10 ft., 6 in. to the center of the light source. This gives an even better illumination than the original 100 candle power installation explained before. These lamps are spaced 150 ft. apart, staggered on opposite sides of the street, giving one lamp for each 75 lineal feet of street. From twenty to thirty of these units are operated in each group, two sizes of transformers being used according to the number of lamps in the group.

The group circuit construction consists of 600-volt, single-conductor, lead-covered jute-covered cable buried in the parkway about 18 in. below the surface. In answer to a question asked at the meeting this afternoon, I might say that we have had a larger number of cases of grounds and opens on this cable than we have experienced on the steel-taped cable installed in a similar manner. We attribute this to the fact that there has been a great impetus to building recently, and that each place where a new building is put up, in digging for the installation of water, gas, etc., the cable is quite apt to become severed. This 600-volt cable is very small and being covered with jute, it gives every appearance of being the small root of a tree, and without thinking the men cut it in two. Also in the installation of parkway fences, our cables quite often are damaged by having stakes driven through them, and it is the opinion of the repairmen that in many cases the steel taped cable would not have been cut where the jute covered cable was.

Only a few lamps being operated in each group, makes it possible to install three, four and even five of these series-series transformers in one manhole. The transformer used is a 6.6 to 6.6 ampere series transformer, having a secondary operating voltage of 300 to 450 volts. From twelve to sixteen of these transformers have their primaries connected in series by means of 500-volt, steel armored cable. The terminals of the transformer are connected to the lead sheath of the cable by means of a wiped joint, similar to the method used in the system described by Mr. Vaughn. This confronted us with the difficulty of having no test point in the primary circuit in the field, and, therefore, we considered it advantageous to install at the termination of the multi-conductor cable a disconnecting pothead by means of which the circuit can be opened and tested in the field.

Individual lamp failures are taken care of by the standard film cut-out socket, and this film puncturing at about 450 volts is practically instantaneous in its operation. The open-circuit voltage of the secondary of the transformer is very high, and would bring us back to the dangerous conditions of the straight series system, if it were allowed to operate at that voltage for any length of time. It is, therefore, necessary to provide some means for short-circuiting the transformer secondary when an open circuit occurs, due to a broken post or a cut cable. This is taken care of in our system by means of a simple film held between two clips, which clips are connected direct across the secondary terminals of the transformer. When the secondary voltage rises beyond the normal value, due to an open in the line, this film will puncture and short-circuit the secondary.

We find that a considerable number of our lamp filaments failed after the circuit is shut down in the morning, due apparently to the cooling of the filament. It, therefore, was necessary to provide that the puncturing voltage of the short-circuiting film be considerably higher than that of the film in the lamp socket. Otherwise if two or more of the lamp filaments had failed after the circuit was shut down in the morning, the film in the transformer would puncture before those in the lamp sockets, and the entire group would be out of service that night. In our system we have installed this secondary short-circuiting device as an integral part of the transformer in order to save the cost of a separate piece of apparatus and the extra cost of labor for installing the same. Our experience has indicated that it would be very desirable if we could develop an automatic circuit restorer which would short-circuit the secondary whenever an open occurred in the group circuit, but which would remove that short circuit as soon as conditions in the exterior circuit had been restored to normal. An instance of the desirability of such a device is when a patrolman in replacing a burned out lamp is careless in his methods; or if the spring on the socket short-circuiting switch has become weak, the film in the transformer punctures due to the momentary open circuit. When this occurs, it is necessary to go to the manhole, open it up, take the top off the transformer, and change the film, and in our system it is even necessary that another class of labor be called upon to do this work, with the result that that group is out of service for a considerable length of time. Up to the present time we have not been able to design anything which would operate satisfactorily, under the severe conditions existing in our manholes, at a cost which would not be prohibitive. So far the cheapest design which has been made up would cost approximately $14.00 to manufacture, and since this is about one-third of the cost of the transformer, we do not feel that its installation would be justified.

As a further measure of safety, the middle point of the transformer secondary is grounded. This brought up a rather interesting point—that the lamps adjacent to the transformer might be seriously damaged if an accidental ground should occur near the transformer. In order to remove this difficulty, the transformer was then designed in two parts having separate primaries, secondaries and cores, in order that the flux linking each primary and secondary might have no effect on that linking with the other primary and secondary. This practically makes two transformers in a single case. This transformer in operation with the middle point grounded is still open to the objection that an accidental ground in the exterior circuit may cause one-half of the transformer to be loaded more heavily than the other half, in just so much as the accidental ground is off the center of the circuit. This, however, has never worked any hardship on the transformer, so far as we know, inasmuch as it is immediately apparent in the reduced candle power of some of the lamps, and the ordinary patrolman will immediately notice the difference and report the trouble to the repairman who removes the ground. These grounds are practically all due to some exterior cause, as I have stated before, and are not due to any trouble in the system itself. This is indicated by reports we have of such cases, where 76 out of 77 such grounds were from exterior causes. These reports cover the operation of 275 groups for one year. Reports on transformer trouble show that 56 out of 59 such cases have been due to leaks in the transformer case. The original transformers were oil-filled, and immediately any water gets into the case, the oil will of course seep out quickly resulting in a burn out. Many of these leaky cases have been due to a patrolman carelessly replacing the cover of the short-circuiting device. Because of these troubles we are at the present time filling each transformer which fails with compound in place of the oil, and on our new contracts we are specifying compound-filled transformers to further reduce this trouble, inasmuch as these transformers operate in manholes where quite frequently the water entirely covers them. This is due to the fact that in some of the outlying territories the sewers are so near to the surface of the ground that it is impossible to give proper manhole drainage.

Except for the change from oil-filled to compound-filled transformers, and the desirability of securing an automatic circuit restorer in place of our present short-circuiting film, the system as originally designed is working satisfactorily in every respect.

DISCUSSION ON "CONSTANT POTENTIAL SERIES DISTRIBUTION FOR STREET LIGHTING" (STEINMETZ), CHICAGO, ILL., JANUARY 9, 1920.

J. R. Cravath: In considering the new systems which we have before us from a practical field one of the first things that is apparent is that there has been a tendency for some time past as a result of the development of the gas filled tungsten lamp to more improved street lighting and more underground work in residence districts. That, in turn, has led to a serious consideration and demand for lower voltage on the actual lighting circuits, I mean the circuits which distribute the energy to the lamps. At the same time the advantages of high-voltage transmission are just as great as ever. In fact, they are more necessary than ever with our large generating plants located a long distance from the point of distribution.

Now the reasons for this tendency to a lower lamp voltage are several. One is that it means cables made for lower insulation and consequently less expensive. Another is that it is undesirable to have high voltages in wires leading up lamp posts. That is, it is much cheaper and safer to have low voltage. It doesn't require high-voltage socket arrangements, and to a large extent does away with the necessity of expensive pot heads in the base of the post. Then there is a question of public safety. I never have heard of very many accidents from series circuits in underground lamp posts, but at the same time one must recognize that frequently they are on concrete bases which may be fairly well insulated from the earth, and in that case there is some possibility of danger to public or to employees.

Another advantage of the low-voltage lamp circuit is that fewer lamps are affected in case of trouble on that circuit. All those reasons have led, as I say, to a tendency to trying to maintain high-voltage transmission and high-voltage distribution up to a certain point near the lamps and from then on have low-voltage at the lamps. There are several variations of the way that has been carried out. One is to have the transformers directly at the lamp posts, as has been spoken of before by Dr. Steinmetz and others, or reactance coils at the lamps.

Another plan, one that has been used here in Chicago recently, and one of the most interesting ones we have to consider tonight, is a series transmission through high-voltage series circuits feeding a small series transformer of relatively low voltage, these series transformers being located in vaults and feeding a group of underground lamps at a comparatively low voltage.

F. A. Vaughn: The Milwaukee street-lighting system was designed about 1914 or 1915, and, with the war coming on at that time, there were certain stoppages in the construction, which set the work back somewhat in respect to the time of completion. It is now about 40 per cent or perhaps 50 per cent completed, and will contain about nine to ten thousand units when complete. The system is operated from constant-potential circuits, that is, from the ordinary 2300-volt alternating-current distribution system, as used by the public utility company for distributing to the houses and factories on the ordinary constant-potential basis. From such feeders are tapped transformers of approximately one-to-one ratio, with taps on the primary and on the secondary, very much as referred to by Dr. Steinmetz, to give a manual adjustment of voltages to the correct constant-potential for the given series-circuit with its particular number of lamps, and their types and sizes. Every complete circuit is grounded in the center, and the lamps are actually fed by series-loops with individual lamp-transformers, or, in certain places, a transformer, as Mr. Cravath has just said, may feed a group of lamps, or, in the case of the double-bracket units, may feed two lamps wired in the three-wire fashion on the secondary of the series-multiple transformers, that is, a three-wire cable runs up the pole from the transformers, each of the transformers is buried in the ground at the base of the pole. The apparatus at the substations consists of means of turning the circuit off and on, in addition to the transformers with the approximately one-to-one ratio of transformation, and induction-type inverse-current relays for the protection of the series circuits against grounds or short circuits, watt-hour meters, and perhaps a potential transformer or a current transformer or a jack from which readings can be taken of the potential and current of the system.

The various types of units on any given series-loop may be grouped promiscuously, incandescent, gas-filled tungsten-filament lamps of any size from 100 candle-power to 1000 candle-power, or multiple units of two or more of these lamps on one post.

Transformer stations are sometimes placed overhead (Fig. 1). What might be termed a more or less temporary type of substation, or a substation which could be used in the suburbs where municipal buildings were not available, or inside types not feasible. Several circuits are grouped into these substations. The substations are placed at the junctions—or, in the case of a group of four, five, six or seven circuits, they are placed at the point where their corners intersect. Thereby, a very few time-switches, which are used for the turning on and off of the lights in this particular case, are utilized—one time switch turning on a group or a multiple number of circuits, if desired.

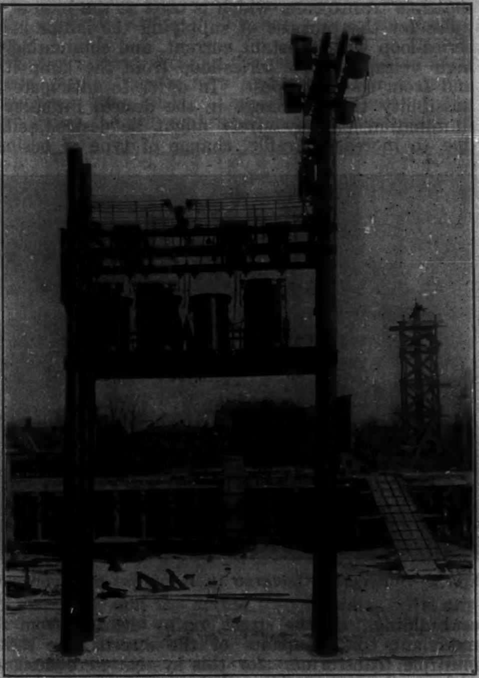

FIG. 1—OUT-DOOR OVERHEAD SUBSTATION ON POLES

In contradistinction to the above particular type of substation, an inside substation is also used with the usual switches and relays, watt-hour meters, etc.

(Fig. 2). This type is located in municipal buildings like the city-hall, the fire-station, school-houses or any other available place where floor space can be secured, and an adequate fire-proof compartment is built for the purpose.

I presume one of the most interesting detail parts of a system of this kind is the type of so-called series-multiple transformer which is used at the base of the poles for the purpose of supplying the lamps in the series-loop with constant current, and eliminating the high voltage of the series-loop from the lamp itself and from the lamp-post. In order to anticipate the possibility that a change in the desired intensity of ilumination on any street might be desired, either due to increased traffic, change of type of business, or buildings on the street, or by demand from the merchants or occupants of the streets, the series-multiple transformers for this system were designed with a view to making their over-load and under-load, as well as the full-load characteristics conform to the possible requirement. Therefore Size I transformer was so designed that its 100 per cent load efficiency point falls between the load of a 100 candle-power lamp and that of two 80 candle-power lamps in series. It, however, was also designed to operate well with the 250 candle-power lamp. So, also, the Size II transformer was built for both the 250 candle-

FIG. 2—SUBSTATION LOCATED IN BASEMENT OF AUDITORIUM

power, 6.6-ampere lamp and the 400 candle-power 6.6-ampere lamp; Size III, for the 400 candle-power, 6.6-ampere lamp and the 600 candle-power, 6.6-ampere lamp; Size X, for the 600 candle-power, 20-ampere lamp lamp and the 1000 candle-power, 20-ampere lamp; and Size XX for the 1000 candle-power, 20-ampere lamp; and an over-load range up to the equivalent of a 1700 candle-power, 20-ampere lamp.

Fig. 3 is a chart showing the per cent load on such series-multiple transformers and regulation of same for various lamp-loads. The left hand half of the chart shows the per cent load on the transformer of any given size and the actual load in watts with any given candle-power rate of lamp for which that transformer was designed. For instance, Size II transformer will operate at approximately 88.5 per cent of full load with a 250 candle-power lamp in the socket and at approximately 140 per cent load with a 400 candle-power lamp in the socket, and so on through the other sizes.

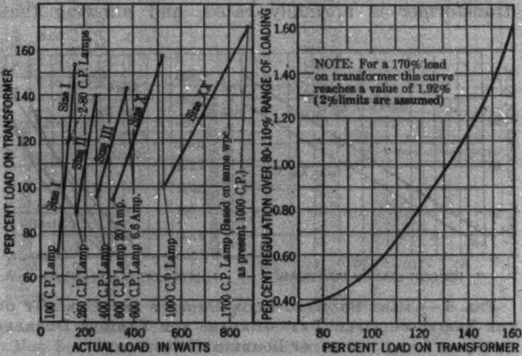

Fig. 3—Chart Showing Per Cent Load on Series Multiple Transformers and Regulation of Same for Various Lamp Loads

I might say that while an anticipation of the change in size of lamps was a matter of foresight in the days when the transformers were designed, the war brought on the fuel conservation problem and that brought on a change in the size of lamp very much more quickly than was anticipated, so that during the past two or three years these characteristics really came into play much more widely and much more quickly than was anticipated. Preparedness was again strongly justified.

The right-hand half of the chart is used as follows: Suppose we select the 400 candle-power lamp, to be used on Size II transformer. We find that this will be a 140 per cent load on the transformer. Now, passing to the right-hand half of the chart, and reading vertically above 140 per cent on the curve, we find that the per cent regulation over 80-110 per cent range of loading with 400 candle-power lamps in the socket will be 1.14 per cent. This means that even when using the size larger lamp than that for which the transformer is normally rated, and allowing these lamps to vary in their individual wattage consuming characteristics over a range of 80-110 per cent of normal, the per cent regulation of this transformer will be within 1.14 per cent.

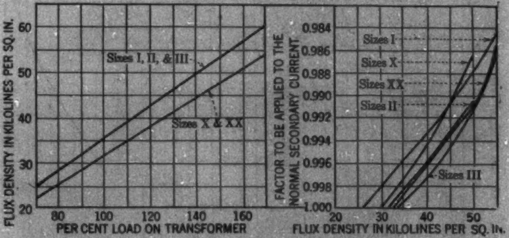

Fig. 4—Chart Showing Flux Density and Current of the Series-multiple Transformers for Various Degrees of Loading

Fig. 4 shows a chart giving the flux density and secondary current of the series-multiple lamp transformers for various degrees of loading. From this chart we may read that with 140 per cent load on the transformer, of either Size I, II or III, the flux density in kilolines per square inch is approximately 50. With this flux density of 50 kilolines per square inch, through the medium of the right-hand curve for Size II, we read a factor of 0.9904. This factor, when multiplied by the normal secondary current of 6.6 amperes, for Size II transformer, will give the actual secondary current of 6.53664 amperes. This is the current that will actually flow through the filament of the 400 candle-power lamp, when burned on the Size II, or 250 candle-power transformer.

After completing the electrical and mechanical design of the coils and deciding upon the actual requirements, as shown by the charts above, as well as the method of testing the manufactured product for compliance with these requirements, the results actually produced by the coils put out by the factory were recorded on "shot-gun" diagrams (which are really not, technically speaking, "shot-gun" diagrams), showing the factory test data for individual transformers.

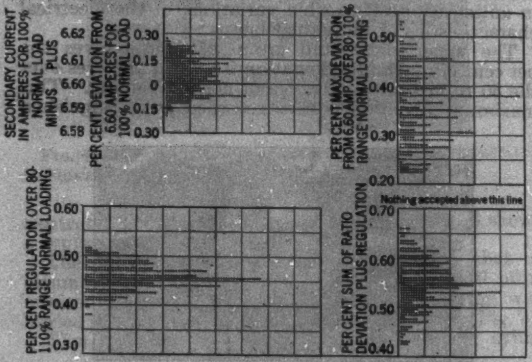

FIG. 5—"SHOT-GUN" DIAGRAM SHOWING FACTORY TEST DATA OF SERIES-MULTIPLE TRANSFORMER, SIZE II

In explanation of these "shot-gun" diagrams, let a Size II transformer be selected. Fig. 5 contains the data for this size. Each dot on the diagram represents a transformer. The compliance of the factory's output, as determined by actual test at the factory, is indicated by the relative positions of these dots in the four groups in this diagram.

Since, according to the table on Fig. 4 the normal secondary current for this size transformer with 250 candle-power lamp is 6.6 amperes, all transformers which are perfect as far as ratio is concerned, will be grouped along a horizontal line indicated by 6.6 amperes, all transformers having any other ratios will be grouped at positions relatively above or below this line, according to their per cent deviation from 6.6 amperes, with 100 per cent normal load.

For the group of transformers shown on Fig. 5 the success of the factory output is indicated. Observe that this scale is very open and the fluctuations, therefore, are really very small, as indicated by the figures representing the per cent deviation from 6.6 amperes.

The lower left-hand group represents the variation in the actual transformers produced by the factory, as expressed in per cent regulation over the 80 to 110 per cent range of normal loading. This is the range named in the specifications which was so specified in anticipation of possible variations in the electrical characteristics of individual lamps.

The upper right-hand group of points gives the per cent of maximum deviation from 6.6 amperes over the 80 to 110 per cent range of normal loading. This maximum deviation is thus recorded as distinguished from the actual per cent regulation shown in the lower left-hand group of points, which latter gives the total range of regulation.

FIG. 6—INDIVIDUAL LAMP-TRANSFORMER AT BOTTOM OF STREET LIGHT POST, STEEL CASE TYPE

In the lower right-hand group of points is plotted the per cent of the ratio deviation plus the regulation, which it was assumed would represent the worst condition that could be anticipated from such a transformer operating with the regular factory production of 250 candle-power lamps.

A maximum limit of this last variation was set at 0.70 per cent, and nothing was accepted above this line in any size of transformer.

These transformers, as indicated earlier in my remarks—or at least the first ones installed in Milwaukee—were steel-cased transformers, connected with steel-taped cable laid in the ground, and they were buried at the base of the transformer in concrete cups and then covered with asphaltum to make them

FIG. 7—BITUMINIZED FIBER TRANSFORMER CASE SPECIALLY DESIGNED FOR MILWAUKEE STREET-LIGHTING SYSTEM

electrolysis-proof, and then covered up. Fig. 6 shows a typical installation of that type of case.

Then another case was utilized, namely, that of bituminized fiber, composed of large and small bituminized fiber ducts, in which the whole thing was buried in the ground without further protection (Fig. 7).

Then wooden pump-log, or similar type of cylindrical wooden cases were also used and installed as shown in Fig. 8. This shows a double transformer used for a two-unit equipment, and it is really two transformers in one case. This is the type of double-bracket equipment.

FIG. 8—INDIVIDUAL LAMP TRANSFORMER AT BOTTOM OF STREET LIGHT POST. WOODEN PUMP-LOG CASE TYPE

Fig. 9 indicates perhaps the most interesting type of unit that we have in Milwaukee, namely, the

30-ft. suspension-height hollow-concrete pole with two 1000 candle-power lamps, and the three-wire system on the secondary of the foregoing series-multiple transformer buried at the base of the pole, fed by under-ground steel-taped cable.

FIG. 9—30-FT. CONCRETE ROUND POST, DOUBLE BRACKET STREET-LIGHT UNIT

On certain of the streets in Milwaukee there is utilized the 30-ft. double-bracket unit spaced about 180 ft. apart, and on both sides of a wide boulevard. A practically uniform illumination of approximately one-half foot-candle is obtained by such installation—four units per block. Fig. 10 indicates a less novel type of mounting, that is, the 15-ft. mounting-height short-post type of unit with a single equipment of

250 candle-power lamps with the harp-type fixture, all of course equipped with the special Milwaukee-bowl-type refractor for the purpose of light distribution. These units are opposite and spaced about six times the mounting height.

FIG. 10—15-FT. MOUNTING HEIGHT SHORT-POST TYPE OF UNIT, WITH A SINGLE EQUIPMENT OF 250-CANDLE-POWER LAMP WITH THE HARP TYPE FIXTURE

Speaking particularly of the lighting of the country roads, I thought it might be apropos to show the idea in Milwaukee County, where we have 150 or more miles of concrete roads, and Fig. 11 is the way some of it is lighted. It shows how by a staggered system, 180 ft. apart, 400 candle-power lamps, a very-close-to-uniform illumination is obtained on those concrete roads in a comparatively simple manner—22½-ft. mounting-height units; with refractors for distribution of the light. The uniformity there was, to the human eye, practically constant, that is, it was practically uniform distribution.

D. W. Roper: The Commonwealth Edison Company have very few arc lamps. Anything that we have is somewhat old-fashioned nowdays, due to the fact,—first, that we have only a year to year contract with the City for a comparatively small number of lamps, and secondly, to the fact that we had anticipated they would take over these circuits at the present time, and they have actually taken over quite a few of them within the last few months. I don't think that there is anything that the Commonwealth

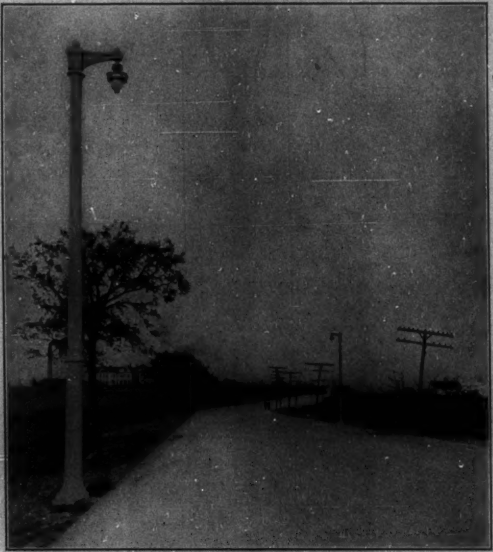

Fig. 11—Milwaukee Type of Installation Used on State and County Highways

Edison has that would qualify with some of these new combinations of series multiple circuits.

Ward Harrison: As I listened to Dr. Steinmetz it became evident that the system which he has described would also allow of the installation of an inexpensive underground construction between the lighting unit and an individual lamp transformer located on a pole at some convenient point. Under such circumstances, however, it probably would be inadvisable to install high-current lamps because the line drop 15 or 20 amperes would run up too fast with any ordinary size wire.

With the low voltage for the underground portion of the circuit, not only is the expense for cable reduced, but it is possible also simply to run out to the curb from the rear lot line at any point where it is desired to place a lamp. As the cable then runs practically parallel to all trenches which might be dug for gas, a sewer, a drive-way, or any other service, there is, therefore, much less likelihood of encountering and possibly cutting through it than where the line runs along the curb. However, good judgment must be exercised in locating such a circuit. If the cable is placed exactly on the property line between two lots it is almost certain to be disturbed if at some future time a fence is erected. On the other hand, if the cable is placed, say, one foot inside of the boundary, and put down approximately a foot below grade, it has a pretty safe resting place; in the suburbs a building is seldom placed to within a foot of the line, and again, very few amateur gardeners are really ambitious enough to drive a spade more than six or eight inches into the ground.

On the constant potential series system, while you can lay out the installations to give the correct number of lamps per transformer, it has been our experience that in nine out of ten of the cases where we have been asked to investigate unsatisfactory lamp performance on such circuits, we have found either the wattage of the lamps or the number of lamps on the circuit changed without making proper adjustment of the transformer taps. Of course, in a series circuit with a constant-current regulator this situation is automatically taken care of.

It seems that the choice between a constant-potential series circuit and the constant-potential multiple circuit will rest quite largely on how simple and inexpensive a method is devised for turning the lamps on and off in the latter system. If a device could be developed that operated without additional wires and cost only three or four dollars per lamp, it is very probable that individual control for a lamp, or two or three lamps, from constant potential circuits, would become the rule. On the other hand, with the phantom circuit control the expense of a switch for each lamp could not be justified and it would be preferable to operate a group of lamps, probably in series, from the transformer.

In this connection I know of one system where it is contemplated to make use of one existing series circuit along one of the main thoroughfares of the city, not only as a series circuit to supply the street lighting for that thoroughfare, but also to turn on individual constant-potential circuits which are adjacent to that thoroughfare, and which light the districts on either side. There will be a series of switches on this system, perhaps a third as many as there are lamps, and these switches will operate the entire street lighting of that vicinity.

F. H. Murphy: In this section of the country the series lighting system, either arc or incandescent, is practically universal, the multiple system being used only where peculiar local conditions dictated such a system and not because it was considered in any way a superior system. In a few instances our own company in some of the smaller towns has resorted to the multiple system where the distances were relatively short and the lamp sizes small, and where the matter of patrolling the circuits was of no very great importance. Special conditions, have, however, caused the installation of a multiple system for the boulevard post system in the business section of Portland. To illustrate this point, I will state that the municipality pays for the maintenance of an arc lamp at each street intersection and has consistently and persistently refused to assist the business district in any way in securing a more decorative and more intensive lighting system. The merchants, therefore, have installed their own white way lighting, which means that the system was built up in relatively small and somewhat disconnected sections. The lighting company supplied this entire district by means of a three-wire Edison system underground and multiple service was therefore readily available on every block. At the time these installations were made cluster posts with small lighting units were considered the best practise. This meant multiple incandescent lamps. The control of the system is block by block and a patrol is necessary to turn the lights on and off. However, this is not such a serious matter in the congested district. In all probability if the system was to be installed today a series system would be used because of the much greater advantage in the patrol of lights.

The real question with us is not whether it shall be the series system or the multiple system, but whether it shall be series incandescent or series arc, with the present feeling somewhat in favor of the incandescent lamp. Our street lighting system at the present time is principally a series magnetite arc system. However, in one of the principal cities which we serve the incandescent series lamp was given a thorough trial, which resulted in replacing all the arcs with incandescent lamps and changing the 4-ampere circuit equipment to 6.6 amperes. The saving effected in the operation and maintenance charges of the incandescent system over the former arc lighting system has returned a very satisfactory income on the investment cost of making the change and the public is better satisfied because they have a steadier burning unit and the outages are much less frequent. In order to give practically the same wide distribution of light as the arc gives, each incandescent unit is equipped with a Holophane band refractor.

It might be added that any city having an installation of a-c. series arc lamps would undoubtedly find it much to their advantage to install a system of series incandescent lamps rather than a series magnetite system because the present transformers could be used with an incandescent system, whereas if the magnetite system is to be used it would be necessary to make a considerable investment in rectifier sets.

We have also invaded the constant-potential series field to some extent. In one of the outlying sections of Portland we have several circuits of 4-ampere series incandescent lamps connected into the secondary of a 2300- to 440-volt constant-potential transformer. In the hood of each lamp is a reactance coil which shunts the lamp and maintains the circuit in case of failure of the lamp for any reason. These circuits have been giving excellent satisfaction. At present they are manually controlled by means of a regular patrolman but there is no serious reason why they could not be controlled by time switches if it was so desired.

On the Interstate Bridge and its approach which connects Vancouver, Washington with Portland, Oregon, we also have about three miles of series incandescent lamps which are operated from constant-potential transformers, connected directly to an 11,000-volt transmission line. The distance covered by the system completely eliminated the multiple lamps and determined that the series system should be selected. The fact that the County would own and maintain the lighting system practically eliminated the arc lamp because of the fact that the County would not have a regular arc man to maintain these lamps. In order not to have to duplicate transformer capacity by supplying constant-potential transformers on 11,000 to 2300 volts, and a constant-current from 2300 volts to the constant-current secondary, it was finally decided to have special constant-potential transformers built with 11,000-volt primaries and approximately 1500-volt secondaries so loaded as to deliver a current of 6.6 amperes and provided with secondary potential taps so as to allow a reasonable amount of regulation. Briefly the reasons for adopting in this case the constant series system were as follows: The line was entirely too long for satisfactory multiple distribution. There was no suitable location for a station type constant-current transformer equipment, the use of standard constant-current transformers would require a duplication of transformer capacity because of the constant-potential transformers necessary to reduce the standard primary voltage of the regular constant-current equipment and by the use of constant-potential series arrangement for the circuit a constant-potential transformer designed for 11,000-volt primary and the necessary secondary voltage to supply 6.6 amperes to the load could be more readily secured. In order that the current might be kept constant on the system under lamp failures at various posts, auto-transformers were used at each lamp outlet and these transformers were of such design that the primary current value through the transformer might increase to 150 per cent of normal value before the secondary current reached the burnout value of the lamp. This installation has been in operation now for over two years and given excellent satisfaction. The installation consists of 67—400 candle power units and 10—250 candle power units, and the breakage for the year 1919 consisted of 12 outside globes and the replacement of 24—400 candle power lamps. One day's time was charged against trouble work on the line due to a loose connection on one of the auto-transformers. This represents the entire expense in maintaining the system for one year. The man in charge of the installation reports that the breakage of globes and lamps was due primarily to malicious shooting. One-half of the installation burns from dusk to midnight and the other half from dusk to daylight, or approximately 4000 hours per annum.

The lights on the draw span and the single span connecting the draw span to the Vancouver, Washington, side of the river, are lighted by multiple lamps connected to the distribution system in Vancouver, Washington. These circuits are controlled by time switches and I wish to call attention to the fact that on the series system these switches have operated with very little trouble whatever, and have proven very satisfactory.

There are many cases, undoubtedly, in which a constant-potential series system has very decided advantages over any other, although the development of the pole type, constant-current transformer with moving coil in the comparatively wide range of sizes which are offered at the present time eliminates some of this advantage of the constant-potential series relation. This new type of transformer, together with a reliable time switch, which it is hoped may be developed in the near future, will undoubtedly prove the solution of a great deal of the outlying distribution for street lighting.

Presented at the Midwinter Convention of the
American Institute of Electrical Engineers,
New York, February 18, 1920.

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ESSENTIAL STATISTICS FOR GENERAL COMPARISON OF STEAM POWER PLANT PERFORMANCE

BY W. S. GORSUCH

Engineer, Economics, Interborough Rapid Transit Co., New York, N. Y.

This paper briefly outlines a method for preparing statistical reports relating to power generation in steam power plants, whereby fairly close comparisons can be made of the efficiencies between different plants, without going into a detailed study of the thermal characteristics of the plants, or the intricate subject of power costs.

The essential items of steam power plant performance that should be recorded and a uniform method of expressing them, are given in tabular form, followed by an illustration demonstrating the advantage of the proposed method.

THE principal difficulty with operating statistics of steam power plants as published today by various State and Governmental bodies is that they are wholly inadequate for the purpose of determining the efficiency of a plant or making general comparisons between different plants. In practically every case the bare figure of coal consumption per kilowatt-hour is given without any reference whatever to the quality of coal and without a clear understanding as to the character of the load on the station. When load factors are given, the figures are often misleading because they are computed upon different bases. No uniformity exists in reporting the output of the power plant, the gross output being used in some cases and the net output in others. Because of these facts the reports as at present published are limited in their use, principally serving to show the total amount of power generated and the total quantity of coal consumed.

The object of this paper is to point out the essential items of steam power plant performance which should be recorded in statistical reports, and to suggest a uniform method of expressing them for the purpose of making a general comparison between different plants, without going into an analysis of the thermal characteristics or cost of generation.

It must be conceded that the proper criterion for comparing power plants is the commercial efficiency, that is, the total cost per kilowatt-hour sent out from the a-c. power station bus, which is the ultimate test of design and operation. To make such a comparison, a uniform system of accounting would have to be adopted, and suitable correction factors would have to be established whereby the operating results and conditions can be reduced to a more or less common basis, all of which is beyond the scope of general statistical reports. For this reason, the discussion of costs in this paper will be limited to that of coal which is the largest single item in the cost of generation.

What is said here refers particularly to steam power plants using coal as fuel and in which all the output is in the form of electrical energy, but the principles relate in their broad application to steam power plants using fuel other than coal and where both steam and electrical energy are supplied.

For the purpose of obtaining a better knowledge of what a plant is doing and for judging the relative merits of different plants, it is recommended that the following items be recorded in statistical reports relating to power generation in steam power plants, and that they be expressed in a uniform way as indicated. These items reflect the influence of station design and arrangement of apparatus, method of operation and management, load, quality and kind of fuel used, and at the same time indicate the character of the load imposed upon the station. For convenience the items are divided into two groups, coal characteristics and load characteristics.

I. *Coal Characteristics of the Plant*

(1) Average B.t.u. supplied to plant per kw-hr. net output.
(2) Thermal efficiency of plant in per cent.
(3) Average B.t.u. per dollar, coal as received (moist basis).
(4) Coal factor or pounds of coal per kw-hr. net output (moist basis).

(5) Average B.t.u. per pound of coal as received (moist basis).
(6) Cost of coal per ton (2240 pound) delivered alongside the plant.
(7) Kind and character of coal.
(8) Cost of coal in cents per kw-hr. net output.

II. *Load Characteristics of the Plant*

(9) Average daily load factor of load
(10) Maximum load for the year.
(11) Yearly load factor of load.

Interval of maximum load 15 minutes or 1 hour depending upon character of load.

(12) Kilowatt-hours net output for the year—(kilowatt-hours sent out from the a-c. bus).
(13) Installed rated capacity, that is, the aggregate maximum continuous rating of the generators in kilowatts.
(14) Average kilowatt-hours net output per kilowatt installed rated capacity.

Items 1, 2, 3, 8, 11 and 14 can readily be computed from items 4, 5, 6, 10, 12 and 13.

Coal Characteristics. The fundamental basis of determining and comparing steam power plant efficiencies as to coal consumption should be the average B.t.u. supplied per kilowatt-hour net output at the power station bus. This will give a basis for discussing and comparing plant efficiencies upon which there can be no misunderstanding. With this factor and a knowledge of the character of load, we have a very good criterion by which to judge the merits of a plant both as to design and operation. The average B.t.u. supplied to the plant per kilowatt-hour net output for the year is determined by dividing the kilowatt-hour net output for the year into the product of the total pounds of coal used during the year by the average B.t.u. per pound of coal as received, (moist basis). This measure of efficiency can be expressed in thermal efficiency or what is generally termed over-all plant efficiency, by dividing it into 3415.

Before a very close comparison can be made between two plants each of which uses a different grade of coal, the coal characteristics of both plants will have to be placed on the basis of coal having the same quality. This involves the effect of quality of coal on the boiler capacity and efficiency which must be taken into account if the object sought is to make an analysis of the thermal characteristics of the plants. However, since the purpose of this paper is to deal with general comparisons which can be made from statistical records without going into a detailed study of the plants, it is proposed to make fairly close comparisons which correct for the difference in the B.t.u. value of the coal itself, but which neglect the effect on the boiler capacity and efficiency. Notwithstanding this inaccuracy which will be relatively small, even with a large difference in the quality of coal, the comparisons as outlined are much more useful than any that can possibly be made from the statistical information published today.

Two plants can be directly compared as outlined above on the basis of the average B.t.u. supplied per kilowatt-hour net output, because both the coal factor and quality of coal are taken into account. A comparison however, cannot be made between the coal factors as given without first correcting for the difference in the B.t.u. value of the coal used. This can be done by multiplying the coal factor of either plant by the ratio of the B.t.u. value of the coal used by that plant, to the B.t.u. value of the coal used by the other plant, thus placing the coal factors on the same basis in so far as the B.t.u. value of the coal itself is concerned.

Because of the lack of laboratory facilities it may not be possible for some small plants to determine the average B.t.u. value of all the coal consumed during the year. In such cases the results of periodic tests should be recorded with an explanatory foot-note, as they will serve to give a general idea of the conditions under which the plant is being operated. There is nothing that upsets the boiler room force so much as changing the grade of coal without notice, since every change necessitates a readjustment of methods of firing. With a knowledge of just what kind of coal is being supplied, the cause of the change in efficiency can more readily be traced.

Generally speaking, the highest thermal efficiency is obtained when burning the highest quality of coal. On the other hand, a low grade of coal purchased at a low price may result in a better commercial efficiency. The B.t.u. per kilowatt-hour net output together with the B.t.u. per dollar will, therefore, give an index of the commercial efficiency of the plant, since coal is by far the largest item in the cost of power production.

Where mixed fuel is used, the relative amount of anthracite and bituminous coal should be recorded, together with the average B.t.u. value of each.

Load Characteristics. The character of load carried has a marked influence on plant efficiency, a fluctuating load and a load with sharp peaks and low load factor require more coal than an equally steady load or a load with a high load factor. In comparing plant performance therefore, it is essential to know the load conditions. These are best indicated by the ratio of the average load to the maximum load for the same period, which is known as the load factor of load and is usually expressed in percentage. Industrial and railway loads are less affected by seasons and daily weather conditions than lighting loads. For this reason it is recommended that the interval for the maximum load be fifteen (15) minutes for stations supplying an appreciable lighting load, and one (1) hour for all other stations.

Much confusion is caused throughout the electrical industry, due to the fact that a uniform method has not been used in determining the load factor. For the purpose of comparing the load conditions of various generating stations, the daily load factor seems to be most suited, and it should be the ratio of average net output during 24 hours to the net maximum load, multiplied by 100 to express in percentage. The load factor should be computed on the basis of 24 hours for the reason that it represents the relation between the actual and possible hours use of the maximum load. In comparing power station statistics which are usually on a yearly basis, the average daily load factor should be used because it is more representative of the operating conditions in the plant itself than a load factor based on the maximum of the month or year.

The fundamental basis of determining and comparing those items of power plant performance that are expressed in terms of unit output of the plant, should be the kilowatt-hour net output. This will eliminate any difference in power consumption of auxiliaries or power used for other purposes in the plant. The result will be that all the factors will be on the basis of the character of load imposed upon the plant.

The plant factor which is the ratio of the average hourly load for the year to the rated capacity of the plant is a measure of the utilization of the installed capacity. This is obtained by dividing 8760 into Item 14.

For the purpose of illustrating the advantage of the proposed over the present method for preparing statistical reports relating to power generation in steam power plants, the following comparison is made between a few of the items of two plants, A and B, which have similar load characteristics, but which use coal differing in quality and price.

A comparison of Items 1 and 2 shows that on the basis of plant *A*, plant *B* has a 12.5 per cent lower plant efficiency and requires 14 per cent more B.t.u. per kilowatt-hour net output at the station bus. If the effect on boiler capacity and efficiency due to quality of coal are neglected, the lower efficiency for plant *B* can be attributed principally to a difference either in station design or method of operation or both since the load characteristics are practically the same.

The equivalent coal factor of plant *B* based on coal having 14,000 B.t.u. per pound, is 1.83 pound or 0.23 pounds more per kw-hr. net output than that required for plant *A*. This means that for the same output and with coal having the same B.t.u. value, plant *B* consumes approximately 14 per cent more coal than plant *A*.

With the present method of recording power plant statistics, Item 4 showing the coal factors would be given, but Items 1, 2, 3, and 5 would not. In this case it would be impossible to tell whether or not any part of the difference in the coal factors was due to the quality of coal. If the relative merits of the two plants were judged from the coal factors alone, the comparison would be misleading for the reason that while the coal

	Plant A	Plant B
I. *Coal Characteristics of the Plant*		
1. Average B.t.u. supplied to plant per kw-hr. net output	22,400	25,600
2. Thermal efficiency of plant in per cent	15.24	13.34
3. Average B.t.u. per dollar, coal as received (moist basis)	5,226,666	5,734,400
4. Coal factor, or pounds of coal per kw-hr. net output (moist basis)	1.60	2.00
5. Average B.t.u. per pound coal, as received (moist basis)	14,000	12,800
6. Cost of coal per ton (2240 lb.) delivered alongside plant	6.00	5.00
7. Kind of coal	Bituminous	Bituminous
8. Cost of coal in cents per kw-hr. net output	0.428	0.446
II. *Load Characteristics of the Plant*		
9. Average daily load factor of load. } Interval of maximum load one hour	50	54
10. Maximum load for the year. } Interval of maximum load one hour	90,000	80,000
11. Yearly load factor of load. } Interval of maximum load one hour	40	38
12. Kw-hr. net output for the year.—(kw-hr. sent out from the a-c. bus)	315,460,000	217,248,000
13. Installed rated capacity, that is, the aggregate maximum continuous rating of the generators in kw	125,000	100,000
14. Average kw-hr. net output per kw. installed rated capacity	2,525	2,173

factor for plant "*B*" is 25 per cent higher than that for plant "*A*", the B.t.u. supplied per kw-hr. net output is only 14 per cent higher. This very clearly brings out the fact that a general comparison cannot be made between the coal factors of two plants without a knowledge of the relative B.t.u. value of the coals used.

Other interesting comparisons can be made from the above tabulations, one of which is the cost of coal per unit of output. The figures given in item 8 show that the cost of coal per kw-hr. net output for plant "*B*" is approximately 4.2 per cent higher than that for plant "*A*".

The above discussion demonstrates the need of a uniform standard method of recording the fundamental factors of steam power station performance in statistical reports, and this paper is presented with the hope that some action will be taken to adopt such a method.

DISCUSSION ON "ESSENTIAL STATISTICS FOR GENERAL COMPARISON OF STEAM POWER PLANT PERFORMANCE" (GORSUCH) NEW YORK, N. Y., FEBRUARY 18, 1920.

E. J. Cheney: I want to give you an idea that I have received as an employee of the Public Service Commission. Instead of objecting because the statistics that we publish are not sufficiently complete and understandable, we get, I suppose, 100 times as many complaints from the companies that make up these statistics, because we ask them for so much stuff, that they do not have time to do anything else than get it ready for us. We would be very glad to improve the reports, but do not like to push the matter too far.

If we started out to get good information—we try to have some now—and go to the logical conclusion, I do not know just where we would end. For instance, a man who is interested in meters, might like to have our report so arranged, and the data so set up by all the companies, that he could tell for each company how much it cost per meter to maintain them, how much per meter it cost to read them, how much per meter it cost to bill the consumers, how much to collect, etc. The distribution man might like to have data which would show the annual cost of pole maintenance, and he would like to get an idea of pole life in the case of all the other companies, and we could go on to an indefinite extent.

I cannot say that these things are not as important as generation, although comparative figures are perhaps more important for generation. I am heartily in favor of the idea of getting statistics which can be interpreted and compared, and I do think we probably can improve our report forms, so that the data we get will be more useful, without making them much more burdensome. Concrete and well thought out suggestions like those in Mr. Gorsuch's paper are most welcome.

W. S. Gorsuch: In regard to the first part of Mr. Cheney's remarks, I wish to say that in preparing data for different reports, the companies are at times put to additional work because in some instances the questions asked lack clearness and uniformity. The result of this is to give out the wrong information which of course will be misleading when comparisons are made.

If the regulating bodies would formulate their questions along the lines suggested in the paper, I believe that the companies would gladly furnish the data, for the reason that such information would be useful for comparative purposes not only to the regulating bodies, but to the companies themselves.

Presented at the Midwinter Convention of the American Institute of Electrical Engineers, New York, February 19, 1920.

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ECONOMICAL SUPPLY OF ELECTRIC POWER

For the Industries and the Railroads of the Northeast Atlantic Seaboard

BY W. S. MURRAY
Consulting Engineer, New York City

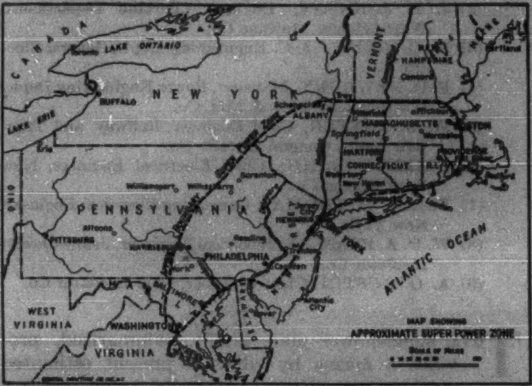
CONTRIBUTIONS TO SYMPOSIUM BY:

(1) W. Le R. EMMET, Consulting Engineer, General Electric Co.
(2) J. F. JOHNSON, Engineer Turbine Department, Westinghouse Machine Co.
(3) H. G. REIST, A-C. Engineering Dept., General Electric Co.
(4) F. D. NEWBURY, Manager, Power Engineering Dept., Westinghouse Elec. & Mfg. Co.
(5) W. B. POTTER, Chief Engineer, Railway and Traction Dept., General Electric Co.
(6) PHILIP TORCHIO, Chief Electrical Engineer, New York Edison Co.
(7) PERCY H. THOMAS, Consulting Electrical Engineer, New York.
(8) W. D. A. PEASLEE, Electrical Engineer, Jeffry-Dewitt Insulator Co.
(9) A. O. AUSTIN, Chief Engineer, Ohio Insulator Co.

THE super-power plan, briefly summarized, provides a means by which a present estimated machine capacity of 17,000,000 horse power—divided 10,000,000 for industrial purposes and 7,000,000 for the railroads—in a region between Boston and Washington and inland from the coast 100 to 150 miles, now operating with a load factor not exceeding 15 per cent, can be lifted to a load factor of greater than 50 per cent and possibly to 60 per cent, and a means by which, conservatively speaking, one ton of coal will do the work of two, and the railroads within the above zone, and those carrying coal into that zone will be relieved of transporting one-half the amount of coal required for power and lighting purposes. In short, the value of machine capacity from a utilization standpoint will be increased three to fourfold, and coal resources for the purposes named conserved twofold.

This means that a present plant capacity of 17,000,000 horse power can be replaced by one of not greater than 5,500,000 horse power, and that not less than 30,000,000 tons of coal per annum can be saved, which at $5.00 a ton will represent $150,000,000 a year.

Besides the above savings, two great departments of economy will be created; one applying both to the railroads and the industries, in the reduced cost of maintenance of machinery, and the other applying to the railroads alone, in the reduction of train miles, by virtue of this plan permitting consolidation of trains with resultant savings in train miles. It is estimated that these latter economies will effect a saving of another $150,000,000 annually thus making a total saving of $300,000,000.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LAKE ONTARIO
Toronto
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BUFFALO
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NEW YORK
Schenectady
Troy
ALBANY
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NEW HAMPSHIRE
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MAINE
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Pittsfield
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R.I.
PROVIDENCE
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New Bedford
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DELAWARE
Dover
WEST VIRGINIA
WASHINGTON
VIRGINIA
ATLANTIC OCEAN
MAP SHOWING
APPROXIMATE SUPER POWER ZONE
SCALE OF MILES

The above are the direct savings as estimated from data collected from actual past operations of the specific order contemplated to be put into force in the zone under consideration.

The railroad situation in this country is too well known to hold any brief upon it in this paper. Not secondary to the concern of those private interests from whom the money has come to build and maintain them is that, now, of the people to whom their service means so much. Forgetting for the moment the much-mooted question as to who will own and control them, we cannot afford however, from a national standpoint, irrespective of ownership, operation or control, to overlook any means, large or small, which will be conducive toward reducing expense keeping the standard of American transportation upon its past high plane, and which will operate to hold the roads solvent.

This plan offers immediate relief from the present intolerable congestion by automatically increasing rail capacity without increasing track mileage. It also operates to reduce power equipment investment to a minimum. By the creation of an overhead common carrier system of power the present cargo space now required for industrial coal will be cut in half; train equipment in all classes will have its service factor doubled, and the present steam power equipment, replaced by electrical equipment, can be transferred to other divisions, where it is so vitally needed.

The super-power plan has lately been given much space in the pages of the technical and public press, and except to say that it contemplates the general application of electricity wherever economically possible in the factories and on the railroads, in the zone described, it will not be necessary to add length to this paper by a more detailed description. It will doubtless engage your interest however, to know the following facts:

1. The plan has the unanimous endorsement of Engineering Council representing the American Institutes of Electrical, Mechanical, Civil and Mining Engineers.

2. It has been presented to and unanimously endorsed by the

a. Western Massachusetts Engineering Association
b. Bridgeport Chamber of Commerce
c. Connecticut Chamber of Commerce

3. It is now in Committee with the U. S. Chamber of Commerce, and so far as I know has found only favor in their considerations for the amount of time they have been able to give it among their many other deliberations.

4. Among the many prominent men with whom I have had opportunity to discuss it, including U. S. Senators and Representatives, Engineers, Central Station Operators and Railroad Officials, and Financiers, all have expressed the keenest interest in it.

5. An appropriation of $250,000 forms a part of the sundry civil bill to provide the money necessary to an investigation, under the direction of the Department of the Interior. This investigation will be followed by a report allocating the losses incident to labor and material by virtue of the present inefficient form of power production and distribution with recommended procedure, to cause their elimination.

While satisfied in my own mind that the true economy of power production, and its distribution in this great industrial and railroad field lies in an organized power policy, it is far more important that we should hear the opinions of those men to whom is largely due the great advances of electricity's application. Therefore, the following letter addressed to Messrs. Emmet, Johnston, Reist, Newbury, Porter, Storer, Torchio and Thomas will explain itself:

(I would add that the address referred to in this letter is one which was delivered to the Connecticut Chamber of Commerce and is reprinted in the JOURNAL of the A. I. E. E. for January 1920).

It has been said that the clash of rival systems has delayed the electrification of railroads. This, in my opinion, is true; but I question whether such a result, after all, has not been beneficial.

Fifteen years has been the period over which we have had opportunity to analyze the theory and practise of railway electrification.

Had we known fifteen years ago what we know today about electrification, and therefore could have decided the proper system to apply to each case, the financial stress under which the railroads were laboring would have prevented electrification.

Today the industrial demand for power has grown so prodigiously that faced with the old methods of power production which we have been perpetuating, we find ourselves faced with two factors, greatly militating against securing and maintaining supremacy in world's trade. Viz.: (1) throttled production, and (2) waste of the nation's natural resources.

The past fifteen years of education have opened the eyes of the "opposing camps" to certain fundamental factors which will now permit a prompt unanimity of opinion regarding what system applies. The railroad managers have in the past justly taken the ground that since we could not make up our minds which system was correct, they would wait until we could. But, as stated before, I believe that finance and not system has been the real cause of lack of action on the part of the railroads to electrify and that the delay has been truly beneficial since we can now stand on more solid ground for decision.

I am enclosing copy of an address made before the Connecticut Chamber of Commerce, November 19th, 1919.

While the super-power line as described in that address is an important adjunct to the plan proposed, the central idea is the indorsement of central station expansion through means of which national production may be increased and the conservation of our natural resources made secure.

As the major demand for power in the territory between Boston and Washington is for industrial purposes, I believe that I will be supported in the following conclusion:

(a) That prime generating machinery should be three-phase.

(b) That the power to be applied to railroads should be through synchronous motor generators with the synchronous motor fields designed to relieve to a maximum degree the direct-current excitation of the prime generating machinery.

(c) That the question of whether train propulsion be by direct or by alternating current should be settled purely upon the economic relations existing between the generating side of the motor-generator substations and the driving wheels of the motive power. In saying this I realize that this robs the single-phase system of the argument to which it has justly laid claim by virtue of the higher efficiency of static versus rotative substations.

Railroad electrifications, in the past, have been individual to themselves. Today I believe that you will agree that they will form only a part of a whole and indeed be a smaller part, the industrial load being the greater—both receiving power from the same source.

I would like to be first under the above conditions in recognizing two serious limitations to single-phase generation.

(a) Size of generators.

(b) Low power-factor regulation.

The Paper's Committee have assigned the 18th of February during the Midwinter Convention in New York as the date for the presentation of a composite paper bearing upon the subject as presented in the accompanying pamphlet.

It is respectfully requested that you, as expert authorities in the design and application of the apparatus in your respective fields, keeping in mind the central idea of zone application of power, contribute your ideas, and while doing so, eliminate all thought of the past battle of systems.

I further believe that you will agree that the hour has struck for a united front and that such a paper will help the advance of electricity's rightful use in the industrial and railway field.

Since writing this letter contributions have been asked from Messrs. Peaslee and Austin, and their valuable opinions touching upon line and insulation form a part of this presentation.

To write a symposium of the contributions forming the principal part of this paper is at once both a delightful and easy task for there treads through such an effort the supporting element of unanimity of opinion. I shall not undertake to comment individually upon these authors' opinions, for that would be but a discussion of their papers. I will rather treat the subject as a whole, shaping its individual parts from the contributed opinions of the authors, using some cement of my own for coherence, and thus, I trust, arrive at a true composite summary.

I am immediately struck by what might be termed the composite voice of these authors. It is as though they say in unison, "We are ready and the conditions are ready to put this thing together." It is opportune here to quote from the address delivered by Past President Charles F. Scott before the New England Section of the National Electric Light Association, September 24, 1919: "The water powers which Nature has bestowed upon New England will be wastefully used until our best engineering abilities combine them into one great regional power system." Again, "The goal is the super-power system, the interconnection of hydraulic and steam stations—a single comprehensive power system, and the universal use of electric power"; and again, "A still larger view shows the power problem from New York to Washington to be closely related to that of New England. High-voltage transmission makes the region from Portland and Boston on the North to Baltimore and Washington on the South a single electrical area. The wide and more varied the field, the more favorable the diversity factor. Power plants at water powers, at coal mines and on tide-water at the cities, combined into one system insure the highest economy in power production."

Professor Scott shows by undeniable records of past growth of central stations, that if the present rate of increase is maintained in New England, its power requirement, now 1,500,000 kw. will be 5,500,000 kw. in ten years. What is true of New England is true throughout all the centralized industrial and railroad districts of the United States. The super-power system now so urgently needed here in the northeast will find demand elsewhere and related in time of application to the industrial and railroad density of the regions to be considered.

We have indeed come to the realization that electricity solved the problems of high-speed economic production and has become the agent of a tremendously diversified utilitarianism. I heard a representative of one of the larger electrical companies say he had just closed an order for a million motors for electrical washers. Think of this as a million kilowatt, and then think also of the diversity factor of such a load! This is but one illustration. The electric range, the toaster, curling iron—all these little implements depending upon electricity, integrate great power loads upon central stations. The amount of power a one-kw. washer takes in an hour would move a 40-ton freight car half a mile. Insignificant as the individual power application may be, their combination makes the central force to drive them immense.

Mr. Emmet more than confirms the statements made to the Connecticut Chamber of Commerce relative to the great economy of electric over steam locomotives and draws deserved attention to the more reliable operating characteristics of the electric engine over that of the steam during the winter months. Traction requirements in such season "go up" and the electric locomotive's capacity automatically "goes up" to meet them, while the steam locomotive's capacities when they are most needed "go down." Who has not had the experience when upon a steam train during bitter cold weather, of having his question, "What's the matter?" answered by the conductor, "We can't make steam."

Mr. Emmet's contribution, though brief, not only brings out the fact that individual turbine sizes are now available for zone power application, and his closing paragraph, while summating important conditions to be satisfied, points to his belief that real results can be accomplished by cooperating in the art of power production—this to my mind is the crux of the situation, for if plant *A* is inter-connected with plant *B*, then the customers of district *A* and those of district *B* both can enjoy the fruits of economical power production, as upon those joint districts could be impressed the voltage from selected machines of high efficiency synchronized upon a common transmission bus.

In comparing the economy of present power production throughout the Boston-Washington zone, with the proposed super-power system, I have stated elsewhere that the former will average at least, if not more than, 40 lb. of steam per kw-hour, and that the average consumption per kw-hour for the super-power line will not be greater than 15 lb. In this connection it is most interesting to note Mr. Johnson's figures of ten, or even less than ten pounds of steam per kw-hour.

As Mr. Johnson has pointed out, "Constant output at as nearly as possible full normal capacity is the first requisite to high efficiency or its equivalent, low cost of production." This is but another way of saying that there can be selected from the inter-connected plants of those now operating and those to be built, the high efficiency units, their size and location related to required distribution, to supply the base load, with the governors of each machine set to contribute its power at highest efficiency.

It is indeed encouraging to find no qualifications in Mr. Johnson's viewpoint regarding the super-power system, but rather a reference to the specific nature of the apparatus necessary to produce satisfactory results, and especially his reference to the adaptability of the multi-cylinder type. It is quite clear that Mr. Johnson's shop specifications are ready and this department of the super-power system only awaits orders to construct. Mr. Emmet and Mr. Johnson have given us the assurances regarding the steam element in the super-power generator. What do Mr. Reist and Mr. Newbury say regarding the other half of the turbo-generators? Mr. Reist says: "So many generators of from 30,000 to 50,000 kv-a. are at present in operation, giving satisfactory results, that there need be no hesitancy in considering generators such as those now used or larger."

Mr. Newbury says: "The generating element in the proposed Boston-Washington power supply system does not involve anything new or untried. The individual power stations need not, and probably would not, be any larger than stations now in operation, or under construction: Certainly any probable station involved in carrying out this super-power development could be designed with steam and electric generating units of size now available."

Mr. Reist's remarks regarding the standardization of potential, methods of insulation and the closed cycle of ventilation are indeed illuminating and all point to that reliability of construction so necessary to continuity of service. It is surely a step in the right direction, and a mark of determination to save thermal losses when, as Mr. Reist points out, the generator itself is made to function as a feed water heater. It may be said indeed that this will be a clean and efficient way to accomplish generator ventilation.

Mr. Reist has opened up an interesting suggestion regarding the consideration of fifty cycles as a standard for world's frequency. Personally, I cannot offer a friendly port to this thought. I believe our generating and transformer investment in the two generally accepted standards of frequency *viz.* 25 and 60 cycles is of an order too large to change now. Certainly, generating equipment of 40 and 62½ cycles should be discouraged. In Mr. Thomas' presentation of the transmission and distribution problem for the super-power zone much valuable suggestion is offered to confirm the standard for prime generating power at 60 cycles.

I think Mr. Newbury has pointed out a fact, not to be denied, in saying that the "super-power scheme will have to be developed by building up local central stations. Its growth must be outward from these centers; but I believe he would agree in the importance of inter-connection of plants through high-voltage transmissions to secure not only diversity factor of load, but *economic* generation and transfer of power from one district to another. The operator who has tried to secure for his company the maximum number of kw-hours per annum from the minimum number of tons of coal burned, has indeed, many times been burdened with the thought of the low efficiency of some of the turbines in his plant that he has had to run on base load when a neighboring plant has had a machine lying idle whose efficiency was the equal of his best machine. Diversity factor though important pales in value when compared to this. That is one of the operating conditions I infer Mr. Emmet had in mind when he made reference to cooperation. I can easily conceive of a tie line between two plants without a single kilowatt of additional capacity paying a handsome return on the investment necessary.

Mr. Potter's able contribution on the traction side of the problem needs but a small amount of cement to bind it to this symposium. He has furnished the cement along with his paper. In short, he tells us that every form of railroad service, passenger, freight or switching can be performed and better performed, by the electric locomotive. The record of electric motive power now in service doing terminal, level and mountain service, and the mileage electrified, bespeaks the practicability of electric traction in the zone under contemplation. Table II is indeed a consolidation of interesting facts and I am pleased to note that he arrives at 14,000,000 tons of coal as the amount to be saved by electric traction over-steam which is 2,000,000 tons greater than the figures I have named in an approximate preliminary study of the situation. On the other hand, I think Mr. Potter has been conservative on his estimate regarding the maintenance figures as I believe the savings here will be *much* more rather than "more" in the amount he has named.

An illness has robbed us of Mr. Storer's direct contribution to this paper, but we find his ample support in the traction field by the recent remarks he made before the Western Railroad Club, December 15, 1919. A copy of these remarks is before me. In them he refers to President Townley's paper, read before the Boston Branch of the A. I. E. E. Mr. Townley's paper was epoch-making in its insistance that the railroad man and the electrical man view the railroad problem *together* and not *apart*. Fifteen years of association between the railroad and the electrical man has brought this amalgamation. Mr. C. L. Bardo, General Manager of the New Haven road was one of the first to recognize this necessity. Mr. Bardo recognized that the requirements necessary to efficient operation by electric or steam locomotive were equal—*but opposite*! He observed that an electric engine would haul twice the tonnage for which it was designed and then apparently without rhyme or reason go out of business three weeks later. Whereas, he noted later, if the designed service duty of the electric engine was strictly adhered to it would do its work on half the coal and half the maintenance, and would double or treble its mileage per engine failure.

Quoting from Mr. Storer's paper, "Actually, electricity has been tried in every known kind of railroad service from street railways to high-speed passenger locomotives; from mining locomotives to the heaviest freight locomotives for mountain grade and tunnel service and from switching in the great classification yards to fast freight trains on trans-continental lines. In every case electricity has shown its ability to do its work as well as, and in most cases, it has done better than the steam locomotive has done and would do it."

Mr. Storer closes his paper by saying: "The big question of capacity, fuel conservation, shortage of labor and the improvement of terminals, will exert all the pressure toward electrification that the railways can stand"; all of which points to the necessity of the super-power system which should be ready to contribute its economies to such an end.

High tribute is due Mr. Torchio for his able and intensely interesting contribution regarding the relative value of water versus steam power development. Looking at his study of the situation from conservation standpoint, brings to mind certain remarks made by Mr. Frederick Darlington, whose able services rendered the War Industries Board during the war are well known and appreciated throughout the country. Mr. Darlington said:

"In the central power station business so much of the cost of service goes to pay for invested capital that it is most essential that the interest rates should be kept low which can be done only by making the investment safe, that is; by so regulating rates that principal and interest will be secured. From a national point of view, as effecting the general industrial efficiency of our manufacturing and power-consuming districts, it is relatively unimportant whether the cost of power is one mill or two mills per unit more or less than some established rate, whereas it is of vital importance for the conservation of resources, for economy of production and for general industrial efficiency that the bulk of the power used should be made by central systems as against isolated plants; therefore let us try to get our law-makers and public executives, national, state and municipal, to take the Government point of view, in other words, to think in terms of the war which are also terms of peace from a Government standpoint, and uniformly and rationally to encourage central power development, provide for a just return to capital in electric power business and grant monopolies under regulations that will foster co-ordination and interstate operations."

The point made by Mr. Torchio regarding the decreasing advantages of water powers on rivers without regulative storage, in proportion to their seasonal variation in flow and especially where their energies can be applied only to independent systems carrying low load factor, has been one, and rightly so, to discourage their development.

It is right here where care must now be used to differentiate between the past small and confined conditions as compared with the present large and unconfined conditions. Where, in the past, rivers whose energies were to be applied to a confined system, and whose load curve for one day could be superimposed upon that of the following with near coincidence, were developed for one second-foot per square mile of drainage area, these same rivers, today, are being developed for two, three and even four second-feet per square mile of drainage area, and we have arrived at this condition because of the tremendous progress made in the inter-connections of systems through the means of which extra load capacity and diversity factor are offered. Today water power development is looking quite as much, if not more, for kilowatt-hours as it is for kilowatts, and therefore the more the energy requirement and diversity factor in inter-connected systems, the greater is the opportunity for unregulated streams to pour their kilowatt-hours into such a composite reservoir of power. This is only saying in another way what Mr. Torchio has already said.

Mr. Torchio's table consolidating a "Review of supply and consumption of coal by classes and territories, and the relation which the potential water powers bear to the amount of coal used in the United States," brings out and confirms my statement that in the east we can never expect a greater contribution from water than 10 per cent of the total power requirement, and it is to the economy of steam produced power that we must earnestly address ourselves. On the other hand, we must be as assiduous in our effort to avail ourselves to a maximum degree of the little that nature has bestowed upon us.

Of great interest comes the check by Mr. Torchio on Mr. Potter's figures of coal to be saved by the electrical operation of the railroads. As stated before, the computed horse power of locomotives in the zone under consideration is 7,000,000. Mr. Torchio shows a total saving of 94,000,000 tons, should the 50,000,000 horse power of steam locomotives in this country be turned into electrical operation. Seven-fiftieths of 94,000,000 would be 13,160,000—a figure lying between my own estimate and that of Mr. Potter's. Again, on the industrial side, Mr. Torchio shows a saving of 61,700,000 tons saved if the present plants in the country, aggregating 25,250,000 horse power, were operated from the lines of high-efficiency central stations. As the computed horse power of the zone under contemplation calls for an aggregate capacity of 10,000,000 horse power, this would accordingly indicate a saving of 10/25 of 61,700,000 or 24,500,000 tons, which added to the above figure, for the railroads, of 13,160,000 tons, makes a total saving of approximately 38,000,000 tons. Both Professor Breckenridge of Yale, Chairman of the Fuel Conservation Committee, Engineering Council, and Director George Otis Smith, head of the department of Geological Survey, have accused me of being too conservative on these savings, and I am glad to see them showing more than less.

We have discussed the generation and utilization of power. The link between its generation and utilization is transmission and distribution. Mr. Percy H. Thomas has contributed a most valuable and illuminating discussion upon this very vital factor of consideration, and it forms a most important part of this presentation. Again we see the objective brought out by Mr. Thomas in that the great base load of the super-power system must be carried by the large, high-economy steam plants supplemented by water power; peak loads and regulative features of the system being taken care of in the main, by "present large generating stations."

Mr. Thomas's comprehensive grasp of the transmission and distribution problem of the super-power zone leaves me with but one recommendation, and that is, your most careful perusal of his paper. The functioning of capacity against inductance in the respective square relation between voltage and current must commend itself to you in the matter of maintaining unity power factor for transmission. Here we see transmission economy maintained and automatic protection against the evil effect of short circuit. A point of almost fascinating interest is the one brought out by Mr. Thomas in the capacity effect on the main line between Boston and Washington to preserve continuity of high power factor and the ability to transfer power north or south without undue loss. The main line will be a veritable static condenser to the whole system, and with such excellent distribution of generating centers, magnetization will provide means of stabilizing and maintaining a constant voltage throughout the whole system.

The simplicity of construction and reduction to a minimum of the switch gear applicable to the 220,000-volt primary transmission, making use to the maximum of present equipment installed, robs the proposed new generating equipment and super lines of much complication and cost.

Mr. Thomas touches on the cost of super-power equipment to the extent of 900,000 kw., this inclusive of generating and transmission equipment up to 700 miles, and names a figure of $150,000,000. Twice that capacity, in combination with the present large plants, would go a long ways toward handling the present industrial and railroad loads in the zone named, Mr. Potter's figures for the railroad requirements being only 750,000 kw.

Mr. Austin's and Mr. Peaslee's contributions regarding the characteristics of line and insulation for high-voltage transmission carry conviction. Particularly interesting is Mr. Peaslee's appeal for ruggedness in the insulator, and Mr. Austin's appeal for simplicity of construction throughout, again touching on many points brought out in Mr. Thomas's paper.

Their papers may be interpreted briefly by saying that in the department of insulation, which is second to none in importance to maintain both economy and continuity of operation, the problem is already solved. Indeed, I think that it can be well said that so great are the returns portended by such a proposed system of organized power policy, that like the great bridges built to sustain a high density of transportation, so may these trunk and tie transmission lines be built to carry their great electrical loads. Their very largeness will bring unusual factors of safety both electrical and mechanical.

With these savings in sight, and with the means at hand to effect them, these means being but the placing in operation of intensive practises long tried out and demonstrated as sound in principle—can this nation afford to be classed as one drunk with the wealth of her natural resources, and rolling in a criminal debauch of their treasure? Is it to be the Americanism of the future to permit this intolerable record of waste to go on?

It is hardly within the province of this paper, should the investigation proposed be made and confirm these projected results, to make suggestions regarding the methods of procedure looking toward the financing and construction of the regional plant required. The following points however, are significant:

1. Conservation in fuel, labor and materials reduced to money equivalents represent approximate savings of $300,000,000 per annum.

2. The highest representative engineering talent in the country agrees that the plan is entirely practical.

3. The securities which represent an investment in such a property (said property to be confined to the features of generation and inter-connecting transmissions) whether underwritten by the

a. States, or

b. Federal Government, or

c. Private Interests, or

d. Private Interests guaranteed by the states or Government will carry a low interest by virtue of their great national importance and financing should not be difficult.

4. Our northeast Atlantic seaboard is the natural finishing shop of American industry, populated with skilled labor to which should be available a cheap and reliable power and by it will be secured high-speed production for shipment in American bottoms (our Merchant Marine) to maintain our supremacy in world's trade.

5. Industrial concerns are far behind in their power requirements and a delay in a supply of electrical power to them will force them to resort to and perpetuate the present wasteful method of supplying it by plants built by themselves. The investment necessary

to their motor requirements will be gladly borne by them and financed immediately.

6. The railroads in this zone are ready for electrification as only by it will capacity be increased: first by the relief from congestion of coal traffic, and second by higher speed and tractive efforts offered in the use of electrical locomotives, thus obviating the immediate necessity of disproportionate increase in track mileage and equipment. The one alternative or the other must be chosen and the right one is apparent.

The following is a summation of approximate investment cost and return for the super-power zone:

SUPER-POWER SYSTEM

(a) New machine capacity 2,700,000 kw.
High-tension transmission mileage 2100
(b) Unit cost per kw. inclusive of transmission lines, $164
(c) Total cost super power system
2,700,000 × $164 = $442,800,000

RAILROADS

(a) Present steam horse power 7,000,000
(b) On account increased speed, traction and machine factors 100 per cent steam horse power can be replaced by 80 per cent electrical horse power
7,000,000 × 80 per cent = 5,600,000 h.p.
(c) Cost of electrical locomotives per horse power = $80(high)
5,600,000 × 80 = $448,000,000
(d) 30,000 miles (single track) to be electrified
Cost per mile $7,500
30,000 × 7,500 = $225,000,000
(e) Total cost railroads $673,000,000

INDUSTRIAL PLANTS

(a) Total horse power 10,000,000 of which 5,000,000 to be changed to electrical drive.

(b) Cost per horse power inclusive of transformers and local distributions $25

(c) Total cost industrial plants 5,000,000 × $25 = 125,000,000

Total zone investment	$1,240,800,000
Total savings per annum	300,000,000
Average return on investment	24 per cent

1. W. L. R. EMMET

LARGE STEAM TURBINES

In discussing the broad expediency of power distribution from central steam stations, it is not necessary to consider the relative merits of turbines of different types. The value of the product of such machines is so great in proportion to their cost that wide variations of efficiency cannot be tolerated, and it may be said that all large turbines are good as compared with other apparatus for obtaining power from fuel. In large turbine units, a variation of 10 per cent in efficiency might justify the scrapping of one machine and the purchase of another, unless the inferior machine could, as is usually the case, be used as a reserve or peak load unit.

The best steam turbine station equipments, operating under favorable conditions, can deliver a horse power hour in the form of electricity with an expenditure of one pound of coal, while four pounds are required to deliver a horse power hour to the drawbar of a good locomotive. The locomotive is further subject to many disadvantages, its fuel supply must be delivered and stored on a relatively small scale in many inconvenient places, and its efficiency is greatly affected by conditions of temperature. Its water supply is also a matter of much trouble and expense. While the comparison of efficiency between the large power station and the smaller engine or turbine equipment used in small stations and isolated power plants is less striking than that with the locomotives, it is nevertheless highly unfavorable to the small plant, and this is fully demonstrated by the rapid growth in the use of central station power in industrial plants large and small. The important requirements for economical power production in turbine stations are: large and continuous demand, facilities for economical purchase, handling and storage of fuel, and an ample supply of condensing water.

Power delivered near centers of demand where load factor is high has a much higher value than that produced in remote places, that is, it pays to transmit power long distances for occasional or irregular demand, while for large and continuous demand it is profitable to produce it locally.

Our large rivers with their fertile valleys have naturally focused our centers of population and of transportation facilities. They afford the requisites for economical power production, and the lines of railways correspond generally to the natural arteries for its distribution. To accomplish ideal results in power production for general uses over large areas will require broad cooperation, and dependable financial conditions under such franchises as will give reasonable security against political attacks, and which will give scope for the many progressive individual activities which are gradually developing the electrical art and educating the public to its uses.

2. J. F. JOHNSON

LARGE STEAM TURBINES

The economy benefits resulting from the proposed consolidation and expansion of electric power production, distribution, and application in a portion of the New England and middle Atlantic section, properly organized and managed, and amply financed, would undoubtedly be very great. Problems involved would require the services of our greatest engineers of finance, organization, distribution, and production, but the writer can see no reason why these should not be satisfactorily and profitably solved.

The problem affecting the steam turbine designer is only a very small portion of the general problem of production. With an electric power generating station, as with a manufacturing establishment, constant output at as nearly as possible full normal capacity is the first requisite to high efficiency or its equivalent, low cost of production.

Studies of the load curves of the various districts served will, no doubt, be made to determine the probable load factor obtainable by the proposed consolidation and expansion.

The greatest amount of saving in cost of production of power is to be made by the discontinuance of the many small and medium sized generating stations, many of which contain apparatus which is either obsolete, incomplete, or uneconomically arranged, and replacing them with large capacity stations designed for maximum reliability and efficiency. This would include the development of available water powers to the maximum degree feasible, and the addition of such steam power stations as would be necessary. Against this saving would, of course, have to be charged the costs and losses incident to the increased distribution system required.

The design of the steam generating stations, such as would be required, should probably not differ materially from those being employed for our largest and most modern stations now building, and their efficiency would not differ materially from those now obtainable except as affected by the higher load factor under which they would operate.

Reliability would, of course, have to be the first aim of the designer, and this requisite would preclude the adoption of experimental apparatus or conditions widely removed from those now used and known to produce satisfactory results.

Generating stations of from 200,000 to 300,000 kw. capacity, employing generating units of from 50,000 to 75,000 kw. capacity, each operating on steam conditions of 300 lb. pressure, 200 deg. fahr. superheat, and 29 in. vacuum referred to a barometer of 30 in., would involve no difficulties of design, construction, nor operation, and from such stations a steam consumption rate of 10 lb. or less per kw-hr. and total station rate of less than 1½ lb. of good quality coal per kw-hr. output should be obtained.

Steam turbines of the multi-cylinder type, similar to those now in use and under construction would possess advantages because of their reliability, efficiency, and flexibility, and also because of their adaptability to other operating conditions, should changes become advisable by reason of further experience or development. Only the high pressure element of a multi-cylinder unit would require alteration to meet such a change of conditions.

3. H. G. REIST

Large A-C. Generators

The following brief discussion has reference to generators needed in a proposed super-power zone extending through the southern part of New England and the southeastern part of New York and Pennsylvania to Washington, D. C. In this area there would naturally be a number of power houses located at points where condensing water was available and where coal could be delivered with the least haulage by rail. Probably most of these power houses would be located on tide-water; others, in the vicinity of the coal mines.

The size of the generating units for such a project will undoubtedly be determined largely by the output required from individual power houses. Probably there would be at least five generating units in each installation, so that repairs and inspection of the units could be made without much reduction in output at any point. Any reduction in output would have to be supplied from neighboring power houses and the transmission of large masses of power should naturally be avoided to limit the line losses. Nor should there be an excessive number of machines, as this would increase the cost of attendance and installation. A second consideration in determining the size of the units is the economical maximum size of the steam turbine. This subject was discussed at length at a recent meeting of the A. I. E. E. and we may assume that three-phase generators can readily be built for the limiting sizes of the steam units suggested at that time. It seems probable that whatever frequency is selected for this system of distribution, turbines and generators can be supplied to meet the most desirable number of units to be placed in each installation. So many generators of from 30,000 to 50,000 kv-a. are at present in operation, giving satisfactory service, that there need be no hesitancy in considering generators as large as those now in use, or larger.

The potential of these generators should be within the limits of our experience, that is, not above 13,200 volts and preferably lower, if this does not cause inconvenience in the lines leading from the generators to the step-up transformers. The efficiency of large modern generating units is very high. The losses may be expected to be less than 2 per cent of the output of the machine, but even with these small losses, careful attention must be paid to the design of the machine to get satisfactory ventilation so as to keep all parts of the machine reasonably cool. While it is permissible to allow a high temperature on the rotating element, with normal maximum potential of 125 or 250 volts in the windings it is better to restrict the heating in the stationary element to a temperature which will not impair the materials used for insulating the windings. It is recognized that nearly all our flexible insulating materials, such as, paper, cloth, fibre and varnishes, deteriorate at a temperature slightly above 100 deg. cent., with greater or less rapidity. The well-known exception to this rule is mica, but an insulation of mica as applied in electrical machinery, consists of a multitude of thin flakes held together by shellac, or other varnishes which also seal the interstices between the flakes. The electrical resistance of all such varnishes decreases with increase in temperature. With some of these materials, the resistance decreases rapidly at temperatures as low as 75 to 80 deg. cent. Other varnishes, especially some that have been developed in recent years, retain their resistance at considerably higher temperatures. When the subject is considered from all standpoints, it seems undesirable to operate such important machinery at a higher temperature than is allowed by "Class A" insulation in the American Institute Rules. Nor is there much objection to insisting on low temperature on such large machines, since the increase in size and cost of cool machines is negligible as compared with those operating at higher temperatures. Moreover, the mechanical construction should be such as has been tested and found to be reliable. The machines should be designed to have minimum losses and an efficient system of ventilation.

Big generators are cooled by circulating a very large amount of air through the various ducts provided for this purpose in the machine. Any soot or dirt in the air, and the air always carries some of this material, is liable to be deposited on the surfaces of the ventilating ducts and on the exposed surfaces of the coils, thus greatly reducing the efficiency of these surfaces in transmitting the heat to the circulating air. To avoid this deterioration, the air is generally passed through air-cleaning apparatus which removes about 98 per cent of the dirt contents, but even with this cleaning, a considerable amount of dirt will accumulate. An improved method of cooling which has been adopted in a few cases and which has excited the interest of engineers in many other installations, is to enclose the air used in cooling the generator completely, and to re-circulate it, thus not taking in any new air. The circulating air may be cooled by the familiar spray system, but in many cases it is preferable to cool it by a system of radiators, similar to those used on automobiles, only they function to cool the air passing through, instead of cooling the water, as in a motor car. Under certain conditions, the water from the condensed steam may be used as a cooling medium in these radiators. This offers a method of returning part of the heat to the boilers, thus saving a small amount of heat which ordinarily is lost. In this way there is no opportunity for dirt to get into the system.

A closed system of ventilation offers some advantage in extinguishing internal fires which sometimes occur, since it will be possible to retain gases or vapors, which may be used in putting out the fire. It is difficult to do this with the ordinary system of ventilation, since the rapid circulation of the air, of which there is a continued new supply, blows gases out of the machine very rapidly even if enclosing dampers are provided.

On account of the great capacity of the lines in such a large system, there may be some advantage in the use of induction generators. Such machines cannot readily be built in as large sizes as synchronous generators, for the following reasons:

1st. Such machines, having smaller airgaps, are more difficult to ventilate than standard synchronous generators;

2nd. Magnetizing current must be supplied through the armature winding. Therefore, larger current is required in the windings which have to be insulated against the potential of the machine;

3rd. The rotor must be at least partly built of laminated iron, making it impossible to design as stiff a structure as otherwise, thus reducing the critical speed.

Roughly, the size of an induction generator would probably be from 25 per cent to 30 per cent larger than a synchronous generator and the cost of the machine in proportion. Since the present limit in the design of steam turbine generator sets is due to the turbines and not to the generator, except possibly at 3600 rev. per min., I believe that it is possible to construct induction generators in sizes comparable with the most efficient operation of the turbine, that is, perhaps 20,000 to 25,000 kw. at 1800 revolutions, and 40,000 kw. at 1200 revolutions, for 60-cycle machines. We might assume that the same conditions hold in connection with 25-cycle generators. The efficiency of induction generators would probably be about 1 per cent lower than of synchronous machines.

In the territory to be supplied by the super-power system, power is at present being distributed at a number of frequencies, such as, 25, 40, 60, 62½. I will not attempt to explain the reason for these various frequencies, but no doubt all of them were, or seemed, justified when they were adopted. The present tendency is to standardize 60 cycles, rather than 25 cycles, since the latter cannot be used for lighting and higher frequency gives greater flexibility to the speed of industrial motors. Synchronous converters and motor-generators can more readily be operated from 60 cycles for transforming to direct current, than from 25 cycles, since there is greater choice in the number of poles of the motors. The use of 60 cycles will be a handicap on the transmission line, but probably the line will be sectionalized and very long transmission of power will be the exception, rather than the rule, so that it is possible that these inconveniences may be overcome. Since much machinery has to be replaced, and a large amount of new machinery added, and since the transmission conditions would be better at a frequency lower than 60 cycles, I think it is well to give consideration to the use of 50 cycles on such a project. This frequency is entirely satisfactory for lighting, for commercial motors of all sizes, and for almost all other work, and it would probably be found that ultimately its adoption would be a great advantage to this country commercially, since we should then have what is going to be the world frequency outside of the United States. It might be well to consider the adoption of this frequency now, rather than to let the matter drift and suffer the inconveniences of the continued use of a frequency different from the rest of the world, and ultimately to find it desirable to change our standard at much greater inconvenience and expense than if it were done now.

4. F. D. NEWBURY

Large A-C. Generators

I have given this subject, as presented in Mr. Murray's several letters and in his paper printed in the January A. I. E. E. Journal, careful consideration. I have not been able to give this matter the time it deserves, but trust the following notes will give my point of view, in regard to the generating end of this subject.

The generating element in the proposed Boston-Washington power supply system does not involve anything new or untried. The individual power stations need not, and probably would not, be any larger than stations now in operation, or under construction. Certainly, any probable station involved in carrying out this super-power development could be designed with steam and electric generating units of a size now available.

Suppose we assume a station of 300,000 kw. to 500,000 kw. total capacity. We have available single-shaft generating units up to 40,000 kw., with generators of 50,000 kv-a. capacity. The preferred speed for such a unit would be 1200 rev. per min., 60 cycles. Eight to twelve such units would give the assumed total station capacity.

There has also been developed and built triple-shaft cross-compound units of 60,000 kw. Five to eight such units would constitute as large a station as has been suggested. This triple-shaft unit consists of three 20,000 kw. turbine cylinders and generators. There is one high pressure element at 1800 rev. per min. and two low pressure elements at 1200 rev. per min.

All of the generating units referred to are actually conservative in size, and by no means represent the largest units of the speeds in question that could be designed. Information concerning this latter point is given in the recent papers by Eskil Berg and J. L. Johnson, on Steam Turbines, and the writer on the subject of Turbo Generators.

As regards the general subject of the single source of power supply within the zone outlined, I believe that the scheme will have to be developed by building up local central stations. It would seem that the greater part of the gain in economy can be secured if all the power requirements of the given local territory, such as Boston or New York, or Philadelphia, or Baltimore, were supplied from local central station systems. The problem of connecting up such local systems in one super-power scheme and interchanging power in both directions from each local system presents a problem of considerable engineering difficulty, and questionable economic advantage in the present state of the art, and in the present state of central station development. There remains so much to be done in developing each local territory that it would seem wisest to carry this on before attempting linking up several local power systems.

5. W. B. POTTER

HEAVY TRACTION LOCOMOTIVES

The suggested system of interconnected economic power generation and distribution throughout the proposed super-power zone should adequately and advantageously provide for the electric operation of the railways within this zone, as well as for the power required for industrial and other purposes. The electrification of these railways would not only insure a substantial reduction in the amount of coal otherwise consumed by the steam locomotives, but would also materially reduce the cost of maintaining the motive power units. Electrification would also provide a more reliable service for all classes of traffic and be a welcome improvement to the traveler as passenger trains would be less frequently late, especially during the winter. The colder the weather the greater is the reserve power of the electric locomotive, which is a much better characteristic than that of the steam locomotive whose power under similar conditions is correspondingly diminished.

There are numerous illustrations of electric operation, which are comparable to the service within the zone under consideration, as well as many other examples of railway electrification throughout the country and abroad, which afford conclusive evidence as to the successful operation of railways with electric power. In fact, a large number of railway electrifications are already embraced within the limits of the proposed zone, and while they do not represent a large proportion of the total mileage, their traffic statistics are available and can readily be studied as a basis for determining the demands of the whole area. A tabulation of these electrifications shows that in this area there are already 380 miles of electric route, embracing 1450 miles of single track and operating 230 electric locomotives and about 1000 motor cars for multiple unit suburban service. A table showing the data of the various roads embraced in this statement, is presented herewith.

TABLE I

Steam Railroad Electrification in the Super-Power Zone

Railroads	Date of electrification	Route Miles	Total mi. of track	No. of locos.	Motor Cars
Balt. & Ohio R. R.	1895	3.6	8.	9	0
L. I. R. R.	1905	88.63	218.	0	477
N. Y. C. & H. R. R. R.	1906	54.00	258.	73	221
W. J. & Seashore R. R.	1906	74.60	150.26	0	109
N. Y. N. H. & H. R. R.	1907	81.63	527.49	106	27
Penn. R. R. (New York)	1910	18.73	97.49	33	8
Boston & Maine R. R.	1911	7.97	21.50	7	0
N. Y. West & Boston	1912	18.23	54.41	1	40
Penn. R. R. (Phila.)	1915	30.5	116.3	0	115
		377.89	1451.45	229	997

In order to obtain a general picture of the railroad traffic which would be affected by the power supply of the super-power zone, we have made a study of the traffic conditions of the territory covered by the zone. In making this study, we have taken data from the Operating Reports of the United States Railroad Administration, extending over the months of 1919, for which comparable figures are available.

The reports of the Railroad Administration do not give separate traffic statistics of the various divisions of the roads embraced in their report, and there is necessarily some uncertainty in estimating the portion of each road and the traffic which would be embraced in the zone. We have tabulated the mileage of those divisions of each road which would presumably be included in the proposed super-power zone, thus determining the percentage of the total mileage of that road lying within the zone. This percentage, or ratio, we have applied to all other data of the road in order to determine the traffic within the zone. This factor, therefore, determines the number of locomotives, the amount of traffic which would be handled electrically instead of by steam, and the tonnage of coal which would be replaced by electric power. In view of these assumptions as to probable area of the super-power zone and the amount of included traffic, the estimate as given can only be an approximation.

As the detailed figures obtained from the Operating Reports do not apply to switching service, we have added 20 per cent to the mileage and tonnage to cover switching service, and as the power requirements per ton-mile for switching are approximately double those for main line service, we have added 40 per cent to cover the coal consumed in the switching.

On this basis we estimate that the railroad traffic in the region covered by the zone can be represented, approximately, by the following table:

TABLE II

Railroad Traffic in the Super-Power Zone (Passenger, Freight and Switching)

Miles of route	12,000
Miles of single track	30,000
Locomotives in service	8,100
Locomotive miles annually	185,000,000
Gross ton miles annually, including main line and switching movements of passenger trains, freight trains and locomotives	170,000,000,000
Tons of coal consumed annually	21,000,000

Considering railway electrification broadly throughout the whole country and including only those lines which handle freight and passenger service with electric locomotives, we find there are about 700 electric locomotives operating over 5000 miles of route.

There have been some data published on the results of heavy electrification. The papers to which we would particularly refer are,—

Murray—*Electrification Analyzed and its Practical Application to Trunk Line Roads.* TRANS. A. I. E. E., Vol. XXX, 1911.

Cox—*Electrical Operation of the Butte, Anaconda & Pacific Rwy.* TRANS. A. I. E. E., Vol. XXXIII, Sept. 1914.

Beeuwkes—"Operating Results from the Electrification of the Trunk Line of the C. M. & St. P. Ry." New York Railroad Club—March 16, 1917.

From data available, it would appear that the ton-miles moved by 6½ pounds of coal in a steam locomotive is approximately equal to that which can be moved by one kilowatt-hour delivered from the power station. Applying this ratio to the last item in Table No. 2, the electric energy required to handle the traffic now handled by the 21,000,000 tons of coal to steam locomotives would be approximately 6,500,000,000 kw-hr.

If we assume 40 watt-hours per ton-mile at the power station, which checks fairly with the records of a mixed service of main line and switching, the total energy for moving the assumed traffic of 170,000,000,000 ton-miles, would be approximately 6,800,000,000 kw-hr.

The actual requirements would, however, be something less. It has been estimated that of all the tonnage moving over the railroad, approximately 12 per cent is taken up with the movement of railroad coal to points of distribution, including a second movement of the same coal in the locomotive tenders. Making an allowance for railroad coal that would still be required, a reduction of 10 per cent would seem a fair estimate. This would correspondingly reduce the yearly power requirements to about 6,000,000,000 kw-hr.

On the basis of probable load factor, this would call for about 1,250,000-kw. of power station equipment.

CONCLUSIONS

The conclusions to which this discussion point may be summarized as follows:

1. Of the whole mileage included in the zone, a not very large proportion has been electrified, but main line electrifications now in operation are of sufficient extent and carry tonnage of a character to present data which can be fairly applied to the traffic of the whole district.

2. The traffic within the zone now handled by steam locomotives, if handled electrically, would require an average output of less than 750,000 kw. and if produced entirely by coal-burning electric power stations, would reduce the coal requirement for transportation purposes from 21 million tons to 7 million tons annually.

3. As a certain proportion of the electric power will be produced from hydraulic power stations, this coal requirement will be reduced in proportion as advantage is taken of hydraulic operation.

4. The reduction in cost of maintaining the motive power units would be a large amount which estimated from the locomotive mileage would be in the order of $15,000,000 or more, annually.

6. PHILIP TORCHIO

ROLE OF WATER POWER AND COAL IN SUPER-POWER SYSTEMS

Summary. A review of the resources and consumption of coal and the availability of potential water powers in the United States shows that:

1. The *Western States* (Mountain and Pacific) have resources in both coal and water powers to meet indefinitely all their heat and power requirements from either source of supply; the potential water powers alone being large enough to supply over six times all heat and power requirements of 1915.

2. The *other states*, with corresponding heat and power requirements 40 times greater than the Western States, have actually smaller resources in coal and potential water powers, the latter capable of supplying only 8 per cent of their total heat and power requirements. It follows that these states must indefinitely, so far as present human knowledge can foresee, depend upon the use of coal to supply the great bulk of these needs.

3. These states, comparatively so deficient in water powers, consumed in 1915, 528,000,000 tons of coal, about one-half for generating power and one-half for generating heat. They obtained an average efficiency from coal of 5 per cent for the power and 50 per cent for the heat. With present methods of generating power in large, modern central stations the efficiency from coal has been raised to 19 per cent. If all the power had been so generated, the coal saving would have amounted to 185,000,000 tons and the railroads relieved of two thirds of this kind of freight.

4. As the investment cost of a steam plant is only about one-third or one-quarter that of a waterpower development with its transmission connections and steam reserve, and as all other items of investment for the electrification of industrial plants would be the

same with power generated either by steam or waterfalls, it follows that every dollar invested in modern steam plants will make available several times more power for the industries than a dollar invested in hydroelectric plants.

5. The offset for the greater overhead charges of hydroelectric power is the saving in coal. The value of this saving will be more or less, according to whether the power is used more or fewer hours every day of the year, and according to the unit price of coal.

Under American conditions, the value of coal saving derived from the use of water power is usually smaller than the difference in overhead charges between steam plants and hydro-plants, except in cases where the power is used continuously every hour of the year, or where the development is connected to a large steam power system which will absorb the entire possible output.

6. With certain industries like electro-chemical processes and the refining of metals, using immense amounts of power continuously, the net cost of hydroelectric power may be considerably less than steam power. There is a great advantage to the whole Nation of devoting as much as possible of the economical water powers, particularly those which are continuous or nearly so and those without regulatory daily storage to intensify the development of these industries which can prosper and render their full benefit to the Nation only under conditions of mass production and most favorable cost of power.

7. In considering the technical and practical features of the special problem affecting all the states (exclusive of the Western States) having insufficient potential water power to meet their present and larger future demands, emphasis should be laid on the importance of concentrating the national efforts to secure the largest possible coal savings in co-ordinating modern methods of power generation by steam. Heavy losses would follow a policy of handicapping these developments by any method of water power development which would involve diverting from them the few but most economical large commercial loads,

and duplication of plant and substitution of water power, and relegating the steam plants to supply the larger in aggregate but less desirable loads of small commercial and domestic services, with poor utilization of the apparatus on account of low average use of power each day during the year.

Where water powers are used as auxiliaries to general power distribution systems, the most economical conditions for both steam and hydroelectric generation can be fulfilled only where a relatively small proportion of the installed generating capacity is hydroelectric.

8. These conclusions would not apply to the Mountain and Pacific States, where the potential water powers are enormous and of cheaper development and can be depended upon to furnish all local power requirements to a very distant future.

THE UTILIZATION OF WATER POWERS AS A MEASURE OF FUEL CONSERVATION

The harassing experiences of coal famines have brought forth repeatedly pressure that immediate efforts be directed to the development of the potential water powers of the country. While from the general standpoint of conserving the national coal resources the proposition deserves the most favorable consideration, it is questionable whether quicker and greater results could not be obtained by concentrating the effort upon improving and changing wasteful methods of utilizing coal.

This we shall discuss under two main headings, one surveying the value of "Potential Water Powers as Sources of Saving Fuel;" the other reviewing the "Technical and Practical Features Attending the Utilization of Water Powers;" so that with the aid of the first survey we may arrive at a clear understanding of the logical lines of development and application of potential water powers for the best interests of the Nation.

1. *Potential Water Powers as Sources of Saving Coal.*

In order to appreciate the situation with respect to utilization of waterpower, it may not be amiss to give a bird's-eye view of the supply and consumption of coal by classes and territories and the relation which the potential water powersbear to the amount of coal used in the United States.

In Table I are given, in detail, for the year 1915, the amount of coal consumed under the two main classifications of coal used for:

Primary power	267,000,000	tons:
Heating	275,000,000	"
Total	542,000,000	"

On the basis of estimates made by the United States Geological Survey, and assuming that only about 60 per cent of the total supply will be recoverable, we find that the present supply of coal in the United States, exclusive of Alaska, if consumed at the rate of the 1915 yearly consumption, will last *four thousand years.* The supply in the Western States amounts to about two-thirds of the total of the United States, exclusive of Alaska. The supply in the remaining states, if consumed at the 1915 rate of about 500,000,000 tons per year, would last *one thousand years.* The greatest shortage is in the supply of anthracite, which, if consumed by the anthracite using states at the rate for 1915, would last only *one hundred years.* In any case, therefore, the saving of coal during one generation is not of vital importance.

This disposes of the immediate urgency of considering the conservation of coal per se, at least for the time being, while more pressing subjects demand attention. The vital importance in conserving railroad facilities, need not be considered provided similar coal savings are made by one means or another.

The next point to visualize is the relation between the national coal requirements and the extent to which they could be supplied by the utilization of water powers in place of coal. We find, using Table I, that the 542,000,000 tons of coal produce a total energy figured in terms of million British thermal units of 3,908,320,000 units,

121,620,000 in the Western States, and
3,786,700,000 in all other states.

TABLE I—REVIEW OF SUPPLY AND CONSUMPTION OF COAL BY CLASSES AND TERRITO

	H.P.	H.P.hrs. (Millions)		Yearly consumption tons	
Primary Power					
Steam railroads	50,000,000	60,000	128,000,000		
Central Station & Electric Railways			18,000,000		
Steam	8,000,000	16,000	....		
Hydraulic	4,000,000	17,000			
Manufacturers & Isolated Plants					
Steam { Electrical—44 per cent Mechanical—56 per cent	25,250,000	46,800	87,750,000		
Hydraulic	4,000,000	12,000			
Mines & Quarries	6,000,000	9,600	18,000,000		
Steam & Naval Vessels	5,000,000	10,000	10,000,000		
Miscellaneous Mills & Irrigation	1,750,000	2,800	5,250,000		
Total Power—Steam	96,000,000	145,200	267,000,000	Western states Other states	7,0 260,0
Hydro	8,000,000	29,000	....		
Total	104,000,000	174,200	267,000,000	Western States Other states	7, 260,
Heating					
Industrial steam	....	....	90,000,000		
Domestic & small steam	....	....	118,000,000		
Coke—Beehive	....	....	42,000,000		
" By-product	....	....	20,000,000		
Coal Gas	....	....	5,000,000		
Total	....	....	275,000,000	Western states Other states	7, [illegible]
Grand Total—Steam	....	....	542,000,000	Western states Other states	14 [illegible]
Hydro	....	....			
Total	....	....	542,000,000	Western states Other states	14, 528,
Potential Water Power		100 per cent load factor Assumed			
Average Continuous 90 Per Cent Efficency					
Western states	33,422,000	293,000	....		
All other states	13,458,000	118,000	....		
Total—United States—Average	46,880,000	411,000	....		
" " " —Minimum	32,082,000	....			
" " " —Maximum	61,678,000	....			

E RELATION WHICH THE POTENTIAL WATER POWERS BEAR TO THE AMOUNT OF COAL USED

Total energy in coal est. 12,000 b.t.u. per lb. (In millions b.t.u.)		Total energy utilized (In millions b.t.u.)		Per cent efficiency	Possib efficien with mod improvem applie
					Per ce
3,070,000,000		153,000,000		4.99	18.9
432,000,000		40,800,000			
....		43,300,000		9.45	18.9
				....	....
2,105,000,000		119,200,000		5.66	18.98
....		30,600,000		....	....
432,000,000		24,450,000		5.66	18.98
240,000,000		25,470,000		10.60	18.98
126,000,000		6,500,000		5.15	18.98
6,405,000,000		369,420,000	Western states 9,420,000 Other states 360,000,000	5.75	18.98
....		73,900,000	Western states 22,900,000 Other states 51,000,000	....	....
6,405,000,000		443,320,000	Western states 32,320,000 Other states 411,000,000	....	18.98
2,160,000,000		1,297,000,000		60	70
2,840,000,000		995,000,000		35	60
1,010,000,000		657,000,000		65	85 + By-pr
480,000,000		408,000,000		85	85
120,000,000		108,000,000		85	85 + By-pr
6,610,000,000		3,465,000,000	Western states 89,300,000 Other states 3,375,700,000	52.5	
13,015,000,000	Western states 335,000,000 Other states 12,680,000,000	3,834,420,000	Western states 98,720,000 Other states 3,735,700,000		....
		73,900,000	Western States 22,900,000 Other states 51,000,000		....
13,105,000,000	Western states 335,000,000 Other states 12,680,000,000	3,908,320,000	Western states 121,620,000 Other States 3,786,700,000		....
					....
		748,000,000			
		300,000,000			
		1,048,000,000			

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If all the water powers of the country were utilized they could replace 1,046,000,000 or 27 per cent of the above stated units, of which are available
746,000,000 in the Western States
= 614 per cent of total energy used, and
300,000,000 in all other states
= 8 per cent of total energy used,

The water powers already developed represent 73,900,000 units of which
22,900,000 in the Western States
= 19 per cent of total energy used, and
51,000,000 in all other states
= 1.4 per cent of total energy used,
while the Western States possess large waterpower resources to furnish several times their total power and heat requirements. All other states, are inadequately supplied with water powers. Considering the present commercial limitations of distances for electric power transmission, they could not receive assistance from the excess water powers of the Western States. Their own water powers, even if fully developed, could not replace more than 8 per cent of the energy required. As about one-sixth of their waterpower is already utilized, the *balance would give only about* 6 *per cent of their power and heat requirements.* From this arises the necessity of an enlightened national policy encouraging and protecting the efforts of such industries as could be instrumental in realizing savings of coal with all other incidental economies.

While the resources of these states are very great, the coal consumption has been steadily increasing from year to year at the rate of about 10 per cent, and if this increase continues, even the present immense reserve would be exhausted in a small fraction of the 1000 years, without large water power resources to fall back upon in a future not distant as measured by the life of nations.

Coal is used to produce either *primary power* or *heating.*

Referring to Table I we find that all of the coal used in the United Stated approximately *one-half* is for producing power and *one-half* for producing heat.

We find further that in producing power only about 5 per cent of the total energy in the coal is utilized, while in producing heat about 50 per cent is utilized. (These figures do not take into account additional losses met in the transmission and utilization of power or heat, as the case may be).

It is evident from these figures showing the low economy of coal for producing primary power that in this field the utilization of water powers would produce the greatest savings in coal.

The economic value of water powers in conserving coal resources is therefore manifest, but *one of the questions to be considered* is whether at the present time quicker and greater results could not in most instances be obtained by the alternative proposition of securing equivalent or even greater economies through improvements in present methods of generating power or heat from coal.

Analyzing the classified means of power production by services, we find that central stations and electric railways realize the highest efficiency from coal, while all other classes of steam power producers obtain from coal about one-half as much as the central stations. Furthermore, with modern equipment the central stations could obtain nearly twice as much work per pound of coal as the present average from their existing equipments. By supplying with electric power all railroads and industries using steam power this could be done, if generated by modern equipment in central stations, for about one-third the present consumption of coal. The total saving would represent 185,500,000 tons of coal, and the railroads would be relieved of two-thirds of this kind of freight.

To realize these coal economies, it would be necessary to make investments for new steam or hydraulic generating equipment to replace the discarded existing steam locomotives on railroads and steam engines in private plants. In all other cases, with the exception of 11,000,000 horse power in existing private plants which already generate electricity for their operation, it would be necessary to introduce a universal electrification of services, substituting electric motors for mechanical drive in plants and electric locomotives on railroads.

In ultimate analysis, each individual case would have to be studied on its economic merits, but, aside from many technical advantages which the electric operation presents over the operation by steam, which advantages may be of greater importance than the mere saving of coal, in general, for a rough approximation, one may assume that a proposition for electrifying an existing steam-driven plant or a steam railroad will be economically advantageous or not according to whether or not the *fuel saved* will *balance the carrying charges of the cost of electrification.* It will appear, therefore, that in general the unit price of fuel will be the fundamental determining factor of the solution. As an illustration, it has been found that in Italy, where the coal price before the war was ten dollars per ton delivered to the locomotive, the saving from electrification of two-track railroads, with electricity at 1 cent per kw-hr., balanced the extra carrying charges of electrification when the roads consumed 700 tons of coal per mile—equivalent to $7000 cost of coal per mile of road.

I repeat that there are many other considerations to take into account in any specific problem, but the broad fundamental principle holds that the unit price of fuel will determine the net economy of transformation from one power service to another of superior fuel economy. While, therefore, by electrification of railroads and manufacturing plants and substitution of modern efficient apparatus in central stations, we could today, reduce the yearly coal consumption for primary power from 267,000,000 tons to 82,000,000—*a saving of* 185,000,000 *tons*—it is doubtful whether the coal saving would balance the carrying charges for the extra investment required.

In different cases the economic advantages would often times be lacking. This would apply particularly to many railroads, but, on the other hand, the centralization of supply for manufacturers and isolated plants, mines, quarries, etc., would in the majority of cases come within economic limits, and the major part of the indicated possible savings for these services of 74,300,000 tons could be realized in a relatively short time. To accomplish this most economically the central station power undertakings already covering the country with a network of transmitting lines, distributing stations and service lines:

1. Should further centralize their production and increase the interconnections between their own and neighboring generating plants, and possibly locate new plants in the coal fields or at by-product gas and coke works; and

2. Should install modern generating apparatus to obtain the highest coal efficiency, and operate this new apparatus more or less continuously, reserving the existing less economical equipment for operation during occasional short periods of high demand for power, such as the evening hours of the winter months. In this manner the added modern equipment would not only serve to carry the new load of the industries transferred to central station service, but would also make a *large saving of coal for the existing load of the central stations.*

It is necessary that we fully realize the importance of this double saving from the addition of modern generating apparatus to existing steam central stations and also the fact that if hydraulic power be so utilized as to appropriate the most economical portion of the steam central station load, the operating costs of the remaining load on these stations, which will usually be required to supplement the hydraulic power, would increase perhaps more than the saving due to the lower cost of the hydraulic power. In addition, the new apparatus for a steam station could be located nearer to the customers who are to use the power, thereby saving the cost and losses of long transmission lines, and the safety of operation and reliability of the electric service would be incomparably greater under all conditions of weather or season or possibility of inimical interference with the lines.

In the foregoing we established the facts that:

All potential water powers are insufficient to provide the total heat and power requirements of the Nation;

The shortage is particularly striking in the case of the states east of the Mountain and Pacific States, hence

The railroads and industries of these Eastern States must always depend largely for their power upon coal;

If, in these states, steam central stations are confined mainly to supplementing hydraulic power from extensive hydro-electric systems, their power costs would increase, offsetting the savings from hydraulic power;

If, on the contrary, central station systems, unified on a large scale, a scale commensurate with that which would prevail in the case of hydroelectric systems, were developed with modern apparatus and correlated with new plants at coal fields, and by-product gas and coke works, etc., the savings in coal would in a few years fully equal the savings from the total potential water powers in the states east of the Mountain and Pacific States. Incidentally, these savings could be realized with a considerably smaller outlay of capital and within a shorter time than that required for the hydraulic development. Also, this program would avoid duplication of plants and conserve an important industry, already splendidly organized with plants and trained men, all confirmed in the enthusiastic faith that their work is the most material and influential factor in the upbuilding of the efficiency of the Nation.

In this respect, after all is said and done, one must admit that the men who are accomplishing the most in saving coal are the central station companies with their power engineers and the manufacturing companies with their motor salesmen, who are achieving the real results by substituting the wasteful small plants operating at 5 and 10 lb. of coal per horse power-hour with the central station supply operating at a coal economy to save over two-thirds of the coal for the same service. It may not be amiss to point out that the merits of waterpower or steam power are of only relative importance to a public utility operating under governmental regulation. Once the prices of service are regulated upon the basis of a reasonable return on investment values, it will be found that the relative savings from hydraulic power over steam power are of very small order as affecting the total cost of service to a customer. These small differences, with a unified system of steam-power generation as suggested, would entirely disappear for practically all domestic and manufacturing classes of service. The only possible exception would be for those rare instances where the customer makes use of a constant power almost continuously throughout the year.

This leads us to the study of the relative value of waterpower and coal power according to conditions of use and applications. We shall find, for instance, that for certain products which the Nation needs the producer must make use of cheap, continuous power, as these necessities could not be commercially obtained in competition with the same products obtained by the use of coal or from importation from other countries, except by power at an extremely low cost. These products, necessary to the Nation, consume in bulk enormous amounts of energy continuously throughout the year, and to these necessities it is obvious that hydraulic power could be applied at greater advantage than to almost any other application which does not make a similar large and continuous use of the available waterpower. These technical and practical points are covered in the following second part of this presentation.

II *Technical and Practical Features Attending the Utilization of Water Powers*

In normal times the exploitation of a water power is justifiable only when the expected economic saving equals or exceeds the results that could be obtained from steam power. The saving must also be predicated upon the existence of an assured market for the sale of the hydraulic power developed. In a few instances, hydraulic power has been secured as incidental to improvements in navigable rivers. It is impossible to generalize and give the value resulting from such double utilization, but it may be said that none of these improvements has been economically justifiable purely from the value of the hydroelectric power developed.

In general, in the study of a water power development one must consider, besides the costs of the development,

The amount of water flow and its variation at different seasons, and

The maximum possible seasonal and daily utilization of the water powers by the industries to be served.

With the exception of a few rivers, like those fed from the Great Lakes, the water flow and consequently the power available for most of the potential water powers in the United States east of the Western States is very variable and without facilities for building in the mountain regions suitable reservoirs to impound the water for the low periods. This is practicable only in situations like those existing in the West. On this account, it has been found necessary to install in conjunction with water powers, large auxiliary steam plants to supplement the deficiency of the water power at times of low water flow, except in instances where water powers are used as auxiliaries to steam systems of relatively large capacity.

The second point under consideration, that of daily utilization of the power by the industries served, is very important and is often overlooked by men not familiar with the operation of power plants. Only certain special industries, like electrochemical industries producing aluminum, carborundum, special steels, caustic soda, nitrate, calcium carbide, etc., can use available power continuously every hour of the day and every day of the year. The average power user, on the contrary, utilizes the maximum power for, say, eight hours a day only, and more or less irregularly even for that period, with no use on Sundays, holidays, etc. This characteristic of individual users makes the utilization of the maximum available power very low. By combining a great number of users, all supplied from one power plant, the conditions are ameliorated in respect to the utilization of the maximum power demand upon the central plant, as the resultant maximum demand is considerably less than the sum of the individual demands of every user. This results from the fact that all users do not reach their maximum power demand at the same time. But, notwithstanding this averaging of individual demands upon the central plant, the net utilization of the resultant maximum demand is still about eight hours a day or 2920 hours out of the possible 8760 hours in a year. In the case of water power plants without regulatory daily storage: the unused power between 2920 and 8760 hours in the year is entirely wasted.

Without further emphasizing this point of the poor utilization of the maximum power in communities, it appears that *where continuous water power is available*, the greatest usefulness is derived by applying it first to special industries and processes which require large amounts of electric power and can be operated continuously, or nearly 8760 hours each year, so that the waste through unused power is reduced to a minimum. If that is not possible, then the best means of utilizing the water power to the greatest advantage is to have it exploited by large power undertakings which can either so arrange their operations that they can utilize all the water power for carrying that portion of their load which is nearly constant throughout the year, leaving the heavy season peak loads to be supplied by their reserve steam stations, or can otherwise distribute the electric power over large territories, including cities, mining fields, agricultural sections requiring electric pumping for irrigation and other uses, so that the resultant utilization of the water power distributed over different places and periods of day or season is high, and the waste of unused available waterpower is reduced. This method of exploitation is economically feasible only where the price of coal is relatively high.

In the case of water powers with variable flow, presenting large variations between minimum and maximum available water power, the necessity for extensive operation of auxiliary steam plants greatly reduces the economic value of the hydro development as an independent source of power, a condition which exists with most potential water powers of rivers east of the Pacific States, where impounding of water is usually impracticable. Economic utilization of such water powers is frequently contingent upon their use in connection with steam power systems so large that the water power is a relatively small proportion of the total.

The economic value of a potential water power is calculable by comparing the investment and operating costs of the water development, plus the steam plant reserve when required, with the cost of an equivalent steam plant or plants. If the power is not to be utilized *in situ* and must be transmitted over long-distance transmission lines, the cost of these lines and the cost of their maintenance and operation must be added to the cost of water power.

To visualize the relative values of water powers under different conditions of water flow and utilization of power, we may make a few comparisons to approximately cover conditions existing in different localities. As costs figures for present prices are lacking I have used pre war prices.

To arrive at a basis for comparing costs, we have reviewed several existing hydroelectric developments and found that the costs per *kilowatt maximum* of power development were as follows:

One case—continuous power = $167
One case— " " = $198
One case—non-continuous power = $188
One case— " " = $238
One case— " " = $222

The costs of these developments would be materially higher under present prices of labor and materials, but I shall neglect this feature and assume a cost of $180 per kw. maximum—a figure smaller than the average of the above five costs—and also neglect the extra cost for transmission lines where required. We shall also assume the low rate of 8⅛ per cent for interest, taxes and up-keep.

On the other hand, for the cost of steam plants, I shall assume $60 per kw. maximum—as the corresponding cost before the war—and the rate of 11 per cent for interest, taxes and up-keep.

The estimates for six illustrations are based in each case on the development of a 100,000-kw. plant. In the several cases the plant investments were:

Hydro without steam auxiliary.......	$18,000,000
Hydro with 50 per cent steam auxiliary	21,000,000
Hydro with 100 per cent steam auxiliary	24,000,000
All steam.........................	6,000,000

These figures clearly illustrate the point that for the same amount of power the hydroelectric development requires several times the capital outlay of an equivalent steam plant.

From the detailed estimates of power costs I have abstracted the results in the following tables of comparison.

Before proceeding with the presentation of these comparisons, it may perhaps be well to caution the reader that the figures apply to the *increment costs* of securing power from new plants under the conditions given in the assumptions, and would not apply, for instance, to the cost of power from the average steam station operating apparatus considerably more expensive and less efficient than large turbo units of modern design.

It may also not be amiss to state here that the steam central station industry has made almost revolutionary progress in raising the efficiency to a maximum of about 19 per cent of the energy in coal, while older plant and the average isolated plant of today utilize less than 6 per cent. It is also reasonable to expect that, with time, still further progress will be made.

On the other hand, in the case of water powers, hydraulic plants have started with, say, 80 per cent recovery of the potential energy, and efficiencies of even 90 per cent have been reached so that no material progress may be expected in increasing the efficiency of hydraulic plants.

As a corollary to these considerations, it follows that with the free play of economic laws in industry, everyone must recognize the latent weakness of extensive water power development, except under the most favorable conditions of cost of development, marketability of product and liberal terms of water grant.

The following tables give comparisons of the *relative values* of hydroelectric and steam power for a number of specific conditions of service which cover all possible ranges of conditions that may exist for different situations of power production from very large plants equipped with modern apparatus.

I again note that the costs given are only theoretical relative increment costs based on pre-war prices of plant installation and operating labor. Actual costs were higher before the war and would be considerably higher at present both for hydroelectric as for steam power, but relatively higher for hydroelectric on account of greater investment charges.

I. WATER POWER CONTINUOUS—33⅓ PER CENT

Average Yearly Use of Kw. Maximum demand = 2920 Hours—Auxiliary Steam Plant 50 Per Cent of Capacity of Hydro Plant.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydroelectric	0.504 cent	0.564 cent	0.564 cent	0.564 cent
Hydro with 50 per cent steam	0.701	0.711	0.731	0.751
All steam	0.393	0.460	0.593	0.727

II. WATER POWER CONTINUOUS—66⅔ PER CENT

Average Yearly Use of Kw. Maximum Demand—5840 Hours—Auxiliary Steam Plant 50 Per Cent of Capacity of Hydro Plant. Similar estimates Have Been Made, from Which We Abstract the Results Comparing the Yearly Unit Costs and Cents Per Kilowatt Hour for the Different Sources of Power.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydro-electric	0.297 cent	0.297 cent	0.297 cent	0.297 cent
Hydro with 50 per cent steam	0.367	0.372	0.382	0.392
All steam	0.250	0.316	0.450	0.584

III. WATER POWER CONTINUOUS—91 PER CENT

Average Yearly Use of Kw. Maximum Demand = 8000 Hours. In this Case, as the Power is Practically Used Continuously, We Assume that, when Transmitted to Long Distances, it Will Require a 100 Per Cent Reserve Steam Station—Instead of the 50 Per Cent Assumed in the Two Previous Cases.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydro-electric	0.218 cent	0.218 cent	0.218 cent	0.218 cent
Hydro with 100 per cent steam	0.298	0.302	0.322	0.329
All steam	0.199	0.265	0.398	0.530

In the case of water power with variable flow, a condition prevailing on almost all rivers except those fed from the Great Lakes, we find that it is necessary to install a 100 per cent steam plant auxiliary for carrying the deficiency of low water flow and as a reserve against interruptions of transmission lines if the water power is utilized at considerable distance from the power plant.

The amount of energy output required from the auxiliary steam station depends upon the load factor of the system and the period of drought on the hydro plant. For average conditions we obtain the following comparative results for similar conditions of service as figured in the case of *a constant water flow*.

IV. WATER POWER VARIABLE FLOW—33⅓ PER CENT

Average Yearly Use of Kw. Maximum Demand = 2920 Hours (2190 W.P —730 Steam)—100 Per Cent Reserve Steam Station.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydro with 100 per cent steam	0.825 cent	0.841 cent	0.876 cent	0.910 cent
All steam	0.393	0.460	0.594	0.726

V.—WATER POWER VARIABLE FLOW—66⅔ PER CENT.

Average Yearly Use of Kw. Maximum Demand = 5840 Hours (4380 W. P.—1460 Steam)—100 Per Cent Reserve Steam Station.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydro with 100 per cent steam	0.448 cent	0.465 cent	0.499 cent	0.532 cent
All steam	0.250	0.317	0.450	0.585

VI—WATER POWER VARIABLE FLOW—91 PER CENT

Average Yearly Use of Kw. Maximum Demand = 8000 Hours (6000 W. P. —2000 Steam)—100 Per Cent Reserve Steam Station.

	Cents per kw-hr. for coal at			
	$1 per ton	$2 per ton	$4 per ton	$6 per ton
Hydro with 100 per cent steam	0.341 cent	0.356 cent	0.386 cent	0.416 cent
All steam	0.199	0.265	0.399	0.530

Under present prices these estimated costs would be larger. This would influence especially the overhead charges for plants, which would make the relative increases larger for hydroelectric than for steam power. It appears from these comparisons that hydroelectric power offers material advantages only when it is continuous and is so used, but that, on the other hand, the cost of steam power is less than that of hydroelectric power for ordinary conditions of service, load factors and price of coal. For such conditions it may also be noted that even relatively large differences in unit cost are of a trifling amount in comparison with the total cost of service including all other items, common both to steam as well as to hydroelectric power, covering costs of transmission, distribution, metering, billing, etc.

Recognizing these economic conditions, it becomes evident that in shaping the policies of utilization of potential water powers to the best end of preserving investment and conserving fuel resources, hydroelectric developments yielding relatively large amounts of continuous power should be co-ordinated primarily to supply energy for the production of products which require large amounts of continuous power such as can best be generated from water powers. Following this policy, it is conceivable to see large centers of electrochemical industries developed in a not distant future to produce nitrates for chemicals, munitions and agricultural fertilizers, caustic soda, electrolytic copper, electrolytic zinc, aluminum, calcium carbide, carborundum, graphite, etc. As for other applications, while today it may appear visionary to state that eventually no steel will be produced without passing through the electrical refining process, some steel men are firmly convinced that electrical steel will ultimately become standard, because of the saving that all users of such steel will be able to make in reduction of material owing to its great elastic limit and its uniformity.

Some of these industries are in their infancy; others are now either consuming great quantities of coal or contending against great odds in their development because of shortage in cheap electric power within commercial distance of the raw products.

By concentrating waterpower developments into these intensified fields of electrochemical and refining products, there would result the attendant economies of mass production which are an absolute necessity to enable any American industry to flourish or meet foreign competition with its cheaper labor. If, for instance, the utilization of the major part of the Niagara Falls power could be arranged for between the United States and Canadian Governments, it would not be a great stretch of imagination to forsee that center become the Pittsburgh of electrochemistry and electro metallurgy with railroads and waterways reaching out to the most inland points and to the oceans.

Developments on rivers with irregular flow can in general only be used with full economy when employed as auxiliaries to or in connection with steam power systems of relatively much larger capacities.

In the mountain and Pacific States, where the potential water powers are enormous, the development of these resources, to save the wasteful use of fuel oil, should be fostered on the widest possible scale.

7. PERCY H. THOMAS

THE SUPER-POWER PLANT

The following discussion is intended to cover the chief factors underlying the economy and operative feasibility of the proposed scheme for interconnecting the electric power systems of the district of the country lying along the north Atlantic seaboard and putting them in reach of the cheapest sources of steam and water power. With the interconnection is associated the electrification of the principal railways. The existing power companies in the district and the railroads now electrified or which it is proposed to electrify constitute the principal power consumers and distributors and must form the nucleus of the system.

The most fundamental object and the most important advantages of the project from the community's point of view, are the conservation of coal and the relief of the railroads from the burden of hauling coal for their own use and for power supply, which coal haulage constitutes more than one-third of the total traffic. Other advantages are mutual support and interchange of power among power companies, more favorable diversity factor and cheaper generation of power.

To conserve fuel most effectively requires both the development of as much water power as may be economically justified and the burning of coal in the most economical manner, as well as the use of low grades of coal. To relieve the railroads of the burden of freight, requires the burning of coal as near the mines as feasible. Furthermore, economical generation of steam power requires use of large power houses. To meet all these conditions requires long transmission and a very great premium is thus put on low priced and economical transmission lines. In fact, operative practicability and low cost of transmission are the essential features of this project. This means the simplest practicable layout and the greatest possible power capacity.

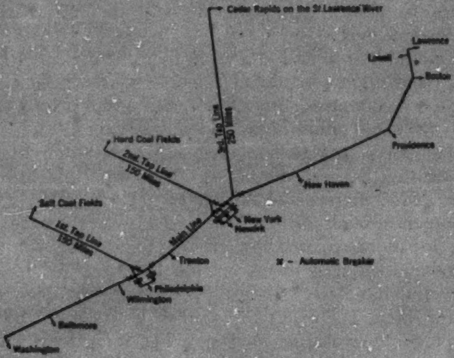
Cedar Rapids on the St. Lawrence River
Lawrence
Lowell
Boston
3rd. Tap Line
250 Miles
Hard Coal Fields
2nd. Tap Line
150 Miles
Providence
New Haven
Soft Coal Fields
1st. Tap Line
150 Miles
Main Line
New York
Newark
Trenton
x – Automatic Breaker
Philadelphia
Wilmington
Baltimore
Washington

FIG. 1

The system shown in outline in Fig. 1 is suggested as one solution of this problem and as a suitable basis for discussion.

This system consists of a main 250,000-volt line connecting Washington or Baltimore with Boston, via Wilmington, Philadelphia, Newark, New York, New Haven and Providence. This line is fed from a group of large stations at the nearest soft coal fields—perhaps on the upper branches of the Susquehanna, but in any case where condensing water can be obtained and from a second group at the hard coal fields, probably supplemented by the Susquehanna and Delaware River water powers. Each group of such powers would feed through a tap transmission line. If a large plant or plants on the St. Lawrence River should turn out to be feasible, another tap line could be run to connect with the main line probably where this line crosses the Hudson River. The length of this line would be about 250 miles, this distance being taken to a junction point with the main line. The total distance to New York probably will be 300 miles or somewhat over.

I would like to call attention to the fact that I have not fixed upon any definite locations for actual coal mine power houses, but have made assumptions as to the probable minimum distances involved. The actual selection of a site or group of sites might add materially to these distances. The only effect, however, of an increase of 25 per cent to 35 per cent in the transmission distance would be to increase the line losses approximately proportionally to the distance and the cost in proportion to the added length. It would have very little effect in the final conclusions. Since the exact location of a site must be determined by suitable cooling water it will obviously be necessary to make use of a certain amount of railroad haulage of coal. This would be necessary in any case since coal must be collected from a number of mines. This is not objectionable if the coal does not have to be hauled long distances nor over main trunk lines. The west branch of the Susquehanna around Williamsport and Lockhaven is one possible site for a large plant. This plant would be fed from the Clearfield coal district. As an alternative, a power station might be located on the Potomac River somewhat below the Maryland coal fields and the transmission from there to Baltimore would be less than 150 miles. The latter alternative would probably work out to about the equivalent of the other as to cost and as to operating quality.

These two or three generating groups would supply the base load or the maximum continuous 24-hour power for all the territory fed by the main line—amounting to perhaps one-third of the peak load. To carry out this idea with the assumed conditions, the single-circuit tap lines should have the maximum possible capacity, which is here taken as 350,000 to 400,000 kw. per circuit. To secure such a capacity at 250,000 volts, requires a number of innovations, which will be discussed later.

It will not be desirable for consumers except the very largest to be connected directly to the 250,000 volt lines on account of complications and the expense. Local consumers in any district can be supplied from the nearest large distributing company. Small water powers or groups of neighboring powers up to 25,000 to 50,000 kw. would be connected with the nearest distributing system without connection to the 250,000-volt system.

As the load in the district grows, additional 250,000-volt circuits can be added from suitable generating sites—probably all more or less widely separated—preferably single-circuit lines.

It will be desirable to make use of and depend upon the present large generating stations to supply all peak and breakdown service. Voltage would be independently controlled at the principal points of the system, as for example, Philadelphia, Baltimore, New York, Boston, as will be later explained.

For example, Philadelphia would receive the first tap line and control its voltage and feed the surplus up and down the main line. Similarly with Newark and New York for the second tap line, and also for the third line to the St. Lawrence if this should be later added.

General Features. The new generating plants should be designed for maximum simplicity and low cost. They should be based upon full-load operation at 90 to 100 per cent load factor. Turbines and generators should be designed for best efficiency at this load. Boilers should be built for high rating and long continuous runs—with distilled water makeup. Very likely coal should be burned in powdered form and the largest units should be used.

The highest steam pressure, probably 500 lb., with at least 200 deg. superheat and economizers with air heaters for preheating air for combustion from exhaust gases and the heat in the generator cooling air should be used. A house turbine operating condensing with the condensate of the main units for cooling water would be used having as large a capacity as can be handled with the amount of condensate available. In addition, atmospheric exhaust auxiliaries would be used to the maximum extent permissible without too high a feed water temperature.

Generators would be synchronous type with governors for controlling the field current, the governors being set for a definite load and the fields controlled automatically to keep a definite power factor—about 98 per cent to 99 per cent lagging and also to take a certain definite amount of charging current at no-load. The reason for this will be explained later.

There would be great advantage in the use of non-synchronous or induction type generators, but their bad power factor and high cost are too much of a handicap to be overcome, unless the characteristics of such generators can be made much more favorable in large sizes than in present sizes; which unfortunately does not seem to be the case.

Large power houses must be located near large and reliable supplies of cooling water—400,000 kw. would require approximately 1500 second-feet flow—or a large lake whose waters could be used over and over, cooling meanwhile.

If a large enough power plant could not be gotten at one point, several closely adjacent plants might be used, all feeding at perhaps 15,000 volts to some central point for stepping up.

High-tension transformers would be connected directly to the line—with opportunity of uncoupling the line dead—but no standard disconnect switches. Something in the nature of a very long enclosed fuse might be introduced for short-circuit protection.

Synchronizing could be done on either end, but on the low-tension side, preferably in the new power houses, however.

No means for automatic circuit opening would be provided, except at the six points (7 points if the third tap line be supplied) shown in Fig. 1. At points where one line connects with another, appropriate towers should be placed close together and facilities provided so that the lines could be cut apart in case of emergency.

The transmission lines proposed have a capacity power something like three or four times as large for a given voltage as heretofore proposed. They have the further advantageous property that they transmit normal power with a very small loss of energy or voltage and at the same time will permit only a relatively small current to flow in short circuit. This is a rather remarkable and very useful property. For example, either 150 mile tap line will permit only about three or four times full load with one end short-circuited.

This favorable design of line is obtained only by careful proportioning and if the load on the line be properly chosen. The load is so taken that the charging current of the line neutralizes the lagging effect of the load current in the reactance of the line.* The proper load for this balance can be made to approximate the desired load of 300,000 to 400,000 kw. by using such construction as to give an abnormally large line capacity and a correspondingly small line reactance. The use of aluminum and the dividing of the line conductor into several separated cables or parts as described hereinafter, sometimes called the "Split Conductor," have a very marked effect to increase capacity and reduce inductance.† This construction also helps greatly in reducing corona and skin effect. Obviously if load and line constants are so chosen that the inductive effect of the line current is balanced by the capacity current at normal voltage, this balance will be lost when a short circuit occurs and the charging current largely disappears. As the charging kv-a. go down as the square of the voltage, and the inductive effect goes up as the square of the current, the

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*See *Output and Regulation in Long Distance Lines*, by Percy H. Thomas. TRANSACTIONS A. I. E. E., 1909.

†TRANSACTIONS A. I. E. E., 1909, loc. cit.

capacity effect will be negligible on short-circuit conditions, leaving the reactance drop the controlling factor. This explains the favorable behavior of this line to limit short circuits above stated.

The layout of Fig. 1 is favorable, since the general effect of the long interconnecting or main line is to increase the system power factor. Power in large quantities is not regularly transmitted in any one direction over the whole length of the main line but blocks of power will pass in alternate directions in adjacent parts and the drop voltage along this line is practicably negligible. For this reason the capacity will not be neutralized in the main line by the effect of the load current in the line inductance but will be available for power factor correction.

It has just been stated that the drop between any two neighboring large stations on the main line will be very small. This is not only desirable but necessary for when power is to be quickly supplied in reverse direction from normal flow, as for example, on a sudden call for help, this reversal of power could hardly be accomplished practically, if there were a large line drop, since the voltage delivered to the station ordinarily sending out power would be too low to be useful when the direction of power was reversed.

But with a line of very low ohmic drop, like the lines here proposed, transfer of energy backward or forward on the same line may be easily accomplished, for by manipulating the field strengths of the generators on both ends of the line, out of phase or circulating current between the generators can be made to counteract the ohmic drop of the load current. This increases line losses somewhat but line loss would be unimportant at time of emergency and would be small in amount in any case. As a net result of these considerations power can be sent in either direction between two points without changing the voltage at either end, the necessary flow of power being caused by suitable change in the power factor of the passing current.

OPERATION

As already stated, under normal conditions the new power houses would send current into the main lines in a constant rate 24 hours a day and no changes would be made for load or seasonal conditions. In case of accidents to line,—which should be very rare,—it is proposed that a certain amount of automatic protection would be provided. The transformers at the station which receives power from one tap line are divided into two parts, one connected to the tap line, the other to the adjacent main line. In case of trouble on the tap line, the tap line circuit breaker would open leaving the connection between sections only through the banks of transformers, stepping down through one and up through the other, and then the main line transformers or the tap transformers would be automatically cut off on the low-tension side as might be called for, leaving the good line operating. The bad section of the system having been cut out, the good parts of the plant could be operated without the bad part. This would be accomplished by relays.

In testing the insulators and making changes of insulators, it is assumed that this work will be done alive by the method used by Johnson in Georgia. While this sounds chimerical to a person not familiar with what has already been accomplished, I believe it to be entirely possible and safe.

Starting Up. Starting could be readily accomplished on account of the subdivision of the high-voltage lines. Philadelphia will excite the section of the line to Newark by the low-tension switches through one set of transformers. This is too short in length for the charging current to be important and will furthermore merely relieve the lagging power factor of the Philadelphia system without increasing load or current. New York can then synchronize with Philadelphia on the low side. Philadelphia will then excite first tap line on its second bank of transformers. This will call for no increase of current at Philadelphia as before on account of the great excess of load lagging current. By connecting the power house transformers at the coal mines before exciting the line, no synchronizing on high voltage would be necessary. Newark or New York would similarly excite the second hard coal tap line (and the third if installed). The line in Baltimore may then be energized at Philadelphia and Baltimore synchronized on the low-tension side. Similarly with the line to Boston, and the system is operating ready for load. The two tap lines which so far are connected only by step up transformers would be connected to the main line at leisure by the high-tension circuit breakers, the system being already synchronized.

The switches or breakers shown in Fig. 1 can be used for automatic cutting apart of the sections of the high-tension system as already explained. On account of the characteristics of the transmission lines as here laid out, these coal mine stations will add little to the short-circuit currents; and one center like New York will supply little short-circuit current to another as Philadelphia.

Spare Lines. This scheme of Fig. 1 offers no spare lines. It is not believed that the expense of a spare line is warranted. It will be noticed that no accident at any one point of the line can cut out more than a small part of the system except momentarily—nor more than one of the mine power plants. If the system grew it would be possible to add other tap lines at suitable points, which would reduce the power shortage due to any failure without providing any idle lines. It would probably be more economical for the community as a whole to make the best shift possible to a certain extent in case of a serious line failure than to pay the charges and endure the disadvantage of the complication of spare lines and additional switching facilities.

Transmission Line. The conductor has been chosen as consisting of three 600,000-cir. mil. aluminum with steel cores. These would be placed in a horizontal plane, 18 to 24 in. apart each hanging from two strings of insulators suspended at an angle, as shown in Fig. 2. This would reduce side swinging and help clearances. Furthermore, if one insulator string should be broken the other would hold up the conductors.

The principal strain on the towers, according to the usual test specification, is due to a broken conductor causing torsion. To reduce this requirement and so lighten the towers as far as possible—suspension towers should be arranged with the conductors free to slide in the hanger, or held only by a moderate grip, having a maximum positive hold. The conductor will tend normally to equalize longitudinal stresses in the adjacent spans automatically and not to develop unevenness. The one danger is the uneven collecting or falling of ice and sleet. A moderate grip on the hangers such as would be justified by the strength necessary in the tower for other requirements would be sufficient to prevent any permanent slipping except in case of a conductor broken in all three cables.

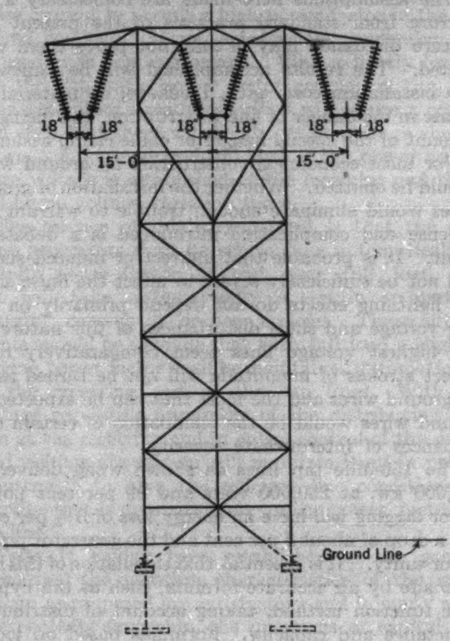

FIG. 2

In this case the conductor will slip in one or more hangers and drop on the ground. The drag on the ground will prevent the line slipping in many towers. Repair in any case will be a big undertaking. The breaking of a single cable of the three of any phase would cause an electrical ground but no slippage.

The assumptions here made are confessedly a departure from standard practises of the present and mature discussion may or may not prove them warranted. The results accomplished will be somewhat less installation cost, with, I believe, no material increase in the danger of line interruption, this being on account of the special hanger or cable clamp assumed.

For most economical construction all ground wires should be omitted. Whether the installation of ground wires would eliminate enough trouble to warrant the expense and complication introduced is a debatable point. It is probable that indirect or induced surges will not be sufficiently severe to affect the lines, since the lightning effects do not depend primarily on the line voltage and since disturbances of this nature on our highest voltage lines seem comparatively rare. Direct strokes of magnitude will not be turned aside by ground wires and the most that can be expected of ground wires would be the elimination of certain disturbances of intermediate severity.

The 150-mile tap lines as shown when delivering 300,000 kw. at 250,000 volts and 99 per cent power factor lagging will have an energy loss of 3½ per cent and a drop of about 5 per cent and the generator power factor unity. It is essential that calculation of this line be made by an accurate formula, such as the hyperbolic function method, taking account of distributed inductance and capacity. Formulas based on localized capacity will give misleading results. The critical condition to cause this failure of the approximate formulas is the one here existing, *viz.*, that the leading and lagging effects of capacity and inductance balance, leaving substantially ohmic losses. With the reactance seven times the resistance the importance of this consideration is evident.

With 400,000 kw. delivered, the energy loss would be 5 or 6 per cent and the line drop about 9 per cent. With 300,000 kw. and a lower power factor the efficiency would go down and the line drop up, but a considerably lower power factor would not give unduly unfavorable conditions. It is here assumed that the existing generating apparatus when coupled up with these lines will be able to supply the necessary out-of-phase current to permit a high power factor in the delivered line load. If not, synchronous condensers can be used as is now customary. With a lightly loaded line the automatic power factor control on the coal mine generators already described will secure a condition such that there will be almost no voltage change between light load and full load at the power house. Since voltage is established at the receiving end there will be no voltage variation there.

The essence of this arrangement is that the coal mine generators shall be set to take automatically a definite power factor at and near full load and yet will take automatically a certain amount of charging current at or near no load. These adjustments should be so made that with full load a slightly lagging power factor shall be maintained automatically at the receiving end of the tap lines. With the voltage regulating apparatus of the distributing system at the receiving end in entire control of the delivered voltage the system will maintain itself in proper adjustment to give the efficient line conditions required—that is approximately unity power factor in the line.

Costs. It having been established in the general terms that the operating characteristics of the super system as proposed are satisfactory and in many ways unexpectedly favorable, the questions of capital and operating costs remain.*

Capital Cost. It may be assumed that a 400,000-kw. power station may be built, under reasonably favorable price conditions for $100 a kw. including overhead necessary for design and construction; that the

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*In this present discussion very free use has been made of the excellent paper, *Problems of 220-Kv. Power Transmission* by A. E. Silver, A. I. E. E., 1919.

transmission lines as proposed, *viz.*, two 150-mile 250,000-volt, 400,000 kw. tap lines and 100 miles of similar capacity line for the central part of the main connecting line, and 250 miles of 200,000-kw. line, can be built at an average cost of $47½ a kw. generated, including overhead. Also that the necessary 250,000-volt step-up and step-down transformers with 250,000-volt switches could be installed for $15.00 a kw., giving the following table:

	Per kw.	
2-400,000 kw. power houses	$100.00	
700 miles 250,000-volt transmission lines	47.50	
250,000-volt transformers and switches	15.00	
Miscellaneous	1.50	$164.00
Total for 900,000 kw., allowing 12½ per cent spare		$150,000,000.00

Taking into account the added cost of real estate, the congestion of fuel handling in the large cities and the complication of handling large blocks of power underground from city stations, a fair estimate of the cost of installing plants to generate this same amount of power at the several large centers, supposed to be supplied, would be about $125.00 per kw., leaving an excess capital cost on account of the transmission scheme of $40.00 per kw., a very small amount for the benefits offered. The value of the 800,000 kw. delivered to the large distributing companies should be in the neighborhood of $20,000,000 to $25,000,000 a year, with present or slightly better conditions.

These cost figures are fairly liberally estimated on the basis of routine construction, probably 10 to 20 per cent should be added to cover additional cost due to the unusual character of much of the work.

Comparing the cost of transmitting the power over transmission lines with the cost of hauling coal by rail, we have to compare the freight on five tons of coal (a sufficient quantity to produce a kilowatt for a year) against the cost of 10 per cent additional coal (required to cover losses in transformers and transmission) plus $3.50 fixed charges on excess capital cost of one kilowatt for the transmission system. That is with coal at $1.00 per ton at the soft coal power station, five and one-half dollars worth would be required for each kilowatt-year. Adding $3.50 excess or fixed charge, gives $9.00. For coal hauling to be on an even basis, coal freight would be 80 cents a short ton. Most of this is on account of interest on the excess investment. This, however, charges the transmission system with 700 miles of line while the actual distance of transmission is only 2 × 150 miles. If the cost of 150 miles, the direct transmission only, is considered, the capital cost may be taken as 12½ per cent less, giving a fixed charge of $2.00 a kw. and a corresponding freight rate of 50 cents a ton. All the above figures are in short tons.

I wish to say that all the above dimensions and estimates are based on the assumption, that the interconnecting system will be characterized by extreme simplicity and that very little so-called flexibility will be provided. This is necessary to reduce costs and to simplify operation and is justified by the completely equipped plants already operating in the inter-connected district.

These approximate figures give an idea of the actual and relative cost and of some of the results that might be obtained by interconnecting the district and supplying from the coal fields the base load for two or three million kilowatt of maximum load now existing. With the future growth of the load and the electrification of the railroads, the St. Lawrence development, or additional coal mine plants, might well add another half million kilowatts in one or two circuits, on a basis more or less on a par with the above, except that if the new power should be hydraulic, it would probably call for a somewhat larger capital expenditure and would show a lower operating cost.

Attention is again called to the fact that the present discussion is based on a hypothetical typical case and not on an actual group of definitely located power houses. Variations of 25 to 50 miles in the length of transmission will, however, make little difference in the net result.

It is to be hoped that as the cost of generating electric power is reduced, partly by the development of such large scale systems as here discussed, partly by the natural development of the art, and partly also by the freer use of local distribution mains, that the use of electric power for heating and cooking and small power will be enormously increased, which will warrant the building of the largest systems, becoming a great source of comfort to the community.

So far no account has been taken of the benefit from the inter-connecting system here under discussion in its opportunity to take advantage of diversity factor in general power consumption nor of its very great advantage in raising the load factor of a railway load, which is at best very low. The relay and breakdown service of the inter-connecting line would be of great benefit. Another advantage accruing is the ability of the system to apply a surplus of power existing in any one system, either temporary or permanent, to any load that may be available. This ability is of much importance in a growing district. Furthermore, the ability to locate very large consumers of power at the most favorable location would be of importance. For example, a large electric furnace plant can now only be placed near some large system, without installing a special power plant. Other similar advantages of the inter-connected system exist, but enough has been said to warrant a careful study of its advantages, disadvantages, possibilities and limitations.

8. W. D. PEASLEE

HIGH-TENSION INSULATORS

The utilization of 220,000 volts or higher for the transmission of power is not a problem that holds any terrors for us as insulator manufacturers.

Due to the size of the cable necessary on such aline and to the fact that it seems rational to design theline for more severe storm loading conditions than standard for an ordinary line, the standard disk suspension insulator as at present manufactured is not entirely satisfactory from a standpoint of mechanical strength.

An insulator for 220,000 volts will have to stand very severe operating conditions as to mechanical stresses and dielectric flux concentrations, and it is very important that certain features of insulator design be followed carefully in any insulator presented for this service. Due to the large size conductors necessary and the spans, the mechanical stress will be considerably in excess of that met with in high-voltage lines now in operation.

The ratio of puncture to flashover in any unit is a very important feature of its design and one which unfortunately has been neglected in most conventional designs of insulators. Due to the effect of transients added to the normal frequency voltage of the line and their time lag or impulse ratio, it is possible to stress the end unit of a string to very high values, and for that reason its actual puncture resisting power should be very high. The puncturing voltage of a disk insulator for such service should never be less than 2½ times the dry flashover voltage of the unit.

In order to reduce corona formation as much as possible, the unit should be designed to reduce the flux concentration and in such a way that the corona forming voltage will be considerably above the operating voltage impressed upon the unit.

For an insulator for service of this kind strength is an important factor. The lines are very important and the ability of the insulator to withstand power arc shock and other events, is of the greatest importance to the continuity of service of the line. We have made a very careful study of the requirements of this insulator.

We are at work at the present time on a disk insulator to meet this requirement. It will be a disk insulator probably of larger diameter than the present design, with a mechanical strength sufficient to meet the strains imposed by the larger loadings necessary in such a line; dry flashover in the neighborhood of 125,000 volts. It will be designed in accordance with the correct theory of surface design between the air and porcelain, and with this design the corona-forming voltage will be very high. The approximate puncture voltage per disk will be in the neighborhood of 300,000 volts, thus securing the high ratio of puncture to dry flashover now recognized as a vital point in correct insulator design.

From these characteristics you will see that this disk will be very well suited for this particular work and our research work has progressed far enough so that we feel confident that by the time such a line is an active proposition, we will be able to undertake the insulation problem. Not more than eight to ten units will be required for the 220,000-volt grounded *Y* line and this will give a string of insulators from 65 to 80 in. in length.

9. A. O. AUSTIN

HIGH-TENSION TRANSMISSION SYSTEMS

It would seem that the super-power project such as Mr. W. S. Murray has outlined, must become more and more of a necessity with each succeeding year. The project would necessarily change in character in time, but the sooner it is started, the greater the saving that would be effected, both as to the economy of operation and the saving of natural resources. The very magnitude of a system of this kind tends to postpone a start on the project, even though the possible economies increase with the size and completeness of the system.

The project must necessarily depend upon an efficient transmission system, and as the cost of the connecting transmission line will be an important factor, it is necessary that we consider the state of the art with a view of adopting a transmission system which will show a maximum economy. Anything effecting the cost of the transmission system should be given careful consideration, for the greater the economy shown the more quickly will a system of this kind be put in operation. Where a project of this magnitude is involved it would seem that a departure from the usual practise is well warranted, particularly where an increase in reliability and a decrease in cost can be effected.

It is possible that at the start, a large part of the system could be connected to advantage with a transmission line not exceeding 110 kv. It is probable, however, that power will be transmitted from large hydroelectric or steam plants located at a considerable distance. The increasing cost of coal will make it highly advisable that the future additional generating equipment be located where the greatest economy can be effected, rather than split up into a lot of smaller, less efficient stations.

The adoption of a high line voltage for the main trunk system would show a saving which might hasten the project several years, consequently it is important that we consider various factors which may be taken advantage of, to increase the efficiency or lower the cost of the line.

The real problem on a large transmission system operating at a high voltage, has usually been that of regulation, and it would seem that the present project is no exception to the general rule. It would seem that the future demands for power in the zone covered by the project would permit the installation of equipment without additional cost, which would give the necessary regulation to insure the success of the system All that would be necessary would be to form a comprehensive plan and install future equipment so as to make the scheme effective.

A transmission line operating at 220 kv. or slightly above will involve large conductors, which would give the system a high degree of mechanical reliability not afforded in the smaller systems. In addition the insulation which would be necessary for the line and equipment, would make the system free from lightning trouble. These two factors would be important in eliminating the necessity for stand-by plants, and should hasten the adoption of the project. The large amount of power would permit of apparatus of a high degree of strength and reliability, so that shut-downs from failure of same would be negligible.

A careful analysis of operating systems would probably indicate that the greatest danger to a system of this kind would be that due to suddenly dropping a large portion of the connected load. This hazard could probably be entirely eliminated by tieing in the main transformers connected to the system, so that they could not be disconnected by the operator unless the voltage of the system was lowered or a defective unit alone be disconnected. Since apparatus would probably be disconnected only in case of failure, a shunted fuse or other inexpensive device could be used to disconnect the defective unit. This would eliminate a considerable cost for oil switches and the freedom from lightning would probably permit the elimination of lightning arresters. This would effect a considerable saving and might advance the installation of the completed project several years.

It is highly desirable to make the main system simple to give the highest degree of reliability and it would seem that in carrying this out that economies could be effected which would more than off-set the increased cost for the higher voltage transformers necessary to permit an economic transmission system for supplying power for a considerable distance. The large conductors necessary for a system such as this would require heavy working stresses for the insulators. The insulator art, however, is such that high ultimate insulators with longer life can be produced at a much lower price than they could previously. It is now possible to procure insulators such that a single string will carry the line and at a cost probably not more than 50 to 60 per cent of that required for a 110-kv. line. From this it will be seen that the cost of installation for a 220-kv. project would probably be less than the cost of some of the 150-kv. lines previously installed. The transmission art has advanced rapidly and many economies could be effected by carrying out a comprehensive scheme, which are not possible in many of the existing systems.

Large additional amounts of power will be required in the district effected by the project and much could be saved by adopting a plan as soon as possible. It would seem that there is no new or untried principle involved, and that by making a survey it would be possible to show the economies which would be effected not only for the present, but for some years to come. It takes considerable time to put large projects into operation, hence it is important that we do not confine our attention entirely to the immediate economies, but look more to the future.

NOTE: For the complete discussion of this paper see JOURNAL of A. I. E. E., August, 1920, page 766.

Presented at the 8th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 19, 1920.

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PRINTING TELEGRAPH SYSTEMS

JOHN H. BELL

Telegraph Engineer, Western Electric Co., New York

Abstract of Paper

This paper describes the Creed, Murray Automatic, Siemens & Halske, Baudot and American Multiplex Printing Telegraph Systems, and their methods of operation. A discussion of the operating features of the systems described is dealt with under the following headings:—

Accuracy Under this heading it is shown that printing systems are more accurate than Morse working and that the printing system which permits of sufficient time for checking of messages is likely to be the most accurate.

Speed of Service. It is shown wherein the multiplex systems are superior to the high-speed systems.

Operator Output. Under this heading reference is made to actual results, both at home and abroad.

Maintenance. Outlines method proposed for detecting incipient faults in apparatus and securing uninterrupted service.

Line Economies. The advantages of using the five-unit code are discussed.

Flexibility. Examples are given of the flexibility of multiplex systems in linking up a number of cities, by means of forking repeaters, thereby resulting in either line or operator economies.

THE object of this paper is to describe briefly those printing telegraph systems in use today which are designed for handling traffic at over 100 words per minute, and to discuss their operating features. This minimum speed limitation has been chosen as it effectively divides the various printing systems into two classes as regards traffic carrying capacity, each of which classes is of sufficient interest to warrant treatment in a separate paper. Practically all the printing telegraph systems not considered in this paper have a carrying capacity of from 50 to 70 words per minute, and are consequently in a class by themselves. The systems capable of carrying heavy traffic loads may be divided into two classes.

(1) High-speed systems.

(2) Multiplex systems.

By a high-speed system is meant a system in which the sending and recording machines operate at the same speed as the signals are transmitted.

By multiplex, is meant a system in which a plurality of sending machines and a plurality of receiving machines are directly connected in the telegraph circuit and individually operate at a lower speed than that at which the signals are being transmitted.

The systems to be discussed are as follows:

High-Speed Systems. Creed (continental Morse code) tape transmission. Signals received on perforated tape which is fed through a printer. Messages are printed on a tape.

Murray automatic (Baudot or five-unit code) tape transmission. Signals received on a perforated tape which is fed through a printer. Messages are printed in page form.

Siemens and Halske (Baudot or five-unit code) tape transmission. Signals received in printed form on a tape.

Multiplex Systems. Baudot (Baudot or five-unit code) direct keyboard transmission. Signals received in printed form on a tape.

American Multiplex (Baudot or five-unit code) tape transmission. Signals received as printed message in page form.

Several years ago there was a diversity of opinion as to the relative merits of high-speed and multiplex systems, and the advantages of the five-unit code over other telegraph codes were not generally appreciated. The tape-transmitting, page-printing multiplex system using a five-unit code, however, has demonstrated its superiority over all others and one object of this paper is to indicate the reasons for this result.

Perhaps the most elementary form of electric signaling is that in daily use by most executives, where one depression of a push button summons a messenger boy, two depressions means stenographer, and so on, a bell or buzzer being the telegraph instrument of call. To send the 26 letters of the alphabet in this fashion, where one beat represents *a*, two beats *b*, three beats *c*, and so on, would, obviously, be a tedious process. By arranging that some beats are short and some long and forming combinations of these short and long beats, a code can be built up. This is the basis of the Morse code. For actual telegraph purposes the push button is replaced by a suitable key and the bell or buzzer, by a sounder. 'Fig. 1A.

The sounder is an instrument which requires a considerable amount of energy to actuate it. Consequently, when a line exceeds a few miles in length it becomes economical to introduce for the reception of the signals, a more sensitive instrument than a sounder, namely, a relay, and to cause the sounder to be actuated locally as shown in Fig. 1B. The relay alone is unsuitable because the noise produced is not sufficiently loud to be heard with distinctness at all times. With such a system, telegraph signalling can be carried on over several hundred miles provided the insulation of the line is good. Under wet weather conditions, however, the strength of the operating current at the receiving end of the line is reduced and the relay must be re-adjusted.

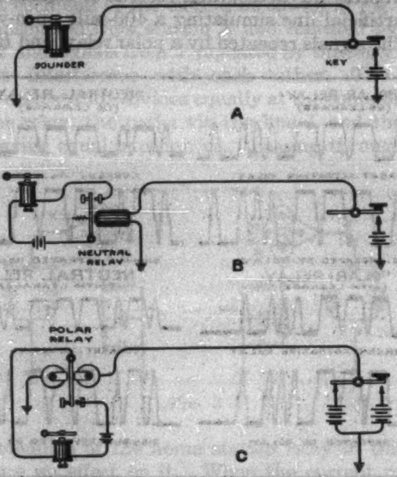


Fig. 1

If, instead of a relay of the simple "neutral" type, and a single-pole battery, there be used a polarized relay and a two-pole battery, Fig. 1C, the need for readjustment of the relay to meet variable insulation conditions is eliminated. The armature of the polar relay is acted upon in both directions. An impulse from the negative or "marking" battery causes the tongue of the polarized relay to make contact with its right hand or marking stop, whilst an impulse from the positive battery reverses its movement.

The positive or "spacing" battery takes the place of the retractile spring in the neutral or non-polar relay. As the current from both batteries is affected equally by variations in the line insulation, this is the equivalent of automatically varying the strength of the tension spring of a neutral relay to the exact degree of adjustment necessary.

Fig. 2 represents oscillograph records of signals transmitted over an artificial line simulating a 400-mile open-wire circuit. *A* shows the signals repeated by a polar relay, and *B* the signals repeated by a neutral relay when the insulation of the line was perfect. *C* and *D* show the repeated signals by the polar and neutral relays respectively, when the insulation resistance of the line is approximately one quarter of its conductor resistance. No readjustment of the relays was made.

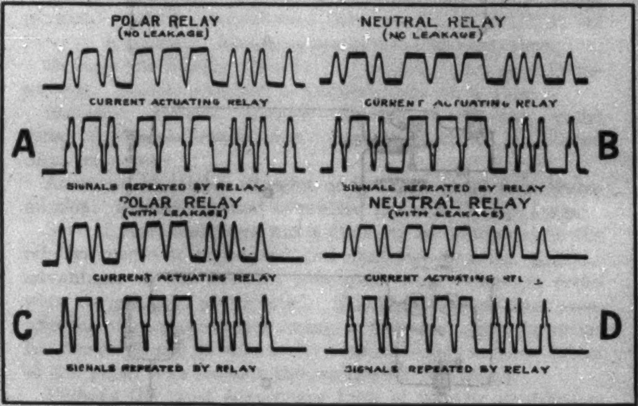


FIG. 2

It will be seen that the repeated signals in the case of the polar relay are the same under the two conditions, whilst in the case of the neutral relay reduced current has produced "pin-pointed" dots. The superiority of the double current or polar method is evident.

A simple circuit of this type, when operated by hand Morse, provides for the transmission of one message at a time at a speed of approximately 30 words per minute. The next advance is to increase the carrying capacity of the circuit by transmitting two messages simultaneously—one in each direction. This was achieved by the invention of the duplex system.

By providing a polar relay having two equal windings, and an assembly of resistances and condensers to simulate the real line, at each end of the circuit, as shown in Fig. 3, signals may be transmitted from station *X* and repeated by the relay at station *Y* and signals from *Y* repeated by the relay at station *X* without interference with each other. The outgoing currents at each station divides equally at the common terminal of the home relay, the paths via the line *L* and the artificial line *A L* having equal impedances. Passing through the two balanced windings of the home station relay in this manner, they produce no effect on it. When the current reaches the distant station, it passes through the line side winding of the relay *R* and then divides between the two paths to ground, one via the transmitting key and battery, the other through the artificial line *AL*. That part of the current through the artificial line winding of the relay passes in the direction to assist the current through the line winding. The relay, therefore, is actuated in accordance with the polarity and duration of the incoming signals.

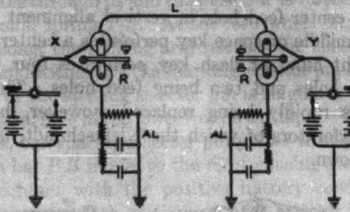


Fig. 3

Except in the case of the longest duplex lines, it is possible to transmit signals at a speed much in excess of the fastest hand operation. In order, therefore, that a line shall be made to carry an increased load, an automatic transmitter for sending and a high-speed tape-recording instrument for reception, are used. Wheatstone designed such instruments in 1858 and these have undergone improvements from time to time, until today, they are capable of operation at a speed of over 400 words per minute. That is to say, the limit of speed of the apparatus is over 400 words per minute. The complete Wheatstone system comprises a perforator, an automatic

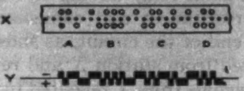


FIG. 4

transmitter and a tape recorder. The original type of perforator has three keys which are struck by small mallets held in the hand. The left hand or dot key perforates two signal holes and a center feed hole in vertical alignment across the tape. The middle or space key perforates a center feed hole, and the right hand or dash key perforates four holes, two being signal holes and two being feed holes. This type of perforator is rapidly being replaced, however, by modern keyboard perforators of which the Kleinschmidt, is probably the best known.

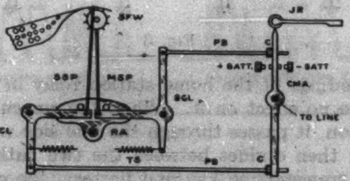


FIG. 5

Fig. 4x shows a piece of tape with the letters *A*, *B*, *C*, *D* perforated.

The automatic transmitter for operation in conjunction with this tape, (Fig. 5) has two vertical selecting pins, each pivoted to a bell crank lever *B C L*. The two bell crank levers control the movements of the contact-making arm *C M A* by means of the push bars, each of which has a collar or flange *C*. Each push bar extends beyond the collar, through a hole in the contact lever which acts as a guide or sliding support for that end of the push bar. When pushed beyond the dead center of the jockey roller, *J R*, the jockey roller ensures firm contact being made and also retains the contact lever in the position placed whilst one pushbar is being withdrawn and the other advanced.

The upward movement of the selecting pins *S S P* and *M S P* is controlled by the rocking arm *R A* and by the perforations in the tape. Power is transmitted to the rocking *R A* and the sprocket feed wheel *S F W* from an electric motor and these units of the driving mechanism are so geared that the marking selecting pin *M S P* rises so as to strike the tape exactly opposite a feed hole. When no signal hole occurs the selecting pin is not free to rise, consequently, the contact-making arm remains in the spacing position.

When a signal hole does occur the "marking" selector pin *M S P* passes through it, and the push bar is forced to the right by the tension spring *T S*. As the feed wheel rotates, moving the tape along, the rocking arm releases the spacing selecting pin *S S P* and withdraws the marking selector pin *M S P*. The upper end of the spacing selector pin is so guided as to engage with the signal hole in the lower row on the tape. When a signal hole occurs, the spacing selector pin passes through it and the lower push bar *P B* moves to the right causing the lever *C M A* to make contact with the positive battery contact point.

In the case of a dash or a space, both selector pins are prevented from rising once owing to the absence of signal holes. During these periods, the contact-making arm remains at rest.

During a dash it rests against the negative battery contact and during a space against the positive battery contact. The signals passing to the line are shown at *Y* in Fig. 4.

For a speed of 120 words per minute each selecting pin is freed to rise through the perforations in the paper tape at the rate of 54 times per second, the rest of the mechanism operating at a like rate. Obviously, the adjustments in such a machine must be extremely accurate.

This method of transmission is used in the Creed printing system which was designed to supplement the Wheatstone system and thereby to effect labor economies and reduction of delay at the receiving end, two features in which the Wheatstone system compares unfavorably with printing systems.

CREED SYSTEM

This system has been in commercial use in England and several other countries for the last 12 years. Actually it may be described as a development of the Wheatstone system. So far as the sending station is concerned there is no change from the apparatus and method of operation used in the Wheatstone. The messages are first handled by operators at keyboard perforators who prepare them in the form of perforated tapes. These operators are generally seated close to the transmitter, and as they complete each length of tape containing two or three messages, pass it forward to an operator who feeds the tape into the Wheatstone transmitter.

At the receiving station the incoming signals are made to actuate a "receiving perforator" which reproduces a perforated tape identical in all respects to that which is fed into the Wheatstone transmitter at the sending station. This perforated tape is then fed into an automatic printer which prints the messages in bold Roman characters on a paper tape.

The general practise is to have the receiving perforator and printer mounted conveniently near each other so that the perforated tape is fed into the printer with minumum delay.

As the printed tape comes from the printer it is fed through a small gumming device, or if gummed tape be used, over a moistening roller, thence to the gumming operator. The duty of this operator is to paste the tape neatly on the standard telegraph blanks. Where the speed is low, such as on circuits feeding or fed by a submarine cable this operator checks the messages as well, but on circuits operated at 100 words per minute the gumming operator is able to perform only the work of pasting; the checking being done by a second operator. This system is an improvement over the Wheatstone system in that it eliminates the laborious process of tape translation.

Receiving Perforator. Fig. 6 shows in diagrammatic form the construction of the Creed receiving perforator which reproduces a perforated tape identical with that fed into a Wheatstone transmitter at the distant end of the line.

A represents a polarized relay of the British Post Office type. No electrical contacts are used, and in place of the contact-making tongue, a light tongue *R T*, having a small slide valve *S V*, at its free end, is provided. Movements of this valve *S V* control a supply of compressed air to the small relay engine *R E*, causing its piston to move from side to side. The rod of this piston carries two circular disks which act as valves controlling the air inlets to a larger double-acting piston (Fig. 7). This larger piston has a rod projecting from

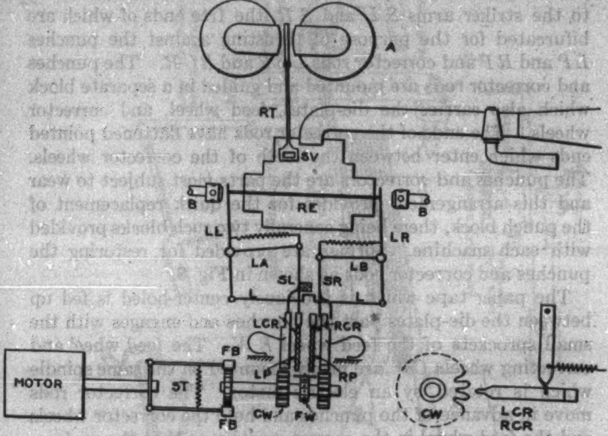

A
RT
SV
B
RE
B
LL
LR
LA
LB
SL
SR
L
L
LCR
RCR
FB
MOTOR
ST
LP
RP
FB
CW
FW
CW
LCR
RCR

FIG. 6

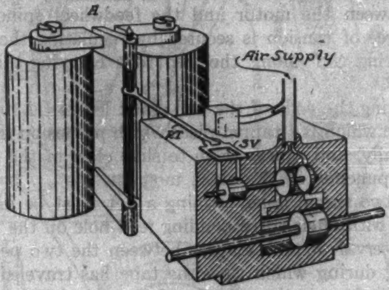

A
Air Supply
RT
SV

FIG. 7

each side of it, the rods projecting through the cylinder cover, and acting upon the bell crank levers *L L* and *L R*, (Fig. 6). The adjustable buffers *B* regulate the travel of the piston and piston valves. The arms *L A* and *L B* transmit the movement to the striker arms *S L* and *S R*, the free ends of which are bifurcated for the purpose of thrusting against the punches *L P* and *R P* and corrector rods *L C R* and *R C R*. The punches and corrector rods are mounted and guided in a separate block which also carries the die-plates, feed wheel, and corrector wheels. The ends of the corrector rods have flattened pointed ends which enter between the teeth of the corrector wheels. The punches and correctors are the parts most subject to wear and this arrangement provides for the quick replacement of the punch block, there being generally two such blocks provided with each machine. Springs are provided for restoring the punches and corrector rods as shown in Fig. 6.

The paper tape which is previously center-holed is fed up between the die-plates past the punches and engages with the small sprockets of the feed wheel *F W*. The feed wheel and correcting wheels *C W* are firmly mounted on the same spindle which is rotated by an electric motor. The corrector rods move in advance of the punches and hold the corrector wheels and the feed wheel in the correct position so that the perforations made by the punches shall be in correct vertical alignment with the feed holes. This ensures that the perforated tape will properly actuate the printer. As the tape is momentarily stopped at the instant of perforating, a friction drive is introduced between the motor and the feedwheel spindle. The exact degree of tension is secured by regulating the position of the spring *S T* along the two rods carrying the friction clocks *F B*.

Connecting the ends of the bell crank levers *L L* and *L R*, is a link *L* on which is a tappet piece which pushes on the striker bars laterally, and causes them to slide clear of the corrector rods and punches at the right moment.

As the two signal holes forming a dot must be in vertical alignment with their corresponding feed hole on the tape and a small interval of time elapses between the two perforating operations, during which time the tape has traveled a little, the right-hand correcting wheel and the right-hand punch are given a lead equal to one-half tooth of the corrector wheel.

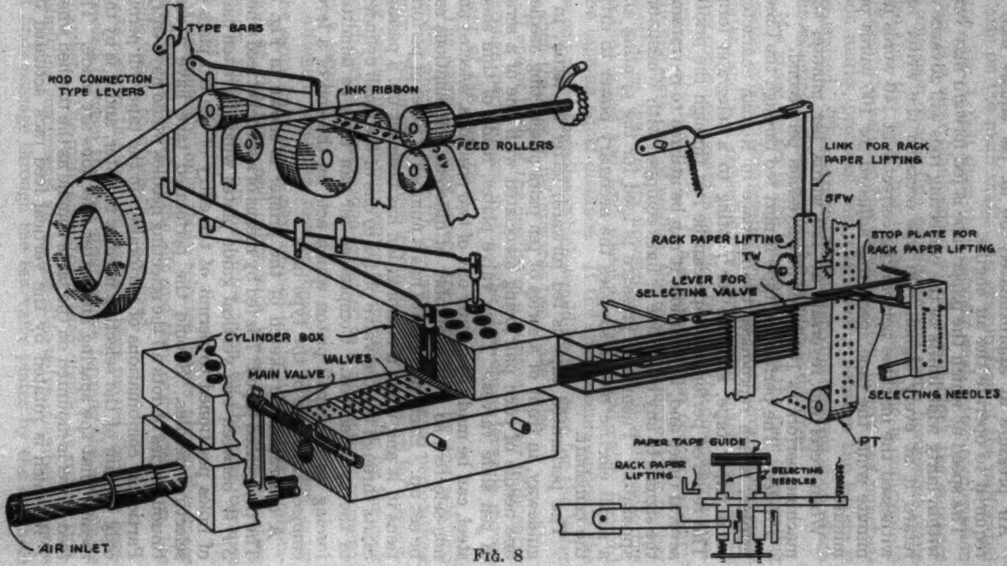

TYPE BARS
ROD CONNECTION TYPE LEVERS
INK RIBBON
FEED ROLLERS
LINK FOR RACK PAPER LIFTING
SFW
RACK PAPER LIFTING
TW
STOP PLATE FOR RACK PAPER LIFTING
LEVER FOR SELECTING VALVE
CYLINDER BOX
VALVES
MAIN VALVE
SELECTING NEEDLES
PT
PAPER TAPE GUIDE
RACK PAPER LIFTING
SELECTING NEEDLES
AIR INLET

FIG. 8

Air pressure at approximately 30 lb. per square inch is required to operate the air engine.

The speed at which the paper tape is fed forward may vary within fairly wide limits as the correcting rods will advance or retard it at every signal. On one occasion the writer has witnessed the machine perforating correctly at 140 words per minute while the tape speed was adjusted for 100 words per minute.

When receiving reversals at 120 words per minute, this machine perforates 108 holes in succession per second.

Printer. The perforated tape is next fed through the Creed printer, which reproduces the message in Roman characters on a paper tape, suitable for pasting to a telegraph form. The received perforated tape can be run through the printer several times and, in addition, it can be utilized to re-transmit the signals to any other line or lines equipped with Wheatstone apparatus. Pneumatic pressure is used for the printing and a small electric motor supplies the necessary power to drive the camshaft.

The perforated tape is fed forward, letter by letter, in a guideway, in front of a series of ten pairs of selecting needles, one needle of each pair being associated with one of a series of ten slide valves. These valve plates, made of thin sheet steel, can be made to occupy one of two positions, selected and non-selected, so that a number of different combinations can be set up. Each valve plate has a number of holes or perforations in it and when a combination has been set up, a passage exists through the ten valve plates to one of the small cylinders in the cylinder box. Compressed air can thus be admitted forcing up the small piston which acts on the end of a lever connected to one of the type bars.

A diagrammatic sketch of the printer is shown in Fig. 8. The perforated tape *P T* is fed by means of a sprocket feed wheel *S F W* between two vertical guide plates which have ten pairs of holes positioned to permit of the entry of the ten pairs of selecting needles. Should the perforated tape have a continuous series of dots all the selecting needles would have entry. Particular combinations of perforation in the tape will permit only certain of the selecting needles to enter, and it is by this means, that the spacing and letter selections are made. The feed wheel is mounted on the same spindle as the toothed wheel *T W* which is rotated by the vertical feed rack actuated by a cam on the main shaft. An examination of the perforated tape, Fig. 4, will show that between each letter combination there is a clear blank space which would block the entrance of one pair of selecting needles in the same horizontal plane also that the lower row of signal holes alone is sufficient to differentiate between the various letters, figures and punctuation marks, provided that the spacing between the letters is recognized. These are the two underlying features upon which the selecting mechanism of the printer is designed.

If a space occurs in the tape neither of the pair of selecting needles opposite this position will find entry through the guide plates, and will consequently permit the space lever which they control to remain in position to block the downward movement of the feed rack. If a space position occurs opposite, say, the sixth pair of needles, the rack will move down to that position so that on its upward movement it will cause the toothed wheel and feed wheel to rotate and carry forward the tape an equal distance. If the space occurs opposite the second pair of needles, which would be the case for the letter *e*, then the tape is fed upwards two space holes.

If one or both of the needles of a horizontal pair finds entry through perforations in the tape, the space lever is moved clear of the feed rack.

The valve selecting levers are controlled by the selecting needles which engage with the lower row of signal perforations only. When one of these levers is pushed forward by a selecting needle, it is in position to be moved laterally by the rack which has a lateral motion imparted to it at the right moment, by a cam.

Although selecting needles below a space position enter the signal holes of the next or part of the next letter combination on the tape, and carry with them the valve selecting levers, these levers are not acted upon by the rack because it does not descend so far.

Consequently, only those selected levers above the space position are moved to the left and they carry with them the slide valves to which they are fixed.

As soon as the slide valves have been moved they are compressed between the two stout plates forming the slide-valve chamber. The main air valve is then opened by cam action, admitting compressed air to the slide-valve chamber, whence it finds passage through the ten valve plates by the one hole made available by the particular valve setting, and forces up the small piston which actuates the corresponding type lever.

The slide-valve chamber is then opened, and the valves are returned to normal position. Meanwhile the paper-lifting rack is drawn up by cam action and in its upward moment it engages the toothed wheel *T W* so that the paper tape is drawn up the proper distance for the selection of the next letter. The path of movement of the rack forms a narrow rectangle, the length of which is determined by the length of space occupied on the tape by the letter combination printed.

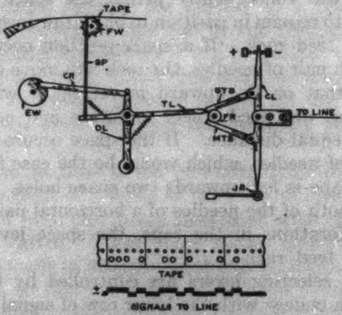


Fig. 9

It will be seen that the differential feed of the tape involves a selecting mechanism which is not required in the case of systems using an equal unit code, which are described later.

This printer has been operated in laboratory tests up to 130 words per minute, but for commercial use it is found best to work it at a speed not exceeding 100 words per minute.

The air pressure required is the same as for the perforator.

MURRAY AUTOMATIC SYSTEM

The Murray Automatic System is not widely used, but where it has been installed it has rendered good service.

At the transmitting station the method of operation closely resembles the Creed. A number of operators perpare the messages on paper tapes by means of keyboard perforators. This tape has feed holes identical with those on the Wheatstone tape, but with signal holes in alignment with the feed holes, on the lower side only. The perforated tapes are passed to the transmitter operator who feeds them into the transmitter.

At the receiving station the messages are received on a receiving perforator which produces a perforated tape identical with that fed into the transmitter at the sending station. This tape is then passed into an automatic printer which prints the messages in page form ready for delivery. In addition to the advantage of the shorter code used this system eliminates the gumming process as used in the Creed.

Fig. 9 shows the appearance of Murray automatic tape. In this case, every letter and symbol occupy the same length of tape (one-half inch) so that a differential tape feed feature is eliminated in the perforator, the tape being fed forward through the perforator letter by letter. Only five punches are needed, all combinations being formed by various groupings of these.

The vertical marks on the tape after every fifth hole are for the purpose of enabling the operators to read the signals.

The transmitter is driven by a phonic wheel motor which is controlled by a vibrating reed. The unloaded reed is designed to vibrate at a certain speed and by loading it with weights, which can be fixed at any position along the reed, the speed can be reduced. The speed of the transmitter can be adjusted by this means over a considerable range.

In the Murray automatic transmitter (Fig. 9) only one selecting pin *S P* is needed. The tape feed wheel *F W* is coupled direct to the phonic wheel motor whilst the eccentric wheel *E W* is geared to rotate at ten times the speed of the feed wheel, that is, once for each sprocket of the feed wheel. For each rotation of the eccentric wheel *E W*, the oscillating lever *O L* makes one complete movement, carrying with it the thrust lever *T L*. If there be no signal hole in the tape, the selecting pin *S P* does not rise so that the thrust lever *T L* strikes the upper or spacing thrust bar *S T B* and carries it past the dead center point of the jockey roller *J R*. The contact lever *C L*, therefore, makes contact with the right-hand battery contact point.

At the next step of the feed wheel *F W* the eccentric wheel *E W* makes another complete revolution and the thrust lever *T L* again moves to the right. If no signal hole exists movement of the thrust lever *T L* will be identical with its previous movement. The contact lever, however, remains in the position set by the previous thrust, due to the jockey roller *J R*.

If the next step of the feed wheel brings a signal hole in line with the selecting pin *S P* the thrust lever *T L* will be tilted downward and will strike the lower or marking thrust bar *M T B* and cause the contact lever to make contact with the other pole of the battery.

With this transmitter operating at a speed of 120 words per minute the pin selecting mechanism is required to operate at the rate of 60 times per second, that is, the eccentric wheel *E W* makes 60 revolutions and the thrust lever oscillates 60 times per second.

Murray Automatic Receiving Perforator. The circuit arrangements of the Murray receiving perforator are shown in Fig. 10.

The vibrating reed *V R* is provided with a weight *W* fixed at some point along the reed so that the rate of vibration of the reed will be slightly faster than twice the frequency of the incoming signals. The reed is kept in constant vibration by means of the local battery *L B*, the circuit being via the contact *V*, the vibrating magnet and the back or front contact of the governing relay. The amplitude of vibration is limited by the buffer springs *B S*. If the amount of energy acting upon the reed be increased, the rate of vibration increases. Conversely, if the amount of energy be reduced, the rate of vibration is reduced. The punching and governing relays are connected in series from the *M* or marking contact of the line relay. The punching magnet is operated when the tongue of the punching relay is pulled up and the contact *P* on the reed is closed. Consequently, if the tongue of the line relay remains in the marking position the punching magnet will be operated each time the vibration of the reed closes the contact *P*. The spacing magnet is operated, when the *S* contact of the reed is closed. If no signals are passing the spacing magnet continues to feed forward the tape, but after about a foot of blank tape has passed an automatic switch *A S*—a small mechanical device—opens the circuit of the spacing magnet *S M* and the tape is stopped. The first marking signal closes this switch and it remains closed so long as signals are passing. The actuation of the spacing magnet operates the escapement mechanism on the motor drive shaft *M D S*. It will be obvious that the punching and spacing magnets

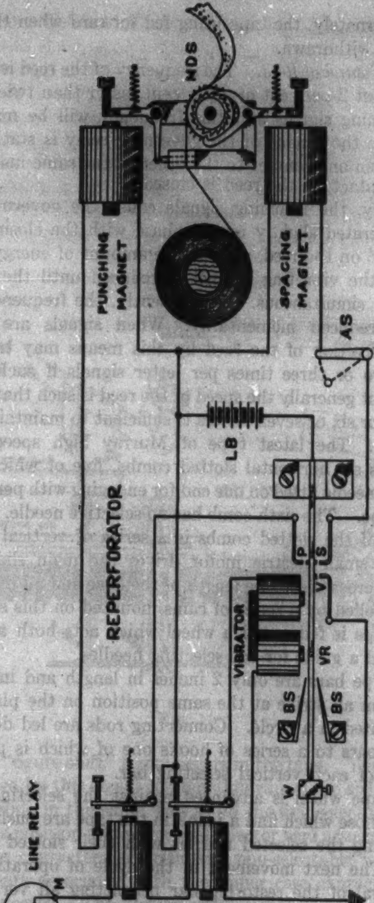

NDS
PUNCHING MAGNET
SPACING MAGNET
AS
LB
REPERFORATOR
P
S
V
VIBRATOR
VR
BS
BS
W
LINE RELAY
M
PUNCHING RELAY
GOVERNING RELAY

FIG. 10

work alternately, the tape being fed forward when the punch has been withdrawn.

Speed Control of Reed. The frequency of the reed is adjusted to be about 2 per cent or 3 per cent faster than twice that of the incoming signals and this frequency will be maintained so long as the tongue of the governing relay is stationary or moves from one contact to the other at the same instant that the *V* contact on the reed is closed.

Actually, the incoming signals cause the governing relay to be operated slightly out of phase with the closing of the contact *V* on the reed, so that the amount of energy passing through the vibrating magnet is reduced until the contacts are again simultaneous. Consequently, the frequency of the reed is reduced momentarily. When signals are passing, speed correction of the reed by this means may take place about two or three times per letter signals if such be necessary, but generally the speed of the reed is such that one correction per six or seven letters is sufficient to maintain unison.

Printer. The latest type of Murray high speed printer comprises six horizontal slotted combs, five of which have a selecting needle fitted on one end for engaging with perforations in the tape. The sixth comb has no selective needle. Mounted in front of the slotted combs is a series of vertical selecting bars. A small electric motor drives the main shaft which extends across the whole length of the base and all movements are controlled by a series of cams mounted on this shaft.

The tape is fed round a wheel which acts both as a feed-wheel and a guide for the selecting needles.

The type bars are only 2 inches in length and in order to have them all strike at the same position on the platen they are mounted in a circle. Connecting rods are led down from the typebars to a series of hooks one of which is positioned in front of each vertical selecting bar.

The tape wheel is advanced against the selecting needles so that those which find a blank in the tape are pushed to the left leaving the selected needles with their slotted combs in place. The next movement in the cycle of operation is the withdrawal of the restoring bar permitting all vertical bars to move forward, but only one vertical bar finds a free path to move, the others being stopped by the blank portions of the slotted combs. The bar so selected, pushes forward the hook immediately in front of it, placing it in the path of the striker bar. As soon as the striker bar comes down, and engages with the hook the restoring bar pushes back the vertical selecting bar so that the setting up of the next letter combination may be proceeded with. This overlapping of the cycle of operation is a distinct advantage permitting a slower operation of moving parts and consequently less wear and tear.

With five slotted combs only 31 combinations can be made, but by adding a sixth comb, which can occupy one of two positions, the number of selections can be doubled. Only 56 selections, however, are required. The sixth bar is moved when the "figure shift" signal occurs in the tape. The vertical bar selected by this signal combination causes the sixth slotted comb to slide in the same way as do the five selecting combs and in this position all the vertical selector bars associated with lower case characters are rendered non-selectable whilst the vertical bars associated with the upper case characters may now be selected. The "letter shift" signal restores the sixth slotted comb to normal.

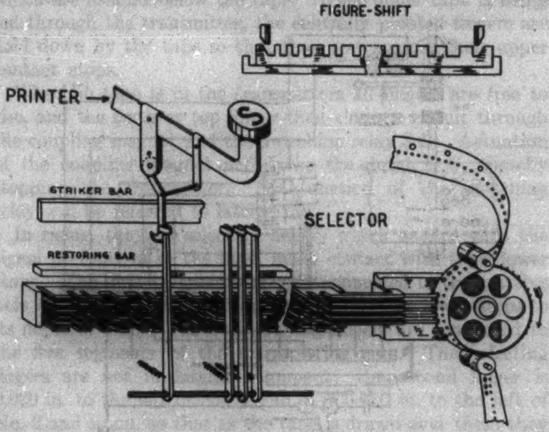


Fig. 11

Fig. 11 shows the selecting mechanism of the old Murray automatic printer, which does not differ fundamentally from that of the new high-speed printer. In the new machine, however, typewriter keys are not provided.

SIEMENS & HALSKE HIGH SPEED SYSTEM

The Siemens and Halske is another type of high-speed system. At the sending end the operations are practically identical with those of the Creed and Murray Automatic in having several perforating operators and one transmitting operator. At the receiving station the perforated tape is eliminated, the messages being printed on a tape direct from the line signals. This tape is pasted on the message blanks.

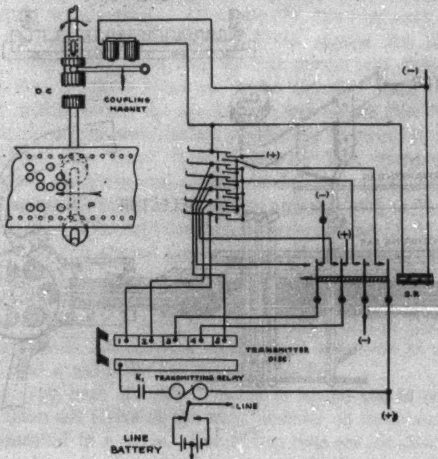


FIG. 12

This system shows a further development in that the intermediate process of perforating a tape at the receiving station is eliminated, but the gumming process is continued.

Fig. 12 shows the high-speed transmitter, used in the Siemens and Halske printer system. The tape in this system is perforated on a keyboard perforator. The five-unit code is used, so that a maximum of five perforations is required per letter combination. The tape, however, differs from the Murray Automatic tape in this respect, the signal perforations are made across the tape instead of along it, so that a space of only 0.1 in. per letter is required, a wider tape being used.

With this form of perforated tape the transmitting mechanism must include means for securing the proper sequence of signals. This is achieved by using a rotating distributor which consists of two metal rings over which contact brushes pass at the rate of one revolution per letter. One ring is solid, the other is divided into five equal segments.

Fig. 12 shows the general arrangement of the transmitting circuit. The perforated tape has two rows of feed holes for ensuring its accurate movement over the six contact fingers which are located below the tape. When blank tape is being fed through the transmitter, the centrally pivoted fingers are held down by the tape so that they rest against their upper contact stops.

When no tape is in the transmitters all fingers are free to rise, and the sixth or top finger then closes a circuit through the coupling magnet and the switching relay *S R*. Actuation of the coupling magnet withdraws the clutch *D C*, thereby stopping the transmitter. The function of the switching relay will be referred to later.

In rising, the five selecting fingers which engage with the signal perforation in the tape, make contact with their lower stops. The upper contact stops are connected to the positive pole of the local transmitting battery and the lower stops to its negative pole. The five selecting fingers are connected to the five segments of the transmitting ring. The selecting fingers are not in lateral alignment; the second finger is 0.020 in. to the left of No. 1, No. 3 is 0.020 in. to the left of No. 2 and so on, so that as the tape is drawn over them they engage with the perforations in sequence and No. 1 engages immediately after No. 5 has engaged with a perforation of the preceding lateral group of perforations. The distributor brushes are so geared that at the moment finger No. 1 engages with a perforation the brushes start to sweep over segment 1; when finger No. 2 engages with a perforation the brushes start to sweep over segment 2 and so on. The solid ring which is in connection with the segmented ring through the rotating brushes is connected through a condenser to the transmitting relay. Consequently, when a perforated tape is inserted and the selecting fingers are set up in different combinations, positive and negative impulses will pass through the transmitting relay and actuate it.

When no tape is in the transmitter the transmitting relay would remain at rest, sending out either a steady positive or negative current. As the system is a synchronous one and the maintenance of synchronism between the two ends of the line is dependent upon the reception of signals, provision is made for sending a signal once per revolution in the following manner:

The switching relay is actuated when there is no tape in the transmitter, as has already been described. The actuation of this switch removes the normal battery connection to the selecting fingers and connects negative battery direct to segment 3 and positive battery to segment 4 of the distributor.

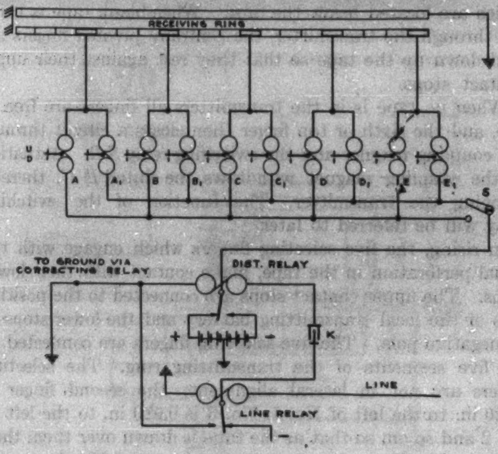


FIG. 13

Consequently, the transmitting relay will send out a reversed pulse each time the brush sweeps over segment 3.

Receiving. The receiving apparatus of the Siemens and Halske system comprises a distributor with rotating brushes which make one revolution per letter.

One noteworthy feature of the system is the use of storing relays which provides an overlap arrangement.

Fig. 13 shows the line and local relays with the receiving ring of the distributor and the storing relays. The storing relays are polarized and are arranged in pairs as shown. The switch *S* is reversed mechanically at each revolution so that relays *a, b, c, d,* and *e* are connected in sequence to the tongue of the distributor relay for one revolution and relays *a′, b′, c′ d′* and *e′* for the next.

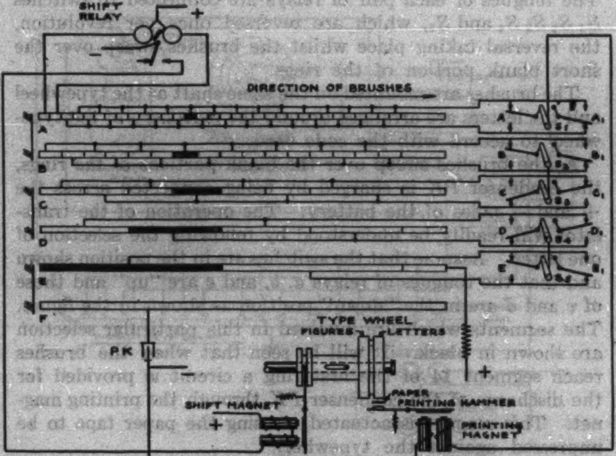

SHIFT RELAY
+
−
DIRECTION OF BRUSHES
A
B
C
D
E
F
A
A1
S1
B
B1
S2
C
C1
S3
D
D1
S4
E
E1
S5
TYPE WHEEL
FIGURES
LETTERS
P.K
−
+
PAPER
PRINTING HAMMER
SHIFT MAGNET
+
PRINTING MAGNET

Fig. 14

The distributor relay is actuated from the tongue of the line relay through the condenser *K* and follows the movements of the line relay. During the reception of signals forming one letter combination the brush makes one complete revolution. Assuming the armature of the relays move to the "up" or *U* position when a positive impulse passes, and to the "down" or *D* position when a negative impulse passes, then a letter combination, say + + − − +, would cause the armature of relays *a b* and *e* to move to the up position and the tongues of relays *c* and *d* to the down position. Being polar relays the tongues remain in the position set until reversed by an impulse of opposite polarity.

At the next revolution the next combination would be set up on relays, a', b', c', d' and e'. Meanwhile, the set-up on the relays a, b, c, d, and e is used to effect the selection and printing of the previous letter.

Fig. 14 shows the translator portion of the distributor. It comprises six segmented rings. These are connected to the contacts of the storing relays shown in the preceding figure. The tongues of each pair of relays are connected to switches S_1 S_2 S_3 S_4 and S_5, which are reversed once per revolution, the reversal taking place whilst the brushes sweep over the short blank portion of the rings.

The brushes are mounted on the same shaft as the typewheel and the letters are arranged around the periphery of the typewheel to accord with the code used.

As the brushes sweep over the blank portions of the rings, the condenser *PK* is charged by being connected across the + and − poles of the battery. The operation of the translator will readily be understood by following the selection of one letter. Assume that the switches are in the position shown and that the tongues of relays a, b, and e are "up" and those of c and d are in the "down" position as shown in the figure. The segments which are involved in this particular selection are shown in black. It will be seen that when the brushes reach segment 14 of the first ring a circuit is provided for the discharge of the condenser *PK* through the printing magnet. This magnet is actuated, causing the paper tape to be impressed against the typewheel.

The typewheel carries two series of characters, one being letters, the other figures and the usual punctuation marks. In the normal position the letter series is in position for printing and by sliding the wheel along its shaft the figure series is brought into the printing position.

The positioning of the typewheel for printing letters or figures is effected by a shift magnet which is controlled by a differentially wound polar relay. Two of the 32 segments of the first ring are connected to the windings of this polar relay so that the position of its tongue is determined by the "figure shift" and "letter shift" signals which are transmitted over the line.

Maintenance of Synchronism. Between each of the five short receiving segments of the receiving ring, there are three correcting segments as shown in Fig. 15. As the signals are being received by the line relay, the brush which sweeps over this ring should be on one of the *C* segments, at the moments when the line relay tongue moves from one contact to the other. Under this condition, the center portion of each signal impulse is utilized for the actuation of the storing relays. If the brush is on an *R* correcting segment, it is in advance of its correct position and must be retarded, if on an *A* segment it means that the motor has been running slow and must be accelerated.

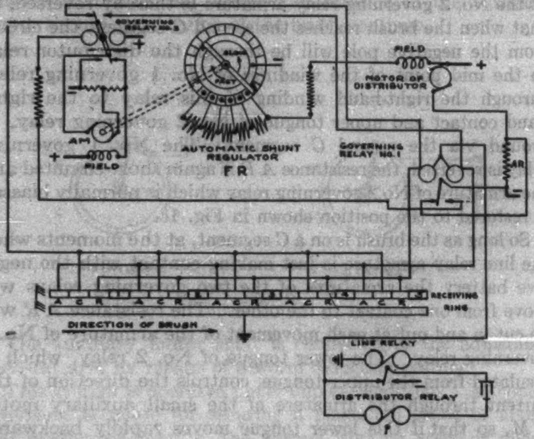

GOVERNING RELAY No. 2
+
−
SLOW
FAST
−
FIELD
+
MOTOR OF DISTRIBUTOR
AM
+
FIELD
AUTOMATIC SHUNT REGULATOR
F R
GOVERNING RELAY No. 1
AR
−
1
2
3
4
5
A C R A C R A C R A C R A C R
RECEIVING RING
DIRECTION OF BRUSH
−
LINE RELAY
DISTRIBUTOR RELAY

Fig. 15

The speed of the motor which carries the brushes is governed by a resistance *A R* in series with its armature and an automatically adjusted resistance *F R* in series with its field winding. If the motors at the two stations are running at approximately the same speed, the intermittent shunting of the resistance *AR* is sufficient to maintain synchronism. The field resistance *F R* is used to control wider speed variations.

When the two motors are running in synchronism the operation of the correcting circuit is as follows:

Assuming the armature of the line relay just makes contact with its right-hand contact when the brush reaches the first *C* segment of the receiving ring a negative current impulse will pass through the distributor relay to the mid point of the windings of the No. 1 governing relay, through the left-hand winding of this relay to the left-hand contact and upper tongue of the No. 2 governing relay via contact *C* to ground. The armature of No. 1 governing relay will be moved to the right, thereby closing a circuit from the negative pole of the battery through the No. 2 governing relay and also inserting the resistance *A R* in series with the armature winding. The position of the No. 2 governing relay armature is thereby reversed, so that when the brush reaches the second *C* segment the circuit from the negative pole will be through the distributor relay to the mid point of the windings of No. 1 governing relay, through the right-hand winding of this relay to the right-hand contact and upper tongue of No. 2 governing relay, to ground via the second *C* segment. The No. 1 governing relay is reversed, the resistance *A R* is again short-circuited and the armature of No.2 governing relay which is normally biassed is restored to the position shown in Fig. 15.

So long as the brush is on a *C* segment, at the moments when the line relay armature is just making contact with the negative battery the armatures of the two governing relays will move from one contact to the other. The resistance *A R* will be cut in and out at each movement of the armature of No. 1 governing relay. The lower tongue of No. 2 relay, which is insulated from the upper tongue, controls the direction of the current through the armature of the small auxiliary motor *A M*, so that if this lower tongue moves rapidly backwards and forwards the armature of the motor merely oscillates. If, however, the power connections to this motor remain undisturbed for a few seconds, the armature commences to rotate and cause a change in the resistance *F R* which is in series with the field of the larger motor.

Should the brush be on an *R* segment, the current impulse through the condenser and the distributor relay will be via the left-hand winding of the No. 1 governing relay to the *R* segment, across the brush to ground. The tongue of the No. 1 governing relay will move to the right, opening the shunt across the resistance *A R* and thereby producing a slight reduction in the speed of the motor.

If the brush is on an *R* segment, when the next correcting impulse is generated the two governing relays will remain undisturbed. Consequently, the resistance *A R* remains in circuit. Should this condition continue for several successive revolutions of the brush, the auxiliary motor will have time to rotate and to aid in retarding the speed of the motor by cutting out resistance in series with the field.

In the event of the brush being on *A* segments, when correcting impulses are generated, the No. 1 governing relay armature will move to the left-hand contact and shunt the resistance *A R*, whilst the position of the No. 2 governing relay will reverse the direction of rotation of the auxiliary motor should the lagging of the brush continue.

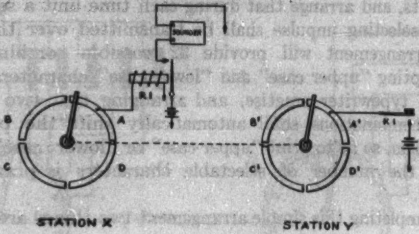


Fig. 16

The auxiliary motor shaft is connected to the adjustable rheostat arm by means of a worm reduction gearing.

This system is operated under duplex conditions. Although provision is made for securing a maximum speed of about 160 words per minute, the normal working speed is about 120 words per minute in each direction.

MULTIPLEX SYSTEM

Multiplex printing telegraph systems are all based on the Baudot system which was invented in 1875 by Emile Baudot, a Frenchman.

The principle of the multiplex system is illustrated by Fig. 16.

If two metal rings be each divided into four segments of uniform length, and contact brushes, connected by a wire, arranged to rotate in synchronism, then segment *A* at *X* will be in electrical connection with segment A^1 at station *Y*, during one quarter of each revolution of the brushes; segment *B* will be connected to B^1 for a like interval of time and so on.

Communication by automatic Morse could be carried on, but it is obvious that the speed of rotation would have to be such as to permit of the longest Morse signal combination being transmitted from each segment. There would consequently be considerable loss of time when sending the short length signal combinations. Transmission of one word per segment would be equally unsatisfactory as words vary in length. Obviously, if there is to be no lost time, each signal combination representing a letter or symbol should occupy the same length of time in transmission. In order to obtain sufficient signal combinations to represent the letters of the alphabet, it is necessary to divide this time interval into five units, and arrange that during each time unit a selecting or non-selecting impulse shall be transmitted over the line. This arrangement will provide 32 possible combinations. By adopting "upper case" and "lower case" characters, as is done in typewriter practise, and arranging that two of the signal combinations shall automatically shift the printing mechanism, so as to print "upper case" or "lower case" characters, the number of selectable characters is practically doubled.

In completing this simple arrangement two things are essential:

(1) Means for keeping the two rotating brushes in synchronism.

(2) Means to change the keyboard set-up so as to send a fresh letter combination at each revolution.

Synchronism may be maintained in the following manner: Assuming 20 segments are to be used for actual signalling, two extra segments are provided at each end. In Fig. 17, these extra segments are numbered 21 and 22. Segment 21 at the sending station is connected to a battery. Segment 22 at the receiving station is connected to a relay.

The brushes are driven by electric motors, the speeds of which are so adjusted that the brush at station *X* rotates slightly faster than that at station *Y*, say about ¾ deg. or 1 deg. per revolution. When the brushes are started off in synchronism, they will sweep over like numbered segments at the same moment. The brush at station *X*, however, will gradually gain so that whilst the *Y* brush is still on segment 21, brush at *X* will be on segment 22. When this condition exists, an impulse will flow from the correcting battery at *Y* connected to segment 21 through the correcting relay at *X*. If it be arranged that the actuation of this relay shall slow down momentarily the speed of the motor at *X* or else by mechanical means retard the brush, the two brushes will again be in synchronism. By this arrangement the duration of the braking effect can be made proportional to the amount of retarding required.

With the arrangement of segmented rings, as shown in Fig. 17, it will be obvious that, as the brush at the corrected station must be slightly out of phase with the incoming correcting impulses, before correction can take place, it will be out of phase with the incoming signal impulses. Consequently, an impulse which should flow say, via segment 17 to the selecting relay connected thereto, may overlap and flow via segment 18 to another selecting relay. The overlapping portion might be sufficient to actuate this selecting relay and thus produce defective selection. This is obviated by using small segments for the receiving ring so that only the middle portion of each signal impulse is utilized to actuate the selecting relays. Fig. 18 shows receiving ring segments of half the length of the sending segments; *a* represents signals in synchronism in which case the middle half of each signal impulse is picked out for actuation of the selecting relays, *b* represents the case where the brushes are ahead of their correct position, and *c* represents the reverse condition. It will be seen that only approximate synchronism is necessary to ensure that the impulses shall be switched to the correct selecting relays.

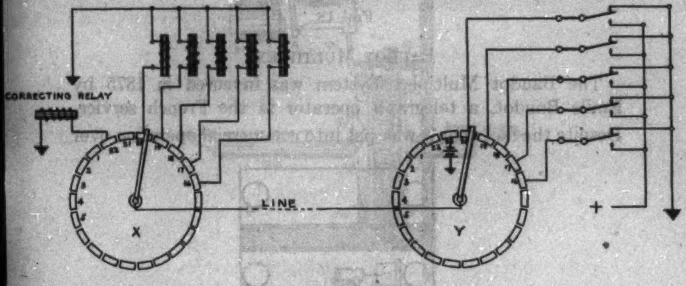


Fig. 17

This feature of utilizing only the middle portion of each signal impulse is advantageous also in permitting the system to be operated with signals which are considerably distorted. Such distortion may be caused by the electrical characteristics of the line, or by extraneous interference such as induction from other circuits.

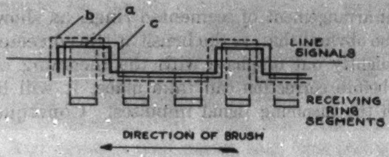


Fig. 18

BAUDOT MULTIPLEX

The Baudot Multiplex System was invented in 1875 by Emile Baudot, a telegraph operator in the French service. Despite the fact that it was put into commercial operation over 40 years ago it held the field against all competitors until recent years.

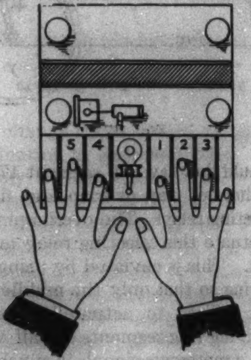


Fig. 19

Next to the conception of the multiplex principle as embodied in the Baudot, perhaps the most important development was the doubling of its carrying capacity by adapting it for duplex operation, which was successfully accomplished in 1910 by Mr. A. C. Booth of the British Service.

Other systems, closely resembling the duplex Baudot, and incorporating features borrowed from the typewriter have since been developed, and these are likely to supplant the Baudot in the future. At the present time a large precentage of the telegraph traffic in France is carried by the Baudot system.

Transmitter. Transmission on the Baudot system is effected by means of small keyboards each having five keys similar to piano keys, four such keyboards (Fig. 19) being connected to a four channel or quadruple set. As the keys are depressed they are automatically locked by small hooks and remain in the depressed position until the brush has swept over the five segments of the sending ring connected to the particular transmitter. An electromagnet mounted in the keyboard is then energized by an impulse through a local ring on the distributor, and releases the hooks, at the same time producing an audible click in the transmitter. This gives a signal,

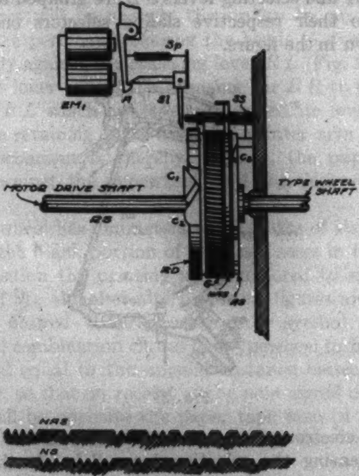


Fig. 20

termed the Cadence signal, to the operator who immediately depresses one or more of the keys setting up the next letter combination.

Printer. There are several types of Baudot printer, all of which are entirely mechanical except the initial actuation of the selecting magnets. In this paper it is proposed to describe briefly the salient features of the "Carpentier" type.

In Fig. 20 $E\ M_1$ represents one of the five selecting magnets connected to the receiving segments on the distributor. These five selecting magnets are mounted close together so that their armatures A and selecting levers $S\ L$ are grouped together and adjacent to their respective sliding selectors one of which $S\ S$ is shown in the figure.

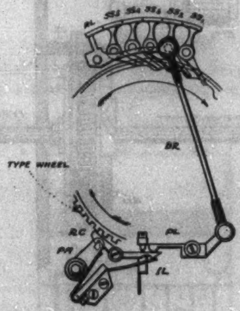


Fig. 21

When an electromagnet is actuated its armature A is pulled forward releasing the selecting lever $S\ L$. As the rotating disk $R\ D$ moves round a selecting lever which has been selected is caught by the cam C_1, and thereby is caused to press against its corresponding sliding selector $S\ S$ forcing the toe piece of the sliding selector into the channel marker $A\ S$. A selector lever which has not been actuated remains on the left side of the cam C_1, as it passes, so that its sliding selector remains with its toe piece in the channel marker $N\ A\ S$.

Immediately a selecting lever has performed its function of causing the sliding selector to be moved it is restored to normal by the cam C^2 and its upper arm is caught by the armature A.

The rotating shaft driven by an electric motor carries the disk *R D*, the gear *G*, the disks *N A S* and *A S* and the typewheel. The disks *N A S* and *A S* are notched as shown in Fig. 20. A front view of the sliding selectors is shown in Fig. 21. Normally these sliding selectors rest with their heads abutting against each other, irrespective of whether they have been selected or not. As the disks rotate notches will pass under their toe pieces, but no movement will take place until a notch is under each toe piece. All toe pieces will then drop into the notches simultaneously, causing the heads to move together towards the right. As the disks continue to rotate the toe pieces will be sharply thrown up by an unnotched portion of the periphery, causing the heads to overshoot and strike sharply against the releasing lever *R L* (Fig. 21). Actuation of this lever causes the releasing bar *B R* to pull up the pedal lever *P L* which strikes the intermediate lever *I L*, and removes the retaining pawl from the printer arm *P A*. The printing mechanism is thereby released, the paper tape is impressed against the typewheel, and the desired character printed.

The typewheel has characters on 31/40ths of its periphery, and whilst the blank portion of 9/40ths passes in front of the printing position the printing arc is restored to normal.

Letters of the alphabet alternate with figures and symbols. When it is desired to print a figure or symbol the "figure shift" signal combination causes the typewheel to be displaced from normal equal to the angular distance between a figure and a letter, so that on receipt of the next signal combination a figure shall be opposite the paper tape instead of a letter. The typewheel is restored to normal by the "letter shift" signal combination.

The speed of rotation of the typewheel must correspond approximately to that of the distributor brushes, and provision for maintaining approximate synchronism between these instruments must be made.

The Baudot system is generally operated at 180 revolutions of the distributor per minute so that an operator sets up letter combinations at the rate of three per second. Assuming six letters per word, this is equal to a speed of 30 words per minute per transmitter or 120 words per minute for the four transmitters. The system may be operated duplex, giving a total carrying capacity of 240 words per minute. One circuit has been in operation for several years between London and Birmingham, giving six channels in each direction—a total carrying capacity of 360 words per minute.

Synchronism. In the Baudot system the brush holder is connected to the motor shaft through gears, and a mechanical device actuated by the correcting relay causes one of the intermediate gears to be rotated one tooth which has the effect of stepping back the brushes with respect to the driving shaft.

MULTIPLEX WITH TAPE TRANSMISSION AND PAGE PRINTING

There are two multiplex systems of this type—the Murray and that developed in the research laboratories of the Bell System in collaboration with the engineers of the Western Union Telegraph Company. The Murray system was the forerunner in showing the advantages to be gained by the use of keyboard perforators instead of direct transmission, as in the Baudot system, and in the use of a page printer, which turns out finished messages ready for delivery, thereby eliminating the gumming process.

A two-arm Murray multiplex installation has been in commercial use in England for about ten years. The war, however, prevented the manufacture of further installations; otherwise, it is probable that complete quadruple duplex installations of this system would have been in service by this time.

The multiplex system designed in this country resembles the Murray as regards the use of perforators, page printers, and rotating distributors, but differs from it in many important details. What will be said in this paper concerning the operation advantages of the American multiplex applies generally to the Murray multiplex.

The American multiplex system, like the Murray system, is based fundamentally on the duplex Baudot system, and was developed primarily to meet the needs of the Western Union Telegraph Company. The further development of the American system, providing greater flexibility by means of forking and extending the individual arm or channel, has been developed in the research and development laboratories of the Bell System.

This system differs from the Baudot system in the following respects:

Messages are first prepared on keyboard perforators, instead of being transmitted direct from five key transmitters. No special signals are transmitted over the line for the purpose of maintaining synchronism. Messages are printed in page form instead of on tape, thereby eliminating the gumming operation. For a complete quadruple duplex equipment, that is, one which provides for the simultaneous transmission of eight messages—four in each direction—there is required at each end of the line:

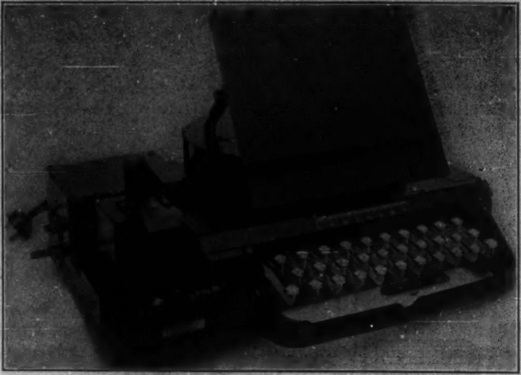


FIG. 22

Distributor with driving fork and impulse motor

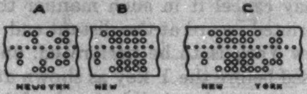


FIG. 23

Four transmitters
Four keyboard perforators
Four automatic control units
Four printers
Line and local relays, and duplex balancing equipment.

Messages are first prepared as perforations on a tape, by means of a keyboard perforator as shown in Fig. 22. The perforations for each character are made across the tape and occupy a space of 0.1 in. per letter character. Hence a 30-word message occupies a length of only 18 inches of tape.

Perforator. The perforator comprises three rows of keys arranged according to standard typewriter practise, which control the selection of the punches. The depression of a key, in addition to effecting the selection of a group of punches, causes a circuit to be closed through an electromagnet whose armature drives the selector punches through the paper tape.

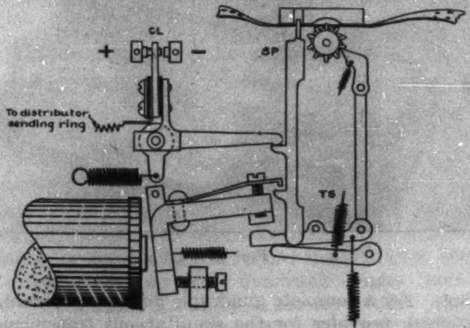
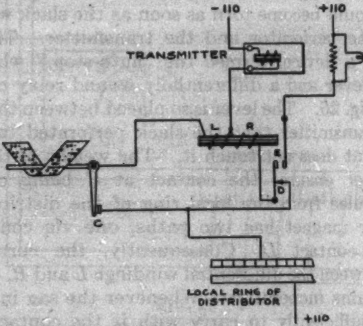


Fig. 24

Provision is made for back spacing the tape, so that in the event of an error having been made, and observed by the operator, she may cancel it in such manner that it does not appear on the printed record at the distant end. For instance, if the word New York is to be perforated, and in error, the *O* key is depressed before the *Y* key as shown at *A* Fig. 23, the tape would be back spaced and all letters back to and including the *Y* would be cancelled as shown at *B*. Then the word would be continued commencing at the *Y* as shown at *C*. This feature is referred to as the "rub-out" and permits the operator to erase errors from the tape which prevents them from appearing in the printed message.

Transmitter. The transmitter, Fig. 24, has five contact levers, *C L*, insulated from each other and controlled by five selecting pins, *S P*. The contact levers normally rest against a busbar connected to one pole of the line battery. These contact levers are connected to five consecutive segments on the sending ring of the distributor. When a selecting pin rises through a signal hole in the perforated tape, its contact lever moves over and makes contact with another busbar connected to the other pole of the line battery.

Fig. 25



-110
+110
TRANSMITTER
L
R
E
D
C
LOCAL RING OF DISTRIBUTOR
+110

Fig. 26

Normally each selecting pin tends to move upwards due to a tension spring, *T S*. Only those pins which find a signal hole in the tape, however, are free to rise, the others being held down by the blank section of the tape.

The contact levers associated with those pins which rise through the signal holes move over to the opposite busbar. Consequently, the five segments of the distributor connected to the transmitter are placed in contact with one or other of the two poles of the line battery according to the perforations in the tape. As soon as the brush on the distributor has swept over these five segments, a current impulse which may be termed the "stepping pulse" is made to pass through the electromagnet which depresses all the selecting pins and by means of a ratchet and tape feed wheel similar to that in the perforator, causes the tape to be stepped forward ready for the next letter combination to be set up. Fig. 25 shows the transmitter with covers in place.

Auto Stop. Although a skilled operator has no difficulty in maintaining an ample length of perforated tape between the perforator and the transmitter, conditions arise in normal operation when a perforating operator may have to suspend work temporarily. The transmitter, however, continues to feed forward the perforated tape and were no provision made, the tape would become torn as soon as the slack was taken up between the perforator and the transmitter. This is taken care of by a device, termed the "auto-stop," which consists of a light lever and a differentially wound relay connected as shown in Fig. 26. The lever is so placed between the perforator and the transmitter that the slack perforated tape sags beneath it, but does not touch it. The weight of the long arm of the lever ensures the contact at *C* being closed. The stepping pulse from the local ring of the distributor to the transmitter magnet has two paths, one via contact *C*, the other via contact *D*. Consequently, the current divides equally between the differential windings *L* and *R*, of the relay, which remains inoperative. Whenever the sag in the tape is taken up sufficiently to carry with it the contact lever, the contact at *C* is opened, so that the next stepping inpulse from the local ring of the distributor passes through one winding only of the differential relay and actuates the relay. Its armature causes contact at *E* to be made and that at *D* to be broken. The relay is therefore locked up and the magnet of the transmitter is energized. So long as this condition is maintained the selecting pins of the transmitter are held in the position for transmitting the "blank" signal which has no effect whatever upon the printer at the distant station.

As soon as the operator commences perforating and the slack tape allows the contact lever to drop, the first "stepping pulse" will cause the unlocking of the differential relay and normal transmitting conditions to be restored.

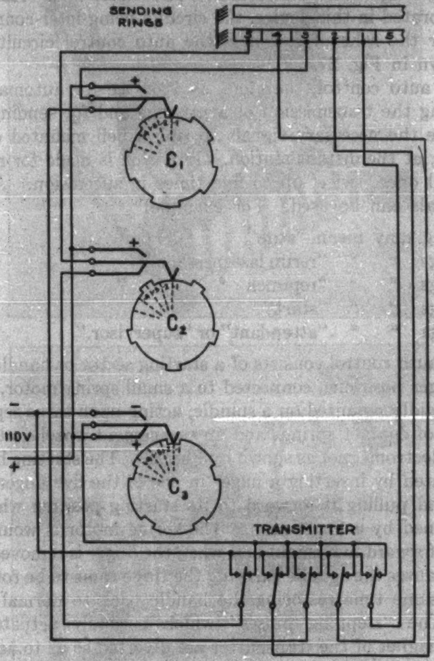


FIG. 27

Auto Control. While not an essential element of the system, the auto control is a desirable adjunct. In order to save time, it is advantageous to be able to signal the distant station operator without having to wait until a message in course of transmission has been completed, and it is essential that such signals shall not involve mutilation of the printed record of such message.

This is secured by means of the "auto control" device which is connected in the circuit between the transmitter and the sending ring of the distributor. The "auto stop" feature is incorporated in this device, the circuits being inter-connected, but for the sake of clearness the auto control circuit alone is shown in Fig. 27.

The auto control, therefore, is a device for automatically stopping the transmission of a message and for sending over the line the necessary signals to ring a bell mounted on the printer, at the distant station. Provision is made for ringing the bell once, twice, up to five times in succession. A code of signals can be used. For example:

1 ring may mean "stop"
2 rings " " "rerun last message"
3 rings " " "repunch " "
4 rings " " "start"
5 rings " " "attendant" or "supervisor."

The auto control consists of a starting sector or handle with five finger positions, connected to a small spring motor, three cams rigidly mounted on a spindle, acting upon three separate groups of contact springs and an escapement wheel controlled by an electromagnet as shown by Fig. 28. The starting handle is actuated by inserting a finger in one of the five finger positions and pulling it forward to its starting position which is determined by a finger stop. The spring motor is wound up by this forward movement and when the finger is removed, the motor causes the spindle carrying the three cams to be rotated, at the same time restoring the handle back to normal position. The "stepping pulses" which normally actuate the electromagnet of the transmitter are diverted so as to actuate the electromagnet controlling the escapement wheel of the auto control device so that the cams are rotated one step for each "stepping pulse."

In order that the circuits may be readily traced, dotted lines have been drawn to show the position of the middle springs on the cams at each step. The normal position of the springs is shown in Fig. 27.

The first step, that is, to position marked No. 2; causes the middle spring of cams 1 and 3 to be raised. The polarity of the right hand contacts on the transmitter is changed from positive to negative by the action of cam 3 whilst the actuation of the spring on cam *C*1 causes segment 3 on the sending ring to be connected to the positive line battery, irrespective

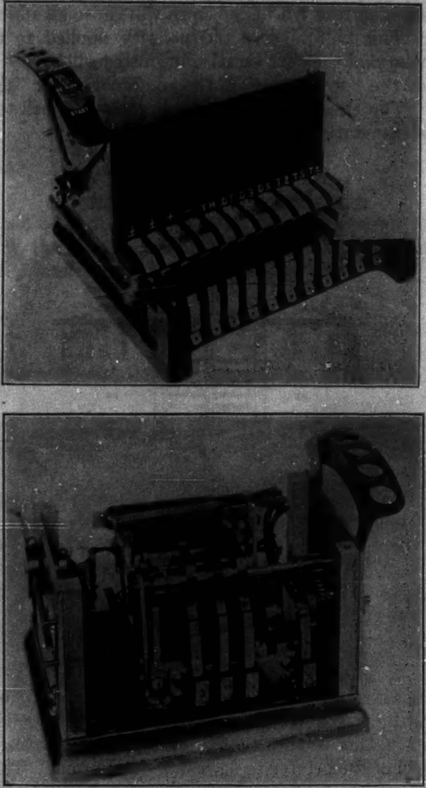


FIG. 28

of the polarity applied to the third contact lever of the transmitter. Consequently, the five segments will send – – + – –, which is the "figure shift" signal. This will actuate the printer at the distant station and adapt it for "upper case" operation. The next step forward of the cams will cause all springs to be raised. This will reverse the polarity applied to the fifth segment so that the next signal transmitted will be – – + – +, which in the "upper case" is the combination for "signal" or "bell." The effect of this will be to cause the bell to ring at the distant station printer.

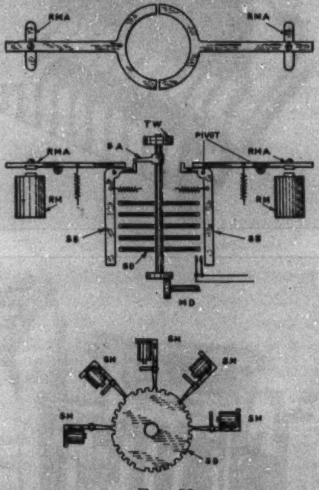


Fig. 29

At the third step of the cams, the middle spring of cam 3 only will be raised, so that negative voltage is applied to all segments. This is the "letter shift" signal and causes the typewheel of the printer to be restored to normal. The fourth step restores the circuit conditions to normal ready for transmitting direct from the transmitter.

This cycle of operation is repeated one, two, three or fou times, according to the setting of the handle. None of the signals transmitted by the auto control device actuates the spacing or lateral movement of the paper carriage so that the auto control may be operated whilst the printer is in the middle of a word. When the auto control signals have been sent, the normal transmission of the message continues, and the printer completes the unfinished word.

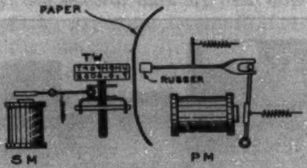


FIG. 30

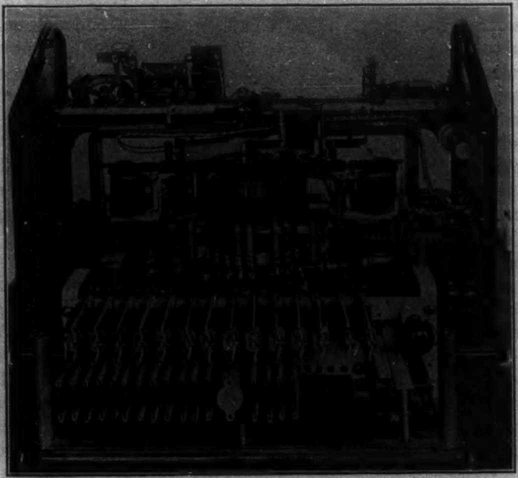


FIG. 31

Printer. The printer is of the movable paper carriage type, and all operations such as causing the paper carriage to be returned for the printing of a new line and moving the paper upwards equal to the distance between lines, are performed automatically under control of signals from the sending station.

FIG. 32

FIG. 33

A typewheel *TW*, Fig. 29, mounted on a vertical shaft, is made to rotate by means of a small electric motor. Projecting from the typewheel shaft is a short arm *S A*, the free end of which just clears the ends of the selector bars *S B*. When stopped by a raised selector bar, the typewheel is so positioned as to print the particular letter associated with that selector bar. Normally, the selector bars are held down by the semicircular ends of the armature levers *R M A* of two release magnets *RM*, Fig. 29.

The five horizontal selector disks are mounted around a hollow drum or shaft enclosing the typeshaft. Each disk has notches so arranged around its periphery, that when one of the disks is moved through a small angle, a notch on each of the disks will be in vertical alignment so as to permit the vertical leg of one selector bar to move inwards. If two disks are moved, the alignment of notches will occur opposite another selector bar, if three disks are moved the alignment will take place opposite still another selector bar and so on. With five selecting disks, 32 possible selections are available. There are 31 selecting bars, each one of which is associated with a character to be printed, or contacts to be closed to cause one of the several functions which the printer has to perform. One selection is not used, since it is desirable to have one combination which will cause no operation in the printer. The selector bar on the right, Fig. 29, is shown as a function or "stunt" bar. These functions are "space," "carriage return," "line feed," "figure shift," "letter shift" and "bell."

The typewheel is mounted on the typeshaft in such a manner that it may be raised so as to bring a second row of characters opposite the printer hammer. This operation is performed by means of the shift magnet *SM*, as shown in Fig. 30. When the "figure shift" signal is received, a "stunt" bar is selected and closes the contacts of the shift magnet *SM*. When the "lettershift" signal is received, another "stunt" bar is selected, which opens the circuit of the "shift magnet" and the typewheel is brought down to its normal position.

The paper on which the messages are printed is fed round the platen of the paper carriage which slides laterally on a square shaft. Fig. 31 shows the printer with all covers removed. Fig. 32 shows an operating table with all equipment in position.

Distributors. The distributor face plate consists of eight rings, four being solid and four segmented, as shown in Fig. 33. The rings are associated in pairs, one segmented and the other solid. The two brushes which sweep over the two rings of a pair are in electrical contact with each other so that the solid ring is connected in sequence to all the segments of its associated ring, once per revolution. The outer or receiving ring has 40 segments and is associated with the fifth ring. The second or correcting ring (40 segments) is associated with the sixth ring. No. 3 is the sending ring (20 segments) and is associated with No. 7. The innermost segmented ring (20 segments) is termed the "local" and is associated with

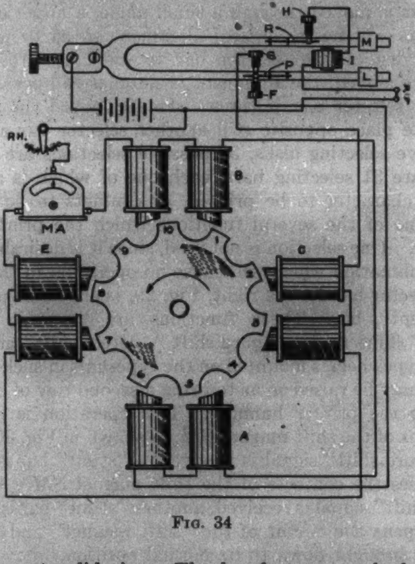


FIG. 34

the innermost solid ring. The brushes are made from thin copper wire and make contact on one segment only at a time. They are mounted on a balanced brush holder which is carried by a spindle projecting through the center of the face plate.

Two types of distributors have been designed, one in which the brush holder spindle is directly connected to the shaft of the driving impulse motor and the other in which intermediate gearing is introduced between these parts, for a purpose which will be described later.

The La Cour or phonic wheel motor together with the electrical driving fork is shown in schematic form in Fig. 34.

The fork carries two contact springs *R* and *P* in electric contact with the fork itself. Mounted below the tines of the fork is an electromagnet *I* for causing the fork to vibrate. When the tines are not loaded with weights *M L* the fork vibrates at the rate of approximately 55 cycles per second. If a slower rate is desired, weights are fixed on the tines as shown in the figure. The rate of vibration can be varied between 25 and 55 cycles by this means. Provision is made for effecting small changes in speed, by means of additional weights near the base of the fork, which are so mounted as to be slidable along the tines by an adjusting screw operated by a handle or knob at the end of the fork. This adjustment can be made whilst the fork is in vibration. A photograph of the fork is shown in Fig. 35. In the case of a quadruple or a four arm installation, the speed of the fork in cycles per second corresponds to the speed of operation of each channel in words per minute, so that if the fork vibrates at 50 cycles per second each transmitter operates at a speed of 50 words per minute.

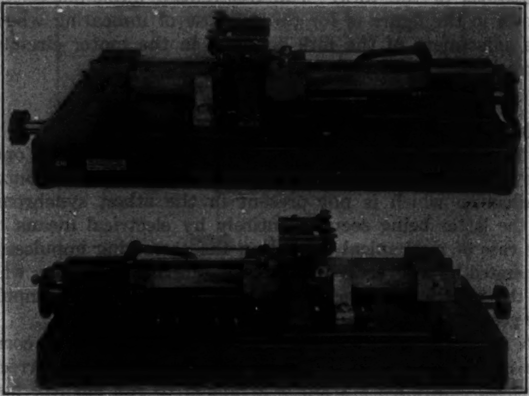


FIG. 35

The phonic wheel motor has an armature so shaped as to form ten teeth or polefaces, equally spaced, around the periphery. Mounted 90 deg. apart are four electromagnets with polefaces shaped so as to be concentric with the edge of the polefaces or teeth on the armature.

The electromagnets diametrically opposite are connected so that *A* and *B* (See Fig. 34) are energized together and *C* and *E* together. Consequently, when the fork is vibrating, *A* and *B* will be energized alternately with *C* and *E*. The number of the polefaces on the armature is such that the armature makes one complete revolution during ten complete vibrations of the fork. The differential milliammeter *MA* shown in the figure is for the purpose of indicating whether the adjustment of the fork contacts in the motor circuits is correct.

SYNCHRONISM

Mechanical Correction. As previously stated two types of distributors are used. They differ only in so far as one is provided with a mechanical correcting or synchronizing mechanism which is not present in the other, synchronism in the latter being secured entirely by electrical means. In the case of mechanical correction the correcting impulses are generated by a reversal of current in one direction only, whilst in the electrical correcting method the correcting impulses are generated by current reversals in both directions.

A schematic diagram of the circuit for mechanical correction is shown in Fig. 36. The distributor motor at the corrected or speed controlled station is arranged to run slightly faster than the motor at the correcting or speed controlling station. If reversals of current in the line signals take place when the brushes which sweep over the correcting rings at the controlled station are on a dead or unconnected segment, as at the point indicated by the arrow marked *X*, no correction takes place. But the brushes at the speed controlled station will gain over those at the speed controlling station so that after one or two revolutions the brushes at the former station will be leading, and be on a "live" segment at the moment when the corrector relay tongue which follows the movements of the line relay, moves to its *M* contact.

Before the "break" relay *B R* has had time to pull up its armature, an impulse passes via the back contact of *B R* to the solid correcting ring, across the brushes, the "live" segment, switch *S* through the correction relay *C R* to the −110 *V*. The corrector relay is actuated and is locked up due to current from the +110 *V* flowing via the armature and front contact of *C R* and the back contact and lever on the stepping pawl *P*. A path is also provided from the +110 through the corrector

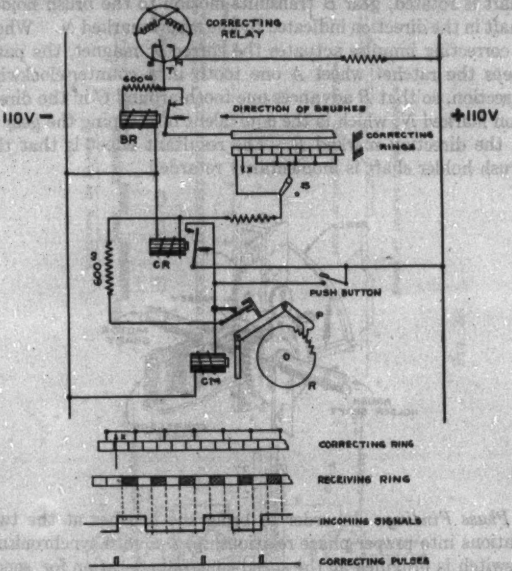

CORRECTING RELAY
T
600Ω
110V −
+110V
DIRECTION OF BRUSHES
BR
CORRECTING RINGS
S
600Ω
CR
PUSH BUTTON
P
CM
R
CORRECTING RING
RECEIVING RING
INCOMING SIGNALS
CORRECTING PULSES

Fig. 36

magnet *C M* which is energized. Actuation of the armature of *C M* causes the pawl *P* to step the ratchet wheel one tooth. As it completes its stroke the pawl lever causes the circuit from the +110 to be opened so that the corrector relay *C R* and the corrector magnet *C M* are restored to normal. The stepping of the ratchet wheel has the effect of momentarily stopping or retarding the movement of the brushes so that they are again in sufficiently close phase relationship with the incoming signals to insure the correct distribution of the electing impulses to the printer selecting relays.

The mechanical feature of the correcting device is shown in Fig. 37. The motor shaft carries with it the frame *F* on which is mounted the corrector magnet, the ratchet wheel *A*, gear wheel *B*, the pawl and jockey roller.

A and *B* are fixed on the same spindle and are prevented from rotating by the jockey. Consequently, when the motor shaft is rotated, gear *B* transmits motion to the brush holder shaft in the direction indicated by the arrow marked *N*. When a correcting impulse actuates the corrector magnet, the pawl steps the ratchet wheel *A* one tooth in a counter-clockwise direction, so that *B* advances one tooth around *C* in the direction marked *N*, which is the equivalent of stepping the gear *C* in the direction marked *R*. The resultant effect is that the brush holder shaft is momentarily retarded.

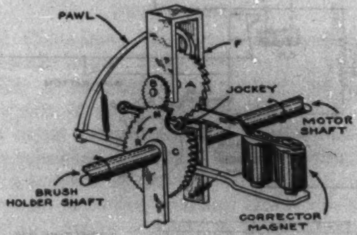


FIG. 37

Phase Finding. In order to bring the brushes at the two stations into proper phase relationship, *i. e.* into synchronism, a switch is provided at the speed-controlling station for sending a long negative pulse from sending segments 1 to 19 and a positive pulse from sending segment 20. The switch marked *S*, Fig. 36, at the speed-controlled station is thrown to the right, disconnecting all the "live" segments but one. Consequently, only one correcting pulse will be generated per revolution of the brushes and it will be inoperative until the brushes at the corrected station overtake the incoming signals, at the position marked *X*.

Electrical Correction. Fig. 38 shows in schematic form the circuit arrangements for electrical correcting. A distributor arranged for electrical correction has the impulse motor directly connected to the brush holder and correction is effected by altering the speed of the fork which determines the speed of the motor. When the resistance *R* is in series with the electromagnet of the driving fork at the speed-corrected station, the speed of the fork is slightly faster than that of the fork at the speed-controlling station, and when *R* is short-circuited the speed of the fork is slightly lower than that of the fork at the other station. The function of the

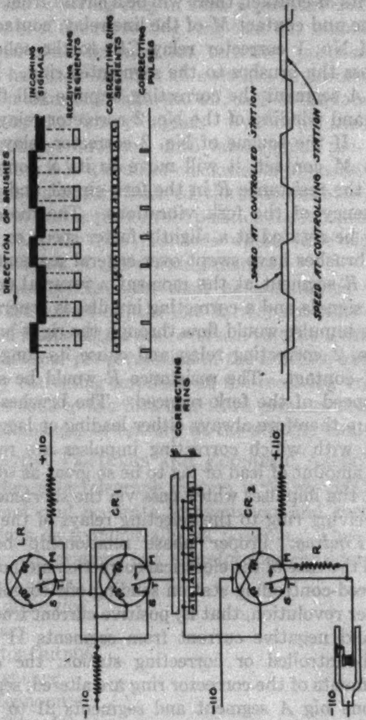

DIRECTION OF BRUSHES
INCOMING SIGNALS
RECEIVING RING SEGMENTS
CORRECTING RING SEGMENTS
CORRECTING PULSES
SPEED AT CONTROLLED STATION
SPEED AT CONTROLLING STATION
CORRECTING RING
LR
CR1
CR2
+110
+110
-110
-110
-110
R
M
S
T

FIG. 38

correcting impulses which are generated from the signals themselves, is, therefore to control the speed of the fork by means of the resistance *R*. The tongue of the corrector relay CR_1 follows the movements of the tongue of the line relay *L R*, but with a slight lag. Consequently, if the line relay tongue moves to its *M* contact, there will be a circuit from the + 110*V*, the tongue and contact *M* of the line relay, contact *S* and the tongue of No. 1 corrector relay CR_1 to the solid correcting ring, across the brushes to the segmented ring. If the brush is on an *A* segment the correcting impulse will flow through the left-hand winding of the No. 2 corrector relay CR_2 to the − 110*V*. If the tongue of No. 2 corrector relay happens to be on its *M* contact, it will move to its *S* contact, thereby inserting the resistance *R* in the fork circuit and speeding up the frequency of the fork vibrations. The motor will consequently be rotated at a slightly faster speed so that by the time the brushes have swept over several segments, they will be on an *R* segment at the moment a reversal occurs in the incoming signals and a correcting impulse is generated. Such correcting impulse would flow through the right-hand winding of the No. 2 correcting relay and cause its tongue to move to the *M* contact. The resistance *R* would be shunted out and the speed of the fork reduced. The brushes of the distributor are therefore always either leading or lagging but the frequency with which correcting impulses are received prevents the amount of lead or lag to be so great as to detrimentally affect the impulses which pass via the shortened segments of the receiving ring to the selecting relays of the printers.

Phase Finding. Proper phase relationship between the brushes in the case of the electrical correction method is secured by the speed-controlling station sending one complete current reversal per revolution, that is, positive current from segments 1 to 10 and negative current from segments 11 to 20. At the speed-controlled or correcting station, the connections to the segments of the corrector ring are altered, segments 1 to 20 form one big *A* segment and segments 21 to 40 one big *R* segment. The connection to the *M* contact on the No. 1 correcting relay is opened so that correcting impulses will be generated when the signal current polarity changes from negative to positive only. Consequently, there is only one position on the correcting ring where correction will take place and the brushes will either gain or lose with respect to the incoming signals, according as they are on the *A* or the *R* segment, until they are in synchronism, when correction will commence. All switches are then turned to normal.

Margin. The outer or receiving ring is mounted so as to be free to be rotated. This is for the purpose of placing the segments of the receiving ring in such a position with respect to the segments of the correcting ring that the middle portion of each signal shall be used for selecting purposes. Once synchronism has been secured, all switches at normal and the distant station transmitters turned off, each of the four printers will be receiving the "blank" signal. By moving the receiving ring in a clockwise direction until false signals are received, and then moving it in a counter-clockwise or opposite direction until false signals are again received, the midpoint between these two positions indicates the correct position for reception. The distance between the two positions where false signals are received is an index of the working margin.

In order that synchronism shall be maintained when all transmitters are turned off, the line battery connections of the first and third transmitters are reversed, so that four reversals of current will occur per revolution of the brushes, and provide the necessary correcting impulses at the receiving end.

The connections to the first and third printers are adapted to meet these changes of battery at the sending end.

OPERATING FEATURES AND ECONOMIC ASPECTS

The function of a telegraph system is to provide a means for the prompt and accurate transmission and reception of communications as economically as possible. The merits of various telegraph systems can therefore be considered under the following heads:

1. Speed of Service
2. Accuracy
3. Operator Output
4. Maintenence
5. Line Economics
6. Flexibility of System

SPEED OF SERVICE

By this term is meant the average time taken in handling telegrams from the moment an operator at Station *A* commences to perforate or to send a message until it is ready at Station *B* for transference to some other part of that office, either for delivery or for retransmission over another circuit. Messages for delivery by messenger have to undergo further handling. As such further handling, however, is not directly associated with or affected by the system of transmission over the wires, time occupied in this connection is not included.

Reviewing the methods of operation of the various systems under consideration, it will be shown wherein time losses, which occur in the high-speed systems, are avoided in the multiplex.

At the Sending End. In the high-speed systems messages are first prepared by perforating operators, seated conveniently near to the transmitting operator who feeds the perforated tapes into a high-speed transmitter. In order that as little time as possible be lost by the perforating operators in passing the message forms with the corresponding tapes to the transmitting operator, it is the usual practise to perforate two or three messages on each tape. If such tapes are immediately fed into the transmitter, the first message has already undergone a delay equal to the time taken to perforate all three messages and the second message has undergone a delay equal to the time taken to perforate the second and third messages. The transmitting operator, however, has no time to sort out the tapes from the various operators so as to transmit messages in their exact order of priority. It usually happens, therefore, that a certain proportion of the messages undergo further delay before actual transmission takes place. It may be taken that the average delay in the case of high-speed systems is not less than three minutes.

With a multiplex system transmission over the lines takes place almost immediately the keys are actuated. In other words a multiplex provides practically direct connection between operator and operator, as is the case with Morse working.

At the Receiving End. There is not much difference between the high-speed and multiplex systems as regards speed of service, a slight advantage, however, resting with the multiplex.

Unfortunately, telegraph traffic does not glide along as smoothly as has been outlined above. Errors creep in and must be corrected. With the high-speed systems, a correction operator is required at each end of the circuit. Messages dealing with the correction of errors pass between these operators and although given priority over other traffic several minutes elapse in their transmission in each direction. Therefore, before an inquiry regarding a message can be received, a considerable number of other messages will have been transmitted, and unless these messages have been arranged in numerical order, some time is needed to trace the required message. Further delay will occur in the transmission of the answering message so that the total delay to a message in which a correction is required may amount to many minutes.

With the multiplex systems the mode of working admits of such corrections being asked and obtained almost instantly.

In a paper read before the Post Office Telephone & Telegraph Society, London, on November 24th, 1913, Mr. John Newlands, Controller of the Central Telegraph Office, states

> There is no doubt that all existing systems of telegraphy which involve the preliminary punching of each message tend to create delay, because even when the punching operation is completed, the message is not yet on its way, the process of transmission through the transmitter has still to be carried out, and when there are two messages on each slip, and each separate slip must perforce await its turn—there is always an appreciable loss of time, which does not take place, for example, on the Baudot System nor even in Morse Keying.

It is interesting to note that this advantage of multiplex operation has been recognized in England and an effort has been made to adapt the Wheatstone System to the multiplex principle, so far as transmission is concerned. The arrangement is as follows:

Beside each perforating operator is a motor-driven Wheatstone transmitter. These transmitters are connected to an electromagnetic rotary switch which is arranged to switch the line to each transmitter in sequence. The switch is actuated by a signal from the transmitter to which it is connected, each actuation transferring the connection to the next transmitter. When perforating operator No. 1 completes a message, he depresses a special key on the perforator which perforates a hole in the tape, not in alignment with the signal holes. As this hole passes through the transmitter, it causes actuation of the switch, which stops the motor of the No. 1 position transmitter and starts the motor of the No. 2 position transmitter, at the same time switching the line from No. 1 to the No. 2 transmitter. Similar operation at the end of the message transmitted by the No. 2 transmitter switches the line to the No. 3 transmitter and so on in rotation.

It is doubtful if such a scheme will prove satisfactory, however. Messages are not of uniform length, nor do operators perforate at exactly the same speed. It effects the elimination of the transmitting operator, but it is difficult to see wherein much improvement can be secured towards providing a quicker speed of service. The use of two or more high-speed transmitters which are operated intermittently involves added complexity and cost to the sending end apparatus.

ACCURACY OF PRINTING TELEGRAPHS

Whatever may be the system of telegraphy employed, the standard of accuracy is dependent upon the care with which the messages are checked. In Morse working, the receiving operator is expected to check the accuracy of a message whilst performing the mental operation of translating the dots and dashes into letters and the physical operation of recording them on a typewriter. Actually three functions are performed simultaneously:

1. Translating telegraphic signals into letters, mentally.
2. Operating a typewriter, selecting correct keys.
3. Scrutinizing the typed copy to see that it contains no errors.

Although it has been claimed that the two first operations are performed entirely automatically by skilled operators, such is not the case except in rare exceptions. If a function is being performed automatically, that is, requiring no attention of mind, then the mind can be employed on some other function. It is only necessary to request a Morse operator to read some message other than that he is in the act of receiving, or to engage him in conversation to show how little attention he is able to give it, without interfering with his main task. Psychological authorities agree that if the mind be engaged in two or more independent actions the principal action is hampered by the side issues.

Apart from its being a slower process than Morse sounder working, the translation of telegraph signals from a tape as in the Wheatstone system involves a division of attention between the tape and the sheet on which the message is being written or typed with the result that the liability to making errors is increased. When continued for a long period it also tends to produce eye strain.

A page-printing system necessitates only the checking of the messages, and consequently that work can be given un-divided attention by the operator, with the result that it is done better.

It may be of interest to mention that a European telegraph administration, desirous of securing data in regard to undetected telegraphic errors, caused a careful examination to be made of a large number of messages which were handled by different telegraph systems. This examination showed that the number of undetected errors per 100 messages was as follows:

Morse tape recording . .	0.11
Morse sounder	0.09
Hughes printer	0.04

It is understood that the introduction of the multiplex printing system in the Western Union Service has resulted in a proportionate improvement as regards accuracy, closely approximating these figures.

When it is realized that telegraphic errors oftentimes result in financial loss, as well as involving considerable expense in official correspondence and investigation, the greater accuracy secured by a printing system must be regarded as an important factor in its favor.

OUTPUT

The introduction of printing telegraphs in place of the Morse has not resulted in so great a percentage gain in operator output as has been achieved by the use of automatic machines in other fields of activity, nor can they ever be expected to do so. The human factor cannot be eliminated entirely; every message must be handled—letter by letter—by the sending operator, and be checked at the receiving end by the receiving operator.

The keyboard perforator, however, has been responsible for a considerable increase in speed of working at the sending end, whilst at the same time its operation involves less mental and physical fatigue on the part of the operator.

At the receiving end of a printing system, the operating duties are less exacting than those of the Morse operator. Although messages must be handled without delay the nature of the work permits of a freedom to the operator which cannot possibly be felt by Morse operators. Consequently, the operators are under less mental strain and can handle a larger volume of business.

It has been found that, in general, printing systems have shown some improvement over the Morse and Wheatstone systems, as regards labor economies.

The following statement, extracted from Mr. John Newlands' paper previously referred to, gives the comparative results of tests of various systems of telegraphy in England. As British messages average 24 words, whilst American messages average 30 words, I have equated the figures to American messages, as shown below:

System	Average per Operator Hour British Messages	American Messages
Baudot Multiplex	43	34.4
Wheatstone with Creed	35	28
Morse Sounder	31	24.8
Wheatstone	26	20.8

The Siemens & Halske and American Multiplex Systems have been installed in the London office since Mr. Newlands' paper was written. No results have yet been published concerning these systems.

In the foregoing statement are shown the four systems of telegraphy by means of which probably 95 per cent of the British telegraph traffic is carried.

The multiplex Baudot shows an output per operator of 23 per cent greater than the high-speed system—the Creed, despite the fact that the Baudot sending operators are limited to a maximum speed of 30 words per minute. Assuming an increase in output pro-rata with the increase in speed resulting from the use of the keyboard perforators and automatic transmitters, say 45 words per minute, the output per operator would be 64 British messages or 51.6 American messages per hour.

All these foregoing figures refer to team work on one or more circuits.

In the 1919 April issue of the "Western Union News," some interesting records of work done on keyboard perforating machines by female operators are given. From 100 to 130 messages per hour have been perforated by a number of operators for short periods, up to eight hours. For periods covering from one month to one year, the hourly average range between 60 to 79 messages.

In one particular case a female operator maintained an average of 74 messages per hour for a period of eight hours during the first month after graduation from the training school.

In the same magazine is given data pertaining to record performances of Morse operators which compare very favorably with the work of the keyboard perforator operators. In order to achieve such results, however, it is necessary that the operators at both ends of the circuit shall be experts whereas in automatic operation the transmitting operator output is not limited by the skill of the receiving operator.

MAINTENANCE

Available information regarding the cost of maintaining installations of the various systems described is unfortunately too meagre to be of value in making direct comparisons.

Printing telegraph machines have not yet reached that stage of perfection where they never get out of adjustment and never have broken parts. Consequently, provision should be made for the removal of faults due to such causes as promptly as possible. What is even more important, however, is the provision of means for the detection of incipient faults before they have developed sufficiently to interfere with operation and for the quick restoration of service when it has been interrupted by an apparatus fault. In this respect, the multiplex systems are superior to the high-speed systems. For a quadruple duplex installation, it is necessary to provide only one spare instrument of each kind, except the distributor. In the case of the American multiplex system, such spare apparatus is mounted on a test table, which is provided with a small local distributor so that the machines may be tested in operation under conditions even more severe than in actual service. Spring clips are used for effecting the electrical connections on the operating and test tables and any machine can be removed and another placed in its stead in a few seconds. If the test table is located in close proximity to the operating tables, interruption to the flow of traffic due to machine trouble can therefore be almost entirely eliminated. A fault in any of the machines, except the distributor, will stop the flow of traffic in one direction over one channel only. Faults in the distributor are of rare occurrence. Generally a spare distribution face is available, and can be substituted in a few minutes.

In the case of high-speed systems an apparatus fault in the transmitting or receiving apparatus causes total suspension of traffic flow in one direction, and generally involves temporary interruption to the traffic in the other direction whilst the distant station is being advised of the trouble.

The percentage of spare apparatus must obviously be greater than in the case of the multiplex systems, except where four or more high-speed installations are in service in one office, and in such cases it is practically impossible to effect apparatus changes so quickly as can be done with multiplex machines. Where a complete spare set of machines is provided, it is possible, of course, to switch over to the spare set without much delay.

One important factor in connection with the maintenance of apparatus is the periodical inspection and overhaul. Such inspections reveal incipient faults. The test table and local distributor of the American multiplex system are particularly valuable for this purpose, as the more severe conditions under which the machines can be tested will reveal small defects or faulty adjustments before such defects make themselves felt under normal working conditions.

For example, if a printer operates at 65 words per minute during one inspection test, and only 60 words per minute at the next, it is an indication that some slight fault exists, which can be detected and remedied before it affects the machine at normal working speed of 50 words per minute.

Provided a properly organized system of periodical cleaning and testing is introduced, it is possible to maintain a service practically free from interruption due to apparatus troubles.

The most satisfactory method of maintaining such apparatus in correct adjustment is to specify spring tensions, distance of movement of relay armatures, airgaps etc. and to use suitable gauges for indicating whether the adjustments of the various units are within the specified limits.

LINE ECONOMIES

In the synchronous type of printing telegraph systems no spacing between signal combinations is required and consequently the code is considerably shortened as compared with the Morse where the spacing between letters is equal to three dot lengths. From studies which have been made it has been found that the average signal combination in the American Morse code is equal to eight dots, and that in the continental

Morse code nine dots. In the synchronous systems the signal combinations are all five-dot lengths. As the dot is the unit of length these codes are generally referred to as eight, nine and five-unit codes. The advantage of a shorter code can perhaps best be illustrated by example. Assuming that over a particular telegraph circuit a maximum transmission speed of 45 cycles per second can be obtained. This speed of transmission will provide a carrying capacity of 100 words per minute for a nine-unit code, 112.5 words per minute for an eight-unit code and 180 words per minute for a five-unit code. Should the traffic load be such that 100 words per

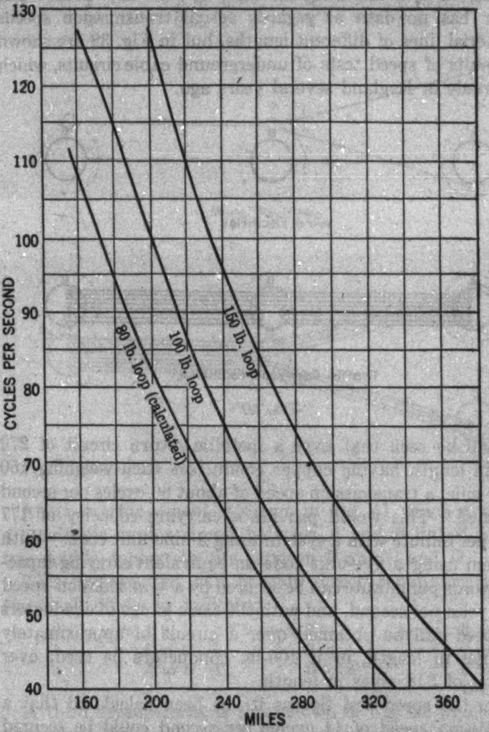
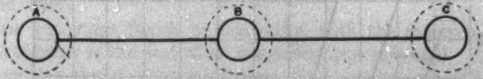
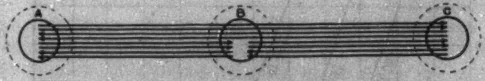


FIG. 39—METALLIC LOOP CONDUCTORS
Curves showing relation between conductor weight, mileage and speed of transmission.

minute will be sufficient to carry it then the transmission speed of the five-unit code system may be reduced to 25 cycles per second. This reduction in speed will permit of operation under adverse weather conditions which tend to disturb the duplex balance, and which would necessitate a reduced speed of working on systems using an eight or nine-unit code. The author has no data as regards actual transmission speeds over aerial lines of different lengths, but in Fig. 39 are shown the results of speed tests of underground cable circuits, which were made in England several years ago.



Wire Facilities



Traffic Carrying Facilities

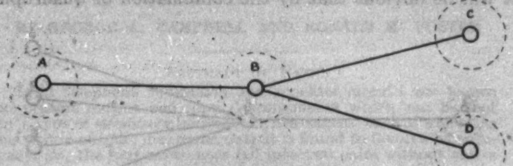
Fig. 40

It will be seen that over a metallic return circuit of 270 miles in length having copper conductors each weighing 150 lb. per mile, a transmission speed of about 80 cycles per second is secured. This would provide a carrying capacity of 177 words per minute with a system using a nine-unit code. With a system using a five-unit code an equivalent carrying capacity in words per minute can be secured by a transmission speed of 44 cycles per second, and with the same gauge of conductors this speed can be obtained over a circuit of approximately 364 miles in length, or if 100-lb. conductors be used, over a circuit of 318 miles in length.

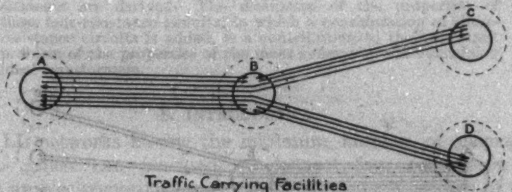
From the speed test figures it has been calculated that a transmission speed of 44 cycles per second could be secured over a 270-mile metallic circuit having conductors of about 75 or 80 lb. weight per mile. Assuming the wire to cost 20 cents per lb., there would be a capital saving in copper of $7560 on a 270-mile circuit, by using a five-unit code instead of a nine-unit code.

Flexibility. An outstanding feature of the multiplex system is the flexibility which it provides in linking up a number of offices with a minimum of line plant. Fig. 40 shows two quadruple duplex circuits radiating from a repeater at station *B*, and providing the following communications:

2	Channels	*A* to *B*	2	Channels	*B* to *C*
2	"	*A* to *C*	2	"	*C* to *B*
2	"	*C* to *A*	2	"	*B* to *A*



Wire Facilities



Traffic Carrying Facilities

FIG. 41

Another arrangement is shown in Fig. 41 which represents a quadruple duplex circuit between *A* and *B* linked up with two double duplex circuits between *B* and *C* and *D*.

This arrangement will provide direct communication between all offices as indicated:

2	Channels	*A* to *B*	2	Channels	*B* to *A*	1	Channel	*B* to *C*
1	"	*A* to *C*	1	"	*C* to *A*	1	"	*C* to *B*
1	"	*A* to *D*	1	"	*D* to *A*	1	"	*B* to *D*
						1	"	*D* to *B*

A third arrangement is that shown in Fig. 42 in which the four arms of a quadruple are shown extended to four duplex printer circuits radiating from one terminal station.

By this means direct communication can be provided between station *A* and four small stations *C*, *D*, *E* and *F*, neither of which perhaps may have sufficient traffic to warrant a direct wire to station *A*.

The duplex printer referred to is a two-channel system using practically the same apparatus as the multiplex, and a small distributor in which the brushes rotate once per letter combination transmitted. No synchronising arrangements are required, and as the brushes are momentarily stopped when they reach the normal or rest position the system has been appropriately termed "The start-stop system."

It will be obvious that by the combination of quadruple or double-duplex multiplex and "start stop" installations it should be possible to effect considerable economies.

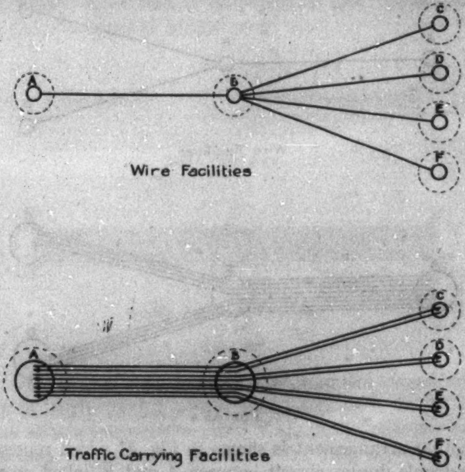


FIG. 42

In addition to the financial savings such circuit combinations help to accelerate the service by reducing the number of times a message is handled, and this in turn tends towards a higher standard of accuracy.

We cannot anticipate the transmission of all telegrams by connecting the originating office with the office of destination as is done in telephone practise but the multiplex in conjunction with the "start stop" will materially help towards developments along such lines.

Presented at the 8th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 10, 1920.

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MAXIMUM OUTPUT NETWORKS FOR TELEPHONE SUBSTATION AND REPEATER CIRCUITS

BY GEORGE A. CAMPBELL AND RONALD M. FOSTER

ABSTRACT OF PAPER

Ideal telephone substation and repeater circuits are shown to present output and input requirements which can be met by a type of circuit containing four resistances, each of which has maximum output; maximum output is found to involve biconjugacy, *i.e.*, the four resistances fall into two pairs without active electrical connection between the two resistances of either pair; all circuits meeting these requirements with the minimum number of elements are enumerated; the necessary formulas for proportioning these circuits and also circuits having superflous elements are derived. The discussion of the properties of these four-resistance circuits, to which a consideration of three-resistance circuits is added, is a contribution to the important problem of the properties of the most general circuit containing ideal transformers.

1. INTRODUCTION

ALL networks having the minimum number of elements which can be used to connect four resistances so that an electromotive force, placed successively in each of the resistances, will have the maximum possible output, $E^2/4R$, into the remaining three resistances, and also some circuits with superfluous windings, are enumerated and briefly discussed in this paper. These networks are called "maximum output networks," a term which will naturally be used to include also all networks connecting any number of resistances when a given source of energy in any resistance has its maximum output into the remaining resistances. The term "maximum output circuit" is also employed, and while no hard and fast distinction is made, it is usually used to include the resistances together with the connecting network. The maximum output circuit, consisting of two resistances connected by a transformer having the turns in the two windings in the same ratio as the square roots of the resistances, is well known. With three or any odd number of resistances it can be shown that a maximum output circuit is impossible.

The outstanding property of four-resistance maximum output circuits is that the resistances are conjugate in pairs; the circuits are said to be biconjugate. Thus if (A, B) is one conjugate pair of resistances and (C, D) the other conjugate pair of resistances an electromotive force in A, B, C, D will produce no current in B, A, D, C, respectively. This remarkable property is, as we shall see, the precise condition which is especially called for in telephone substation and repeater circuits. The biconjugacy condition from a practical standpoint may be more important than that of maximum output, but the latter has been chosen as the term for designating this class of circuits since maximum output involves biconjugacy, whereas biconjugacy does not necessarily involve maximum output.

The maximum output and conjugate properties may be illustrated quite simply by taking a Wheatstone bridge having the same total resistance in each of the six branches. An electromotive force in any branch will then have its maximum output into the remaining branches, divided equally between the four adjacent branches, the non-adjacent branch receiving nothing and thus being said to be conjugate. The circuit is thus a maximum output circuit with six resistances and three pairs of conjugate branches.

It is found that the minimum number of elements which the network may contain, provided no unnecessary restrictions are permitted between the input ratios, as defined in Section 4, and the ratios between the four resistances, is six, which may be six windings, three on each of two transformers. The enumeration of the distinct networks which can be formed with two three-winding transformers shows a total of 57,340, all of which have the same theoretical possibilities on the assumption that ideal transformers are employed. In practical applications it is usually allowable to employ a network which imposes one or more restrictions between the input ratios and the resistance ratios. A complete enumeration of such networks shows that there are 23,664 distinct networks involving a three-winding transformer and a two-winding transformer; 2,361 distinct networks made up of two two-winding transformers; 136 networks employing but one three-winding transformer; and 38 distinct networks made up of but one two-winding transformer. The complete list of possible networks thus totals 83,539; and no other networks are possible which do not include a greater number of elements for accomplishing the same end.

Each of these distinct networks presents the possibility of from one to eight distinct proportionings, all of which have the same energy inputs but different combinations of current directions in the branches. The resulting total number of distinct cases is over half a million distributed as shown in Table I.

TABLE I

Transformers	Windings	Restrictions	Distinct networks	Total cases
2	3+3	0	57,340	446,234
2	3+2	1	23,664	124,536
2	2+2	2	2,361	6,596
1	3	2	136	316
1	2	3	38	40
			83,539	577,722

2. Substation Circuits

The telephone substation circuit consists primarily of a transmitter and a receiver with a network for connecting them to a line. The primary requirements are—

1. Sending efficiency the greatest possible consistent with the receiving efficiency.

2. Receiving efficiency such that line noise will interfere as little as possible with telephone conversation.

3. Elimination of side-tone (the disturbance produced in the talker's own receiver).

4. No restrictive limitations as regards the impedances of the transmitter, receiver, and line.

It will be assumed in this paper that any given transmitter has a fixed resistance and when spoken into generates an electromotive force the magnitude of which is independent of the circuit to which the transmitter is connected. A given receiver is also supposed to have a fixed resistance independent of the magnitude or frequency of the current passing through it and its reactance is assumed to be annulled by a suitable impedance associated with the receiver. Similarly, the line reactance will be assumed compensated, leaving a fixed resistance containing a fixed electromotive force arising from the line noise. These fixed resistances of the transmitter, receiver, and line will be designated by T, R, and L.

The substation conditions may now be put in the following mathematical form: Three arbitrarily chosen resistances T, R, L are to be so connected by a network that L will have the maximum possible input into T and R;* the ratio between these two inputs to have an arbitrary value y as found experimentally to be the best for reducing line interference; and the input between T and R to be zero. These conditions are identically those stated in proposition 6 of Appendix I, and from that proposition it follows that the ideal substation circuit must have at least one auxiliary resistance X and if there is but one, making a total of four resistances, they are connected by a maximum output network.

In the engineering design of substation circuits a large number of practical requirements must be met; such as transmitter current supply, signaling to and by the subscriber, proper allowance for the departure of the line, transmitter, and receiver impedances from the fixed reactanceless resistances assumed in this elementary mathematical treatment, the allowance for the deviations between actual and ideal transformers, and the question of costs. These practical requirements assume different forms, depending upon the particular use to which the substation circuit is to be put, and the best answer can be reached only on the basis of a detailed study of a large amount of experimental data. This side of the problem will not be entered into in this paper, since it is a large problem and does not form a part of a purely mathematical discussion of the available circuits. This paper is limited to the problem of discussing, as fully as may be, all of the possible ideal circuits which should form the starting point for the practical discussion of substation circuits.

3. REPEATER CIRCUITS

One of the applications of telephone repeaters is for two-way amplification of telephone conversations on long lines, the repeater being placed near the midpoint of the line. When the person at the east end of the line is talking, the speech wave passes from the line marked E, Fig. 1, to the receiving element R of the telephone repeater, which produces an amplified effect in the transmitting element T which is passed along to the line marked W. When the person at the western end of the connection is talking, the path taken by the transmission is not reversed in every particular, since it cannot be reversed through the repeater, but must traverse the repeater from the receiving to the transmitting element in order to be amplified. Its course is thus from *W* to *R* to *T* to *E*. To make the system as efficient as possible the transfer of energy between *E* and *R*, *T* and *E* must each be as large as possible, and since the input ratio is a reciprocal property, the combined input from *E* into *R* and *T* must be a maximum. Similarly, the input from *W* into *R* and *T* must be a maximum. Moreover, none of these four inputs can be zero, otherwise there would be no transmission in at least one direction. These conditions are identical with the requirements specified in proposition 7 of Appendix I; the network required for connecting the four elements *E*, *W*, *R*, *T* is therefore a maximum output network for use with four resistances. The necessary biconjugacy of such networks satisfies two additional requirements for repeater circuits. There is no direct transmission between *E* and *W*, which might happen to be superposed on the amplified output in such a way as to produce a certain amount of confusion. Finally, and most important of all, the circuit is non-singing, that is, there is no feeding back of energy from *T* into *R* through the network and connected lines *E* and *W*, and thus the presence of the source of energy presented by the repeater does not lead to the automatic production of an alternating current which might interfere with the easy reception of speech in case it fell within the audibility range.

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*The restatement in this form of condition 1 is possible by virtue of the reciprocal property of the input ratio as explained in section 6 of Appendix I.

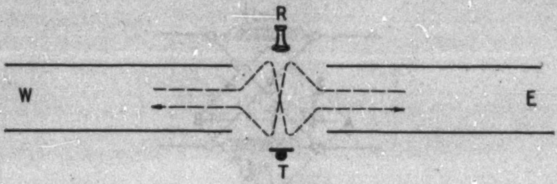

Fig. 1

Repeater networks must not unbalance the lines to which they are connected, and they present, of course, a large number of other practical conditions. These will not be taken up here, but this paper will, however, contribute towards meeting them by discussing the ideal circuits satisfying the above fundamental conditions, without imposing any restrictions upon the magnitudes of the four resistances presented by the two lines and the receiving and transmitting elements of the amplifier.

4. General Properties of Maximum Output Circuits

This investigation of maximum output networks was actually carried out by first determining, some years ago, the possibilities of one two-winding transformer, then of one three-winding transformer, and showing that no further increase in the number of windings on a single transformer would change the restrictions which were imposed on the input ratios and the resistances. When it came to extending the investigation to include all cases where no unnecessary restrictions should be imposed, it became highly desirable, on account of the large number of circuits which must be included, to find a general circuit including all cases, so that the properties of individual circuits could be deduced as special cases. Upon looking at the problem from this viewpoint it was found that the comparatively simple circuit of Fig. 4 presented the general problem; it consists merely of four separate closed loops, each containing one of the four resistances and one winding from each of two four-winding transformers. To distinguish this circuit from others it has been called the master circuit,

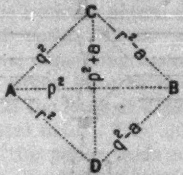

Fig. 2

Certain properties which had been noted in the earlier circuits were found to hold for the master circuit, and the question was raised as to whether they were not fundamental and deducible directly from the conservation of energy. This proved to be the fact, and these general properties will now be considered before proceeding to the master and other concrete circuits.

We assume that one of the four resistances is the source of activity, and that it contains an invariable sinusoidal electromotive force of amplitude E, and has a fixed effective resistance R. Its maximum possible output or external activity has then the definite value $E^2/4R$. In general, the actual output will be only a part of this, and the ratio of this part to the possible maximum is called the output ratio. Each of the other resistances in the circuit will absorb a certain amount of the energy and the ratio of this energy to the maximum possible output $E^2/4R$ is called the corresponding input ratio. The input ratios are positive and not greater than unity, and the sum of all the input ratios for a given impedance paired with every other impedance in the circuit cannot exceed unity.

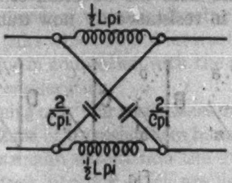

Fig 3

In case the input ratio between two resistances, neither of which vanishes, is zero, the two branches containing the resistances are said to be conjugate. The input ratio possesses the reciprocal property, being unchanged when the electromotive force is transferred so that the driving and driven points change places, the passive network remaining unchanged.

A circuit containing four resistances presents thus only six distinct input ratios between pairs of resistances. These six input ratios are not entirely independent but must satisfy conditions imposed by the conservation of energy as is shown in Appendix II. These conditions assume very simple form when the circuit is a maximum output circuit: the input ratios for any two mutually exclusive pairs of resistances are equal, so that there are but three distinct input ratios; the sum of the three input ratios is equal to unity. One of the three input ratios must be equal to zero and thus a maximum output circuit has two pairs of conjugate resistances. Such a circuit is said to be biconjugate, and the fact that biconjugacy accompanies the maximum output of four resistances is the outstanding characteristic of these circuits. The formal proof of biconjugacy is given in Appendix I in connection with formula (2) which is the mathematical expression of biconjugacy.

There is also a restriction as to the direction of current flow which is given by formula (5) which assumes the simple form $q\,q_1\,r\,r_1 < O$ of (7) if all driving-driven point impedances of the network are pure resistances. This means that three of the four quantities q, q_1, r, r_1 may have their signs arbitrarily determined, but the sign of the fourth must be chosen so that the product of all four will be negative. Physically, this may be explained as follows: Insert a positive electromotive force in resistance A, and note the direction of the current flow which is produced in resistance C; now transfer the electromotive force to C making its positive direction correspond with the direction of current flow, and note the direction of the current produced in resistance B. Proceed in the same way to transfer the electromotive force to B, and then to D; it will be found that the current which flows in A will have its direction opposed to the electromotive force as originally placed in A.

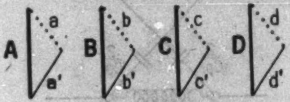

Fig. 4

The three arbitrary sign choices give eight sign combinations, and thus there are for every maximum output circuit eight proportionings which leave the distribution of energy unchanged, but alter the relative direction of current flow in the four resistances. Where the resistances occur in two, three, or four essentially disconnected portions of the circuit the essentially different sign combinations are reduced from eight to four, two, and one, respectively. Thus with the master circuit of Fig. 4, which consists of four entirely distinct loops, the positive direction in each branch is unrelated to the positive direction in the other branches, and the eight sign combinations are not essentially distinct. Because of the alternative sign combinations which are always involved, irrespective of whether they lead to circuits which are practically distinct or not, the mathematical formula for the proportioning of the transformers is complicated when expressed in terms of the input ratios and involves radicals, as is seen by reference to (**30**).

A maximum output circuit with four resistances which does not impose unnecessary restrictions must have at least four degrees of freedom, as is stated in proposition 5 of Appendix I. This is the starting point for the enumeration of all possible circuits employing the minimum number of elements. By permitting two or three restrictions a three-degree circuit may be employed in place of a four-degree circuit, but it is not possible to have a four-resistance maximum output circuit of two degrees of freedom without allowing the resistances to separate into two pairs which do not interact, and such circuits are thus of no value in connection with the applications which we have under consideration.

A circuit containing only four resistances must be a maximum output circuit if three of its resistances have maximum output, or if two have maximum output into the other two, or if one has maximum output into two which are mutually conjugate, as is proved in propositions 6, 7, 8 of Appendix I. These propositions bring the substation set and the repeater circuit within this class of circuit, as was stated above.

Maximum output circuits having more than four resistances present questions of considerable complexity, and a discussion of them is not included in this paper since they are not required for the particular applications under consideration.

5. THE MASTER CIRCUIT

The mathematical treatment of the master circuit is contained in Appendix III, where a rule is given for proportioning the winding turns when the magnitudes of the four resistances, the three input ratios, and the current directions are assigned. The procedure is to calculate the values of P, P_1, Q, Q_1, R, R_1 by formulas (**30**), (**31**), or (**32**), and then use any two of the ratios (**29**), or any linear combinations of the four ratios (**29**) which give distinct proportions. Thus, all eight windings may be retained, or one winding may be dropped from each of the two transformers; but at least one winding must remain in series with each of the four resistances.

In practical applications, it may be necessary to consider the effect of either accidental or intentional variations in the magnitudes of the resistances, or of the turns, which will cause the circuit to depart from exact conformity with the maximum output and biconjugate conditions assumed as the basis of the above rule. Accordingly, a generalized rule is given in formula (**41**) which may be employed for discussing the behavior of the master circuit under any conditions.

In the master circuit we have what is probably the highest possible degree of symmetry among the resistances and windings. This may be of importance in practical applications where the electrical balance of the circuit calls for symmetry, or where a symmetry of the circuit will reduce the effect of the departure of practical transformers from ideal operation.

6. UNRESTRICTED MAXIMUM OUTPUT NETWORKS EMPLOYING THE MINIMUM NUMBER OF WINDINGS

The master circuit with two windings eliminated, is an unrestricted maximum output circuit employing the minimum number of windings; permutation of the resistances gives a total of only six distinct circuits. That permutation of the resistances in the remaining fifteen four-degree geometrical circuits should increase this total to 57,340, or 446,234 cases if attention is paid to the direction of current flow, is one more striking illustration of the large number of possible combinations presented by even a few elements.

All of the distinct master circuits can be made up by the use of two ideal transformers each having six windings with turns in the proportion $P : P_1 : Q : Q_1 : R : R_1$ as is shown by the rule (**29**). Two 18-winding ideal transformers, having the same six winding turns in triplicate, are sufficient to give all of the 446,234 cases, as is shown by the formula (**50**) employed in stating the rule in Appendix IV for calculating the winding turns for all of these cases. Each winding may involve from one to six of the windings connected, either aiding or opposing, but it never requires two windings such as P, and so each set suffices for a winding in every possible case.

The complete list of these circuits is presented by Table V in the form of 16 charts, one for each of the four-degree circuits of Fig. 5. The interpretation of the chart will be understood by considering the meaning of the insert in the chart for circuit **M**, the first line of which reads *M* 2468 *AC* 56.12 = 672. The first group of four figures comprises the four branches

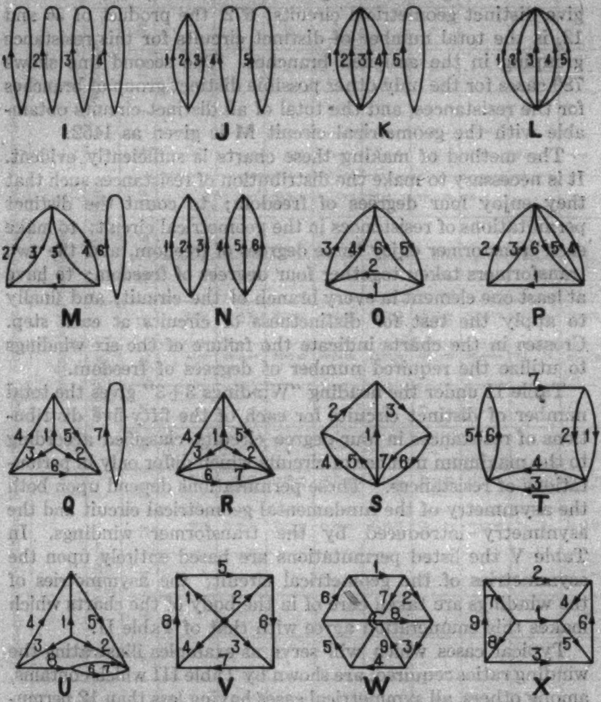
1 2 3 4
I
1 2 3 4 5
J
1 2 3 4 5
K
1 2 3 4 5
L
2 3 5 4 6 1
M
1 2 3 4 5 6
N
3 4 6 5 2 1
O
2 3 6 5 4 1
P
4 1 5 3 2 6 7
Q
4 1 5 3 2 6 7
R
2 3 1 4 5 7 6
S
7 5 6 2 1 4 3
T
4 1 5 3 2 8 6 7
U
5 1 2 8 6 4 3 7
V
1 6 7 2 8 5 9 3 4
W
2 7 4 9 6 8 5 1 3
X

FIG. 5—GEOMETRICAL CIRCUITS WITH FOUR DEGREES OF FREEDOM

in which the four resistances are placed; *A* is a reference letter used in the body of a chart to show every distribution of transformer elements with which this resistance distribution occurs (for example the first entry of *A* shows that the two transformers may have their windings in branches 1, 2, 5 and

1, 2, 6); the next part of the insert is 56, which is the number of times the reference letter A, or C which includes A in combination with other reference letters, occurs in the chart; 12 is the number of permutations of the four resistances, considered non-interchangeable, among the four branches, **2**, **4**, **6**, **3** which give distinct geometrical circuits; 672, the product of 56 and 12, is the total number of distinct circuits for this resistance grouping in the assigned branches. The second line shows 780 cases for the only other possible distinct group of branches for the resistances, and the total of all distinct circuits obtainable with the geometrical circuit **M** is given as 1452.

The method of making these charts is sufficiently evident. It is necessary to make the distribution of resistances such that they enjoy four degrees of freedom; to count the distinct permutations of resistances in the geometrical circuit; to make each transformer enjoy three degrees of freedom, and the two transformers taken together four degrees of freedom; to have at least one element in every branch of the circuit; and finally to apply the test for distinctness of circuits at each step. Crosses in the charts indicate the failure of the six windings to utilize the required number of degrees of freedom.

Table II under the heading "Windings 3+3" gives the total number of distinct circuits for each of the fifty-five distributions of resistances in four-degree circuits, classified according to the maximum number of circuits which differ only in permutations of resistances. These permutations depend upon both the asymmetry of the fundamental geometrical circuit and the asymmetry introduced by the transformer windings. In Table V the listed permutations are based entirely upon the asymmetries of the geometrical circuit; the asymmetries of the windings are taken care of in the body of the charts which makes this enumeration agree with that of Table II.

Typical cases which will serve as examples illustrating the winding ratios required, are shown by Table III which contains, among others, all symmetrical cases having less than 12 permutations. The first line, for example, shows that taking the four-degree circuit **L** and placing the resistances and windings in the branches whose numbers appear immediately below the letters designating these resistances and windings, *viz.*, **1234**, **125**, **345**, we have an unrestricted maximum output circuit provided the windings on one transformer are in the ratio $(P_1 + Q - R)$: $(P_1 - Q_1 + R_1)$: P_1 and on the other

TABLE II

Enumeration of Distinct Maximum Output Networks. With Resistance Permutations.

Four degree circuit .5	Resistances in branches	Windings in branches (part)	Windings and Permutations: Windings 3 + 3					Windings 3 + 2				Windings 2 + 2				Windings 3 + 1			Windings 2 + 1		
			3	4	6	12	24	4	6	12	24	3	6	12	24	4	12	24	3	12	24
I	1234	1234			1																
J	4125	3415				4	4			1	2										
K	1253	4512				5	3	1		3	2						1				
L	1234	5123	1			3		1		2						1					
M	2463	1526				4	26			2	15				1						
	1	3561,2				7	29			1	16			1	2						
N	1245	3614	1			6	7			2	4			1							
O	1235	4612,3			2	11	19			5	17			2	3		1	2			1
P	2451	3645,2,1				11	41			6	28				6						
	3	1624			1	7	18			4	11				1		1				
	6	1345				8	9	1		7	6				1						
Q	4572	6137				7	31			5	14			1	1						
	6	12374				2	9			3	2			1							
R	6732	45167,3				5	21		1	2	9			1	1						
	31	4526				10	53			3	30				3						
	35	4216			1	10	22		1	4	14			1	1						
	6324	7516,3,2					131				76				13		1	1		1	
	1	7524,1				12	63			6	31			3	4						
	11	7426,5,1				14	65			8	36			3	4		1		1		
S	4623	1574,2				5	67			1	34			1	4						
	57	12346				4	8			1	3										
	27	1356				7	52			2	25				2						
	21	3571,2,4					127				67				11						
	51	2374				7	49			2	27				2						
T	1352	7461				8	22			1	13				1						
	7	2467,1		1		10	18			1	11			1	1						
U	6732	45136			1	4	14			3	3										

TABLE II (*Continued*)

Enumeration of Distinct Maximum Output Networks. With Resistance Permutations.

Four degree circuit Fig. 5	Resistances in branches	Windings in branches (part)	Windings and Permutations																		
			Windings 3 + 3					Windings 3 + 2				Windings 2 + 2				Windings 3 + 1			Windings 2 + 1		
			3	4	6	12	24	4	6	12	24	3	6	12	24	4	12	24	3	12	24
	41	53286				2	31				8										
	42	52186			1	5	15			3	3										
	6832	45176,5,3				7	46			6	12			1	1						
	42	5217				13	49			8	18			1	1						
	41	5327					111				44				3						
	6341	52874,1				4	40			3	9			1	1						
	15	4287				10	48			9	18			1	1						
	42	5127					107				45				3						
V	5123	46785,1,2					86				25				3						
	34	2678					108				47				3						
	5614	2378					112				46				3						
	17	2348					117				45				3						
	13	24786,1				2	49				16			1	1						
	5712	34685,1				9	44			2	12			3							
	13	2468				9	52				23			3							
	5678	12345			2	6	11			2	3	1	1								
W	1234	56789				2	22			1	4										
	5	46789				7	22			2	4										
	9	456782				2	3														
X	1247	356891,4				2	7														
	48	35679					47				9										
	69	34578				7	22			2	4										
	68	34579				3	24			2	4										
	1697	23458					50				10										
	47	23589					45				9										
	48	23579					46				9										
	4786	12359				2	21			1	4										
	9	123567,8					12														
Total.................			2	1	9	263	2,255	3	2	116	927	1	1	27	85	1	5	3	1	1	1

T

Examples of Maximum Outpu

Circuit Fig. 5	Resistances $ABCD$	Windings xyz	Windings $x'y'z'$	Limitations imposed on A, B, C, D, p, q, r	Wind		
					x	y	z
L	1234	123	345	..	$P_1 + Q - R$	$P_1 - Q_1 + R_1$	P_1
	1245	126	345	..	$Q - R$	$-Q_1 + R_1$	P_1
T	1387	135	246	..	R	Q_1	P_1
I	1234	123	124	..	R	Q_1	P_1
O	1235	124	126	..	$P_1 + R$	$P_1 + Q_1$	$-P_1$
O	1235	146	246	..	$P + Q - Q_1 + R - R_1$	$-P_1 + R_1$	$P_1 + Q_1$
P	2453	145	236	..	$-R$	$-P - Q_1$	$-P_1 - Q$
R	6735	124	167	..	$P + Q + Q_1 - R - R_1$	$P + Q - R_1$	$P + Q_1 - R$
U	6732	167	438	..	P_1	$Q - R$	$-Q_1 + R_1$
U	6742	167	368	..	P_1	$P_1 + Q - R$	$P_1 - Q_1 + R_1$
V	5678	125	347	..	P_1	R_1	$-P_1 + Q + R_1$
V	5678	127	345	..	$Q_1 + R$	$-P + Q_1$	$P_1 - Q - R_1$
						Sub Totals (including all unrestricted circuits not in	
L	1234	123	45–	$P - Q + R_1 = O$	R	Q_1	P_1
K	1253	123	45–	$P + Q_1 - R = O$	Q	R_1	$-P_1$
P	2456	456	13–	$P_1 - Q_1 + R_1 = O$	P	Q	R
R	6732	167	45–	$P + Q + Q_1 - R - R_1 = O$	P_1	$Q - R$	$-Q_1 + R_1$
R	6735	167	24–	$P + Q + Q_1 - R - R_1 = O$	$-P_1$	$P_1 + Q + R$	$P_1 + Q_1 + R_1$
						Sub Totals (including all circuits with one restrictio	
V	5678	13–	24–	$P - P_1 + Q - Q_1 = Q + Q_1 + R + R_1 = O^1$	R	R_1	..
V	5678	12–	34–	$P + Q + R = P_1 - Q - R_1 = O^1$	P_1	R_1	..
						Sub Totals (including all circuits with two restricti	
L (*F*)[7]	1234	123	5––	$P - Q + R_1 = P + Q_1 - R = O^2$	R	Q_1	P_1
K (*E*)	1253	125	4––	$P + Q_1 - R = P_1 + R_1 = O^3$	R	Q_1	P_1
O (*G*)	1235	124	6––	$P_1 - Q = P_1 - R_1 = O^3$	$P_1 + R$	$P_1 + Q_1$	$-P_1$
P (*G*)	2453	124	6––	$P - Q - R_1 = P_1 - Q_1 - R_1 = O^2$	$-P_1$	$P_1 + Q$	R_1
R (*H*)	6324	125	7––	$P_1 + Q = P + Q_1 + R = O^3$	$P + R$	$P - Q$	$-R$
R (*H*)	6351	246	7––	$P_1 - Q - R = P + Q + R_1 = O^1$	$Q_1 + R_1$	$-P_1 - R_1$	$P - Q - R$
O (*G*)	1235	134	6––	$P_1 - Q = P_1 - R_1 = O^3$	$-Q_1 + R$	$P_1 + Q_1$	Q_1
O (*G*)	1235	145	6––	$P_1 - Q = P_1 - R_1 = O^3$	$P + Q - R_1$	R_1	$-P_1 - Q_1$
R (*H*)	6324	156	7––	$P_1 + Q = P + Q_1 + R = O^3$	$-P_1 + Q_1 + R_1$	$P_1 - Q_1$	$P - Q$
R (*H*)	6351	24–	7––	$P - Q - R = P - P_1 = P + Q + R_1 = O^4$	P	$-Q$	..
R (*H*)	6324	15–	7––	$P - Q = P_1 + Q = P_1 - Q_1 - R = O^6$	$R - R_1$	$-R$	..
O (*G*)	1235	14–	6––	$P_1 + Q_1 = P_1 - Q = P_1 - R_1 = O^6$	$R + R_1$	$-R_1$	..

1 $A + B = C + D$

2 $\dfrac{AB}{A + B} = \dfrac{CD}{C + D}$

3 $D = \dfrac{AB}{A + B}$

4 $A = D,\ B = C$

5 $C = A + B,\ D = \dfrac{AB}{A + B}$

6 $A = \dfrac{D}{C}(C + D),\ B = C + D$

7. After the branch of the four-degree circuit has been opened b
Fig. 6 as shown in parentheses, except in numbering and direction of
8. "etc." indicates that the three other circuits in the same colu
9. One of these cases gives an imaginary solution and is not inc

TABLE III

utput Circuits with Minimum Number of Windings

	Winding Ratios						
	x' :	y' :	z'	Symmetries	Permutations of resistances	Sign combinations	Total cases
	$P-Q+R_1$ P $-P-Q-R$ Q $-P_1+Q$ $-P-Q+Q_1-R+R_1$ $P-Q-R_1$ $-P_1$ $-Q+R_1$ $-Q+R_1$ P $-Q_1-R_1$	$P+Q_1-R$ $-Q+R_1$ $P-Q_1-R_1$ R_1 $-P_1+R_1$ $-P_1+Q$ $P_1-Q_1-R_1$ P_1+Q+R $-Q_1+R$ $-Q_1+R$ $-R$ $-P_1-Q_1$	P Q_1-R $-P_1+Q+R_1$ $-P_1$ P_1 P_1+R R_1 $P_1+Q_1+R_1$ $-P-Q-Q_1+R+R_1$ $-P+Q-Q_1+R-R_1$ $-P-Q-R$ $P+Q+R$	(1, 2); (3, 4); (12, 34) (1, 3); (4, 5); (123, 456) (12, 34, 56) (1, 3); (3, 4) (1, 2); (34, 56) (1, 3); (34, 56) (2, 3); (4, 5) (6, 7); (23, 45) (6, 7); (24, 35) (6, 7); (25, [illegible]) (136, 245); (126, 437) (136, 245); (126, 437)	3 3 4 6 6 6 6 6 6 6 6 6	8 4 8 1 8 8 8 8 8 8 8 8	24 12 32 6 48 48 48 48 48 48 48 48
ot in this table)…………					87,340		440,224
	$P+Q_1-R$ Q_1 $-P_1+Q-R$ $-Q+R_1$ $P+Q-R_1$	P $-P_1-R_1$ $-Q+R$ $P+Q-R_1$ $P+Q_1-R$	.. ..	(1, 2, 3) (1, 2, 3) (4, 5, 6) (6, 7); (24, 35) (6, 7); (23, 45)	4 4 4 6 6	6 3 6 7–8 7–8	24 12 24 44 44
riction not in this table)…………					23,664		124,536
	P P	$-P_1$ $-R$	.. ..	(136, 245); (126, 437); (137, 256) (136, 245); (126, 437)	3 6	4 4	12 24
rictions not in this table)…………					2,561		6,596
	Circuits on Fig. 7						
	I 13–16 II 1–3 etc.[8] II 19, 22, 25, etc.[8] I 1–12 II 8, 13, 18, etc.[8] I 17–28 II 4, 5, 9, 10, 14, 15, etc.[8] II 6, 7, 11, 12, 16, 17, etc.[8] II 20, 21, 23, 24, 26, 27, etc.[8]			(1, 2, 3) (1, 2) (1, 2) (23, 45) (13, 56) (23, 46)	4 12 12 12 12 12 24 24 24	4 1 2 4 2 4 2 2 2	16 12 24 48 24 48 48 48 48
				Sub Totals…………	136		316
	V 1, 2 III 1, etc.[8] IV 1, 13, etc.[8] III 2, 3, etc.[8] IV 2, 3, 14, 15, etc.[8]			(12, 34); (13, 35); (13, 56) (13, [illegible])	3[9] 12 24	2 1 1	4 12 24
				Sub Totals…………	36		40
				Grand Totals…………	[illegible]		[illegible]

[CAMPBELL & FOSTER]

ed by the infinite impedance of x', the circuit becomes the three-degree circuit of
n of branches.
column of Fig. 7 are included.
included in the total.

transformer in the ratio $(P - Q + R_1)$: $(P + Q_1 - R)$: P. Moreover this circuit is symmetrical with respect to branches 1 and 2 which may be interchanged without giving a distinct circuit; similarly with respect to branches 3 and 4; and finally there is a third symmetry which does not alter the circuit, if branches 1 and 2 are changed with branches 3 and 4, respectively. This leaves three distinct permutations for the resistances, and since circuit L has eight sign combinations for the currents, the total number of different cases is 24.

In place of two transformers each having three windings, we might employ one transformer having a two-mesh core and six windings, since the two types of transformers are shown by Appendix VI to be exactly equivalent when they may be regarded as ideal transformers. If two windings, one from each of the two transformers, are in series, the two-mesh transformer may be arranged to replace them by one winding; a total of five windings will then be sufficient. With actual transformers there are practical differences between single-mesh core and multi-mesh core types, which has led to a proposal that multi-mesh core transformers be used, the original suggestion having been made by Mr. K. S. Johnson.

7. MAXIMUM OUTPUT CIRCUITS EMPLOYING ONE TRANSFORMER

If certain relations hold between the resistances and the input ratios, one or two of the windings of one or both of the three-winding transformers may require only zero turns, when certain circuit connections are employed; these special cases have been included in Tables I, II, and III, and the restrictions or relations imposed are given by Table III. The more important, of the special cases are those in which one of the two transformers may be entirely eliminated; this happens when two of the three windings vanish, leaving the other winding to open effectively the branch in which it occurs on account of the infinite impedance it introduces, and thereby reduce the degree of the circuit from four to three.

All of the 174 distinct maximum output circuits with four resistances and one transformer having no superfluous windings are shown in Fig. 7*, distinct symbols being employed to represent the transmitter T, the receiver R, the line L, auxiliary resistance X, and the transformer windings. The geometrical circuits involved are of the third degree and are designated as **E, F, G, H** on Fig. 6, but the circuits of Fig. 7 do not happen to be grouped according to the geometrical circuits, since the drawing was taken from the original study of substation circuits made several years ago, which did not start from the geometrical circuit idea, but from the different possible groupings of the resistances and the various arrangements of windings connecting selected resistance terminals. The correlation between the circuits of Fig. 7 and the three-degree circuits of Fig. 6 is supplied by the insert headed "Circuits on Fig. 7" of Table III.

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*The correctness of this enumeration of the 174 circuits has been independently checked by Mr. F. W. McKown.

In Table III all the information required for making particular applications of the maximum output circuits of Fig. 7

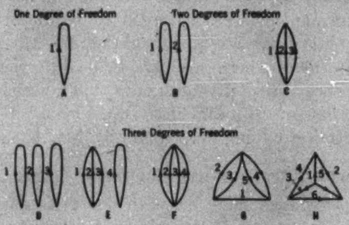


FIG. 6— GEOMETRICAL CIRCUITS

is given; this includes the limitations imposed on the resistance and input ratios A, B, C, D, p, q, r and the ratios of the windings expressed in terms of the quantities P, P_1, Q, Q_1, R, R_1 which are defined by formulas (**30, 31, 32**). The remaining columns show the symmetries presented by the circuit, the number of permutations, sign combinations, and total cases which have been explained above in connection with the first group of circuits in Table III.

The limitations imposed by the employment of only one transformer for a maximum output circuit are the most important practical aspect of these circuits. These limitations, given in the fifth column of Table III, are repeated in expanded form in Table IV employing the notation $T/L, R/L, y = q^2/r^2$ for the resistance ratio of transmitter to line, resistance ratio of receiver to line, and the ratio of the inputs from the line

TABLE IV—EQUATIONS FOR FIG. 8

				Additional Curves for Three Resistance Circuits
A	$\sqrt{\frac{L}{R}} = \sqrt{y\frac{L}{T}} = \sqrt{1+y}$		A′	$\sqrt{\frac{R}{L}} + \sqrt{y\frac{T}{L}} = \sqrt{1+y}$
B	$\frac{R}{L} = \frac{1}{y}\,\frac{T}{L}$		B′	$\frac{R}{L} = y\frac{T}{L}$
C	$\frac{R}{L} = \frac{1}{1+y}$		C′	$\frac{R}{L} = 1+y$
D	$\frac{T}{L} = \frac{y}{1+y}$		D′	$\frac{T}{L} = \frac{1+y}{y}$
E	$\frac{T}{L} = y\,(1+y),$	$\frac{R}{L} = 1+y$		
F	$\frac{T}{L} = \frac{1+y}{y},$	$\frac{R}{L} = \frac{1+y}{y^2}$		
G	$\frac{T}{L} = \frac{1+y}{y},$	$\frac{R}{L} = 1+y$		
H	$\frac{T}{L} = \frac{y}{1+y},$	$\frac{R}{L} = \frac{y^2}{1+y}$		
I	$\frac{T}{L} = \frac{y}{1+y},$	$\frac{R}{L} = \frac{1}{1+y}$		
J	$\frac{T}{L} = \frac{1}{y\,(1+y)},$	$\frac{R}{L} = \frac{1}{1+y}$		
K	$\frac{T}{L} = \frac{1+y}{y},$	$\frac{R}{L} = \frac{1}{1+y}$		
L	$\frac{T}{L} = \frac{y}{1+y},$	$\frac{R}{L} = 1+y$		
M, M′	$\frac{T}{L} = 1,$	$\frac{R}{L} = (\sqrt{1+y} + \sqrt{y})^2$		
N, N′	$\frac{T}{L} = \frac{(\sqrt{1+y}+1)^2}{y}$	$\frac{R}{L} = 1$		

into the transmitter and receiver, respectively. These limitations are also given in graphical form by Fig. 8 where the two resistance ratios are plotted for each of the circuits for both $y = 1$ and $y = 1.5$. The curves occurring on this drawing are marked with reference letters *A, B. . . . M, M′, N, N′* which also occur on Fig. 7, by means of which the curve applying to any one of the 174 circuits may be readily found. It will be noted that a few of the curves have more than one branch for a given value of y, *e. g.*, *A, N, N′, M, M′*, the last of which falls off the drawing.

Fig. 8 shows, for example, that if $(T/L = 1,\ R/L = 2)$ no

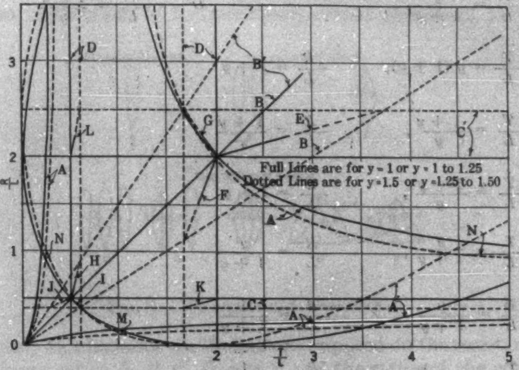
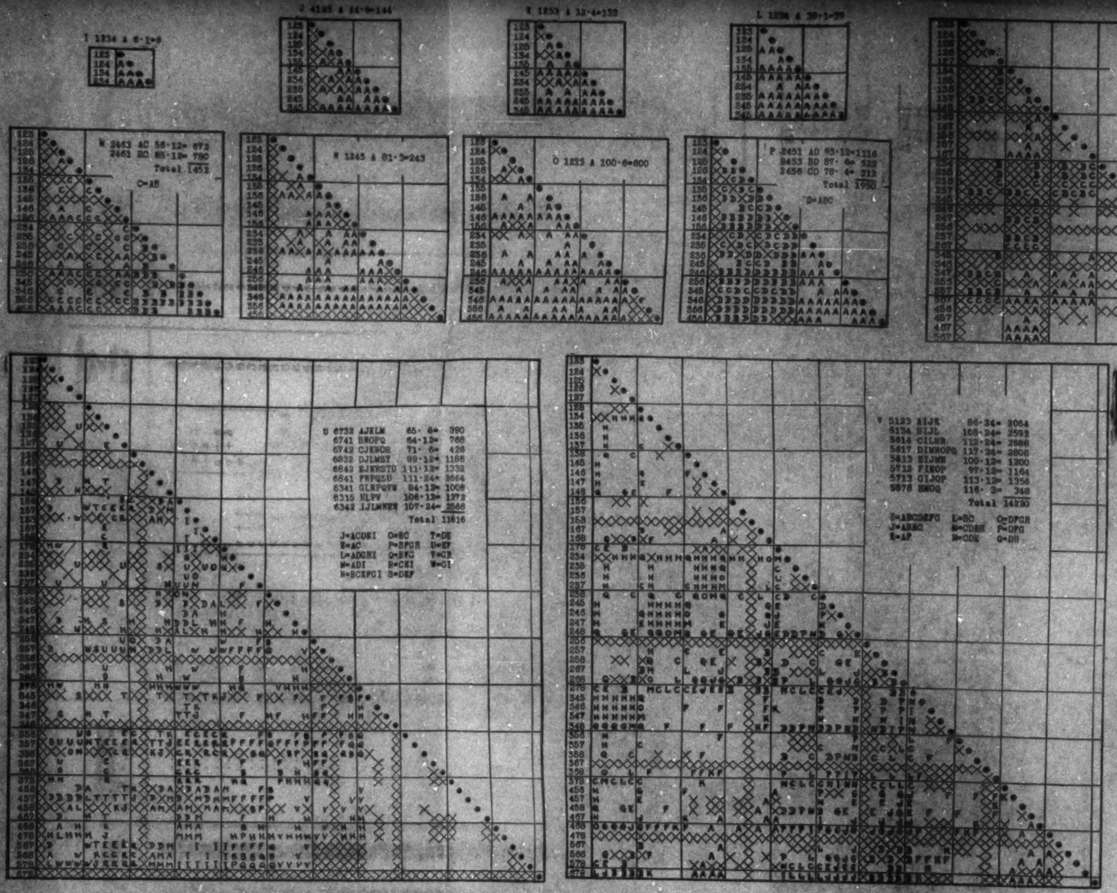


Fig. 8

one-transformer circuit can be adapted to give maximum output with any value of y in the range from 1 to 1.5. If, however, $(T/L = 2,\ R/L = 2,\ y = 1)$ all of the circuits bearing the reference letters *A, B, E, F,* and *G* may be employed; these are circuits *I*, 1-28; *II*, 1-54; *III*, 1-3; *IV*, 13-24 of Fig. 7, a total of 97 distinct circuits from which to choose in making practical application.

8. MAXIMUM OUTPUT CIRCUITS WITH SUPERFLUOUS WINDINGS

The enumeration of maximum output circuits, with the minimum number of windings, has been carried through for


I 1234 A 6·1=6
J 4125 A 24·6=144
K 1253 A 33·4=132
L 1236 A 39·1=39
M 2463 AC 56·12= 672
2461 BC 65·12= 780
Total 1452
C=AB
N 1245 A 81·3=243
O 1235 A 100·6=600
P 2451 AD 93·12=1116
2453 BD 87·6= 522
2456 CD 78·4= 312
Total 1950
D=ABC
Q 4573 AC 69·12= 828
4576 BC 60·4= 240
Total 1068
C=AB
U 6732 AJKLM 65·6= 390
6741 BNOPQ 64·12= 768
6742 CJKNOR 71·6= 426
6832 DJLMST 99·12= 1188
6842 EJNRSTU 111·12= 1332
6841 FNPQSU 111·24= 2664
6341 GLNPQVW 84·12= 1008
6315 HLPV 106·12= 1272
6342 IJLMNRW 107·24= 2568
Total 11616
J=ACDEI
K=AC
L=ADGHI
M=ADI
N=BCEFGI
O=BC
P=BFGH
Q=BFG
R=CEI
S=DEF
T=DE
U=EF
V=GH
W=GI
V 5123 AIJK 86·24= 2064
5134 BIJL 108·24= 2592
5614 CILMN 112·24= 2688
5617 DIMNOPQ 117·24= 2808
5613 EIJMN 100·12= 1200
5712 FIKOP 97·12= 1164
5713 GIJOP 113·12= 1356
5876 HMOQ 116·3= 348
Total 14220
I=ABCDEFG
J=ABEG
K=AF
L=BC
M=CDEH
N=CDE
O=DFGH
P=DFG
Q=DH

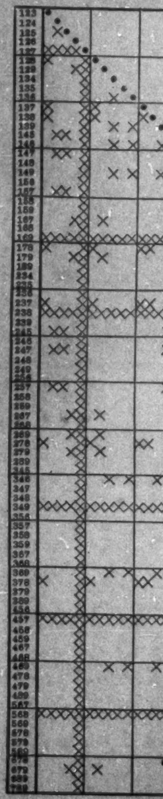
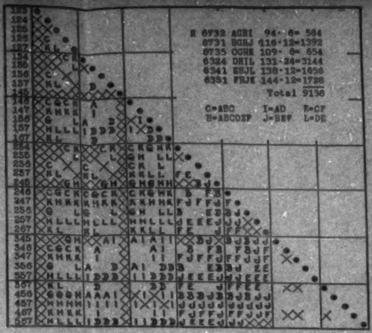
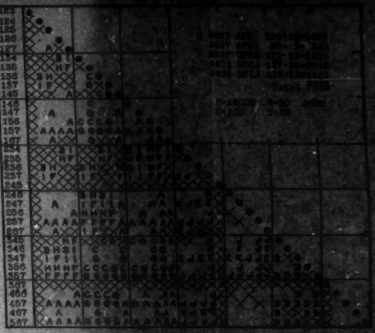
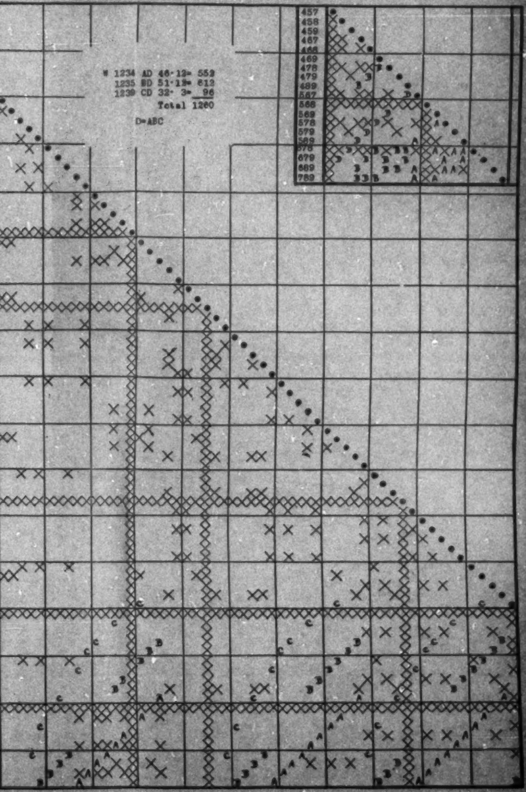


TABLE V

At the left of each chart is a list of all possible sets of three numbers, or triads, representing three distinct branches of the corresponding geometrical circuit on Fig. 5. Each chart is a double-entry table providing entries corresponding to every pair of triads listed. Each column of a chart refers to the same triad as the corresponding row, but the headings of the columns are omitted, the diagonal line of black dots serving instead. Thus each entry refers to two triads, one in the horizontal row with the entry and the other in the horizontal row with the black dot vertically above the entry. For example, the first entry *H* in the chart for *X* refers to the triads **135** and **129**.


R 6732 AGHI 94· 6= 564
6731 BGHJ 116·12=1392
6735 CGHK 109· 6= 654
6324 DHIL 131·24=3144
6341 EHJL 138·12=1656
6351 FHJK 144·12=1728
Total 9138
G=ABC I=AD K=CF
H=ABCDEF J=BEF L=DE


Total


W 1234 AD 46·12= 552
1235 BD 51·12= 612
1239 CD 32· 3= 96
Total 1260
D=ABC

Total 6744

THRE
One Re

Circuit Fig. 6	Resistances *ABC*	Windings *xy*	*x'y'*	Limitations imposed on *A, B, C, p, q, r*	
D	123	12	13	..	*b*
E	124	13	14	..	*a* + *b*
		12	34	..	*b*
		13	24	..	*a* + *b*
		13	34	..	*a* + *b*
		14	34	..	*c*
F	123	12	14	..	*b*
		12	34	..	*b*
		14	24	..	*a* + *b* +
G	124	13	15	..	*a* − *b*
		12	35	..	*b*
		13	25	..	*a* − *b*
		15	23	..	*a* + *c*
		13	35	..	*a* − *b*
		23	25	..	*a* − *b*
		23	35	..	*a* − *b*
		23	45	..	*a* + *b*
		24	35	..	*c*
		25	34	..	*a* − *c*
		25	35	..	*a* − *c*
G	234	12	15	..	*a*
		12	25	..	*a*
		12	35	..	*a*
		15	23	..	*a* + *b* −
		12	45	..	*a*
		14	25	..	*c*
		15	24	..	*a* + *b* −
		15	25	..	*a* + *b* −
H	124	13	56	..	*a* + *b*
		16	35	..	*a* + *b* +
		23	56	..	*a* + *b*
		25	36	..	*a* + *c*
		26	35	..	*a* + *b* +
		35	36	..	*a* + *c*
		36	56	..	*a* + *b* +
H	125	13	46	..	*a* + *b*
		16	34	..	*b* + *c*
Sub-Totals....................					
E	124	14	3 −	$a + b = 0$	[illegible]
F	123	12	4 −	$a + b + c = 0$	*b*
G	124	13	5 −	$a - c = 0$	*a* − *b*
G	124	23	5 −	$a - c = 0$	*a* − *b*
Sub-Totals....................					
G	124	3 −	5 −	$a = b = c$	
Sub-Totals....................					
Totals—Series Type....................					
D	123	123		..	*a'*
E	124	134		..	*a'* − *b'*
F	123	124		..	*a'* − *c'*
G	124	135		..	*a'* + *b'*
G	124	235		..	*a'* + *b'*
G	234	125		..	*b'* + *c'*
H	124	356		..	*a'* − *c'*
Sub-Totals....................					
E	124	34 −		$a' - b' = 0$	*b'*
F	123	14 −		$b' - c' = 0$	*a'* − *c'*
G	124	35 −		$a' + b' + c' = 0$	*b'*
G	234	15 −		$a' - b' = 0$	*b'* + *c'*
Sub-Totals....................					
F	123	4 −		$a' = b' = c'$	
Sub-Totals....................					
Totals—Parallel Type....................					
Grand Totals....................					

$a = pq$ | $\sqrt{B\,C}$

$b = rp$ | $\sqrt{A\,C}$

$c = qr$ | $\sqrt{A\,B}$

TABLE VI

HREE RESISTANCE CIRCUITS

Resistance Having Maximum Output

Winding Ratios x : y		x′ : y′		Permutations of resistances	Sign combinations	Total cases	Circuit number on Fig 9
	−a	c	−a	3	1	3	A1–3
	a	c	−a	6	2	12	4–9
	−a	c	a + b	3	2	6	10–12
	a	c	−b	6	2	12	13–18
	a	c	a + b	6	2	12	19–24
	−a	c	a + b	6	2	12	25–30
	−a	a + b + c	a	6	4	24	31–36
	−a	a + b + c	c	3	4	12	37–39
+ c	a	a + b + c	b	3	4	12	40–42
	−a	a − c	−a	3	4	12	43–45
	−a	a − c	−a + b	6	4	24	46–51
	−a	a − c	−b	6	4	24	52–57
	−a	a − b	−b	6	4	24	58–63
	−a	a − c	−a + b	6	4	24	64–69
	−b	a − c	−b	6	4	24	70–75
	−b	a − c	−a + b	6	4	24	76–81
	−b	a − c	−c	3	4	12	82–84
	−b	a − c	−a + b	3	4	12	85–87
	−b	c	−a + b	3	4	12	88–90
	−b	a − c	−a + b	6	4	24	91–96
	−a − b	a + b − c	−a − b	6	4	24	97–102
	−a − b	a + b − c	−a	6	4	24	103–108
	−a − b	a + b − c	−b	6	4	24	109–114
− c	−a − b	b	−a	3	4	12	115–117
	−a − b	a + b − c	−c	6	4	24	118–123
	−a − b	a + b − c	−a	6	4	24	124–129
− c	−a − b	c	−a	6	4	24	130–135
− c	−a − b	a + b − c	−a	6	4	24	136–141
	a	a + b + c	a + c	6	4	24	142–147
+ c	a	a + c	a + b	3	4	12	148–150
	b	a + b + c	a + c	6	4	24	151–156
	−b	a + b + c	−a − b	6	4	24	157–162
+ c	b	a + c	a + b	6	4	24	163–168
	a + b	a + b + c	−a − b	6	4	24	169–174
+ c	−a − b	a + b + c	a + c	3	4	12	175–177
	a	b + c	−a + c	6	4	24	178–183
	a	a − c	−a − b	3	4	12	184–186
				186		681	
	−a	..	..	3	1	3	A187–189
	−a	..	..	3	3	9	190–192
	−a	..	..	3	2	6	193–195
	−a	..	..	3	2	6	196–198
				12		24	
	..	..	..	1	1	1	A199
				1		1	
				199		706	
	−b′	c′		1	1	1	B1
	−b′	c′		6	2	12	2–7
	b′ − c′	−c′		3	4	12	8–10
b′ + c′	−b′	−c′		3	4	12	11–13
+ c′	a′ + c′	−c′		6	4	24	14–19
	a′ − b′	−c′		6	4	24	20–25
	−a′ + b′	−a′ + b′ + c′		3	4	12	26–28
				28		97	
	−c′	..		3	1	3	B29–31
	c′	..		3	2	6	32–34
	a′	..		3	3	9	35–37
	−c′	..		3	2	6	38–40
				12		24	
	..	..		1	1	1	B41
				1		1	
				41		122	
				240		828	

$pq = r$ if A has maximum output

with $pr = q$ " B " "

$qr = p$ " C " "

the purpose of facilitating the selection of the simplest possible circuit for practical applications. In many cases, however, conditions will be presented which cannot be adequately met by the use of just these circuits, and it is necessary to turn to others which have a somewhat larger number of windings. When no limit is placed on the number of windings, there is also no limit to the number of possible maximum output circuits.

A group of circuits which seem to present certain broad practical possibilities are the fifty-five four-degree circuits with four resistances of Table II, when employed with a winding in each branch from each of two transformers. This gives from two to twelve extra windings and permits the introduction of additional symmetries. The circuit having two superfluous windings is the master circuit which is important on account of the high degree of symmetry it presents.

Maximum output circuits may be obtained from any geometrical circuit with four or more degrees of freedom by inserting four resistances in four branches which utilize four degrees of freedom, and then proportioning two or more ideal transformers so as to meet the master circuit requirements for the desired choice of conjugate pairs of resistances, input ratios, and directions of current flow. A possible way of proportioning the transformers is given in Appendix IV.

9. CIRCUITS COMPOSED OF THREE RESISTANCES AND IDEAL TRANSFORMERS

The ordinary telephone substation employs a transformer, or induction coil, to connect the three resistance elements, transmitter, receiver, and line, and includes no balancing resistance. In order that a comparison of these circuits with the maximum output circuits may readily be made by any one interested in substation circuits, these circuits will be discussed and all possible cases involving the minimum number of elements will be listed.

Appendix V gives the mathematical discussion from which it appears that there are two entirely distinct types of circuit, for one of which the plus sign of (**54**) holds, and for the other the minus sign. These will be called the series and parallel types of circuit, respectively, since the two types have the directions of current flow corresponding to the simple series and parallel connection of three resistances. This distinction was first pointed out, and the terminology adopted by, Mr. K. S. Johnson, who made a systematic investigation and enumeration of the 54 circuits obtainable with three resistances and not more than one transformer. While the two types of circuit differ as regards the directions of current flow, the energy relations which depend upon the squares of p, q, r are the same for both types. One resistance may be made to have its maximum output divided arbitrarily between the other two resistances; but it is not possible to have maximum output from all three resistances, or from two resistances, unless the third resistance is effectively disconnected from the system.

The enumeration of all circuits containing the minimum number of elements which impose no restrictions, in addition to those which are inherent between the input ratios and the resistance ratios, has been made in the same matter as for the four-resistance maximum output circuits. The following totals were found:

TABLE VII

Type	Transformers	Windings	Restrictions	Distinct circuits	Total cases
Series	2	2+2	0	186	681
Parallel	1	3	0	28	97
Series	1	2	1	12	24
Parallel	1	2	1	12	24
Series	0	0	2	1	1
Parallel	0	0	2	1	1
				240	828

All 240 distinct circuits are presented by Fig. 9 where the three resistances are drawn in heavy lines, the windings of one transformer in light lines and of the other transformer dotted. On Fig. 9 series type circuits appear above, and parallel type circuits below; all circuits imposing no restrictions occur in the left blocks, those imposing one restriction in the middle blocks, and those imposing two restrictions in the right. Several different restrictions are involved which are designated by the letters A, A', B, B', C, C', D, D', G, I placed at the left of each row of circuits to which they apply. The same letters are found on Fig. 8 where the restrictions between the values of R/L, T/L, and $y = p^2/q^2$ are shown graphically, on the assumption that $p^2 + q^2 = 1$ so that the line L has maximum output.

All the information necessary for calculating the turn-ratios of the transformer windings, to give maximum output of any one of the three resistances divided in any desired proportion between the other two resistances and with any possible directions of current flow, is given in Table VI. This table is similar to Table III and does not require special explanation. Since Table VI gives the complete rules for proportioning each of the circuits, it is not necessary to go back to the formulas of Appendix V in making the calculation for a particular circuit.

Without the restriction to the minimum number of elements there is no limit to the total number of possible circuits. Any circuit with three or more degrees of freedom, having the three resistance elements in branches utilizing three degrees of freedom, and a winding from each of one (two) or more ideal transformers in every branch, may be proportioned to be an unrestricted parallel (series) type circuit. Every such circuit is equivalent to one of the 214 unrestricted circuits shown on Fig. 9, so far as the electrical behavior in the resistance elements is concerned, as is readily seen from the properties of ideal transformers stated in Appendix VI.

Two two-winding transformers so connected into a circuit that one, and only one, closed path is formed by windings, are equivalent to a three-winding transformer, and for some purposes may be regarded as of the same order of complexity. Accordingly it may be noted that there are 186 unrestricted parallel type circuits each employing two two-winding transformers, equivalent in electrical behavior to the 28 unrestricted parallel type circuits using a three-winding transformer shown on Fig. 9.

APPENDIX I

GENERAL PROPERTIES OF MAXIMUM OUTPUT NETWORKS AND CIRCUITS

Without going into details with respect to the network to be employed, a number of important properties of maximum output networks and circuits can be deduced from the conservation of energy and other fundamental considerations. These properties will first be stated as eleven propositions and will then be proved.

1. The network connecting the four resistances can contain

no resistances, or other sources of dissipation, and must thus be made up entirely of reactances.

2. Between the six pairs of the four resistances ($A, B; C, D$) ($A, C; B, D$) ($A, D; B, C$) there are but three input ratios p^2, q^2, r^2; their total is equal to unity, and at least one of the three must vanish, *i. e.*,

$$p^2 + q^2 + r^2 = 1 \tag{1}$$

$$p\,q\,r = 0 \tag{2}$$

3. The driving point impedances S_{11}, S_{22}, S_{33}, S_{44} and driving-driven point impedances S_{12}, S_{13}, etc., or one-point and two-point impedances between points 1, 2, 3, 4 located in the four resistances A, B, C, D have the following magnitudes;

$$\left.\begin{aligned} S_{11} = 2\,A,\ \ S_{22} = 2\,B \\ S_{33} = 2\,C,\ \ S_{44} = 2\,D \end{aligned}\right\} \tag{3}$$

$$\left.\begin{aligned} |\,S_{12}\,| &= \frac{2\,|\,\sqrt{A\,B}\,|}{|\,p\,|},\ |\,S_{34}\,| = \frac{2\,|\,\sqrt{C\,D}\,|}{|\,p\,|} \\ |\,S_{13}\,| &= \frac{2\,|\,\sqrt{A\,C}\,|}{|\,q\,|},\ |\,S_{24}\,| = \frac{2\,|\,\sqrt{B\,D}\,|}{|\,q\,|} \\ |\,S_{23}\,| &= \frac{2\,|\,\sqrt{B\,C}\,|}{|\,r\,|},\ |\,S_{14}\,| = \frac{2\,|\,\sqrt{A\,D}\,|}{|\,r\,|} \end{aligned}\right\} \tag{4}$$

4. The phase angles of the two-point impedances are subject to one condition, provided four of these impedances are finite, *viz.*, the angles of the products of non-adjacent finite two-point impedances differ by 180 degrees, *e. g.*,

$$a\,r\,g\,(S_{13}\,S_{24}) = a\,r\,g\,(-\,S_{14}\,S_{23}) \tag{5}$$

a condition which is also contained in the relation

$$\frac{S_{13}\,S_{23}'}{S_{14}\,S_{24}'} = -\,\frac{C}{D}\text{, where } S_{23}' = \text{conjugate of } S_{23} \tag{6}$$

If all S's are real and $p^2 = 0$ we may conveniently use the expressions

$$\left.\begin{aligned} S_{13} &= \frac{2}{q}\left|\sqrt{A\,C}\right|,\ S_{24} = \frac{2}{q_1}\left|\sqrt{B\,D}\right| \\ S_{14} &= \frac{2}{r}\left|\sqrt{A\,D}\right|,\ S_{23} = \frac{2_1}{r}\left|\sqrt{B\,C}\right| \\ &q^2 + r^2 = 1,\ \ q_1 = \pm\,q,\ \ r_1 = \mp\,r \\ &q\,q_1\ r\,r_1 < 0. \end{aligned}\right\} \tag{7}$$

where q, q_1, r, r_1 are real, either positive or negative.

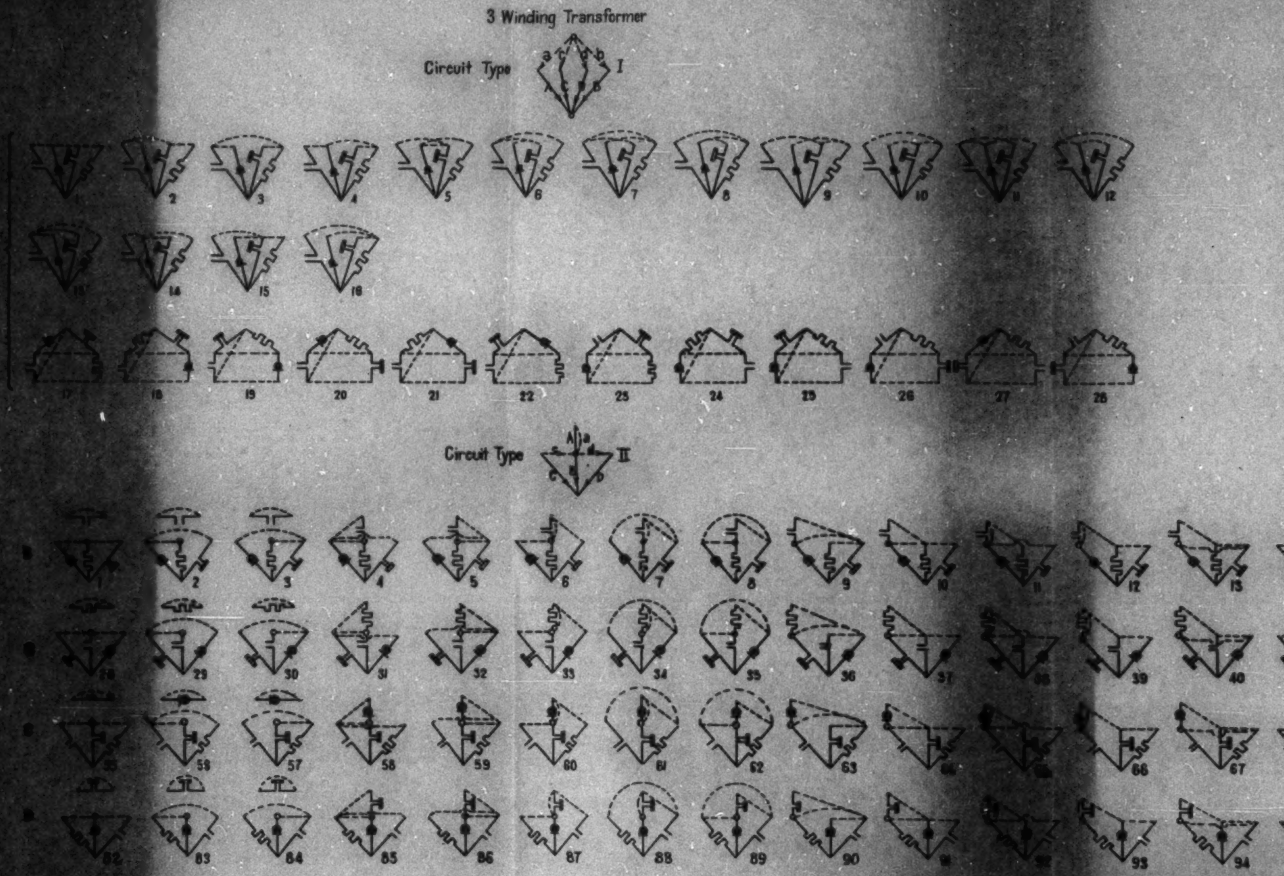
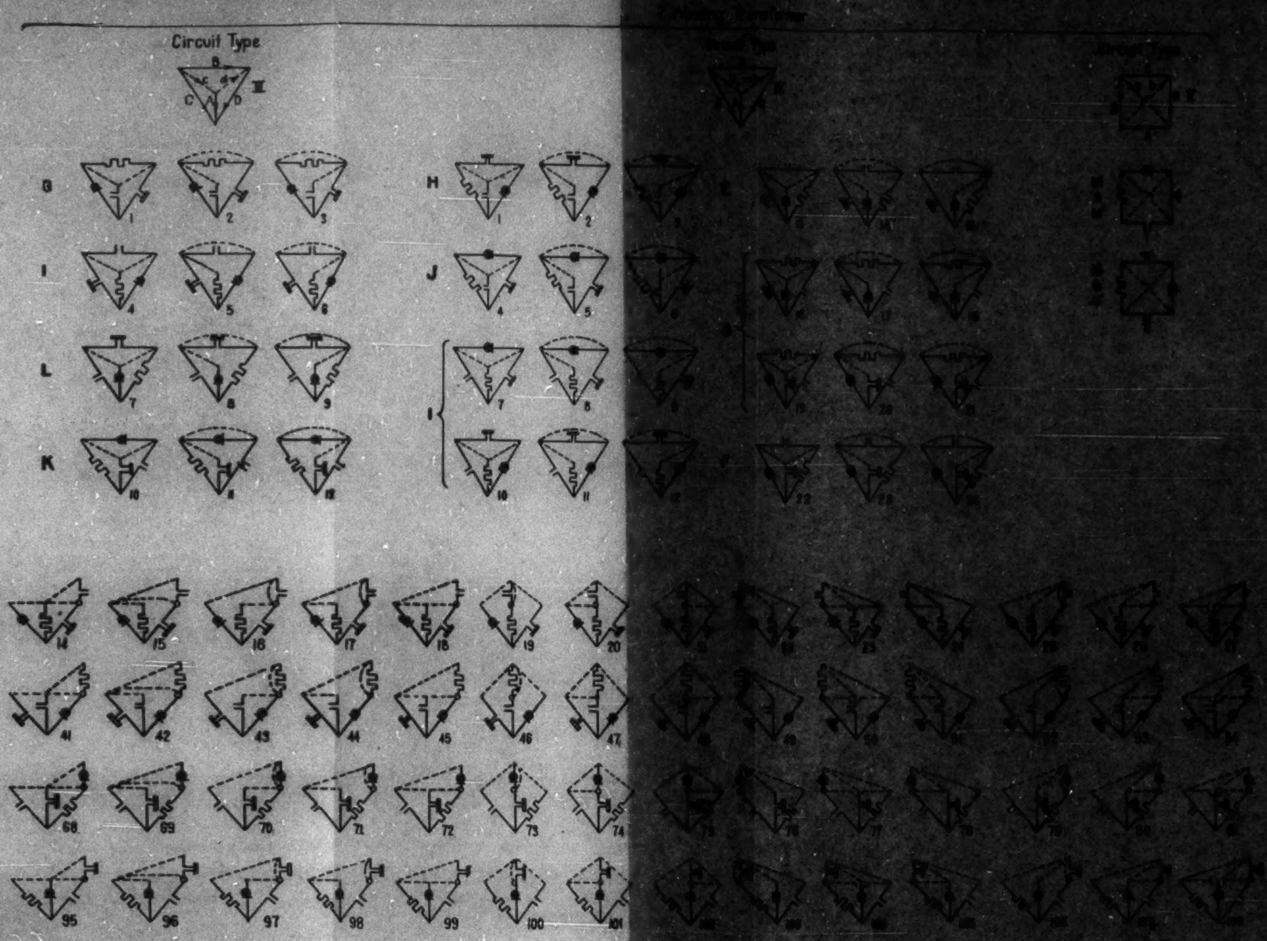

3 Winding Transformer
Circuit Type
I
A
Circuit Type
II

FIG. 7—IDEAL INVARIABLE SUBSTATIONS EMPLOYING O



2-Winding Transformer
Circuit Type
B
C
A
D
III
IV
Circuit Type
V
G
H
I
J
K
L
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2
3
4
5
6
7
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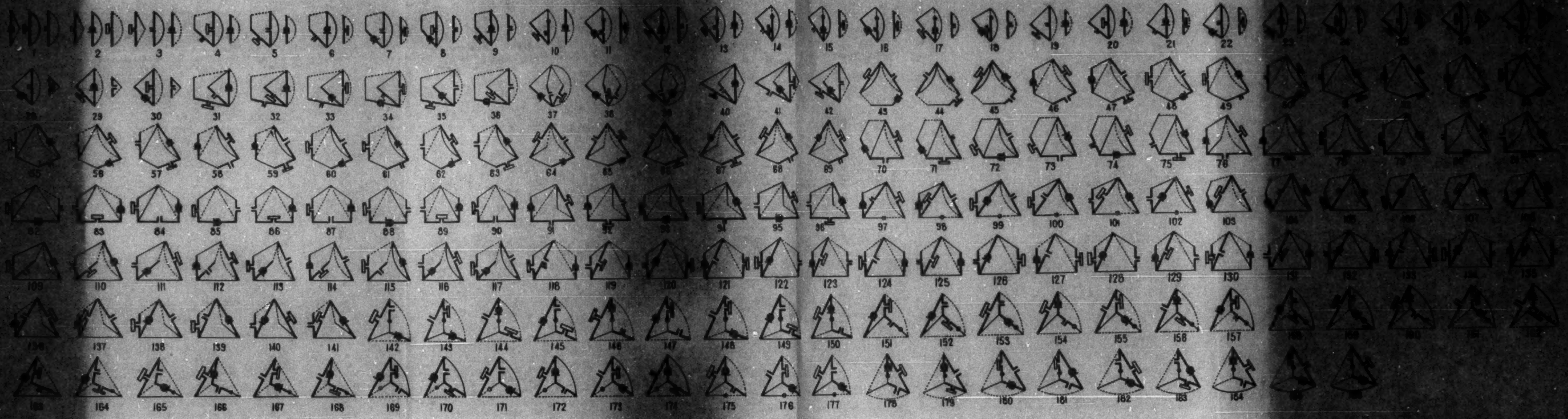
101

One Resistance and One Transformer

(CAMPBELL & FOSTER)

Elements effectively in series



Elements effectively in parallel.

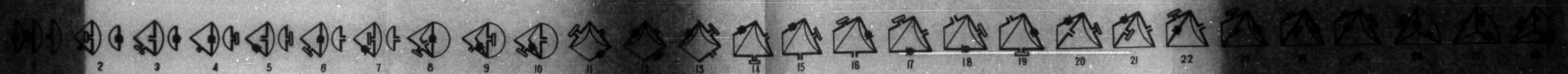
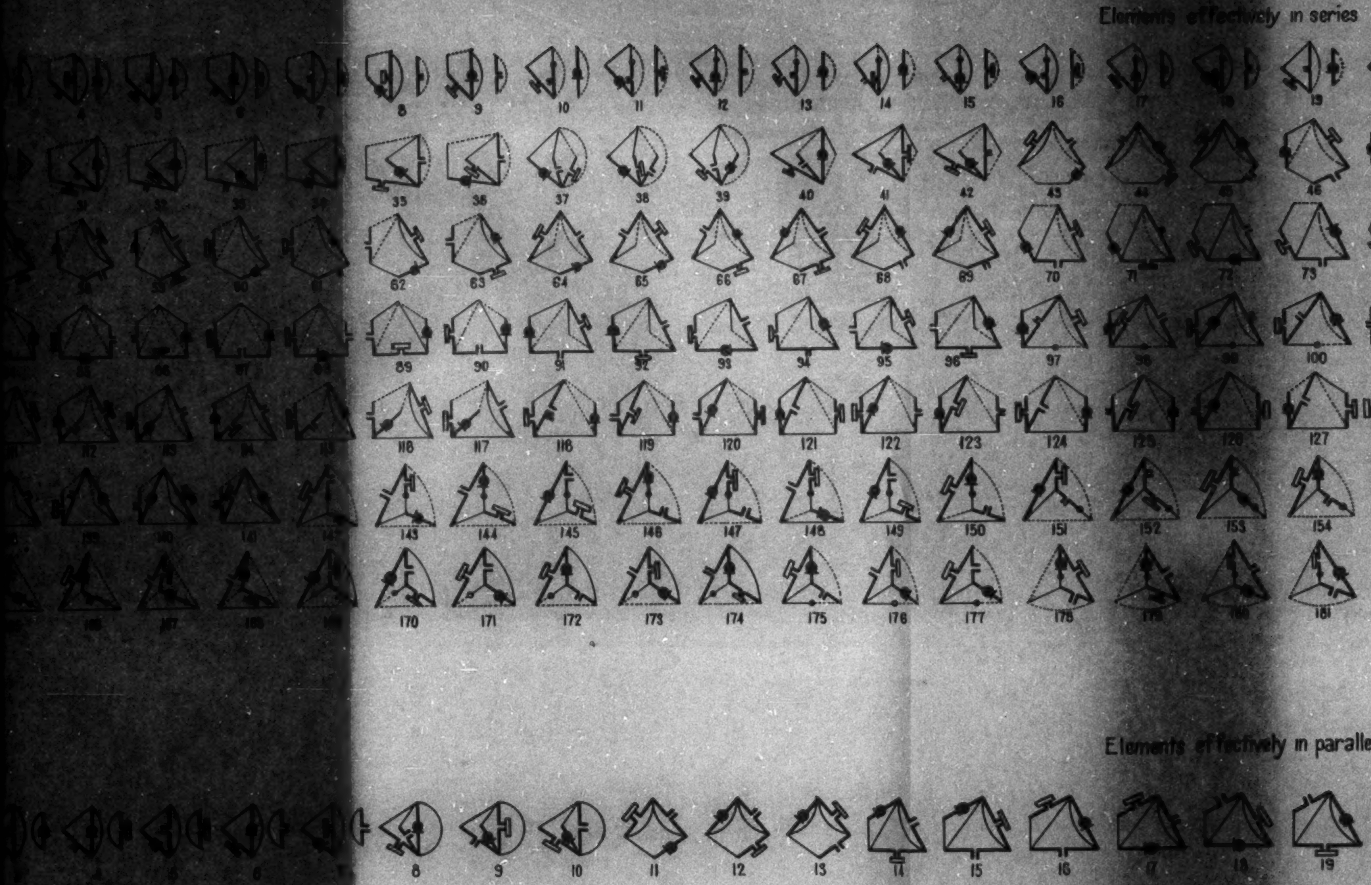


Fig. 9—Substation Circuits with Three Resistance Elements



Elements effectively in series

Elements effectively in parallel

8 9 10 11 12 13 14 15 16 17 18 19
35 36 37 38 39 40 41 42 43 44 45 46
62 63 64 65 66 67 68 69 70 71 72 73
89 90 91 92 93 94 95 96 97 98 99 100
116 117 118 119 120 121 122 123 124 125 126 127
143 144 145 146 147 148 149 150 151 152 153 154
170 171 172 173 174 175 176 177 178 179 180 181
8 9 10 11 12 13 14 15 16 17 18 19

Fig. 9—Substation Circuits with Three Resistance El

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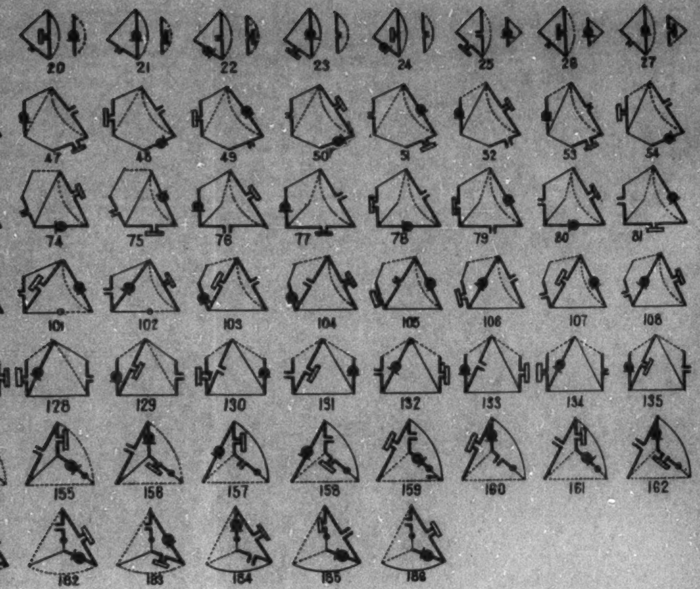

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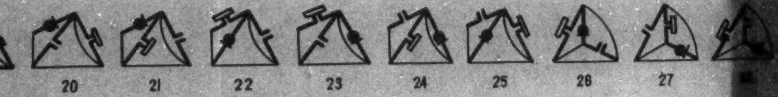

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[CAMPBELL & FOSTER]

5. A maximum output circuit which imposes no restriction upon the values of the four resistances, and only the necessary restrictions (1) and (2) between the input ratios, must have at least four degrees of freedom; for given resistances and given input ratios the two effective degrees of freedom remaining in a four-degree circuit containing two ideal transformers are sufficient for maximum output.

6. If a circuit contains three resistances, one of which has its maximum output into the other two, (and not into one alone), these two being mutually conjugate, the circuit must contain at least one additional resistance, and if these four resistances are the only resistances, the connecting network is a maximum output network.

7. If two resistances each have their maximum output into the same two resistances, the circuit can include no resistance except these four, and the connecting network is a maximum output network.

8. If a circuit contains but four resistances, of which three have maximum output, then the fourth resistance must also have maximum output.

9. In case all two-point impedances are real, increasing the value of all four resistances in the same ratio leaves the maximum output property unaltered; increasing the value of but two resistances which are conjugate in the same ratio w destroys the maximum output property, but all one-point and two-point impedances are increased by the common ratio $\dfrac{(w+1)}{2}$; increasing the values of two conjugate resistances (A, B) in the ratios (v, w), respectively, changes the value of p_1 from zero to $\dfrac{2\,q\,r\,(w-v)}{(w+1)\,(v+1)}$.

10. A maximum output network made up entirely of ideal transformers will have real two-point impedances, and will hold for all frequencies.

11. The two-point impedances of a given maximum output circuit may all be made real at a given frequency, or be given any assigned phase angles consistent with the phase conditions (5) by inserting sections of ideal artificial crossed-capacity lines between the resistances and the network. The artificial

sections leave the maximum output property standing for all frequencies if it held for the original network.

Proofs. It is known that a given electromotive force E, in a given resistance A, cannot have a greater output than $\frac{E^2}{4A}$ into the circuit outside of A, and that it will have this output into any resistance B, provided the two resistances are connected by an ideal transformer of suitable ratio. Since the total possible output from a resistance can be transferred to any second resistance, it follows that the maximum output network, as defined above, cannot itself absorb any of the energy, and in consequence cannot contain any resistances or any other dissipative sinks; this is proposition 1.

Proposition 3 giving the magnitude of the S's follows from the definitions of p, q, r and the S's. The values given in (7) follow by condition (5) as the most general possible if the S's are real.

In Fig. 2 let p^2, q^2, r^2 be the input ratios for the energy received in B, C, D, respectively, from an electromotive force in A, and therefore $p^2 + q^2 + r^2 = 1$, since by assumption the total output into B, C, D is the maximum possible. These input ratios enjoy the reciprocal property, that is, if the electromotive force is transferred from A to B, the amount of energy received in A will still be a p^2 part of the maximum which any network could transfer from B to A. With this reciprocal property recognized it can now be shown, as a condition imposed by energy conservation, that the remaining three input ratios of the maximum output circuit must also be p^2, q^2, r^2. For suppose that, instead of p^2 between C and D, we had $p^2 + a$; then to make the total output of D and C each equal to unity we must have $q^2 - a$ between D and B and $r^2 - a$ between C and B. But B must also have its maximum output and this requires $a = 0$.

The conservation of energy will be satisfied by virtue of the preceding conditions for an electromotive force in any one of the four resistances, but it must also be satisfied when two independent electromotive forces are simultaneously active in different resistances. For this the conditions are given by (18) with $t_{11} = t_{22} = t_{33} = t_{44} = 1$ since each of the four resistances is to have maximum output. Rewrite the first three of the six conditions (18) as follows:

$$\left.\begin{aligned} t_{13}\,t_{23}' &= -\,t_{14}\,t_{24}' \\ t_{12}'\,t_{23} &= -\,t_{14}'\,t_{34} \\ t_{12}\,t_{13}' &= -\,t_{24}\,t_{34}' \end{aligned}\right\} \qquad (8)$$

and their product is

$$|\,t_{12}\,t_{13}\,t_{23}\,|^2 = -\,|\,t_{34}\,t_{24}\,t_{14}\,|^2$$

which requires that at least one t on each side vanish, and reference to the set of three equations shows that the vanishing pairs must be t_{12}, t_{34}; t_{13}, t_{24}; or t_{23}, t_{14}; thus p, q, or r must vanish for they are of the same magnitude as the t's. This was expressed above by the condition $p\,q\,r = 0$. Proposition 2 with its noteworthy biconjugacy condition is thus proved as a necessary consequence of the conservation of energy.

All but two of the six conditions (18) contain any one of the pairs t_{12}, t_{34}; t_{13}, t_{24}; or t_{23}, t_{14}, and thus all but two are satisfied by the vanishing of any one pair. The remaining two conditions would be satisfied by a second pair of vanishing t's, but we shall assume that this is not the case since the circuit would then fall into two electrically unconnected parts, each containing a pair of resistances, and this arrangement has no interest for us. Let the one and only vanishing pair of t's be t_{12}, t_{34}. The remaining two conditions are, if written in terms of the S's,

$$\frac{S_{13}\,S_{23}'}{S_{14}\,S_{24}'} = -\,\frac{C}{D}$$

$$\frac{S_{31}\,S_{41}'}{S_{32}\,S_{42}'} = -\,\frac{A}{B}$$

both of which are consistent with the necessary magnitudes of the S's as given in (4) and both of which require the angular condition (5).

Proposition 5 may be proved as follows: In case it is p that is zero and all two-point impedances are real, and there is but one electromotive force, which is located in one of the four resistances, the currents in the four resistances must stand in the proportion

$$\left.\begin{array}{lccccccc} & |\sqrt{CD}| & : & 0 & : & q\,|\sqrt{AD}| & : & r\,|\sqrt{AC}| \\ \text{or} & 0 & : & |\sqrt{CD}| & : & r_1\,|\sqrt{BD}| & : & q_1\,|\sqrt{BC}| \\ \text{or} & q\,|\sqrt{BC}| & : & r_1\,|\sqrt{AC}| & : & |\sqrt{AB}| & : & 0 \\ \text{or} & r\,|\sqrt{BD}| & : & q_1\,|\sqrt{AD}| & : & 0 & : & |\sqrt{AB}| \end{array}\right\} \quad (9)$$

according as the electromotive force is placed in A, B, C, D, respectively, which follows from the values for the one-point and two-point impedances given in (3) and (7). These proportions are not completely independent, for the sum of the first three multiplied by $q\,|\sqrt{B}|$, $r_1\,|\sqrt{A}|$, $-\,|\sqrt{D}|$, respectively, vanishes, with similar linear relations for any three of the four proportions. The circuit taken as a whole has, therefore, but two effective degrees of freedom. The linear relations between the proportions are, however, not independent of q, r, A, B, C, D, and hence the circuit must have inherently at least four degrees of freedom; the effective degrees of freedom may be reduced to two by ideal transformers or other means which impose fixed relations between the currents in different branches. The number of independent transformers, could not be more than two, since each transformer imposes one linear relation among the currents. It is shown in Appendices III and IV that any circuit having four degrees of freedom may be made a maximum output circuit by properly distributing the resistances and the six windings of two suitably proportioned three-winding transformers among the branches, which completes the proof of proposition 5.

To prove proposition 6 let q^2, r^2, $p^2 = 0$ be the input ratios between A, C; A, D; and C, D, where $q^2 + r^2 = 1$ and neither q nor r vanishes. Assume that C does not have its maximum output and insert reactance and an ideal transformer between C and the network, and so proportion them that the output of C shall be a maximum, which must be possible since it has initially an input into A, and it is necessary only to annul the reactance of the remainder of the system and step its impedance up or down so as to make it equal to the resistance C. All inputs from C will be increased in the same ratio and thus q^2 must be increased; the input from A to D will not be changed since D is conjugate to C; the total input from A into all resistances other than C and D may be increased, but cannot be decreased, since it is zero to start with. The total output from A has thus been increased; but this is impossible since A was assumed at the start to have its maximum output. It follows that C also had originally its maximum output. Similarly for D. But by assumption neither q^2 nor r^2 is equal to unity, and therefore there must be at least a fourth resistance in the circuit to receive the output from C and D which is not absorbed by A. If we assume that there is but one additional resistance B, the inputs of C and D into this resistance must be r^2 and q^2, and this ensures that the entire circuit possesses the maximum output property.

To prove proposition 7 let q^2, r^2, $q^2 + a$, $r^2 - a$ be the input ratios between the resistance pairs A, C; A, D; B, D; B, C, respectively, where $q^2 + r^2 = 1$. These are the most general relations which meet the conditions of the proposition and they make the output of C and D equal to $q^2 + r^2 \pm a +$ (input into other resistances than A and B). Since no output can be either greater than unity, or negative, a must vanish, and there can as well be no input into any resistances other than A and B. The circuit is therefore a maximum output four-resistance circuit.

Proposition 8 is proved by substituting $t_{11} = t_{22} = t_{33} = 1$ in (18), which reduces the first three of the conditions to the form (8) used above, and making one of these substitutions in the remaining three conditions which shows that $t_{44} = 1$, and therefore the fourth resistance also has maximum output.

To prove proposition 9 note that multiplying the magnitudes of two conjugate resistances, say C, D, by the factor w, multiplies the associated one- and two-point impedances $S_{33}, S_{44}, S_{31}, S_{32}, S_{41}, S_{42}, S_{34}$ (equals infinity) by the common factor $\frac{(w+1)}{2}$ since the outgoing current for given electromotive forces in these resistances is cut down in this ratio. To determine the new values of the three remaining impedances S_{11}, S_{22}, S_{12}, substitute the values of the seven impedances just found in (16) and (18) the first three of which then give $S_{11} = A\,(w+1)$, $S_{22} = B\,(w+1)$, $S_{12} =$ infinite. Thus all of the one- and two-point impedances have been multiplied by the same factor $\frac{(w+1)}{2}$ as stated in the second clause of the proposition; one consequence is that the biconjugate relation is left unchanged. Now increase resistances A, B to $A\,w$, $B\,w$ which changes the one-point impedances at these resistances to $2\,A\,w$, $2\,B\,w$ and leaves them conjugate; since they have maximum output the circuit is now a maximum output circuit by proposition 7. Multiplying the magnitudes of two conjugate resistances A and B by the factors v and w, respectively, multiplies the associated one- and two-point impedances S_{11}, S_{31}, S_{41} and S_{22}, S_{32}, S_{42} by the common factors $\frac{(v+1)}{2}$ and $\frac{(w+1)}{2}$, respectively, S_{12} remaining infinite. Substituting these values in (**16**) and (**18**) we find that

$$t_{34} = p_1 = \frac{2\,q\,r\,(w-v)}{(w+1)\,(v+1)}.$$

No formal proof of proposition 10 is necessary at this point since networks composed of ideal transformers are investigated in considerable detail in Appendices III and IV.

Proposition 11 follows from the fact that the ideal, artificial crossed-capacity line, Fig. 3, has a pure resistance iterative impedance $\sqrt{L/C}$ which is independent of frequency and may be given any phase angle $2\tan^{-1} 1/2\,p\sqrt{L\,C}$ per section. Three artificial line sections are sufficient, but four, one for each resistance, may be used to give symmetrical expressions for all two-point impedances. The iterative impedance of any section of line is equal to the square root of the product of the open and closed circuit impedances. The phase difference, that is, the lag of the current at the far end behind the current at the transmitting end, is the imaginary part of the propagation constant, which is equal to the anti-hyperbolic tangent of the square root of the quotient of the closed-circuit impedance divided by the open-circuit impedance. For the artificial line section of Fig .3 the open- and closed-circuit impedances are

$$\frac{1}{2}\left(\frac{1}{2}\,L\,p\,i + \frac{2}{C\,p\,i}\right) \text{ and } \frac{2\,L}{C\left(\frac{1}{2}\,L\,p\,i + \frac{2}{C\,p\,i}\right)} \qquad (10)$$

respectively. Using these we obtain the values stated above for the iterative impedance and phase angle.

APPENDIX II

CONSERVATION OF ENERGY

In any network made up of resistances, self inductances, mutual inductances, capacities (R_q, L_q, M_{qr}, C_q) with instantaneous impressed electromotive forces (e_q) and currents (i_q, i_r) the conservation of energy demands

$$\sum e_q i_q = \sum R_q i_q^2 + d/dt \left\{ \sum \frac{1}{2} L_q i_q^2 + \sum M_{qr}\, i_q\, i_r + \sum \frac{1}{2\, C_q} \left(\int i_q\, dt \right)^2 \right\} \tag{11}$$

and in the special case of steady cisoidal oscillations with impressed electromotive forces E_q cis $(p\,t)$, currents I_q cis $(p\,t)$, self impedances Z_q, mutual impedances Z_{qr}, this condition becomes, after dropping the common factor cis $(2\,p\,t)$,

$$\Sigma\, E_q\, I_q = \Sigma\, Z_q\, I_q^2 + 2\, \Sigma\, Z_{qr}\, I_q\, I_r \tag{12}$$

By introducing one- and two-point impedances, the electromotive forces and currents may be eliminated and two sets of conditions of the following forms obtained:

$$\frac{1}{S_{jk}} = \sum Z_q \frac{1}{S_{jq}\, S_{kq}} + \sum Z_{qr} \left\{ \frac{1}{S_{jq}\, S_{kr}} + \frac{1}{S_{jr}\, S_{kq}} \right\} \tag{13}$$

$$\frac{1}{2} \left(\frac{1}{S_{jk}} + \frac{1}{S_{jk}'} \right) = \sum \frac{R_q}{S_{jq}\, S_{kq}'} \tag{14}$$

or $$t_{jk} + t_{jk}' = \Sigma\, t_{jq}\, t_{kq}' \tag{15}$$

where $$t_{jk} = \frac{2\, |\sqrt{R_j\, R_k}\,|}{S_{jk}} \tag{16}$$

and a prime accent indicates "conjugate of."

With a total of four resistances, there are ten relations of the form (15) which it is convenient to have written out in full:

$$\left. \begin{aligned} |t_{11} - 1|^2 + |t_{12}|^2 + |t_{13}|^2 + |t_{14}|^2 &= 1 \\ |t_{12}|^2 + |t_{22} - 1|^2 + |t_{23}|^2 + |t_{24}|^2 &= 1 \\ |t_{13}|^2 + |t_{23}|^2 + |t_{33} - 1|^2 + |t_{34}|^2 &= 1 \\ |t_{14}|^2 + |t_{24}|^2 + |t_{34}|^2 + |t_{44} - 1|^2 &= 1 \end{aligned} \right\} \tag{17}$$

$$\left.\begin{aligned}
(t_{11}-1)\,t_{12}' + t_{12}\,(t_{22}'-1) + t_{13}\,t_{23}' + t_{14}\,t_{24}' &= 0\\
(t_{11}-1)\,t_{13}' + t_{12}\,t_{23}' + t_{13}\,(t_{33}'-1) + t_{14}\,t_{34}' &= 0\\
t_{12}\,t_{13}' + (t_{22}-1)\,t_{23}' + t_{23}\,(t_{33}'-1) + t_{24}\,t_{34}' &= 0\\
(t_{11}-1)\,t_{14}' + t_{12}\,t_{24}' + t_{13}\,t_{34}' + t_{14}\,(t_{44}'-1) &= 0\\
t_{12}\,t_{14}' + (t_{22}-1)\,t_{24}' + t_{23}\,t_{34}' + t_{24}\,(t_{44}'-1) &= 0\\
t_{13}\,t_{14}' + t_{23}\,t_{24}' + (t_{33}-1)\,t_{34}' + t_{34}\,(t_{44}'-1) &= 0
\end{aligned}\right\}\qquad (18)$$

In case all t's are real, and we write p, p_1, q, q_1, r, r_1 for t_{12}, t_{34}, t_{13}, t_{24}, t_{14}, t_{23} and u_j for $(t_{jj}-1)$ then (17) and (18) may be written

$$\left.\begin{aligned}
u_1^2 + p^2 + q^2 + r^2 &= 1\\
u_2^2 + p^2 + q_1^2 + r_1^2 &= 1\\
u_3^2 + p_1^2 + q^2 + r_1^2 &= 1\\
u_4^2 + p_1^2 + q_1^2 + r^2 &= 1
\end{aligned}\right\}\qquad (19)$$

$$\left.\begin{aligned}
p\,(u_1+u_2) + (q\,r_1 + r\,q_1) &= 0\\
q\,(u_1+u_3) + (p\,r_1 + r\,p_1) &= 0\\
r_1\,(u_2+u_3) + (p\,q + p_1\,q_1) &= 0\\
r\,(u_1+u_4) + (p\,q_1 + p_1\,q) &= 0\\
q_1\,(u_2+u_4) + (p\,r + p_1\,r_1) &= 0\\
p_1\,(u_3+u_4) + (q\,r + q_1\,r_1) &= 0
\end{aligned}\right\}\qquad (20)$$

Excluding the cases for which

$$p\,p_1 = q\,q_1 = r\,r_1 \qquad (21)$$

which are series, parallel, and other connections of the four resistances that cannot give a maximum output circuit, we find that in all remaining cases the following, of which the first three are independent, are among the consequences of (19) and (20)

$$t_{11} + t_{22} + t_{33} + t_{44} = 4 \qquad (22)$$

$$p\,q\,r + p\,q_1\,r_1 + p_1\,q\,r_1 + p_1\,q_1\,r = 0 \qquad (23)$$

$$p^2 + p_1^2 + q^2 + q_1^2 + r^2 + r_1^2 = 2 - \frac{(q_1^2 - q^2)(r_1^2 - r^2)}{2(p_1^2 - p^2)}$$

$$- \frac{(r_1^2 - r^2)(p_1^2 - p^2)}{2(q_1^2 - q^2)} - \frac{(p_1^2 - p^2)(q_1^2 - q^2)}{2(r_1^2 - r^2)} \qquad (24)$$

$$= 2 + \frac{p^2 + p_1^2}{2}\left(\frac{q\,q_1}{r\,r_1} + \frac{r\,r_1}{q\,q_1}\right)$$

$$+ \frac{q^2 + q_1^2}{2}\left(\frac{r\,r_1}{p\,p_1} + \frac{p\,p_1}{r\,r_1}\right)$$

$$+ \frac{r^2 + r_1^2}{2}\left(\frac{p\,p_1}{q\,q_1} + \frac{q\,q_1}{p\,p_1}\right) \qquad (25)$$

$$p^2 + q^2 + r^2 = 1 - \frac{[\,\Sigma\,(p_1^2 - p^2)(q_1^2 - q^2)\,]^2}{4(p_1^2 - p^2)(q_1^2 - q^2)(r_1^2 - r^2)} \qquad (26)$$

$$[r^2(p^2 + p_1^2 + q^2 + q_1^2)(p\,q_1 + p_1\,q)^2]^2$$
$$= 4\,r^4(p\,p_1 - q\,q_1)^2 + 4\,r^2(p\,q_1 + p_1\,q)^2 \qquad (27)$$

$$2\,p^2 + 2\,q^2 + r^2 + r_1^2 = 2 - \frac{p^2(r_1 + r)^2}{2\,q^2} - \frac{q^2(r_1 - r)^2}{2\,p^2}$$

if $p_1 = p,\ q_1 = -q$ (28)

The proof of (14) may be made by inserting in branches j, k the direct and retrograde electromotive forces $e_j = E_j \operatorname{cis}(p\,t)$, $e_k = E_k' \operatorname{cis}(-p\,t)$ which together produce in any branch q the current

$$i_q = \frac{E_j}{S_{jq}} \operatorname{cis}(p\,t) + \frac{E_k'}{S_{kq}'} \operatorname{cis}(-p\,t)$$

Substituting these values for e, i in (11) gives a homogeneous second degree equation in the E's and since the E's are independent, the coefficients of E_j^2 $E_k'^2$ and E_j E_k' must vanish individually. The coefficient of the product $E_j\,E_k'$ is independent of t, since $\operatorname{cis}(p\,t)\operatorname{cis}(-p\,t) = 1$, consequently that part which involves differentiation with respect to t contributes nothing, the other terms give (14), or (15) after multiplication by $4\,|\sqrt{R_j\,R_k}\,|$. Similarly, condition (13) is found by substituting two direct electromotive forces E_j, E_k with the resulting currents $I_q = E_j/S_{jq} + E_k/S_{kq}$ in (12) and equating the coefficient of $E_j\,E_k$ to zero.

Multiply equations (20) by p_1, q_1, r, r_1, q, p and add in pairs giving the three equations:

$$\begin{aligned} pqr + pq_1r_1 + p_1qr_1 + p_1q_1r &= pp_1\,(u_1 + u_2 + u_3 + u_4) \\ &= qq_1\,(u_1 + u_2 + u_3 + u_4) \\ &= rr_1\,(u_1 + u_2 + u_3 + u_4) \end{aligned}$$

whence either (21) or both (22) and (23) must hold. Assuming the latter alternative, taking the sum of (19) and reducing by means of (22) we find

$$\begin{aligned} &2\,(p^2 + p_1^2 + q^2 + q_1^2 + r^2 + r_1^2) \\ &\quad = 4 + (u_1 + u_2)\,(u_3 + u_4) + (u_1 + u_3)\,(u_2 + u_4) \\ &\qquad + (u_1 + u_4)\,(u_2 + u_3) \end{aligned}$$

and substituting for each term in parentheses its value as given by (20) we obtain (25). Combining terms in (25) which have the same denominator and reducing by (23) we may obtain (24), or by a rearrangement of terms (26); (27) is obtained by eliminating r_1 between (23) and (24).

If $p_1 = \pm p \neq 0$ then (23) may be written

$$p\,(q \pm q_1)\,(r \pm r_1) = 0$$

which requires that either $q_1 = \mp q$ or $r_1 = \mp r$, or in other words if one of the three pairs p, p_1; q, q_1; r, r_1 have the same absolute values then another pair must have the same absolute value and the two pairs will have opposite sign relations.

That condition (21) does not include any circuits having maximum output possibilities is a consequence of proposition 2 above which demands that of the three products $p\,p_1$, $q\,q_1$, rr_1 at least one shall be finite and at least one shall be zero.

A more general discussion of the conditions imposed by the conservation of energy will be presented in another paper.

APPENDIX III

DISCUSSION OF THE MASTER CIRCUIT

Rule: The winding turn ratios a : b : c : d of each transformer of the master maximum output circuit must be equal to the sum, after multiplication by arbitrary parameters α, β, γ, δ of any two or more of the four ratios

$$\left.\begin{array}{rrrr} 0: & P: & Q: & R \\ -P: & 0: & R_1: & Q_1 \\ -Q: & -R_1: & 0: & P_1 \\ -R: & -Q_1: & -P_1: & 0 \end{array}\right\} \qquad (29)$$

where P, P_1, Q, Q_1, R, R_1 are given by

$$\left.\begin{array}{l} |\sqrt{CD}|P \ :|\sqrt{BD}|Q \ :|\sqrt{BC}|R \ : \\ |\sqrt{AB}|P_1 :|\sqrt{AC}|Q_1 :|\sqrt{AD}|R_1 \end{array}\right\}$$

$$= \left\{\begin{array}{l} q_1 : q\,r \ : -q^2 : \\ q \ : q_1 r : q\,q_1 \end{array}\right\}, \text{ if } p = 0 \tag{30}$$

$$= \left\{\begin{array}{ccc} r^2 & : -r_1 : & -p\,r \ : \\ -r\,r_1 : & r \ : & p\,r_1 \end{array}\right\}, \text{ if } q = 0 \tag{31}$$

$$= \left\{\begin{array}{lll} q^2 & : -p\,q & : -q_1 : \\ q\,q_1 : & p\,q_1 : & q \end{array}\right\}, \text{ if } r = 0 \tag{32}$$

and A, B, C, D are the resistances

$p^2 = p_1^2$, $q^2 = q_1^2$, $r^2 = r_1^2$ are the input ratios and are subject to the further conditions—

$p^2 + q^2 + r^2 = 1$

$p\,q\,r = 0$

$p\,q = -p_1 q_1$, $qr = -q_1 r_1$, $r\,p = -r_1 p_1$ which restrict the current directions. The parameters α, β, γ, δ and α', β', γ', δ' for the two transformers must be such that corresponding turn ratios are not identical; distinct sets of parameters not being necessarily sufficient since but two of the four ratios (**29**) *which they multiply are distinct.*

Proof: For the case $p = 0$ the current proportions (**9**) must all satisfy the ampere-turn condition for each of the two ideal transformers which gives two and only two of the three conditions required to fix the turn ratios $a : b : c : d$ for a transformer since but two of these proportions are independent. The third condition is optional and it is convenient to employ one of the same form as the other two but with arbitrary parameters α, β, γ, δ :

$$a\,\alpha + b\,\beta + c\,\gamma + d\,\delta = 0 \tag{33}$$

and if use is made of the last two proportions of (**9**) the two ampere-turn conditions are

$$a\,q\,|\sqrt{BC}| + b\,r_1\,|\sqrt{AC}| + c\,|\sqrt{AB}| = 0$$

$$a\,r\,|\sqrt{BD}| + b\,q_1\,|\sqrt{AD}| + d\,|\sqrt{AB}| = 0$$

Solving these three equations we find that $a : b : c : d$ is equal to the sum after multiplying by α, β, γ, δ of the four proportions:

$$\begin{array}{ccccccc}
0 & : & |\sqrt{A\,B}| & : & r_1\,|\sqrt{A\,C}| & : & -q_1\,|\sqrt{A\,D}| \\
|\sqrt{A\,B}| & : & 0 & : & q\,|\sqrt{B\,C}| & : & r\,|\sqrt{B\,D}| \\
r_1\,|\sqrt{A\,C}| & : & -q\,|\sqrt{B\,C}| & : & 0 & : & (q\,q_1 - r\,r_1)\,|\sqrt{C\,D}| \\
q_1\,|\sqrt{A\,D}| & : & -r\,|\sqrt{B\,D}| & : & -(q\,q_1 - r\,r_1)\,|\sqrt{C\,D}| & : & 0
\end{array}$$

respectively, which is one of the numerous forms in which the above rule (**29**) might be written as is proved by multiplying this form by q_1 and reducing by means of $q\,r = -q_1\,r_1$, $q^2 = q_1^2$, $q^2 + r^2 = 1$.

The four proportions of (**29**) are given for the sake of symmetry, but the solution derived from any two, say the first two:

$$\left.\begin{array}{l}
a : b : c : d \\
\qquad = -\beta\,P : \alpha\,P : \alpha\,Q + \beta\,R_1 : \alpha\,R + \beta\,Q_1 \\
a' : b' : c' : d' \\
\qquad = -\beta'\,P : \alpha'\,P : \alpha'\,Q + \beta'\,R_1 : \alpha'\,R + \beta'\,Q_1
\end{array}\right\} \quad (\mathbf{34})$$

is perfectly general.

The three formulas (**30**), (**31**), (**32**) may be combined into a single formula in various ways, for example:

$$\left.\begin{array}{l}
|\sqrt{C\,D}|\,P : |\sqrt{B\,D}|\,Q : |\sqrt{B\,C}|\,R : \\
|\sqrt{A\,B}|\,P_1 : |\sqrt{A\,C}|\,Q_1 : |\sqrt{A\,D}|\,R_1
\end{array}\right\}$$

$$= \left\{\begin{array}{l}
\sqrt{1-p^2} : \sqrt{1-q^2} : \sqrt{1-r^2} : \\
\sqrt{1-p^2} : \sqrt{1-q^2} : \sqrt{1-r^2}
\end{array}\right\}$$

$$= \left\{\begin{array}{lll}
1 - p^2 & : p_1\,q_1 - r_1 & : p_1\,r_1 - q_1 : \\
q\,q_1 - r\,r_1 & : r - p_1\,q & : q - p_1\,r
\end{array}\right\}$$

The first formula fails to give the signs and neither formula applies to the circuit except when proportioned for maximum output of all four resistances; a general formula is given below.

The necessary and sufficient conditions in order that the master circuit may be a maximum output circuit are:

$$\left.\begin{array}{l}
A\,B\,[c\,d]^2 = C\,D\,[a\,b]^2 \\
A\,C\,[b\,d]^2 = B\,D\,[a\,c]^2 \\
A\,D\,[b\,c]^2 = B\,C\,[a\,d]^2
\end{array}\right\} \quad (\mathbf{35})$$

and, since all factors must be positive, this may be put in the equivalent form

$$A^2 : B^2 : C^2 : D^2 = [a\,b]^2\,[a\,c]^2\,[a\,d]^2 : [a\,b]^2\,[b\,c]^2\,[b\,d]^2 \\ : [a\,c]^2\,[b\,c]^2\,[c\,d]^2 : [a\,d]^2\,[b\,d]^2\,[c\,d]^2 \qquad (36)$$

The six quantities $[a\,b]$, $[a\,c]$, $[a\,d]$, $[b\,c]$, $[b\,d]$, $[c\,d]$ are abbreviations for $(a\,b' - a'b)$, $(a\,c' - a'c)$, etc., and satisfy the identity

$$[a\,b]\,[c\,d] - [a\,c]\,[b\,d] + [a\,d]\,[b\,c] = 0 \qquad (37)$$

but are otherwise unrestricted as is readily seen by making $d = a' = 0$, $b' = 1$.

Proof: To establish (35) it is sufficient to show that these conditions are necessary and sufficient to make the one-point impedances at A, B, C, D equal to $2\,A$, $2\,B$, $2\,C$, $2\,D$ as required by (3). The determinant* of the network and its principal cofactor are:—

$$\Delta = \begin{vmatrix} A & 0 & 0 & 0 & a & a' \\ 0 & B & 0 & 0 & b & b' \\ 0 & 0 & C & 0 & c & c' \\ 0 & 0 & 0 & D & d & d' \\ a & b & c & d & 0 & 0 \\ a' & b' & c' & d' & 0 & 0 \end{vmatrix} \qquad (38)$$

$$= \Sigma\,A\,B\,[c\,d]^2$$

$$= A\,B\,[c\,d]^2 + C\,D\,[a\,b]^2 + A\,C\,[b\,d]^2 + B\,D\,[a\,c]^2 \\ + A\,D\,[b\,c]^2 + B\,C\,[a\,d]^2 \qquad (39)$$

$$\Delta_{11} = B\,[c\,d]^2 + C\,[b\,d]^2 + D\,[b\,c]^2$$

$$= \frac{(\text{terms of } \Delta \text{ containing } A)}{A} \qquad (40)$$

The one-point impedances at A and B are

$$S_{11} = \Delta/\Delta_{11} = 2\,A \text{ by } (3)$$

$$S_{22} = \Delta/\Delta_{22} = 2\,B$$

---

*Each ideal transformer introduced into a network adds a border to the network determinant; each term in the border equals the number of turns of the ideal transformer in the circuit corresponding to the row or column in which the term occurs, as is proved in Appendix VI.

and require $\Delta - A\,\Delta_{11} - B\,\Delta_{22} = 0$

or $A\,B\,[c\,d]^2 = C\,D\,[a\,b]^2$

and by symmetry the other two conditions of the set (35) follow; from these three relations (36) is obtained.

Generalized Rule: *Rule* (29) *for the winding turn ratios of the master maximum output circuit may be generalized to permit any assigned values of* p, p_1, q, q_1, r, r_1 *which are real and satisfy the fundamental conditions* (23) *and* (24) *by replacing the right side of* (30, 31, 32) *by*

$$\left\{\begin{array}{l} r\,(q + q_1 v) : r\,(p_1 v - p) : - v\,(p\,q_1 + p_1\,q) : \\ r\,(q\,v + q_1) : r\,(p\,v - p_1) : - (p\,q_1 + p_1\,q) \end{array}\right\} \qquad (41)$$

where v

$$= \frac{r^2\,(p^2 + p_1^2 + q^2 + q_1^2) + (p\,q_1 + p_1\,q)^2 + 2\,r\,(p\,q_1 + p_1\,q)}{2\,r^2\,(p\,p_1 - q\,q_1)}$$

Proof: From (7) by symmetry

$$p = \frac{2\,|\sqrt{A\,B}\,|}{S_{12}} = \frac{2\,|\sqrt{A\,B}\,|\,\Delta_{12}}{\Delta}$$

$$= - \frac{2\,|\sqrt{A\,B}\,|}{\Delta}\Big[\,C\,[a\,d]\,[b\,d] + D\,[a\,c]\,[b\,c]\,\Big]$$

where Δ is the determinant (38) and Δ_{12} is the cofactor of the element in the second column of the first row. To abbreviate set

$$\left.\begin{array}{l} L\ = [a\,b]\,|\sqrt{C\,D}\,|\,,\ M\ = [a\,c]\,|\sqrt{B\,D}\,|\,, \\ \qquad\qquad\qquad\qquad N\ = [a\,d]\,|\sqrt{B\,C}\,| \\ L_1 = [c\,d]\,|\sqrt{A\,B}\,|\,,\ M_1 = [b\,d]\,|\sqrt{A\,C}\,|\,, \\ \qquad\qquad\qquad\qquad N_1 = [b\,c]\,|\sqrt{A\,D}\,| \end{array}\right\} \qquad (42)$$

which by virtue of (37) must satisfy the identity

$$L\,L_1 - M\,M_1 + N\,N_1 = 0 \qquad (43)$$

By (39)

$$\Delta = L^2 + L_1^2 + M^2 + M_1^2 + N^2 + N_1^2 \qquad (44)$$

and from the cofactors we derive:

$$\left.\begin{aligned}
p &= -\frac{2}{\Delta}(M N_1 + M_1 N)\\
p_1 &= -\frac{2}{\Delta}(M N + M_1 N_1)\\
q &= -\frac{2}{\Delta}(N L_1 - N_1 L)\\
q_1 &= -\frac{2}{\Delta}(N L - N_1 L_1)\\
r &= -\frac{2}{\Delta}(- L M_1 - L_1 M)\\
r_1 &= -\frac{2}{\Delta}(L M + L_1 M_1)
\end{aligned}\right\} \qquad (45)$$

Let $N = v N_1$ and we find that the ratios $L : M : N : L_1 : M_1 : N_1$ have the values given by the right side of (41). The expression for v is determined by the condition that the value (44) for Δ must be such as to satisfy any one of the equations (45). Substituting we find that this requires

$$(F - G) v^2 + 4 E v - (F + G) = 0 \qquad (46)$$

where
$$\begin{aligned}
E &= r^2 (p p_1 - q q_1)\\
F &= 2 r (p q_1 + p_1 q)\\
G &= r^2 (p^2 + p_1^2 + q^2 + q_1^2) + (p q_1 + p_1 q)^2\\
G^2 &= 4 E^2 + F^2 \quad \text{by (27)}
\end{aligned}$$

Equation (46) proves to be essentially a perfect square, giving

$$v = \frac{2 E}{G - F} = \frac{G + F}{2 E}$$

which upon substituting the values of E, F, G gives the value of v stated in the above rule. This value of v satisfies (43) which is, in the present notation,

$$E v^2 - G v + E = 0 \qquad (47)$$

By substituting in succession $a = 0$, $b = 0$, $c = 0$, $d = 0$ in (42) and comparing with (29), we find that $L : M : N : L_1 : M_1 : N_1$ is the same as the left side of (30) which completes the proof of the generalized rule (41).

From (41) the special cases (30, 31) may be derived by first making $p = \pm p_1, q = \pm q_1, r = \mp r_1$, dividing through by the common factor $2\,p\,q$ and then setting $p = 0$ or $q = 0$. The ratios (**41**) become indeterminate if $r = 0$ or $p = \pm p_1$, $q = \mp q_1$ but on evaluating by means of (**23**) we obtain (**32**). As a practical rule for proportioning the maximum output circuit (**30, 31, 32**) is much simpler than (**41**) and has, therefore, been presented as the working rule.

The behavior of the circuit of Fig. 4 has been shown to depend solely upon the six quantities (**42**) but since the ratio of these quantities only is involved in (**45**) and also in the expressions for t_{11}, t_{22}, t_{33}, t_{44} which correspond to the one-point impedances and since (**43**) must hold, there are but four independent parameters involved. This is also the number of independent quantities among p, p_1, q, q_1, r, r_1 in view of conditions (**23**), (**24**). The ratios of the turns of two three-winding transformers is also four. The number of independents is thus the same number four in each of the three aspects of the circuit, as it should be.

APPENDIX IV

GENERAL RULES FOR PROPORTIONING IDEAL TRANSFORMERS IN MAXIMUM OUTPUT CIRCUITS

In Appendix III a general rule for proportioning the winding turns for the master circuit has been derived; this rule may now be extended to all other maximum output circuits, by observing that whatever the circuit connections, the operation of the circuit is determined by the ratio of winding turns connecting every two pairs of resistance terminals, as explained in Appendix VI, where it is shown how ideal transformers may be substituted for each other, without changing their effect in the circuit.

Take any geometrical circuit having at least four degrees of freedom. Pick out four branches opening four separate degrees of freedom. Insert in these branches the four resistances and any set of windings on two transformer cores which meet the requirements (**29-32**) of the master circuit. Add, quite arbitrarily, any number of sets of transformer windings, each set comprising a winding in every branch connected to a selected branch-point, the number of turns counted outward from the branch-point being the same in each winding of the set and the windings all being on one core which may or may not carry other sets of windings including one of the sets corresponding to the master circuit. The transformers may be combined into multi-mesh core transformers, or separated into two-winding transformers as explained in Appendix VI. If a transformer winding is not placed in every branch of the geometrical circuit that branch must be short-circuited.

That this procedure always gives a maximum output circuit follows from the fact that the added sets of windings each give an algebraic total of zero turns in every closed path, and are thus without effect; with these windings short-circuited, the circuit becomes a master circuit, with an added network of short-circuits which have no effect, and may be eliminated.

If the circuit is of the fourth degree, it has been shown that there are 57,340 distinct unrestricted circuits employing two three-winding transformers. The proportioning of these windings might be carried out by means of the above directions, but it is convenient to have a more specific rule such as the following:—

The circuit, the values of the four resistances A, B, C, D, and the values of p, q, r are supposed given, and the values of P, P_1, Q, Q_1, R, R_1 determined by (**30, 31,** or **32**). Select one of the transformers with windings x, y, z; trace the four paths in the circuit which contain no winding of this transformer, no winding but x, no winding but y, and no winding but z, respectively. Assign positive directions arbitrarily for the first path and concurrently with that of the branch containing the winding in the other paths. Form the array

$$\begin{matrix} A_0 & B_0 & C_0 & D_0 \\ A_x & B_x & C_x & D_x \\ A_y & B_y & C_y & D_y \\ A_z & B_z & C_z & D_z \end{matrix} \qquad (48)$$

where the element of the array in the ith row and kth column is $+1$, -1, or 0, according as the ith path traverses the kth resistance in the positive direction, the negative direction or not at all. Make the turns in winding x equal to the sum of all the elements in the following array after multiplying each by the corresponding elements in the border row and column which are taken from (**48**)

$$\begin{array}{cccc|c} 0 & P & Q & R & A_0 \\ -P & 0 & R_1 & Q_1 & B_0 \\ -Q & -R_1 & 0 & P_1 & C_0 \\ -R & -Q_1 & -P_1 & 0 & D_0 \\ \hline A_x & B_x & C_x & D_x & \end{array} \qquad (49)$$

or, in expanded form,

$$\left.\begin{aligned} &(A_0 B_x - A_x B_0) P + (A_0 C_x - A_x C_0) Q + (A_0 D_x - A_x D_0) R \\ &\quad + (C_0 D_x - C_x D_0) P_1 + (B_0 D_x - B_x D_0) Q_1 \\ &\qquad\qquad\qquad\qquad + (B_0 C_x - B_x C_0) R_1 \end{aligned}\right\} \qquad (50)$$

This rule may be proved by adding the four proportions of (**29**) after multiplying them by A_0, B_0, C_0, D_0, respectively, and putting the resulting number of turns with signs reversed directly in series with the four resistances, when the four paths considered above may be shown to have each a total of zero turns; the added windings and the windings resulting from the rule are therefore mutually equivalent.

It may be shown that the expressions of the form $(A_0 B_x - A_x B_0)$ do not assume the values ± 2 since the two terms cancel if both paths contain both resistances which require both to be in the common part of the two paths.

To illustrate the use of this rule, take the circuit which is listed on the eighth line of Table III, *viz.*, circuit **R** with resistances in branches **6, 7, 3, 5** and the two sets of windings in branches **1, 2, 4** and **1, 6, 7.** The arrays (**48**) for the two transformers are found by inspection to be

$$\begin{array}{cccc} -1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 1 \\ 0 & -1 & -1 & 0 \\ -1 & 0 & 0 & 1 \end{array} \qquad \begin{array}{cccc} 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}$$

and substituting these quantities in formula (**50**) for each of the windings gives

$$x : y : z = (P + Q + Q_1 - R - R_1) : (P + Q - R_1) : (P + Q_1 - R)$$

$$x' : y' : z' = (-P_1) : (P_1 + Q + R) : (P_1 + Q_1 + R_1)$$

as listed in Table III.

APPENDIX V

CIRCUITS WITH THREE RESISTANCES

The circuits in question have real one- and two-point impedances; (**24-27**) all apply if $p_1 = q_1 = r = 0$ so as to separate the fourth resistance completely from the other three, and if the subscript from r_1 which is no longer needed is dropped (**24**) becomes:

$$\left.\begin{aligned} p^2 q^2 + q^2 r^2 + r^2 p^2 \pm 2\,p\,q\,r &= 0 \quad \text{or} \\ \frac{r}{p\,q} &= \frac{\sqrt{1-(p^2+q^2)} \mp 1}{(p^2+q^2)} \end{aligned}\right\} \qquad (53)$$

$$= \mp 1, \text{ if } (p^2+q^2) = 1 \qquad (54)$$

$$= \mp\left[\frac{1}{2} + \frac{p^2+q^2}{8}\right] \text{ or } \mp\left[\frac{2}{p^2+q^2} - \frac{1}{2}\right]$$

$$\text{if } (p^2+q^2) \text{ is small} \qquad (55)$$

In these forms the condition gives the information which is needed in considering side-tone reduction since the energy involved in the side-tone and the energy which reaches the receiver at the listening end of the line (transmission attenuation being neglected) are in the ratio $r^2 : p^2 q^2$ if L, T, R are associated with A, B, C. For a given value of $(p^2 + q^2)$, the total output of A, there are two possible values for the ratio $r^2 : p^2 q^2$; we are interested primarily in and will confine our attention to the smaller value. For very small values of the total output $(p^2 + q^2)$, the value of the ratio is shown by (**55**) to be 1 : 4, which is also its minimum possible value; the ratio increases continuously with increasing values for $(p^2 + q^2)$, becoming equal to 1 : 1 when $(p^2 + q^2)$ attains its maximum value, unity, as is shown by (**54**). The amount of energy then appearing as side-tone is equal to the amount of energy delivered to the receiver at the listening substation (the effect of line attenuation being ignored). This is not the nearest approach to side-tone elimination, for, as we saw above by making $(p^2 + q^2)$ small, the amount of energy appearing as side-tone can be reduced to one quarter of this amount, but this improvement is not enough to be practically satisfactory, and it is obtained only at a serious sacrifice in the absolute value of the volume of receiving at the listening station. The mathematical work follows that for four-resistance circuits, so only an outline and the results are given below:

Series Type Circuit. Circuit D, Fig. 6, symmetrical in $A, B, C, a, b, c, a', b', c', r, q, p$.

$$p^2 q^2 + q^2 r^2 + r^2 p^2 = 2\, p\, q\, r \tag{56}$$

$$\Delta = \begin{vmatrix} A & 0 & 0 & a & a' \\ 0 & B & 0 & b & b' \\ 0 & 0 & C & c & c' \\ a & b & c & 0 & 0 \\ a' & b' & c' & 0 & 0 \end{vmatrix}$$

$$= \Sigma A\, [b\, c]^2 \tag{57}$$

$$\Delta_{11} = [b\, c]^2 \tag{58}$$

$$\Delta_{12} = -\, [a\, c]\, [b\, c] \tag{59}$$

$$\left.\begin{aligned} p &= -\, 2/\Delta \,|\, \sqrt{A\, B} \,|\, [a\, c]\, [b\, c] \\ q &= +\, 2/\Delta \,|\, \sqrt{A\, C} \,|\, [a\, b]\, [b\, c] \\ r &= -\, 2/\Delta \,|\, \sqrt{B\, C} \,|\, [a\, b]\, [a\, c] \end{aligned}\right\} \tag{60}$$

Hence $[b\, c] : [c\, a] : [a\, b] = p\, q \,|\, \sqrt{B\, C} \,| : r\, p \,|\, \sqrt{A\, C} \,|$

$$: q\, r \,|\, \sqrt{A\, B} \,| \tag{61}$$

and the special two-winding cases are:

$$\begin{aligned} a : b : c = \quad 0 \quad &: \quad q \,|\, \sqrt{B} \,| : -\, p \,|\, \sqrt{C} \,| \\ -\, r \,|\, \sqrt{A} \,| : \quad &0 \quad : \quad p \,|\, \sqrt{C} \,| \\ r \,|\, \sqrt{A} \,| : -\, q \,|\, \sqrt{B} \,| : \quad &0 \end{aligned} \tag{62}$$

of which but two are independent. For maximum output of A, $(p^2 + q^2) = 1$ and (56), (57) become

$$p\, q = r \tag{63}$$

$$\Delta \quad = 2\,A\,[b\,c]^2$$

Parallel Type Circuit. Circuit D, Fig. 6, symmetrical in *A, B, C, a, b, c, r, q, p*.

$$p^2\,q^2 + q^2\,r^2 + r^2\,p^2 = -\,2\,p\,q\,r \tag{64}$$

$$\Delta \quad = \begin{vmatrix} A & 0 & 0 & a \\ 0 & B & 0 & b \\ 0 & 0 & C & c \\ a & b & c & 0 \end{vmatrix}$$

$$= -\,\Sigma\,A\,B\,c^2 \tag{65}$$

$$\Delta_{11} = -\,B\,c^2 - C\,b^2 \tag{66}$$

$$\Delta_{12} = C\,a\,b \tag{67}$$

$$\left.\begin{aligned} p &= 2/\Delta\,|\sqrt{A\,B}\,|\,C\,a\,b \\ q &= 2/\Delta\,|\sqrt{A\,C}\,|\,B\,a\,c \\ r &= 2/\Delta\,|\sqrt{B\,C}\,|\,A\,b\,c \end{aligned}\right\} \tag{68}$$

$$a : b : c = \frac{|\sqrt{A}\,|}{r} : \frac{|\sqrt{B}\,|}{q} : \frac{|\sqrt{C}\,|}{p} \tag{69}$$

For maximum output of *A*, $(p^2 + q^2) = 1$, which makes (**64**) and (**65**) assume the forms

$$p\,q = -\,r \tag{70}$$

$$\Delta \quad = -\,2\,B\,C\,a^2 \tag{71}$$

APPENDIX

IDEAL TRANSFORMERS

Circuits containing ideal transformers may be solved for cisoidal oscillations by using the ampere-turn and volt-per-turn properties of ideal transformers, or by solving first for ordinary transformers, and then evaluating the expressions on passing to the ideal transformer limit, but in general discussions, such as that of the master circuit, it is more convenient to employ the following symmetrical bordered determinant:—

$$\begin{vmatrix} Z_{11} & Z_{12} & . & . & Z_{1n} & N_{11} & . & . & N_{1m} \\ Z_{21} & Z_{22} & . & . & Z_{2n} & N_{21} & . & . & N_{2m} \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ Z_{n1} & Z_{n2} & . & . & Z_{nn} & N_{n1} & . & . & N_{nm} \\ N_{11} & N_{21} & . & . & N_{n1} & 0 & . & . & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ N_{1m} & N_{2m} & . & . & N_{nm} & 0 & . & . & 0 \end{vmatrix} \tag{72}$$

where Z_{jk} is the mutual impedance between closed paths j, k with all transformer windings short-circuited and N_{jk} is the number of turns in the closed path j on the core k. The bordered determinant is used in exactly the same way as the ordinary network determinant for determining impedances, currents and voltages.

For transformers which closely approximate to the ideal it is convenient to use the following amplified form of (72):—

$$\begin{vmatrix} Z_{11} & Z_{12} & . & . & Z_{1n} & N_{11} & . & . & N_{1m} \\ Z_{21} & Z_{22} & . & . & Z_{2n} & N_{21} & . & . & N_{m2} \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ Z_{n1} & Z_{n2} & . & . & Z_{nn} & N_{n1} & . & . & N_{nm} \\ N_{11} & N_{21} & . & . & N_{n1} & Y_{11} & . & . & Y_{1m} \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ N_{1m} & N_{2m} & . & . & N_{nm} & Y_{m1} & . & . & Y_{mm} \end{vmatrix} \tag{73}$$

where $Y_{jk} = i\, \mathcal{R}_{jk}\, h^{-1}\, p^{-1} = i\, 0.7958\, \mathcal{R}_{jk}\, p^{-1}$ has the dimensions of an admittance; $\mathcal{R}_{jk}$ is the mutual reluctance between magnetic meshes j, k; h is the factor for converting ampere-turns into gilberts.

This determinant is obtained by writing Kirchhoff's electromotive force equations for the n independent closed electrical paths, and the corresponding magnetomotive force equations for the m independent closed magnetic meshes, which are of the forms:

$$\left.\begin{aligned} Z_{11} I_1 + Z_{12} I_2 \ldots + Z_{1n} I_n + N_{11}\dot{\phi}_1 \ldots& \\ + N_{1m}\dot{\phi}_m = E_1& \\ -h N_{11} I_1 - h N_{21} I_2 \ldots - h N_{n1} I_n + \mathcal{R}_{11}\phi_1 \ldots& \\ + \mathcal{R}_{1m}\phi_m = 0& \end{aligned}\right\} \qquad (74)$$

Divide the magnetomotive force equations by $-h$ and for the magnetic fluxes substitute their time rates, using the relation for cisoidal oscillations $\dot{\phi}_j = i\,p\,\phi_j$. The complete set of $(n + m)$ equations are now linear in the currents $I_1, \ldots I_n$ and the time rates of flux $\dot{\phi}_1, \ldots \dot{\phi}_m$ and (73) is the determinant of the set of equations.

The discussion of (73) would include the application of complex quantities so as to include transformers having any self and mutual impedances, due to winding and core losses, but this falls outside the scope of this appendix.

Returning to (72), which is derived from (73) by making all Y's zero, we will briefly indicate the results which follow from applying the theory of linear transformations to this determinant, and then state, without formal proof, a number of theorems which apply to ideal transformers in general, finding a particular application in the case of the maximum output circuits.

Multiplying both the $(n + 1)$st column and row of (72) by a_{11} ($\neq 0$) and adding to them a_{12} times the $(n + 2)$nd column and row, respectively, leaves the impedances, currents, and voltages unchanged, since the determinant and the minors involved all remain unchanged except for the same factor a_{11}^2. Generalizing this we find that each of the m bordering columns and rows of (72) may be replaced by linear combination of these columns and rows, respectively, without changing the impedances, currents, and voltages, provided the determinant of the transformation

$$\begin{vmatrix} a_{11} & \cdot & \cdot & a_{1m} \\ \cdot & \cdot & \cdot & \cdot \\ a_{m1} & \cdot & \cdot & a_{mm} \end{vmatrix} \tag{75}$$

does not vanish; the transformation is then called non-singular. There are thus ∞^{m^2} changes which may be made in m ideal transformers without any resulting modification in the electrical behavior of the network.

By subtracting bordering rows and columns one from another, after multiplication by suitable constants, it is evidently possible to reduce one or more of the windings to zero turns, unless there is but one linearly independent transformer. This simplifies the set of transformers and thus it is of importance to know how many windings may be reduced to short circuits in this way. It may be shown that $(m-1)$ windings on each of the m transformers may be reduced to short circuits, provided the m transformers are linearly independent. In this case at least one of the m-rowed determinants of the transformer windings will not vanish, and by transposing paths this may be made the determinant

$$D = \begin{vmatrix} N_{11} & \cdot & N_{1m} \\ \cdot & \cdot & \cdot \\ N_{m1} & \cdot & N_{mm} \end{vmatrix}$$

which will be transformed to

$$D' = \begin{vmatrix} 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & 1 \end{vmatrix}$$

by means of the linear transformation (**75**) if the parameters have the values

$$a_{ij} = D_{ji}/D \tag{76}$$

where D_{ji} is the cofactor of the element N_{ji} in the determinant

D. Each transformer will have, after this transformation, $w = (n + 1 - m)$ windings where n, m are the number of degrees of freedom of the geometrical circuit and the number of independent transformers. But each independent transformer introduces an independent ampere-turn condition between the currents; the number of effective degrees of freedom is thereby reduced to $e = n - m$. Thus if n, e are given the number of transformers and the necessary windings on each are $m = n - e$, $w = e + 1$. For the unrestricted maximum output circuit $n = 4$, $e = 2$ which makes $m = 2$, $w = 3$; in other words, two ideal transformers each with three windings are required.

IDEAL TRANSFORMER THEOREMS

1. An ideal transformer is an electrical device which may be used to transfer energy, but itself can neither absorb, store, nor supply energy. Its action may be expressed in terms of "winding turns" upon one or more magnetic core meshes. A single-mesh magnetic core is assumed in theorems 2–14.

2. For an ideal transformer the total ampere-turns equal zero and the voltage drop is the same for every winding turn; therefore, n windings impose a total of n distinct conditions, *viz.*, the total ampere-turns equal zero and the potential differences between terminals are proportional to the algebraic number of turns connecting the terminals.

3. An ideal transformer may be replaced by a fictitious closed loop, having self impedance equal to zero, and mutual impedance with each real branch of the circuit equal to the algebraic number of turns contained in this branch.

4. Introducing an ideal transformer into a network adds a border to the network determinant, each term of which equals the number of winding turns in the closed path corresponding to the row, or column, in which the term occurs.

5. Each independent ideal transformer reduces by one the effective degrees of freedom of the network; the greatest number of independent transformers which can be introduced into an n-degree circuit without effectively opening every closed path is thus $(n - 1)$. The number of transformers and the necessary windings on each are $m = n - e$, $w = e + 1$ if n, e are to be the geometrical and the effective degrees of freedom of the network.

6. With m independent ideal transformers connected into

any portion of a network, presenting n degrees of freedom, the general case demands but $(n+1-m)$ windings on each of the transformers.

7. Two sets of ideal transformers which are mutually derivable, the one from the other, by a non-singular linear transformation, are electrically equivalent.

8. The electrical effect of a set of ideal transformers will not be altered by adding one or more transformers, each of which is a linear combination of the original set of transformers, or by removing one or more transformers, each of which is a linear combination of the reduced set of transformers. In every case the determinant (**72**) to be used for determining the impedances is one for which all linearly dependent transformers have been eliminated; the value of the determinant is otherwise zero and the mathematical expression for the impedances would require evaluation.

(9) An ideal transformer with n windings, none of which has zero turns, may be replaced by $(n-1)$ transformers each with two windings in any way which leaves unchanged the ratio of the number of turns which are upon any core and directly connect any two pairs of points of the network.

10. Short-circuiting one winding of an ideal transformer, for which the number of turns is not zero, short-circuits all windings.

11. Each winding of a transformer v may be short-circuited provided one or more of its windings, which do not have an algebraic total of zero turns, form part of a closed path p completed by windings on other transformers, each of which contributes a total of zero turns to the closed path, or is made to do so by the linear transformation of adding to the transformer t the winding turns of transformer v, after multiplication by $(-N_{tp}/N_{vp})$, which is the ratio, with sign reversed, of the total algebraic turns in the closed path p on transformers t and v.

12. Any closed path, made up of windings from ideal transformers, may be opened by omitting any one branch of this closed path, provided every transformer contributes an algebraic total of zero turns to the closed path.

13. One or more windings of an ideal transformer, which can be cut once only by a closed surface, that cuts no other part of the electrical system, may all be increased or decreased by the same number of turns counted outward through the surface;

in this way any one of these windings may be reduced to zero turns, *i. e.*, to a short circuit.

14. A two-winding ideal transformer connected between an element having impedance and the point of measurement, multiplies impedances, electromotive forces, and currents by the factors a^2, $-a$, and $-a^{-1}$, respectively, where $a = N_1/N_2$ is the ratio of the number of turns on the measuring side N_1 to the number of turns on the element side N_2 each counted in the assumed positive direction for the closed path.

15. An ideal transformer with an n-mesh magnetic core, may be replaced by n ideal transformers with single-mesh magnetic cores; the above theorems, which have been intentionally phrased to apply specifically to single-mesh core ideal transformers, may thus be applied to multi-mesh core transformers with slight modifications.

APPENDIX VII

GEOMETRICAL CIRCUITS WITH FOUR DEGREES OF FREEDOM

By distinct electrical circuits we mean those which may have different internal electrical properties, demanding in the first instance, different conditions of continuity. For the purposes of this paper it is necessary to determine the distinct geometrical circuits having four degrees of freedom.

For geometrical circuits, the geometrical condition of distinctness may be put in the following concrete form: Assume that wire or rubber thread models have been made of two circuits and (a) that the models may be translated and rotated, (b) that the wires may be bent or stretched, (c) that wires or groups of wires, which have a series connection may be interchanged, and (d) that any two disconnected portions may be connected at a single point. Then the circuits are not distinct if the wire models may be brought into geometrical coincidence by these four transformations; otherwise they are distinct. After the geometrical circuits have been replaced by physical electrical circuits, it is to be understood that what is required is in every case the coincidence of interchangeable elements.

Geometrical circuits having four degrees of freedom must have not less than four, nor more than nine, branches, provided branches which are in series, and must thus carry the same current, are excluded from the count. The lower limit is evident; the upper limit is proved by noting that in any connected portion of the circuit having more than one branch-point, at least three branches terminate at every branch-point, and for any circuit whatever $p = b - f + s$ where f, b, p, and s are the number of degrees of freedom, of branches, of branch-points, and of separate portions of the circuit, respectively. Set $f = 4$, since we are dealing with a four-degree circuit and $s = 1$, corresponding to a singly connected circuit, as we wish the largest value of b for any given p. In this case, $2b \geqq 3p$, since each branch has two ends, and each branch-point has connected to it at least three of these ends. It follows that $b \leqq 9$.

By trial, it is readily found that **W** and **X** of Fig. **5**, are the only possible nine-branch circuits in which three branches terminate at each of the required six branch-points. Circuits having eight branches are now found by allowing first one and then another branch of **W** and **X** to shorten and vanish. In **X** it is sufficient to consider the vanishing of branches **1** and **9** since all other branches are by symmetry equivalent to one or the other of these two. In **W** there is complete symmetry and it is sufficient to consider the vanishing of branch **1**. With the understanding that **X − 1 = V** means that circuit **X** after the vanishing of branch **1** becomes equivalent to circuit **V** (ignoring the numbering and the positive direction of branches) the deduction of all of the sixteen four-degree circuits is shown by the following table:

X − 1 = V,
X − 9 = U,
W − 1 = V,

V − 1 = S,
V − 5 = R,
U − 1 = T,
U − 3 = S,
U − 6 = Q,
U − 8 = R,

T − 1 = M,
T − 7 = O,
S − 1 = N,
S − 2 = P,
S − 4 = M,
R − 1 = O,
R − 2 = P,
R − 6 = M,
Q − 1 = M,

P − 1 = L,
P − 2 = K,
P − 4 = J,
O − 1 = K,
N − 1 = J,
M − 1 = K,
M − 2 = J,

L − 1 = I
K − 1 = I
J − 1 = I

It is thus shown that the only possible four-degree circuits are the sixteen shown by Fig. 5, where each branch is numbered for convenience in reference and has an assigned positive direction. Fig. 6 gives all circuits having one, two, and three degrees of freedom.

DISCUSSION ON "PRINTING TELEGRAPH SYSTEMS" (BELL), AND "MAXIMUM OUTPUT NETWORKS FOR TELEPHONE SUBSTATION AND REPEATER CIRCUITS" (CAMPBELL AND FOSTER), NEW YORK, N. Y., FEBRUARY 19, 1920.

General George O. Squier: I believe that the printing telegraph is the thing, and coming to be more and more the method of the future, and I find that is the opinion in Europe where I have recently made a visit of inspection.

The printing telegraph was used a very great deal in the war and we early found that the only hope of handling the large amount of traffic was to get some sort of a hopper that we could pour it into, ad lib., and get it out at the other end. The volume of traffic in operating a large army in France is unbelievable, and luckily we were able to take over this very same system to Europe and put it into operation. This same system described today, was in extensive operation. In my inspection I saw poured into this hopper messages between the headquarters of General Pershing at Chaumont and the great business end of the army down at Tours. And without that method of carrying the vast traffic, I am perfectly certain the whole machine would have collapsed. It was absolutely necessary to have a perfect system of communication at all times, and that solved it. We also put this system in between London and Paris, and Tours and Chaumont, so that you could get a page printed record immediately out in France. If the war had gone on, we proposed to have that all over between the different headquarters.

So, the printing telegraph really went to the battlefield for the first time, and we got away with it, eight messages on the line. It proved for the first time that this sort of technical apparatus could go to the battlefield. That was encouraging and made us think that this apparatus as well as other can be operated by the ordinary soldier.

In regard to Little Silver, the Government Laboratory for radio research work. There we have large research laboratories, and are carrying out various forms of signalling, of all sorts, and of all the different methods of signalling that we have to consider in the army, I think it is safe to say that the method of signalling I am now employing is probably the best, that is, the human voice.

That is the reason why the telephone is so superior to all other methods and we can afford therefore, in any case, to exert every technical effort to extend the range of the human voice. Every one, by ordinary conversation of course, is an expert signal man. You do not have to teach anybody to operate this puncher or to learn the Morse code, but you make a signal man at your mother's knee. However, that appeals to the ear only, and we must have records. So that the eye method must come in and of those, the typewriter, telegraph printing system is the best. All these other things are substitutes. We learn to speak directly and that is best from the oral standpoint. We learn to read and that is best from the optical standpoint; those two methods go hand in hand, and they have each forged to the front. That is why the printing system is ahead of any other system.

Now that science has advanced to such a state that we really can produce a large number of channels, reliable channels, along one conductor, we are within a measureable distance, I believe, of extending the telegraph system, the eye system, and beginning to approach the method that we now have for telephoning. The fundamental difference between the operation of the telephone and telegraph is that in the telephone each operator is exclusively connected with the other man, and has the line all to himself. In the telegraph you have to write something out then hand it in to somebody else, or have it sent or phoned. In that respect the telephone has gone much further than the telegraph, and I believe we are now within a measurable distance, of having both systems in the house, so that if you call an individual residence, if the line is busy, you need not be told that, but just leave the message. That would be ideal.

It is not possible now due to wire difficulty. We have already the Telautograph. That is perhaps useful in its way. I look for something not so far away due to the fact that we can have so many independent channels.

General Edgar Russel: I can speak of the multiplex telegraph with positive affection. It was such a good friend to us at the time of our severe need, and it helped us out in such a marked way, when difficulties seemed to be mounting beyond the possibility of our meeting them.

I have been greatly interested in Mr. Bell's paper today, and I join with General Squier in hoping we may hear more of the engineering considerations that brought out this remarkable instrument. I was very much pleased to get some statements about comparative traffic possibilities. I might have gained the idea from what I saw of the operation of the multiplex in France, that its capacity was unlimited already, and that it needed no further improvement. I feel that we owe a tremendous debt to the engineers that developed this remarkable telegraph system that served an invaluable purpose when our needs in the A. E. F. were so severe.

At the time when the telegraph business in France was growing beyond all bounds, and the mounting traffic curve seemed as if it were going to fall over backwards, these instruments we succeeded in getting over in the midst of vast tonnage and personnel difficulties, rescued us from a very difficult situation. After these instruments were put in operation our very greatest difficulty, the lack of Morse operators, became much less acute. When the multiplex was established between our headquarters at Tours and

Paris and Chaumont, traffic difficulties immediately gave way, and our concern about insufficient telegraph service between those main points practically vanished.

The possibility of putting, as we did by composite devices on the telephone lines, a six-way multiplex transmission was, I believe, first demonstrated in France. I do not understand that it has been the practise in this country to composite for six-way transmission. The system succeeded wonderfully. When the American cable was laid across the English channel, we operated it with this six-way system between our headquarters in Chaumont, France, and London.

Our multiplex telegraph system in France was a standing wonder to our French friends. As stated by the lecturer, the Baudot printer system is carrying the bulk of traffic over there, and of course they were well acquainted with the old types of tape printers. Although a few of the French experts connected with their telegraph system had seen the American multiplex, most of them would stand and watch with keen interest this page printer in operation with its fascinating automatic shifting device at the end of each line.

When I went to London, I found in the Great British Post Office Central four or five multiplex instruments in operation, and I was told by those in charge of the systems there, that whereas they recognized its many excellent points, they had a very elaborate set of reasons why the Baudot tape printer system was better. I confess I was unable to answer many of their arguments, for today I got for the first time a clear understanding of the reasons why the American system of multiplex is far superior to the Baudot and kindred systems.

I think that the speedy application of the American multiplex to our needs in France was the outstanding feature of our telegraphic accomplishments. The way in which we arrived at our decision to adopt the multiplex may be of interest. Before I started to France with General Pershing on May 28th, 1917, I had only a few days in which to decide. But I had been in consultation with General Squier at Washington, who had assured me he was going to support us with every possible assistance that could be secured in this country, and when I came to New York and saw Colonel Carty, and some of his associates in the great companies controlling the supply of personnel and material, I realized that with such support as that, I would be justified in recommending that we go ahead with the installation of the modern multiplex and the modern telephone repeater systems.

As you understand, we were far from our base, in a strange country, and where we could not get spare parts or the facilities for repair. Besides there were tonnage difficulties and difficulties of manufacture in this country owing to the conflicting activities of war. But I now feel as I look back on that critical time, when we had only a day or two to decide, what a serious mistake we would have made, if we had assumed the conditions of war would not permit us to avail ourselves of this wonderful modern invention. In the midst of the enormously growing demands upon us we could say to officers "Gentlemen, bring us all the telegrams you want. You do not need to limit your telegraphic correspondence in any degree."

Those who have been over in France know of the delays we had in getting mail; in fact, the difficulties in the mail service seemed insuperable, but by the free use of the telegraph service, which this instrument gave us, we were enabled to transmit promptly this vast volume of messages, which the mail service was inadequate to handle expeditiously.

C. E. Davies: In Canada we are using automatic telegraphs to handle approximately 65 per cent of our 24-hour load. I can only add my evidence to what has been said in the paper as to the accuracy compared with Morse. Our records, dealing with complaint cases from customers, actual complaints of errors with the automatic, show approximately a ratio of 3 to 7, with Morse. It is the intention of the company to extend the automatic principle throughout, as soon as the equipment is available.

I can remember twenty years ago, as a Morse operator, the amused tolerance with which we looked on anything in the nature of a suggestion that we could handle business by automatics. That feeling was gradually wiped out through improvements in the automatic system until today it is well recognized that the automatics will never be replaced by the Morse system.

The problems that we have to face in Canada are somewhat different than in the United States, inasmuch as the transcontinental lines are from east to west and cover great distances. We have been enabled to operate circuits which have been affected with power induction satisfactorily, but as yet we have not been able to find any solution to a difficulty presented by frequent troubles due to the Aurora Borealis. Some time ago I suggested to our chairman it might be a matter of interest to the engineers of the A. I. E. E. to make an investigation as to these disturbances. I do not know whether anything has been accomplished along that line or not. The Canadian lines are interrupted considerably more, I believe, than the American lines.

The disturbances seem to be principally confined to circuits running east and west, and as we are now installing triple channel multiplex circuits from Montreal and Toronto to Winnipeg, and Vancouver, that is going to be one of our principal problems, how to operate during the periods of interruption due to the Aurora Borealis.

H. A. Emmons: I have been greatly interested in Mr. Bell's paper, primarily in connection with outputs. What he has said in comparing the various methods checks our own experience in the Western Union Service. We find that our accuracy corresponds very closely to what the British figures show, and that our output is going up steadily.

We started in 1915, installing multiplex apparatus on a wide or rather on a broad plan of equipping all trunk circuits throughout the United States and Canadian connections with printing telegraph apparatus of multiplex type. Today the trunk circuit traffic handled by printing telegraphs amounts to about 38,000,000 messages a month in round figures. That is about 70 per cent of the trunk circuit traffic that the Western Union Company handles. Those are not originating messages; many messages are handled more than once due to difficulties of reaching various isolated places, many of which are small places.

It may interest the engineers to know that the telegraph company operates a New York-San Francisco direct multiplex circuit which is about 3200 miles long, and I received a report the other morning, stating that the operators on this circuit have run 90 messages an hour. Each operator had run 90 messages an hour for the 12 o'clock to 8 a. m. tour. That is a most remarkable output on a circuit that is 3200 miles long. It is an equivalent of 180 messages an hour, in both directions, between New York and San Francisco. We do not expect to get that during day hours. There are too many interruptions over such a great mileage of wire.

One of the most remarkable things in regard to the multiplex is its freedom from interruption. Figures which I have been getting together largely indicate that the maintenance is of a high order; the men handling the apparatus appear to become so familiar with it that interruptions over any extended period are very few. It is most gratifying to the engineers who have fathered this device to find that this is true. We had a great many misgivings initially that the personnel detailed to the maintenance of the apparatus might not measure up to the high standard that would be required. I am happy to say that it is the exception rather than the rule that these men fall down on their jobs.

It shows what men can do if they specialize, and these men are specialists. If they were required to do many things besides maintain the apparatus, it would not be possible. As we all know, this is an age of specialists, and I feel sure that if printing telegraphs are to be a success in any company, the men that handle the apparatus and the operators that operate it, must be specialists. We must not expect that they will be all around men. Some of them will be, many of them will be, but the average man, making the personnel of a telegraph organization must be a specialist when it comes to printing telegraph apparatus.

General Squiers spoke of the personnel that went overseas.

Here in America the printing telegraphs are operated almost exclusively by young women. When the war came on we knew that there was going to be a great need for telegraph operators overseas. There were few multiplex operators who were men, most of them were women, and the army had no provision at that time for handling women. We learned very early that multiplex operators were going to be required overseas. Many of them volunteered, they got into various organizations outside of the signal corps, and it was with some difficulty that they were transferred to the signal corps.

I think that was accomplished, however, and the first lot of young men that went overseas, if my recollection is correct, more than 25 per cent of them were thoroughly experienced operators, attendants, and test-board men, making up the necessary personnel for operating that overseas multiplex, and I am exceedingly glad to learn that they came through under the difficult war conditions which they met.

P. M. Rainey: One difficulty which the telegraph engineer experiences in his development work is that of securing complete information to enable him to determine the requirements of the field. By this I mean information which will enable him to anticipate as many as possible of the problems which are likely to arise in the field. As I see it the greatest success will result from the close coöperation of those who are doing this laboratory or development work, and those who best know conditions in the field.

J. P. Edwards: I have noticed only one item of statistics that I consider out of line insofar as the Western Union Traffic is concerned, and that is the statement of the Chairman that 80 per cent is handled by automatics. That of course you found corrected in Mr. Emmons' statement. Approximately 47 per cent of the total traffic handled is handled automatically, principally by the multiplex system. The facility with which the traffic has been handled for a number of years on account of its growth could not have been accomplished except with the development of the multiplex in the Western Union, because generally speaking we are doing business today on the same wire plant that we had five years ago, and approximately the same wire plant we had ten years ago.

The Morse operators have been quite concerned for a number of years that the automatic was displacing them, and Mr. Davies has pointed out his experience of the concern of the men with whom he has been associated for years over the approaching disappearance of their occupation. As a matter of fact, the Western Union is in need of more Morse operators today than ever in the history of the company.

E. R. Shute: In training the staff for the operation of the printing telegraph system, we have established schools in the principal cities where we teach the students the touch system of keyboard manipulation and our operating practises.

The students qualify and handle approximately the capacity of the circuits, as operated by the Western Union Telegraph Company, in about three months. After reaching this stage, that is, graduating from the training school, they are given further instructive supervision for from three to six months, when they become fully qualified printing telegraph operators.

There is nothing that I think of that I can add to the paper presented by Mr. Bell. The large number of engineers who do not know that there is a printing telegraph system in extensive use is surprising to one who knows of the work done in this line and I think we should have more papers on this subject in the future.

R. E. Chetwood: There has not been very much change in line construction, on account of the use of automatic apparatus. Of course, the larger output per wire, due to the multiplex has made it desirable that we have more reliable line construction, and more reliable wires than we did previously. When a wire is handling from 250 to 400 messages an hour, you can afford more expensive and stronger construction.

The result of that has been that in our line construction we have strengthened our pole lines considerably; we have also replaced some of the smaller sized iron wires when they reached their normal life with heavier iron wires. We are using a great deal of No. 4 and No. 6 B. W. G. iron wire, where previously, No. 8 and No. 9 B. W. G. wires were used. That gives us greater continuity of service and the wires have not been subject to the troubles which wires of less strength receive in winter, fall and spring, due to the combined effect of wind and ice.

In general, as Mr. Edwards pointed out, the wire mileage has not increased and that is due entirely to the use of the multiplex apparatus. There was a tremendous increase in telegraph business during the war. It came at a time when we could not have obtained new wire to string on the lines, to handle the increased business, and it was the multiplex and other printing telegraph systems that allowed us to handle the tremendous increase in business, without increasing the wire plant.

George R. Benjamin: Regarding the Aurora Borealis and its effect on transmission we have arranged to operate the circuits metallically. Ordinarily the telegraph circuits are operated with a grounded return. To take care of such time as we are seriously affected on the more important circuits, we have arranged to connect them up and operate them metallically.

The Western Union service is using, and other companies are using a considerable number of Morkrum sets, stop and start method, and I regret very much Mr. Bell did not say something about the Morkrum apparatus. As regards the multiplex, it is really a development of the Baudot but there are many important changes, as Mr. Bell suggested. The changes are all intended to increase the efficiency of multiplex operation. The Baudot system is limited as to speed, the speed ranges being between 30 and 40 words per channel. In some cases it is 30 words and in exceptional cases they have arranged to operate at 40. The use of perforated tape for transmission increases the efficiency because it does not admit ordinarily of any stoppages or periods of wait. The method of synchronism saves in line time, about fifteen or twenty-five per cent; that is, with a lower line frequency, it is possible to get the same output as the Baudot operating at say, 30 or 40 words a minute. The page printer was devised primarily to meet American requirements. It is possible that tape printing should be taken into consideration. The page printer of course is very much more complex, and it requires more attention and has many more parts. It also increases the maintenance. The American people have not been accustomed to tape printing and probably will not take to it very kindly at first, but I think eventually they will go into it and turn out a good job, and they will be satisfied with the good service.

Lloyd Espenschied: The phase of Mr. Campbell's paper which is probably of most immediate interest to the engineer is that of the conjugacy obtaining between the terminals of the network. This conjugacy is the property of mutual balance which is familiar to the communication engineer as employed in separating the oppositely directed channels of two-way transmission circuits.

His work should find application in all branches of communication, for example, to direct and alternating-current telegraphy, to telephony, to high-frequency currents transmitted by wires or by radio, and so on. To expand Dr. Campbell's paper into one which would feel more familiar to the engineer, would require approaching it from the engineering or application aspect. There is here a fertile field and I trust that somebody will find it within their province to pursue the subject further by taking up particular problems, in coordination with the present paper, such as that of substation circuits to which the paper so directly applies. Reference is made to the telephone repeater but the repeater as commonly practiced today makes use of only a very limited form of Dr. Campell's circuit arrangements.

O. B. Blackwell: It will be of interest in connection with Dr. Campbell's paper to point out that in telephone and telegraph transmission we have a class of problems which do not occur in power transmission. These problems arise from the fact that a telephone or telegraph circuit is used in general not only for transmitting messages going, let us say, from east to west, but also messages going from west to east. At certain points in the circuit such as at the subscribers' sets and at repeater points, it becomes desirable to branch the circuit into two parts or "channels" one of which carries transmission east to west, and the other of which carries transmission west to east. At these points it becomes very important that transmission going in one direction shall cause as small currents as possible in the channel arranged for currents going in the opposite direction. It is these factors that make Dr. Campbell's work of the greatest value to us in that he discusses broadly the problems which arise in thus dividing a telephone or telegraph circuit into an east-going and west-going channel, with maximum efficiency and with minimum mutual interference between the two channels.

I think perhaps the paper does not sufficiently emphasize one fact. Dr. Campbell in his discussion refers to his four elements as resistances. This does not mean, of course, that the application of this work is limited to those cases in which the elements are actually pure resistances. In the practical cases in which we apply it the elements have both resistances and reactances.

John H. Cuntz: A word as to the training of multiplex operators for service overseas might be of interest. When the first lot of multiplex men were sent over, as Mr. Emmons has told you, it was very soon found that it would be extremely difficult to obtain trained multiplex men in this country; in fact, it was absolutely impossible to spare any more. So it devolved upon the Signal Corps to train the necessary men.

Up at the College of the City of New York we already had a Signal Corps school, where men were being trained in radio and general Signal Corps work, and it was decided to change that over, a part of it at least, into a multiplex school. The order came through and the multiplex school was started immediately, and we realized from the beginning that in order to get men ready as soon as possible, it was best not to take radio men, or men skilled in electrical work, but to train touch typists as multiplex operators. We got hold of these touch typists as fast as possible, and we trained them and sent them over within the required time.

Orders subsequently came through for more and more men, and the supply of touch typists gave out, so we had to get rather raw material and train them as well as possible. At the same time, the Western Union Telegraph Company coöperated with us in the most remarkable and able manner, and the whole country was scoured for possible multiplex men. We secured them from the Pacific Coast, from Florida and from everywhere in between. A great many multiplex men unfortunately had already been transferred to other branches of the service, but we got them from the infantry, and artillery and various other branches; and they were concentrated at the City College and sent overseas in quick time.

By the fall of 1918, we had already sent over several hundred, and as it was contemplated to enlarge the multiplex system on the other side, plans were already under way for training several thousand, which would have gone through if the armistice had not come along just then. As far as we can hear, the multiplex men we sent over, after a little necessary further training, acquitted themselves very favorably, and did particularly well.

John H. Bell: The only remarks which seem to call for any attention are those by Mr. Benjamin. He mentioned that I do not refer to the Morkrum System. I do by inference on the first page,—"practically all the printing telegraph systems not considered in this paper have a carrying capacity of from 50 to 70 words per minute, and consequently are in a class by themselves." It is for that reason I did not include the Morkrum System. If I had included the Morkrum, I would have had to include a description of the Western Electric start-stop, the Hughes, and the Harrison, and that would have made the paper very much too lengthy.

G. A. Campbell: No questions have been raised which need any word from me. Of course, the paper, as it is presented, is a mathematical one, and no attempt has been made here to go into the physical problem, which I hope, will be presented by some one else at another time.

Presented at the 8th Midwinter Convention of the
American Institute of Electrical Engineers,
New York, February 19, 1920.

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A METHOD FOR SEPARATING NO-LOAD LOSSES IN ELECTRICAL MACHINERY

BY CARL J. FECHHEIMER
Designing Engineer, Westinghouse Electric and Mfg. Co.

The method proposed makes use of idle operation of the machine as a motor, the voltage being varied, and speed kept constant. After deducting the armature $I^2 R$ losses from the watts input, the remaining watts are plotted against the voltage. A formula is derived based upon the assumption that the watts are equal to constant windage and friction loss plus core loss which latter varies as a constant power of the voltage. In applying the method, tangents to the curve are drawn at two points, and from the slopes of these tangents, the voltages and watts at these points, the exponent of the core loss curve, the core loss, and the windage and friction may be calculated with the use of the equations derived. An example is given of the close agreement with the test curve in the case of an induction motor; and other examples are cited of close agreement of the core loss and windage and friction losses with the losses measured by means of the usual belted method. The fact is pointed out that in some machines more accurate results are obtainable by means of the proposed method than with the usual belted method.

THE usual method of measuring iron, and friction and windage losses in direct-current and synchronous alternating-current machinery consists in driving by belt from a small direct-current motor and measuring the power input. This method has three disadvantages: (a) The losses in the belt, and the losses in the driving motor are somewhat uncertain, especially the former; (b) the driving motor belt, and possible other auxiliaries require extra floor space, and necessitate equipment which may not be available or difficult of application; and (c) in driving some machines, especially turbo-generators with large stored energy in rotating parts, it is difficult to average with accuracy the current input to the driving motor.[1]

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[1] The author believes that the belt friction loss may vary while taking observations when driving large turbo generators, which may introduce an additional error.

The method outlined below may at times necessitate equipment which is not available, or it may require more changes than would warrant its use.

The method is one that has been used frequently. We believe, however, that the method of separation of iron from friction and windage losses is new. It consists in operating the machine idle as a motor, at constant speed, with various voltages impressed across the armature, the watts input to the armature being measured. In the case of a d-c. machine, the excitation and voltages are simultaneously changed so as to maintain constant speed. With synchronous a-c. machines, it is desirable to maintain approximately unity power factor by altering the excitation with the voltage, although slight departure from minimum current will not introduce appreciable errors. With induction machines, the method is the usual idle running saturation curve—the voltage being varied and frequency kept constant. It is advisable to employ integrating instruments for the power, although in some cases indicating instruments will give satisfactory results.

In d-c. and a-c. synchronous machines, the armature $I^2 R$ loss is usually negligible, but in induction machines, this loss is, in general, of appreciable magnitude, and should, therefore, be deducted from the power input.

If we plot volts as abscissas, and power input to the armature[2] (or primary, if an induction motor) as ordinates, the curve will be like that shown in Fig. 1. It is well known that, in many machines, if the laminated iron subject to cyclic changes in flux densities is worked above the knee[3] of the *B-H* curve, there is stray flux produced which may, by its rate of change, induce electromotive forces in parts external to the laminated iron structure, and such differences of potential may cause currents to circulate, and thereby give rise to by no means negligible losses. If this condition obtains, the total so-called iron losses augment at a higher rate than ordinarily and we shall, therefore, consider only that part of the loss curve below the point at which the high loss gradient begins. We may then write the equation of the curve:

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[2] It is understood that if the $I^2 R$ losses in the armature or primary are appreciable, they must be deducted from the power input.

[3] We appreciate that "knee" of curve is liable to be misleading. It appears to us, that with ordinary iron, the departure referred to begins around 12,000 lines per sq. cm. in the core, and 16,500 lines per sq. cm. in the teeth. In giving the method of separation to our Testing Department we have asked them to select the high point at approximately 90 per cent of normal voltage.

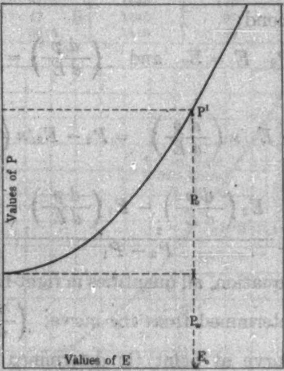

FIG. 1

$$P = P_w + P_0\,(E/E_0)^n \qquad (1)$$

Where E = any voltage,

E_0 = voltage below "knee" of B-H curve,

n = constant exponent which indicates the rate at which iron loss increases with voltage,

P_0 = iron loss at voltage E_0,

P_w = friction and windage loss (constant at constant speed),

P = total loss = sum of friction and windage and iron losses.

Or, transposing P_w and taking natural logarithms,

$$\log_e (P - P_w) = \log_e (P_0/E_0)^n + n \log_e E \qquad (2)$$

Differentiating with respect to E, and remembering that $(P_0/E_0)^n$ is a constant,

$$\frac{1}{(P - P_w)} \frac{dp}{dE} = \frac{n}{E} \tag{3}$$

Or, solving, $P_w = P - \frac{E}{n}\left(\frac{dp}{dE}\right)$ (4)

Select two points[4] on the curve;

For the first:

$$P = P_1 \quad E = E_1 \quad \text{and} \quad \left(\frac{dp}{dE}\right) = \left(\frac{dp}{dE}\right)_1 ;$$

For the second:

$$P = P_2 \quad E = E_2 \quad \text{and} \quad \left(\frac{dp}{dE}\right) = \left(\frac{dp}{dE}\right)_2$$

Then,

$$P_w = P_1 - E_1/n\left(\frac{dp}{dE}\right)_1 = P_2 - E_2/n\left(\frac{dp}{dE}\right)_2 \tag{5}$$

Whence

$$n = \frac{E_2\left(\frac{dp}{dE}\right)_2 - E_1\left(\frac{dp}{dE}\right)_1}{P_2 - P_1} \tag{6}$$

In this equation, all quantities in right-hand member may be determined from the curve; $\left(\frac{dp}{dE}\right)_1$ is the slope of curve at point (1) determined by drawing tangent to curve at that point, etc.

Having found n, we may substitute in (**5**), and evaluate P_w; or we may solve for P_w without solving for n, thus:

$$P_w = \frac{E_2 P_1\left(\frac{dp}{dE}\right)_2 - E_1 P_2\left(\frac{dp}{dE}\right)_1}{E_2\left(\frac{dp}{dE}\right)_2 - E_1\left(\frac{dp}{dE}\right)_1} \tag{7}$$

Finally, if P' = power input at normal voltage, E_0

$$P_0 = P' - P_w \tag{8}$$

We now have all of the constants in equation (**1**), and the curve may readily be checked by calculating a number of points.

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[4] See previous foot-note in regard to "knee" of curve for selection of higher point.

We give below data from a typical test curve of a three-phase induction motor; the equivalent single-phase resistance (1.5 × resistance between terminals) = 0.0438 ohm.

TABLE I

Volts	Amperes	Watts input	I^2R	Watts-I^2R
220	104	3640	475	3165
200	93	3210	380	2830
175	80	2730	280	2450
150	68	2320	202	2118
125	57.5	1940	145	1795
100	47	1640	97	1543
75	36.5	1410	59	1351

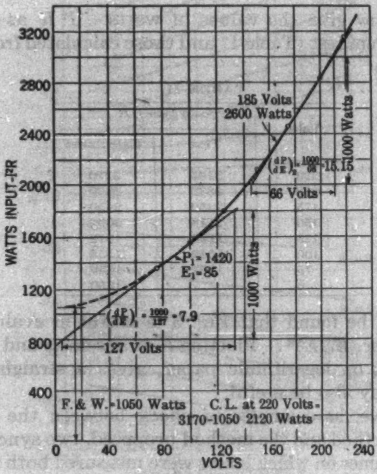

FIG. 2

We shall plot (Watts $-$ I^2R) against volts as in Fig. 2. If we draw tangent to curve at point $E_2 = 185$ volts, we obtain $\left(\frac{d\,p}{d\,E}\right)_2 = 15.15$, and $P_2 = 2600$ watts. Similarly, for point 1, we take $E_1 = 85$ volts, $P_1 = 1420$ watts, $\left(\frac{d\,p}{d\,E}\right)_1 = 7.9$.

Substituting in equation (6)

$$n = \frac{185 \times 15.15 - 85 \times 7.9}{2600 - 1420} = 1.81,$$ the exponent for iron loss.

Then from equation (**5**),

$$P_w = 2600 - \frac{185}{1.81} \times 15.15 = 1050$$ watts friction and windage.

From equation (**8**), $P_0 = 3170 - 1050 = 2120$ watts iron loss at 220 volts. Substituting in equation (**1**),

$$P = 1050 + 2120 \left(\frac{E}{220}\right)^{1.81}$$ is the equation of the curve.[5]

We now give the values of watts $- I^2 R$ as determined from test (Table I) and those calculated from the equation:

TABLE II

Volts	Watts-I^2R	
	Test	Calculated
220	3165	3170
200	2830	2830
175	2450	2450
150	2118	2092
125	1795	1812
100	1543	1558
75	1351	1350
0	....	1050

It will be found that Fig. 3 is useful in evaluating $(E/E_0)^n$ or $(E_0/E)^n$. Plotting K as abscissas and K^n as ordinates on logarithmic paper, gives a straight line which may also be useful.

We give, as examples of check between the usual belted method and the method proposed, two synchronous machines on which losses were measured both ways. (See following table.)

It is interesting to note that slight inaccuracies in drawing the tangents to the curves at the two points do not introduce great errors. For example, in Fig. 2, the tangent at point 2 can be drawn but little, if any, different from that indicated; but at point 1, we might, as extreme with not careful work, draw the tangent so that its slope $\left(\frac{d\,p}{d\,E}\right)$ is 8.3 instead of 7.9. This gives for the exponent "n" 1.774 instead of 1.81 and for P_w 1020 watts instead of 1050 and for P_0 2145 instead of 2120. The difference in friction and windage is thus 3 per cent and in core loss 1.2 per cent. Although this example does not so indicate, the exponent "n" usually changes by a larger percentage with slight changes in tangents than does the friction and windage or core loss. Also in the example, the exponent is less than 2,

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[5] In this case, the iron is worked below "saturation" at normal voltage, so that we have taken E_0 to correspond to normal voltage of 220.

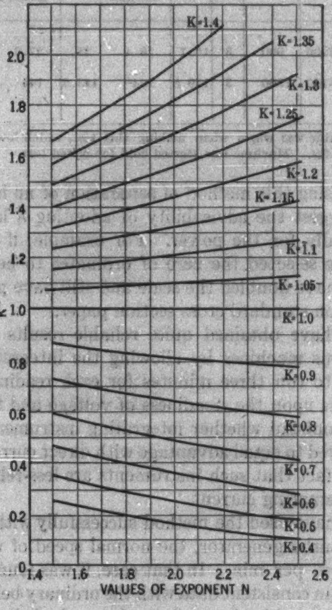

FIG. 3

whereas in those synchronous machines which we have investigated, the exponent is slightly above 2.[6]

Machine rating				Kw. core loss at nor. voltage		Kw. friction and wind		
Kv-a.	Volts	R.P.M.	Ph.	Belt	Proposed method	Belt	Proposed method	Remarks
1340	2200	900	3	16.1	16.4	28.	27.6	Part of m-g. set
10,000	4160	1800	3	129.5	124*	143.5	144	Turbo generator

*Machine was warm when making this test, which lowered the iron loss slightly. Machine was started cold for belted test.

In using this method of separation of no load losses, we suggest the advisability of adopting a large scale, especially for the power. For example, if in Fig. 2, had we selected the zero of ordinates to be 1000, we could have doubled the scale and still have plotted the curve on standard cross section paper.

We have obtained quite reliable results with synchronous machines by allowing the integrating wattmeter to run three minutes for each reading. Much depends upon the steadiness of voltage and frequency. It is doubtful whether integrating instruments can be employed to equal advantage with direct current, owing to the fact that such instruments are less reliable than for alternating current.

We have used the method successfully with a 13,500-kv-a. turbo generator, the normal speed of which was 2185 rev. per min. In that case, it was found difficult to obtain consistent data with the ordinary belt method, owing probably, in part, to change in belt loss. We have not experienced difficulty in starting polyphase synchronous machines, having simultaneously brought up to speed the turbo generator and the machine supplying power to it.

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[6] Several years ago, the writer observed when plotting on logarithmic paper, core loss curves of synchronous machines (taken by belted method), that the exponent was greater than 2, even below those densities which correspond to beginning of saturation. It would be interesting to have explanations from Institute members.

While the foregoing application is outlined for separation of iron loss from friction and windage loss, and determination of the equation of the iron loss curve below saturation, it is obvious that it may be applied to any problem in which the general equation is similar in form.

DISCUSSION ON "A METHOD ON SEPARATING NO-LOAD LOSSES IN ELECTRICAL MACHINERY", (FECHHEIMER), NEW YORK, N. Y., FEBRUARY 19, 1920.

V. Karapetoff: Equation (1) in Mr. Fechheimer's paper is of the form that has interested me for a number of years. Now and then a problem arises in which a quantity varies (or may be assumed to vary) according to the *n*th power of an independent variable, plus a constant term. The expression of the no-load losses in an electrical machine is one such case, and unfortunately the logarithmic paper cannot be used for the determination of the exponent *n*, because of the presence of the other constant term in the equation.

The separation of the core loss from the friction by the method described is not new in principle, although I do not recall having seen the use of a fractional exponent in just this way. In my "Experimental Electrical Engineering", in the chapter entitled "Direct-Current Machinery—Efficiency and Losses", I give a similar formula, with the one exception that the exponent *n* is assumed to be equal to 2. This is nearly correct for machines working at a slightly higher degree of saturation than that selected for the numerical example in the paper under discussion. In many cases it will be simpler to obtain an approximate value of the friction losses using the exponent 2, and then obtain more accurate values of P_w and *n* by a few trials, using logarithmic paper. This possibility does not detract, of course, from the ingenuity of the method as used by Mr. Fechheimer in application to large machines.

The introduction of the quantities P_0 and E_0 does not seem to be necessary, since they do not appear in the final expressions (6) and (7). It may be preferable to write (1) in the form

$$P = P_w + K E^n \qquad \text{(a)}$$

The application of this formula is limited to the range of flux values for which *n* is approximately constant; this statement takes the place of P_0 and E_0.

In the place of expression (6) I wish to suggest a different formula. Taking a derivative of (a) with respect to *E*, and introducing for the sake of brevity

$$p = d\,P/d\,E \qquad \text{(b)}$$

we obtain

$$p = k\ n\ E^{n-1} \qquad \text{(c)}$$

Writing this expression for two points, 1 and 2, on the curve in Fig. 2, we get

$$\frac{p_2}{p_1} = \left(\frac{E_2}{E_1}\right)^{n-1} \tag{d}$$

$$n - 1 = \frac{\log\left(\dfrac{p_2}{p^1}\right)}{\log\left(\dfrac{E_2}{E^1}\right)} \tag{e}$$

Whether formula (6) or eq. (**e**) is used it is advisable to compute n for more than one pair of points, in order to see if it remains approximately constant.

In order to increase the accuracy of the method, it may be advisable to compute the friction loss P_w from the data for several points on the curve, and to take the average value. This may be done as follows: Multiplying both sides of equation (**c**) by E we get

$$p\,E = n\,K\,E^n \tag{f}$$

Substituting the value of $k\,E^n$ derived from this expression in to equation (a), we obtain

$$P = P_w + \frac{p\,E}{n} \tag{g}$$

Let (g) be written for several points, say, on the curve. Adding these expressions term by term we get

$$\sum P = q\,P_w + \frac{1}{n}\sum p\,E \tag{h}$$

$$P_w = \frac{1}{q}\left(\sum P - \frac{1}{n}\sum p\,E\right) \tag{i}$$

The value of P_w so computed will represent the true average value for the whole curve and for the average value of n.

A much closer approximation to the real shape of the experimental $P - E$ curve is obtained by the addition of a quadratic term to expression (**1**), thus:

$$P = P_w + a\,E^2 + k\,E^n \tag{j}$$

The addition of this term, which represents the eddy current loss, and possibly some of the hysteresis loss, makes it possible to carry the analysis to much higher degrees of saturation than is permissible with the less accurate formula (1). Moreover formula (**j**) permits a simultaneous use of four points on the $P - E$ curve instead of two, as in Mr. Fechheimer's method. To determine the unknown coefficients P_w, a, k, and n, we differentiate equation (j) with respect to E and multiply the result by E. This gives

$$E\,p = 2a\,E^2 + n\,k\,E^n \qquad \text{(k)}$$

Eliminating $k\,E^n$ between (j) and (k), as before, we obtain

$$n\,P + E^2 a\,(2-n) - n\,P_w = E\,p \qquad \text{(l)}$$

We now take two points, 1 and 2, on the given curve, write equation (l) for both, and subtract one expression from the other to eliminate $n\,P_w$. The result is

$$n\,(P_2 - P_1) + a\,(2-n)\,(E_2^2 - E_1^2) = E_2\,p_2 - E_1\,p_1 \qquad \text{(m)}$$

A similar equation is then written for two other points on the curve, say 3 and 4. These two equations may then be solved together for n and $a\,(2-n)$, considering the latter expression as one variable. Knowing a and $n\,P_w$ may be computed from equation (l). Finally k may be determined from equation (j), should this quantity be deemed to be of interest.

While the actual computations are more involved than with the simpler formula (l), the resultant curve passes through four points on the given curve, instead of two only. This resultant curve will also probably fit the given curve so closely that check computations for per cent error will be unnecessary in most cases.

In an exceptionally important case involving a dispute, or with a large number of experimental data of comparatively small accuracy, it may be advisable to use the method of least squares, so as to obtain not one of the possible solutions but *the most probable curve* and *the probable error*. Since one of the unknown quantities, n, is an exponent, an expansion according to Taylor's theorem is necessary. A similar case in thermodynamics has been treated in detail.*

In some cases, especially with large machines, the *retardation method* † for determining the friction loss may prove preferable to the one described in the paper under discussion; or else the two methods may be used to check each other. The machine under test is brought to a speed say 10 per cent above that at which the friction is to be measured. Then the power is shut off and the machine is allowed to slow down to a speed about 10 per cent below the desired speed. While the machine is slowing down, a speed-time curve is taken, using an electric tachometer; or else a deceleration-time curve may be taken using a magneto in series with a galvanometer and a condenser. This test is performed with the field current on, and then again with the field circuit open. The field current must have exactly the same value as during a run at the rated speed at which the total loss is determined with a wattmeter, as in the test described in Mr. Fechheimer's paper. The slopes of the two retardation curves will give the relative proportion of the friction in the total stray loss, and knowing the numerical value of this loss from the wattmeter reading, the amount of friction loss is readily computed.

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*V. Karapetoff, "Engineering Applications of Higher Mathematics", Part III, p.62.

†V. Karapetoff, "Experimental Electrical Engineering", Vol.I Cap. on "D-C. Machinery—Efficiency and Losses".

Thus, with a simple additional test the hypothesis such as equation (1) becomes unnecessary, and the computations are reduced to dividing a wattmeter reading in a known ratio. The retardation test has been further improved by Professor Robertson* who has described a very accurate stroboscopic method for speed-time measurement, and by D. Kapp† who has shown how to vary the retardation rate at will by keeping the machine connected to the line.

I believe that Mr. Fechheimer is wrong in his statement that the retardation method requires the determination of the moment of inertia of the revolving part, and I do not think any of us would care to use this method if it were based on the accuracy of the calculation of the moment of inertia. There are at least four methods whereby the moment of inertia can be eliminated out of the computations, one method is to put a brake on the shaft as an additional known friction loss, and to perform one run with the brake and one without. From the slopes of the two curves the moment of inertia may be eliminated.

Another method is to change the unknown moment of inertia by adding a mass of a known moment of inertia, and again to make two runs, one with the additional mass, and one without it. There is also a purely electrical method proposed not long ago by Dr. Kapp (see reference above), in which he allows the machine once to retard and once to accelerate, while the machine is connected to the source of power through a large resist-

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*Professor D. Robertson, "Separation of the No-Load Stray Losses in a Continuous-Current Machine by Stroboscopic Running-Down Methods", *The Journal of the Institution of Electrical Engineers* (British) Vol. 53, 1915, pp. 308 to 328.

†Dr. Gisbert Kapp, "Experimental Determination of the Moment of Inertia of a Continuous Current Armature", *The Journal of the Institution of Electrical Engineers*, (British) Vol. 44, 1910, pp 248 to 254.

ance. This also gives two equations, from which the moment of inertia is eliminated. Perhaps the simplest method for eliminating the moment of inertia is to perform one run by the retardation method, and another at a constant speed, taking one wattmeter reading. From the two runs the moment of inertia may be eliminated and this is probably the method most used.

In the particular class of machines with which Mr. Fechheimer is concerned, it seems to me that the retardation method is especially proper, and particularly with the refinements that have been brought into the art within the last few years. One of the refinements is the use of very accurate measuring devices for determining instantaneous speeds. Another refinement is the use of Dr. Robertson's stroboscopic method referred to above.

William F. Dawson: Evidently the main object of Mr. Fechheimer's paper was to bring out the advantage of this method of separating no-load losses. I may say for the sake of historical accuracy, that the method is not new. I have known about it for twelve or fifteen years and it was in use before that. The method is very convenient and is highly to be commended. Anybody who has ever had to do with using belts for getting core losses must realize their limitations. Unless they are specially prepared and glazed joints take the place of lacings there are many difficulties, and for turbo-generators, it is practically impossible to curve any belt around a small shaft or available pulleys so as to drive the load evenly and uniformly.

My own experience with respect to testing turbo-alternators began eight or nine years ago. With the then available methods, it was difficult to get core losses. We could take them by retardation, but as Mr. Fechheimer says, they were not satisfactory. It is possible, as Dr. Karapetoff says, to take them without knowing the moment of inertia. At that time however, I could not be sure of the speed. Tachometers were not satisfactory, and counting machines, while suggested, were not available. Therefore we adopted the method described by Mr. Fechheimer. The turbo alternators, with the turbine wheels connected to them, were driven as synchronous motors, and by varying the electromotive forces supplied by the generator, and taking a full curve for each voltage reading, we obtained the minimum input or unity power factor.

As a rule, we found that while we got very excellent readings in the upper part of the curve, we could not get steady readings much lower than one-third the normal voltage and so had to extrapolate the curves but the results were very satisfactory. I never thought that the exponent was constant, and I have run much above the saturation and found exponents as high as five. I think—I do not know —that the explanation is as Prof. Karapetoff has pointed out. Core losses of one group of materials are superimposed upon the core losses of another group of material. Judging by the saturation curve of the machine, one may be below the knee, but there are a number of elements which enter into the saturation, the saturation of the armature core itself, which is the principal thing, and saturation of the teeth which will often saturate more quickly than any other part of the magnetic circuit. The magnetic length is short and it has small effect upon the saturation curve, but a large effect on the core loss. Machines with large air gaps straighten out the saturation curve enormously. In some machines the focault current losses are generated by magnetic potential difference across the slot and these introduce a new element, depending on the width of the conductor.

There is one interesting thing about this scheme. One can test the turbo-alternator, and separate these no-load losses, even with the turbine wheels on. But one must be cautious, because in running a turbo-alternator either as a synchronous motor or synchronous condenser, without steam passing through the turbine, the wheels and buckets may become so hot as to damage them. For this reason it is a standard practise with us to test with the wheels removed, or else to lead low pressure steam into the turbine, in such a way, as not to cause any motoriug.

Those interested in the steam end of the set may at the same time measure the turbine rotation losses, at various degrees of vacuum. Do not try to maintain such a high vacuum that it cannot be maintained steadily. We found we could do very well with a vacuum of 27 or 28 inches (2 inches to 3 inches absolute).

There is another method of separating these no-load losses. I will not go into that method now but the article I have prepared will appear in the February number of the *General Electric Review*.

W. I. Slichter: The discussion of the variation of the exponent *B*, which causes variation in core loss, is most interesting. We have not only the cause of this variation, which we find in simple samples of iron; but we have this variation which has been alluded to as the flux seeking different paths and causing additional eddy currents in the solid masses. There is also a third cause, which I should contribute to this, that as we go up in saturation, the wave shape in our flux will be shaped, and the relation which we figure between voltage and flux density at one point will not be the same at another. The matter, therefore, must be handled with great care by one not experienced with it.

P. L. Alge (by letter): Mr. Fechheimer's method of extrapolating an incomplete curve by assuming it to be of a simple logarithmic character has proved useful in many fields of experimental science, notably that of ballistics, and his application of it to the separation of no load losses is very appropriate.

Mr. C. MacMillan, has proposed a somewhat similar method, which we use to arrive at the same end. This method consists in plotting the excitation watts against voltage squared, thus bringing the lower experimental points much nearer to the axis of zero voltage, and drawing a representative straight line, or at worst a very flat curve, through the experimental points by eye. If the core loss did obey a square law, the points would fall exactly on a straight line and this line would be very accurately determined. In practise, the law is so nearly a true square law that the points over the middle range of voltage do fall nearly on a straight line and it is not difficult to choose a representative line. The method amounts to assuming the value of n to be 2 and taking a mean value of the coefficient of V^n. It has the advantages of requiring no calculation except squaring the voltage, of throwing the excitation curve so near to the axis of zero volts as to require but a very short extrapolation, and of giving the curve to be drawn the simplest possible shape.

The chief difficulty we have met with in using this method has been that the experimental points frequently give indications of obeying two quite different laws of variation in different parts of the curve, thus making any extrapolation inaccurate. (Both parts are frequently straight lines when watts are plotted against volts squared). However, this difficulty applies to any method of extrapolation and it can only be said that orderly methods of extrapolation like Mr. Fechheimer's and Mr. Macmillan's, will give much better results than the usual hit or miss method of drawing the watts-volts curve by eye.

The question raised by Mr. Fechheimer, as to the causes of variations of core loss with voltage at a higher rate than the square is a very interesting one. I believe there are four principal reasons which may cause n to be greater than 2:

1. Lessening of the effective area of flux path with

increase in density. As the flux density rises, saturation sets in in some places notably at the tooth corners and at unannealed edges of the punchings, so that the flux is crowded into a smaller area of iron the higher the density is. This lessening of area of path causes the flux density to rise more rapidly than the voltage, thus causing the losses to go up faster than the square of the voltage.

2. Increasing distortion of wave form with increase in density. The wave form of flux variation at any particular point of the iron is not usually sinusoidal, even though the total flux varies sinusoidally: since the distribution of flux between available parallel paths changes markedly with increase of density. Thus eddy currents of triple and higher frequencies exist in the iron even at low densities and with sinusoidal variation of total flux. These higher frequency eddy losses may increase at a considerably faster rate than the square of the voltage.

3. Lessening of effective resistance of the eddy current paths with increase in voltage. Any eddy current losses caused by imperfect insulation between stampings may be expected to rise at a higher rate than the square of the voltage since the contact resistance between stampings will decrease as the eddy currents increase.

4. Crowding of flux into iron outside the laminations. As Mr. Fechheimer, has observed, at high densities some flux will be crowded into the solid iron frame or other magnetic parts contiguous to the laminations. These parts will have a much higher specific loss than the laminations, so that after they begin to receive flux the total losses will rise quite rapidly.

C. J. Fechheimer: The trend of much of the discussion is in regard to the variation of the exponent "n", which I took to be constant. We must remember that in commercial testing, there is no desire to secure laboratory precision. I have plotted core loss curves of a large number of machines on logarithmic paper, and the curves were always a straight line indicating the constancy of the core loss exponent "n", provided there was no saturation in any parts of the magnetic circuit undergoing cyclic change. I do not mean to define just what the point of the beginning of saturation is other than as stated in the third footnote in the paper. The shape of the no-load saturation curve is not necessarily a means for determining the point of departure, as the saturation may be in the parts undergoing cyclic change, such as the armature core, or it may be in the poles in which the flux is constant. Inasmuch as the tests are usually made in the testing department of the manufacturing company the designer could indicate to the testing department what is the highest point on the curve to which the tangent might be drawn. In all cases in which the method has been used, the curves calculated by means of equation (1) after "n" and "$P\ w$" had been evaluated by means of equations (6) and (4) were almost in exact coincidence with the test curves. I therefore, do not believe it would be wise to ask our testing departments to employ the more tedious formulas which Prof. Karapetoff has so nicely worked out.

As I understand Mr. Dawson's statement in regard to "foucalt current losses generated by magnetic potential difference across the slot", such losses at no-load are either due to saturation of the teeth to which I have just referred, or arise from some of the main flux entering the slot; in the latter case, these losses are proportional to the square of the voltage and are included as part of the so-called "iron" loss, regardless of the method of measurement.

Prof. Slichter's reference to the change in wave shape of flux at different points on the saturation curve, I understand to be a change in flux wave shape in the air gap. That is modified if the teeth are saturated, and so long as we keep below high tooth densities, no error is introduced.

I agree with Mr. Dawson that the exponent "n" may be as high as five, well above saturation. The losses increase at a very high rate, once the flux does not remain in those parts to which it is supposed to be confined.

In abstracting the paper, I made mention of the retardation method of measuring no-load losses, and pointed out its disadvantages, difficulty in securing accurate measurement of speed, and in calculating the moment of inertia. Prof. Karapetoff tells us of refined methods of taking speed-time curves and of means of evaluating the moment of inertia experimentally. Such improvements undoubtedly have advantages, especially if employed in laboratories; but our commercial testing departments, which today are loaded with more instruments than they would like to have, would very reluctantly introduce such devices as Prof. Robertson's stroboscope or additional brake (the friction loss in which is probably subject to variation). I am not entirely familiar with some of the methods of measuring moment of inertia about which Prof. Karapetoff tells us, and I believe that he has given us a number of suggestions which may prove helpful.

Presented at the 8th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 19, 1920.

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INHERENT REGULATION OF CONTINUOUS CURRENT CIRCUITS

BY A. L. ELLIS
Thomson Laboratory, General Electric Company, Lynn, Mass.

AND

B. W. ST. CLAIR
Standardizing Laboratory, General Electric Co., Lynn, Mass.

This paper discusses the voltage changes, inherent in d-c. circuits, upon change of load. These variations are independent of $i\,r$ drops or speed of prime movers. A simple means of mitigating their effect is given.

INTRODUCTION

IN developing a device to measure time to one millionth of a second we were at one time confronted with the question, how is the voltage of a "flat" compounded generator affected by an increase or decrease of load? We were not interested in what happened as indicated by a voltmeter, (as the generator was carefully compounded for the range of load under consideration), but what happened at the instant the load was changed. We were surprised at the extent of the voltage drop, indicated by the oscillograph, when the load was increased. The very interesting oscillograms and data resulting from further investigation of this drop form the basis of this paper.

The principles governing this drop are set forth in elementary text books on engineering, and while simple in this form their application to a network of conductors supplied with current by a generator is difficult to follow, and when mathematically stated lead to very complex equations that can be comprehended only by a painstaking study of the problem.

The result is that the busy engineer with insufficient time to concentrate upon this subject frequently encounters, in practise, manifestations of this drop without realizing the cause or recognizing the principle.

We find very little written on the subject; a suggestion of it here and there in articles referring to transients, and with the exception of reference made to it in Lamme's paper[1] on commutation, no comprehensive statement of the case has come to our notice.

The simple explanation of this dip in voltage given in Lamme's paper follows verbatum: "Assume, for instance, a 100-volt generator supplying a load of 100 amperes—that is, with one ohm resistance in circuit. The drop across the resistance is, of course, 100 volts. Now, assume that a resistance of one ohm is thrown in parallel across the circuit. The resultant resistance in circuit is then one-half ohm. However, at the *first instant* of closing the circuit through the second resistance, the total current in the circuit is only 100 amperes, and therefore the line voltage at the first instant momentarily must drop to 50 volts. However, the e. m. f. generated in the machine is 100 volts, and the discrepancy of 50 volts between the generated and the line volts results in a very rapid rise in the generator current to 200 amperes. If the current rise could be instantaneous, the voltage dip would be represented diagrammatically by a line only; that is, no time element would be involved. However, *the current cannot rise instantaneously in any machine,* due to its self-induction, and therefore, the voltage dip is not of zero duration, but has a more or less time interval. The current rises according to an exponential law, which could be calculated for any given machine if all the necessary constants were known. However, such a great number of conditions enter into this that it is usually impracticable to predetermine the rate of current rise in designing a machine; and it would not change the fundamental conditions if the rate could be predetermined, as will be shown later."

Lamme points out that, "If a given load is thrown on a machine, the dips will be relatively less the higher the load the machine is carrying. Also, if the *same percentage* of load is thrown on each time, then the dips should be practically the same, regardless of the load the machine is already carrying." He also gives a table of test results to substantiate this taken from oscillograph curves which were considered too faint to be reproduced in the paper.

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[1] B. G. Lamme, "Physical Limitations in D-C. Commutating Machinery." TRANS. A. I. E. E. 1915, Part II.

In our investigation of the "dip" we were able to get very good oscillograms though the phenomenon is a difficult one for the oscillograph to record. It is the purpose of this paper to extend the evidence by oscillograph records and plots drawn up from a mathematical consideration of a range of ideal circuits, which give at a glance a better understanding of the relative importance of various conditions than would be disclosed in pondering over a word picture.

LOAD INCREMENTS

Several types of generators were examined and the "dip" found in all in accordance with the theory within the limitations of the oscillograph.

The following oscillograms were obtained from the same generator under various conditions of load and may be considered to represent fairly the performance of any continuous-current generator, making due allowance for generator constants.

The performance of any continuous-current generator in this regard can be more easily understood by considering what happens when

A non-inductive load is added to a generator carrying no load........................(Case one);
An inductive load is added to a generator carrying no load........................(Case two);
A non-inductive load is added to a generator carrying a non-inductive load............(Case three);
An inductive load is added to a generator carrying a non-inductive load..............(Case four);
A non-inductive load is added to a generator carrying an inductive load..................(Case five);
An inductive load is added to a generator carrying an inductive load..................(Case six).

CASE ONE

If an ideal non-inductive resistance is thrown on the circuit of a separately excited generator carrying no load, there will be an instant demand for current of value $\frac{E}{r_1}$ where E is the voltage at the terminals to which load is applied and r_1 is the resistance of the applied load.

Owing to the inductance of armature and leads, current cannot be supplied instantly, and as no current was flowing before load was applied and did flow afterward, obviously it must have increased from zero value. As $E = IR$ the voltage must drop to zero at the instant the load is applied, and increase along a simple exponential curve depending only upon the time constant of the circuit, to the value equal to the generated voltage minus the internal IR drop.

The drop to zero is therefore, fundamental and is independent of the type of the generator, its size, its generated voltage, or the value of the applied load. These variables influence only the time of recovery to normal voltage and the form of the recovery curve.

Considering the inductance of the circuit to be confined to the armature: The instant demand for current gives rise to an e. m. f. of self-induction which opposes any increase of current (at that instant zero) and with respect to the generated voltage is therefore a counter e. m. f. The value of this counter e. m. f. with respect to the value of the generated voltage at any instant during the transient, is just sufficient to allow current to flow into the circuit at a rate such that the e. m. f. of self-induction $(L\frac{di}{dt})$ at any instant added to the IR drop at that instant equals the generated voltage. Therefore, at the instant when load was applied and the current was zero the counter e.m.f. of self-induction just equaled the generated voltage in value, consequently the voltage of the circuit reduces to zero.

If, as in all practical cases, part of the inductance is in the leads of the external circuit, the voltage at the generator terminals should drop instantaneously to a value approaching zero as the inductance of the external circuit approaches zero.

Capacity effects greatly complicate matters and their consideration precludes a simple presentation of the phenomena. This is particularly so in the case of underground cable distribution where we find distributed capacity and inductance to a considerable extent. However, in the case of isolated plants, overhead distribution, and branches of underground systems shut off, as it were, by the inductance of protection devices, instruments, meters, etc., the available capacity is, in general, so small as compared to the instant demand for energy that capacity effects have been neglected. Those who wish to consider capacity effects in connection with this phenomenon can refer to such standard works as "Transient Phenomena" by C. P. Steinmetz, and "Propagation of Electric Waves Along Telegraph Conductors" by J. A. Fleming.

The following conclusions and equations assume lumped constants that remain constant during the time necessary for the readjustments to take place, and are applicable also to battery circuits where there is a series inductance that would correspond to the inductance of the generator armature, as for instance, the series coils of a circuit breaker or watt-hour meter.

As previously pointed out, applying a non-inductive load to an unloaded generator results, at the generator terminals, in an instantaneous drop of voltage to zero. From this point the voltage returns to normal value according to an exponential curve.

$$e = E\,(1 - \epsilon^{-\alpha_1 t}) \qquad (1)$$

where

e is voltage at any instant
E = normal voltage
t = time since application of load.

$$\alpha_1 = \frac{r}{L_a}$$

where

r = complete resistance of circuit.
L_a = inductance of armature.

Equation 1 can be developed from

$$e = ir_1 \qquad (2)$$

where i is instantaneous value of current
r_1 = resistance of applied load.
and

$$i = I\,(1 - \epsilon^{-\alpha_1 t}) \tag{3}$$

where $$I = \frac{E}{r_1}$$

At $t = 0$, that is at the instant of closing the switch $\epsilon^{-\alpha t}$ reduces to 1, and equation **3** becomes

$$i_{(t=o)} = 0 \tag{4}$$

which substituted in (**2**) reduces (**2**) to

$$e_{(t=o)} = 0$$

From the foregoing it will be seen that the voltage drop is independent of the value of the resistance applied, also that the velocity of return to normal is a function of the resistance, and an inverse function of the inductance.

The natural question as to the whereabouts of the voltage generated by the rotating armature of the machine, which is dependent only upon the speed of rotation and the strength of field, can be answered by a study of the reactions of the armature itself, the instant of closing the circuit.

Because the armature is inductive it resists any attempt to change the value of current flowing in it by a counter e. m. f. = e_c

$$e_c = L_a \frac{di}{dt} \tag{5}$$

which by differentiating (**3**) with respect to t, and substituting in (**5**) becomes

$$e_c = rI\,\epsilon^{-\alpha_1 t} \tag{6}$$

$$= E\,\epsilon^{-\alpha_1 t} \tag{7}$$

which at $t = o$ becomes

$$e_{c_{\,t=o}} = E$$

From **7**, **3**, and **2** it is seen that the sum of the external ir_1 voltage and the armature reactance voltage neglecting armature resistance, is always equal to the generated voltage, that is

$$ir_1 + L_a \frac{di}{dt} = E$$

Fig. 1 is a plot from calculated values and illustrates how a change in the value of the applied non-inductive load affects the shape of the voltage recovery curve and the time of recovery. The solid lines represent current increase as well as generator e. m. f. The broken lines represent the decrease in counter e. m. f. as the current approaches its steady value. The sum of the generator voltage and the counter e. m. f. at any instant during the transient equals 100 per cent of the generator voltage when the transient terminates.

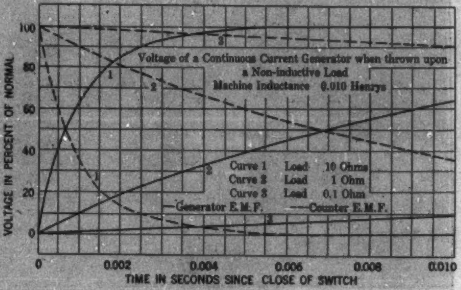

FIG. 1

Some oscillograms from actual tests are shown in Fig. 2. None of these show the voltage drop completely to zero, as should be expected, for a simple calculation is sufficient to show that in every case, the rate of rise of voltage was such that, assuming a natural period of 4000 cycles per second for the oscillograph vibrator, the voltage would be sufficiently great to give measurable deflections before the vibrator could return to zero. The table of test results given in Lamme's paper, taken from oscillograph curves, does not indicate sufficient drop in cases where the added load was a large part of the combined load, particularly where a load of 417 amperes was added to a 1200-volt

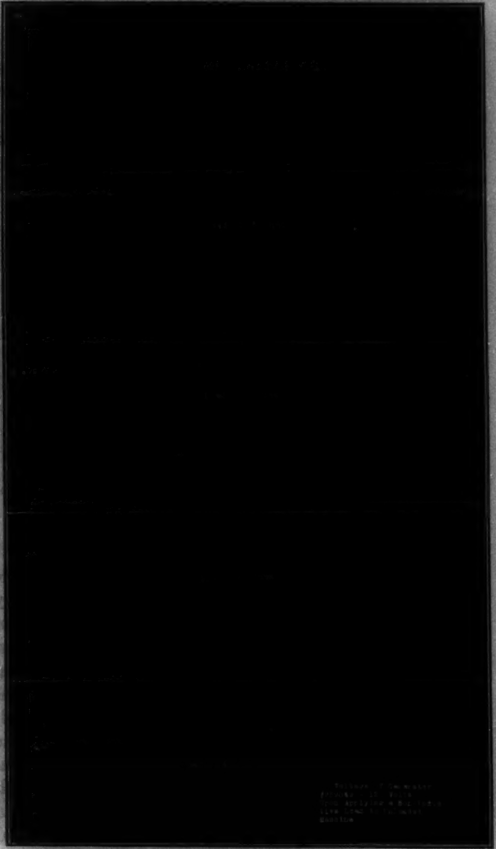

FIG. 2

generator carrying no load. The volt "dip" from 1200 to 500, there recorded, we believe due to the limitations of the oscillograph (mentioned by Lamme), should have been substantially to zero volts.

The oscillograms show clearly the variation in the rate of recovery and time of recovery with varying demand upon the generator. The distance between each peak represents an elapsed time of approximately 0.001 second. Some of the detail in the original oscillograms has disappeared due to the several photographic processes necessary to reduce them to print. None have been retouched for any purpose.

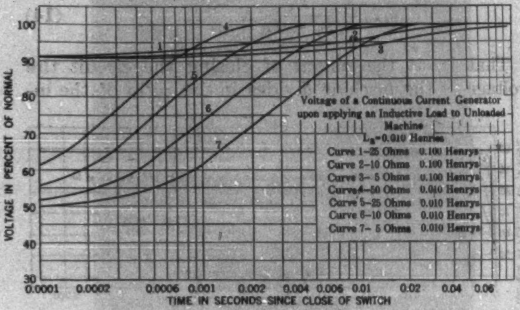

FIG. 3

CASE TWO

An inductive load thrown upon an unloaded machine.

Under these conditions the voltage can never drop to zero, since

$$e = ir + \frac{di}{dt} L_1 \tag{9}$$

where L_1 is inductance of load.

As in equation (3) the current will rise according to an exponential law,

$$i = I(1 - \epsilon^{-\alpha_2 t}) \tag{10}$$

where $$\alpha_2 = \frac{r_1}{L_a + L_1}$$

differentiating we get

$$\frac{di}{dt} = \frac{r_1 I}{L_a + L_1} \epsilon^{-\alpha_t t} \tag{11}$$

$$= \frac{E}{L_a + L_1} \epsilon^{-\alpha_t t} \tag{12}$$

substituting (10) and (12) in 9 we get

$$e = E\,(1 - \epsilon^{\alpha_t t}) + \frac{L_1 E}{L_a + L_1} \epsilon^{-\alpha_t t} \tag{13}$$

which at $t = o$ becomes

$$e_{t=o} = \frac{L_1}{L_a + L_1} E \tag{14}$$

That is, at the instant of closing the circuit the voltage of the machine divides in the ratio of the inductance of the external and internal circuits, and there is an instantaneous drop to this value. The instantaneous voltage drop depends upon the ratio of the inductances, and is independent of their absolute value, or of the resistance of the circuit. The velocity of return to normal voltage, however, depends, upon the resistance, since at the instant of closing the switch

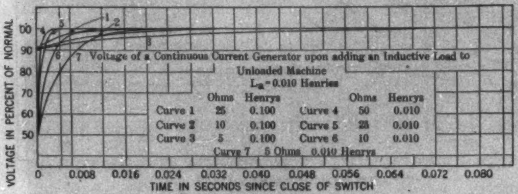

FIG. 4

$$\frac{dt}{dt} = \frac{rI}{L_t}$$

where $$L_t = L_a + L_1 \tag{15}$$

Fig. 3 is a plot of calculated values on semilogarithmic paper and shows how a change in the value of resistance and inductance of applied load affects the extent of voltage drop and the rate of rise to normal.

Fig. 4 gives the same data plotted on cross section paper.

Oscillograms from tests under several conditions of load are shown in Fig. 5.

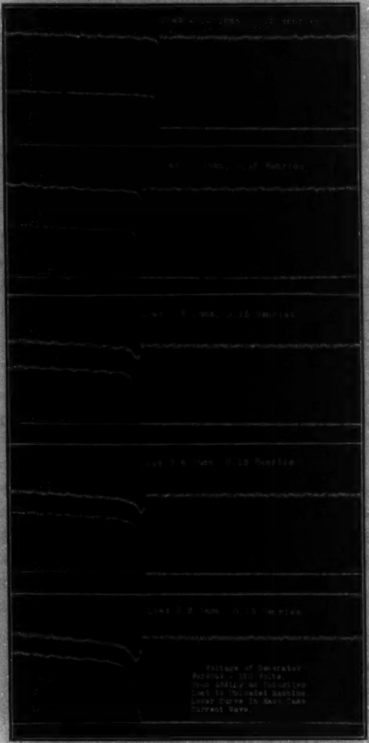

Fig. 5

Case Three

A non-inductive load added to a machine carrying a non-inductive load.

In this case there is an instantaneous drop of voltage to a value that is equal to the combined resistance of the circuits, times the current furnished by the machine at the instant of closing the switch,—that is

$$e_{t=0} = \frac{I_1 (r_1 + r_2)}{r_1 r_2} \tag{16}$$

where

r_1 = steady load.
r_2 = added load.

$$I_1 = \frac{E}{r_1}$$

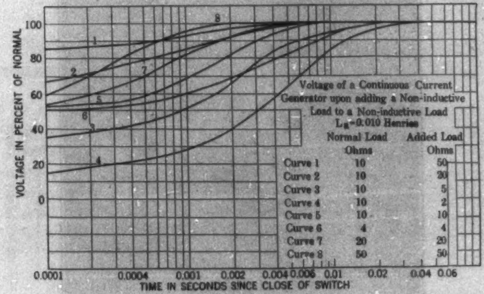

FIG. 6

The current builds up along an exponential law.

$$i = I_1 + I_2 (1 - \epsilon^{-\alpha_3 t}) \tag{17}$$

where

$$I_2 = \frac{E}{r_2}$$

$$\alpha_3 = \frac{(r_1 + r_2)}{r_1 r_2 L_a}$$

and the voltage is

$$e = iR \tag{18}$$

where

$$R = \frac{r_1 + r_2}{r_1 r_2}$$

$$e = I_1 R + R I_2 (1 - \epsilon^{-\alpha_3 t}) \tag{19}$$

Fig. 6 is a plot of calculated values to logarithmic abscissas for several values of steady load and added load.

Fig. 7 gives the same data plotted to rectilinear coordinates.

Oscillograms from tests under several conditions of load are shown in Fig. 8.

Oscillograms for similar values of load added to a generator under no load (case one) are shown for comparison.

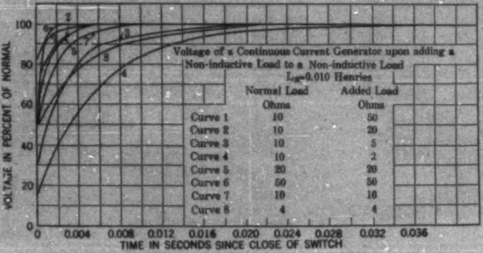

FIG. 7

CASE FOUR

The machine carrying a non-inductive load, and an inductive load added.

In this case there is no instantaneous drop of voltage but the voltage falls on an exponential curve, and rises on a similar curve of smaller exponent value. The voltage across the two branches is alike at every instant and is

$$e = i_1 r_1 = i_2 r_2 + \frac{d i_2 L_2}{dt} \tag{20}$$

where i_1 is the instantaneous value of current in branch 1, i_2 the instantaneous value of current in branch 2. L_2 = the inductance of branch 2.

$$i_1 = I_1 + K\epsilon^{-\alpha_4 t} - K\epsilon^{-\alpha_5 t} \tag{21}$$

where

$$K = (r_2 - \alpha_4 L_2)B$$

where

$$B = \frac{\alpha_5 I_2 r_2 + E}{r_1 L_2(\alpha_5 - \alpha_4)}$$

where

$$\alpha_4 = \frac{S - q_1}{2 L_2 L_a}$$

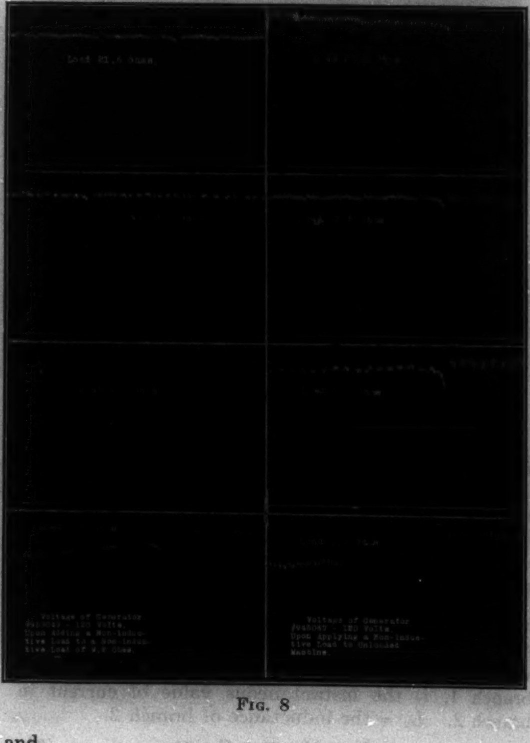

FIG. 8

and

$$\alpha_5 = \frac{S + q}{2 L_a L_2}$$

where

$$q_1 = \sqrt{S - 4\, r_1 r_2 L_a L_2}$$

where

$$S = L_a (r_1 + r_2) + L_2 r_1$$

At $t = o$ (**21**) of course reduces to

$$i_{1_{t=o}} = I_1 \text{ and the voltage becomes} \qquad (22)$$

$$e_{t=o} = I_1 r_1 = E \qquad (23)$$

Fig. 9 is a plot of calculated values for several conditions of steady and added load.

Oscillograms from tests under several conditions of load are shown in Fig. 10 in comparison with similar values of non-inductive load added to a generator carrying non-inductive load. (Case three).

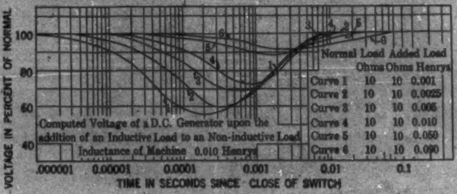

FIG. 9

CASE FIVE

The machine carrying an inductive load, and a non-inductive load added.

In this case there is an instantaneous drop to zero and a building up from there on an exponential curve.

$$e = i_1 r_1 + L_1 \frac{di_1}{dt} = i_2 r_2 \qquad (24)$$

where

$$i_2 = I_2 + K_2\, \epsilon^{-\alpha_6 t} + K_3 \epsilon^{-\alpha_7 t} \qquad (25)$$

where

$$\alpha_6 = \frac{S_2 - q_2}{2\, L_a L_1}$$

$$\alpha_7 = \frac{S_2 + q^2}{2\,L_a\,L_1}$$

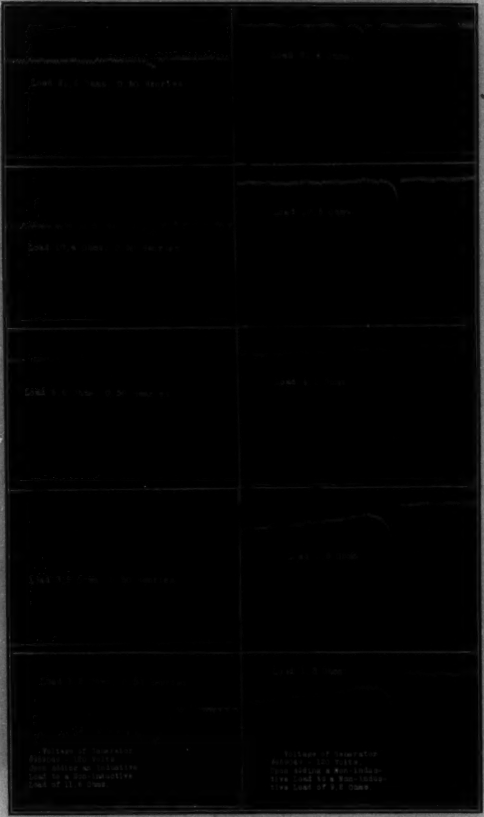

FIG. 10

$$S_2 = L_a\,(r_1 + r_2) + L_1\,r_2$$

and

$$q_2 = \sqrt{S^2 - 4\, r_1\, r_2\, L_a\, L_1}$$

$$K_2 = B_2\,(r_1 - \alpha_6\, L_1)$$

$$K_3 = (I_2 + K_2)$$

where

$$B_2 = \frac{I_2}{L_1\,(\alpha_6 - \alpha_7)}$$

Substituting (**25**) in (**24**) we get

$$e = r_2\,(I_2 + K_2\,\epsilon^{-\alpha_6 t} + K_3\,\epsilon^{-\alpha_7 t}) \tag{26}$$

which on equating $t = o$ becomes

$$e_{t=o} = 0 \tag{27}$$

since in (**26**) both K_2 and K_3 are negative numbers and $-K_2 - K_3 = I_2$.

Fig. 10-A is a calculated plot of several values of load added to a 10-ohm load of variable inductance.

Oscillograms from tests under several conditions of load are shown in Fig. 11 in comparison with similar values of non-inductive load. (Case three.)

CASE SIX

An inductive load added to an inductive load.

There is in this case an instantaneous drop in voltage, but never to zero. The instantaneous voltage is approximately equal to the voltage of the machine, times the ratio of the added inductance to the sum of the added load and machine inductances. The voltage builds up from this value along an exponential curve of four terms. The current builds up from zero according to an exponential curve of two terms.

$$e_1 = e_2 = i_1\,r_1 + L_1\,\frac{di_1}{dt} = i_2\,r_2 + L_2\,\frac{di_2}{dt} \tag{28}$$

$$i_1 = I_1 + K_4\,\epsilon^{\alpha_8 t} - K_4\,\epsilon^{-\alpha_9 t} \tag{29}$$

where

$$K_4 = B_3\,(r_2 - \alpha_9\,L_2)$$

$$B_3 = \frac{-\,I_2\,(r_2 - \alpha_8\,L_2)}{(r_1\,L_2 - r_2\,L_1)\,(\alpha_9 - \alpha_8)}$$

$$\alpha_8 = \frac{S_3 - q_3}{2\,L_2}$$

$$q_3 = \sqrt{S_3^2 - 4 L_2 r_1 r_2}$$

$$S_3 = L_a (r_1 + r_2) + L_1 r_2 + L_2 r_1$$

$$L_z = L_1 L_2 + L_a L_1 + L_a L_2$$

Differentiating (**29**) we get

$$\frac{di_1}{dt} = - \alpha_8 K_4 \epsilon^{-\alpha_8 t} + \alpha_9 K_4 \epsilon^{-\alpha_9 t} \qquad \textbf{(30)}$$

Substituting (**29**) and (**30**) in (**28**) reduces it to

$$e_1 = r_1 \{ I_1 - K_4 [\epsilon^{-\alpha_8 t} - \epsilon^{-\alpha_9 t}] \} + L_1 K_4 (\alpha_8 \epsilon^{-\alpha_8 t} - \alpha_9 \epsilon^{-\alpha_9 t}) \qquad \textbf{(31)}$$

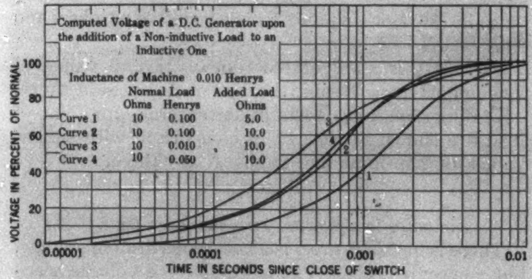

Fig. 10-A

This at $t = 0$ becomes

$$e_{1\,t=0} = r_1 I_1 + L_1 K_4 (\alpha_8 - \alpha_9) \qquad \textbf{(32)}$$

$$= E + L_1 K_4 (\alpha_8 - \alpha_9) \qquad \textbf{(33)}$$

The second term of this expression is always a negative quantity since α_9 is always greater than α_8.

Fig. 12 is a plot of calculated values for several conditions of steady and added load.

Oscillograms from tests under several conditions of load are shown in Fig. 13 in comparison with similar values of inductive load added to a generator carrying non-inductive load. (Case four).

LOAD DECREMENTS

A load removed from the circuit produces effects opposite to those when load is added, namely, a voltage rise instead of dip.

It is well-nigh impossible to express mathematically such cases or analyze them experimentally, due to our

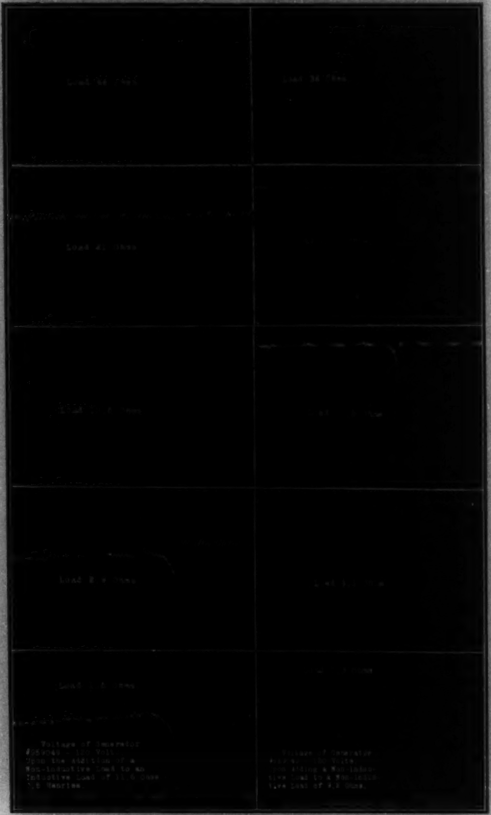

FIG. 11

inability to assign proper value to the increasing resistance of the arc that forms when the load is removed.

CONCLUSION

The foregoing equations can of course be at best but first approximations especially at values well along the recovery curve. No allowance has been attempted for the variation of permeability with flux density, or for the individual characteristics and reactions of machines themselves. However, the equations will be found valuable and reliable for the computation of voltages near the start of the transient, which should make them of value in the determinations of those conditions that would cause flicker of lamps upon the loading of a generator or circuit.

The principles and equations already laid down lead to some rather unusual conclusions regarding the means of improving the instantaneous regulation of circuits where flickering of lamps is objectionable. The voltage changes that have been discussed are inherent in such circuits, and in general, in large measure are independent of the cross section of feeder or service lines. No matter how large the cross section there will be a voltage dip that is inherent in the circuit, and if the inductance of the series circuit is large enough the application of a load will cause flicker. Being dependent upon the inductance of the circuit for their duration, attempts at their elimination by the compounding of

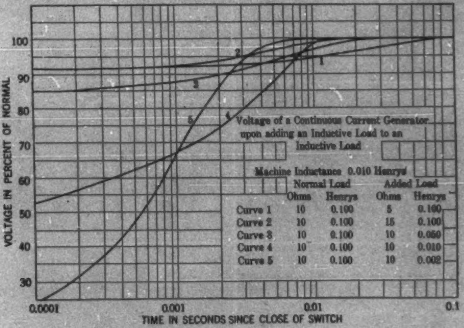

FIG. 12

machines are obviously steps in the wrong direction. Better instantaneous regulation will be obtained from

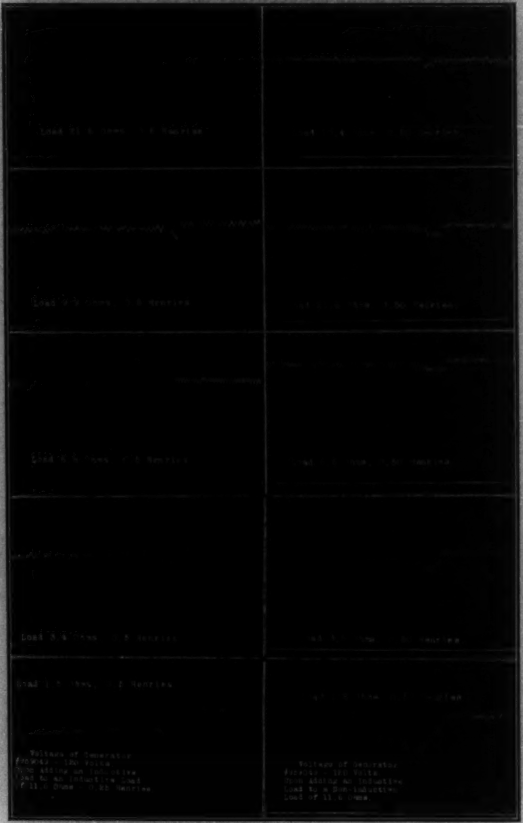

FIG. 13

shunt machines, than from compounded ones due to the lesser inductance of the former. Not only does compounding lead to a greater duration of transient conditions but the extra windings have no effect on the generated voltage of the machine until the current has risen well along the recovery curve.

The disturbances of large loads applied to lines can be mitigated by the insertion of suitable values of inductance coils of low resistance in series with the applied loads. In general the larger the value of the inductance applied the smaller will be the effect of a given load on the rest of the line. The fundamental principles involved in this application were discussed under the second case of load increments. It was there shown that the greater the ratio of the applied inductance to the machine inductance, the less will be the instantaneous drop. The other cases, that of inductance added to loads already on the lines, follow this same principle though the analytical treatment of them is not so simple.

DISCUSSION ON "INHERENT REGULATION OF D-C. CIRCUITS" (ELLIS AND ST. CLAIR), NEW YORK, N. Y., FEBRUARY 19, 1920.

V. Karapetoff: I should like to suggest to the authors of the paper that they mark their specific cases more distinctly. For example, when an inductance is added to the resistance, it is understood from the text that the inductance is put in series with the resistance. In another case they add a resistance and an inductance in parallel, and in one of the cases the machine is supposed to be carrying an inductance load. I do not know how a direct-current machine can carry a purely inductive load; perhaps it may, but certainly the paper needs a more explicit marking of the cases.

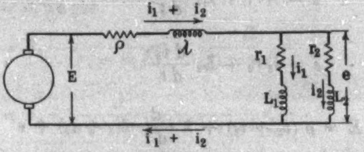

FIG. 1

Some years ago I read a paper on the concentric method in education in which I advocated going from the simple to the more complicated and from the concrete to the abstract. Now, this paper is written according to the concentric method, beginning with the simplest case and going to complicated cases. It may therefore seem inconsistent on my part to criticise a paper for being so written. In a professional paper I would rather see a more general case presented, first—I mean a case that admits of mathematical treatment without undue complication, and from which other cases may be derived as specific examples. Otherwise we can go on multiplying these cases, and yet not get a bird's eye view of the whole situation and of the possibilities of the method itself. I should like to show that all these cases and many others are comprised in one, which permits of a very simple integration.

Fig. 1. shows a d-c. generator loaded on two branch loads in parallel. Each load consists of a resistance r and of some inductance L in series with it. The resistance ρ and the inductance λ of the armature and of the leads to the load are concentrated in the line, thus giving an ideal armature with a constant induced e. m. f., E. The change in the armature reaction, due to the sudden change in the load, is disregarded as it is in the original paper.

At the instant $t = o$ the original values of the three resistances and of the three inductances are *suddenly* changed, and the values shown in the figure are the *new* values. The values of inductance previous to $t = o$ are of no interest since the currents then are steady; instead of giving the values of the resistances previous to the change, we shall assume that the steady currents up to the time $t = o$ are known.

After the instant $t = o$ the transient currents in the circuit are expressed by the simultaneous differential equations:

$$e = i_1 r_1 + L_1 \frac{d\,i_1}{d\,t} \tag{1}$$

$$e = i_2 r_2 + L_2 \frac{d\,i_2}{d\,t} \tag{2}$$

$$E = \rho\,(i_1 + i_2) + \lambda \frac{d}{d\,t}(i_1 + i_2) + e \tag{3}$$

where e is the transient load voltage. The result of the elimination of any two out of the three unknown functions, *viz.*, i_1, i_2, and e, leads to a linear differential equation of the second order. Hence, the solution is of the well-known form

$$i_1 = I_1 - \frac{\alpha}{r_1 - m\,L_1}\,\epsilon^{-mt} - \frac{\beta}{r_1 - n\,L_1}\,\epsilon^{-nt} \tag{4}$$

$$i_2 = I_2 - \frac{\alpha}{r_2 - m\,L_2}\,\epsilon^{-mt} - \frac{\beta}{r_2 - n\,L_2}\,\epsilon^{-nt} \tag{5}$$

In these equations I_1 and I_2 are the *final* values of the currents when $t = \infty$, and α and β are the constants of integration to be determined from the initial currents at $t = o$. The decrements (or the increments) m and n, according to which the transient currents vary from their initial to the final values, are functions of the six given constants of the circuit.

Substituting the values of i_1 and i_2 from (4) and (5) into (1), (2), and (3), and equating to zero the coefficients of ϵ^{-mt} and ϵ^{-nt}, since the results must be identically true at any instant of time after $t = o$, we get

$$\frac{1}{r_1 - m\,L_1} + \frac{1}{r_2 - m\,L_2} + \frac{1}{\rho - m\,\lambda} = 0 \tag{6}$$

and a similar expression for n. Since the expressions for m and n are identical, and since equation (6) is a quadratic with respect to m, we conclude that m and n are the two roots of equation (6) which thus can be readily computed. Both roots of equation (6) are always real.

Let the steady currents in the two load branches, at the time preceding the sudden change, be ρ_1 and ρ_2 respectively. Because an electric current possesses electromagnetic inertia and cannot be changed suddenly the values of the transient currents at the instant immediately following $t = 0$ are also equal to ρ_1 and ρ_2. Substituting these values into eqs. (4) and (5) when $t = 0$, we get

$$\rho_1 = I_1 - \frac{\alpha}{r_1 - m\,L_1} - \frac{\beta}{r_1 - n\,L_1} \tag{7}$$

$$\rho_2 = I_2 - \frac{\alpha}{r_2 - m\,L_2} - \frac{\beta}{r_2 - n\,L_2} \tag{8}$$

From these expressions the values of α and β can be easily computed.

Thus, the whole problem is reduced to the following four simple steps:

(1) Compute the initial steady currents ρ_1 and ρ_2 by Kirchoff's laws;

(2) Compute the final currents I_1 and I_2 at $t = \infty$ by Kirchoff's laws;

(3) Solve the quadratic equation (6) for m, and take the two roots of it as the values of m and n.

(4) Solve the simultaneous linear equations (7) and (8) for α and β.

The transient currents are then given by the equations (4) and (5) and the variable voltage across the load is

$$e = E_0 - \alpha\,\epsilon^{-mt} - \beta\,\epsilon^{-nt} \tag{9}$$

where

$$E_0 = I_1\,r_1 = I_2\,r_2 \tag{10}$$

is the final load voltage at $t = \infty$.

H. R. Summerhayes: The work of Messrs. Ellis and St. Clair has a very practical application in that it refers to the phenomenon observed when elevator loads are thrown on to d-c. generators, and the flicker in lights, of course, is observed when the generators are relatively of small size. There is a dip in the voltage, when the elevator load is thrown on, observable even when d-c. service is obtained from the largest central stations, with capacity of a million amperes, more or less, and that is due to the self-inductance of the whole circuit, which is relatively very low, so that the dip has very little effect on the lamps. But one reason that an elevator load is so particularly objectionable in small plants, as far as lighting is concerned, is that you not only add the motor load, that is, the inductive load, but in many instances, to secure a better control, a non-inductive resistance is thrown in shunt across the circuit at the same time as the elevator motor in series with its starting resistance, and that may account for the rather large dips that are observed.

The result of many investigations has been summarized in what you might call a rule of thumb formula that with a d-c. generator of any size, you can throw on about one-sixth load without objectionable flicker being caused on the light supplied from that generator. That is a rough rule, of course, but it appears to hold for many sizes and types of generators, but if you try to throw on more than one-sixth load, the flicker in the lights is objectionable, and has to be met by such expedients as inducing inductance in the motor circuit or by other methods.

L. W. Thompson: A second method to overcome the dip that occurs in d-c. generators is to inductively couple the power and light circuits, through a one to one ratio transformer, so that at each sudden application of power load, a compensating pulse of energy is added to the lighting circuit at the instant the drop occurs.

Good results have been obtained from this method and it is readily applicable, for it is only necessary to add the equivalent of a 15 kw., 60-cycle lighting transformer for every 75-kw. generator capacity.

B. W. St. Clair: The general equations for transients are, of course, well known. To reduce them to fit specific cases requires more time than many engineers can afford and also demands mathematical transformations that are difficult for many practical engineers to follow, though they are simple enough to mathematically inclined engineers and physicists. The simpler cases require very simple equations, quite different in appearance from the general equations. It is much easier to substitute numerical values into these simplified equations for these cases than it is to substitute in the general equation. It is for this reason that the paper is written as it is. The paper does not attempt to cover all possible cases, but it does cover some of the more usual ones.

Most of the oscillograms reproduced in the paper do not show the voltage drop completely to zero in those cases where the theory carries it to zero. This is not due to a breakdown of the theory, but to limitations of the oscillograph. We made numerous tests of the effect of the damping of the oscillograph vibrator upon the faithfulness of the records of these extremely rapid transients. The value of damping used for these records was one that gave quite accurate results in most cases after the return from zero voltage was 25 per cent completed. A few simple computations are sufficient to show that it is impossible in most cases for the vibrator to reach zero where the change from full voltage to zero was instantaneous before the voltage recovery was well along on its cycle.

I am afraid that Dr. Karapetoff has misread those portions of the paper where the six specific cases are described. There is no case where the machine is supposed to be carrying a pure inductance. What is meant by an "inductive load" is, of course, one that has finite resistance. This is the usual meaning in engineering literature for the term. The curves and oscillograms for these inductive cases give the resistance and the inductance of the oncoming load.

Our case 1—the most simple one possible corresponds to a load of lamps added to a machine otherwise unloaded. Case 2—to a motor load in place of a lamp load added to an unloaded machine. Case 3—is similar to a lamp load added to a lamp load and this is one that may cause much trouble from flicker in office buildings where isolated plants are used. Case 4 corresponds to a motor load added to an existing lamp load. This is a case that has been the cause of numerous complaints and investigations in this city where elevators and lights are operated from the same generator. The fifth case is that of lamps added to a motor load and the last that of a motor load added to a motor load. This is Dr. Karapetoff's general case. A comparison of its equations and those of some of the earlier cases will show the desirability of using the specific rather than the general equations for numerical computation.

To use the general equation requires first of all the solution of at least two and generally more simultaneous equations before the main equations can be attacked.

The impulse transformer described by Mr. Thompson is a very ingenious device. It is an attempt to add to the lamp circuit another transient that will be of opposite polarity to the first so that if exactly the counterpart of the first it will neutralize the first one. It is, however, impossible to make this correction exact. The transients change with the lamp load, the starting currents required by the elevator, etc. It can be looked upon as a special case 6. There is always a drop of voltage in the lamp circuit. There is also added to that circuit a sudden rise of voltage. However, as the two transients generally have very different duration times, the correction is never complete. The voltage will change. Whether this change will cause perceptible flicker depends upon the rate of change, the size of the lamps, etc. The resulting flicker will be lessened in general if in addition to the transformer a large inductance of low resistance is added to the elevator circuit.

Philip Torchio: How would this be complicated if you took into account the temperature coefficient of lamp filaments?

B. W. St. Clair: This is something we tried to work out but found it a difficult thing to do. Dr. Langmuir has worked out the change in temperature and candle power as functions of filament sizes for sinusoidal voltages but not for impulse voltages. It is difficult to get an expression for the rate of cooling of the filament during these voltage shifts.

Presented at the 8th Midwinter Convention of the
American Institute of Electrical Engineers,
New York, February 19, 1920.

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THE MEASUREMENT OF PROJECTILE VELOCITIES

BY PAUL E. KLOPSTEG
Physicist, Sales Department, Leeds and Northrup Co.
AND
ALFRED L. LOOMIS
Formerly Major, Ordnance Dept., U. S. A.

The paper discusses the requirements imposed by proving ground practise upon a chronograph which is intended for general ammunition testing. Instruments of the standard pre-war pattern were entirely inadequate in number for testing the immense quantities of ammunition contracted for by the Government during the war. The development work undertaken to cope with the whole problem of velocity testing was first directed towards devising methods for speeding up measurements with the existing instruments. Conditions made it necessary to lay aside this work and concentrate upon the development of a totally new device for obtaining the results both speedily and accurately. The instrument which was developed, and adopted as a standard ordnance chronograph, is designated "Aberdeen chronograph." An account of its development is given. It is an assembly of many standard parts, with a few necessary special ones. Rapid production of the instruments was thereby made possible. The Aberdeen chronograph, and the procedure in determining velocities by this means, are described in detail. For the sake of comparison, the Boulengé chronograph is briefly described. Comparative results as to speed and accuracy of measurement are given.

INTRODUCTORY

ONE of the very important measurements which must be made at an army proving ground is that of projectile velocity. Not only is it essential that such measurements be made, in connection with ammunition testing and development, but it is equally essential that the results be reliable. Data of the greatest accuracy are needed in certain problems of experimental development. Velocity measurements for the computation of range tables must be as free from error as possible. In making powder tests, seven "uniformity" rounds are fired; the measured velocities, for acceptance of the powder, must agree within narrow limits. Rejection of a lot usually means the rejection of thousands of dollars' worth of material.

The difficulty of making these speed tests is increased by the fact that the portion of the flight of a projectile with which we are directly concerned, is an extremely transient phenomenon. The interval during which the shell lends itself to the measurement is of the order of hundredths of seconds. This fact adds a third difficult requirement to the two above mentioned, namely, certainty of obtaining the record when the projectile passes. Failure to do so necessitates the firing of an extra round, sometimes at a cost of hundreds of dollars. During the emergency there were several additional requirements. The apparatus had to fulfill the demand of giving good results day after day, under all conditions of weather, and this in the hands of operators without technical training.

It was recognized in the early summer of 1917 that when deliveries of the immense quantities of ammunition contracted for by the government would commence, the matter of making the necessary velocity tests quickly and accurately would prove an embarrassing problem. The existing chronographs, of the Boulengé pattern, were limited in number, and manufacture of enough of these instruments in time to meet the situation was, because of their mechanical complexity, out of the question. In order to represent adequately what the problems were and how they were met, we shall first make brief mention of some of the characteristics of the Boulengé chronograph. A fuller description is contained in War Department Circular No. 1682, and in the 5th edition of Foster's Handbook, page 1128.

In all the various practical instruments devised for velocity determinations, the measurement is that of the time interval required by the projectile to traverse a measured section of its path. With the Boulengé chronograph, this section is included between two wire screens, the breaking of which by the projectile actuates the recording mechanism. The latter utilizes two electromagnets, each in series with one of the respective screens. These magnets, by virtue of the current through them so long as the screens are intact, support, respectively, a rod to receive the record, and a weight, to release a marking "knife" by falling upon a trigger. The rod and weight are intended to fall freely under the action of gravity. The origin of readings on the rod is the mark made by the knife when the rod is suspended. A simultaneous break of circuits of the two magnets, produced by a device called a disjunctor, produces a second mark, somewhat higher up, which represents the time required by the instrument to function. When the projectile is fired through the screens, it breaks, first, the circuit of the magnet supporting the rod, then that of the circuit supporting the weight, thus making a third mark, still higher up on the rod. Measurement of the distances from the origin of readings to the two other marks mentioned gives the data from which the time interval may be turned up in tables. From the interval so found, the velocity is determined by the usual formula.

For the sake of a later comparison, statements should be made as to the accuracy, and as to the best conditions for precision of measurement with the Boulengé chronograph. Cranz in his "Lehrbuch der Ballistik"—which is probably the most authoritative work on ballistics yet produced—states that a given Boulengé chronograph is capable of repeating its reading of a certain time interval to 0.0001 second; but that, because of certain constant errors, which are very difficult to avoid or to determine with certainty, the absolute value of the time interval as measured by the instrument is, at best, reliable to 0.001 second. This statement is fully borne out by experience. The interval most favorable for precision of measurement being about 0.11 to 0.12 second, the distance between screens should be variable so that the distance best suited to a particular gun may be used. With the 155 mm. gun, for instance, this distance is 200 feet; on account of the blast, the first screen is placed about 100 feet from the muzzle. As a result, the second screen must be very high, even with moderate elevations of the gun. Thus, with an elevation of but thirty degrees, the second screen must be about 175 feet above the level of the gun. It is needless to point out, perhaps, that a movable screen support which can rigidly hold a screen at a height of 175 feet, has not yet been built. Even were this possible, the time consumed in hauling down a screen from a height of 175 feet, repairing or replacing it, and hauling it up again, would be so considerable as to render rapidity of testing quite impossible. The immediate remedy may be sought in a greatly shortened screen distance. In this case, of course, a more accurate recording instrument would be desirable.

In addition to the undesirable feature of requiring so much time for replacement, the wire screens have another serious drawback, which has repeatedly been mentioned in the writings of the Germans, the French and others upon the subject, and which thrusts itself upon the attention of anyone making a careful study of them. This is the very considerable uncertainty as to whether the recorded interval actually and accurately corresponds to the interval taken by the projectile to travel the measured distance between screens. The uncertainty is due in part to persistence of contact of the wires with the projectile, even after they are broken; in part also to the fact that the instant of break depends somewhat upon whether a wire is squarely hit by the tip of the projectile, or whether the tip strikes through the screen between neighboring wires. It happens occasionally—in spite of the fact that the wires are ordinarily spaced about one-third of a caliber apart—that the projectile slips through the screen without breaking a wire.

With the idea of eliminating the wire screens as the signaling end of the Boulengé arrangement, one of the writers in the summer of 1917 presented to the Sandy Hook Proving Ground a plan for "wireless" screens which had occurred to him some months before, in connection with other work upon the measurement of short time intervals upon which he had been engaged. Essentially, a screen of the type proposed was to be a mutual inductance coil of sufficiently large diameter to permit firing a projectile through it without great risk of injuring the windings. If a steady magnetic field is set up within such a coil by a current through one of the windings, it follows that any sudden disturbance of the field—by the passage of a projectile of magnetic material, for example—will produce a corresponding momentary electromotive force in the other winding of the coil. Preliminary tests showed the supposition to be correct.

For the purpose of developing this idea into a routine method, and to work upon other related problems, one of the present writers entered upon work at the Sandy Hook Proving Ground as Electrical Engineer in the Ordnance Department. The preliminary work was carried on at the Bureau of Standards, where facilities were kindly offered, and where much splendid cooperation was given, particularly by Dr. P. G. Agnew and Mr. W. H. Stannard. At the same time other possible solutions of the problem of speeding up the tests were being considered, and apparatus was being built. Frequent trips were made to Sandy Hook for field tests, which had to be made under very adverse conditions on Saturday afternoons and Sundays, so as not to interfere with routine firing.

The inductor scheme as partially developed was put to actual test, with very promising results; the Boulengé chronograph used with two such coils to all appearances functioned in the same manner as though the projectile had broken the usual type of wire screens. Unfortunately the work had to be discontinued before the development could be considered through the experimental stage, because of the removal of the proving ground from Sandy Hook to Aberdeen, Md. Here experiments with the coils became prospectively impossible for some months, and the work was concentrated upon another device which seemed to hold considerable promise, and with which tests could more readily be made. This device had been constructed at the Bureau of Standards for the purpose of recording signals produced by the "bow wave" of the projectile. The instrument which was finally evolved was called the Aberdeen chronograph. Work upon the inductor screens was not resumed because the newly developed instruments met all the needs which necessitated the development work, and their rapid production, as well as other problems occupied attention until the close of the war. It has been reported recently, however, that the inductor method is being applied with success in England.

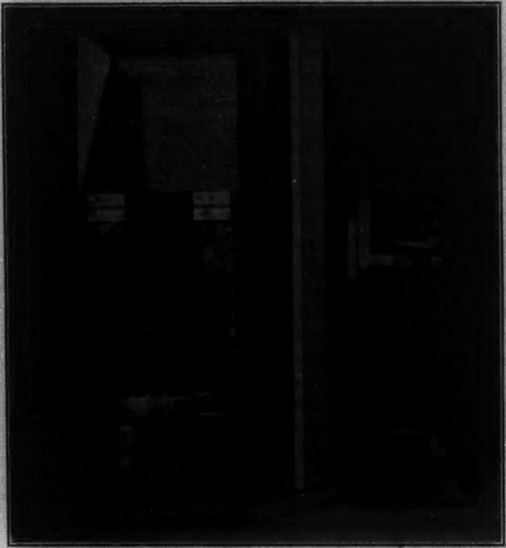

Fig. 1—Aberdeen Chronograph, Fall Apparatus and Direct-reading Velocity Scale. Instruments in Operating Position.

Development of the Aberdeen Chronograph

The device mentioned in the preceding paragraph consisted of a Leeds & Northrup governed motor, on the vertical shaft of which was mounted a flywheel with a smooth rim. A strip of paper could be fastened upon the rim to receive the records produced by the projectile when it actuated the signaling device. Up to this point the instrument differed from previously described ballistic chronographs only in that a constant speed motor was used. All such chronographs, however, utilized the breaking of wires for the signals, the interruption of the current producing a spark from a point in close proximity to the rapidly revolving drum. In this case, the recording system was designed for open-circuit operation, *i.e.*, the spark was to occur when a circuit was closed by the projectile. In connection with such circuits, it had been planned to use automatic contacts which should close when the bow wave of the projectile impinged upon them. The task of constructing such switches so that they would operate unfailingly and under all conditions proved formidable. The greatest possible speed in getting something usable remained imperative, and the urgency became greater as time passed.

It then occurred to one of us to try a new form of screen, or better, a form of contact target, in which contact should be established mechanically by the tip of the projectile. This target in its simplest form consists of a pair of metal sheets, of suitable thickness, separated by an insulating layer. Experiments immediately demonstrated the success of such an arrangement. Thus, the recording mechanism arranged for "make" circuit signals, together with the contact targets, became a most promising basis for further work of development.

On March 2, 1918, a modified design of the Mark I instrument* was taken to the Leeds & Northrup Company, and on March 15th three of the new Mark II instruments had been completed and were ready for operation at Aberdeen. These three instruments were used daily in routine measurements, in a shanty of rough boards on the gun platform, and so successful were they that it was decided to proceed immediately with the construction of the instruments in quantity. A new design, Mark III, including valuable suggestions from the manufacturers, was worked out which, with the exception of minor details, became standard. In these, of course, the faults revealed by experience with the Mark II instruments were eliminated. Much experimenting in the laboratory and under service conditions was carried on in the meantime on both the electrical and mechanical components of the apparatus. Several further improvements were made, and from this point the production of Mark IV Aberdeen chronographs in lots of 25 proceeded without delay.

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*Mark I, Mark II, etc., is the War Department designation of successively developed types of ordinance material.

FIG. 2—ABERDEEN CHRONOGRAPH, SIDE VIEW, WITH CASE OPENED FOR INSPECTION OF PARTS

An important factor in the success of the Aberdeen chronograph in the emergency, aside from questions of its excellence as an instrument, was the fact that from the time production began until the last instrument needed had been delivered, the average rate of production per working day was one complete instrument. This became possible, even with manufacturing facilities already overtaxed, because the design included many standard parts.

DESCRIPTION OF MARK IV ABERDEEN CHRONOGRAPH

Fig. 1 shows the chronograph ready for operating, and Fig. 2 is a side elevation, with the case open for inspection of the circuits. The size of the apparatus may be judged by comparison with the 12 inch scale, shown in the latter view. In Fig. 3 the case of the chronograph is shown closed, with the carrying handle attached.

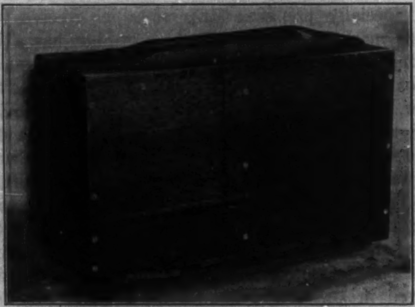

FIG. 3—CHRONOGRAPH IN CLOSED CASE WITH CARRYING HANDLE

The mechanical part of the device is seen by reference to the photographs to consist of a shallow cylindrical drum of aluminum, mounted on the vertical shaft of a small (110-volt direct-current) motor. On the lower end of the same shaft is carried the constant speed governor, upon which depends, in large measure, the accuracy of the instrument. It is a development of the governor used in the Leeds & Northrup temperature recorder. The function of the governor is to control the power input into the motor. Its *modus operandi* may be understood by reference to Fig. 6. A thorough analysis of the device resulted in the design illustrated, which is capable of holding the speed so constant that the mean deviation from average speed, as determined in repeated tests over a fixed 0.2 second interval, was less than 1/10 per cent. Another way of representing this fact is in showing the results of a series of observations. Taking 100 measurements of a 0.2 second interval, 62 were within 0.04 per cent of average speed; 70 within 0.05 per cent; 77 within 0.06 per cent; 95 within 0.07 per cent; 97 within 0.08 per cent; and 100 within 0.09 per cent. In these measurements, therefore, it is safe to assume a maximum probable error of ±0.1 per cent. A speed of 25 rev. per sec. is thus maintained; the method for checking this value will be described later.

A specially prepared strip of paper receives the record marks. The paper, which is blue, has a thin coating of white paraffin on one side. When a spark passes through the paper, it not only perforates the latter, but also melts a tiny bit of the paraffin, leaving a bright blue, easily distinguishable spot. The width of the strip is equal to the depth of the cylindrical drum, and its length to the inner circumference of the drum. It is held in position very firmly against the smooth inner surface of the drum, on account of the rapid rotation of the latter. By this arrangement, much saving of time is effected, since the record strip may be easily inserted in the drum, and, with a little practise, removed while the motor is running at full speed. With the speed mentioned, and an inner circumference of the drum of 500 mm., the paper strip is given a linear speed of 12,500 mm. per second. Thus a very open and uniform time scale is secured. Three spark points (Fig. 6), in the same vertical line, are held within 1 mm. of the record strip by an insulating block. A spring latch holds the points either in the recording position or at some distance from the paper.

Reference to Fig. 6 shows that one of the electrical spark-recording units consists essentially of a condenser, and the primary of a transformer, both in a circuit with the contact target. Up to the instant of firing, the condenser is kept charged through high resistances, from the supply line which furnishes the power for the motor. The supply line of 110- or 220-volts direct current, is connected at *AB*. When the tip of the projectile strikes the target, contact is established between the plates, and a discharge passes through the transformer resulting in a spark from the corresponding recording point. The main condenser has a capacity of 10 micro-farads. Later work showed that an auxiliary condenser of 6 microfarads, connected in parallel with the plates of the contact target, greatly intensified the spark. The complete circuits are shown in the diagram. Three recording units were provided, as shown. Normally two only were used, the third being for emergency, or for use in cases where average velocity values over two adjacent sec-

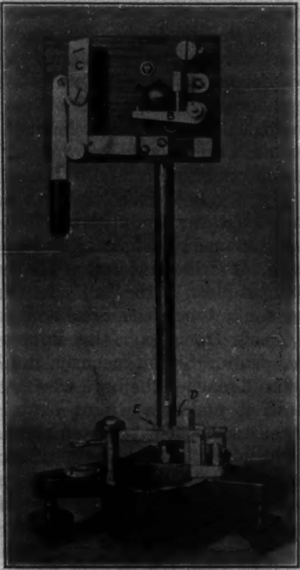

FIG. 4—FALL APPARATUS FOR VERIFYING ACCURACY AND CONSTANCY OF MOTOR SPEED

tions of the trajectory were to be determined on the same instrument.

A ready means for verifying the speed of the rotating drum is essential, to give the operator confidence in the performance of his instrument if for no other purpose. If we could, in some manner, produce two sparks, separated by an accurately-known short interval of time, we should have a ready means for checking the accuracy and constancy of the motor speed. A device which could be utilized for this purpose had previously been worked out in connection with psychological time measurements.* This is a "fall apparatus," the modified form of which for use with this chronograph is seen in Fig. 1, and shown in detail in Fig. 4. A ¾-inch steel ball *A* rests upon a spring-supported horizontal lever *B*. When the counterpoised hammer *C* which is under spring tension is released, it strikes the end of the lever a smart blow, knocking it clear of the ball. At the instant of striking, the contact between the hammer and lever produces a spark from, say, the uppermost spark point in the drum. Simultaneously the ball begins to fall. When it reaches the bare wire *D*, held on an insulated post, it makes a wiping contact with the latter, and then touches the tungsten plate *E*, between which and the wire it closes the second circuit. At the instant contact is made at *E*, a second spark jumps, say from the next lower spark point. Obviously we may adjust with great exactness the distance between the lowermost point on the ball, when it is in position on the upper lever, and the plate *E*, so that the time of fall between these two points is 0.2 second. This corresponds to five revolutions of the drum. If the speed of the latter is correct, two marks will be found upon the strip, one directly above the other. With slightly higher or lower speed than the correct value, the two marks will be displaced with reference to each other. From the amount of this displacement the constant error in speed becomes easily known. Thus, when the fall apparatus is kept permanently connected as shown in Fig. 1, it is a simple matter at any time dur-

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*Klopsteg, *Journal of Experimental Psychology*, II, 253, 1917.

ing the course of the day to calibrate the chronograph. In practise, the speed remained so constant in a day's run that calibrations were made only two or three times during the day.

Fifty feet was chosen as the standard distance between targets for all velocities from 500 to 2500 feet per second. With a fixed distance between the contact targets, and fixed speed and dimensions of the drum, there is a one-to-one correspondence between distance between record marks and velocity. Thus it becomes possible to construct a direct-reading scale for velocities which is suitable for all the values between the limits mentioned. The scale was so designed that, when the shot was fired, regardless of the relative positions between the spark points and the abutting ends of the record strip, the velocity readings could be made without possibility of mistake. The scale is shown in Fig. 1, and again in the photograph of the accessories case (Fig. 5). This picture also shows the fall apparatus taken apart and in the case.

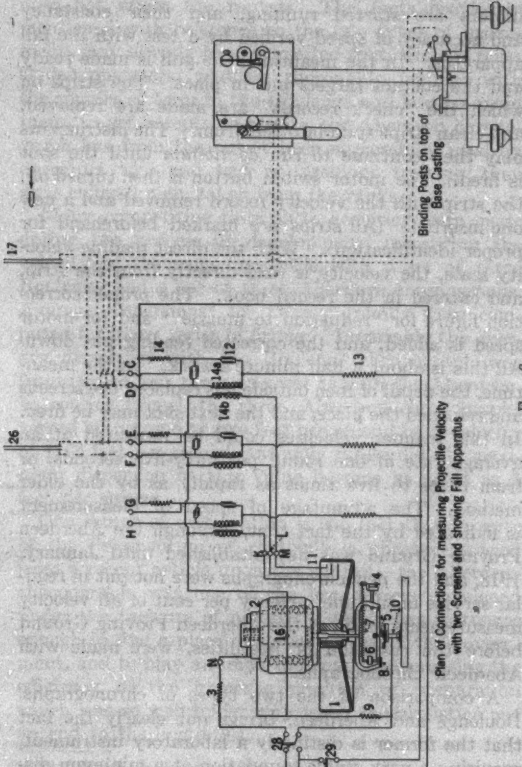
In its contact target arrangement, already described, the Aberdeen chronograph is unique among the ballistic chronographs. The fact that the recording spark occurs when the tip of the projectile establishes contact between the metal plates has the consequence that the otherwise great uncertainty in the distance between the points at which the signals occurred is practically eliminated. It may here be repeated that such uncertainty is probably the chief point of weakness, so far as accuracy is concerned, in the ordinary wire screens used with the Boulengé chronograph. That such errors are truly minimized with the contact targets is shown by the fact that the distance between targets could be reduced to 25 or 30 feet without seriously affecting the agreement among several instruments measuring the same time interval.

PROCEDURE IN VELOCITY MEASUREMENTS

The application of the Mark IV chronographs to velocity measurements in such a way as to secure the data with minimum loss of time was in itself a development problem. Three instruments were used with the same pair of contact targets, and the observations were independently made and entered in the data book by three respective observers. The averages were then entered in a fourth book, together with the necessary accompanying data as to the gun, kind of projectile, etc.

The contact targets were made up beforehand in quantity. For the three-inch guns they consisted of lead foil strips about 3-in. by 12-in. which were attached with hot paraffin to the opposite sides of a sheet of paraffined building paper 5-in. by 17-in. Much of the routine firing was done with constant elevation of the guns, so that permanent supports for the screens could be used. These supports consisted of uprights, each of which had two cross-arms at the proper height, extending out on one side. The lower cross-arm was provided with a spring clamp, on the insulated inner surfaces of the jaws of which there were contact buttons which made connections with the lead foil sheets. The upper arm also carried a clamp which supported the contact target in a vertical position, and between the two clamps the target was held with sufficient firm-

FIG. 5—ACCESSORIES CASE WITH NECESSARY AUXILIARY APPARATUS


17
26
C
D
E
F
G
H
14
12
14a
14b
13
K
L
M
11
2
16
N
3
4
5
6
7
8
9
10
1
28
29
A
B
Binding Posts on top of Base Casting

Plan of Connections for measuring Projectile Velocity with two Screens and showing Fall Apparatus

FIG. 6

ness to prevent its being moved appreciably by the wind. From the contact buttons on the lower clamp, wires were run in conduit to the instrument building, just behind the gun platform.

In undertaking a series of measurements, the motors are started running, and their constancy and accuracy of speed verified by a test with the fall apparatus. In the meantime, the gun is made ready and the contact targets put in place. The strips on which the "check records" are made are removed, and clean strips spun into the drum. The instruments may then continue to run *ad libitum* until the shot is fired. The motor switch button is then turned off, the strip with the velocity record removed and a new one inserted. (All strips are marked beforehand for proper identification). With the direct reading velocity scale, the velocity is read directly from the strip, and entered in the record book. The proper correction figure for "reduction to muzzle"* and for motor speed is added, and the corrected reading put down. All this is about a half minute's work. In the meantime, the detail of men outside has replaced the screens and reloaded the piece, and the next shot may be fired. In this manner velocities could be measured at an average rate of one round per forty-five seconds, or from three to five times as rapidly as by the older method. The advantage of speed of measurement is indicated by the fact that, although the Aberdeen Proving Ground was not established until January, 1918, and the new chronographs were not put in regular service until June, seventy per cent of all velocity measurements made at the Aberdeen Proving Ground before the cessation of hostilities, were made with Aberdeen chronographs.

A comparison of the two types of chronographs, Boulengé and Aberdeen, brings out clearly the fact that the former is distinctly a laboratory instrument, requiring a very stable foundation at a minimum distance of several hundred yards from the gun platform, while he latter is a field instrument, which may be and is being used immediately behind the gun platform; or it may be set upon the ground as near the gun as may seem desirable. Proximity to the gun does not affect the records. The facts previously mentioned with reference to the two types of instruments may already have suggested that, while it takes skilled operators, who are thoroughly familiar with the sources of error in the Boulengé and how to avoid them, to get acceptable results, the Aberdeen chronograph has from the outset been successfully used by operators who had had no previous training; they were enlisted men, taken from the ranks.

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*By "reduction to muzzle" is meant the process of transforming the observed velocity to that which obtained at the muzzle, on the assumption that the projectile experienced uniform retardation from the instant it left the gun.

Many competitive tests as to accuracy were made during the process of development, measuring the velocity of the same projectile with a set of three Boulengé and a set of three Aberdeen chronographs. Average dispersions* for twenty rounds were computed for many series of firings. Invariably the average dispersion with the Boulengé chronograph was from two to four times as great as with the Aberdeen. In routine firings, the three Aberdeen chronographs agreed within one or two feet per second on a velocity of 1700 feet per second, and only rarely did the maximum dispersion at this value become as great as 5 feet per second.

One source of gratification is the fact that in spite of the "momentum of custom"—a phrase borrowed from a recent article upon the subject—the Aberdeen chronograph was one of the few devices developed under the pressure of necessity, which was fortunate enough to find a place in actual service of the government, and to play an important part in increasing the efficiency of the immense program of preparation which was so well under way when the armistice put an end to the hostilities.

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*By dispersion is meant the maximum difference between any two of the three instruments used. The average dispersion is the sum of the dispersions divided by the number of rounds.

DISCUSSION ON "THE MEASUREMENT OF PROJECTILE VELOCITIES" (KLOPSTEG AND LOOMIS), NEW YORK, N. Y., FEBRUARY 19, 1920.

E. E. F. Creighton: There are some questions that come up that possibly may not be quite in line with the paper, but perhaps Mr. Klopsteg would be willing to answer them. First, in regard to the velocity of the projectile inside the gun—are those instruments adapted to such measurements? Since in all photographs we have seen the projectiles are ahead of the gases, why is it necessary to move the screen out about 100 feet to get rid of the effect of the gases?

Another question is in regard to the bow wave—can the bow wave break the screen wires, and does the factor of wire drawing enter into the problem? If the projectile can pass between wires that are less than the caliber, that means the wires are slipped aside without breaking. Would it not be quite possible to have considerable wire drawing without breaking the wire at the time the projectile passed through.

Regarding the insulation between the two plates—to me it is an entirely new way of making contact. If a paper was placed between the two plates, how could the contact be made permanent? Would not the insulation remain between the two plates?

Regarding the absolute time—it must be evident to all of those who handle instruments that the relative time of making contact would be extremely accurate, but there enters into the time between the contact and the spark a certain time element in the inductance of the circuit.

Clayton H. Sharp: Once in the course of business a problem came to me to determine the relative velocities of flight of different kinds of golf balls. The question was the effect of the character of the rubber core in the golf ball. This introduced some of the problems of projectile flight. In the first place, of course, it was necessary to hit the balls a uniform blow. An attempt was made to do that by taking a d-c. motor and putting the equivalent of a golf club on the face of a pulley on the end of the motor. The club was arranged to slide longitudinally in guides and had two positions corresponding to slightly different circles of swing, and was pulled by a spring from the "in" position to the "out" position. In the "in" position the club cleared the golf ball which was set up on a tee. With the motor in operation a trigger could be tripped allowing the club to go to the "out" position and to strike the ball. The strength of the blow depended on the linear velocity of the head of the club which could be computed from the radius and the rev. per min. of the motor.

The ball passed through two wire screens quite close together. In order to measure the time we took a synchronous motor and put a wooden pulley on it, and put a circular chart on the end of the pulley. An electromagnet connected up with the screens and actuating a stylus on the chart left a record of the times of passage through the screens. It had to be done indoors, with a limited space, and did not succeed quite as well as we might have wished, but at least it was the application of the principle of constantly driven motor to the measurement of very short time intervals.

Roy Kegerreis: About the blast reaching the screen before the projectile; the problem of the Helmholtz sound resonator is somewhat the same. If one calculates anything about that it is necessary to calculate the magnitude and direction of the velocity of each particle of air at every point, both within and without the hollow spherical resonator. That problem has been solved and leads to a pretty complicated sort of mathematics. Somewhat the same thing happens in the case of the gun.

A rough concept of the action of the blast before the gun may be had by imagining a row of blocks set up rather near to each other. If the one at the end were pushed, the next would soon move, and the next, and so on until a large number of blocks would be pushed ahead, the number of moving blocks being ever on the increase.

The force which the gases from the gun exert may be compared to the force which pushes the first block and the moving blocks may be compared to the air particles before the gun. This shows why the actual front of the compression wave may be ahead of that shown on a photograph, since the compressed air before the products of combustion from the gun is so nearly transparent.

P. E. Klopsteg: We made no tests to determine projectile velocities inside the gun. The primary effort in this development work was to secure a device which would quickly and accurately measure the external velocity, and the work was carried on constantly with that in view. It was never considered feasible to adapt this device to measuring the extremely short intervals involved in the velocities inside the bore.

The Bureau of Standards, however, has been carrying on very extensive and elaborate investigations for the Navy Department, using an oscillograph driven at very high speed, something like ten meters a second, for making determinations of that kind.

The effect of the blast is merely this. Although in the photograph the projectile is always shown ahead of the blast, it has been shown very clearly that at the instant the projectile leaves the muzzle, some of the blast does get ahead of the projectile for a short distance, and that the projectile is then actually accelerated by the pressure of the blast behind it, for a distance of some few feet after it leaves the bore of the gun. However that may be, the blast from the large gun does continue along after the projectile has left, and if the first screen was not placed far enough away, the blast might do damage to the screen support rather than the wires.

As to the effect of the bow wave on the breaking of wires, I am unable to answer the question, and to the question as to whether there is wire-drawing, I am unable to give a definite answer.

The effect of the projectile going through the screen without breaking the wires was observed in work on the 37 mm. gun, which required a screen made of fine wires very close together. These wires were woven back and forth between two pieces of paper glued together, and it was just a curious happening that in one or two cases the projectile got through one of the two screens without breaking the wires.

The insulation between the two plates need not be very high. As I attempted to show in the diagram of connections, the condenser is permanently charged from the line, and since the plates are in parallel with the condenser, the plates are also permanently charged from the line, through a high resistance. The passage of the spark requires only a momentary, not a permanent contact. This makes possible the arrangement used in the fall apparatus, such that when the hammer comes around and hits the lever a blow there is only a momentary contact, causing the spark to jump. With the contact targets, in most cases it is found that there is no contact between the perforated plates after the projectile has passed.

As to relative and absolute time, the accuracy of the measurements for the Aberdeen chronograph depended on the accuracy of the fall device. If we want to measure to accuracies of a few hundredths of a per cent, rather than tenths of a per cent, then we must know accurately the acceleration due to gravity and make allowance for it. In the use of the fall apparatus, as nearly as we could determine with all the experimental facilities we had, the same interval, within 0.00005 second, was required for the ball to fall a given distance from the upper to the lower contact point, and operate the contacts.

As regards the time lag between the passage of the spark and the contact at the projectile, the accuracy of the arrangement is assured by the similarity of the two circuits. One may assume, I think, that if a lag takes place at the first screen, there will be an equal lag at the other. That was assumed all through the measurements, and the results indicated that the measurements were giving satisfactory values.

C. H. Sharp: Could not the drum speeds be accurately calibrated by making it in the form of siren, and blowing through it, or matching it up against a tuning fork?

P. E. Klopsteg: Yes, it could. Again, the reason for not going to a more complicated arrangement for this purpose was that it was necessary in many cases to use the apparatus in the field, and laboratory devices of that kind could not be handled by unskilled men. We had considered at one time driving the drum with a shunt-wound d-c. motor, so wound and supplied with a source of current such that it would run at approximately normal speed, and then to mount the rotor of a synchronous motor on the same shaft. The auxiliary synchronous motor could be driven by means of a tuning fork, thus regulating the speed very precisely. That was given up as a complicated and unnecessary labor. The device used was found in practice to give all the accuracy desired.

C. H. Sharp: Regarding the Boulengé chronograph, the thought strikes me, at least, of its being a cumbersome and awkward and indefensible device. Why is it that it has been so generally accepted in ballistics?

P. E. Klopsteg: That is difficult to answer. One reason is that all military men grew up with the Boulengé chronograph, and it is a case of "momentum of custom," as I have indicated in the paper, which keeps it going. There is presumably a psychological reason for it—a false assurance, perhaps, brought about by the knowledge that when the rods hang, the two circuits are intact, and when the rods fall, the circuits are broken. The fact is too often overlooked that the Boulengé chronograph, although it is an instrument which when properly installed will seldom fail to give a record of some kind, it is nevertheless a delicate measuring instrument, the successful operation of which requires not only great care but considerable manipulative skill and a knowledge of all the possible sources of error. Judging from publications prior to 1915, it appears that the German authorities have conflicting views on the reliability of the Boulengé chronograph.

Presented at the 8th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 20, 1920.

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A NEW FORM OF VIBRATION GALVANOMETER

BY P. G. AGNEW

Secretary, American Engineering Standards Committee

Vibration galvanometers are very useful in null measurements, but have not been much used in industrial laboratories on account of their being sensitive to external vibrations and requiring delicate adjustments. The present instrument, which has a sensitivity higher than other forms of the moving-iron type, but less than that of the most sensitive forms of the moving-coil type, has the advantages of sturdiness, quick responsiveness, and freedom from the effects of external vibration. It consists essentially of a fine steel wire, mounted on one pole of a permanent magnet, and so arranged that the free end of the wire may vibrate between the poles of an electromagnet through which the current to be detected passes.

THE vibration galvanometer is a very useful instrument in alternating-current measurements where null methods can be used as, for example, in an almost endless variety of bridge measurements, in various applications of the alternating-current potentiometer, and in testing instrument transformers. As in the case of direct-current galvanometers, there are two general types of vibration galvanometers, the moving-coil type and the moving-iron type. In each type the moving element is mechanically tuned so that its natural period is the same as that of the alternating electromagnetic forces produced by the current to be detected, thus using the principle of resonance to produce a relatively large motion for a very small current.[1]

The reading is usually made by observing the image of an electric lamp filament reflected in a very small mirror attached to the moving system, by means of a telescope, or by a projection upon a screen. When current passes through the instrument the vibration of the moving element causes the image of the filament to appear to broaden out into a band.

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[1] For a general discussion of vibration galvanometers, see: Laws' Electrical Measurements, 1917, p. 434; F. Wenner, TRANS. Am. Inst. Elec. Engineers, 31, p. 1243; F. Wenner, *Bull. Bureau of Standards*, 6, p. 347, 1910; A. Campbell, *Proc. Physical Society of London*, 26, p. 120, 1914.

It is evident that a vibration galvanometer of either type is simply a specialized form of synchronous motor, the whole mechanical output of which is used in overcoming air friction and elastic hysteresis.

The vibration galvanometer has been used but little in industrial laboratories, its principal use being in physical laboratories, and principally in precision work. The chief reasons for this limited use are that it is easily disturbed by external mechanical vibrations, and that delicate adjustments are necessary. In the present form of instrument these difficulties are greatly reduced. It is not, however, as sensitive as some forms of the moving-coil type.

Principle of Operation. The present instrument is of the moving-iron type. It consists essentially of a fine steel wire, mounted on one pole of a permanent magnet, and so arranged that the free end of the wire may vibrate between the poles of an electromagnet through which the current to be detected passes. If an unmagnetized steel wire, *W*, is held near the pole of an electromagnet, as in Fig. 1, the end of the wire will be pulled toward the pole of the electromagnet during each half wave of the current flowing in the winding of the electromagnet. That is, the wire will vibrate with twice the frequency of the current. If the wire be magnetized by mounting it on the pole of a permanent magnet, the free end of the wire will be alternately attracted and repelled by the alternating flux of the electromagnet. That is, the wire will vibrate with the same frequency as that of the current. But what is of more importance, the alternating mechanical pull will be very much greater than with an unpolarized wire, since the total flux from the wire is much greater.

The permanent magnet plays the same role in increasing the motion of the wire that the permanent magnet in a telephone receiver does in increasing the motion of the diaphragm. The total pull varies as B^2, and the change in the pull varies as the change in

B^2 resulting from the alternating current. Hence if a given small alternating current produces a change in flux, ΔB, the alternating mechanical pull is proportional to $B \Delta B$, instead of to $\overline{\Delta B^2}$, as it would be if the wire were not polarized by the permanent magnet.

Fig. 2 shows one of the first arrangements experimented with. A small electromagnet is mounted on the inside pole face of the permanent magnet, and the fine wire vibrator on the opposite pole face. The operation is readily seen. During one half-cycle the flux from the pole tip A is slightly strengthened, and that through B is weakened. Hence the vibrator moves toward A. During the next half-cycle the conditions are reversed and the vibrator moves toward B, and so on.

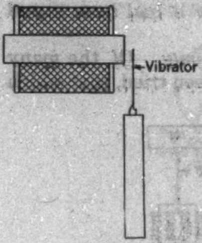

Fig. 1

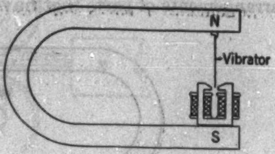

Fig. 2

Considering the arrangement as a motor, it is easy to see how the back electromotive force is generated. If the vibrator is moved back and forth by mechanical means, magnetic lines of force from the end of the vibrator move back and forth between A and B, alternately increasing and decreasing the flux in each, and thus generating an alternating electromotive force in the winding.

The vibrator, which is, of course, mechanically tuned so that its period is the same as that of the current to be detected, is observed by a microscope mounted perpendicular to the plane of the paper and focused on the end of the vibrator. In this way a much smaller motion can be detected than is possible with the unaided eye.

During a set of experiments undertaken in order to determine optima of conditions, such as size and shape of electromagnet, shape of pole tips, length of gap, and distance between end of vibrator and pole tips, an interesting observation was made. As a limiting case, the air gap of the electromagnet was reduced to zero by using a closed core as shown in Fig. 3. Even under this condition the device was nearly a tenth as sensitive as with the best arrangement that could be obtained with the same core after it had been sawed so as to form an adjustable gap.

Arrangement for Greatest Sensitivity. Of the many arrangements of parts that have been tried, that which gives the greatest sensitiveness and general convenience of working is shown in Fig. 4.

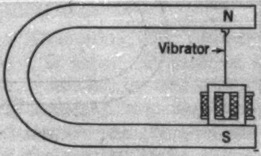

FIG. 3

The electromagnet is placed outside the gap of the permanent magnet, but in a position in which an appreciable amount of flux from the latter passes through the cores of the former. It is to be noted that in principle the arrangement is identical with the earlier one described above, the most important difference in the details of the arrangement being the weaker field to which the core is subjected. The two principal advantages of putting the electromagnet outside are: first, other things being equal, the instrument will be most sensitive when the core is working at the point of maximum differential permeability, and the point of maximum differential permeability occurs at very low values of B, (ideally, at $B = O$); second, with low frequency vibrators, which must be of very small diameter, the effect of the strong magnetic field upon the frequency of the vibrator introduces practical difficulties in tuning, since the magnetic forces combine with the elastic force to change the total restoring force, thus changing the frequency. The relative positions of the permanent magnet and the electromagnet were varied systematically, but, as would be expected, the one shown gives the best results.

A high differential permeability is necessary, as just mentioned, for sensitiveness. This means a high initial permeability in the core. Decidedly the best results have been obtained with cores made of good sheet silicon transformer steel. Results seemed to indicate that the sensitiveness varies as the square of the initial permeability instead of as the first power, as was expected, but sufficiently accurate data was not obtained to make certain on this point.

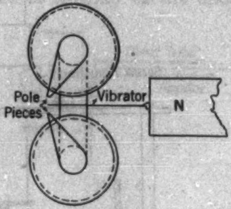

FIG. 4

If it is attempted to increase the sensitiveness by bringing the end of the vibrator close to the pole tips, a condition of instability is reached, the vibrator being pulled over to one or the other of the pole tips. Other practical difficulties arise, such as changes in the tuning of the vibrator. In general, the shorter the gap, the closer can the end of the vibrator be placed to the pole tips without such difficulties being encountered. Considerable changes may be made in length of gap, shape of pole tips, etc., without greatly affecting the maximum working sensitiveness attainable by the various adjustments.

Generally satisfactory results are obtained with the pole tips brought down in the form of truncated pyramids, the faces being about 2 by 0.5 mm., the short edge being parallel with the plane of vibration of the vibrator, a gap of about 1.5 mm., and the end of the vibrator about 1.5 mm. from the pole tips. An improvement by a factor of 1.5 to 2 may be obtained by setting the pole tips at an angle, as the field is intensified by such an arrangement, (See Fig. 4).

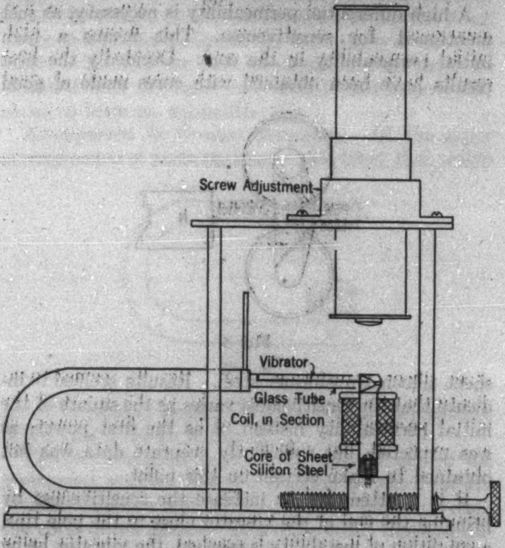

FIG. 5

The Vibrator. The vibrator is mounted on a small base of soft steel, as shown in Fig. 6, and is held in place magnetically, by merely placing the base on the face of the magnet. For convenience in changing vibrators for different frequencies, an aluminum wire is inserted in the base as a handle. For a 60-cycle vibrator a 0.1-mm. wire is convenient, and with this diameter a length of about 33 mm. is required. A 0.04-mm. wire of the same length has a frequency of about

25 cycles. For soft steel wires, the following formula is sufficiently accurate for design purposes:

$$\text{Frequency} = 65{,}000 \, \frac{\text{diameter}}{(\text{length})^2}$$

where the dimensions are in centimeters.[3]
For piano wire the constant is higher, about 70,000.

At first it was hoped that such a vibrator might be permanent enough in its frequency to serve the purpose of a frequency meter as well as that of a galvanometer, but it is not feasible, since the actual resonance frequency depends to a small extent upon the position of the vibrator in the magnetic field. Advantage may be taken of this fact, however, to provide a fine adjustment for tuning the vibrator to the exact frequency desired. If an auxiliary small magnet, or better, a pair of such magnets, is placed near the end of the vibrator, the frequency may be changed several per cent, by changing the position of the auxiliary magnet. While it does not readily provide as large a range of adjustment, a more convenient method is to move an iron rod toward or away from one pole of the permanent magnet by a screw motion, as shown in Fig. 5.

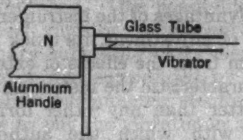

FIG. 6

If the vibrator is polished, and illuminated by a horizontal beam of light, a sharp line of light may be obtained in the microscope, very similar in appearance to the image of an incandescent filament of moderate brilliancy when viewed in a telescope. Satisfactory readings may also be made by viewing the wire in the ordinary way. In comparing sensitiveness it is convenient to use a vibration just sufficient to make the vibrator appear of double diameter.

It is well to keep the vibrator covered with a very thin film of oil to prevent corrosion. The effect of such a film on the frequency is too small to be at all inconvenient. For many purposes it is convenient to shield the vibrator with a glass tube, as in Fig. 6.

Performance. With a magnifying power of 50 to 100, which has been found satisfactory under working conditions, a motion of the vibrator of five microns is easily visible. With a one-ohm winding an electromotive force of three microvolts can be detected. The construction is such that high resistance coils may easily be wound for high current sensitivity. With a 270-ohm winding the sensitivity is such that a current of 0.05 microampere can readily be detected.

The chief advantages of the instrument are its sturdiness, the ease of adjustment, its quick responsiveness, and its freedom from the effects of external vibration. In the last characteristic the instrument is an order of magnitude better than any other form of vibration galvanometer with which the author is familiar. Both the freedom from external vibration and the quick responsiveness are due to the relatively large damping by air friction, and the extremely small mass of the moving element. Although the vibration galvanometer depends upon the principle of resonance, an appreciable amount of damping is necessary to give a reasonably quick response to changes in circuit conditions, and the smaller the mass of the moving element, the smaller the amount of energy necessary for a given amplitude, and the more quickly will the requisite amount of energy be supplied from the circuit.

The "resonance range" is about one per cent. That is to say, if the frequency of the current is one per cent above or below the frequency of resonance, the amplitude of vibration will be half as great as at resonance.

The efficiency of the present instrument as a motor is very low, the back electromotive force being only a

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2. This formula is in the form given by Rayleigh, (Theory of Sound, Vol. 1, art. 171), for the design of the rectangular prong of a tuning fork,

$$N = 84{,}590\ t/l^2$$

where N is the frequency, t the thickness, and l the length.

few per cent of the applied electromotive force. If it should be found possible to increase the electrical efficiency of the device, as a motor, to 50 per cent, the sensitivity would be increased in like ratio, and the instrument would be able to do the work of the most sensitive form of the moving-coil type. Wenner has shown that it is possible to develop an electrical efficiency of over 97 per cent in a vibration galvanometer of the moving-coil type.[3]

The instrument, in various stages of development, has been used in routine testing at the Bureau of Standards for more than three years. While many improvements have been made in detail, a much-to-be-desired radical increase in electrical efficiency has not been accomplished. One possibility is the use of a hardened steel vibrator, permanently magnetized, but a preliminary substitute experiment, in which a soft iron vibrator was surrounded by a magnetizing coil, did not show encouraging results. No attempt has been made to put the vibrator in a vacuum so as to reduce the air friction. It would be necessary to use a fairly high vacuum as the resistance of the air is nearly independent of the pressure down to a pressure of about one mm. of mercury. The sharpness of tuning necessary would increase as the sensitiveness increased, as would also the response to external mechanical disturbances, especially disturbances synchronous with the current. A very promising line of attack which has not been tried is the use of a vibrator made of vacuum process iron, to secure, if possible, a much higher flux in the vibrator.

I am indebted to Mr. J. B. Dempsey for a large part of the experimental work, and for valuable suggestions.

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[3] Wenner, *Bull. of the Bureau of Standards*, 6, p. 364, 1910.

DISCUSSION ON "A NEW FORM OF VIBRATION GALVANOMETER" (AGNEW), NEW YORK, N. Y., FEBRUARY 20, 1920.

John B. Whitehead: To one who has worked with the older forms of vibration galvanometer, involving the string suspension, it is obvious that this instrument presents many advantages. Simplicity of operation is at once conspicuous. I want to ask Dr. Agnew if he will not say a word on the adjustment of the instrument to different values of frequency. The suggestion is made here that with a given standard frequency, such as 60 or 25, vibrators are readily designed, but for frequencies which differ by several per cent from these, it would be necessary to include some provision for accurate tuning.

I have recently had occasion to measure alternating currents of very small values, and have found very great assistance indeed in the use of d-c. galvanometers in combination with hot cathode tubes, or kenotrons. These tubes rectify completely at all frequencies, and the use of the most sensitive forms of d'Arsonval instrument is thus possible. I suspect, from the figures given in the paper, that the sensitivity reached with the vibration galvanometer may not be reached with the d'Arsonval instrument, and if so, it will still be necessary to go to the vibration galvanometer in order to get these extreme limits of sensitivity. However, I want to call attention to the very great ease with which one may measure small alternating currents using kenotrons and galvanometer.

Clayton H. Sharp: I think it should be pointed out once more that the great advantage of this instrument is that it makes, apparently, the vibration galvanometer, an instrument available for many classes of usage to which vibration galvanometers as hitherto constructed have not been applicable. The ordinary type of vibration galvanometer as we have known it in the past has been practically unusable in a building which was subject to vibrations having a periodicity somewhat similar to the natural period of the instrument. If Dr. Agnew has succeeded, as I take it he has in producing an instrument which is not so affected by building vibrations while still retaining a satisfactory degree of sensibility electrically, he has advanced the art a very material step.

W. H. Pratt: Some six or eight years ago I had occasion to determine the moment of inertia of the moving system of an air damping instrument. After giving the procedure sufficient consideration, I attempted to determine the time of vibration of this moving system on the conditions of no damping, by conducting the experiment in a vacuum tank. I immediately discovered what has been pointed out in the paper, that the damping is reduced by a very small amount, until the condition of high vacuum is attained.

P. E. Klopsteg: Another modification of the instrument suggests itself, and that is the use of a tungsten filament mounted, possibly with an elastic mounting, in an exhausted glass tube, to pass the direct current through the filament so as to heat it, and to use the same form of electro-magnet for the alternating current, and observe the heated filament with the microscope. This would be an arrangement somewhat similar to the Einthoven string galvanometer, except that the filament is made incandescent with direct current, and the alternating current passed through the magnet windings. Tuning might be accomplished by using a magnet or pair of magnets to change the tension of the filament. Of course, the process might be reversed, presumably, and the regular Einthoven arrangement used, that is, to use the direct current in the magnet and send the alternating current through the filament. With this arrangement the filament would require outside illumination. The other plan seems more worth trying, namely, to heat the filament with direct current, and allow the alternating flux, due to the current being detected, to vibrate the filament and observe the incandescent filament with a microscope.

P. G. Agnew: Replying to Professor Whitehead's question in regard to the method of tuning, the magnetic control described in the paper gives a frequency adjustment up to 5 per cent. For more than 5 per cent, the vibrator is changed. In practise the change is a simple matter. I usually keep a few pieces of wire hung up in the laboratory, oiled, so as to avoid corrosion, and when it is necessary to use a different frequency, say 30 cycles, one mounts a piece of the wire, and clips it off to the calculated length. If the diameter of the wire is known, the formula gives the length to a few per cent, and so only one or two trials are necessary to get a satisfactory adjustment. The mounting and adjustment of a vibrator can easily be made in ten or fifteen minutes.

The forms suggested by Dr. Klopsteg would actually be entirely new forms of the vibration galvanometer, rather than mere modifications of the present form. The development of Dr. Klopsteg's ideas would certainly constitute an interesting and a valuable contribution to the subject of electrical measurements.

Dr. Sharp has very clearly pointed out the most important advantage of the instrument. At first sight one would expect the present instrument to be affected much more than it is by external vibration, in fact, much more than is the coil type. In an axially mounted coil we would expect, as a first approximation, that the instrument would be independent of external vibration, which could be effective only through accidental variation from axial symmetry, while in the present form the vibrator would be expected to respond to external vibration. Experience shows, however, that the present form is much less affected than is the coil type by external vibration. The principal reason for this is to be found in the extremely small mass of the vibrator, coupled with the damping. Only a minute amount of energy is involved in the vibration and this is quickly dissipated in the damping.

Presented at the 8th Midwinter Convention of the
American Institute of Electrical Engineers,
New York, February 20, 1920.

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A PRECISION GALVANOMETRIC INSTRUMENT FOR MEASURING THERMOELECTRIC E. M. FS.

BY T. R. HARRISON AND PAUL D. FOOTE
Both of the U. S. Bureau of Standards

A new principle has been developed whereby an ordinary millivoltmeter may be converted into an instrument in which the usual errors arising from a variable line resistance are entirely eliminated. The instrument measures true e. m. f. in a simple circuit or if connected across a resistance or network through which a current flows it indicates the potential drop which would have existed had the instrument not been connected. In this respect it functions as a potentiometer, yet it does not operate on the potentiometric principle since it does not require a standard cell (or the equivalent) or an auxiliary battery, the only e. m. f. employed in the adjustment being that of the source measured. No loss in precision of setting results; in fact the adjustment may be readily made to 10 times the scale accuracy. Various wiring diagrams are shown and methods are discussed for constructing instruments of zero temperature coefficient and properly damped. A new deflection potentiometer is described which offers (considerable advantage over the ordinary type for small e. m. fs. in a circuit of variable resistance.

THE maximum e.m.f. developed by a rare-metal thermo-couple is about 15 to 18 millivolts, and that developed by a base-metal couple about 50 to 80 millivolts. To measure such small e.m.fs. accurately by a galvanometric method requires the use of specially constructed millivoltmeters. A millivoltmeter, although graduated to read e.m.f. is fundamentally a current-measuring device. The deflection of the pointer is approximately proportional to the current flowing through the instrument.

If e is the e.m.f. developed by the couple, R_g the resistance of the galvanometer, and R the resistance of the lead wires and couple, the current flowing $= \frac{e}{(R_g + R)}\cdot$ Hence in order that a certain deflection correspond to a definite e.m.f., the scale of the instrument must be graduated for a preassigned value of $R_g + R$. Any change in the resistance R from this standard value causes an error in the observed reading of the e.m.f. Such variations may arise from bad contacts, deterioration of the couple, change in depth of immersion, temperature coefficient of lead wires and couple, etc.

In order to minimize the effects of these variations the resistance R_g of the galvanometer is made large compared to that of the line and couple R. This latter resistance in general practise may vary from 1 to 15 ohms. Hence the resistance R_g must be very great in order that changes in R be negligible. The construction of a millivoltmeter having the combined features of high resistance and robustness sufficient to resist the mechanical shocks to which it is necessarily subjected in any industrial plant is an exceedingly difficult problem. The highest resistance ever employed is about 1200 ohms, and instruments are manufactured having a resistance of only 10 ohms. All questions considered, the most satisfactory pivot instruments from the pyrometric standpoint have a resistance of about 5 to 10 ohms per millivolt.

If an instrument having a resistance of 100 ohms is calibrated to read correctly for a line and couple resistance of 2 ohms, and if the resistance of the latter for one of the various reasons mentioned above changes from 2 to 5 ohms, the indicator will read in error by 3 per cent or 30 deg. at 1000 deg. cent. With a 10-ohm indicator the error would be about 20 per cent, or 200 deg. at 1000 deg. cent. Such errors are of serious importance, and are not easily detected unless measurements of the line resistance are frequently made. These measurements require the use of an auxiliary instrument such as a Wheatstone bridge, and then corrections must be applied to the observed reading of the millivoltmeter to take account of the variation in line resistance from the value for which the indicator was standardized.

In the following discussion an instrument is described in which provision is made whereby the total resistance of the circuit may be easily adjusted to the preassigned value for which the scale of the instrument is graduated without requiring the use of any auxiliary instrument or a source of e.m.f. other than that of the couple being measured. Although the instrument was designed primarily for pyrometric purposes, its usefulness is not confined to this field alone.

DESCRIPTION OF COMPENSATED INSTRUMENT

Fig. 1 illustrates the apparatus in a general form. When the instrument is used as a simple temperature indicator, the resistance of the circuit through which the current from the thermocouple flows is made up as follows: The resistance *T* of the thermocouple, resistance *L* of the line or lead wires, a rheostat of resistance r_x which may be adjusted thus altering the resistance of the circuit, a resistance r_2, a resistance r_3 including the resistance of the moving element, r_3 being shunted by resistance r_4, r_5, and r_6 connected in series. In addition to these parts of the circuit, arranged as shown in the figure, there is a resistance r_7 connected between points *C* and *K*, and a key *K* which when closed, short-circuits r_5, and connects r_4, r_6, and r_7 directly together through the key.

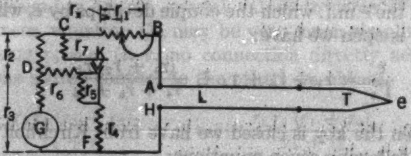

FIG. 1—GENERAL DIAGRAM OF APPARATUS FOR ELIMINATING ERRORS DUE TO UNKNOWN LINE RESISTANCE, FROM WHICH TWO SIMPLE TYPES ARE DERIVED

By adjusting one or more of the resistances until a definite relation is obtained between the deflections of the moving element *G* when the key is open and closed respectively, a definite relation will be established between different resistances of the system. The most convenient relation between the respective deflections is identity, and this relation will be used in the following discussion. In the general case here described, identical readings could be established by adjusting any of the resistances r_x, r_2, r_3, r_4, r_5, r_6, or r_7, but it will appear directly that one desirable method is to adjust r_x, for then the sum of r_x, L, and T is brought to a definite value. The instrument may be so designed that when $r_x + L + T =$ a constant $= r_1$, the total resistance of the circuit is the preassigned value for which the instrument was calibrated. The relation between the resistances may be deduced as follows:

Representing the current flowing through r_3 by i'_3 when the key is open and by i_3 when the key is closed, and the e.m.f. which the couple develops by e, when the key is open we have:

$$\frac{e}{i'_3} = r_1 + r_2 + r_3 + \frac{(r_1 + r_2)\, r_3}{r_4 + r_5 + r_6} \qquad (1)$$

When the key is closed we have from Kirchhoff's laws the following seven equations:

$$\begin{aligned}
&i_1 - i_2 - i_7 = 0\\
&i_2 - i_6 - i_3 = 0\\
&i_3 + i_4 - i_1 = 0\\
&i_7 + i_6 - i_4 = 0\\
&i_2 r_2 - e + i_1 r_1 + i_3 r_3 = 0\\
&i_2 r_2 + i_6 r_6 - i_7 r_7 = 0\\
&i_3 r_3 - i_4 r_4 - i_6 r_6 = 0
\end{aligned}$$

in which i with subscript represents the current through the resistance designated by the corresponding subscript.

Solving these equations we obtain:

$$\begin{aligned}
&\frac{e}{i_3}\left[r_4 \left(1 + r_2/r_6\right) + r_7 \left(1 + r_4/r_6\right)\right]\\
&\quad = r_7 \left[\frac{(r_1 + r_2)\, r_3}{r_6} + (r_1 + r_2 + r_3)\left(1 + r_4/r_6\right)\right]\\
&\qquad + (r_1 r_3 + r_1 r_4 + r_3 r_4)\left(1 + r_2/r_6\right) + r_2 (r_1 + r_4)
\end{aligned} \qquad (2)$$

If the deflection of the galvanometer is the same whether the key is open or closed, the current flowing through the galvanometer coil is equal in the two cases

($i'_3 = i_3$) and we may substitute for e/i_3 in (2) the value given for e/i'_3 in (1). Upon simplification of the equation thus obtained we have (3) which is the general conditional equation for the relation between the resistances necessary in order that the deflection of the galvanometer be not altered by closing the key K:

$$r_2 r_4 + \frac{r_3 (r_1 + r_2)(r_2 r_4 + r_4 r_6 - r_5 r_7)}{(r_2 + r_6)(r_4 + r_5 + r_6)}$$

$$= r_1 r_3 + \frac{r_2 r_6 (r_1 + r_4)}{r_2 + r_6} \qquad (3)$$

By assigning certain special values to some of the resistances, equation (3) may be very much simplified.

Making $r_5 = \infty$ (*i. e.* no connection directly across from one switch contact to the other) we obtain:

$$r_2 r_4 - \frac{r_3 r_7 (r_1 + r_2)}{r_2 + r_6} = r_1 r_3 + \frac{r_2 r_6 (r_1 + r_4)}{r_2 + r_6} \qquad (4)$$

If in addition to making $r_5 = \infty$ we make $r_7 = 0$ the equation reduces to the form:

$$r_2 r_4 = r_1 r_3 + \frac{r_2 r_6 (r_1 + r_4)}{r_2 + r_6} \qquad (5)$$

and finally, upon making $r_6 = 0$ in addition to making $r_5 = \infty$ and $r_7 = 0$, we have the important relation *viz.*:

$$r_2 r_4 = r_1 r_3 \quad \text{or} \quad r_1 = \frac{r_2 r_4}{r_3} \qquad (6)$$

This arrangement will be discussed fully below. If we impose the sole condition that $r_5 = 0$, equation (3) reduces to the form:

$$r_1 = \frac{r_2 r_4}{r_6} \qquad (7)$$

This is also a very simple form which possesses the advantage that it is independent of the value of r_7, and therefore of contact resistance at the key. For high sensitivity in adjusting r_1 to the proper value, r_7 should be practically zero and r_4 should be also low in order that a large current may flow through r_1 when the key is closed.

Fig. 2 shows a diagram of the instrument where $r_5 = \infty$ and $r_6 = r_7 = 0$ as discussed above. The circuit *C D G F* is an ordinary millivoltmeter or galvanometric indicator of resistance $r_2 + r_3$, the latter resistance including that of the moving element *G*. In series with this is an adjustable rheostat *C B*, the maximum value r_1 of which is chosen equal to the maximum value of the resistance of the line and couple likely to occur in practise. The galvanometer is calibrated to read correctly when the total resistance *R* of the circuit has the preassigned value $r_1 + r_2 + r_3$. If in the construction of the instrument $\frac{r_2 r_4}{r_3}$ is made equal to r_1, then when the rheostat *C B* is adjusted to a value r_x such that the deflection of the instrument is the same whether the key is open or closed, the sum of resistances $r_x + L + T$ must equal r_1 and the resistance $r_x + L + T + r_2 + r_3$ of the circuit with the key open must equal the preassigned value $R = r_1 + r_2 + r_3$ for which the scale of the instrument was graduated. Thus it follows, as will be shown more clearly later, that *the instrument measures true e.m.f. in a simple circuit, or if connected across a resistance through which a current flows it will indicate the potential drop which would have existed had the instrument not been connected. In this respect its action is similar to that of a potentiometer. The principle may be also applied to net work conductors, an extension of which leads to a serviceable deflection potentiometer.*

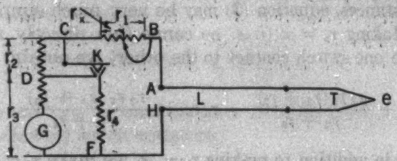

FIG. 2—OPEN SHUNT TYPE FOR HIGH SENSITIVITY, ESPECIALLY SUITABLE FOR THERMOCOUPLES

Equation (**6**) may be deduced in a simple manner as follows: If e represents the e.m.f. developed by the couple, e' the potential drop across $D\,F$ when the key is open, and e'' the potential drop across $D\,F$ when the key is closed, we obtain the equations:

$$e' = \frac{e\,r_3}{L + T + r_x + r_2 + r_3} \tag{8}$$

and

$$e'' = \frac{e\,r_3\,r_4}{(L + T + r_x)\,(r_3 + r_4) + r_3\,r_4} \tag{9}$$

If r_x is so adjusted that these two potential drops and hence the deflections of the indicator are the same, we have on equating (**8**) and (**9**):

$$L + T + r_x = \frac{r_2\,r_4}{r_3} = \text{a constant} = r_1 \tag{10}$$

by construction. Adjustment of the rheostat alters the readings both when the key is open and closed. A simple process for operating the instrument is accordingly as follows:

1. Read the instrument with the key open.
2. Close the key and adjust the rheostat $C\,B$ until the instrument reads approximately the same as in 1.
3. Repeat 1 and 2 if necessary.

MAGNIFICATION OF ERRORS

By making r_3/r_4 equal to from 5 to 10, the ease with which the proper setting can be obtained is greatly improved. That this is true will be seen from the following consideration.

If the instrument is calibrated to read correctly when the resistances are related according to the equation $r_1/r_2 = r_4/r_3$ and if the rheostat $C\,B$ is out of adjustment by an amount $\delta\,r_1$, the instrument will read in fractional error, when the key is open, by an amount:

$$\frac{\delta\,e'}{e'} = -\,\frac{\delta\,r_1}{r_1 + r_2 + r_3}$$

and when the key is closed by an amount:

$$\frac{\delta e''}{e''} = - \frac{\delta r_1}{r_1 + \dfrac{r_3 r_4}{r_3 + r_4}}$$ whence

$$\frac{\delta e'}{\delta e''} = \frac{r_1 r_3 + r_1 r_4 + r_3 r_4}{(r_3 + r_4)\,(r_1 + r_2 + r_3)} \qquad \textbf{(11)}$$

If $r_1 = r_2$ and $r_3 = r_4$, the above expression simplifies to $\frac{\delta e'}{\delta e''} = \frac{1}{2}$ or $\delta e'' - \delta e' = \delta e'$. Therefore when $r_3/r_4 = 1$ the difference between the readings will equal the error in the reading when the key is open. If however, r_2/r_1 and r_3/r_4 are made large, say equal to 10, we find $\frac{\delta e'}{\delta e''} = \frac{1}{11}$ or $\delta e'' - \delta e' = 10\, \delta e'$. Thus if $r_3/r_4 = 10$, the error when the key is open is only 1/11 of the error when the key is closed, or 1/10 of the difference between the readings when the key is open and closed respectively.

In the process for operating the instrument described it is rarely necessary to make a second adjustment when the resistances r_3/r_4 are in as high a ratio as 10. In position 1, the instrument functions as an ordinary galvanometer. The single setting in position 2 reduces the error in the ordinary galvanometer to 1/11 of itself, which is usually sufficient.

The adjustment for the proper external resistance, if desired, can be made with 10 times the precision necessary. Thus, if the galvanometer can be read to 1/10 of a scale division, the line resistance can be set for an error of only 1/100 of a scale division, which is at least 10 times the accuracy possible with any indicating instrument. Hence the factor of variable line resistance which may give rise to very serious errors is easily and accurately controlled by a simple mechanical adjustment.

MULTIPLE INSTALLATION

The device is readily applicable to multiple installations of different line resistances. For multiple point recorders and indicators as many resistances *C B* may be employed as there are couples. These may be inexpensive rheostats having a resistance of approximately 15 ohms each, located in each couple line between the couple and the selective switch. These rheostats may be adjusted in the manner described whenever convenient or necessary. The fact that it is not essential to have the extra resistances adjusted to precisely 15 ohms is of great advantage from the constructional point of view. The instrument may be calibrated for exactly 15 ohms external resistance. If the adjustable resistances accordingly have values of from 16 to 17 ohms, from 1 to 2 ohms more are cut out of the circuit in the process of obtaining a setting than would be the case if resistances of precisely 15 ohms were employed. In either case, however, the final e.m.f. reading is correct.

The following illustrates a suitable proportioning of resistances for a 300-ohm indicator with $r_3/r_4 = 10$:

$$r_1 = 15 \text{ ohms.}$$
$$r_2 = 150 \text{ ohms.}$$
$$r_3 = 135 \text{ ohms.}$$
$$r_4 = 135/10 \text{ ohms.}$$
$$r_1 + r_2 + r_3 = 300 \text{ ohms.}$$

PROPER DAMPING

In designing an instrument of the type discussed above, care must be taken to avoid excessive over damping when the key is closed. The resistances effective in damping are $R = r_1 + r_2 + r_3$ when the key is open and $R_d = r_3 + \dfrac{r_1 r_4}{(r_1 + r_4)}$ when the key is closed. It is evident that R_d is less than R. If we let $K = r_2/r_1 = r_3/r_4$ the following two relations are obtained:

$$r_1 = \frac{R - R_d}{K + \dfrac{R - R_d}{R}} \qquad \text{(a)}$$

$$r_3 = r_1 K \frac{R_d}{R - R_d} \qquad \text{(b)}$$

For the simple indicator R is known as soon as the scale range is chosen. Suitable values may be selected for r_1 and K and equation (a) solved for R_d. If the simple indicator is not seriously overdamped when its total resistance is R_d, this value of R_d is satisfactory, and r_3 may be computed from equation (b). The values of r_2 and r_4 follow from K.

If, however, the simple instrument is too sluggish when its total resistance is R_d, the minimum allowable value of R_d may be determined experimentally, and this value, substituted in equation (a), determines r_1 with proper choice of K; whence r_2 and r_3.

TEMPERATURE COEFFICIENT

In the method so far described, no account has been taken of the temperature coefficient of the instrument. An increase in the temperature of the instrument causes a slight weakening of the springs which tends to allow a given current flowing through the instrument to produce a greater deflection[1]. This tendency is opposed by the effect of a slight decrease in the magnet strength. The effect of the weakening of the springs is almost always in excess of the effect due to the decrease in magnet strength. Thus an instrument becomes more sensitive to current as its temperature rises. In order that a given deflection shall continue to indicate the e.m.f. marked on the scale, the current which this e.m.f. causes to flow through the instrument must decrease with increasing temperature. This requires that the resistance of the circuit increase slightly with increase in temperature. The construction of the instrument is such that its resistance really does so increase. Nearly all of the moving coils used in millivoltmeters are made of copper, the resistance of which increases about 0.4 per cent of its initial value per degree rise in temperature. The majority of instruments require that from 1/13 to 1/40 of the resistance of the galvanometer be composed of copper in order to neutralize the excess of the effect of weakening of springs over decrease in magnet strength and thus give the instrument a zero temperature coefficient.

For an instrument which is constructed with the proper proportion of copper in the circuit, the value of

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[1] See Bureau of Standards Circular No. 20.

r_1 should be the same whatever the temperature of the instrument. This requires that the value of $\frac{r_2 r_4}{r_3}$ also must not vary with temperature. The resistance of r_2 which is of manganin, is practically independent of temperature, but r_3 is composed partly of copper, since it includes the moving element. Therefore in order that r_4/r_3 shall remain constant with changing temperature, r_4 must be composed of the same proportion of copper and manganin as r_3.

Any greater proportion of copper in the instrument than that required as stated above will result in too great a rate of increase of resistance with increase in temperature, and hence the instrument will read too low when its temperature is higher than that at which it was calibrated. Instruments are often made with a greater proportion of copper than is thus required in order to secure a more substantial design suitable for industrial practise and to avoid other difficulties which arise in the construction. One method of making such an instrument read correctly at different room temperatures would be to reduce manually the resistance of the circuit by an amount equal to the increase in resistance of this extra amount of copper in the circuit. The instrument described in the present paper may be arranged so that this compensation is included in the adjustment for variable line resistance without any extra constructional features or manipulation. This is done by allowing the resistance r_4 to have a greater proportion of manganin to copper than r_3. The necessary proportion is found as follows:

Let Q = temperature coefficient of the simple indicator.
P = temperature coefficient of the compensated indicator.
η = temperature coefficient of simple indicator contributed by the copper.
ν = temperature coefficient of simple indicator contributed by springs and magnet.
α = temperature coefficient of r_3.
β = temperature coefficient of r_4.

η' = temperature coefficient of compensated indicator contributed by all changes in resistance.

The total temperature coefficient of an indicator is made up of the two parts, the temperature rate of change per ohm of resistance and the combined coefficient of the springs and magnet. Hence for the simple indicator we obtain:

$$Q = \eta + \nu = \frac{\frac{d}{dt}(r_1 + r_2 + r_3)}{r_1 + r_2 + r_3} + \nu$$

$$= \frac{\frac{d\,r_3}{dt}}{r_1 + r_2 + r_3} + \nu \qquad (12)$$

since r_1 and r_2 are constants. For the compensated indicator, however, $r_1 = \frac{r_2 r_4}{r_3}$ is a function of the temperature since both r_3 and r_4 depend upon the temperature. Hence:

$$P = \eta' + \nu = \frac{\frac{d}{dt}(r_1 + r_2 + r_3)}{r_1 + r_2 + r_3} + \nu$$

$$= \frac{\frac{d\,r_1}{dt} + \frac{d\,r_3}{dt}}{r_1 + r_2 + r_3} + \nu$$

$$= \frac{r_1\left(\frac{1}{r_4}\frac{d\,r_4}{dt} - \frac{1}{r_3}\frac{d\,r_3}{dt}\right) + \frac{d\,r_3}{dt}}{r_1 + r_2 + r_3} + \nu \qquad (13)$$

since $$\frac{d\,r_1}{dt} = \frac{r_2 r_4}{r_3}\left(\frac{1}{r_4}\frac{d\,r_4}{dt} - \frac{1}{r_3}\frac{d\,r_3}{dt}\right)$$

$$= r_1\left(\frac{1}{r_4}\frac{d\,r_4}{dt} - \frac{1}{r_3}\frac{d\,r_3}{dt}\right)$$

If in equations (12) and (13) we substitute

$$\alpha = \frac{1}{r_3}\frac{d\,r_3}{dt} \quad \text{and} \quad \beta = \frac{1}{r_4}\frac{d\,r_4}{dt}$$

we obtain the following:

$$Q = \frac{r_3\,\alpha}{r_1 + r_2 + r_3} + \nu \qquad (12)'$$

$$P = \frac{r_1\,(\beta - \alpha) + r_3\,\alpha}{r_1 + r_2 + r_3} + \nu \qquad (13)'$$

Whence:

$$P = Q + \frac{r_1\,(\beta - \alpha)}{r_1 + r_2 + r_3} = \text{temperature coefficient of compensated indicator.} \qquad (14)$$

In order to obtain the condition for an instrument of zero temperature coefficient put $P = 0$ in the above equation, whence:

$$\frac{r_1\,(\alpha - \beta)}{r_1 + r_2 + r_3} = Q \quad \text{(condition for zero temperature coefficient)} \qquad (15)$$

The coefficients α and β are due to the copper content in the resistances r_3 and r_4. If we assume the temperature coefficient of copper to be 0.004 we obtain:

$$\alpha = \frac{\text{per cent of copper in } r_3}{25{,}000}$$

$$\beta = \frac{\text{per cent of copper in } r_4}{25{,}000}$$

On substituting these values of α and β in equation (**15**) the following equation is obtained for the condition that the compensated indicator have a zero temperature coefficient:

$$(\text{per cent of copper in } r_3) - (\text{per cent of copper in } r_4)$$

$$= 25{,}000\,\frac{r_1 + r_2 + r_3}{r_1}\,Q \qquad (16)$$

If the simple indicator has been so designed that its temperature coefficient is zero, *i. e.* $Q = 0$, equation (**16**) shows that the copper to manganin ratio should be the same in the two resistances r_3 and r_4.

If the simple indicator has a too high copper content so that its temperature coefficient for voltage is positive, *i. e.* $Q > 0$, the experimentally determined value of Q

for the simple indicator and the known copper ratio in r_3 may be substituted in equation (**16**) and the proper copper ratio for the shunt r_4 may be readily computed.

Perfect compensation cannot be obtained if $\frac{r_1}{(r_1 + r_2 + r_3)}$ is small and Q is large. The greatest degree of compensation is then secured when r_4 is made entirely of manganin, *i. e.* $\beta = 0$. Hence on making $\beta = 0$, from equation (**15**), we obtain as a condition for the possibility of perfect compensation that:

$$Q \leq \frac{r_1 \alpha}{r_1 + r_2 + r_3} \tag{17}$$

Many simple indicators have a temperature coefficient Q which is larger than the maximum value expressed by equation (**17**). In this case the shunt r_4 should be constructed entirely of manganin and the relation between the temperature coefficient of the compensated indicator, P, and that of the simple indicator, Q, readily derived from equations (**12**) and (**13**), is as follows:

$$P = Q\left(1 - 0.004\,\frac{r_1}{r_3 Q} \cdot \frac{\text{ohms of copper in } r_3}{r_1 + r_2 + r_3}\right) \tag{18}$$

or

$$P = (\eta + \nu)\left(1 - \frac{r_1}{r_3}\,\frac{\eta}{\eta + \nu}\right) \tag{19}$$

Equation (**19**) is useful for computing directly the value of P on the assumption of a definite value for ν. Usually ν is approximately equal to $- 0.0002$. We will consider the application of the above equations to the instrument already described.

$r_1 = 15$ ohms.
$r_2 = 150$ "
$r_3 = 135$ "
$r_4 = 13.5$ "
$r_1 + r_2 + r_3 = 300$ "
Let $r_3 = 50$ ohms copper + 85 ohms manganin.
$Q = + 0.00047$ (assumed to be determined by experiment).

On applying the test given by equation (**17**) it is found that $Q > \frac{r_1 \alpha}{(r_1 + r_2 + r_3)}$. Hence perfect compensation is impossible for the above instrument. By making the shunt entirely of manganin, we obtain on substituting the proper values in equation (**18**):

$$P = (0.00047)\,(0.84) = 0.00040$$

Thus the compensated indicator has a temperature coefficient 16 per cent smaller than that of the simple indicator. The same conclusion follows also from equation (**19**).

USE OF THE COMPENSATED MILLIVOLTMETER FOR MEASURING POTENTIAL DROP

It is at times desirable to measure a small potential drop across a resistance through which a current is flowing. When an ordinary millivoltmeter is employed for this purpose, the potential drop is altered to some lower value which is indicated by the instrument, provided it is graduated to read potential difference at its terminals or at the ends of calibrated lead wires. A similar reading may be obtained with the instrument here described if the rheostat CB is set at a marked position such that $r_s = r_1$, since with this adjustment the instrument measures potential drop across its terminals.

Often however, the measurement actually desired is the potential drop which existed across the resistance without the millivoltmeter being connected. This value which may be obtained potentiometrically is also readily measured by the compensated indicator, as will be seen in the following section. When the rheostat CB is so adjusted that the deflection of the instrument is the same with the key open or closed, the reading is the potential drop which would exist were the instrument disconnected. One method employed for compensating for irreproducibility of the temperature-e.m.f. relation of different thermocouples of a given type is by the use of shunt and series resistance in the head of the couple.[2]

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[2] See Foote, Harrison and Fairchild, "Thermoelectric Pyrometry," Pyrometer Symposium, Chicago, 1919.

An ordinary millivoltmeter reads the potential drop existing across the shunt when the instrument is connected. This value depends upon the resistance of the indicator. The reduction in the observed e.m.f. of the couple (potential drop across the shunt) is greater the lower the resistance of the meter. With the compensated indicator however, the ratio of the shunt and series resistances may be definitely adjusted and the instrument, regardless of its resistance, will always indicate, similarly to a potentiometer, that proportion of the e.m.f. e developed by the couple which is represented by the expression

$$e \cdot \frac{\text{shunt resistance}}{(\text{shunt resistance} + \text{series resistance})} \cdot$$

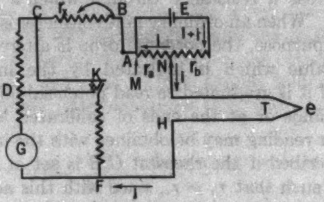

Fig. 3—In Conjunction With any Potentiometer of low Resistance the Instrument Produces a Perfectly Compensated Deflection Potentiometer

Deflection Potentiometer

The compensated indicator may be used as a galvanometer in conjunction with an ordinary potentiometer of low resistance forming a perfectly compensated deflection potentiometer, the compensation being effected by the adjustment of r_x in the manner already described.

This is shown in the following discussion, a special case of which resolves itself to the condition above mentioned in which the instrument may be used to measure the potential drop, which existed before the instrument was connected, over a resistance carrying a current.

Fig. 3 illustrates the use of the instrument as a deflection potentiometer. The potentiometer circuit consists of the battery E and the resistances r_a and r_b the latter including the internal resistance of the battery. When the indicator or galvanometer circuit is broken, for example at A, the potential drop e_1, across r_a is given by equation (**20**):

$$e_1 = \frac{r_a}{r_a + r_b} E \qquad \textbf{(20)}$$

Upon connecting the instrument to the potentiometer so that the e.m.f. e of the thermocouple (or any source of e.m.f.) T opposes e_1, the currents flowing through r_a and r_b are unequal and different from the original value. Assume that the currents now flowing are as indicated in the figure, I being the current in r_a, i the current in the indicator or galvanometer circuit, and $I + i$ the current in r_b. Since the sum of e.m.fs. and potential drops around any closed circuit is zero, the following two equations may be written:

$$E = (r_a + r_b)\, I + r_b\, i \qquad \textbf{(21)}$$

$$e \;= r_a I - R\, i \qquad \textbf{(22)}$$

in which R equals the resistance of the circuit from M, through $A\,B\,C\,D\,F\,H$ etc. back to N. Substituting the value of I from equation (**21**) in equation (**22**) and subtracting this value of e from the value of e_1, given by equation (**20**) we obtain:

$$e_1 - e = i \left(R + \frac{r_a\, r_b}{r_a + r_b} \right) \qquad \textbf{(23)}$$

This is the equation of a deflection potentiometer.[3]

If now the resistance r_2 is so adjusted that the reading of the instrument is the same whether the key K is closed or open, we have the following relations: Representing the values of i with the key open and closed respectively by i' and i'', we have for the potential drops across $D\,F$ in the two cases, $i'\, r_3$ and $i'' \dfrac{r_3\, r_4}{r_3 + r_4}$.

Equating these we obtain:

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[3] For general theory and design of deflection potentiometers see Brooks, Bull. Bu. Stds. Vol. 8, 1911, p. 395 (Sci. paper 172) and p. 419 (sci. paper 173).

$$i'' = i' \frac{r_3 + r_4}{r_4} \qquad (24)$$

the condition necessary for equal deflections.

We also have the values R' and R'' for R with the key open and closed respectively.

$$R' = r_x + L + T + r_2 + r_3 \qquad (25)$$

$$R'' = \frac{(r_x + L + T)(r_3 + r_4) + r_3 r_4}{r_3 + r_4} \qquad (26)$$

From equation (**23**) it follows that:

$$e_1 - e = i'\left(R' + \frac{r_a r_b}{r_a + r_b}\right) = i''\left(R'' + \frac{r_a r_b}{r_a + r_b}\right)$$

since both e_1 and e are independent of R, e being the e.m.f. of the couple and e_1 being defined by equation (**20**). If the values of R' and R'' from equations (**25**) and (**26**) and the value of i'' in terms of i' from equation (**24**) are substituted in the above equation, we obtain:

$$\frac{r_2 r_4}{r_3} = r_x + L + T + \frac{r_a r_b}{r_a + r_b} \qquad (27)$$

But by the construction of the instrument $r_1 = \frac{r_2 r_4}{r_3}$.

Hence from equation (**25**), $R' = \frac{r_1 + r_2 + r_3 + r_a r_b}{(r_a + r_b)}$.

If this value of $R = R'$ is substituted in equation (**23**) we obtain:

$$e_1 - e = i\,(r_1 + r_2 + r_3) \qquad (28)$$

The compensated indicator is calibrated for a total resistance $r_1 + r_2 + r_3$ so that the current i flowing through the instrument produces a deflection which indicates directly the difference between the e.m.f. e of the couple and the potential drop e_1 existing across r_a with the instrument disconnected. The potentiometer is graduated to read the value of e_1. *Thus when the compensated indicator is used as the galvanometer of an ordinary potentiometer, a deflection potentiometer is obtained which requires no compensation other than that provided by the indicator.*

Evidently the instrument has the disadvantage for continuous use as a deflection potentiometer that a new adjustment is required whenever the dial setting of the potentiometer is changed enough to cause a material difference in the value of $\frac{r_a r_b}{(r_a + r_b)}$. In any form of a deflection potentiometer however, changes in the resistance of the couple and leads, unless compensated for, will introduce errors in the galvanometer readings. The ordinary deflection potentiometer may be provided with a rheostat, in series with the couple and galvanometer, which is adjusted until the full scale range is equivalent to one step on the dial. The instrument here described compensates simultaneously for changes in both the couple resistance and potentiometer resistance. This adjustment is simpler than the adjustment for scale sensitivity of the ordinary deflection potentiometer, and hence if the resistance of the couple varies seriously, the new type of deflection potentiometer appears to be the more easily operated.

If in equation (**28**) we let $e = 0$, we obtain the condition which applies when the compensated indicator is connected across a resistance carrying a current. *It follows directly from the reasoning presented above that the instrument indicates the potential drop, e_1, which would exist if the instrument were not connected.*

CALIBRATION OF THE COMPENSATED MILLIVOLTMETER.

Another useful application of this feature may be employed in calibrating the scale of the indicator. The scheme of connections is the same as that shown in Fig. 3 except that there is no thermocouple T in the circuit. The instrument is connected directly, with the proper polarity to give positive deflections, to the e.m.f. terminals of a potentiometer of sufficiently low resistance that, for all settings, $\frac{r_a r_b}{(r_a + r_b)}$ plus the resistance L of any leads employed is not greater than r_1, and the galvanometer terminals of the potentiometer are short-circuited. When the resistance $C\ B$ is so adjusted that the deflection of the indicator is unaltered by depressing the key K, the reading of the potentiometer gives the value of the e.m.f. which should be indicated by the millivoltmeter. The other obvious method of calibrating the instrument in terms of the potential drop across its terminals when the resistance $r_2 = r_1$ is not as simple experimentally, and the former method is applicable at any temperature when the shunt resistance r_4 is constructed with the proper ratio of manganin to copper (see equation **16**) necessary to give a zero temperature coefficient. With the latter method however, the calibration should be made at the temperature for which a definite marked position of the rheostat $C\,B = r_1 = \frac{r_2 r_4}{r_3}$. This position can be a definite one at only one temperature for a compensated instrument, if the simple indicator has a temperature coefficient. However, the errors likely to arise from not considering this factor are usually of negligible importance.

USE OF COMPENSATED MILLIVOLTMETER FOR MEASUREMENT OF POTENTIAL DROP ACROSS A NETWORK

It follows from the above discussion that the compensated indicator may be used to measure the true potential difference between any two points of a complicated network existing were the instrument not connected. In this respect the instrument acts as a potentiometer. This is more clearly demonstrated as follows:

In Fig. 4 let r = resistance of network between M and N, excluding branch $M\,P\,N$.

R = resistance of branch $M\,P\,N$.

e_1 = potential drop across $M\,N$ for condition 1. Primed letters refer to condition 2, unprimed letters to condition 1.

The current i_{MN} which flows from M to N when an e.m.f. E is introduced in the branch $C\,D$ of the network is equal to the current i'_{CD} which flows from C to D when the same e.m.f. is introduced in the arm $M\,N$[4].

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[4] Jeans Math. Theory of Elec. and Mag. 3rd Ed. p. 327. See also Maxwell.

These two conditions are shown by (a) and (b) of Fig. 4.

$i_{MN} = i'_{CD}$ by the above theorem. (29)

$i_{MN} = \dfrac{E}{R + r}$ from Ohm's law. Hence:

$i'_{CD} = \dfrac{E}{R + r} K$ where $K < 1$ is a constant depending upon the values and the grouping of the separate resistances of the network. Also,

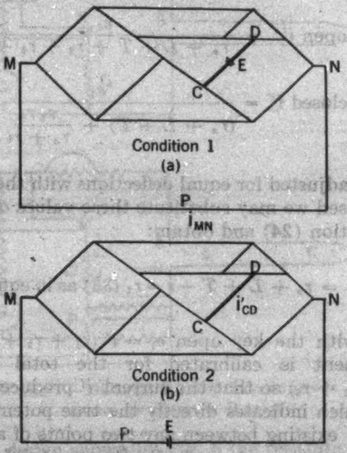


FIG. 4—SHOWING GENERAL DEFLECTION POTENTIOMETER PRINCIPLE, OR POTENTIOMETRIC NATURE OF READINGS BY THE COMPENSATED INSTRUMENT

$$i_{MN} = \frac{e_2}{R} = \frac{E}{R + r} K \quad \text{or} \tag{30}$$

$$e_2 = \frac{E}{1 + r/R} K \tag{31}$$

When $R = \infty$ let $e_2 = e_1$, whence from equation (31) $e_1 = E\,K$. Substituting this value of $K = e_1/E$ in equation (30) we obtain:

$$i_{MN} = \frac{e_1}{R + r} \tag{32}$$

Thus the current flowing in the branch R is equal to the potential drop across $M\,N$ with the branch circuit open, divided by the total resistance of the circuit.

Suppose the branch circuit to be the compensated indicator. The current flowing through the instrument is given by equation (**32**), and the two values R' and R'' of R (key open and closed) are expressed by equations (**25**) and (**26**). Hence:

$$\text{Key open } i' = \frac{e_1}{r_x + L + T + r_2 + r_3 + r}$$

$$\text{Key closed } i'' = \frac{e_1}{(r_x + L + T) + \frac{r_3 r_4}{r_3 + r_4} + r}$$

If r_x is adjusted for equal deflections with the key open and closed we may substitute these values of i' and i'' in equation (**24**) and obtain:

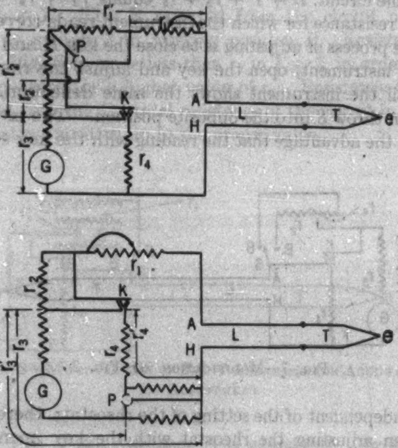
$$\frac{r_2 r_4}{r_3} = r_x + L + T + r = r_1 \text{ (33) as in equation (27)}$$

Thus with the key open $e_1 = i' (r_1 + r_2 + r_3)$. The instrument is calibrated for the total resistance $(r_1 + r_2 + r_3)$ so that the current i' produces a deflection which indicates directly the true potential difference e_1 existing between any two points of a network. If there are several e.m.fs. E_1, E_2, etc. in different arms of the network, it readily follows that the instrument still measures the true potential drop, on open external circuit, between any two points to which it is connected.

DOUBLE SCALE RANGE

Figs. 5 and 6 illustrate modifications of the simple design shown in Fig. 2 to permit the use of two different e.m.f scales. In Fig. 5 the resistances r_1 and r_3 are increased in the same ratio and by such an amount that the total resistance $r_1' + r_2' + r_3$ gives the desired sensitivity for the high range scale. Thus $r_1/r_2 =$

$r_1'/r_2' = r_4/r_3$ and the factor for the magnification of errors is the same with either range. With the plug in at *P*, $r_1' - r_1$ and $r_2' - r_2$ are short-circuited and the instrument operates on the low range scale. With the plug out r_1 is increased by the fixed resistance $r_1' - r_1$ and r_2 is changed to r_2', and the instrument operates on the high range scale.



FIGS. 5 AND 6—DOUBLE RANGE INSTRUMENTS, COMPENSATED IN EITHER RANGE

In the design shown by Fig. 6 the resistances r_3 and r_4 are increased in a similar manner such that $r_1/r_2 = r_3/r_4 = r_3'/r_4'$.

USE OF AN UNADJUSTABLE VALUE FOR r_1

In the foregoing discussion adjustment of the resistance r_1, which consists in part of the line and couple, has been considered. As has already been pointed out, if any of the four resistances in Fig. 2 is adjusted to give equal galvanometer deflections with the key open and closed, the relation $r_1/r_2 = r_4/r_3$ will be established. In Fig. 7, r_3 is made equal to r_4, and its value is chosen such that the instrument reads correctly when the total resistance of the circuit equals $r_2 + r_3 + r_x$ where $r_2 + r_x$ is the total resistance of the rheostat. With this arrangement $r_1 = L + T$. When switch *S* is in position *B'*, adjustment for equal deflections makes $r_1 = r_2$, (since $r_4 = r_3$). After this adjustment switch *S* is thrown into position *B*, and the resistance of the circuit, $L + T + r_x + r_3$ equals $r_2 + r_x + r_3$, the resistance for which the instrument reads correctly. The process of adjusting is to close the key *K* and read the instrument, open the key and adjust the rheostat until the instrument shows the same deflection, and then throw *S* into the opposite position. The method has the advantage that the reading with the key closed is independent of the setting of the rheostat; therefore, when adjusting the rheostat with the key open, the reading to which the instrument should be brought is accurately known. It is immaterial which way switch *S* is set during the adjustment, provided its position is changed after the adjustment has been made. Three disadvantages will be noted. (1) Since $r_3 = r_4$, there is no multiplication of errors in setting; (2) an extra switch *S* is required, and this switch must be operated each time an adjustment is made; (3) any contact resistance existing in the rheostat will alter the final resistance of the circuit by double this amount. Such contact resistances constitute no error in the instrument shown in Fig. 2.

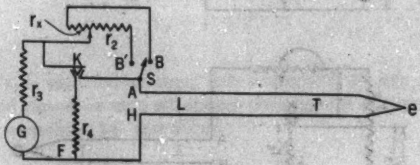


FIG. 7—MODIFICATION OF FIG. 2

Fig. 8 shows a design, somewhat similar to that illustrated by Fig. 7, which is modified to give a multiplication of errors on setting. Resistance r_4 is made equal to r_3/m, where m is any desirable factor for multiplication of errors. The total resistance $r_x + r_y$ of the upper rheostat is made equal to the maximum allowable line and couple resistance. The total resistance of the rheostat of which r_2 is part is made equal to $m\,(r_x + r_y)$. Adjustment for equal deflections with switch S at B' makes $r_2 = m\,(L + T)$, whence, since the two rheostats are adjusted simultaneously, $r_y = L + T$. This is removed from the circuit upon setting switch S in working position B. Resistance r_3 in Fig. 8 is made less than r_3 in Fig. 7, in order to compensate for a higher value of r_2. Resistance C is added to bring the total resistance of the circuit in the operating position up to the proper value.

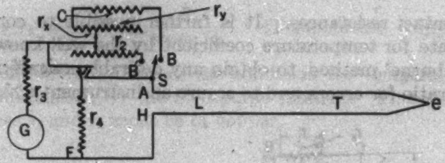


Fig. 8—Modification of Fig. 7 Arranged for Accuracy of Adjustment

In Fig. 9 the resistance r_4 is adjustable. By construction r_2 is made equal to r_3. Switch S is closed and r_4 is adjusted until equal deflections of the galvanometer are established. This makes $r_4 = r_1 = L + T$. Opening switch S adds resistance r_x to the circuit, making the total resistance equal to $r_2 + r_3 + x$, the resistance for which the instrument is calibrated. With S closed and K open the reading is independent of the setting of the rheostat. This reading is noted. With switches K and S closed r_4 is adjusted until the reading is the same as that determined above. The factor m increases rapidly as resistance $L + T\ (= r_1)$ decreases. Its value is given by the expression $m = \dfrac{r_2 + r_3 + x}{2\,r_4\,(1 + r_4/r_3)}$

COMPENSATED MILLIVOLTMETER WITH $r_5 = 0$

The circuit for an instrument designed according to equation (7) in which $r_5 = 0$ is shown by Fig. 10. The instrument possesses many advantages, one of these being the complete elimination of objectionable effects of contact resistances. It is further possible to compensate for temperature coefficient by the well known Swinburne[5] method, to obtain any desirable magnification ratio for errors, and to secure an instrument which is satisfactorily damped either with the key open or closed. Where a desirable magnification ratio cannot be otherwise obtained a battery may be inserted between *C* and *K*, Fig. 10. The proportioning of the various resistances is illustrated by the following discussion:

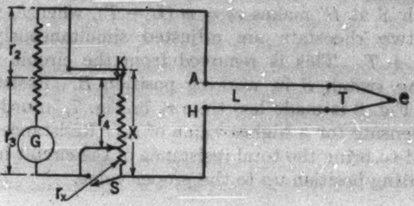


FIG. 9—ANOTHER MODIFICATION OF FIG. 2

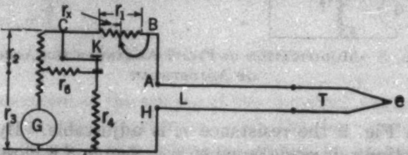


FIG. 10—CLOSED SHUNT TYPE OF COMPENSATING INDICATOR

Let i' = current through the galvanometer coil necessary to produce full scale deflection.

e' = maximum e.m.f. measured = highest scale reading.

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[5] Hallo and Land, Electric and magnetic measurements and measuring instruments, 1906, p. 260.

R_g = resistance of the moving copper coil.

m = magnification ratio for errors $\left(\dfrac{\text{key closed}}{\text{key open}}\right)$.

R_d = proper damping resistance.

Hence:

$$\frac{(r_3+r_4+r_6)\,(r_1+r_2)}{r_4+r_6}+r_3=e'/i'=S$$

(Scale range) **(34)**

Equation (**34**) is the condition for scale range such that an e.m.f. e' produces a full scale deflection.

$$r_1/r_2=r_4/r_6$$

(Key adjustment) **(35)**

Equation (**35**) is deduced as shown by equation (**7**) and expresses the condition that the deflection of the galvanometer is the same with the key K open or closed.

$$r_1+r_2=(r_4+r_6)\frac{R_g}{r_3-R_g}$$

(Temperature compensation) **(36)**

Equation (**36**) imposes the condition for temperature compensation by the Swinburne method. The resistances r_4 and r_6 must be of copper.

$$\frac{1}{m}=\frac{\delta e'}{\delta e''}=\frac{{}_1+r_4-r_4\dfrac{r_4}{r_3+r_4+\dfrac{r_2\,r_6}{r_2+r_6}}}{r_1+r_2+\dfrac{r_3\,(r_4+r_6)}{r_3+r_4+r_6}}$$

(Errors) **(37)**

Equation (**37**) shows the relation between the error in a setting with the key open and with the key closed. This is derived in a manner similar to that employed in the discussion of Fig. 2. For this relation r_7 is roughly zero.

$$R_d=r_3+\frac{(r_1+r_2)\,(r_4+r_6)}{r_1+r_2+r_4+r_6}=r_3+\frac{(r_1+r_2)\,r_4}{r_1+r_4}$$

(Damping) **(38)**

Equation (**38**) gives the total resistance of the galvanometer circuit with the key open. This should equal the proper damping resistance which is slightly greater than the critical damping resistance. It will be noted that when the resistance r_6 is adjusted for identical deflections with the key open and closed the resistance effective in damping is the same with the key in either position, and hence if the instrument is properly damped with the key open, it will be properly damped with the key closed.

The above 5 equations serve to determine the 5 variables r_1, r_2, r_3, r_4, and r_6. However, the condition for proper damping, equation (**38**), need not be satisfied exactly. The behavior of an instrument usually allows considerable tolerance in the value of R_d. Hence in order to obtain wide limits for construction, the first four equations may be solved as indeterminants and the best values of the resistances consistent with a suitable damping resistance may be chosen for the final design. The third term in the numerator of the right hand member of equation (**37**) is usually small. In place of equation (**37**) we shall assume the following relation:

$$\frac{1}{K} = \frac{r_1 + r_4}{r_1 + r_2 + \dfrac{r_3\,(r_4 + r_6)}{r_3 + r_4 + r_6}} \tag{39}$$

The solution of equations (**34**), (**35**), (**36**), and (**39**) is as follows:

$$r_1 = \frac{R_g\, S}{K\, r_3\left[1 - \dfrac{r_3 R_g}{(r_3 - R_g)\,(S - r_3)}\right]} \tag{40}$$

$$r_2 = S - r_1 - r_3 - \frac{r_3 R_g}{r_3 - R_g} \tag{41}$$

$$r_4 = \frac{r_1\,(r_3 - R_g)}{R_g} \tag{42}$$

$$r_6 = \frac{r_2 r_4}{r_1} \tag{43}$$

The following example illustrates the method of applying the above equations: It is desired to convert an ordinary millivoltmeter, scale range 0 – 15 millivolts into the above compensated instrument having a scale range 0 – 60 millivolts. The resistance of the simple indicator is 250 ohms of which the copper content $R_s = 60$ ohms. The instrument if suitably designed is accordingly properly damped on $R_d = 250$ ohms. Since the scale range is to be increased by the factor 4, the value of $S = 4 \times 250 = 1000$ ohms. Let $K = 10$. This is closely equal to m the magnification ratio for sensibility to changes in r_1 with the key closed. For various assumed values of r_3 compute r_1 from equation (**40**). For corresponding values of r_3 and r_1 compute r_2 from equation(**41**). Similarly for equations (**42**) and (**43**).

The following table illustrates the results thus obtained:

Values of Resistances
for $r_5 = 0$

r_3	r_1	r_2	r_4	r_6	$r_3 + \frac{(r_1 + r_2)(r_4 + r_6)}{r_1 + r_2 + r_4 + r_6}$	m
65	15.3	140	1.27	11.7	77	10.0
70	47	463	7.84	77.3	143	
80	55.5	626	18.5	208.5	250	10.2
90	53.5	676	26.7	338	333	
100	50	700	33.3	466	400	10.3
200	26.8	688	62.5	1605	700	
300	17.85	607	71.4	2428	800	10.7

Any of the above designs gives a millivoltmeter of scale range 0 – 60 millivolts, compensation for temperature coefficient by the Swinburne method and complete elimination of the error due to uncertain line drop. The values corresponding to $r_3 = 80$ furnish the proper damping, 250 ohms, characteristic of the particular instrument selected. If we substitute the values of the various resistances in equation (**37**) we obtain $m = 10.2$ which gives the ratio of sensitivities with the key closed and open.

If equations (**40**) to (**43**) do not furnish satisfactory values for the resistances with a suitable choice for K, equation (**36**) may be omitted and equations (**34**), (**35**) and (**39**) may be solved indeterminately similar to the manner above outlined. If the final choice of values for the various resistances results in an excessive temperature coefficient of the compensated instrument, this can be minimized by making r_4 and r_6 of unequal temperature coefficients, thus allowing the adjustment of r_1 to vary with the temperature as was done with the instrument illustrated by Fig. 2.[6]

The instrument on the right in Fig. 11 is an ordinary millivoltmeter. The other instrument is the compensated millivoltmeter constructed according to the wiring diagram shown in Fig. 2. It will be noted that the additional wiring requires no alteration in the size of the finished instrument.

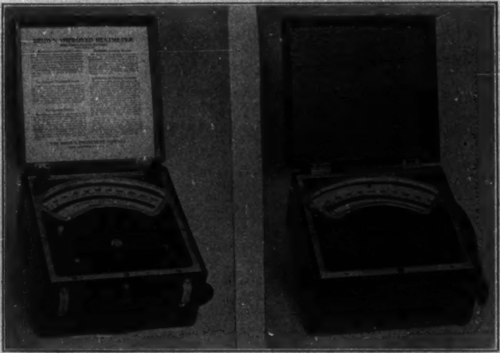

FIG. 11—COMPARISON OF INSTRUMENTS WITH AND WITHOUT COMPENSATING DEVICE. COMPENSATED INSTRUMENT SHOWN ON LEFT

SUMMARY

A new principle has been developed whereby an ordinary millivoltmeter may be converted into an instrument in which the usual errors arising from a variable line resistance are entirely eliminated. All of the several modifications described afford some simple means by which the total resistance of the galvanometer circuit, internal and external, is adjusted to a preassigned value for which the scale of the instrument is calibrated. In one form now manufactured the instrument consists of an adjustable resistance r_x in series with the moving coil and swamping resistance of the millivoltmeter. On depressing a key part, r_2, of the swamping resistance is short-circuited and the remaining part together with the moving coil, r_3, is shunted by a resistance r_4. The instrument is calibrated in terms of the potential drop across its terminals for a maximum value of $r_x = r_1$. In the construction the resistances are proportioned according to the relation $r_2 r_4 = r_1 r_3$. If the resistance r_x is so adjusted that the deflection of the pointer is unchanged by depressing the key it is shown that the total resistance of the circuit is that for which the instrument is calibrated, the sum of r_x and all external resistance being thus made equal to r_1. Hence it follows that the instrument measures true e.m.f. in a simple circuit, or if connected across a resistance or network through which a current flows it indicates the potential drop which would have existed had the instrument not been connected. In this respect it functions as a potentiometer, yet it does not operate on the potentiometric principle since it does not require a standard cell or an auxiliary battery, the only e.m.f. employed in the adjustment being that of source measured.

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[6] This method of treatment has recently been developed more fully and will appear in an early issue of the *Journal Wash. Acad. of Sci.* In this treatment the compensation for coefficient of springs and magnet is considered. This condition is more easily satisfied than is that required by eq. (**36**). This method is useful in making low range instruments.

By constructing r_3/r_4 equal to from 5 to 10 it is possible to adjust r_x with 5 to 10 times the precison necessary. Thus if the galvanometer can be read to 1/10 of a scale division the line resistance may be adjusted with a precision equivalent to 1/100 of a scale division, which is at least 10 times the accuracy possible with any indicating instrument. This principle of magnification of errors greatly facilitates the proper adjustment of r_x. By varying the copper to manganin ratio in r_4 it is possible to produce a compensated instrument of zero temperature coefficient from a millivoltmeter having an excessive copper content.

The instrument may be used as a galvanometer with an ordinary potentiometer, forming a deflection potentiometer which requires no compensation other than that provided by the indicator. Such an instrument is especially serviceable in thermocouple work.

The compensated millivoltmeter may be used in multiple installations of thermocouples having different line resistances, as many resistances r_x being employed as there are couples. These may be inexpensive rheostats, one located in each line between the couple and selective switch, and still the accuracy of adjustment will be as high as though precision rheostats were employed.

Methods are described for constructing instruments of double scale range with compensation in both ranges.

Specifications are given for an instrument which compensates for line resistance, which has a zero temperature coefficient, and which is properly damped either with the key closed or open.

DISCUSSION ON "A PRECISION GALVANOMETRIC INSTRUMENT FOR MEASURING THERMOELECTRIC E. M. Fs." (HARRISON AND FOOTE), NEW YORK, N.Y., FEBRUARY 20, 1920.

J. B. Whitehead: In the abstract the statement is made: "The instrument measures true e. m. f. in a simple circuit, or if connected across a resistance or network through which a current flows it indicates the potential drop which would have existed had the instrument not been connected;" which suggests that the instrument takes no current, or functions, as does the electrostatic voltmeter. This, however, cannot be true—hence my question. It is quite obvious, that even if limited to correction of drop in the leads, the principle has great value for refined measurements.

A. E. Kennelly: It is very important to be able to secure potentiometer apparatus, so compensated that no errors are produced by introducing it into the circuit it measures. The principle represented here is a very general application and it ought to prove very valuable to laboratory men.

W. D. A. Peaslee: In connection with instruments for measuring thermo-electric currents of this kind, I recently ran into an interesting phenomenon that may be of interest to you. In using some rather high temparature kilns in the manufacture of porcelain, we began to get, under certain conditions, very freaky readings on the thermocouples. They were used in certain parts of the kiln, some base metal couples, and in other parts rare metal couples. On the recommendation of some men who we thought knew more about the matter than we did, we were using switches, and installed indicating and recording instruments, and got some very peculiar readings. We got readings after the apparatus had been in operation a little while, that were the sum of the base metal and rare metal readings, and got a reading that would be the difference in these readings and then some that were some integral of the two. Then we would have readings for a while that did not seem to mean anything. We finally got into it, and I came to the conclusion that there was some sort of ionic conduction through the terrifically hot gases of the kiln that gave us a leakage combined with a dust leakage, which was in the switch. The manufacturers of these instruments stated that it was absolutely impossible. By the simple experiment of closing the dampers, in some of the gas chambers, we proved that in these high temperatures it was possible, and it was possible also in these high temperatures to get electrified drift of some kind that made your errors in these instruments of too great a magnitude to be permissible in that work.

We had a very interesting thing occur which the paper brings to mind. We have indicating instruments on the kilns by which the kiln operators do their firing, and they read a log from these indicating instruments. If the men happen to play a game of cards in the middle of the night, and neglect their duty, it results in rather serious consequences in the kiln, and everything is lovely in the morning, except you have lost $10,000 or $12,000 worth of porcelain. We rigged up a means in our research laboratory by which we could plug the recording instrument on to any indicating instrument at will, so that the men did not know what indicating instrument was on, but through some rather subterranean channels, I found the men immediately knew—we could not figure it out. When we plugged the recording instrument and indicating instrument in parallel, to a thermo-coupled circuit, whenever the man reads his indicating instrument, he gets a static kick on the indicating instrument every time he starts reading across. He has eighteen couples on which he presses a button to take his readings, and when he comes to the one to which the recording instrument is coupled he gets a kick, and he says—"The old man is watching me on that one," and we always get results on that instrument. I am anxious to get some system whereby we can get our recording instrument on the indicating instrument without their knowledge.

It really is a very delicate problem. In firing porcelain in tunnel kiln, you fire to very nice limits, and not only that, the time curve to be arrived at is very important and it is desirable that the firing process should be very strictly under control.

That is the problem on which I would like to keep some of the men interested in thermocouple indicators and recording instruments—to give us a system where we can put in 30 or 40 indicating and a few recording instruments, and be sure of our results, and also to be sure that the men we are keeping check on are not as well posted on what we are doing as we are.

E. M. Hewlett: If you put a dummy resistance near to the instrument itself, you might know these instruments at any time, without this kick, I should say.

W. D. A. Peaslee: I have tried that—I will admit that none of the manufacturers agreed with me, but I have become used to that, and it does not bother me so much anymore. I believe most of it, some sort of a charge, due to the friction of the paper roll in the recording instrument, giving you a static disturbance of some kind, because we have tried all sorts of arrangements, and resistances now manufactured, and have not been able to prevent a snappy kick when the button is pressed in. We have tried one thing which will be put in sometime during this week, in which we disconnect completely the recording instrument, and allow it to discharge itself, and after that is done connect the thermo-couple to the indicating instrument.

E. M. Hewlett: What I meant was, instead of letting the man pick out the particular instrument, you could contrive something that would not let him do that, something else that could go on at the same time.

W. D. A. Peaslee: That is what I was aiming to do, but they have me outguessed all the time.

Clayton H. Sharp: I have examined with a great deal of interest the diagrams which are given showing how an ordinary milli-ammeter may be converted into one of the compensated type. There are many times when one of these plans of connection might be used to advantage, and as I look this paper over, the Fig. 7 seems to offer a scheme whereby a very simple and ordinary milliammeter can be arranged so that it will work on this compensated principle. Of course, one cannot go ahead and convert any instruments if there are legal restrictions on the use of this plan, but inasmuch as the development has come from the National Bureau of Standards, I take it that that condition is fulfilled. I think, however, we ought to be warned if that is not true.

H. B. Brooks: In regard to Dr. Whitehead's question as to the way the chief advantage is reached. Of course, at the start one recognizes that network matters are not always evident by mere inspection—it is necessary to trust to Kirchhoff's laws, and rely on your mathematics, to a certain extent, but so far as I can I will illustrate the matter.

Suppose we have a source having zero internal resistance, for example, a storage cell, connected to a voltmeter which consists of a moving coil of relatively low resistance in series with a large resistance. Suppose further that we connect one terminal of a shunt resistance to the terminal of the storage cell which is connected directly to the moving coil of the instrument. Now if we connect the other end of the shunt resistance to the other pole of the storage cell, the voltmeter will show no change in reading. This is because our shunt resistance has gone on in such a way as to simply load the source a little more. If we connect the same shunt resistance in parallel with the moving coil, we have not appreciably changed the current through the series resistance but part goes through the shunt and the deflection is decreased.

Instead of connecting the end of the shunt circuit to the junction of the series resistance and the moving coil, we may connect it at some intermediate point on the series resistance. We will find it necessary to cut out some resistance from the unshunted part of the series resistor to maintain the deflection at the original value. When the proper relations exist for the instrument to measure true e. m. f., the additional current drawn from the source is so divided that the current which flows through the moving coil is exactly equal to the current before we applied the shunt. It is not evident what the proportions must be, but it is necessary to depend on the mathematics of the case.

J. B. Whitehead: Is more load taken from the source?

H. B. Brooks: It is; more current is supplied by the thermocouple when the key is depressed. In spite of this, the instrument measures the true e. m. f. of the couple.

J. B. Whitehead: It is suggested in the second sentence of the abstract that that is so.

H. B. Brooks: That is the way we get a magnification of the rheostat adjustment error. We have an adjustable rheostat which compensates for the resistance in the source. The instrument may be so designed that if the reading changes by one-tenth of a division upon closing the key, the error due to incorrect adjustment of the rheostat with the key open will be only from one-fifth to one-tenth of this amount.

Referring to the question regarding legal matters, the authors requested that any question of that sort that might be asked should be written into the minutes of the convention, and that the question be transmitted to them in order that they might reply in their own words.

Presented at the 8th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 20, 1920.

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NOTES ON THE SYNCHRONOUS COMMUTATOR

BY J. B. WHITEHEAD

Professor of Electrical Engineering, Johns Hopkins University

AND

T. ISSHIKI

Engineer, Shibaura Engineering Works

In the use of the synchronous commutator in series connection as a suppressor, serious errors may arise due to relatively small amounts of capacity in the commutator and galvanometer circuits. The magnitude of these errors is studied for a number of different connections, and methods for eliminating them are pointed out. A number of wave forms are given, indicating the nature of the errors.

Used as a shunt suppressor the commutator is far more reliable and this method of connection is always to be preferred.

An appendix gives a theoretical analysis of two cases investigated and shows a close agreement with the experimental observations.

In certain methods of measurement of the crest values of high alternating voltages it is necessary to measure accurately the average values of very small alternating currents. One of the commonest of these methods is that in which a condenser, in series with a low resistance, is placed across the voltage and the voltage drop over the resistance, due to the charging current, measured on an instrument of D'Arsonval type with high series resistance, through the use of either a synchronous commutator or rectifying vacuum tubes.

In these methods, if the commutator is used the instrument receives either a rectified alternating current, made up of both half waves, or a uni-directional pulsating current, made up of alternate half waves with the intermediate opposite half waves cut out either by opening the circuit (series commutator), or by short-circuiting the instrument (shunt commutator). When the commutator is used so as to eliminate alternate half waves, F. Bedell[1] has called it a *suppressor*. If the rectifying vacuum tube is used the current in the measuring instrument is always pulsating with alternate half waves cut out. In any of the above mentioned cases the instrument reads the average current in terms of its calibration by continuous current. For the cases in which the current is pulsating, the instrument reading is multiplied by two to obtain the average value of the condenser charging current, it being assumed that the active half wave of charging current is accurately reproduced in the instrument and that in the intervening half wave interruption no current flows in the instrument.[2]

Measurements of crest value by these methods are probably the best so far obtained, but apparently no special effort has been made to put them on a precision basis. When a sensitive galvanometer is used, the pulsating character of the current raises questions as to the influence of capacitance and reactance in the instrument circuit, and as to the validity of the factor 2 mentioned above. The observations recorded below show that under certain circuit conditions serious errors may arise.

THE APPARATUS

The commutator used in the experiments was six inches in diameter and had six brass segments ½-in. wide, insulated from each other by $^1/_{32}$-in. built-up mica, and from the hub by standard V-shaped commutator bushings. Two brass rings, 6-in. in diameter and ½-in. wide, were mounted one on each side of the central commutator segments and insulated from them by intermediate bakelite rings. Screws holding the outer brass rings in place, passed through the bakelite connecting the outside continuous rings to opposite sets of three alternate commutator segments. The commutator was mounted on the end of the shaft of the 6-pole generator. Four brushes of $^1/_{64}$-in. thick spring bronze, stiffened with backings of steel spring, were carried on a graduated bakelite disk arranged for complete rotation about the axis and clamping in any position. The commutator was machine-constructed throughout. It was frequently tested for insulation and showed throughout a resistance of the order 10^3 megohms.

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[1] F. Bedell, *Journal of Franklin Institute*, No. 176, 1913, p. 385.

[2] W. E. Sumpner, *Philosophical Magazine*, p. 155, January 1905.

The 6-pole generator was single phase and had a capacity of 5 kw. at 60 cycles and 120 volts. It was driven by an adjustable-speed continuous-current shunt motor, permitting a range of frequency from 20 to 90 cycles. The armature was surface wound, giving a smooth wave approximating closely to sine shape.

Two D'Arsonval galvanometers were used, both of suspension type, read by telescope and scale. One, No. 24515, had a resistance of 115 ohms, a sensitivity of 40 megohms, an undamped period of 9.5 seconds, and a critical damping resistance of 560 ohms. The corresponding figures for the other galvanometer, No. 23518, were 1680 ohms, 1280 megohms, 22 seconds, and 3400 ohms. The sensitivities for the two instruments when critically damped therefore were 33 and 420 megohms, respectively.

COMMUTATOR AS SERIES SUPPRESSOR

Obviously the most rigid test of the circuit and instrument for pulsating current is to apply a continuous electromotive force, chopping it up into alternate intervals at full value and zero value by means of the series commutator, as indicated in full lines in Fig. 1. The complete interruption of the current and the sudden application of the full electromotive force accentuate any conditions tending to upset the perfect rectangular half wave alternating with a half period of complete interruption. The test of this performance is the ratio of the readings of the galvanometer with commutator at standstill and when running. This ratio should be 2. This arrangement uses only two brushes and will be spoken of as the series connection of the commutator.

Fig. 1 shows the commutator in series connection; that is, as a simple make and break device in the circuit containing a single dry cell E, resistance $R = 10^6$ ohms and $r = 10^4$ ohms and the galvanometer shunted with its critical damping resistance. Several different types of resistance were used for R; a series of carbon lightning arrester rods aggregating 200,000 ohms, a series of so-called "lavite" units aggregating 10^6 ohms, and a precision set of non-inductive, capacity-free, manganin units aggregating 10^6 ohms, in which the wire was wound in single layers on thin micanite plates, suspended on glass rods.

A number of observations were made at different speeds of the commutator C, and with S open and closed, corresponding to running and standstill conditions for C. In every case it was found that the ratio of galvanometer deflections for running and standstill was greater than 0.5. A typical set of observations is shown in Table I and plotted in Fig. 2 as curve No. 2. The speed, in terms of the frequency of the generator, is plotted as abscissa, and the ratio of running to standstill deflection of the galvanometer as ordinate.

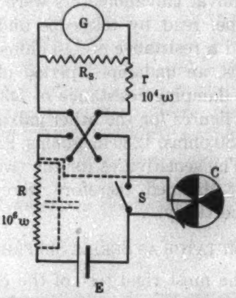

FIG. 1—COMMUTATOR CONNECTED AS SERIES SUPPRESSOR

The increase of the ratio of running to standstill deflections with the frequency obviously suggests the presence of capacity as a disturbing element. This was confirmed by a rearrangement of the apparatus and connections so as to reduce the capacity in circuit. In Fig. 2, curve No. 1, the resistance R was connected into the circuit over a pair of leads consisting of twisted lamp cord about 100-ft. long and having a capacity of about 0.005 microfarads. In curve No. 2, R^1 was a small compact resistance of the same value as that pertaining to curve No. 1, but connected to the circuit with short leads. The remaining curves showing decreasing departure of the ratio of running to standstill values from 0.5, show the successive improvements resulting from shortening and separating the various connections of Fig. 1. It should be noticed, however, that under the very best conditions

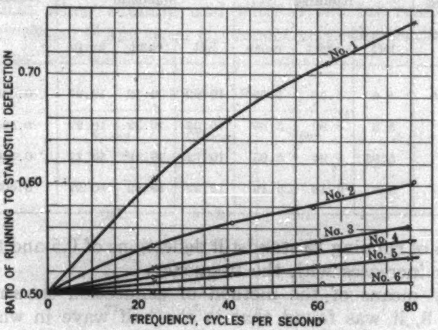

Fig. 2—Ratio of Running to Standstill Deflection for Various Speeds of Commutator. Commutator in Series.

No. 1—Leads for 10^6 ohm Resistance: 100 ft. Twisted Lamp Cord (Capacity about 0.005 microfarads), Leads for Commutator: 15 ft. Twisted Lamp Cord (Capacity about 0.0007 microfarads), Brush Narrower than Insulation between Segments.

No. 2—Leads for 10 ohm Resistance: Short Separated Wires, Leads for Commutator: Same as No. 1. Brush Wider than Insulation between Segments.

No. 3—Same as No. 1 except that Leads for 10^6 ohm Resistance Were Separated to Two 100 ft. Wires.

No. 4—Same as No. 2 except that Brush was narrower than Insulation between Segments.

No. 5—Same as No. 3 except that Commutator Leads Were also separated.

No. 6—Same as No. 4 except that Commutator Leads Were Separated of short, well-separated connections, with the arrangement of Fig. 1, there is always an error in assuming a ratio of running to standstill deflections of 0.5 and this error increases with the frequency.

TABLE I

Speed cycles	Deflection of galvanometer						
	Running			Standstill			Running / Standstill
	left	right	mean	left	right	mean	
23.5	5.6	5.59	5.595	10.22	10.32	10.27	0.545
41	5.8	5.80	5.80	10.22	10.32	10.27	0.565
59	5.95	5.95	5.95	10.22	10.32	10.27	0.580
81	6.2	6.18	6.19	10.22	10.32	10.27	0.603

By means of the method of connection shown in Fig. 3, it was found that in the half wave in which the circuit is closed by *C*, the current is greater than the standstill value, and during the half wave in which the circuit is opened by *C* there is still some current in the positive direction. *B* is a second com-

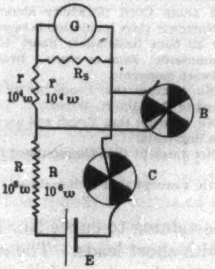

FIG. 3—SERIES AND SHUNT SUPPRESSORS IN THE SAME CIRCUITS

mutator on the same shaft with *C* and is connected in shunt with the galvanometer, as indicated. By proper setting of the brushes of *B* it is a simple matter to short-circuit the galvanometer for the periods of opening or closing by *C*, and thus read the current for the opposite half wave. For example, at standstill, with *B* open and *C* closed, the galvanometer reading was 9.9; when running and with *B* set for reading during closed *C* and open *C*, respectively, the readings were as follows:

TABLE II

Speed cycles	Closed *C*	Open *C*
23	5.78	1.1
41	6.15	1.38
60	6.5	1.7
77	6.7	....
80	....	1.9

Since in each case the galvanometer is receiving current only one-half the time, the readings must be multiplied by 2 to obtain the full values. They show therefore that during the half wave of closed *C* the current is greater than 9.9, and during the half wave of open *C* the current is greater than 0. Moreover the excess of current value increases with the frequency.

A further study of the influence of capacity with the connection of Fig. 1 was made by inserting additional capacity in shunt to the resistance *R*, between the commutator leads, and between commutator segments. In the upper part of Fig. 4 curves are plotted showing the variation of the ratio of running to standstill deflection with the frequency for series commutator and for different values of capacity connected in shunt to the resistance *R*. The lower half of Fig. 4 gives curves taken with the commutator in shunt connection. These curves will be referred to below. Similar curves are obtained when the capacity is connected between the leads to the commutator.

In Fig. 5 the two lower curves were taken at 60 cycles with various values of capacity connected between the opposite sets of segments of the commutator.

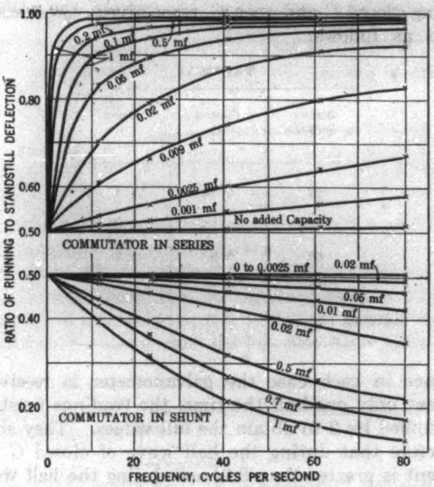

Fig. 4—Influence of Capacity in Shunt to 10^6 Ohm Resistance upon Ratio of Running to Standstill Deflection. Brush Narrower than Insulation between Segments

This connection is made by connecting the capacity in parallel with the two outer collector rings of the commutator. Curve No. 2 is taken with a brush which is thicker than the insulation between segments and curve No. 3 is taken with a brush which is thinner than the insulation between segments. It is noteworthy that the use of the thick brush, short-circuiting as it does the capacity shunted around the commutator and thus discharging the capacity, is the cause of a wide error in the value of the ratio of running to standstill deflections. Curve No. 1 shows that the effect of adding capacity between commutator leads is similar to that of adding capacity between the terminals of *R*.

In view of the foregoing results, it appeared of interest to study the wave form of the current in the galvanometer. The current values being too small for the oscillograph, the method shown in Fig. 6 was adopted. *C* is the rectifying commutator already described arranged for series connection, that is, using only two brushes and operating as a half cycle make and break. Switching is also provided so that *C* may be arranged in shunt connection or cut out entirely. *D* is a metal disk also mounted on the generator shaft and equipped with three narrow insulating sectors set in its outer surface. Two brushes on *D* short-circuit the galvanometer, as indicated, during a complete cycle except for the brief interval when the short circuit is interrupted by one of the insulating segments, that is, once in each cycle of make and break. The particular instant in the cycle at which the galvanometer receives current is varied by rota-

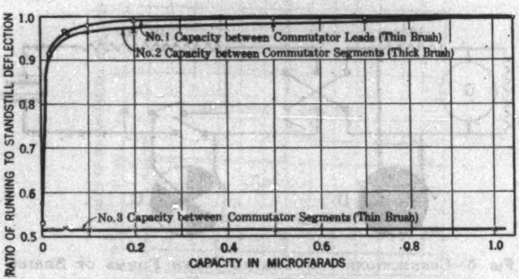

Fig. 5—Influence of Capacity between Commutator Leads and between Commutator Segments upon Ratio of Running to Standstill Deflection. Commutator in Series. 60 Cycles

ting the brushes on *C*. The arrangement constitutes a point by point method for studying the current in the circuit of the commutator *C*.

Observations were taken with *C* both in series and in shunt. A typical set is given in Table 3 and plotted in Fig. 7. It was found that with *D* operating and *C* cut out completely, that is, with steady current in the battery circuit, the deflection of the galvanometer had a slow and irregular variation and usually different values for opposite positions of the reversing switch. These variations were due to thermal electromotive forces or other variable contact conditions at the brushes of *D*. In order to eliminate this trouble right and left readings of the steady current (*C* not in circuit) and of the instantaneous current (*C* in circuit) were taken in quick succession and their mean values used. The wave form is obtained by taking for each brush setting the ratio of the mean of the instantaneous readings to the mean of the steady readings.

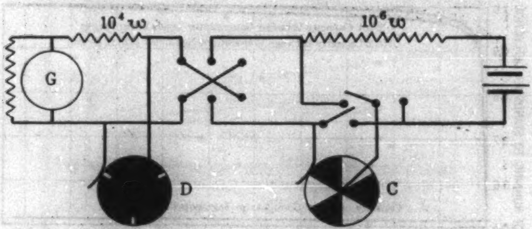

FIG. 6—CONNECTIONS FOR TAKING WAVE FORMS OF SERIES AND SHUNT SUPPRESSORS

The more interesting results are plotted in the curves of Figs. 7, 8 and 9, showing several forms for both series and shunt connections of the commutator. In the case of Fig. 7 the frequency was 60 cycles and the 1-megohm resistance *R* and the commutator *C* had leads consisting of twisted lamp cord 100-ft. and 15-ft. long (capacities about 0.005 and 0.0007 microfarads) respectively, the brush was thinner than the width of the insulating segments in the commutator *C*. In Fig. 8 the conditions were the same at a frequency 24 cycles, and in the lower curve a brush thicker than the insulating commutator segments was used. In Fig. 9 short separated leads of minimum capacity were used to *R*, and the 15-ft. twisted leads to *C*.

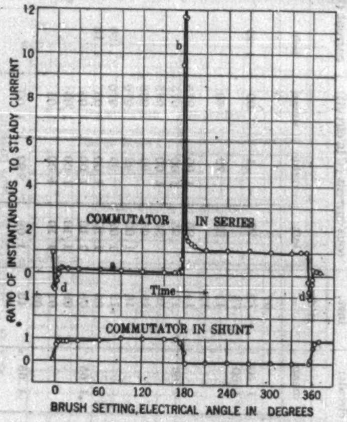

Fig. 7—Wave Form of Current Through Galvanometer 60 Cycles. Brush Narrower than Insulation between Segments. Leads for 10^6 Ohm Resistance: 100 ft. Twisted Lamp Cord (Capacity about 0.005 Microfarads. Leads for Commutator: 15 ft. Twisted Lamp Cord (Capacity about 0.0007 Microfarads)

As already indicated, the results shown in the foregoing figures are due to the capacity located in different parts of the connections. For example, in Fig. 7, during open circuit by the commutator, the capacity of the commutator itself charges as indicated at (*a*) and *C′* the capacity around the resistance *R* discharges through the resistance. At (*b*) on the instant of closing, the capacity *C′* charges, giving an excess current through the galvanometer which immediately tends to fall to the value fixed by the resistance *R*.

TABLE III

60 cycles, brush narrower than insulation between commutator segments, leads for 10^8 ohms resistance, twisted lamp cord 100 ft. long, (capacity about 0.005 microfarads), leads for commutator—twisted lamp cord 15 ft. long, (capacity about 0.0007 microfarads)

Electrical Angle degree	Commtutator in Series							Commutator in Shunt						
	Deflection for instantaneous current (*C* in circuit)			Deflection for steady current (*C* not in circuit)			Ratio: inst.	Deflection for instantaneous current (*C* in circuit)			Deflection for steady current (*C* not in circuit)			Ratio: inst.
	left	right	mean	left	right	mean	st'dy	left	right	mean	left	right	mean	steady
-6	1.7	1	1.35				.97							
-3	-.6	-1.4	-1	1.80	1.00	1.40	-.72	.28	-.1	.09	1.22	.98	1.10	.08
-1.5	-.62	-1.88	-1.25	2.08	1.12	1.60	-.78							
0	.20	-.15	-.65				-.41	.88	.72	.80				.73
1.5	.5	-.88	-.19				-.12							
3	.9	-.47	.215				.14	.95	1.00	.975				.89
6	.85	-.32	.265	2.08	1.12	1.60	.17	.95	1.00	.975				.89
9	.92	-.30	.31				.19	.95	1.00	.975				.89
15	.63	-.02	.305				.19	.95	.92	.935				.85
30	.60	-.08	.26				.16	1.02	.92	.970				.89
60	.57	-.17	.2				.12	1.02	.92	.970	1.20	.99	1.095	.89
90	.48	-.22	.13				.08	1.10	1.00	1.05	1.12	.92	1.02	1.03
120	.57	-.32	.125	2.18	1.12	1.65	.08	1.10	1.00	1.05				1.03
150	.7	-.42	.14				.09	1.10	1.00	1.05				1.03
165	.7	-.47	.115				.07	1.10	1.00	1.05				1.03
171	.72	-.47	.125				.08							
174	.78	-.45	.165	2.18	1.12	1.65	.10	1.08	.92	1.00				.98
175.5	.97	1.09	1.03				.62							
177	16.2	15.6	15.9	2.22	1.12	1.67	9.52	.62	.21	.42	1.12	.92	1.02	.41
178.5	19.0	18.6	18.8				11.70							
180	18.6	17.5	18.05				11.63	.18	-.20	-.01				.01
181.5	2.9	2.0	2.45				1.63							
183	2.68	1.82	2.25				1.52							
186	2.42	1.55	1.99	1.78	1.05	1.415	1.40							
195	1.42	2.2	1.81				1.25							

At (*d*) if the brush is narrower than the insulating segment of the commutator, the counter-electromotive forces of the capacities C and C' are in series and therefore greater than the battery electromotive force E; hence, there is momentarily a reverse current after the circuit opens. Then as C' discharges through R the electromotive force falls and the battery charges the capacity of the commutator, resulting in a positive charging current for the interval (a). If the brush on C is wider than the insulating segment, the capacity of C is discharged through the brush and during the following interval of open circuit the charging current of this capacity is higher. (See Fig. 8, lower curve).

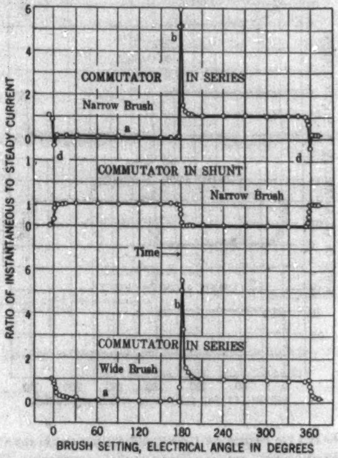

FIG. 8—WAVE FORM OF CURRENT THROUGH GALVANOMETER. 24 CYCLES. ALL LEADS SAME AS FIG. 7

When the leads to the commutator contain capacity with either a wide or narrow brush, the capacity discharges through the commutator and not through the galvanometer at each interval of make, and therefore this capacity charges through the galvanometer during each interval of break; hence there is excess current through the galvanometer at each interval of break, resulting in an increase of the ratio of running to standstill conditions. (See curves of Figs. 5, 7, 8, 9).

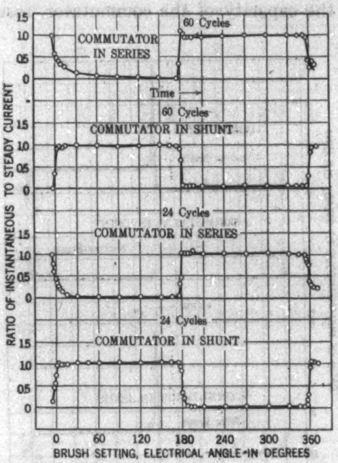

FIG. 9—WAVE FORM OF CURRENT THROUGH GALVANOMETER, BRUSH NARROWER THAN INSULATION BETWEEN SEGMENTS. LEADS FOR 10^6 OHM RESISTANCE: SHORT SEPARATED WIRES. LEADS FOR COMMUTATOR: 15 FT. TWISTED LAMP CORD (CAPACITY ABOUT 0.0007 MICROFARADS)

The effect of capacity between segments in the commutator itself is as follows: This capacity charges during open circuit. If a thick commutator brush is used, the capacity discharges through the brush at the instant the brush bridges the insulating commutator segment, and hence the capacity charges in each interval of open circuit, giving excess galvanometer current and increased ratio of running to standstill current. If, on the other hand, the commutator brush is narrower than the insulating segment, the capacity retains its charge except for loss due to leakage and hence there is no recharge and no excess galvanometer current after the first charging interval, consequently the ratio of running to standstill current in the galvanometer is independent of the value of the capacity and approximately equal to 0.5. (See curves of Fig. 5).

The apparent errors of the series connected commutator may be reduced to a minimum or avoided entirely in two ways: First, by any method of connection whereby the current in the resistance *R* is not interrupted. Two methods of accomplishing this are shown in Figs. 10 and 11. In Fig. 10, by using three brushes on the commutator during the period in which the galvanometer current is interrupted, the current in *R* is maintained at constant value by means of the resistance *R′* of value equal to that in the galvanometer circuit. A number of accurate observations with this method of connection at various values of frequency and capacity were made, and the ratio of running to standstill deflection of the galvanometer was very accurately 0.5 throughout, except for the case of large capacity connected between the points (*b*) and (*c*). In Fig. 11 the current in *R* is also constant and the galvanometer may be shunted around either of the resistances *r*. In a series of experiments with this connection, 120-volts continuous potential was used, and capacity up to 1 microfarad was shunted

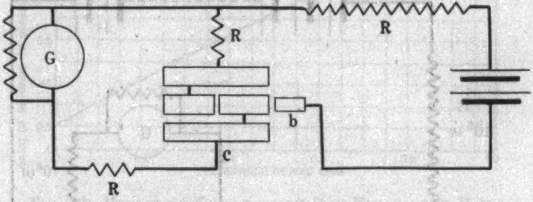

FIG. 10—CONNECTIONS OF SERIES SUPPRESSOR FOR ELIMINATING ERRORS DUE TO CAPACITY

around both resistances *R* and *r*, in order to intensify any disturbances due to charging currents. Both at 24 cycles and 60 cycles the ratio of running to standstill deflection was accurately equal to 0.5 throughout.

A second method for reducing the error in the simple connection of Fig. 1 is a reduction of the value of the resistance *R*. Using the twisted lamp cord leads mentioned above, to *R* and *C*, Fig. 12 shows a series of observations at 60 cycles on the value of the ratio of running to standstill deflections in relation to the value of *R*. As indicated, when *R* is equal to 1 megohm the ratio is 0.72 and the current in circuit is 3.1×10^{-6} amperes. For $R = 5 \times 10^{-4}$ ohms the ratio is 0.51 and the current 6.2×10^{-5} amperes. For $R = 10^4$ and 10^3 ohms, the ratios were found to be 0.501 and 0.4995 respectively. In the last case the current in circuit was approximately 3.1 milliamperes. (See Fig. 12).

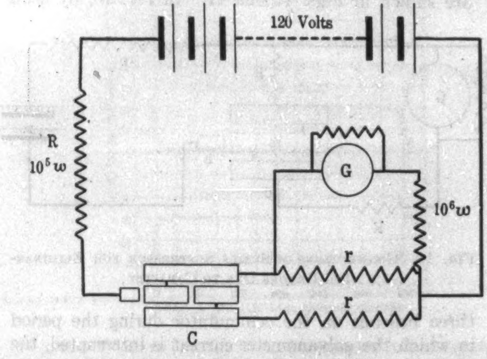

FIG. 11—CONNECTIONS OF SERIES SUPPRESSOR FOR ELIMINATING ERRORS DUE TO CAPACITY

Analyses of the series commutator connection for capacity *C*, shunted around the resistance *R* and across the commutator leads, are given in an appendix. It is shown that the value of the ratio of instantaneous to steady current in the galvanometer for any particular position of the brush setting, as well as the ratio of running to standstill deflections of the galvanometer, are functions only of the product $f\,C\,R$ and of α where f is the frequency and α is the ratio of R to r (see Fig. 1). That is, reductions in the values of frequency, capacity or of the resistance R, all result in a closer approach of the ratio of running to standstill deflections to the value 0.5. Curves are also given showing a close agreement between the observations and the theory.

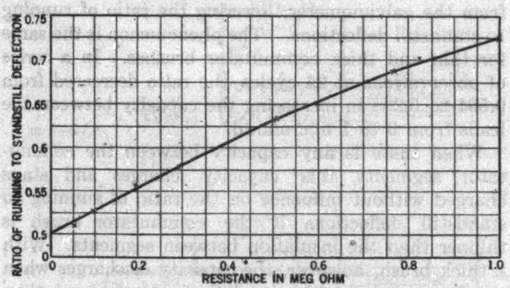

Fig. 12—Effect of Variation of R in Fig. 1 upon Ratio of Running to Standstill Deflections. Commutator in Series. 60 Cycles. Thin Brush

COMMUTATOR CONNECTED AS SHUNT SUPPRESSOR

With the commutator connected in shunt, as indicated by the dotted line in Fig. 1, the errors due to capacity are not so serious as in the series connection. In the shunt connection, since the current through R varies only by the alternate cutting in and out of the relatively small resistance of the galvanometer circuit, the variation of electromotive force at its terminals is correspondingly small. Throughout a number of observations with this method of connection the ratio of running to standstill deflections was very accurately 0.5 even when the ratio R/r was not greater than 10 (values at 23, 40, 60 and 80 cycles all being 0.501). Although the current pulsation in R in this case is relatively great, the low value of R tends to reduce the charging current of any small capacity which may exist. For values of the ratio R/r greater than 100, the current pulsation is small and if the circuit has no capacity, the error is small (values of ratio R/r at the above frequencies being 0.502, 0.501, 0.501 and 0.500).

Capacity, however, soon introduces trouble. When located in the leads to the commutator it charges on open commutator and discharges on a closed commutator. Thus during the open period current is shunted from the galvanometer, lowering the ratio of running to standstill deflections. The phenomenon is the same for thin and thick commutator brushes. In a series of observations at 24 cycles, the ratio decreased from 0.504 to 0.298 in increasing the capacity between the leads from 0 to 1 microfarad.

When there is any capacity between the commutator segments, this capacity charges and stays charged without influence on the ratio of running to standstill deflections if the commutator brush is thinner than the insulation between segments. With a thick brush, however, the capacity discharges when the brush bridges the insulating segment and then charges in the position of open commutator, the performance then being as in the case of the foregoing paragraph. The capacity between commutator segments, as measured, was found to be 0.00126 microfarad. In a series of observations the addition of 0.02 microfarad between commutator segments changes the ratio of running to standstill values from 0.498 to 0.488 with still further lowering for increase of capacity.

When there is capacity in the leads to R on closed commutator, this capacity receives the full battery e.m.f. On open commutator at the instant of opening, the counter e.m.f. of the capacity is equal to that of the battery and no current flows in the galvanometer G. As the capacity discharges through R there is a delay in the rise of current to its full value, thus causing a lowering of the ratio of running to standstill deflections, and in greater amount the greater the capacity shunting *R*. (See Fig. 7, and curves in the lower half of Fig. 4).

PERFORMANCE ON ALTERNATING CURRENT

The errors arising from the presence of capacity in using the commutator with alternating current are smaller than those that have been described. This is to be expected since on commutation the alternating current tends to rise gradually from the zero value and not abruptly as in the case of the pulsating continuous current. In a number of observations taken with alternating current introduced by means of a transformer located in place of the battery of Fig. 1 and with the commutator connected in both series and shunt for reading alternate half waves and also for the reading of full rectification by means of the addition of two more brushes to the commutator in the usual manner, the ratios were found to be very accurately 0.5 under *properly selected* conditions. However, even with careful elimination of capacity

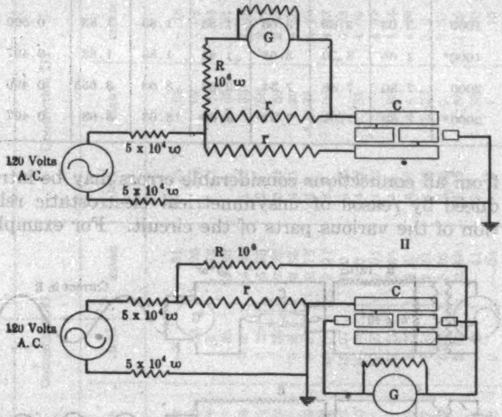

FIG. 13—SERIES COMMUTATOR. ELIMINATION OF ELECTROSTATIC UNBALANCING

TABLE IV

r ohm	Deflection of Galvanometer						
	Full Wave (Fig. 13, I)			Half Wave (Fig. 13, II)			Half wave / Full wave
	left	right	mean	left	right	mean	
1000	3.63	3.69	3.66	1.83	1.83	1.83	0.500
1000*	3.66	3.70	3.68	1.83	1.83	1.83	0.497
2000	7.30	7.38	7.34	3.63	3.68	3.655	0.490
2000*	7.30	7.32	7.31	3.63	3.63	3.63	0.497

from all connections considerable errors may be introduced by reason of unsymmetrical electrostatic relation of the various parts of the circuit. For example,

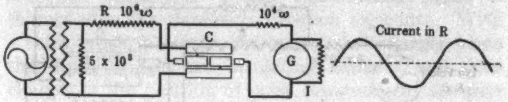

FIG. 14—SERIES AND SHUNT COMMUTATORS. COMPARISON OF VARIOUS TYPES OF PULSATING CURRENT IN SERIES RESISTANCE

TABLE V

Electrical angle degree	Deflection of Galvanometer											
	Connection I			Connection II			Connection III			Connection IV		
	left	right	mean	left	right	mean	left	right	mean	left	right	mean
−3	−.2	−.1	−.15	−.32	−3.	−.31	−.25	−.25	−.25	−.1	−.02	−.06
0	.32	.5	.41	.22	.28	.25	.02	.02	.02	.22	.28	.25
3	.88	1.1	.99	.78	.8	.79	.31	.31	.31	.48	.52	.5
9	1.99	2.1	2.05	1.8	1.85	1.83	.8	.82	.81	.98	1.08	1.03
15	3.0	3.12	3.06	2.88	2.9	2.89	1.32	1.38	1.35	1.5	1.6	1.55
30	5.37	5.5	5.44	5.21	5.28	5.25	2.5	2.58	2.54	2.7	2.78	2.74
45	7.37	7.52	7.45	7.22	7.32	7.27	3.51	3.59	3.55	3.67	3.78	3.73
60	8.8	9.1	8.95	8.8	8.9	8.35	4.31	4.39	4.35	4.4	4.5	4.45
75	9.7	9.98	9.84	9.69	9.81	9.75	4.8	4.88	4.84	4.88	5.00	4.89
81	9.9	10.08	9.99	9.82	10.0	9.91	4.9	4.95	4.93	4.92	5.1	5.01
87	9.98	10.1	10.04	9.92	10.07	10.0	4.95	5	4.98	4.95	5.1	5.03
90	9.98	10.1	10.04	9.95	10.05	10.0	4.97	5.01	4.99	4.95	5.1	5.03
93	9.95	10.07	10.01	9.9	10.07	10.04	4.93	5.	4.97	4.91	5.1	5.01
105	9.6	9.75	9.68	9.6	9.72	9.66	4.8	4.88	4.84	4.75	4.9	4.83
120	8.62	8.68	8.65	8.62	8.72	8.67	4.32	4.4	4.36	4.2	4.4	4.3
135	7.02	7.02	7.02	7.07	7.15	7.11	3.58	3.62	3.6	3.4	3.58	3.49
150	4.88	4.85	4.87	4.98	5.05	5.02	2.53	2.6	2.57	2.32	2.5	2.41
165	2.38	2.38	2.38	2.5	2.62	2.56	1.32	1.38	1.35	1.22	1.1	1.16
177	.28	.2	2.4	.38	.43	.41	2.7	.32	.30	.08	.18	.13
180	−.2	−.2	−.2	−.08	0	−.04	.09	.1	.10	−.2	−.09	−.15
183	−.8	−.97	−.89	−.7	−.68	−.69	−.3	−.2	−.25	−.5	−.4	−.45

introducing alternating e.m.f. in place of the battery in Fig. 11 for obtaining alternate half waves in the galvanometer, and then by a simple change of connections sending the complete rectified alternating current through *G*, the ratio of half wave to full wave deflections was found to be 0.511. When, however, the circuit was rearranged, as shown in Fig. 13, so as to divide the resistance *R* between the two halves of the circuit, the results of Table IV are obtained, showing that the ratio for half wave to full wave deflection of the galvanometer is very close to 0.5. In these circumstances capacity (up to one microfarad) may be shunted to *r* or *R* without affecting the ratio 0.5 of half wave to full wave. The position of the brushes was usually set for maximum deflection, although the method of setting for zero deflection and then shifting 90 electrical degrees was sometimes used.

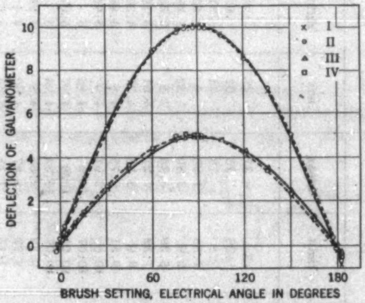

Fig. 15—Deflection of Galvanometer for Various Positions of Brushes, Connection Fig. 14. Capacity Eliminated as Far as Possible

For further study of the influence of capacity, the connections shown in Fig. 14 were used. Connection *I* gives complete rectification and normal alternating current through the resistance *R*. Connection *II* gives complete rectification with pulsating current through *R*. Connection *III* is the simple series connection for both *R* and the galvanometer and therefore has pulsating current in *R* with alternate half waves eliminated. Connection *IV* is ordinary shunt connection with normal alternating current in *R*. In all cases current is introduced into the circuit through a step-down transformer giving very low voltage in the galvanometer circuit.

Table V gives the readings taken with each of the methods of connections shown in Fig. 14, for various

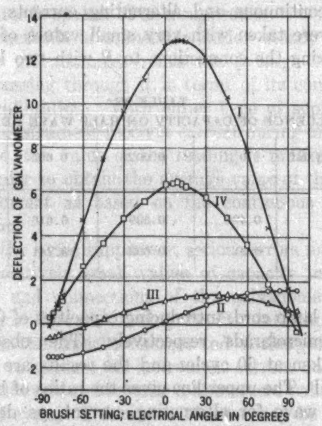

Fig. 16—Deflection of Galvanometer for Various Positions of Brushes, Connection Fig. 14. Resistance *R* Shunted by 0.02 Microfarad Capacity

positions of the commutator brushes. The results are plotted in Fig. 15. It is of interest to note that in this case in which capacity has been eliminated as far as possible, the ratio of the half wave to the full wave deflection is very closely equal to 0.5 for all positions of brushes, independent of the method of connection.

Fig. 16 shows the effect of shunting the resistance *R* with 0.02 microfarad. In this case the ratio of the maximum value for connection *IV* to the maximum value for connection *I* is 0.498, thus indicating the reliability of the shunt method of connection. On the other hand, the ratio of the maximum value for connection *III* to the maximum value for connection *I* is only 0.105, showing the unreliability of the series commutator connection when the circuit contains any capacity.

In order to compare the disturbance introduced in the series connection by the presence of capacity when using continuous and alternating currents, observations were taken with very small values of capacity by making the connections to *R* with two lengths of twisted lamp cord, introducing capacities of 0.002 and 0.006 microfarads respectively. The observations were taken at 60 cycles and the results are given in Table VI .The upper line gives the ratios of half wave to full wave for alternating current, as determined by connections *III* and *II*, respectively, and the lower line the ratio of running to standstill deflection using continuous current.

TABLE VI

INFLUENCE OF CAPACITY ON HALF WAVE READING

Capacity m.f.	0	0.002	0.006	0.02
A. C.	0.496	0.509	0.616	0.849
D. C.	0.51	0.602	0.741	0.907

It will be noticed, while that generally in the same direction, the disturbing effect of the capacity is less for alternating than for continuous current.

DISCUSSION

Small alternating currents may be measured by means of a synchronous commutator with a high sensitivity D'Arsonval galvanometer. The combination may be placed directly in the circuit, or perferably may be used in conjunction with high non-inductive series resistance to measure the voltage drop over a relatively low non-inductive resistance carrying the current to be measured.

The commutator may be connected in series with the galvanometer so as to open and close its circuit during alternating half waves, in shunt so as to short-circuit the galvanometer during alternate half waves, or it may be connected so as to completely rectify the alternating wave, thus making use of both half waves. In the two former methods of connection the commutator is spoken of as a suppressor.

In all methods of connection under steady conditions the galvanometer reads the average value of current passing through it in terms of its continuous current calibration. With either type of suppressor, since the instrument receives current during only one-half cycle, the value of the reading is multiplied by two in order to obtain the average value of the alternating current as based on the continuous current calibration.

Using the series suppressor, serious errors may arise due to relatively small values of capacity in the resistances and connections of the galvanometer and commutator circuit. Owing to the interruption of the circuit the capacity charges and discharges in such a way as to increase the instrument current during the closed half cycle and to raise it above zero value in the open half cycle. This source of error may be avoided if during the open half cycle the source of alternating current or electromotive force is kept closed by a second circuit equivalent to that of the galvanometer. This may be done by using the opposite segments of the commutator. (See Fig. 10).

The shunt suppressor is far more reliable and with careful elimination of capacity from the galvanometer circuit the errors may be reduced to negligible values.

If it is desired to use the series suppressor the circuit conditions may be tested by taking the ratio of the deflections due to the pulsating current, consisting of alternating half waves, and that due to the completely rectified alternating current by means of connections similar to those in Fig. 14. This ratio should be 0.5.

This test should be followed by one applying a continuous e.m.f. with the commutator connected as either a series or a shunt suppressor. The test lies in the ratio of the running to standstill deflections of the galvanometer. This ratio should be 0.5. For the simple series suppressor the value will be always higher than 0.5 unless an auxiliary circuit is used. With the shunt suppressor the value 0.5 may be very closely reached.

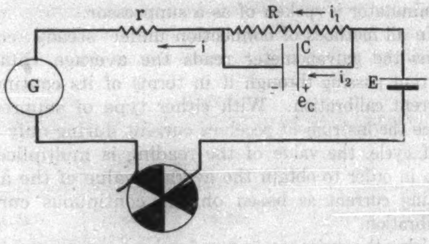

FIG. 17

In using rectifying vacuum tubes in place of a commutator, two tubes and two resistances must be placed in the alternating circuit. The voltage drop over either resistance may be used for measuring the current. In this case the e.m.f. applied to the galvanometer is the pulsating unidirectional e.m.f. due to alternate half waves. Since no commutator is used, the galvanometer circuit with its series resistance is closed at all times and the presence of capacity introduces no error.

Following is a summary of the conclusions from the tests:

(1) In the use of the synchronous commutator as a series suppressor, serious errors may arise due to relatively small amounts of capacity in the commutator and galvanometer circuits.

(2) Used as a shunt suppressor the commutator is far more reliable and this method of connection is always to be preferred.

(3) The galvanometer should be calibrated with continuous current with the commutator both at standstill and running as an ordinary make and break. If the ratio of these two readings is 2, the circuit conditions will introduce no error with alternating current.

(4) Several sources of error due to the commutator are eliminated by the use of vacuum tube rectifiers.

(5) A number of wave forms of alternating current of very low value are given.

APPENDIX

The following mathematical analysis of the case of the series suppressor as described in the paper gives results in close agreement with those observed.

I. Series suppressor in which the high resistance R is shunted by capacity C and continuous electromotive force E applied. No other capacity in circuit.

Referring to Fig. 17, let i, i_1 and i_2 be the instantaneous currents through galvanometer, R, and C (charging current) respectively, and e_c the instantaneous value of the voltage over C, and let their positive directions be those shown by arrows.

Consider the interval (half cycle) during which the circuit is closed by the commutator. Let us call this interval "closed half cycle". During this half cycle C is charged.

We have

$$i\,r + i_1 R = E \tag{1}$$

$$i_1 R = e_c = 1/C \int i_2\,dt \tag{2}$$

$$i_1 + i_2 = i \tag{3}$$

Eliminating i and i_2 from these equations we obtain

$$\frac{d\,i_1}{dt} + \frac{g}{C}\,i_1 = \frac{E}{C\,R\,r} \tag{4}$$

where $$g = \frac{R + r}{R\,r} = \frac{1}{R} + \frac{1}{r}$$

The solution of this differential equation is

$$i_1 = A\epsilon^{-\frac{g}{C}t} + \frac{E}{R + r} \tag{6}$$

where A is an integration constant to be determined by boundary conditions.

From (2) and (6)

$$e_c = A\,R\,\epsilon^{-\frac{g}{C}t} + E\,\frac{R}{R+r}$$

If we take the instant of the closing of the circuit as $t = 0$, $t = \dfrac{1}{f}$ at the instant of opening, f being the frequency, and the value of e_c at these two instants are respectively

$$|e_c|_{t=0} = A\,R + \frac{E}{R+r} \qquad (7)$$

$$|e_c|_{t=\frac{1}{2f}} = A\,R\,\epsilon^{-\frac{g}{2fC}} + E\,\frac{R}{R+r} \qquad (8)$$

Next consider the interval (half cycle) during which the circuit is open. Let us call this interval "Open half cycle."

During this half cycle the charge of C discharges through R and the equation of the circuit is

$$\frac{d\,e_c}{dt} + \frac{e_c}{C\,R} = 0 \qquad (9)$$

the solution of this equation is

$$e_c = A'\,\epsilon^{-\frac{t}{CR}}$$

where A is an integration constant to be determined by boundary conditions.

If we take the instant of the opening as $t = 0$, the instant of the closing is $t = \dfrac{1}{2f}$ and the values of e_c at two instants are respectively

$$|e_c|_{t=0} = A' \qquad (10)$$

$$|e_c|_{t=\frac{1}{2f}} = A'\,\epsilon^{-\frac{1}{2fCR}} \qquad (11)$$

After the steady state of the circuit is established, the values of $|e_c|_{t=0}$ and $|e_c|_{t=\frac{1}{2f}}$ of the closed half cycle must be equal to the values of $|e_c|_{t=\frac{1}{2f}}$ and $|e_c|_{t=0}$ respectively of the closed half cycle, so that (**10**) and (**11**) must be identical with (**8**) and (**7**) respectively, *i. e.*,

$$A' = A\,R\,\epsilon^{-\frac{g}{2fC}} + E\,\frac{R}{R+r}$$

and
$$A'\,\epsilon^{-\frac{1}{2fCR}} = A\,R + \frac{R}{R+r}R$$

Eliminating A' from these two equations, we have

$$A = -\frac{E}{R+r}\,\frac{1-\epsilon^{-\frac{1}{2fCR}}}{1-\epsilon^{-\frac{g+1/R}{2fC}}}$$

Putting this value of A into (**6**) and replacing g by $1/R + 1/r$ we have for the value of i_1 during the closed half cycle

$$i_1 = \frac{E}{R+r}\left\{1 - \frac{1-\epsilon^{-\frac{1}{2fCR}}}{1-\epsilon^{-\frac{1}{2fC}(2/R+1/r)}}\,\epsilon^{-1/C\,(1/R+1/r)t}\right\} \tag{12}$$

From this equation and (**1**), we have for the closed half cycle

$$i = \frac{E}{R+r}\left[1 + \frac{R}{r}\,\frac{1-\epsilon^{-\frac{1}{2fCR}}}{1-\epsilon^{-\frac{1}{2fC}(2/R+1/r)}}\,\epsilon^{-1/C\,(1/R+1/r)\,t}\right] \tag{13}$$

If I be the value of steady current through galvanometer or the value of i when there is no interruption of circuit, then

$$I = \frac{E}{R+r}$$

Therefore the ratio of i and I or instantaneous current to steady current through galvanometer for any particular time t during the closed half cycle is

$$\frac{i}{I} = 1 + \frac{R}{r}\,\frac{1 - \epsilon^{-\frac{1}{2fCR}}}{1 - \epsilon^{-\frac{1}{2fC}(2/R+1/r)}}\,\epsilon^{-1/C\,(1/R+1/r)\,t}$$

$$= 1 + \alpha\,\frac{1 - \epsilon^{-\frac{1}{2fCR}}}{1 - \epsilon^{-\frac{1}{2fCR}(2+\alpha)}}\,\epsilon^{-\frac{1}{2fCR}(1+\alpha)\,2ft} \qquad \mathbf{(14)}$$

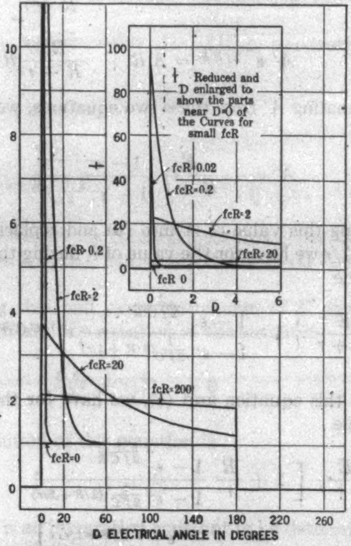

FIG. 18—WAVE FORM OF CURRENT THROUGH GALVANOMETER, *R* SHUNTED BY *C*, COMMUTATOR IN SERIES. CALCULATED BY EQUATION (15) $\alpha = R/r = 100$.

where $\alpha = R/r$

If we denote the time by degrees of the electrical angle *D*,

$$t = \frac{1}{2f}\,\frac{D}{180}$$

Putting this value of t into (**14**) we have

$$\frac{i}{I} = 1 + \alpha \frac{1 - \epsilon^{-\frac{1}{2fCR}}}{1 - \epsilon^{-\frac{1}{2fCR}(2+\alpha)}} \epsilon^{-\frac{1}{2fCR}(1+\alpha)\frac{D}{180}} \tag{15}$$

This equation gives the ratio of instantaneous to steady current through galvanometer for any particular position of brush during closed half cycle. The value of this ratio for the open half cycle is, of course, zero.

This equation shows that i/I for any particular value of D is function only of $f\,C\,R$ and α.

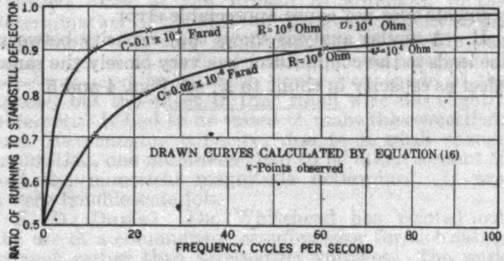

Fig. 19—Curves between Frequency and Ratio of Running to Standstill Deflection. Commutator in Series

In Fig. 18 curves between D and i/I computed from equation (**15**) for $f\,C\,R = 0.02, 0.2, 2, 20$ and 200 farad ohm sec.$^{-1}$ and $\alpha = 100$ are given. Corresponding curves as observed are given in Figs. 7 and 8.

The ratio of mean value of i during one cycle to steady current I, or the ratio of running to standstill deflection, S, is as follows:

$$S = \int_0^{1/2f} \frac{i\,dt}{I\,1/f}$$

$$= f\int_0^{1/2} \left[1 + \alpha \frac{1 - \epsilon^{-\frac{1}{2fCR}}}{1 - \epsilon^{-\frac{1}{2fCR}(2+\alpha)}} \epsilon^{-\frac{1}{2fCR}(1+\alpha)2ft} \right] dt$$

by (**14**)

$$\therefore S = \frac{1}{2}\,\frac{f\,C\,R}{1 + 1/\alpha}\,\frac{\left(1 - \epsilon^{-\frac{1}{2fCR}}\right)\left\{1 - \epsilon^{-\frac{1}{2fCR}(1+\alpha)}\right\}}{1 - \epsilon^{-\frac{1}{2fCR}(2+\alpha)}} \qquad \mathbf{(16)}$$

It is seen from equation (**16**) S is also a function only of $f\,C\,R$ and α.

Fig. 19 shows the close agreement between the observations and theory, the drawn curve shows the values calculated by formula (**16**) and the points marked are those taken from Fig. 4, the results of observation. The values of capacity are 0.02, and 0.2 microfarad which is so large that the capacity between commutator segments does not cause appreciable error.

II. A similar analysis shows that capacity between the leads to the commutator has very closely the same effect as capacity in shunt to R; see Figs. 4 and 5.

DISCUSSION ON "NOTES ON THE SYNCHRONOUS COMMUTATOR" (WHITEHEAD AND ISSHIKI) NEW YORK, N. Y., FEBRUARY 20, 1920.

P. G. Agnew: I have been reminded of an observation I made several years ago in connection with a curve tracer of the Rosa type. The device was quite similar to that of Prof. Whitehead, because it was always used as a mill instrument, and high capacity effects were found to be of importance. Unless the connections are correctly made, there are certain capacity effects induced which shift the whole curve, there is also a very small capacity effect, due to the brushes, as a commutator segment impinges a brush.

Another interesting illustration came up with Dr. Dorsey's work at the Bureau of Standards in the determination of the ratio of electrostatic unity to the importance of a very small capacity effect. He was using a spherical condenser, and of course a very small wire was used to establish connection inside of the sphere, but the effect in that small wire was slightly different. It had to be raised to make the connection; and the changing capacity, due to a small change in position, one millimeter, had to be taken account of and the numerical magnitude determined. It was a very troublesome job.

E. D. Doyle: Dr. Whitehead has pointed out the use of a commutator or suppressor for alternating current rather than alternating voltages. The work we have done at the laboratories has been rather different from this, in that we wanted to commutate low voltages, running down as low as 10 to 50 microvolts. We found that in such cases a commutator will not serve the purpose, in that for the lower values, the commutator seems to develop an infinite resistance. We have had to use butt contacts, rather than sliding contacts a synchronous motor rotating a cam which moves the contacts back and forth thereby effecting the reversals as required.

A. E. Kennelly: In the continuous current circuit, the current becomes steady after a certain time, usually a very short time, and then as a rule no capacitance effect is observed. With an a-c. circuit we all know that capacitance effects reveal themselves in a number of ways and must be taken into account, and if we do not take them into account, we get results which are ridiculous when interpreted, unless we make proper allowances.

This paper calls our attention to the fact that in the d-c. circuit when periodically opened and closed, the capacitance effect may be much more serious than we are apt to realize at first sight. In this case, the difficulties were overcome by reducing the circuit to the continuous-current type.

J. H. Morecroft: I think of one point in respect to Dr. Agnew's paper which is worth while emphasizing and is perceived at once, if one calculates the amount of power necessary to actuate this vibration galvanometer sufficiently to give a reading. If one takes Dr. Agnew's data, he will find that this instrument gives, at the frequency mentioned in the paper, (that is to say, 25 and 60 cycles) a readable deflection with 10^{-12} watt, one millionth of one millionth of a watt. Evidently this gives the experimenter an instrument, which, at the frequency noted, is comparably better than any other type of instrument available.

I have used the vibration galvanometer a great deal in laboratory work, especially at the low frequencies where the telephone was not usable, and have encountered the difficulty Dr. Sharp mentioned—every time a car runs up and down the street outside the laboratory, the galvanometer goes off the scale, and we have to wait until the cars stop running before we can make any determinations. That was with a very sensitive type of vibration galvanometer recently described by Dr. Wenner. I think this galvanometer described by Dr. Agnew is a very wonderful device and I hope soon to have one. There is one point which struck me both in Dr. Whitehead's paper and in his discussion of Dr. Agnew's paper, and that is the use of the vacuum tube in rectifying current in place of a commutator.

I think Dr. Whitehead is aware of the fact—he did not point it out—that an instrument of the type Dr. Agnew tells us about would give a deflection varying with the first power of the impressed voltage, that is, it gives a scheme of sensibility, which does not fall off too rapidly as the voltage goes down. If one tries to use the two electrode vacuum tube in place of the commutator, one will find at once that it has a very bad quality. The amount of rectified current which one can get with the tube varies as the square of the impressed voltage, and it will be appreciated that as the voltage goes down, the rectified part of the current,(which is the only thing the direct-current voltmeter has cognizance of) falls off more rapidly; hence, anyone trying to use a vacuum tube in place of the commutator should have that point in mind, that the sensitiveness of these rectifying tubes falls off very rapidly as the impressed voltage decreases.

This difficulty can be remedied to some extent, by impressing a polarizing voltage on the tube, so that there is a large direct current flowing through the tube all the time, but, of course, one must not have that current flowing through the galvanometer being used as detector, and there must be a balanced circuit put in, otherwise the device is no good, and that makes it somewhat difficult to use the vacuum tube, as a sensitive rectifier. It sounds nice to use a vacuum tube, having no moving contacts—it is an excellent thing to get away from moving contacts—still the vacuum tube does labor under some difficulties.

In our laboratory we noticed also an effect which one of the members, discussing Dr. Whitehead's paper, brought up, that has to do with the use of commutators on very low-voltage circuits. It seems that a contact having very few ohms resistance for ordinary currents may possibly open the circuit for a very low voltage. In the case of sliding metallic contacts in circuits excited perhaps with micro-volts, the contact actually seems to open. Thus, there is a limiting voltage beyond which the contact does show a very high resistance, and is comparatively useless. It will also be found that the rubbing of the two metals (brush and commutator) will itself give more than a micro-volt and hence, unless one is very careful the errors involved due to the rubbing of one metal against another are likely to be greater than the thing being measured.

J. B. Whitehead: I thank Prof. Morecroft for emphasizing the point I meant to bring out when I mentioned the use of the Fleming valve as a rectifier, namely, the difference in sensitivities, the vibration galvanometer having an evident advantage there, for the reasons which Prof. Morecroft has mentioned. There are however, many cases in which extreme sensitivity is not necessary, and in these cases the vacuum tube appears to have many advantages.

In connection with the second point that Prof. Morecroft has raised, namely, the difficulties of the commutator at low values of voltage, we encountered them to some extent, in taking our wave forms, and we have described in one of the paragraphs in connection with Fig. 6, just how the trouble, as we found it, was taken care of by taking rapid reverse readings, with commutator running on closed circuit, and on open circuit. It is necessary to take immediately successive observations in order to correct for conditions at the point of contact of the brush with the commutator.

that there is a large direct current flowing through the lamp at the time, but of course one must [illegible]

[illegible]

[illegible]

[illegible]

Presented at the 8th Midwinter Convention of the American Institute of Electrical Engineers, New York, February 20, 1920.

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OSCILLOGRAPHS AND THEIR TESTS

BY A. E. KENNELLY, R. N. HUNTER AND A. A. PRIOR

ABSTRACT OF PAPER

A method and technique for the testing and calibration of oscillographs is described, using an auxiliary vibrator or "oscillographmeter" for the production of Lissajous optical figures, whereby the resonant frequency f_0 of the tested oscillograph may be readily ascertained. From this and one other test, which is preferably a comparative calibration at 60 ~ and at the resonant frequency, the bluntness of resonance B of the oscillograph is determined.

From the two essential constants f_0 and B of an oscillograph, its indications at any assigned frequency can be corrected for the inertia of its vibratory system. At high frequencies, the correction may be relatively large.

A number of oscillographs have been tested for their f_0 and B. The principal results obtained are reported in the paper.

ALTHOUGH oscillographs have come into very general electrical engineering use, and are invaluable in many laboratory investigations, no methods for testing them seem to have been published. It is important to develop methods for the testing of oscillographs, because all those oscillographs which employ mechanical vibratory systems behave differently to alternating currents of different frequencies. If calibrated at say 60 ~, their calibration at other frequencies, and especially at high harmonic frequencies, will be different. If an oscillographic waveform is analyzed into its Fourier components, the apparent value of each component requires to be corrected, both as to amplitude and as to phase. In some cases, these corrections may be very large; while in others they may be insignificant; but in all cases the oscillographer should know, and be able to determine, the magnitude of the correction to be applied to the terms of his Fourier analysis, if only for the purpose of assuring himself that these corrections are too small to be worth taking into account, for those purposes to which his oscillographic records are to be applied. Strictly speaking, no oscillogram is complete, as a technical record, without an appended index of correction. It has already been pointed out that these corrections are due*; but no technique has been published, so far as the authors are aware, for determining the range of frequency errors in any particular instrument. It is the object of this paper to supply a technique for the testing of oscillographs, as the outcome of laboratory researches conducted to that end for the last two years.

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*Bibliography 7, 12.

Before discussing the technique of oscillograph testing, it is desirable to consider certain preliminary and underlying principles relating to the operation of the mechanically vibrating oscillograph. These principles are by no means new†; but the forms in which they are here presented are believed to be distinctive, and relatively easy for the student to apprehend. They depend upon a close analogy between the mechanics of a simple vibratory system and the electromagnetics of a simple alternating-current system.

Vector Current Impedance of an L. R. S. Branch. Fig. 1 represents a simple branch circuit $A\,B$, between a pair of alternating-current mains $m\,m'$, which are maintained at a constant r.m.s. voltage, by means of voltmeter V. The branch contains a condenser of capacitance C farads, a resistance of R ohms and an inductance of L henrys. Any resistance associated with the inductance is supposed to be displaced into, and segregated with, the resistance R. The values of C, L and R, are assumed to remain fixed and unchanged.

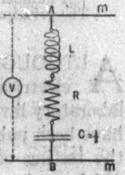

Fig. 1—Single *LRS* Branch Circuit

The a-c. generator connected to and supplying the mains $m\,m'$, is supposed to be able to supply, at will, any desired frequency, from the lowest to the highest, without altering the r.m.s. e.m.f. E volts. This e.m.f., while being varied in frequency from nearly zero to a very high value, remains simply sinusoidal throughout. In other words, the impressed e.m.f. is supposed to be pure, and devoid of harmonics. Then, at any impressed frequency f cycles per second, and impressed angular velocity $\omega = 2\,\pi\,f$ radians per second, the impedance of the branch circuit will be

$$Z = R + j\left(L\,\omega - \frac{1}{C\,\omega}\right) \qquad \text{ohms}\,\angle \qquad (1)$$

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†Bibliography 2, 4, 5, 7a, 8, 12.

or, if $S = 1/C$, is the elastance of the condenser in darafs, this becomes

$$Z = R + j\,(L\,\omega - S/\omega) \qquad \text{ohms} \angle \qquad (2)$$

the graph of this impedance, as is well known, is the straight line $A\,B\,C\,D\,E\,F$, Fig. 2, in the impedance plane, parallel to the Y axis, and distant R ohms therefrom. Commencing with frequency zero, the impedance $j\,L\,\omega$ of the reactor is indefinitely small, but the impedance $-\ j\,S/\omega$ of the condenser is infinite; so that the impedance Z then has the size of infinity, in the direction of $B\ A$ produced, and with a slope α of $-\ 90$ deg. As the impressed angular velocity is increased, both the size and the slope of Z diminish; until, at a certain frequency, which may be denoted by ω_1, the size of the reactance $(L\,\omega - S/\omega)$ is negatively just equal to the size of the resistance R. The impedance will then be $O\,B$ Fig. 2, or

$$Z_1 = R - j\,R = R\,\sqrt{2}\ \overline{\angle}\ 45 \qquad \text{ohms} \angle \qquad (3)$$

the slope α being then $-\ 45$.

Increasing the angular velocity up to the resonant value, which may be denoted by ω_0, the reactance $L\,\omega - S/\omega = 0$, and the impedance becomes the real value $O\,D = R$ ohms, of slope 0 deg.

Increasing the angular velocity up to a certain value ω_2, when the reactance has a size that is positive and just equal to R, the impedance reaches the vector value $O\,E$, and is:

$$Z_2 = R + j\,R = R\,\sqrt{2} \angle 45. \qquad \text{ohms} \angle \qquad (4)$$

The values of angular velocity ω_1 and ω_2, may be called the lower and upper *quadrantal angular velocities*, respectively. Similarly, Z_1 and Z_2, corresponding thereto, are the *quadrantal impedances*.

Finally, if the impressed angular velocity is increased towards infinity, the reactance of the condenser C becomes indefinitely small; but that of the inductance L indefinitely great; so that the impedance of the $C\,L\,R$ branch approaches infinity in size, and $+\,90$ deg. in slope, and lies in the vector direction from O, towards $E\,F$ indefinitely produced.

The impedance diagram of Fig. 2 has been prepared for the case in which $R = 100$ ohms, $C = 0.5 \times 10^{-6}$ farad and $L = 5 \times 10^{-3}$ henry. Each unit on the scale of the diagram then represents 100 ohms. The resonant angular velocity ω_0 is thus 20,000 radians per second, corresponding to a frequency of $f_0 = 3183 \sim$. If we denote by u, the ratio of the impressed to resonant frequency, or

$$u = \omega/\omega_0 = f/f_0 \qquad \text{numeric} \qquad (5)$$

the small circles along the impedance locus, Fig. 2, are marked with their corresponding values of u, for this particular case, from $u = 0.2$ or $\omega = 4000$, to $u = 5$ or $\omega = 100{,}000$.

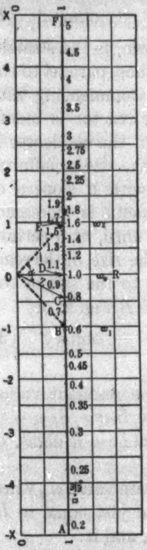

FIG. 2—RECTILINEAR GRAPH OF IMPEDANCE TO CURRENT FOR CASES OF

$$\begin{Bmatrix} R = 10^2, & L = 5 \times 10^{-3}, & S = 2 \times 10^6 \\ r = 10^{-2}, & m = 5 \times 10^{-7}, & s = 2 \times 10^2 \end{Bmatrix}$$

$\omega_0 = 20{,}000$ $\Delta = 10{,}000$ $B = 0.5$

IN THE ELECTRIC CASE, UNIT LENGTH = 10^2 OHMS.

IN THE MECHANIC CASE, UNIT LENGTH = 10^{-2} MECHANIC ABSOHMS, OR DYNES-PERP. CM. PER RADIAN PER SEC.

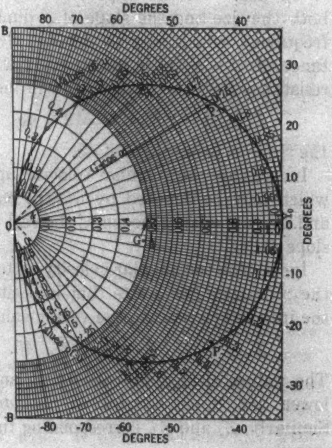

FIG. 3—CIRCULAR GRAPH OF ADMITTANCE TO CURRENT OR TO VELOCITY FOR CASES OF

$$\begin{Bmatrix} R = 10^2, & L = 5 \times 10^{-3}, & S = 2 \times 10^6 \\ r = 10^{-2}, & m = 5 \times 10^{-7}, & s = 2 \times 10^2 \end{Bmatrix}$$

$\omega_0 = 20{,}000$ $\Delta = 10{,}000$ $B = 0.5$

IN THE ELECTRIC CASE, UNIT LENGTH REPRESENTS 10^{-2} MHO.

IN THE MECHANIC CASE, UNIT LENGTH REPRESENTS 10^2 MECHANIC ABMHO OR RADIANS PER SEC. PER UNIT V.M.T.

The lower quadrantal value of impedance OB, occurs at $u = 0.618$, or $\omega_1 = 12{,}361$ radians per second. The upper value $O E$, occurs at $u = 1.618$, or $\omega_2 = 32{,}361$. The range between these two quadrantal values of impressed angular velocity may be described, for convenience as the quadrantal range.

As will be subsequently discussed, the quadrantal range of u in each case corresponds to the sharpness of resonance. In this case, the quadrantal range of u is from 0.618 to 1.618, or amounts to 1.0.

If we form the graph of the admittance Y, of the branch $L R S$, by forming the successive planevector reciprocals of the impedance Z, we obtain the circle $O Y_1 Y_0 Y_2$, Fig. 3, around which advance is made clockwise, as the impressed frequency is increased. $O Y_1$ and $O Y_2$, are the quadrantal admittances at the lower and upper quadrantal angular velocities ω_1 and ω_2 respectively. Any point in the Z graph, Fig. 2, characterized by the slope $-\alpha$ deg., of the planevector $O P$, corresponds to a point P, in the Y graph Fig. 3, having for its planevector $O P$ the slope $+\alpha$ deg, and the size $G \cos \alpha$, where G $(= 1/R)$, is the conductance of R in mhos.

It is easy to show that the two quadrantal angular velocities ω_1 and ω_2, whose vector admittance slopes are 45 deg. plus and minus respectively, have, as their geometrical mean, the resonant angular velocity, or

$$\sqrt{\omega_1 \omega_2} = \omega_0 \qquad \frac{\text{radians}}{\text{sec.}} \qquad (6)$$

Moreover, this relation is not confined to the quadrantal pair of angular velocities. Any pair ω_1' and ω_2', characterizing the vectors $O P$ and $O P'$ in Fig. 3, whose slopes are respectively $+\alpha$ deg. and $-\alpha$ deg., are subject to the relation expressed in (6), or have ω_0 as their geometric mean.

Vector Maximum Cyclic Current in a C L R Branch. It will be evident from a consideration of the vector admittance graph Fig. 3, that if a maximum cyclic constant voltage E is impressed, at standard phase, on the $C L R$ branch, at varied frequency, the vector graph of maximum cyclic current in the branch will be a circle, similar to that of Fig. 3, since

$$I = E Y \qquad \text{max. cyc. amperes} \angle \qquad (7)$$

where I is the maximum cyclic vector current. In fact, Fig. 3 may then be interpreted as a vector current graph, if the scale of the diagram be properly chosen. Consequently, as the frequency is increased from zero to infinity, the maximum cyclic current changes from $0 \angle 90$ deg. to $0 \scalebox{1}[-1]{$\angle$} 90$ deg. amperes, through the resonant value $E\,G \angle 0$ deg.

Corresponding Mechanical Vibrating System. Fig. 4 represents the elements of a simple bifilar, oscillographic vibrator, in which two similar parallel fine wires or strips, $A\,B$ and $C\,D$, carry the same alternating current in a uniform powerful magnetic field. The direction of the field lies substantially in the plane $A\,B\,D\,C$. At the center of the rectangle $A\,B\,D\,C$, the small mirror m is fastened symmetrically. The two wires are supposed to be equally stressed in tension. The vibrator is immersed in a damping fluid, such as air, oil, or glycerine.

When a maximum cyclic current I absamperes passes through the vibrator, it produces a maximum cyclic vibromotive torque on the mirror

$$F = A\,I \qquad \text{max. cyc. dyne-perp.-cm.} \angle \qquad (8)$$

This is a planevector alternating torque, proportional to I, and measurable in dynes acting perpendicularly to a radius arm of 1 cm., supposed to be erected perpendicularly to the plane of the mirror. The coefficient A is a real numeric, depending on the strength of the magnetic field, the length and the tension of the vibrator wires, etc. The phase of the torque will be the same as the phase of the current, if the wires are of non-magnetic material. The angular displacements produced in the vibrator mirror are assumed to be small; *i. e.*, of the order of a few degrees only.

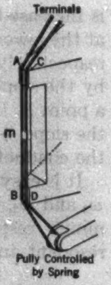

FIG. 4—OSCILLOGRAPH VIBRATOR

The alternating vibromotive torque F (abbreviated v.m.t.), acting on the vibrator of Fig. 4, corresponds to the alternating e.m.f. E, acting on the $C\,L\,R$ branch of Fig. 1. F tends to produce an alternating angular velocity of the mirror, just as E tends to produce an alternating current in the $C\,L\,R$ branch. Just as E is opposed by an electric impedance Z, expressed as in (2), so F, is opposed by a mechanical impedance z, expressed by a similar equation

$$z = r + j\,(m\,\omega - s/\omega) \qquad \frac{\text{dynes perp. cm.}}{\text{radians per sec.}} \angle \qquad (9)$$

Here r is the frictional resistance of the vibrator $A\,B\,C\,D$, Fig. 4, to alternating angular velocity, which causes energy to be dissipated as heat in the damping field. The quantity $(m\,\omega - s/\omega)$ is that mechanic resistance of the vibrator to alternating angular velocity, which causes energy to be shifted cyclically from the resilient wires to the inertia of the vibrating system, and which, therefore, occupies mechanical power in cyclic storage and release, but without dissipation. The quantity m corresponds to the inductance L of the branch circuit, and is the moment of inertia of the vibrating system, expressible in $gm - cm^2$, or what is equivalent thereto, in $\dfrac{\text{dynes-perp. cm.}}{(\text{radians per sec.})^2}$. The quantity s corresponds to the elastance S of the branch circuit, and is the elastic stiffness, or resistance to angular displacement, of the vibrator, expressible in dynes-perp.-cm. per radian. The complex quantity Z, is thus a mechanic impedance, analogous to electric resistance, and capable of being expressed in mechanic absohms $\angle$.

Vector Angular-Velocity Impedance. In view of the corresponding relations between an electric branch circuit, and a mechanic vibrator, the impedance graph of Fig. 2 may be regarded as representing the graph of impedance to angular velocity of the vibrator mirror in Fig. 4. At a very low impressed angular velocity, this impedance z commences at ∞ ⦣ 90 deg. mechanic absohms. At a very high impressed ω, z ends at $\infty \angle 90$ deg. mechanic absohms. At resonant angular velocity, ω_0, z passes through the value $r \angle 0$ deg.

Vector Angular-Velocity Admittance. In view of the same electric-mechanic analogy, the circular vector admittance graph to alternating current, Fig. 3, may also be regarded as a vector admittance graph to alternating torque. As the angular velocity impressed on the vibrator increases from zero to infinity, the admittance $y = 1/z$, varies from $0 \angle 90$ deg., through $g \angle 0$ deg., to 0 ⦣ 90 deg. mechanic abmhos, where $g = 1/r$, is the "mechanic conductance" of the system, expressible also in $\dfrac{\text{radians per sec.}}{(\text{dynes perp. cm.})}$.

The circular admittance graph of Fig. 3, corresponds not only to Fig. 2, by inversion; but also to either of the following cases: Electrically, $R = 100$ ohms, $G = 10^{-2}$ mho, $L = 5 \times 10^{-3}$ henry, $S = 2 \times 10^{6}$ darafs.

Mechanically, $r = 10^{-2} \dfrac{\text{dynes perp. cm.}}{\text{radian/sec.}}$,

$$g = 100 \frac{\text{radian/sec.}}{\text{dyne perp. cm.}}, \; m = 5 \times 10^{-7} \text{ gm.} - \text{cm.}^2,$$

$$s = 2 \times 10^2 \; \frac{\text{dyne perp. cm.}}{\text{radian}}$$

In either case, the resonant angular velocity $\omega_0 = 20{,}000$ radians/sec.

In the electric case, unit size on the graph corresponds to 10^{-2} mho.

In the mechanic case, unit size corresponds to

$$10^2 \; \frac{\text{radian/sec.}}{\text{dyne perp. cm.}}.$$

The resonant range in u is from $u_1 = 0.618$, through 1.0 to $u_2 = 1.618$.

The size of the resonant admittance is G mhos electrically; or $g \dfrac{\text{radians/sec.}}{\text{dyne perp. cm.}}$ mechanically; or what may be described as "mechanic abmhos", since the mechanic units are all referred, for convenience, to the C. G. S. system.

The size in Fig. 3 of the admittance Y or y, at any slope α deg., is $G \cos \alpha$ electrically, or $g \cos \alpha$ mechanically.

We may express the maximum cyclic angular displacement of the vibrator mirror by Θ radians $\angle$. The instantaneous angular velocity will then be $\dfrac{d\theta}{dt}$ or $\dot{\theta}$ radians per sec. and the maximum cyclic value of the mirror's angular velocity may then be denoted by $\dfrac{d\Theta}{dt}$, or $\dot{\Theta}$ radians per sec. Here $\dot{\Theta}$ corresponds to I.

Folliowng the same analogy with reference to (**6**),

$$\dot{\Theta} = F y \qquad \text{max. cyc. radians per sec.} \angle \quad (\mathbf{9})$$

where the v.m.t. F is taken as of standard phase, or zero slope. Consequently, Fig. 3 may be interpreted as the graph of angular velocity produced by an exciting current I max. cy. absamperes in the vibrator, at varied angular velocity, the scale of the diagram being suitably chosen. As ω varies from zero through ω_1, ω_0, and ω_2, to infinity, $\dot{\Theta}$ varies from $0 \angle 90$ deg., through $F\,y_1 \angle 45$ deg., $F\,g \angle 0$ deg., $F\,y_2$ ⦧ 45 deg. to 0 ⦧ 90 deg., radians per sec. $\angle$. This means that at the angular velocity of resonance, the angular velocity of the mirror is greatest, and is in phase with the exciting alternating current. At a very low frequency, $\dot{\theta}$ is insignificantly small, but in leading quadrature to the exciting current. At a very high frequency, $\dot{\theta}$ is again very small, but in lagging quadrature to the exciting current. At the quadrantal frequencies f_1 and f_2, the size of the angular velocity will be 0.707, or $2^{-1/2}$ of that at resonance, and will be in semiquadrature with the exciting current.

Thus in Fig. 3, interpreted electrically at 10^{-2} mho per unit of length, with impressed frequency $\omega = 10{,}000$ radians per second, or $u = 0.5$, $Y = 0.555 \times 10^{-2} \angle 56.3$ deg.; so that 1 volt max cy. e.m.f at this frequency and standard phase applied to the $C\,L\,R$ circuit, of the constants chosen above, would establish a current of $0.555 \times 10^{-2} \angle 56.3$ deg. max. cy. ampere, or 5.55 max. cy. milliamperes, leading the e.m.f by 56.3 deg. If, however, the mechanic application of Fig. 3 is considered, with 100 mechanic abmhos to the unit of length, one max. cy dyne-perp-cm. applied to the vibrator, at $\omega = 10{,}000$ or $u = 0.5$, would establish a max. cy. angular velocity of $55.5 \angle 56.3$ deg. radians per sec. As the mirror crossed the zero point on its scale, it would be moving with the angular velocity of 55.5 radians per second, or 3180 degrees per second, and this zero would be crossed 56.3 deg. ahead in phase of the impressed v.m.t.

In the electric system, resonance occurs at

$$\omega_o = \frac{1}{\sqrt{C\,L}} = \sqrt{S/L} \qquad \frac{\text{radians}}{\text{sec}} \qquad (11)$$

In the mechanic system, resonance correspondingly occurs at

$$\omega_o = \sqrt{s/m} \qquad \frac{\text{radians}}{\text{sec}} \qquad (12)$$

Damping Constant of Oscillating Systems, Electric and Mechanic. If we energize either the condenser or the reactor or both these elements in Fig. 1, and removing, from the mains, the branch circuit $C\,L\,R$, close it on itself, the damping factor of its current, assuming no loss of power in either S or L is known to be

$$\Delta = \frac{R}{2\,L} \qquad \text{hyps per sec.} \qquad (13)$$

This may be regarded as the hyperbolic angular velocity of decay in the circuit, expressible in hyperbolic radians per second. Similarly, if we energize either the wires, or the mass of the vibrator, or both these elements in Fig. 4, the damping factor of the angular velocity, assuming no loss of power in either s or m, is:

$$\Delta = \frac{r}{2\,m} \qquad \text{hyps per sec.} \qquad (14)$$

This is also the hyperbolic angular velocity of decay in vibration. This quantity Δ is very important in oscillographic theory. The reciprocal of Δ may be called the *oscillatory time constant* of either system, or

$$\tau_o = 1/\Delta = \frac{L}{(R/2)} = \frac{m}{(r/2)} \qquad \frac{\text{seconds}}{\text{hyp. radian}} \qquad (15)$$

This is equal to the time in which free oscillations fall to $1/\epsilon$ th in size. The damping factor may also be obtained from the quadrantal angular velocities, by the relation

$$\Delta = \frac{\omega_2 - \omega_1}{2} = \frac{f_2 - f_1}{\pi} \qquad \frac{\text{hyp}}{\text{sec.}} \qquad (16)$$

or from any angular velocity ω, at which the slope of the vector admittance is α deg., by the more general relation:

$$\Delta = \frac{\omega^2 - \omega_o^2}{2\,\omega \tan \alpha} \qquad \frac{\text{hyps.}}{\text{sec.}} \qquad (17)$$

In the case represented by Figs. 2 and 3, $\Delta = 10{,}000$ hyps. per sec.

Sharpness and Bluntness of Resonance. The sharpness of resonance of a circuit, or of a vibrator, may be defined as the ratio of the resonant angular velocity to the damping factor, or

$$\blacktriangle = \omega_o/\Delta = \frac{\omega_o}{\left(\dfrac{\omega_2 - \omega_1}{2}\right)} = \frac{2\,\omega_o\,\omega_1}{\omega_o^2 - \omega_1^2} = \frac{2\,\omega_o\,\omega_2}{\omega_2^2 - \omega_o^2}$$

$$= \frac{f_o}{\left(\dfrac{f_2 - f_1}{2}\right)} = \frac{2\,\pi\,f_o}{\Delta} = \frac{m\,\omega_o}{(r/2)} = \frac{s}{(r/2)\,\omega_o}$$

$$= \frac{\sqrt{m\,s}}{r/2} = \frac{\sqrt{L/C}}{R/2} \qquad \text{numeric} \qquad (18)$$

Very sharp resonance thus implies a large ratio of ω_0 to the semi-quadrantal range $\frac{(\omega_2 - \omega_1)}{2}$. Resonance sharpnesses in designedly resonant electric circuits are commonly of the first order, (near 10^1) and occasionally exceed the second order (10^2). In vibrators, the magnitudes encountered are of similar orders. In air-damped vibration galvanometers, a resonant sharpness as high as the third order, (1000) may be met.

The reciprocal of the sharpness of resonance may be called B, the bluntness of resonance

$$B = 1/\Lambda = \Delta/\omega_0 = \frac{\left(\frac{\omega_2 - \omega_1}{2}\right)}{\omega_0} = \frac{\omega_0^2 - \omega_1^2}{2\,\omega_0\,\omega_1}$$

$$= \frac{\omega_2^2 - \omega_0^2}{2\,\omega_0\,\omega_2} = \frac{\left(\frac{f_2 - f_1}{2}\right)}{f_0} = \frac{\Delta}{2\,\pi\,f_0} = \frac{(r/2)}{m\,\omega_0}$$

$$= \frac{(r/2)\,\omega_0}{s} = \frac{(r/2)}{\sqrt{m\,s}} = \frac{R/2}{\sqrt{L/C}} = \frac{u_2 - u_1}{2} \quad \text{numeric} \quad (19)$$

When the bluntness of a $C\,L\,R$ circuit, or of a vibrator, attains the value unity, the system is critically damped, or is just aperiodic. If it exceeds unity, the system is overdamped, or ultraperiodic. If the bluntness is less than unity, or the sharpness greater than unity, free oscillations can occur. With high sharpness, numerous oscillations accompany oscillatory decay. Air-damped oscillographic vibrators commonly have a sharpness Λ of 100 or more, or a bluntness B of about 0.010 or less. Oil damped vibrators may easily have a bluntness B greater than unity. These numerical coefficients Λ and B are important in oscillographic theory.*

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*The resonance sharpness thus defined is double that used in some publications. See Bibliography 11, 13, 17. As defined in (18), it may be distinquished as Λ_0, and has certain algebraic advantages over the lesser value $\Lambda_s = \frac{\omega_0}{2\,\Delta}$. In this paper Λ stands for Λ_0, and its reciprocal B for B_0. Λ_0 may be regarded as an oscillatory sharpness, depending on the oscillatory time constant $\tau_0 = \frac{2\,L}{R}$ seconds, and as distinguished from the sharpness of sustained alternation Λ_s, depending on the time constant of continuous-current application $\tau_s = L/R$ seconds. Hence $B_s = 2\,B_0$, $\Lambda_s = \Lambda_0/2$; or $B_0 = B_s/2$ and $\Lambda_0 = 2\,\Lambda_s$.

In the case represented in Figs. 2 and 3, $\Lambda = 2$, and $B = \frac{1}{2}$. This means, therefore, that in the case of those figures, the resistance to either electrical or mechanical motion is just half of that which would render the system aperiodic, or critically damped.

Rapidity of Change in Phase Angle α, of I or $\dot{\Theta}$, with respect to ω. In sharply resonant systems, $(\Lambda > 1)$, a small change in the impressed angular velocity will involve a rapid change in the phase angle α of current I, or angular velocity $\dot{\Theta}$, Fig. 3, near resonance. In cases of very sharp resonance, or small damping, a change of a few radians per second in ω, near to ω_0, may carry the vector I, or $\dot{\Theta}$, around the quadrantal range in slope (from $\alpha = +45$ deg. to $\alpha = -45$ deg.). On the other hand, in bluntly resonant systems $(B > 1)$, it may require a relatively large change in ω to carry the vector I or $\dot{\Theta}$ over the same range. In the neighborhood of the resonant angular velocity ω_0, we may notice that

$$\frac{d\,\alpha}{d\,\omega} = -\,1/\Delta = -\,\tau_0 \qquad \text{seconds} \quad (\mathbf{20})$$

so that the phase α changes near resonance by $-\tau_0$ radians per single radian per second in ω. By (**14**), this change is rapid in a system of large L and small R, or of large m and small r. With a large frictional resistance r, this rate of change in α becomes small.

Again, the quadrantal range in α is from α_1 to α_2, or -90 deg. or $-\pi/2$ radians (see Fig. 3). The corresponding change in angular velocity is $\omega_2 - \omega_1 = 2\,\Delta = 2/\tau_0$. Consequently, the average rate of change in α with respect to ω, over the whole quandrantal range is

$$\frac{\alpha_2 - \alpha_1}{\omega_2 - \omega_1} = \frac{-\,\pi/2}{(2/\tau_0)} = -\,(\pi/4)\,.\,\tau_0 = -\,0.7854\,\tau_0 \qquad \text{seconds} \quad (\mathbf{21})$$

or the average rate of change in α with respect to ω is 78.5 per cent, over the whole resonant range, of what it is at resonance, where the rate is a maximum. Above and below the quadrantal frequencies, at which the reactive factor

$$\frac{m\,\omega - s/\omega}{r} = \pm\,1,$$

the rate of change in α with change in ω diminishes. At large reactive factors, it diminishes approximately as the inverse square of the reactive factor.

Impedance to Displacement. The foregoing discussion relates to the evaluation of the maximum cyclic current I in a $C\,L\,R$ branch, or to the maximum cyclic angular velocity $\dot{\theta}$ of a vibrator mirror. This maximum cyclic angular velocity occurs at the moment when the mirror is passing through its zero or central position. The oscillographer is only indirectly interested, however, in the angular velocity of his mirror. He is directly interested in the angular displacements, or elongations, of his mirror. In the electric-circuit analogy, the mirror displacement corresponds to the alternating displacement of electricity, or electric quantity, Q coulombs $\angle$. The quantity Q is the time integral of the current I. In the assumed condition of a simple harmonic or purely sinusoidal current, the quantity Q is also sinusoidal. It is in direct proportion both as to magnitude and phase with the voltage across the condenser, or is in lagging quadrature with the current. We are therefore led to study the impedance to displacement or "displacement impedance" of a branch or a vibrator, in order to evaluate Θ, the maximum cyclic angular displacement of a vibrator mirror.

It is easily shown that the displacement impedance Z', for the simple alternating electric circuit, is

$$Z' = j\,\omega\,\{\,R + j\,(\,L\,\omega - S/\omega)\,\} = (S - L\,\omega^2) + j\,R\,\omega \quad \frac{\text{volts}}{\text{coulombs}}\angle \qquad (\mathbf{22})$$

and similarly for the mechanical vibrator:

$$z' = j\,\omega\,\{r + j\,(m\,\omega - s/\omega)\,\} = (s - m\,\omega^2) + j\,r\,\omega \quad \frac{\text{dyne perp. cm.}}{\text{radian}}\angle \qquad (\mathbf{23})$$

$$= s\,\{\,(1 - u^2) + j\,2\,B_0\,u\} = s\,\{\,(1 - u^2) + j\,B_s\,u\} \quad \frac{\text{dyne perp. cm.}}{\text{radian}}\angle$$

As u varies from 0 to infinity, z' traces a parabola. When $B_0 = 0$, the parabola collapses into a straight line along the x axis. When $B = 8$ it nearly coincides with a straight line parallel to the Y axis. Consequently, the max. cyclic displacement of electricity in the branch circuit, and accumulated in the condenser, from an impressed max. cyclic e.m.f. E volts $\angle$ on the branch, will be

$$Q = E/Z' \qquad \text{max. cy. coulombs } \angle \qquad (\mathbf{24})$$

Similarly, the max. cyclic displacement of the mirror of the vibrator, under a max, cyclic v.m.t. F dynes-perp.- cm., will be:

$$\Theta = F/z' \qquad \text{max. cy. radians } \angle \qquad (\mathbf{25})$$

Fig. 5 is a graph of displacement impedance, for a case of $\Lambda = 2$, or $B = 0.5$. In such a case, the resistance r to motion is just equal to the surge resistance $\sqrt{m\,s}$, or

$$R = \sqrt{L/C} \qquad \text{ohms} \qquad (\mathbf{26})$$

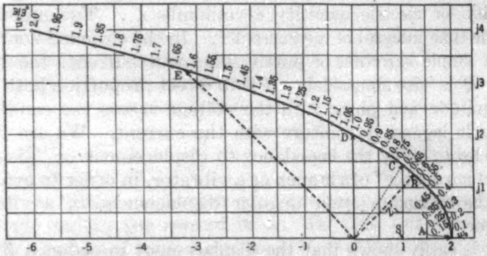

FIG. 5—GRAPH OF IMPEDANCE TO ELECTRIC OR MECHANIC DISPLACEMENT FOR CASES OF

$$\left\{\begin{array}{lll} R = 10^2, & L = 5 \times 10^{-3}, & S = 2 \times 10^6 \\ r = 10^{-2}, & m = 5 \times 10^{-7}, & s = 2 \times 10^2 \end{array}\right\}$$

$$w_0 = 20{,}000 \quad \Delta = 10{,}000 \quad B = 0.5$$

IN THE ELECTRIC CASE, UNIT LENGTH REPRESENTS 10^6 VOLTS PER COULOMB.

IN THE MECHANIC CASE, UNIT LENGTH REPRESENTS 10^2 DYNES-PERP. CM. PER RADIAN.

and the resistance is just 0.5 critical. In the current impedance graph of Fig. 2, similar relations between the ohmic and surge resistances were chosen; so that the graph of Fig. 5 may be derived from that of Fig. 2, according to (**21**), by applying to each vector, at angular velocity ω, the factor $j\,\omega$, which rotates the vector counter-clockwise through 1 quadrant and extends it in the numerical ratio ω, which ratio increases steadily as the frequency is increased. We thus rotate the diagram of Fig. 2 through $+ 90$ deg., and apply the factor ω throughout.

It will be evident that at $\omega = 0$, the initial displacement impedance is S or s, as the case may be, taken as $O\,A$ Fig. 5, along the axis of real quantities. At the lower quadrantal angular velocity ω_1, this impedance is $O\,B$, at a slope of 45 deg. At resonance, or ω_o, this impedance has reached $O\,D$, along the j axis. At ω_2, it has reached $O\,E$, at the slope of 135 deg., and thence, as the angular velocity is increased indefinitely, the displacement impedance tends to an infinite size, at slope 180 deg. It will be observed that the curve of displacement impedance is a parabola $A\,B\,C\,D\,E$ Fig. 5, with its axis along the line of real quantities.

The displacement impedance is a minimum at that point C in the curve, where the tangent is perpendicular to the radius vector.

Displacement Admittance. The reciprocal of displacement impedance, or "displacement admittance," is of still greater interest and practical importance from the oscillographic viewpoint. From (**23**), considering only the mechanical case, the displacement admittance is

$$y' = 1/z' = \frac{1}{(s - m\,\omega^2) + j\,r\,\omega} = \frac{1}{m\,(\omega_o^2 - \omega^2) + j\,r\,\omega}$$

$$= \frac{1}{m\,\{\,(\omega_o^2 - \omega^2) + j\,2\,\Delta\,\omega\,\}} \qquad \frac{\text{radians}}{\text{dyne perp. cm.}}\ \angle \qquad (\mathbf{27})$$

$$= \frac{1}{s\,\{\,(1 - u^2) + j\,2\,B_o\,u\}} = \frac{1}{m\omega_0^2\,\{\,(1 - u^2) + j\,2\,B_o\,u)\}}$$

$$= \frac{y}{j\,\omega} = -\,j\,y/\omega \qquad \frac{\text{radians}}{\text{dyne perp. cm.}}\ \angle \qquad (\mathbf{27a})$$

At $\omega = 0$, $y' = y_o' = 1/s\ \angle 0$ deg.

$$\frac{\text{radians}}{\text{dyne perp. cm.}}\ \angle \qquad (\mathbf{28})$$

At $\omega = \omega_1, y' = y_1' = \dfrac{1}{\sqrt{2}\,r\,\omega_1}\ \overline{\angle}\ 45$ deg.

$$\frac{\text{radians}}{\text{dyne perp. cm.}}\ \angle \qquad (\mathbf{29})$$

$$\text{At } \omega = \omega_o,\ y' = y_o' = -j\,\frac{1}{r\,\omega_o} = \frac{1}{r\,\omega_o} \overline{<} 90 \text{ deg.}$$

$$\frac{\text{radians}}{\text{dyne perp. cm.}} \angle \qquad (30)$$

$$\text{At } \omega = \omega_2,\ y' = y_2' = \frac{1}{\sqrt{2}\,r\,\omega_2} \overline{<} 135 \text{ deg.}$$

$$\frac{\text{radians}}{\text{dyne perp. cm.}} \angle \qquad (31)$$

$$\text{At } \omega = \infty,\ y' = y'_\infty = 0 \overline{<} 180 \text{ deg.}$$

$$\frac{\text{radians}}{\text{dyne perp. cm.}} \angle \qquad (32)$$

Displacement Admittance Graph. Referring to (**27**a), and to Fig. 6, it will be seen that if we draw the circular graph of $\dot{\Theta}$, $O\,B\,C\,D$, and at any particular value of ω, or $u\,\omega_o$, such as $\omega = 10{,}000$, draw the vector $O\,P$, of size 0.555×10^2 $\dfrac{\text{radians per sec.}}{\text{dyne perp. cm.}}$, at slope $\alpha = 56.3$ deg., we may then draw perpendicularly thereto the vector $O\,p$, and dividing 55.5 by ω obtain, to a correspondingly altered scale, the displacement admittance of 5.55×10^{-3} radian per dyne-perp.-cm., at a slope of β deg. $= -33.7$ deg. This means that if unit max. cy. v.m.t., or 1 dyne-perp.-cm., be applied to the vibrator of this case, at $\omega = 10{,}000$, the max. cy. displacement of the mirror will be 5.55×10^{-3} radians $= 0.32$ degree. This elongation, or max. cy. angular displacement, would occur 33.7 deg. in phase behind the corresponding elongation in torque or exciting current.

Proceeding in this manner, we might map out the entire displacement admittance* graph a, p, b, c, d, O. At or near zero frequency, denoted by ω_o, the displacement admittance would be the vector $O\,a$, in phase with the exciting current of the vibrator. At the lower quadrantal frequency ω_1, it would be the vector $O\,b$. At the upper quadrantal frequency, it would be $O\,d$. At resonance, it would be $O\,c$, in lagging quadrature with the excitation.

The maximum admittance would occur at ω_d, in this case at $u = 0.707$, or $\omega = 14{,}140$. This vector will vary in position with different values of B. *It is to be noted, however, that the maximum admittance never occurs at resonance, although in sharp vibrators, it will lie close to resonance.*

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*Bibliography 18, p. 505 and 511.

If we consider Fig. 6, in terms of its electric representation, the scale being 10^{-2} mho. per unit length, then at $\omega = 10{,}000$

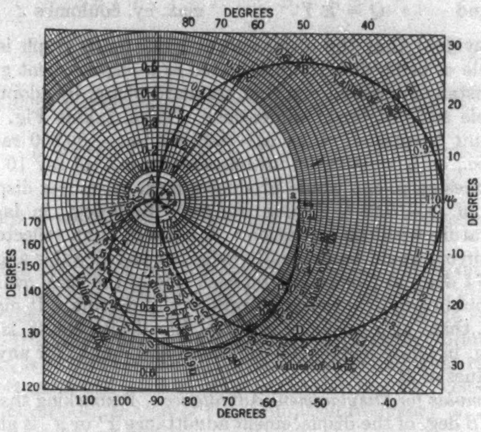

FIG. 6—GRAPH OF DISPLACEMENT ADMITTANCE FOR CASES OF

$$\left\{\begin{array}{lll} R = 10^2, & L = 5 \times 10^{-3}, & S = 2 \times 10^6 \\ r = 10^{-2}, & m = 5 \times 10^{-7}, & S = 2 \times 10^2 \end{array}\right\}$$

$$w_0 = 20{,}000 \quad \Delta = 10{,}000 \quad B = 0.5$$

FOR CIRCLE $A\ B\ C\ D$
IN THE ELECTRIC CASE, UNIT LENGTH = 10^{-2} MHO.
IN THE MECHANIC CASE, UNIT LENGTH = 10^{-2} MECHANIC ABMHOS, OR $10^2 \left(\frac{\text{RADIANS}}{\text{SEC.}}\right)$ PER UNIT V.M.T.

FOR GRAPH $a\ b\ c\ d$
IN THE ELECTRIC CASE, UNIT LENGTH = 10^6 COULOMB PER VOLT.
IN THE MECHANIC CASE, UNIT LENGTH = 10^{-8} RADIAN PER UNIT V.M.T.

and $u = 0.5$, the electric displacement admittance would be $0.555 \times 10^{-6} \nwarrow 33.7$ deg. coulomb per max. cy. volt on the CLR branch terminals.

It is also easy to show that the displacement admittance graph $a\ b\ c\ d$, Fig. 6, is obtainable, by direct vector inversion, from the displacement impedance graph *A B C D E*, of Fig. 5.

Displacement:

Since

$$\Theta = F\, y' \qquad \text{max. cy. radians} \ \angle \qquad \textbf{(33)}$$

and

$$Q = E\, Y' \qquad \text{max. cy. coulombs} \ \angle \qquad \textbf{(34)}$$

it is evident that the displacement admittance graph is also capable of being directly interpreted as a displacement graph, to constant impressed v.m.t. at standard phase, by adopting a suitable scale of linear dimensions. Thus, taking Fig. 6, at resonant frequency; *i. e.*, $u = 1.0$, and $\omega_o = 20{,}000$ radians per sec., or $f_o = 3183 \sim$, an impressed v.m.t. of say 10 max. cy. dynes-perp. cm. would produce a max. cy. mirror displacement of $\Theta = 0.05 \diagdown 90$ deg. radian = 2.87 deg., in lagging quadrature with the excitation. A beam of light, reflected by the mirror, would be cyclically deflected to twice this angle or ± 5.74 deg., at elongation, on each side of the zero, owing to the doubling optical reflecting property of mirrors. It may be noted that, at resonance ($u = 1$), the displacement is just 90 deg. in phase behind the excitation and v.m.t., for any and all values of *B*.

Formulas for Displacement Admittance. Remarking that the slope β deg. of the displacement admittance Y' or y', is always negative, lying between the values $-$ 0 deg. and $-$ 180 deg., we have

$$\omega = \sqrt{\omega_o^2 + \Delta^2 \cot^2 \beta} + \Delta \cot \beta$$

$$= \omega_o \left\{ \sqrt{1 + B^2 \cot^2 \beta} + B \cot \beta \right\} \qquad \frac{\text{radians}}{\text{sec.}} \qquad \textbf{(35)}$$

$$\text{or} \quad u = \omega / \omega_o = \sqrt{1 + B^2 \cot^2 \beta} + B \cot \beta \qquad \text{numeric} \qquad \textbf{(36)}$$

The polar equation of displacement admittance is

$$| y' | = \frac{-\sin \beta}{r\, \omega} = \frac{-\sin \beta}{r\, \omega_o \left\{ \sqrt{1 + B^2 \cot^2 \beta} + B \cot \beta \right\}} \qquad \text{numeric} \qquad \textbf{(37)}$$

$$= \frac{-\sin \beta}{2\, B\, s \left\{ \sqrt{1 + B^2 \cot^2 \beta} + B \cot \beta \right\}} \qquad \text{numeric} \qquad \textbf{(37a)}$$

The corresponding vector equation is

$$y' = \frac{-\sin\beta}{r\,\omega}\angle -\beta \text{ deg.}$$

$$= \frac{-\sin\beta\ \overline{\diagdown}\ \beta \text{ deg.}}{2Bs\,\{\sqrt{1+B^2\cot^2\beta}+B\cot\beta\}}$$

$$= \frac{-\sin\beta\ \overline{\diagdown}\ \beta \text{ deg.}}{2Bsu}\qquad \frac{\text{mirror radians}}{\text{unit v.m.t.}}\angle \qquad \textbf{(38)}$$

The slope β, pertaining to any assigned value of ω, is

$$\beta = \tan^{-1}\left(\frac{2\,\Delta\ \omega}{\omega^2-\omega_o^2}\right) \qquad \text{degrees} \qquad \textbf{(39)}$$

$$\text{or}\quad \beta = \tan^{-1}\left(\frac{2B_o u}{u^2-1}\right) = \tan^{-1}\left(\frac{B_s u}{u^2-1}\right) \quad \text{degrees} \qquad \textbf{(40)}$$

In the testing of oscillographs, the calibration is ordinarily conducted at a relatively low frequency, say 60 ∼, which, for practical purposes, may be regarded as of the same effect as zero frequency, or ω_s. By (**28**), the displacement admittance at ω_s is $1/s$, at zero slope. It is desired to find what ratio the admittance will bear at any impressed angular velocity ω, (or its ratio u), to that at the calibration frequency ω_s. From (**38**) and (**28**) we obtain

$$D = y'/y_s' = \frac{-\sin\beta\angle-\beta \text{ deg.}}{2B_o u} = \frac{-\sin\beta\ \overline{\diagdown}\ \beta \text{ deg.}}{B_s u}$$

$$= \frac{-\sin\beta\ \overline{\diagdown}\ \beta \text{ deg.}}{2B_o\{\sqrt{1+B_o^2\cot^2\beta}+B_o\cot\beta\}}$$

$$\text{numeric}\angle \qquad \textbf{(41)}$$

This is an important formula. It gives the vector *deviation factor D*, for any oscillograph at any impressed ω, when the values of ω_o and B are known for the instrument. Thus, in the case represented by Fig. 6, where ω_o is 20,000, and $B = 0.5$, if the instrument is calibrated at 60 ∼ $= \omega_s$, it may be required to know what will be the deviation factor at say $\omega = 5000$ or $u = 0.25$. Here, by (**40**),

$$\beta = \tan^{-1}\frac{0.25}{-0.9375} = \tan^{-1}(-0.2667) = -(14 \text{ deg. } 56 \text{ min.})$$

Then by (**41**)

$$D = y'/y'_s = \frac{-\sin(-14 \text{ deg. } 56 \text{ min.}) \overline{\diagdown} (14 \text{ deg. } 56 \text{ min.})}{2 \times 0.5 \times 0.25}$$

$$= \frac{0.2577}{0.25} \overline{\diagdown} \, 14 \text{ deg. } 56 \text{ min.} = 1.031 \overline{\diagdown} \, 14 \text{ deg. } 56 \text{ min.}$$

This means that the max. cy. displacement produced at 796 ~, or 5000 radians/sec. will be 3.1 per cent greater than that produced by the same impressed torque at 60 ~, and will lag 14 deg. 56 min. in phase behind the impressed torque; whereas at 60 ~, the phase lag of displacement would be insignificant in such a vibrator. The *correction factor* will be the reciprocal of the deviation factor, or in this case 0.969 ∠ 14 deg. 56 min., at 796 ~. The frequency of 796 ~ would be 13 times 61.23 or would be the thirteenth harmonic of 61.23 ~. The lag of mirror displacement to this harmonic would thus be nearly 15 deg. of the harmonic frequency. The position of the recorded 13th harmonic would thus be retarded by nearly 15 deg. of its own cycle, and the harmonic, as recorded photographically should be advanced through more than 14° of its own cycle, in order to place it in proper relation to the fundamental. Its recorded or apparent magnitude should also be reduced 3 per cent, in order to have the same scale of amplitude as the fundamental and calibrated frequency.

The following Table of important displacement frequencies may be convenient for reference.

Of the above 8 important angular velocities, Nos. 1, 2, 5 and 7 are always presented, at the successive negative slopes of 0 deg., 45 deg., 90 deg., and 135 deg. No. 3, the maximum admittant value of ω, is always to be found when $B < 1$, or $\Lambda > 1$. For values of B equal to, or greater than, unity, which we may call blunt cases, ω_d has no independent existence, because the greatest admittance is then found at $\omega = \omega_s = 0$. Angular velocity No. 4, ω_f, is always presented when $B < 1$; but in blunt cases, the system is too heavily damped to permit of free vibration. No. 6 is always presented, but its phase position varies with B. It is midway in angular velocity between ω_1 and ω_2; but is not midway between them in phase. No. 8, the upper angular velocity at which the displacement has the same size as at ω_s, is not found when $B > 0.707$. In

TABLE I.
PRINCIPAL IMPRESSED ANGULAR VELOCITIES OF A VIBRATORY SYSTEM.

	Angular Velocity radians/sec.	Displacement Admittance Mirror radians/dyne p. cm.	
1. Initial, (near zero), ω_s		$y_s' = 1/s \angle 0^\circ = c \angle 0^\circ$	(42)
2. Lower quadrantal,	$\omega_1 = \sqrt{\omega_0^2 + \Delta^2} - \Delta = \omega_0 \{ \sqrt{1 + B^2} - B \}$	$y_1' = \dfrac{1}{\omega_1 r \sqrt{2}} \angle 45^\circ = \dfrac{g}{\omega_1 \sqrt{2}} \angle 45^\circ$	(43)
3. Max. admittant,	$\omega_d = \sqrt{\omega_0^2 - 2\,\Delta^2} = \omega_0 \sqrt{1 - 2\,B^2}$	$y_d' = \dfrac{1}{r\,\omega_f} \angle \tan^{-1} (\omega_d/\Delta)$	(44)
4. Free vibrational,	$\omega_f = \sqrt{\omega_0^2 - \Delta^2} = \omega_0 \sqrt{1 - B^2}$	$y_f' = \dfrac{1}{r\sqrt{\omega_f^2 + (\Delta/2)^2}} \angle \tan^{-1} \dfrac{\omega_f}{(\Delta/2)}$	(45)
5. Resonant,	$\omega_0 = \sqrt{\omega_0^2 - 0} = \omega_0 \sqrt{1}$	$y_0' = \dfrac{1}{r\,\omega_0} \angle 90^\circ = \dfrac{-j\,g}{\omega_0}$	(46)
6. Midquadrantal,	$\omega_{12} = \sqrt{\omega_0^2 + \Delta^2} = \omega_0 \sqrt{1 + B^2}$	$y_{12}' = \dfrac{1}{r\sqrt{\omega_0^2 + 5\,\Delta^2/4}} \angle \tan^{-1} \left\{ \dfrac{\omega_{12}}{(\Delta/2)} \right\}$	(47)
7. Higher quadrantal,	$\omega_2 = \sqrt{\omega_0^2 + \Delta^2} + \Delta = \omega_0 \{ \sqrt{1 + B^2} + B \}$	$y_2' = \dfrac{1}{r\,\omega_2 \sqrt{2}} \angle 135^\circ = \dfrac{g}{\omega_2 \sqrt{2}} \angle 135^\circ$	(48)
8. Duplicate initial,	$\omega_{s2} = \omega_d \sqrt{2} = \omega_0 \sqrt{2\,(1 - 2\,B^2)}$	$y_{s2}' = c \angle \tan^{-1} \left(\dfrac{2\,\Delta\,\omega_{s2}}{\omega_d^2 - 2\,\Delta^2} \right)$	(49)

sharp cases, it is always present. Its phase position, or slope, varies with B.

A displacement admittance graph presenting all of these 8 angular velocities is seen in Fig. 7, for the particular case of $\omega_0 = 25,000$, or $f_0 = 3979 \sim$, and $B = 0.225$. Here y_s' is taken as 1, corresponding to an oscillographic standard calibration at or near zero frequency. We find y_1, y_0 and y_2, all at their corresponding slopes of $-$ 45 deg., $-$ 90 deg. and $-$ 135 deg., with the respective sizes of 1.9642, 2.22 and 1.2571, at the frequencies 3182, 3979 and 4973 $\sim$. The maximum admittance value y', at $\omega_d = 23,701$, or $f_d = 3,772 \sim$, is 2.2807, occurring at $\beta = -$ 76 deg. 38 min. 55 sec. The free vibrational value y_f', at $\omega_f = 24,359$, or $f_f = 3,877 \sim$, is 2.2657 ∠ 83 deg. 24 min. 50 sec. The midquadrantal value y_{12}, at $\omega_{12} = 25,625$, or $f_{12} = 4078 \sim$, is 2.1551 ∠ 96 deg. 15 min. 49 sec. Finally, the duplicate initial value of y', or y_{s2}', at $\omega = 33518$, or $f_{s2} = 5334 \sim$, is 1.0 ∠ 142 deg. 53 min. 30 sec. In the oscillograph corresponding to this case, the deviation factor at $\omega = 4000$, or $f = 636 \sim$, would be 1.023 ∠ 4 deg. 13 min. and at $\omega = 8000$, or $f = 1272 \sim$, it would be 1.101 ∠ 9 deg. 7 min.

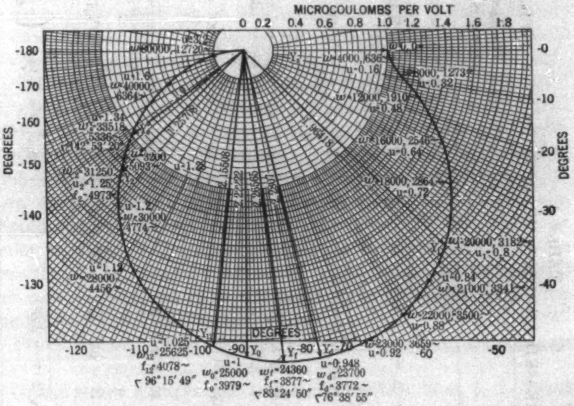

Fig. 7—Vector Displacement Admittances

$w_0 = 25,000 \quad \Delta = 5,625 \quad B = 0.225 \quad s = 10^6 \quad m = 0.0016 \quad r = 18$

Fig. 8 is a vector chart of the deviation factor D for any value of bluntness B between $B = 0.25$ and $B = 1.0$. Enter the chart with u, the ratio of the impressed angular velocity; to the known resonant angular velocity ω_o. The intersection of the u line and the B line will lie on a radius vector whose size and slope make the factor D.

Fig. 9 is a similar vector chart for finding D, when B lies between 1.0 and ∞. There is actually but little change in the form of the curve for values of B exceeding 4. The limiting curve is a semi-circle of unit diameter on the real axis.

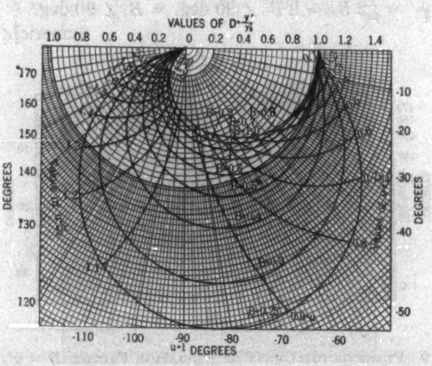

Fig. 8—Planevector Chart of the Deviation Factor D for any Oscillograph whose Bluntness Lies between $B = 0.25$ and $B = 1.0$, and whose Resonant Frequency is Given.

Example: If $B = 0.6$, and $u = 0.3$, then $D = 1.022 \,/\overline{21.6 \text{ Deg.}}$

Fig. 10 is a similar vector chart for D, when B lies between 0.05 and 0.25 inclusive. As B diminishes, the curve of D rapidly approximates to a circle of diameter $\frac{1}{2B} = \Lambda/2$, or radius $\frac{1}{4B}$, and having its center displaced approximately 0.25 unit to the right of the axis $-O\gamma$. At $B = 0$, this circle has an infinitely great diameter.

Constants of an Oscillograph Vibrator. It may be seen from an inspection of (**39**), (**40**) and (**41**), that the deviation factor of any oscillograph vibrator can be either computed or identified from the foregoing curve sheets, if we can ascertain its two essential working constants ω_o and Δ; or what is equivalent thereto, ω_o and B, or f_o and B. The tests of oscillographs thus direct themselves towards the determination of these two constants. If, however, not merely the deviation factor of a vibrator; but also its essential elements are required; then we must seek for the four constants A, m, r and s. This calls, as we shall see, for an additional test.

If we divide (**46**) by (**42**), we find:

$$y_s'/y_o' = j\,2\,B_o = 2\,B_o \angle\ 90\ \text{deg.} = B_s \angle\ 90\ \text{deg.} \qquad \text{numeric} \angle \qquad (\mathbf{50})$$

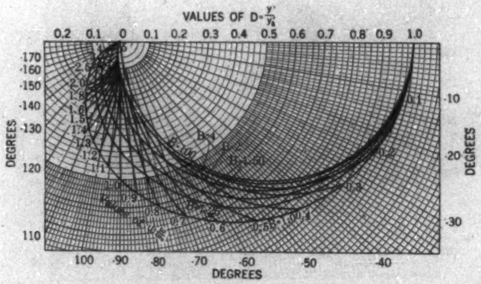

Fig. 9—Planevector Chart of Deviation Factor $D = y'/y'_s$ for any Oscillograph whose Bluntness Lies between $B = 1$ and $B = \infty$, and whose Resonant Frequency is Given.

Example: If $u = 0.5$ and $B = 2.0$, then $D = 0.468\ /\ \overline{69.4\ \text{Deg.}}$

If, therefore, we can (1) identify the resonant frequency f_o, of a vibrator, and (2) compare the calibration of the vibrator at this frequency with that at zero frequency, we shall have; see Figs. 6 to 10:

$$\left|\frac{y_s'}{y'_o}\right| = 2\,B_o = B_s \qquad \text{numeric} \qquad (\mathbf{51})$$

The bluntness B_o *of the vibrator will then be half the ratio of the max. cy. displacement at zero frequency to the max. cy. displacement at resonant frequency,* equal current excitations and impressed torques being used in each test. If the vibrator is sharp, the same fact may be expressed by saying that *the sharpness* Λ_o *will be twice the ratio of the electric exciting* *current needed to produce the same max. cy. displacement, or deflection, at zero frequency, as will produce it at resonant frequency.*

All that is necessary, therefore, for finding the deviation factor of a vibrator at any impressed frequency, is to identify its resonant frequency f_o, and then compare its calibration at that frequency with that at f_s, a low frequency, practically equivalent to zero.

An equivalent alternative formula is:

$$\Delta = \frac{\omega_o}{j^2} \cdot \frac{y_s'}{y_o'} = \pi f_o \frac{y_s'}{y_o'} \angle 90 \text{ deg.} \quad \frac{\text{hyps}}{\text{sec.}} \angle \qquad (52)$$

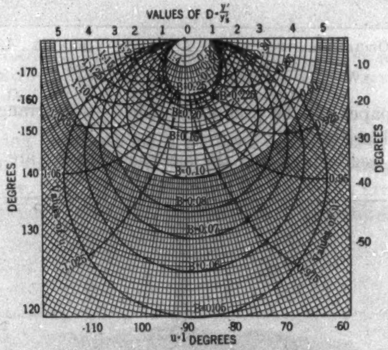

FIG. 10—PLANEVECTOR CHART OF THE DEVIATION FACTOR $D = y'/y'_s$ FOR ANY OSCILLOGRAPH WHOSE BLUNTNESS LIES BETWEEN 0.05 AND 0.25, AND WHOSE RESONANT FREQUENCY IS GIVEN.

EXAMPLE: IF $u = 0.9$, AND $B = 0.08$, $D = 4.17 \;/\overline{37.1 \text{ Deg.}}$

Oscillographmeter. In order to detect and identify the resonant frequency f_o of a vibrator, which, as above pointed out, is not the same as, and may even be remote from, the frequency f_d of maximum displacement, an oscillographmeter has been devised and constructed. It is shown separately in Fig. 11, and mounted in front of an oscillograph under test in Fig. 11-a. It consists of a simple permanent-magnet auxiliary vibrator, air-damped, and therefore of very sharp resonance. This vibrator is firmly set with its vibration axis horizontal, and with its mirror close to that of the tested oscillograph mirror, whose vibration axis is vertical. A strong beam of light is then directed first on the oscillograph mirror, thence on to the auxiliary mirror, and thence vertically upwards to a horizontal sheet of thin graduated paper; so that the path of the vibrating beam on this paper can be examined. The optical arrangement is indicated in Fig. 12.

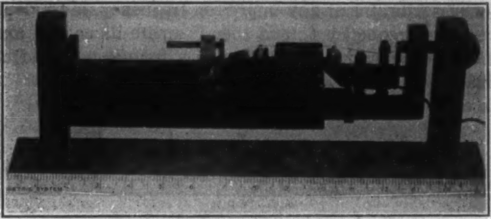

Fig. 11—Oscillographmeter or Auxiliary Vibrator. Length of Mirror 2.3 mm. Breadth 0.9 mm.

If the oscillograph and the auxiliary vibrator are operated from the same audio-frequency source of adjustable frequency; then they will both vibrate with that frequency. Because their vibration axes are set mutually perpendicular, the

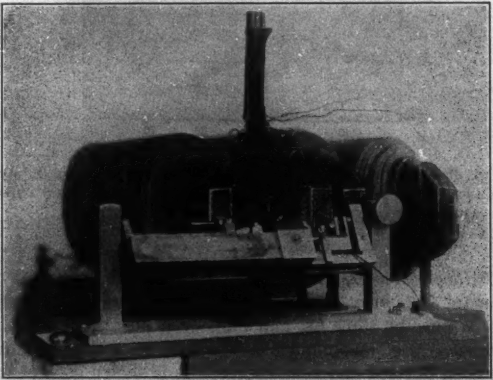

Fig. 11-a—Oscillographmeter Applied to Oscillograph for Test of Latter.

vibratory path of the beam of light on the paper plane *F F*, Fig. 12, will be a Lissajous figure*. If the max· cy. displacements of the two mirrors are cophasal, the figure formed by

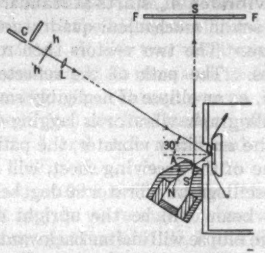

FIG. 12—OPTICAL SYSTEM OF OSCILLOGRAPHMETER *A* AND THE TESTED OSCILLOGRAPH *O*. LENGTH OF *A* MIRROR 2.3 MM. WIDTH 0.9 MM. DISTANCE *O-A* = 1.1 CM. DISTANCE *A-S* = 50 CM.

the moving beam will be a straight line. If they are not cophasal, the figure will be an ellipse. If they are in quadrature, this ellipse will include its maximum area and its two perpendicular axes will be both equal and parallel to the two component displacements. If these two components are equal, the ellipse will, at this quadrature phase relation, become a circle.

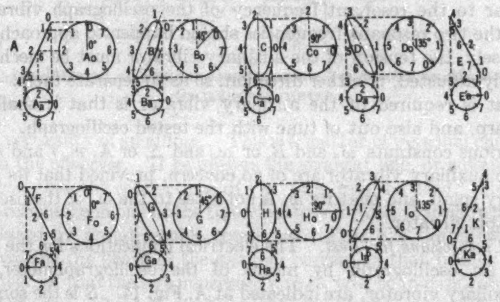

FIG. 13—LISSAJOUS FIGURES WITH COMPONENTS IN THE RATIO OF 2 TO 1 AND VARIOUS PHASE DIFFERENCES

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*Bibliography 1.

The development of Lissajous ellipses is outlined in Fig. 13, for the particular case in which the auxiliary vibrator amplitude is half of the oscillograph vibrator amplitude. Commencing at A, the auxiliary vibrator A_a, starts at standard phase, and the oscillograph A_o, set in mechanical quadrature therewith, also at standard phase. The two vectors then rotate with equal angular velocities. The path of the reflected beam will be straight line A; *i. e.*, an ellipse of negligibly small minor axis.

When the oscillograph vibrator is lagging 45 deg. in phase with respect to the auxiliary vibrator, the path of the beam of light on the plane of the receiving sheet, will be the ellipse B. Again, with the oscillograph vibrator 90 deg. behind C_a in phase, the path of the beam will be the upright ellipse C. With 135 deg. of lag, the ellipse will incline backwards as at D. With 180 deg. of lag, the ellipse will collapse into the straight line E.

The effect of a reversal of phase in either the oscillograph or the auxiliary vibrator, is shown at F, G, H and K. In each case it will be seen that, at quadrature, the ellipse stands upright. The direction of change in inclination of the ellipse is, however, relatively reversed in the two series.

In making the test of the oscillograph, we must first make sure that the resonant frequency of the auxiliary vibrator is not near to the resonant frequency of the oscillograph vibrator. If the two resonant frequencies should happen to approach too closely, the tension of the auxiliary vibrator must be mechanically adjusted, in either direction, so as to separate them. All that is required of the auxiliary vibrator is that it shall be sharp, and also out of tune with the tested oscillograph. The various constants ω_o and B, or ω_o and Δ, or A, m, r and s, of the auxiliary vibrator are of no concern, provided that its B is very small, and that its ω_o is not close to the ω_o of the oscillograph vibrator.

Connections for Test. The electrical connections for the test of an oscillograph by means of the oscillographmeter, or auxiliary vibrator, are indicated at A, Fig. 14. S is the source, or secondary coil of an oscillator, supplying, in parallel, both the oscillograph O, and the auxiliary vibrator A, each through an adjustable resistance. The two supply currents will thus have the same phase, and the two impressed v.m.ts. will be cophasal. The Lissajous figure, obtained from the optical combination of the two vibrators, will then be a straight line, as at A, Fig. 13, if the impressed frequency is low, and remote from resonance in either vibrator. The impressed frequency is now raised, until the resonant angular velocity ω_0, or resonant frequency f_o, for O, is approached. The Lissajous ellipse will now begin to standup. The impressed frequency is now carefully adjusted, until the Lissajous ellipse is upright, and the frequency then noted. If the two vibratory components are equal, the Lissajous ellipse, at resonance in O, will be a circle, as shown at A, Fig. 14. This, however, is an unsuitable adjustment, since it does not lend itself to precise observation. An ellipse of say 2 : 1, or 3 : 1, in major and minor diameters is preferable, as at C, Fig. 13.

If the optical adjustment is fine, so that the beam of light is sharp and thin, it may be more easy to observe the existence of the thin cophase straight line of A, E, F and K, in Fig. 13, than the verticality of an ellipse. This is a matter of personal choice with the observer. If he prefers the straight line, or collapsed ellipse, then the connections of Fig. 14 B, may be used. Here, the resistance in one of the two vibrator circuits, O as shown, is replaced by an adjustable condenser. This has the effect of advancing the phase of the exciting current in that branch by 90 deg., so that the two impressed v.m.ts. will be in quadrature. If the two vibrator displacement slopes are cophasal, the Lissajous figure will then be the circle, or upright ellipse. At displacement slope quadrature, on the contrary, the two actual displacements will be cophasal, and resonance in O will show itself by the Lissajous straight line of Fig. 14B .

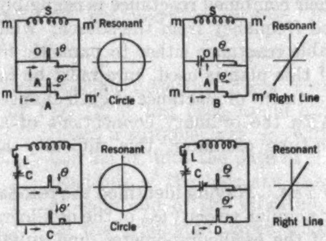

FIG. 14—METHOD OF TESTING OSCILLOGRAPH VIBRATORS BY MEANS OF OSCILLOGRAPHMETER

With the auxiliary vibrator air damped, and reasonably well constructed, its bluntness B will be very small, and its displacement slope will then be negligible, until within a few cycles per second of its resonant frequency. This means that the displacement slope of the auxiliary vibrator will either be substantially 0 deg., or substantially 180 deg., provided that the two vibrators are not closely coresonant.

It has been found that when a Vreeland oscillator is used as the source of impressed frequency, difficulties may arise owing to small parasitic harmonics or impurities in the derived current wave, which may give rise to false resonances, and erroneous results, in the behavior of the sharply resonant auxiliary vibrator. Much trouble was experienced in the first few months of this research, by such parasitic resonances. They can be eliminated, however, by electrically tuning the main testing circuit to the impressed frequency, as indicated in Fig. 14 at *C* and *D*. An adjustable inductor and condenser are so varied, that their combined reactance is negligibly small to the main impressed frequency from the source; while opposing a very considerable reactance either to parasitic frequencies, or harmonics. If this plan is used, care must be taken to make the proper adjustment of reactance at each change of impressed frequency. With the ordinary connections of a pliotron, or triode vacuum-tube oscillator, this difficulty has not been encountered.

Technique of Test. Having identified the resonant frequency f_o, or resonant angular velocity ω_o, of the oscillograph vibrator, with the aid of the oscillographmeter, and suitable Lissajous optical figure, the oscillograph is calibrated for its specific deflection, or maximum cyclic deflection per unit of r.m.s. exciting current, both at a low impressed frequency, f_s, say 60 ~, and at f_o. In this part of the test, the oscillographmeter is either removed, or left out of circuit. As we have already seen in connection with (**51**), the ratio of these two calibrations, or specific deflections, is equal to the value of 2 *B* for the instrument. Having found ω_o and *B*, we are able to find the vector deviating factor, and its reciprocal, the correcting factor, for the instrument, at any or all impressed frequencies that may be used. Defining the specific deflection θ as the vector scale deflection of the oscillograph divided by the r.m.s. value of the exciting current in the oscillograph vibrator, we have, if θ_s is the specific deflection at low frequency, and to standard phase

$$D = \theta / = \theta_s \frac{1 - u_s^2 + j\,2\,B_o u_s}{1 - u^2 + j\,2\,B_o u} = \frac{1 - u_s^2 + j\,B_s u_s}{1 - u^2 + j\,B_s u} \qquad \text{numeric } \angle \qquad (\mathbf{53})$$

Since the ratio ω_s/ω_o is usually a negligibly small real number, this may be written without serious error

$$D = \theta/\theta_s = \frac{1}{1 - u^2 + j\,2\,B_o u} = \frac{1}{1 - u^2 + j\,B_s u}$$

$$= \frac{-\sin\beta \angle -\beta}{2\,B_o u} = \frac{-\sin\beta \nwarrow \beta}{B_s u} \quad \text{numeric} \angle \quad (\mathbf{54})$$

which agrees with (**50**) and (**51**), at resonance, when $u = 1$.

If we seek only the size of this ratio, and ignore the slope or phase deviation, we may write this in the form

$$|D| = \left|\frac{\theta}{\theta_s}\right| = \frac{1}{\sqrt{1 + 2\,u^2\,(2\,B_o^2 - 1) + u^4}} \quad \text{numeric} \quad (\mathbf{55})$$

which is devoid of imaginary quantities. The correcting factor is then

$$K = 1/D \quad \text{numeric} \quad (\mathbf{56})$$

When making specific deflection tests with the optical arrangement of Fig. 12, it has been found advantageous to insert a suitable lens L, with a fine cross hair on it, into the path of the beam. The positions of the hair can then be detected at each end of the vibrating beam of light on the graduated paper scale $F\,F$. When, however, Lissajous figures are being examined, the lens and crosshair are withdrawn.

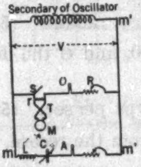

Fig. 15—Method of Determining Quadrantal Points

It should be remembered that the angular deflection of the beam of light reflected from the oscillograph's mirror is double the angular deflection of the mirror. Since the equations refer to the angular displacements of the mirror, this factor of 2 should be taken into account in interpreting the deflections.

Alternative Methods. Instead of determining B by the ratio of specific deflections θ_s and θ_o, it is possible, after finding ω_o, to find Δ, and hence B, by identifying optically one, or preferably both of the quadrantal angular velocities ω_1 and ω_2. The electrical connections for this are shown in Fig. 15. The plan is to introduce such equal resistance and reactance into the auxiliary branch circuit, as shall cause the exciting current and v.m.t., in one circuit, to differ in phase from that in the other by

45 deg., and so to enable quadrantal frequency to be identified optically from a Lissajous diagram. After observing ω_o, and then say ω_1, we can compute ω_2 by (**6**). Again, observing all three values ω_1, ω_o and ω_2, they should check by (**6**). We then find Δ by (**16**), and *B* by (**19**).

This alternative method enables a calibration of the oscillograph at resonant frequency to be dispensed with. On the other hand, it is apt to be less precise than the preferred method; because Δ is found as the difference of two identified frequencies. With sharp vibrators, these two quadrantal frequencies lie close together, and their difference is thus subject to relatively considerable error. In complete and thorough tests of an oscillograph, however, it may be worth while identifying all three values ω_o, ω_1, and ω_2, besides finding θ_s and θ; so as to secure mutual checks.

Another method for finding Δ and ω_o, without the use of the oscillographmeter, when the vibrator is sharp, is to make an oscillogram of a series of naturally decaying oscillations of the mirror, impressing on the same record a known fairly high frequency, from the movement of a second vibrator separately excited. The rate of decay in amplitude is given by the relation

$$\Theta = \Theta_i \, \epsilon^{-\Delta t} \qquad \text{max. cy. radians} \qquad (57)$$

where Θ_i is the initial displacement, at $t = 0$, and Θ the displacement at time t seconds

whence $$\Delta = (1/t) \text{ logh } \Theta_i/\Theta \qquad \text{hyps. per sec.} \qquad (58)$$

The frequency of vibration, as determined from the record, is

$\frac{\omega_f}{2\,\pi}$ cycles per second.

This method can be applied with fair success, to sharp vibrators. It cannot ordinarily be applied when *B* is greater than 0.2. This means that in ordinary liquid-damped oscillographs, this alternative is unsatisfactory.

It might be supposed that since a calibration is desired near to zero frequency, a continuous-current calibration would suffice. It has been found, however, that specific deflections with continuous currents are considerably in excess of those measured with low-frequency alternating currents, apparently because the elastic system has a greater opportunity to stretch under a steady impressed torque than under alternating torques.

A fairly typical and commercial form of oscillograph has its behavior represented in Fig. 16. This oscillograph is damped by a specially prepared liquid, supplied with the instrument. By means of the oscillographmeter, and Lissajous figures, the following frequencies are identified, $f_1 = 2663 \sim$, $f_o = 3300 \sim$, $f_2 = 4077 \sim$. The specific deflection θ_o, at resonance, was observed to be 5.35 mirror radians per r.m.s. absampere. The specific deflection θ_s at 60 ~, was also observed to be 2.25 mirror radians per r.m.s. absampere. The ratio θ_o/θ_s is 2.38; so that $\Lambda = 4.67$ or $B = 0.21$. From ω_o and B, the curve of Fig. 16 has been drawn, by the use of (**40**) and (**41**) or of (**54**). In addition, a number of values of the specific deflection were observed experimentally at different frequencies. These are

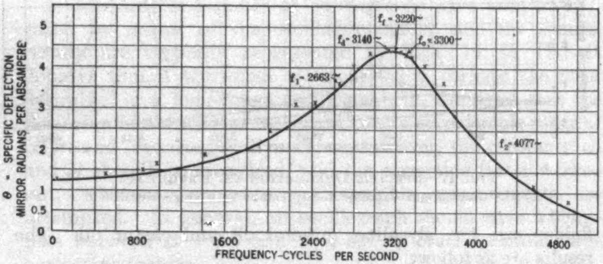

FIG. 16—SPECIFIC DEFLECTION VS. FREQUENCY
h Vibrator in Light Damping Liquid. Δ = 4450 hyps. per second

marked as crosses on the diagram. If these observations were in complete accordance with the theory, the crosses should coincide with the curve. It will be noted that θ_d has the value 5.4 mirror radians per absampere at 3140 ~, which is 160 ~ below f_o. In this instrument, at 1000 ~, the size of the deviation factor is 1.09, or the instrument overindicates 9 per cent at 1000 ~, with respect to the low-frequency calibration. The slope of the deviating factor is also $\beta = -8$ deg. by (**40**).

It may be observed that a curve of specific deflection θ as ordinates, against frequency f as abscissas, always rises to a maximum at f_d, unless B exceeds unity. The curve is always dissymmetrical about the maximum ordinate, being steeper on the high-frequency side. But if the curve of θ as ordinates be drawn against frequency squared, or f^2, as abscissas, the curve will be symmetrical about the maximum ordinate, as far as f_{s2}, *i. e.*, as far as symmetry is possible.

Fig. 17 gives the curves of specific deflection θ, against impressed frequency f, for a vibrator in four successive damping fluids; *viz.*, air, glycoline, mineral oil and castor oil. The results are as follow:

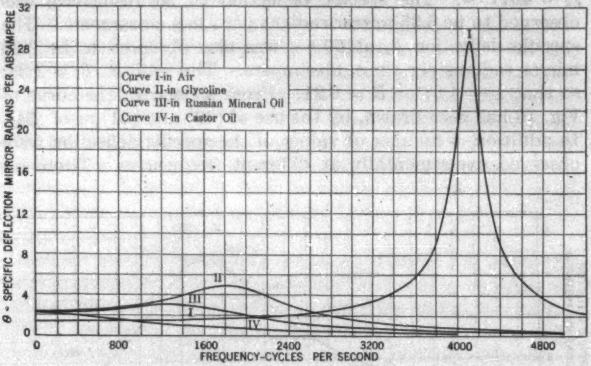

FIG. 17—SPECIFIC DEFLECTION VS. FREQUENCY
Oscillograph Vibrator 0-6. Temp. 26.5 deg. cent.

TABLE II
CONSTANTS OF AN OSCILLOGRAPHIC VIBRATOR IN DIFFERENT DAMPING FLUIDS

Curve	Damping fluid	f_0 cycles / sec	ω_0 rad. / sec.	B_0 numeric	f_1 cycles / sec.	f_2 cycles / sec.	Δ hyps / sec.	fd cycles / sec .	Resonant range cycles / sec.
I	Air	4,110	25,820	0.024	4,013	4,207	609	4,110	194
II	Glycoline	1,891	11,880	0.26	1,523	2,513	3,100	1,827	490
III	Mineral Oil	1,614	10,140	0.43	1,060	2,460	4,400	1,272	1,400
IV	Castor Oil	1,302	8,180	1.1	505	3,365	8,980	0	2,860

Whereas in air, the vibrator had a bluntness of 0.024, or a sharpness of 41.7, in castor oil, the bluntness was increased to 1.1 or a sharpness of 0.9. The effect was also to change the resonant frequency f_o from 4110 ∼ to 1302 ∼.

In case III, the resistance r of the vibrator to motion is 0.43 of that which would render the motion aperiodic. In case IV, however, the resistance is ten per cent greater than that necessary for strict aperiodicity. The case is therefore one of overdamping, or ultraperiodicity. It is developed in greater detail in Fig. 17. The curve is drawn from the formulas already given, and the crosses indicate the observations. It will be seen that $f_d = 0$. At resonance, the specific deflection is only about half that at zero frequency.

It may be noted that except in the air-damped case, the value of θ_s is substantially the same for all the damping fluids. This condition seems to be typical.

A considerable number of oscillographs have been investigated by means of the oscillographmeter, and the general results are typically represented by Figs. 15 to 17. It appears that a bluntness of about 0.6 is perhaps the most desirable, in a long-range oscillograph, in the sense that the deviation factor is then moderate, over a considerable range of impressed frequency. Even then, however, the constants ω_o and B should be measured, and the correcting factor worked out, whenever accurate oscillographic measurements are attempted. In general, an oscillograph should have a bluntness between $B = 0.5$ and $B = 0.8$, in order to avoid large correcting factors.

Tests of the Auxiliary Vibrator. An exploration test of θ versus f for one of the oscillographmeters used in this research, is graphed in Fig. 18. It was also tested by the oscillographmeter method. The vibrator is seen to be very sharp. The three principal frequencies are $f_1 = 2511 \sim$, f_o 2514.5 $\sim$, f_2 2518 $\sim$. The resonant range is 7 cycles per second, and the bluntness $B_o = 0.0014$, $\Lambda_o = 718$. In order to present the peak value of θ on the same sheet as the earlier values, the observations are plotted to two different scales. It is evident that provided the resonant frequency is slightly avoided, the phase departure of the auxiliary mirror from zero or 180 deg. in the v.m.t. will be small.

Precautions to be taken. In supporting the auxiliary vibrator in front of the oscillograph, it is important that the auxiliary steel frame does not come so close to the poles of the oscillograph magnet, as materially to disturb the magnetic field in which the oscillograph vibrator swings. At the same time, the auxiliary and oscillograph mirrors have to be brought within a distance of about 1.3 cm., in order to secure good Lissajous figures, without making the auxiliary mirror unfavorably long. If the magnetic field of the oscillograph is somewhat weakened, this will not appreciably affect the identifiable frequencies f_1, f_o, and f_2. It may affect θ_s and θ_o, however; so that where there is reason to believe that the oscillograph's magnetic field has been affected by the proximity of the oscillographmeter, the specific deflections θ_s and θ_o should be measured with the oscillographmeter removed.

It should be noted that any dissymmetry in the construction of a vibrator, such as unequal dimensions, or unequal tensions in the two strips composing it, or a dissymmetrical mounting of the mirror, may give rise to plural resonances in the action of the instrument. A number of such cases have been observed. A good airdamped oscillographic vibrator requires good mechanical workmanship. When the vibrator is liquid damped these plural resonances often disappear.

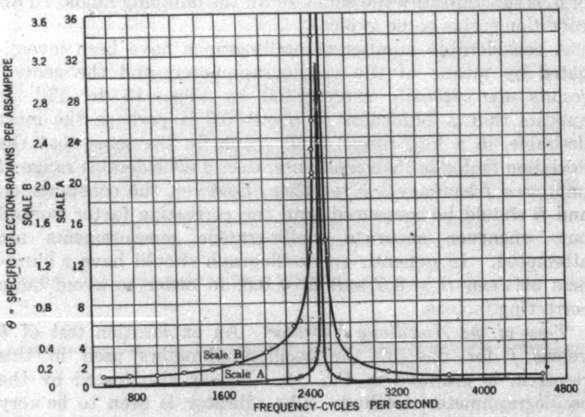

FIG. 18—SPECIFIC DEFLECTION VS. FREQUENCY
Auxiliary Vibrator A-1. Temp. 22.5 deg. cent.

Measurements Necessary for Evaluating A, m, r and s. In order to evaluate the fundamental constants of an oscillograph vibrator, it seems necessary to obtain an additional independent datum, such as is supplied by measuring the motional impedance of the vibrator at resonant frequency. This motional impedance is a pure resistance, and, unlike that of the telephone receiver, has no reactive component at resonant frequency. It is, however, relatively very small. Whereas the resonant motional impedance of a telephone receiver is sometimes over 1000 ohms, and is often over 100 ohms, that of a vibration galvanometer near $f_o = 1000 \sim$ may be near 10 ohms*; that of an ordinary liquid-damped oscillograph is in the neighborhood of 0.025 ohm. Airdamped oscillographs have larger motional impedances.

If we call Z the measured motional impedance of the oscillograph vibrator, at resonant frequency, expressed in absohms, and θ_m the observed max. cy. mirror displacement at resonant frequency per max. cy. absampere; then*

$$A = \frac{Z}{\theta_m\ \omega_o} \qquad \frac{\text{dynes perp. cm.}}{\text{absampere}} \angle \tag{59}$$

$$r = \frac{A}{\theta_m\ \omega_o} \qquad \frac{\text{dyne perp. cm.}}{\text{mirror radian per sec.}} \tag{60}$$

$$m = \frac{r}{2\ \Delta} \qquad \text{gm.} - \text{cm.}^2 \tag{61}$$

$$s = m\ \omega_o^2 \qquad \frac{\text{dyne perp. cm.}}{\text{mirror radian}} \tag{62}$$

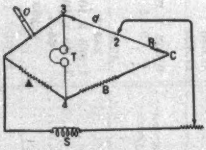

Fig. 19—Slide-Wire Bridge
For measuring the motional impedance of an oscillograph vibrator O

An analysis of several oscillographic vibrators is given in Table III.

Technique for Measuring Motional Impedance. A convenient form of slidewire Wheatstone bridge is indicated in Fig. 19, for the purpose of measuring the small motional impedance of a liquid-damped oscillograph. Two equal anti-inductive resistances A and B have about one ohm each. The oscillator, supplying the adjustable impressed frequency, is brought to slider 2 and junction 1. The slider 2 can be adjusted to a fraction of a millimeter, over a manganin slide wire, having a linear resistance not exceeding 0.01 ohm per cm. A pair of low-impedance head telephones T, enables a re-

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*Bibliography 13.
*Bibliography 13.

TABLE III. DATA FOR SOME OSCILLOGRAPH VIBRATORS TESTED

Vibrator Damping fluid	V_1 Glycoline	V2 Glycoline	V3 Glycerine	V4 Glycoline	V5 Glycoline	V6 Glycoline
Span, cm	....	1.26	1.26	1.18	1.25	1.25
Strip { width, cm	....	0.0350	0.0350	0.0350	0.0372	0.0372
Strip { thickness, cm	....	0.0025	0.0025	0.0025	0.0028	0.0028
Strip spacing, center to center, cm	....	0.055	0.055	0.056	0.052	0.052
Mirror { length, cm	....	0.150	0.150	0.130	0.180	0.180
Mirror { width, cm	....	0.065	0.065	0.080	0.060	0.060
Mirror { thickness, cm	....	0.014	0.014	0.014	0.014	0.014
Strip tension, kilodynes per strip	....	51.5	51.5	74.5	39.1	96.0
Air-gap of vibrator well in cm	....	0.28	0.28	0.23	0.15	0.15
D-c. resistance, ohms	1.591	0.725	0.725	0.670	1.610	1.610
A-c. resistance at the resonant freq., ohms	1.650	0.753	0.745	0.733	1.739	1.739
f_0, ~	2280	2366	1707	2950	2170	2760
f_1, ~	1862	2010	1246	2520	1670	2240
f_2, ~	2790	2780	2340	3480	2815	3390
ω_0, rad/sec	14,320	14,840	10,720	18,520	13,630	17,320
ω_1, rad/sec	11,680	12,620	7,825	15,730	10,480	14,050
ω_2, rad/sec	17,520	17,460	14,705	21,840	17,650	21,300
Z_m, absohms	0.032×10^9	0.0219×10^9	0.0054×10^9	0.02085×10^9	0.0201×10^9	0.030×10^9
θ, rad/absamp	9.15	8.55	2.62	4.41	12.4	8.38
Δ, hyp. rad/sec	2920	2420	3440	3055	3580	3655
A, dyne-p-cm./absamp	244.5	173.0	192.5	255.0	119.0	207.0
r, dyne-p-cm./rad. per sec	1.865×10^{-3}	1.364×10^{-3}	6.85×10^{-3}	3.124×10^{-3}	0.705×10^{-3}	1.428×10^{-3}
m, gm.-cm ^{2}	31.95×10^{-8}	28.3×10^{-8}	99.6×10^{-8}	51.15×10^{-8}	9.85×10^{-8}	19.56×10^{-8}
s, dyne-p-cm./rad	65.5	62.1	114.5	175.3	18.2	58.6
B_0	0.203	0.1295	0.320	0.1645	0.262	0.211

TABLE IV. OSCILLOGRAPHIC RESULTS

			From observations of f_0, θ_0 and θ_S						From Lissajous figures			
Vibrator	Damping fluid	Temp deg. cent.	Δ	f_0 ~	fd ~	ff ~	f_1 ~	f_2 ~	f_1 ~	f_2 ~	w_0	B_0
1	2	3	4	5	6	7	8	9	10	11	12	13
0—1	Castor oil	27.5	5,750	1,486	721	1,170	827	2,657	813	2,570	9,388	0.615
0—3	Air	22.5	1,010	3,235	3,220	3,228	3,079	3,401	3,100	3,420	20,320	0.050
0—4	Air	27	1.430	2,985	2,970	2,978	2,767	3,223	2,760	3,215	18,755	0.096
0—5	Air	28.5	823	2,880	2,875	2,878	2,751	3,013	2,760	3,000	18,095	0.0455
0—5	Castor oil	28	5,700	1,151	Imag	707	561	2,375	513.5	2,475	7,230	0.79
0—6	Air	25	609	4,110	4,110	4,110	4,013	4,207	4,010	4,240	25,830	0.0236
0—6	Glycoline	26	3,100	1,957	1,827	1,891	1,523	2,513	1,464	2,645	12,300	0.252
0—6	Russian Min'l Oil	28	4,400	1,614	1,272	1,454	1,060	2,460	993	....	10,140	0.434
0—6	Castor Oil	27	8,980	1,302	Imag	Imag	505	3,365	517	....	8,172	1.10
0—7	Air	26	92	2,635	2,335	2,635	2,620	2,650	2,615	2,650	16,560	0.0055
0—7	Glycoline	27	1,320	1,674	1,650	1,662	1,475	1,895	1,444	1,957	10,520	0.125
0—7	Russian Min'l Oil	25	3,000	1,614	1,460	1,540	1,202	2,158	1,246	2,133	10,140	0.288
0—7	Castor Oil	27	6,240	1,196	Imag	665	560	2,544	555	2,520	7,515	0.83
0—8	Air	27	1,620	4,210	4,195	4,203	3,962	4,468	3,830	4,570	26,460	0.0613
0—8	Russian Min'l Oil	26	4,150	2,010	1,780	1,900	1,455	2,775	1,404	2,755	12,630	0.33
0—8	Glycoline	27	3,142	2,323	2,215	2,270	1,880	2,880	1,932	2,820	14,600	0.215
0—8	Castor Oil	27	12,400	1,674	Imag	Imag	615	4,565	634	....	10,520	1.18
G	Light damping liquid	....	4,450	3,300	3,140	3,220	2,663	4,077	2,520	4,070	20,740	0.214.

sistance balance to be obtained on the oscillograph vibrator O. An extraneous resistance R, connects the end of the slidewire and junction C. A zero balance is first obtained at the resonant frequency f_o, and then at any frequency well removed from resonance. The change in the setting of the slider thus produced enables the motional impedance Z absohms to be evaluated.

Acknowledgments. The authors desire to express their acknowledgments to the American Telephone and Telegraph Co., under an appropriation from which, the experimental researches here reported were carried out; also to Mr. H. G. Crane of Boston, who has had much experience with air-damped oscillographs, for his care, skill and courtesy in preparing for us the improved form of oscillographmeter shown and described in the paper. It is believed that he is willing to furnish similar instruments on demand.

SUMMARY

1. Every mechanical oscillograph vibrator is subject to a vector correction factor, at other frequencies than that at which it is calibrated, and especially at high frequencies.

2. The mechanical laws governing an oscillographic vibrator are precisely similar to those governing a simple alternating-current branch circuit of fixed C, L and R, under variable impressed frequency. The velocity-impedance graph is a straight line. The displacement-impedance graph is a parabola.

3. Every oscillogram, intended for accurate analysis, is completely defined in regard to its correction for frequency by two constants; namely, f_o the resonant frequency, and B_o the oscillatory bluntness. These two constants, or their equivalent data, should be measured, and recorded on the oscillogram.

4. If the two constants f_o and B_o,—the resonant frequency and bluntness of an oscillograph,—are measured, its behavior and correction factors for all frequencies can be computed relatively to its calibration at any particular frequency.

5. In ordinary liquid-damped oscillographs, f_o commonly lies between 1000 $\sim$ and 5000 $\sim$. The greater the damping, the lower f_o tends to become.

6. The bluntness of resonance B_o indicates the ratio of the frictional resistance present to the resistance necessary for strict aperiodicity. When $B_o = 1$, the vibrator is just aperio-

dic. The bluntness of an air-damped vibrator may be as low as 0.0014. Oscillographs damped in castor oil may readily have bluntness greater than 1, or be ultraperiodic. The best bluntness for a long-range oscillograph, in order to have relatively small corrections to be applied at moderately high frequencies is, perhaps, near $B_o = 0.6$.

7. The value of f_o for an oscillograph may conveniently be determined by means of a simple permanent-magnet auxiliary vibrator, operating from the same source, and so arranged as to supply a critical and recognizable form of Lissajous optical figure at resonance.

8. Having identified f_o for an oscillograph, the bluntness B_o may be determined, by measuring the specific deflection at a low frequency and also at the resonant frequency.

9. A number of actual oscillographs have been tested, and typical results, obtained from them, are reported.

10. In order to obtain the fundamental constants A, m, r and s, of a vibrator, it is desirable to measure its motional impedance at resonance, in addition to securing the measurements for f_o and B.

11. In analyzing the oscillograph of a periodically recurring complex harmonic wave into its Fourier components, the analysis is first made, as though the oscillograph had no error with change of frequency, and then each evaluated harmonic component should be corrected, both in magnitude and in phase position, by applying the correction factor for that frequency, in view of the values of f_o and B for the instrument.

12. A number of curves of deviation factors for different values of B have been worked out, and are presented for use, in order to supply approximate corrections by inspection.

LIST OF SYMBOLS EMPLOYED

A	Torque constant of a vibrator, or the torque produced by unit current $\left(\frac{\text{dyne perp. cm.}}{\text{absampere}}\angle\right)$.
$\alpha = \tan^{-1}\left(\frac{L\,\omega - 1/C\,\omega}{R}\right)$	Slope of an impedance to current (radians or degrees).
$B = \Delta/\omega_o = B_o$	Bluntness of resonance of a vibrator or branch circuit based on τ_o (numeric).

$B_s = 2\,B_0$	Bluntness of resonance of a vibrator or branch circuit based on τ_s (numeric).
β	Slope of a displacement admittance, varying between − 0 deg. and − 180 deg. (degrees)
C	Capacitance of a condenser (farads).
$c = 1/s$	Slackness of a vibrator $\left(\frac{\text{radians}}{\text{dyne perp. cm.}}\right)$.
Δ	Damping constant of a vibrator, or hyp. angular velocity of decay $\left(\frac{\text{hyps.}}{\text{sec.}}\right)$.
$D = y'/y_s'$	Deviation factor, or ratio of vector displacement at ω to that at ω_s (numeric $\angle$)
E	Maximum cyclic e.m.f. impressed on a branch circuit (volts $\angle$).
F	Maximum cyclic torque impressed on a vibrator (dynes-perp-cm.).
f, f_0, f_1, f_2	Frequency, with reference to resonance and quadrantal values (cycles per sec.).
G	Conductance of a branch circuit (mhos).
$g = 1/r$	Mechanical conductance of a vibrator (mechanical abmhos).
I	Alternating current supplied to a branch circuit (max. cyclic amperes $\angle$). also alternating current supplied to a vibrator (max. cyclic absamperes $\angle$).
Θ	Maximum cyclic displacement produced in a vibrator mirror (radians $\angle$).
Θ_i	Initial cyclic displacement produced in a vibrator mirror (radians $\angle$).
$\dot{\Theta}$	Maximum cyclic angular velocity produced in a vibrator mirror $\left(\frac{\text{radians}}{\text{sec.}}\angle\right)$.
$\theta = \Theta/I$	Specific mirror deflection, or max. cy. displacement per r.m.s. absampere, produced in a vibrator $\left(\frac{\text{radians}}{\text{absampere}}\angle\right)$.
θ_s, θ_0	Specific deflections produced at a low frequency, and at resonant frequency $\left(\frac{\text{radians}}{\text{absampere}}\angle\right)$.

θ_m	Max. cy. displacement per max. cy. absampere produced in a vibrator $\left(\frac{\text{max. cy. radians}}{\text{max. cy. absamp.}} \angle\right)$.
$j = \sqrt{-1}$	
$K = 1/D$	Correction factor for the max. cy. displacement at any impressed ang. vel. ω, to that at ω_s (numeric $\angle$).
L	Constant inductance of a branch circuit (henrys).
m	Moment of inertia of a vibrator system (gm — cm.^{2}).
Q	Quantity of electricity cyclically displaced in a branch circuit (coulombs).
R	Constant resistance of a branch circuit (ohms)
r	Resistance to motion of a vibrator $\left(\text{mechanical absohms or } \frac{\text{dyne-perp.-cm.}}{\text{radian per sec.}}\right)$
s	Stiffness coefficient of a vibrator $\left(\frac{\text{dynes perp. cm.}}{\text{radian}}\right)$.
t	Elapsed time (seconds).
$\tau = \tau_0 = 1/\Delta$ $= \frac{2L}{R}$	Oscillatory time constant of a circuit or system $\left(\frac{\text{seconds}}{\text{hyp. radian}} \text{ or seconds}\right)$.
$\tau_s = \tau_0/2 = L/R$	Non-oscillatory time constant of a circuit (seconds).
$u = \omega/\omega_0$	Ratio of an impressed angular velocity to the resonant angular velocity of a system (numeric).
$\Lambda = \omega_0/\Delta$ $= \Lambda_0 = 2\,\Lambda_s$	Sharpness of resonance of a branch circuit or vibrator based on τ_0 (numeric).
$\Lambda_s = \Lambda_0/2$	Sharpness of resonance of a branch circuit or vibrator based on τ_s (numeric).

v.m.t.	Vibromotive torque (dynes perp. cm.).
$Y = 1/Z$	Admittance to alternating current in a branch circuit (mhos $\angle$).
$y = 1/z$	Admittance to angular velocity in a vibrator $\left(\text{mechanical abmhos or } \frac{\text{radians per sec.}}{\text{dynes perp. cm.}} \angle\right)$
$Y' = 1/Z'$	Admittance to displacement in a branch circuit $\left(\frac{\text{coulombs}}{\text{volts}} \angle\right)$.
$y' = 1/z'$	Admittance to displacement in a vibrator $\left(\frac{\text{mirror radians}}{\text{dyne perp. cm.}} \angle\right)$.
Z	Impedance to alternating current in a branch circuit (ohms $\angle$).
z	Impedance to angular velocity in a vibrator $\left(\text{mechanical absohms or } \frac{\text{dynes-perp. cm.}}{\text{radians per sec.}} \angle\right)$.
Z'	Impedance to displacement in a branch circuit $\left(\frac{\text{volts}}{\text{coulombs}} \angle\right)$.
z'	Impedance to displacement in a branch vibrator $\left(\frac{\text{dynes perp. cm.}}{\text{radian}} \angle\right)$.
ω	Impressed angular velocity of circuit or vibratory system $\left(\frac{\text{radians}}{\text{sec.}}\right)$.
$\omega_0, \omega_1, \omega_2$	Impressed angular velocity of resonance and quadrantal values $\left(\frac{\text{radians}}{\text{sec.}}\right)$.
$\angle$	Sign of planevector or complex quantities.

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DISCUSSION ON "OSCILLOGRAPHS AND THEIR TESTS" (KENNELLY, HUNTER AND PRIOR), NEW YORK, N. Y., FEBRUARY 20, 1920.

F. S. Dellenbaugh, Jr.: The paper in addition to a very valuable method of calibrating oscillographs, presents a most interesting change in the philosophy of attacking problems of this sort. It was not so very long ago, 25 years at the most, when electrical circuits were so little understood that mechanical analogies were used as devices to explain electrical action. Now the electrical circuit can be analyzed so simply that the method of attack is completely reversed, and electrical analogies are used to explain mechanical action.

It also is the first time, to the best of my knowledge, that the term Displacement Impedance has been used, and an equation derived for it.

It has been seen from the illustrations that the locus of this impedance is a parabola in the first and second quadrants, with $y = o$ at $x = 2$ and extending to $x = -6$. The upper half of the parabola only is given however, and if we complete the curve and examine the meaning of the additional branch we find a more generalized application of the impedance equation.

The equation of the parabola is:

$$Z' = (S - L\,\omega^2) + j\,R\,\omega$$

So plotted that $(S - L\,\omega^2)$ forms the abscissas and $j\,R\,\omega$ forms the ordinates. Thus any line drawn from the origin to the curve represents the impedance vector and its projections on the axes give its coordinates in the form of a complex number.

It is evident that in order to have the lower part of the parabola, R must become negative, since $j\,R\,\omega$ must be negative, and ω cannot be negative, as that would also affect the abscissas. Therefore the generalized curve involves $-R$. In the electrical circuit this of course means a generated e. m. f. In the mechanical circuit this means negative friction, or an applied external force.

If we substitute a term "e" representing a generated voltage in the electrical circuit, and specify that it must be of the same frequency and in phase with the applied e. m. f. we obtain the equation:

$$V = i\,Z = i\,R + e + j\,i\,(L\omega - S/\omega)$$

From which

$$i\,Z'' = i\,(S - L\,\omega^2) + j\,(R\,i\,\omega + e\,\omega) = i\,(S - L\,\omega^2) + j\,\omega\,(R\,i + e)$$

Where V is the applied voltage and i the resulting current, the whole equation being put in the voltage form in order to introduce e unmodified. It is of course possible to replace any voltage e by a resistance drop $i\,R'$, and if we let:

$$R' = -\,K\,R$$

Then

$$Z'' = (S - L\,\omega^2) - j\,R\,(K - 1)\,\omega$$

If K is greater than 1.

Thus for various values of K we get a family of curves all having the same general form as the locus under consideration and converging from both sides of the X-axis to a straight line coincident with the X-axis when K is unity.

When $K = 2$ we get the lower half of the same curve that is already represented in the positive ordinate portion presented.

The impedance now is:

$$Z'' = (S - L\,\omega^2) - j\,R\,\omega$$

That is, the displacement of the current which will pass through this circuit is equal and opposite to that which would have passed through the circuit represented by the original equation for Z'. In other words the system is delivering power at the same rate at which it previously absorbed it, and all phase relations, must, *a priori*, remain the same, except that the current has shifted 180 deg.

Following out the same line of reasoning as applied in the original analogy we find for the mechanical case that the lower half of the parabola is the locus of the mirror displacement on an oscillograph which must be obtained by applying an external force as a torque producing forced oscillations of the mirror of the same magnitude and the opposite phase as would have held for the free case, the value of the force being twice the friction and always opposite it in direction.

In other words the oscillograph will now deliver as a generator current of the same characteristics as it previously absorbed as an instrument, but opposite in sign.

This does not have any immediate useful application but extends the scope of the analogy. It is interesting to note that a generator similar to this type has actually been proposed, and I believe constructed, in connection with producing small amounts of high-frequency power for testing circuits in radio telegraphy. It is also interesting to note that the efficiency of such a generator may be made quite high, as there is no inherent limitation other than the usual friction and resistance losses.

As a conclusion from this expansion it might be stated that the lower half of the parabola should be added and the complete curve drawn, for although in the immediate application its values are fictitious, other cases of similar mechanical analogy might give it real importance.

B. W. St. Clair: I think that this is one of the best contributions to the oscillograph art we have had in a number of years. Anyone who has had to do with general testing, and especially with development work will appreciate that there are many limitations to our present oscillographs. There are occasions when it has been necessary to give unsatisfactory answers to inquisitive engineers about the faithfulness with which reports represented the happenings in some complicated circuit or machine.

The oscillograph has proven to be one of the finest engineering instruments we possess and it has enabled us to solve many problems that could not have been approached without it. Its range of usefulness, however, has not been as extensive as might be desired on account of the rather low natural period of the vibrator. For that reason we have had to effect a compromise between sensitivity and natural period. In cases where the sensitivity had to be much increased, the natural period was often lowered so far as to make the accurate recording of the higher harmonics of a 60-cycle wave impossible. The normal vibrator takes considerable current for its operation, and, there are many instances where we have wished we might increase its sensitivity a hundred fold. It is possible in extreme cases to increase the sensitivity fifty fold. But this can only be done by a serious lowering of the natural period. This, of course, represented a compromise between accuracy of recording and ability to get measurable deflections. Until the advent of Dr. Kennelly's paper it was impossible to correct the oscillogram for limitations of the vibrator at high frequencies.

There is another application of Dr. Kennelly's work that is going to be very helpful in modern oscillograph practise. It is the ease with which it is now possible to determine the damping factors of our vibrators. Previously, every time it was necessary to make a serious change in the vibrator, either in tension on the strings, or the size of the mirror, or the dimensions of the strings, it was necessary to make elaborate tests to determine how near aperiodic the damping was. It involved the making of films at high speeds showing the way the vibrator returned to zero when suddenly disconnected from a storage battery. Several times in cases where it was necessary to use increased tension, the damping was so much less than critical that the vibrator made appreciable deflections on the negative side of the zero line. In general, it has been our experience that with oil damped vibrators the damping is excessive; that is, there is too much frictional resistance for the vibrator to be aperiodic. It will now be possible to determine how far our damping is from critical by making a determination of Dr. Kennelly's "bluntness constant." This is a much easier thing to do than the older test for damping.

This question of damping is a very important one, especially on impulse work. A number of cases have come to our attention where erroneous results were given because our damping was not sufficiently great. There have been cases where it has been necessary to increase the vibrator tension almost to the breaking point, and to decrease the size of the mirrors almost to the vanishing point in order to get a sufficiently high period for some particular impulse test. Experience showed that every vibrator under these conditions needed checking for damping. The determination of the bluntness constant will be easier than the making of films of the damping. We cannot use Dr. Kennelly's methods in their present form directly for the correction of impulse results, but they will be useful for checking damping, which is so very important in this kind of work.

Again it will be possible to make this determination at the point where the oscillograph is to be used. In many cases it is necessary to use the instrument in hot engine rooms where the damping is far from what it was in the laboratory.

N. E. Bonn: Mr. St. Clair brings out the fact that it requires a tremendous amount of skill to accurately operate the oscillograph. You not only have to take pictures and develop them, but you have got to measure quantities such as B and F in the damping, and the resonant frequency of the oscillograph element. The oscillograph is limited in its usefulness to a certain amount of energy, that is, since we must use the vibrator very near the critical damping point, the use of a varying coil is necessary which means that you cannot detect very small amounts of electric current. The sensitivity is materially decreased. In certain cases I can see how the oscillograph cannot be used, because it may be that the natural frequency of the vibrator, by which you make the measurements, is just as high as, or perhaps higher than, the natural frequency of the oscillograph meter.

It might be well to mention here that there are certain classes of work in which the old point to point method is still preferable to the oscillograph, because it is much more simple, and there is no chance of errors which have to be corrected, especially if you consider some of the more modern types of the point to point methods, such as the Rosa curve tracer. That is an instrument that gives absolutely correct results and at the same time reduces the labor of getting the wave shape to a fraction of its former value. It is capable of plotting curves in a comparatively short time, with very great accuracy, because it follows the potentiometer method of measurement.

We have taken a further step in that direction and now we think we will be able to develop an instrument which will be entirely automatic, which will automatically plot the coil—of course, if the coil is periodic, it must be a periodic phenomenon or must be capable of being made periodic by some external means. The instrument which is being developed is going to be altogether automatic, so that the only thing to be required will be to press a button and throw a switch, and the instrument will automatically give you the true shape of the curve you are looking for within certain frequencies. I think that 2000 frequency is the upper limit of the point to point method of wave-shape determination, because of the impossibility of getting a contact of very short duration.

M. A. Rusher (by letter): I agree with the authors of this paper that for certain extremely accurate tests in which high frequencies are involved, more attention should be given to the matter of calibrations than has usually been given. One who is not well acquainted with the use of the oscillograph is likely, however, to infer from this paper that the operation of the oscillograph is inherently bad, which is not true. New uses are being made of it everyday with perfectly satisfactory results.

Assuming that the apparatus and tests here presented are the best possible, it is still true that it has no application to all of those tests involving transient phenomena. Moreover, it is not necessary to apply any such correction to the wide range of investigations involving direct current and alternating current of low frequencies only. Even in the case of wave shape records, which seems to be the case which has received the most attention, I believe that results will show that the corrections are small except in relatively few cases.

The tests suggested do not seem to be so readily made as indicated. Unless they are made under the exact conditions of test in regard to temperature of damping liquid, etc., I do not see that they are of much benefit. It appears to me that it would be necessary to remove the complete galvanometer from the oscillograph and either place it in a special box with special optical system and the oscillographmeter or else have these properly mounted in a dark room where the necessary Vreeland Oscillator is available for the test. Tests of resonant frequency should be made with the damping liquid at different temperatures and a curve plotted between temperature and resonant frequency.

By knowing the temperature of the damping liquid in any later test, the proper corrections could then be applied.

In order to test the oscillographmeter, advantage can be taken of the fact that the resonant frequency without damping is the same as the natural free period of the vibrator. This can be very easily determined by taking an oscillogram at the instant of disconnecting a direct current from the vibrator circuit, at the same time recording a wave of known frequency from another vibrator. The period of the vibration without damping can be found by comparing the two waves over any given period. It would simplify the matter of corrections if all could be referred to this natural free period of vibration as it can be determined without any special apparatus. I believe that when standard vibrators with the ordinary damping liquid are used, it would only be necessary to make the tests suggested by the authors on a few vibrators at different temperatures of damping liquid and from these, constants could be determined which would apply to any vibrator of the same type. The accuracy would be almost as great as if the tests were taken on each vibrator at sometime previous to using it in the oscillograph.

A. E. Kennelly: Mr. Dellenbaugh has shown us that the parabolic curve which is only a half parabola in the paper, is really part and parcel of a whole parabola, the lower part being the generator part, and the upper part the motor part. Mr. Dellenbaugh is the father of the lower half of this parabola. Cutting the parabola in two in the middle is always an unscientific process, from the geometrical standpoint, and a complete parabola is always preferable if only on theoretical grounds.

Mr. Hunter has contributed largely to this work by a great deal of patient observation, and I am sorry he did not extend his remarks.

Mr. Bonn has referred to the wave tracer, and there is room in electrical laboratories for all kinds of instruments. We want oscillographs and wave tracers, and there is always room for various types, and I am glad to know that a wave tracer is likely to be developed which does not take too long to get around the cycle. As a rule, by the time you get around the cycle, when you start to trace the waves the second time, one is apt to find the wave has changed, and when you start to trace it the third time, one finds that the wave is still further changed. If you could get the tail end before the head changed, you might get a complete curve, and it is gratifying to know that we may get the tail and head to agree with the new instrument that is suggested.

The oscillographmeter is not limited by its particular frequency of resonance. All that is necessary is that its resonance shall not be near to the resonance of the tested instrument. It may be above or below and if it is used above the natural frequency, the results are as good as if used below. We have used it both ways, and it is hard to tell the difference.

The remarks of Mr. St. Clair are very interesting. I am afraid that test of his may still be necessary to determine the damping for impulse work, because the work described in this paper applies mainly to sustained oscillations and is not directed to impulse oscillograms which differ as much from sustained oscillograms and wave forms as free oscillations in electric circuits differ from sustained oscillations. There is another chapter to be contributed by somebody in dealing with impulse oscillograms.

Presented at the 8th Midwinter Convention of
the American Institute of Electrical Engineers,
New York, February 20, 1920.

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THE ACCURACY OF COMMERCIAL ELECTRICAL MEASUREMENTS

BY H. B. BROOKS
Physicist U. S. Bureau of Standards

The paper discusses the accuracy required in commercial electrical measurements, and the means of obtaining it, namely, proper choice, installation, use, and maintenance of instruments. Conditions of use, external disturbing influences, and features of design and construction affecting the accuracy are considered in detail. The best types of instrument for measuring voltage and current are mentioned.

The sources of error in electrodynamic wattmeters are discussed, including the effect of instrument transformer errors. The principal factors affecting the accuracy of watthour meters are given. In conclusion, some improvements which should be soon forthcoming are mentioned.

1. INTRODUCTION; DEFINITIONS

THE object of this paper is to discuss the need for accuracy in practical electrical measurements,* the means by which it may be obtained, and the effect of external conditions and manner of use upon the results. The discussion is limited mainly to commercial measurements, and is intended more especially for the average user who is neither ignorant of the subject nor a laboratory expert. It is hoped, however, that the latter, as well as designers and makers of instruments, may find some points of interest.

It is not often realized to what extent measurement runs through the whole structure which the electrical engineer has created. We have at our disposal such varied means for electrical generation, distribution, conversion and utilization that we are apt to think of measurement only in terms of electrical instruments and meters. Yet almost any device we can mention, as for example an incandescent lamp, requires in its manufacture a great variety of measurements, each of which must be made with some definite accuracy if the lamp is to give a satisfactory performance. The control of a large electric system by a load dispatcher is another example. Here the state of affairs in the various parts of the system is shown by the instrument readings, and we may liken the measuring apparatus in this case to the nervous system by which the brain is kept constantly advised of the condition of the entire body.

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*This paper is written from the laboratory viewpoint, but the effects of service conditions and operation are rather fully discussed. It is not possible to secure as high accuracy in the field as in the laboratory, and in comparing service performance with laboratory performance this fact should be kept in mind.

Accuracy should be clearly distinguished from sensitivity. An instrument is sensitive when a relatively small change in the quantity under measurement produces an appreciable motion of the index; it is accurate when its indications differ only slightly from those of suitable standard instruments. A well-made ammeter is sensitive to 0.1 per cent at or near full-scale reading, but may fall short of being accurate by several per cent because of bad adjustment.[1]

While initial accuracy is important, sustained accuracy in service is what counts in the long run, and it is quite possible in some cases to get better sustained accuracy from certain types of instruments than from others of much higher initial accuracy but of more delicate and intricate construction.

The accuracy of an instrument may be expressed in several ways. The simplest and in many respects the most unsatisfactory is to state that the instrument is accurate within a certain percentage. Such a statement can apply to only a part of the scale. For example, it was at one time customary for the maker to certify that precision direct-current portable voltmeters were accurate to 0.2 per cent. At full scale (say 150 divisions) this amounts to 0.3 of a division, and such an accuracy is quite possible. However, at 50 divisions the same percentage error is only 0.1 division, which is approaching the limit of reading. Such a guarantee should be qualified by stating the part of the scale over which it applies. Even then, this permits errors of increasing amount, expressed in divisions, as the upper end of the scale is approached, which from the user's standpoint is an unnecessary limitation.

A method much superior to the preceding is to certify that the instrument is correct to within a given percentage of the full scale value. Such a guarantee is stated (or understood) to apply over the entire scale, and in uniform-scale instruments amounts to saying that no division mark is out of its proper position by more than a certain linear distance. For instruments which are to be used for general testing or experimental work and will therefore be used over a considerable part of the scale, such a guarantee seems to be a satisfactory one for all concerned.

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[1] For a more detailed discussion of the terms sensitivity and accuracy, see "Variance in Measuring Instruments," by F. J. Schlink, Scientific Paper No. 328 of the Bureau of Standards.

2. Accuracy Requirements

It is not easy to specify just what accuracy is required for a given class of electrical instruments. It is a waste of money to expend labor in adjusting apparatus to a higher accuracy than the work requires, and it is no less so to vitiate the results of an important test by the use of one or more instruments of insufficient accuracy. Aside from their extra cost, instruments of needlessly high accuracy are sometimes less rugged than cheaper ones of sufficient accuracy.

However, it is not usually practicable for the maker to determine which individual instruments will be used for approximate measurements and which for work requiring greater accuracy. In general, therefore, he must adjust all instruments of a given type and grade to an accuracy appreciably greater than the lowest tolerable for some particular purpose. For his own guidance and that of the user, he should have standard requirements for instruments of definite grades, and all instruments should be clearly marked to show their grade. This need was recognized in England ten years ago by the Engineering Standards Committee, which issued the "British Standard Specification for Ammeters and Voltmeters." A revised edition has just been issued, covering also wattmeters, frequency and power factor meters. Graphic ammeters, voltmeters, and wattmeters are the subject of a separate specification. Ammeters, voltmeters, and wattmeters are divided into three grades, namely, Sub-Standards, First Grade, and Second Grade. Each instrument must be marked to show the grade to which it belongs.[2]

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[2] The accuracies required by the British Standard Specification are as follows:

Kind and Grade		Accuracy
Voltmeters, sub-standard	±	0.2% of full scale value.
Ammeters, sub-standard	±	0.5% of full scale value.
Voltmeters, first grade	±	1% of the reading, above half scale;
	±	0.5% of full-scale value, below half scale.
Ammeters, first grade	±	2% of the reading, above half scale;
	±	1% of full-scale value, below half scale.
Voltmeters, second grade	±	2% of the reading, above half scale;
	±	1% of full-scale value, below half scale.
Ammeters, second grade	±	4% of the reading, above half scale;
	±	2% of full-scale value, below half scale.

In the case of ammeters with external shunts, the error of either the shunt or the millivoltmeter with its leads must not exceed one-half the specified tolerance for the given grade.

In an interesting paper on "Guarantees of Accuracy of Instruments for Industrial Electrical Measurements," G. Campos[3] points out that the radically different method of use of voltmeters and ammeters requires recognition in the specification of accuracy for the two. He says:

> While it is desirable for a voltmeter to be accurate within 1 per cent in the very limited region in which it is used, and even a greater accuracy is useful in many cases, it is on the other hand entirely superfluous that such values be maintained when by some accident the voltage drops to half the normal, at which time it is more than sufficient for the operator or the user to follow roughly the variations, either the coming up, or the renewed fall of the voltage; in such cases 2 or 3, or even 4 per cent accuracy is practically sufficient.

Campos's suggestion implies that the cost of voltmeters could be reduced, or the accuracy over the working part of the scale could be increased for the present cost, if a much lower accuracy were permitted outside the working range.

He also points out that the unnecessary requirement of too high an accuracy not only means a useless increase in cost but may lead to the impairment of some essential qualities. He makes the suggestion that in addition to the more precise but more delicate instruments used for the normal operation of a plant there ought to be provided emergency instruments, especially ammeters, which will give rather rough but reliable indications in case of short circuit or similar accidents.[4]

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[3] *L'Elettrotecnica*, vol. 4, pp. 123-127; 1917.

[4] This applies only to abnormal currents which last for an appreciable time. Direct-current ammeters having this feature have been constructed by Jules Richard of Paris. In this ammeter the moving coil is controlled by the usual two spiral springs over, say, the lower two-thirds of the scale. When the coil moves farther, a third spring also opposes the motion, and thus the current required for a given increment of angular motion is increased. For example, the lower two-thirds of the scale may be graduated from 0 to 100 amperes, while the upper third extends from 100 to 300 amperes. This type of ammeter is used, for example, in connection with motors, the running current being observed on the lower two-thirds of the scale and the large momentary starting currents on the upper third. If an ammeter of the usual construction is used for such a purpose, the angular deflection given by the running current is too small for accurate reading.

3. How Accuracy may be Obtained

A required degree of maintained accuracy in service requires attention to all of the following points: (1) choice of the most suitable operating principle; (2) selection of a competent and experienced maker; (3) correct installation; (4) intelligent use; (5) suitable maintenance and testing.

(a) *Choice of Operating Principle.* For nearly all direct-current purposes the permanent-magnet moving-coil instrument is the most desirable. This type is so well known that no description need be given here. It has a practically uniform scale, which facilitates the mechanical sub-division of the scale between the points laid out by comparison with a standard, and also makes it easier to estimate fractions of a division. This type is made in a great variety of sizes and styles, from which it is possible to select instruments for all but very special requirements.

For alternating current the question of choice is not so simple. There are available five types, namely, moving-iron, electrodynamic,[5] induction, hot-wire, and thermocouple instruments. Electrostatic instruments are available for the measurement of alternating voltage and power, but on account of their delicacy they are more suitable for the laboratory than for commercial work, and they are used only when it is essential or very desirable that no current be taken by the voltage element.

In choosing a-c. ammeters and voltmeters, one finds two schools of thought favoring the moving-iron and the induction type respectively. The advocates of the moving-iron type point out that well made ammeters on this principle are useful with only slight changes in accuracy over the whole commercial range of frequency, and that even at say 500 cycles the change in accuracy as compared with that at ordinary frequency is only a few per cent. Those who champion the induction type point out the relatively low torque and torque-weight ratio and the delicacy of the parts of the moving-iron and electrodynamic types, and contrast these with the ruggedness, high torque and torque-weight ratio, and ease of repair by ordinary operators which characterize the induction instrument. They also point to the widespread use of the induction watthour meter and its practical advantages over the electrodynamic (direct-current) watthour meter. The advocates of the moving-iron and electrodynamic instruments, on the other hand, point out that induction instruments can be adjusted for only one frequency (two in the case of induction ammeters), and hence would be relatively more affected by departures from normal frequency. The matter is not one that can be decided off-hand, but needs to be considered in connection with the circumstances of use, including the kind of labor available for maintenance of instruments. However, it is not amiss to point out that for experimental or other work where the frequency must be varied, the induction type is unsuitable.[6] Other features affecting the choice are the effects of stray magnetic fields, ability of the ammeters to withstand short circuits, and scale length per unit area of space occupied on panels, which favor the induction type, and low impedance of the ammeters which favors the moving-iron type.

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[5] The use of the shorter adjective form "electrodynamic" is suggested as preferable to the use of the somewhat clumsy noun "electrodynamometer" as an adjective. The French have used "électrodynamique" for a number of years, setting us a good example.

For laboratory use at usual frequencies, when the desired accuracy is better than the moving-iron type will give on various frequencies and wave forms, the electrodynamic type is preferable because it can be checked on reversed direct current, and its error due to inductance can be readily computed for any given frequency.

For the measurement of high-frequency currents there is a choice between the hot-wire (expansion) instrument and the more recent thermocouple instrument. The hot-wire instrument has a number of failings, including zero shift, liability of damage from overloads and rough handling, and sluggishness of response to changes of the current. The hot-wire voltmeter is practically independent of the frequency of the circuit. The comparison of a 150-volt hot-wire voltmeter with an electrostatic voltmeter of similar range, first on direct current, then on 180,000 cycles, gave no indication of any effect of frequency, within the limit of reading. The thermocouple instrument has a permanent-magnet moving-coil system which is connected to a thermocouple or a bridge

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6. This may seem like a needless warning, but the writer has in mind the case of a laboratory intended for commercial research and investigation which obtained a large outfit of induction instruments because their limitations were not recognized.

arrangement[7] of thermocouples. It appears probable that the operating advantages of the thermocouple instrument will cause it eventually to supersede the hot-wire instrument. For some purposes, however, the hot-wire instrument has an advantage over the bridge form of thermocouple ammeter in that the former indicates correctly the value of a direct current regardless of its direction, while the thermocouple ammeter may even deflect below zero for one direction of the current.

(b) *Scale Law.* This term may be used to denote the manner in which the length of a division varies over the scale. If this length is constant, the scale is said to be linear. If the scale divisions increase in length going up scale, and in such a way that n times the electrical quantity gives n^2 times the angular deflection, the instrument has a quadratic or square-law scale. The latter are found in the precision ammeter and voltmeter made on the Kelvin balance principle, and the torsion head electrodynamic voltmeter. Many instruments have scale laws which are neither linear nor quadratic; for example, some electrodynamic voltmeters have a scale which for the first half of the angular deflection resembles the quadratic, but which shows a decrease in length of a division from about the middle on to the upper end of the scale. The reason for this departure will now be given, with a discussion of the scale law as affecting the initial and sustained accuracy.

Strictly speaking, no instrument scale is exactly linear or quadratic. Neglecting pivot friction, a permanent-magnet moving-coil instrument would give a scale law more and more nearly linear as the angular length of the scale is reduced. Over the whole scale there will be departures from uniformity of length of the divisions, because of variations in the strength of the magnetic field from point to point along the gap. The electrodynamic voltmeter or ammeter will tend more and more toward the quadratic scale as the angular length of the scale is reduced. There are two distinct causes at work; first, the general operating principle, second, the geometrical relations between the fixed and the moving coils which determine for each position of the moving coil the strength and direction of the magnetic field whose interaction with that of the moving element produces the torque. The latter cause may be called the "geometry" of the operating system, and the way in which it operates may be seen from the following illustration.

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[7] The bridge arrangement of thermocouples is described by P. Gossen, *E. T. Z.* vol. 31, pp. 143-4; 1910.

In an electrodynamic voltmeter in which a moving coil turns within a fixed coil, there are two positions of the moving coil in which no current, however large, can produce torque. One occurs when the coil is turned below its normal zero position until the plane of the moving coil is parallel to the plane of the fixed coil. The other position is above the full scale point, with planes parallel as in the preceding. The latter is a stable position with current flowing, the former is unstable. In the stable position the magnetic fluxes set up by the two coils are additive, while in the unstable they are opposing. Between these two extreme positions the torque corresponding to a given current is not constant, but depends on the form and relative dimensions of the coils. If an arrangement of coils could be used such that the torque for a given current is constant over a certain part of the range of motion of the moving coil, and the scale be limited to that range, then the instrument will have a quadratic scale if made as a voltmeter or an ammeter, or a linear scale if made as a wattmeter. Much effort has been spent in devising forms of coils that would give this result, and some unusual and difficult forms have been used; for example those of Bruger and Raps[8]. However, further research has shown[9] that with proper relative dimensions the desired results can be had with sufficient approximation using simple circular or rectangular coils of very reasonable form. In any case the moving coil must not approach too near the limiting positions of zero torque.

By varying the coil geometry it is possible to give the scale special features. For example, a well-known American portable wattmeter is designed so that the divisions near the zero are 2.7 times as long as those near the upper end of the scale. This scale is approximately a logarithmic[10] one, and tends to give equal percentage accuracy on all deflections.

As an extreme case of what can be done in altering the natural scale law by special geometry may be mentioned a form of electrodynamic ammeter and voltmeter[11] made by Hartmann and Braun. In this instrument, which inherently would tend to the quadratic scale, a special arrangement of coils gives a scale which is nearly linear over more than nine-tenths of the range of current or voltage measured. The instrument possesses great interest as showing what can be done in the way of meeting special requirements with relatively simple coil geometry. The scale of such an instrument is shown in Fig. 1. In general, however, instrument scales should follow the natural law of the instrument, if an efficient design is desired.

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[8] Bruger, *E. T. Z.* vol. 15, pp. 331-4, 1894; Raps, *E. T. Z.* vol. 20, pp. 665-670, 1899.

[9] Sack, *E. T. Z.* vol. 28, pp. 268-271, 1907.

[10] Edgcumbe, Industrial Electrical Measuring Instruments, 2d ed., p. 9.

While agreeing with the theoretical basis of the logarithmic scale, the writer wishes to call attention to a practical defect of electrodynamic voltmeters and wattmeters having such scales. The defect consists of an appreciable effect upon the accuracy when the index is slightly bent and the resulting zero error is removed by shifting the spring abutment, as is done by turning the zero adjusting device with which most modern instruments are equipped. If the geometry is such that the instrument has a linear scale as a wattmeter, or a quadratic scale as ammeter or voltmeter, and if further there is a small angular distance α below the zero and a similar distance above the full scale mark which could be used as extensions of the scale, if desired, with no change in length of division as a wattmeter, then a bending of the index by an angle not exceeding α, either above or below the zero, followed by a return of the index to zero by using the zero adjustment, will not affect the accuracy of the instrument. We may define the desired geometry for the electrodynamic type

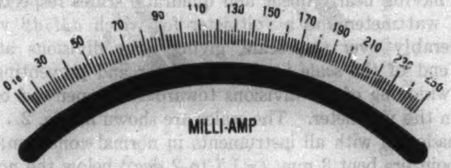

Fig. 1—Linear Scale Obtained in Electrodynamic Instruments by Special Coil Geometry

mathematically by saying that if M is the mutual inductance between the fixed coil and the moving coil and θ the angle by which the moving coil is deflected from its position of rest, then over the whole length of the scale and including the angular margin α at each end of the scale, $dM/d\theta$ should be constant. When a wattmeter has other than a uniform scale[12] $dM/d\theta$ varies from point to point. If in such an instrument the index be bent and the zero error be then corrected by moving the spring abutment, the values of $dM/d\theta$ corresponding to 10, 20, ... divisions of the scale are now different from the normal, and the instrument shows errors which may be serious. The existence of this source of error has been known[13] for a number of years, and its numerical magnitude for two well-known types of instrument is shown by the results of tests recently made in the writer's laboratory. Four instruments were included, namely, a wattmeter and a voltmeter having nearly linear and quadratic scales respectively, and a wattmeter and a voltmeter for which $dM/d\theta$ varied considerably over the scale, giving large divisions at the lower end of the scale in the wattmeter and preventing the usual widening of the divisions toward the upper end of the scale in the voltmeter. The scales are shown in Fig. 2. Test was made (a) with all instruments in normal condition; (b) with pointers bent 3 mm. (=1.7 to 2 deg.) below the normal position; (c) 6 mm. (=3.4 to 4 deg.) below the normal position; (d) 3 mm. above the normal position; (e) 6 mm. above the normal position[14]. In each case the points tested were 0.2, 0.4, 0.6, 0.8, and 1.0 times the full-scale value.

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[11] Bruger, *E. T. Z.* vol. 25, pp. 822-5, 1904, *Phys. Zs.* vol. 4, pp. 876-881, 1903.

The results for the wattmeters are shown in the curves of Fig. 3, from which it will be seen that the errors introduced

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[12] This assumes that the springs give a counter torque which varies directly as the angle through which the moving coil deflects from its position of rest. While this is not strictly true, it is sufficiently close to the facts for the purpose of the present discussion. For data on the variation of springs from Hooke's law, see Bradshaw, *Elec. Jour.* vol. 3, pp. 390-6, 1906.

[13] H. B. Taylor, *Elec. Jour.* vol. 2, pp. 480-1, 1905.

[14] Actually, the pointers were not bent, but the same result was had as would have been given by the process described. The test was made by setting the pointer over marks 3 mm. and 6 mm. above and below each scale point tested, after the pointer had been brought to a rest position 3 mm. or 6 mm. above or below its normal zero position, by shifting the spring abutment.

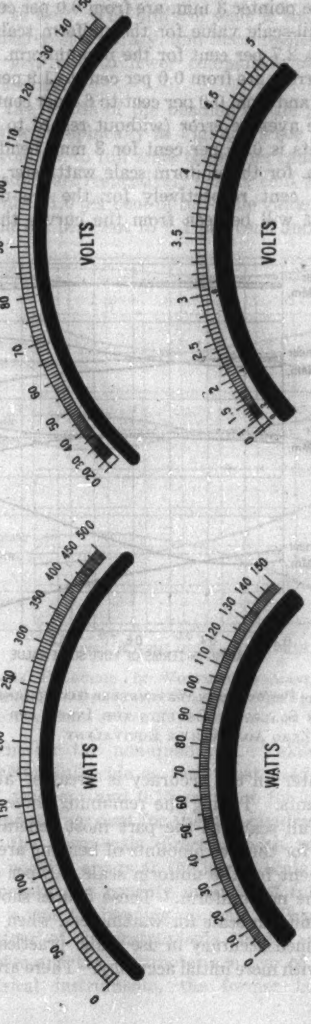
VOLTS
0
20
30
40
50
60
70
80
90
100
110
120
130
140
150
VOLTS
0
1
1.5
2
2.5
3
3.5
4
4.5
5
WATTS
0
50
100
150
200
250
300
350
400
450
500
WATTS
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150

FIG. 2—LINEAR AND NON-LINEAR WATTMETER SCALES, AND QUADRATIC AND NON-QUADRATIC VOLTMETER SCALES, USED IN POINTER-BENDING TESTS

by bending the pointer 3 mm. are from 0.0 per cent to 1.1 per cent of the full-scale value for the uniform scale wattmeter, 0.0 per cent to 2.7 per cent for the non-uniform. For 6 mm. bending, the errors are from 0.0 per cent to 1.9 per cent for the uniform scale, and from 0.0 per cent to 6.2 per cent for the non-uniform. The average error (without regard to sign) for the five scale points is 0.25 per cent for 3 mm. bending, 0.45 per cent for 6 mm. for the uniform scale wattmeter, 1.1 per cent and 2.45 per cent respectively for the non-uniform scale wattmeter. It will be seen from the curves that the effect of a bent pointer on the accuracy is greatest at the highest and lowest points. Taking the remaining three points, from 0.4 to 0.8 of full scale, as the part most generally used, the average errors for the two amounts of bending are 0.1 per cent and 0.15 per cent for the uniform scale, 0.8 per cent and 1.9 per cent for the non-uniform. These figures show the superiority of the uniform scale for wattmeters, when the question is one of sustained accuracy in use under practical conditions, as contrasted with mere initial accuracy. There are advantages

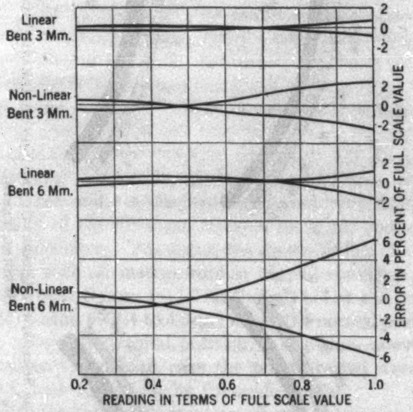

Fig. 3—Errors Produced in Wattmeters having Linear (Uniform) and Non-linear Scales by Bending the Index and Returning it to Zero by the Zero Adjuster or Equivalent

for the maker as well. The uniform scale is easier to lay out with a given degree of accuracy, and instruments with $dM/d\theta$ constant will give rise to fewer complaints of inaccuracies developed in service.

Comparing the results of the bending of the pointer in the voltmeters, shown graphically in Fig. 4, we see that over the whole range from 0.2 to full scale the errors caused by 3 mm. and 6 mm. bending are respectively 0.0 per cent to 0.5 per cent and 0.1 per cent to 1.0 per cent of full-scale value for the quadratic scale, and 0.1 per cent to 2.1 per cent and 0.1 per cent to 2.8 per cent for the non-quadratic. Taking the region from 0.6 to 0.8 of full-scale as that most used, the average errors are 0.1 per cent and 0.15 per cent for the quadratic, 0.2 per cent and 0.5 per cent for the non-quadratic. Bending the index in the down-scale direction with respect to the coil produced positive errors at the lowest scale point tested, changing to negative at roughly one-half of full-scale value. The converse is true for bending up scale. These statements apply to all four of the instruments tested.

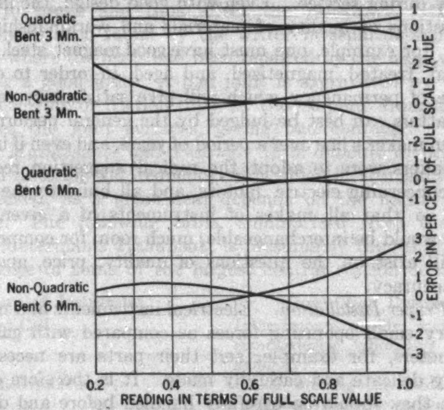

FIG. 4.—ERRORS PRODUCED IN VOLTMETERS HAVING QUADRATIC (SQUARE-LAW) AND NON-QUADRATIC SCALES BY BENDING THE INDEX AND RETURNING IT TO ZERO BY THE ZERO ADJUSTER OR EQUIVALENT

The voltmeter and the wattmeter are two of the most important electrical instruments, the former because of its relation to lamp life and efficiency, the latter because its use in checking standard watthour meters directly affects the revenues. It is the writer's opinion that in future designs makers should strive to realize the condition ($dM/d\theta$ = a constant) which gives the desirable forms of scale.

(c) *Design and Workmanship.* After choosing the proper type of instrument and a suitable scale law, one is still very much dependent upon the maker's knowledge, skill and facilities. For example, poor design in a d-c. voltmeter, such as an abnormally large air gap, may result in a loss of accuracy during service. Even with good design, the important questions of quality of materials and workmanship still remain; for example, one must have good magnet steel, properly heat treated, magnetized, and aged, in order to ensure a degree of permanency which will give satisfactory service. Such matters can best be judged by the general performance of a given maker's line over a period of years, and even if instrument makers were to adopt the radical suggestion recently made[15] concerning electric motors, and all build to the same designs, so that all makes of instruments of a given type and size would be interchangeable, much room for competition could still exist on the questions of quality, price, and sustained accuracy.

(d) *Proper Installation.* Electrical instruments and meters have very small operating forces as compared with gas and water meters, for example, and their parts are necessarily relatively delicate and carefully made. It is therefore essential that they should be carefully handled before and during installation in order to avoid damage to pivots and jewels. Instruments and meters should not be mounted on switchboards until all other work on the board is completed, in order to avoid shocks caused by hammering, drilling, etc. They should be installed where they can be easily read, and where they will not be subjected to extremes of temperature, vibration or shocks, strong stray magnetic fields, or moisture. If it is impossible to avoid all of these, one must expect more frequent repairs or replacements, with reduced accuracy in service.

An important precaution, which is often overlooked, is to have all contact surfaces clean and firmly brought together. Neglect of this precaution in the case of contacts carrying large currents, as in ammeter shunts, results in excessive heating of the shunt and consequent errors. Also, particularly in the case of millivoltmeters and separate-shunt ammeters, bad contacts at the binding posts or the shunt terminals may give rise to large and variable errors. The leads connecting the instrument and shunt are part of the instrument, and may have a resistance equal to 5 to 10 per cent of the total, and any change in their resistance directly affects the accuracy. The joints between such leads and the terminals in which they end should be carefully examined to see that they are securely soldered. When leads for portable millivoltmeters are in frequent use, some of the strands are very liable to break and thus affect the resistance. Excessive errors have been found by the writer in separate-shunt ammeters where the leads had been repaired by the user, without securing a good joint. A bad joint is aggravated by the use of rubber gum or friction tape, as the sulphur from the tape corrodes the joint surfaces, giving bad contact.

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[15] Burke, *Elec. World.* vol. 73, pp. 172-5, 1919.

(e) *Maintenance and Testing.* It is difficult to lay down general rules for the frequency with which instruments should be checked, as a great deal depends on the nature of the service. The following table, summarized from the Electrical Meterman's Handbook, is presumably the result of the experience of some of the largest central station companies:

Name of Instrument	How Often Checked
Standard Cell	Intercompare weekly, and one to standardizing laboratory semi-annually.
Resistance Standard	Check working standards against reference standards at least semi-annually. Latter to standardizing laboratory at least once every two years.
Potentiometer	Once a year.
Volt Box	Once a month.
Secondary Standard Deflection Instruments	Compare with primary standards at least once every two weeks.
Working Standard Deflection Instruments	Every two weeks.
Portable Watthour Meters ("rotating standards")	Direct-current, at least once a week; alternating-current, at least once in two weeks.
Stop Watches.	Daily.
Calibrated Resistances (used in meter testing)	Weekly.

The preceding table does not include switchboard instruments, for which no data on frequency of checking is available, to the writer's knowledge.

Users of musical instruments realize that they must be tuned occasionally if they are to give satisfactory results, and also that some types, such as the violin, require very frequent tuning as compared with others. Users of electrical instruments should realize that these also need occasionally to be brought into accord with standards, and that this should be done more frequently when the service they render is more exacting or important.

4. ACCURACY GUARANTEES

It is the practise of most American makers to publish either in their catalogues or on certificates accompanying the individual instruments, statements or guarantees of the accuracy of their product. The accuracy is usually expressed as a percentage of full scale value, and this method would seem to be quite suitable with the possible exception of switchboard voltmeters, frequency meters, ammeters for constant-current circuits, or other instruments for measuring a quantity which in practise is held at a constant value. The advantage of allowing a greater error outside of the working range in the latter class of instruments is that the maker can give high accuracy over the limited working range at a lower cost.

Instruments with non-uniform scales, such as a-c. voltmeters, have a part of the scale near the zero which is practically useless. For such cases, the guarantee should state to what part of the scale the stated accuracy applies.

The British specification requires a certain percentage accuracy from full scale down to half scale, and a fixed percentage of full scale value below this point.

The wording of the guarantees of American makers varies considerably in definiteness even between different instruments of the same maker. Thus: one maker referring to 9-inch switchboard instruments states that "The d-c. instruments are accurate to within 1 per cent of full scale value under all normal working conditions," while for similar a-c. instruments he writes "The indications of all (a-c.) instruments are guaranteed to be correct within 1 per cent of full scale value when used on circuits of any frequency from 15 to 140 cycles, irrespective of any wave form, power factor or temperature likely to be encountered in commercial practise." Other makers avoid definite statements and use such general expressions as "...... instruments are accurate to a high degree. They may be used on circuits of any frequency, wave form or power factor and are free from heating errors," or in the words of another maker "These instruments will be found of particular value as reference standards in laboratories, or in any place where it is desirable to check portable or switchboard instruments with standards of absolute accuracy."

While the question of specifying the accuracy of an instrument presents some complications, it is easily seen that the accuracy of instrument transformers, especially of current transformers, is still more difficult to cover in any simple manner. The following is a typical form of guarantee of one maker for voltage transformers:

> Are designed to have their actual ratio exactly equal to their marked or nominal ratio for a secondary load of 25 volt-amperes (which is the load resulting when practically the maker's entire line of switchboard instruments is connected to the secondary), but are normally built for a secondary capacity of 50 volt-amperes * * * For any load from no load to full load, the actual ratio will not differ from the nominal ratio by more than 1 per cent for either inductive or noninductive loads.

It is suggested that if makers would each publish curves of ratio and phase angle for their current transformers on at least two standard secondary burdens, comparison of the different makes as to suitability for a given service could more readily be made.

Examination of the catalogues of several American makers shows the following accuracy statements or guarantees expressed in per cent of full scale value: Switchboard instruments, 2 per cent in three-inch and five-inch diameter, 1 per cent for seven-inch and nine inch; medium grade d-c. portable instruments 0.5 per cent; standard d-c. portable instruments, 0.2 per cent; standard portable electrodynamic instruments, 0.25 to 0.4 per cent for single-phase, 0.5 per cent for polyphase; portable moving-iron instruments, 0.5 per cent; portable induction ammeters, 0.5 to 1 per cent; laboratory standard instruments, 0.1 to 0.2 per cent. Instrument transformer accuracies are given as from 0.5 to 2 per cent on stated loads.

5. EFFECT OF CONDITIONS OF USE ON ACCURACY

(a) *Introductory.* All instruments are to some extent susceptible to the influence of surrounding conditions, such as varying temperature, stray magnetic fields, electrostatic effects, etc. Some conditions may affect the accuracy seriously although no permanent change takes place in the instrument, which may show the same performance when checked in the laboratory before and after use, while other conditions may permanently injure the instrument. It is therefore important for the user of instruments to understand their operating principles, the location of their magnets and coils, whether they are magnetically shielded, etc., as only with such knowledge can he most effectively locate and protect the instruments so as to secure the best obtainable accuracy.

In discussing the effects of conditions of use on accuracy, watthour meters are separately treated in section 11.

(b) *Care in Handling.* Because of their necessarily small operating forces and delicate moving parts, electrical instruments and meters should be handled very carefully if sustained accuracy is to be expected. If they are to be shipped, they should be carefully packed. Indicating instruments, which have relatively sharp pivots, require more care in handling than a-c. watthour meters, which have rounded pivots. While the preceding statements may seem trite to many, it is remarkable in how many ways the facts are overlooked by those who deal with instruments. Property clerks, unless restrained, will stamp numbers on instruments, and even experienced laboratory men will hammer on a bench upon which are pivoted instruments, or use instruments directly on tables which are set in strong vibration by motors or other machines. If instruments must be used where such vibration exists, they should be set on pads of felt or folded cloth. It has been suggested[16] that the shelves on which instruments are stored should be provided with a thick felt covering, and that this will improve the service accuracy and reduce repair bills.

(c) *Overloads.* Instruments subjected to overloads may have their accuracy impaired or destroyed. Some of the forms of damage which may result from overloads are: burning out of windings, bending of pointers, alteration of value of resistors or shunts, annealing and deforming of springs, change in shape of moving coils and consequent lack of balance of the moving system. Some of these, such as deformation or annealing of springs, can usually be discovered by inspection, while others may require tests. Wattmeters are especially liable to be damaged because they have not only a watt limit but also limits of current and of voltage. With low power factors, the attempt to get a good reading may result in overloading the current or voltage coil.

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[16] F. P. Cox, *Elec. Rev.* (N. Y.), vol. 52, p. 589; 1908.

To avoid overloads when using portable instruments, it is advisable to lay out the work carefully in advance, calculating the approximate values of current, voltage or power. When instruments have more than one range, it is advisable to try the highest range first, as there will then be the least risk of damage if an abnormal condition exists. All lines and connections should be run and secured in such a way as to avoid accidental contacts which may produce short-circuit currents through the instruments. Leads attached to instruments should not be allowed to hang over the edge of the bench, as this is liable to result in dragging the instrument off onto the floor.

Instruments which have been overloaded should be put under suspicion of inaccuracy, and should be checked before further use. Switchboard instruments should be made on more substantial lines than portable or laboratory instruments, as exposure to momentary overload is unavoidable.

Permanent-magnet moving-coil instruments may suffer magnetic damage by exposure to strong fields, such as those set up by short circuits. This damage leaves no mark, and should be guarded against by proper location of the instruments.

(d) *Fraction of the Scale Used.* It is not possible for the maker to locate each mark on the scale where it should be; also, the user must estimate the position of the pointer between scale divisions. Since the combined error amounts roughly to a constant angular displacement, it will introduce a relative error into the result which increases as the deflection diminishes. For this reason, it is desirable to use as large a part of the full deflection as is practicable. In the use of portable instruments this question may be decided from the requirements at the moment and the ranges available. In choosing switchboard instruments, however, one must be governed also by the unusual requirements which may arise. On railway circuits where heavy overloads are encountered, and where the measurement of light loads is of minor importance, it is necessary to have instruments of larger capacity than would be used, for example, on lighting feeders. Wherever accuracy is essential, however, as in the testing of meters, or plant acceptance tests, ranges should be such as give not less than half and preferably two-thirds of full scale deflection.

An important limitation to the foregoing statement is that in some cases the power required to operate instruments of lower range may occasion errors in other parts of the measuring apparatus which may be difficult to allow for. For example, if a one-ampere ammeter is used with a current transformer in order to get larger deflections for currents below one ampere, it should be remembered that such an ammeter has about 25 times the impedance of a five-ampere ammeter of the same type and make, and that the increase in impedance may affect the performance of the current transformer so as to make the observed results less accurate than if a five-ampere ammeter had been used. Another instance is that of low-range a-c. voltmeters, which are of low resistance. There may be cases where the current they take from the source will introduce errors of more significance than the reading error which would occur if a voltmeter of greater range were used.

(e) *Variation of Room Temperature.* The net effect of a change of room temperature upon the reading of an instrument is the resultant of a number of changes, some of which tend to annul each other, and others, such as those caused by change of dimensions of the parts, are of negligible effect. It is desirable that the user should know the general facts about this matter in order that errors may be avoided or minimized when tests are of necessity made under abnormal temperatures.

In a direct-current voltmeter, an increase of temperature of 1 deg. cent. produces the following effects: The strength of the spring is reduced 0.04 per cent; the magnetic flux density in the air gap is reduced by about 0.02 per cent[17]. The weakening of the spring tends to increase the reading, and the lowering of the flux density tends to reduce it. Hence for a given current through the coil, the effect of 1 deg. cent. rise of temperature is to increase the reading by 0.02 per cent. However, the moving coil is of copper, and its resistance increases 0.4 per cent per degree C, thus tending to reduce the reading for a given applied voltage. If we add to the copper coil a resistance which is 19 times as great as that of the coil and is unaffected by temperature, the resistance of the whole circuit will increase

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[17] This is an average of the values found for six instruments of three makers. The individual values ranged from − 0.01 to − 0.03 per cent.

0.02 per cent per degree cent. A voltmeter so made will therefore be compensated for temperature. If less than the above amount of added resistance be used, as may be the case for low ranges, the reading of the instrument on a given voltage will decrease as the temperature rises, while if more added resistance is used the reading will increase.[18].

In a direct-current ammeter of the shunted type the effects on magnet and springs are as above stated, but there are some limitations not found in the voltmeter. For example, it is not feasible, in general, to add to the moving coil a series resistance 19 times that of the coil because such a construction would require too large a drop of potential between the shunt terminals, thus increasing the size, weight, and cost of the shunt and also the power loss in it. In practise, it is usual to make shunts of relatively low drop (50 to 75 millivolts) for switchboard use, where temperature compensation is not of particular importance, while for portable instruments of greater accuracy drops from 100 to 200 millivolts are commonly used. It might appear that the ammeter of low drop might be compensated for temperature by making the shunt of a material which increases in resistance with temperature at such a rate as to supply the higher drop which the millivoltmeter requires for a given deflection. Such a procedure is not feasible because of the relatively large difference in the temperatures of the millivoltmeter and the shunt when the latter is carrying a load. It is the usual custom to make the shunts of an alloy [19] having a negligible temperature coefficient, and to minimize the temperature effect on the millivoltmeter by reducing the relative drop on the copper coil as much as is consistent with good design in other respects. The important thing for the user to remember is that the temperature errors of d-c. switchboard ammeters, while not of serious consequence for switchboard purposes, are such as to make them unsuitable for accurate work, such as the testing of watthour meters or acceptance tests of apparatus, unless the temperature of the instrument is determined and proper allowance is made for it. It is preferable for such work to use instruments whose temperature corrections are small.

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[18] This assumes the average value of −0.02 per cent for the temperature coefficient of the magnetic field. If exact compensation must be assured, the individual instrument must be investigated. The average value, however, gives results satisfying all but extreme requirements.

[19] It is becoming general practise to use manganin, or similar alloy, for both portable and switchboard ammeter shunts. In some cases where a lower grade of performance is admissible, and first cost is important, switchboard ammeter shunts are made of cast iron.

In the moving-iron ammeter an increase of temperature affects the magnetic conditions in the iron in such a way as to decrease the operating torque by about the same percentage that it reduces the spring strength. These ammeters are thus very nearly independent of temperature changes.[20] The temperature coefficient of a moving-iron voltmeter thus depends mainly upon the temperature coefficient of resistance of its circuit, which depends on the ratio of the resistance of the copper coil to that of the manganin or similar alloy in series with the copper. For example, a well-known American portable moving-iron voltmeter having ranges of 150 and 300 volts has a total resistance of 5100 ohms, of which 130 ohms are in the copper coil. Neglecting the effect of temperature change upon the series resistor, the effect of a rise of temperature of 10 deg. cent. would be to make the voltmeter read lower by $10 \times \frac{130}{5100} \times 0.4$ per cent, or about 0.1 per cent. This applies to the 300-volt range. For the 150-volt range the corresponding effect would be 0.2 per cent.

In an electrodynamic ammeter with spring control and with the fixed coil and the moving coil in series, the only appreciable effect of temperature on the readings is that caused by the change in the strength of the springs, which is 0.04 per cent per degree cent. For any given position of the coils, the operating torque varies as the square of the current; the scale, however, is marked to read the first power of the current. Such an ammeter will therefore read 0.02 per cent higher on a given current for each degree cent. rise of temperature. An electrodynamic voltmeter is simply a low-range ammeter as just described, plus a series resistor of suitable value. If this has 19 times the resistance of the copper, the temperature coefficient of resistance of the instrument will be 0.02 per cent per degree cent. and will just offset the effect of temperature changes on the spring strength. The instrument will be compensated for temperature as a voltmeter.

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[20] This statement is based on the experience of the Bureau of Standards with several instruments of each of two prominent American makers.

In an electrodynamic wattmeter a similar condition exists, but since the torque is proportional to the first power of the quantity indicated on the scale, the effect of the spring change with temperature (0.04 per cent per degree cent.) enters unchanged. If the resistance of the voltage circuit remained unchanged as the temperature varied, the wattmeter would read 0.04 per cent higher per degree cent. of temperature rise. This requires, for compensation, that a series resistor of negligible temperature coefficient be used, having nine times the resistance of the copper. Commercial wattmeters often have more than enough added resistance for temperature compensation, and would actually be better in this respect if a part of the added resistance were of copper. Attention is called to this matter because the effect of temperature on the spring strength has been frequently overlooked, and in consequence the statement has been made that as much series resistance as possible should be used in order to minimize the temperature error. If copper be added to the potential circuit to minimize the temperature coefficient, it should be in the form of fine wire or strip wound on a thin card, so as to respond quickly to temperature changes, and it should be mounted as near the springs as practicable. It is of theoretical advantage, at least, to have both springs above the moving coil so that the ascending current of warm air from the coils will maintain the springs at nearly the temperature of the moving coils.

For induction instruments no general statement concerning temperature effects can be given. In simple instruments on this principle the effect of temperature may be rather large. In the form made by an American maker on the "series-transformer" principle it is stated that approximate independence of temperature changes is inherent in the design of ammeters and voltmeters. Induction wattmeters (now made only for switchboard use) have an inherent temperature error which has been reduced by the use of a rotor of special alloy instead of aluminum. It is not practicable to secure complete compensation in this way without reducing the operating torque to an objectionably low value.

In the hot-wire instrument a change of room temperature changes the length of the working wire, and also changes the distance between the supports of the ends of the wire. If the base plate supporting the wire be made of material having the same coefficient of expansion as the wire, the instrument will theoretically be compensated for room temperature changes. However, the wire takes up the temperature of its surroundings very quickly, but the base plate does not, so that a considerable lag effect occurs. In the best modern hot-wire instruments the performance in this and in other respects has been improved by substituting platinum-iridium for the platinum-silver so long used for the working wire. The platinum-iridium has a much smaller coefficient of expansion, but may be operated at a much higher temperature.

While the use of the hot-wire instrument is limited to laboratory and radio purposes, it may be well to point out that it is subject to an external condition which does not affect other electrical measuring instruments, namely, variation in the atmospheric pressure. While the subject has not been studied with any degree of thoroughness, some experiments made seven years ago by the writer, using a Hartmann and Braun hot-wire voltmeter, showed that a reduction of air pressure from 75.2 cm. to 55.9 cm. (29.6 to 22.0 inches) increased the reading on a given voltage by 2.7 per cent. For intermediate values of pressure, the change in the reading was directly proportional to the change of pressure.

(*f*) *Self-Heating of Instruments.* In the preceding discussion of effects of room temperature it has been assumed that temperatures within the instrument were uniform throughout. Most instruments contain sources of heat in the form of coils traversed by currents, and it is therefore possible for differences of temperature to be set up while the instrument is in circuit. This affects the accuracy of the results obtained.

In d-c. millivoltmeters the heating of the coil is entirely negligible. In d-c. voltmeters of moderate range it is small. For example, the power expended in a 150-volt d-c. voltmeter of 15,000 ohms resistance at full scale deflection is 1.5 watts. Since this is nearly all expended in the series resistor, the effect on the accuracy should be very small.

In a-c. voltmeters and the voltage circuits of wattmeters the resistance for a given voltage is much lower than in d-c. instruments, and the rate of heat liberation is much greater. It is good practise to partition off and ventilate the series resistor, in which nearly all the heat is liberated. In switchboard instruments the series resistor is often supplied in a separate case for back of board mounting.

Direct-current ammeters with separate shunts, being essentially millivoltmeters, have negligible internal heating, but may have errors caused by the heating of the shunts. This may occasion appreciable errors if the resistance material used in the shunts has an appreciable temperature coefficient or thermoelectric force against the material of which the shunt blocks are made. The copper-nickel resistance material variously known as constantan, *Ia Ia*, advance, eureka, and ideal has a very small temperature coefficient of resistance, and would be very well adapted for ammeter shunts if it were not for its high thermoelectric power which makes it unsuitable for the purpose.[21] Manganin meets both requirements.

A considerable part of the heat produced in switchboard ammeter shunts is carried away by conduction to the cables or bus bars, from which it escapes by radiation and convection. Even if the opportunity for escape of heat is the same at each end of the shunt, there will still be a difference of temperature between them. This is caused by the flow of current through a circuit made of dissimilar metals, and is known as the Peltier effect. It causes a thermal electromotive force in such a direction as to increase the reading of the ammeter. In cases of unequal heating of the ends of the shunt, caused by bad contacts or by difference in the size of bus bars, the resultant thermal electromotive force may either increase or decrease the reading of the ammeter. Thermal electromotive force may be detected by breaking the main circuit, when the index will fail to return to zero if appreciable thermal electromotive force is present. This effect may be distinguished from zero shift (due to spring fatigue) by disconnecting one lead from the instrument, when the position of the index will change by an amount proportional to the thermal electromotive force.

The use of copper bus bar as an ammeter shunt is no longer recommended, and the same is true of the use of internal copper shunts in self-contained d-c. ammeters, except for small ranges where the heating is small. In the latter case the effect of room temperature on the shunt and on the copper moving coil is the same, and no error results. Such construction is not advisable for currents exceeding say 15 amperes.

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[21] For the construction of coils of high resistance, such as series resistors and multipliers for voltmeters, constantan has some advantages over manganin.

In the moving-iron ammeter the power loss (in good designs) is small, and the effect of temperature changes is small. In the moving-iron voltmeter the conditions are somewhat less favorable, as the heating of the copper coil increases the resistance of the circuit.

Electrothermic instruments, including the expansion (hot-wire) and the thermocouple types, are unique in that their operation depends on internal heating. The effect of this heating in hot-wire instruments has been referred to. No published data seems to be available on the limitations of thermocouple instruments, such as effect of varying room temperature and of standing in circuit.

(*g*) *Stray Magnetic Fields.* Unidirectional stray fields are set up by conductors carrying direct currents, by permanent magnets, and by direct-current electromagnets. The strength of the field, upon which depends the magnitude of any error produced by it in the indications of an instrument, varies directly as the strength of current in the conductor and decreases with the distance from the conductor to the given point. Thus the necessity for precaution increases as currents are greater and as the instrument is located nearer to the conductor.

It is important to remember that the magnets used in direct-current instruments of the usual unshielded types set up a strong stray field which is capable of appreciably affecting the accuracy of other instruments near by. Conductors carrying moderate or large currents to the instruments set up stray fields which may occasion errors. The iron framework of switchboards, magnetized by large currents in bus bars, may set up strong stray fields at a considerable distance from the bus bars. Even unmagnetized masses of iron placed too near some types of instrument will affect the working field of the instrument and thus introduce errors. The iron pipe framework of testing benches has given rise to disturbances of this kind.

Unidirectional stray field affects the accuracy of the permanent-magnet moving-coil, the moving-iron, and the electrodynamic types of instruments.[22] The first-named type, because of its large value of working field strength, is in general less affected by a given value of stray field than the other two. The smallness of the air gap is also an important feature, and of two otherwise similar direct-current instruments, the one having the smaller air gap will be less affected.

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[22] McCollum has suggested that the hot-wire instrument would theoretically be affected by stray magnetic field of the same frequency as that of the current in the wire. This effect can be observed, but would be very small in most practical cases.

The following table is given by Farmer:[23]

EFFECT OF STRAY MAGNETIC FIELDS ON D'ARSONVAL TYPE VOLTMETERS

Stray field, lines per cm.²	Error at ½ of full-scale deflection, per cent	
	Shielded	Unshielded
5	0.5 to 1.0	2
10	0.75 to 1.75	3.5 to 5.5
15	1.0 to 3.0	6.0 to 7.5
20	1.25 to 3.25	7.5 to 10

Moving-iron instruments are affected by unidirectional stray field, even when operating on a-c. circuits. Moving-iron instruments on the repulsion principle are stated to be considerably more affected by stray field than those on the attraction principle.[24] Electrodynamic instruments are affected by unidirectional stray field only when they are used on direct-current circuits. This is in contrast to the moving-iron instrument, which is affected by either unidirectional or alternating stray fields when used on either d-c. or a-c. circuits.

Alternating magnetic fields are set up by alternating currents in conductors, by a-c. electromagnets, transformers, etc. In general, such fields are not liable to attain the values which occur with direct current, on account of the larger values of voltage and hence smaller currents for the same power in a-c. plants.

Alternating magnetic fields affect the readings of permanent-magnet moving-coil instruments only when the field strength is great enough to reduce permanently the strength of the magnet.[25] Other types, including moving-iron, electrodynamic and induction instruments, have their accuracy affected only during the time of exposure to the field. Moving-iron instruments used on a circuit of a given frequency are affected by alternating fields of this or any other frequency, or by unidirectional fields. Electrodynamic instruments, however, are affected only by fields of the same frequency as that of the circuit on which they are used, with the further restriction that the effect depends on the direction and time-phase of the field with respect to the current in the moving coil. That is, with the moving coil in a given position, there is a direction of the stray field in which no torque is produced on the coil; also, for any position of the coil there will be no torque if the stray field is in time-quadrature with the current in the coil. It is stated[26] that the stray-field error in unshielded non-astatic electrodynamic wattmeters may be as much as 25 per cent with an a-c. field of five lines per cm.^{2}, and 75 per cent with ten lines, and that a suitable shield will reduce the effect of a field of 20 lines per cm.^{2} to practically zero without introducing eddy current or other errors.

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[23] F. M. Farmer, Electrical Measurements in Practise, p. 54.

[24] In a test of nine moving-iron ammeters, reported in *Electrician*, vol. 76, pp. 120-123, 1915, the stray field was set up by two conductors located six inches away from the instrument, each carrying 2000 amperes. The change of reading due to stray field was 0.4, 1.5, and 1.9 per cent in the three attraction-type ammeters, and 3.5, 7.4, 12.3, 9.4, 6.8, and 5.7 per cent in the six of the repulsion-type.

If the stray field is of constant direction, it will be possible to turn an unshielded electrodynamic wattmeter into such a position that no deflection occurs when voltage only is applied. If the position of the index is noted, and the instrument is then used to measure power, no error from stray field will occur if the wattmeter is turned so that the index points along the given direction.

Induction instruments, because of their strong working fields, are only slightly affected by stray fields of the values found in practise.

To avoid errors due to stray field, we may (1) keep instruments at a sufficient distance from heavy conductors or other sources of the disturbance; (2) eliminate the effect of stray field by reversing the connections to the instrument or by turning the instrument through 180 deg. between readings, taking the mean as the correct value; (3) construct the instrument so as to be astatic as to other than its own working field; (4) surround the working parts with a magnetic shield. In the most careful work it is well to make use of more than one of these methods, if practicable. These precautions will now be briefly considered in detail.

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[25] An extreme case of this kind is given by E. P. Peck, (*Elec. World*, 51, p. 1220; 1908) in which a d-c. voltmeter was connected to a 600-volt circuit and placed within 18 inches of heavy bus bars. After a severe short-circuit, which caused very large alternating currents to flow through these bars, the voltmeter read 350 volts when the applied voltage was 600. It should be noted that damage to an instrument of this nature leaves no external mark.

[26] F. M. Farmer, Electrical Measurements in Practise, p. 159.

Opinions differ somewhat as to the separation necessary between permanent-magnet moving-coil instruments in order to avoid error from interaction.[28] Since it is often necessary to assemble a number of instruments in a relatively small space, it is important to know just how far to separate them in order to secure a desired degree of accuracy. For this reason some experiments were made recently in the writer's laboratory on permanent-magnet moving-coil instruments. The results are given in the curves of Fig. 5. In obtaining data for these curves the instrument *L* at the left was tested at two points by a potentiometer while other instruments were removed from its vicinity. The readings so given were taken as the normal or "zero error" values. Another instrument (an unshielded standard portable voltmeter) was then placed alongside the first, and successively at various distances to the right, the tests of the first instrument being repeated for each position of the disturbing instrument. The curves show the error in per cent as a function of the distance in inches between the centers of the moving coils.

With unshielded instruments side by side, (condition 1) the error is 0.8 per cent, and in the worst position, with pole-pieces adjacent, it is 2.0 per cent. As the disturbing instrument is moved away the error drops until it becomes practically zero when the centers of the coils are about 20 inches apart. The same distance applies to conditions 2 and 3. For the iron-cased instruments, curves 4 and 5 show that the maximum error with instruments as close together as possible is much less than in the unshielded instrument, and drops to a negligible value with a separation of the coil centers of about eight inches. It is usually recommended that when portable instruments must be used in places subject to stray magnetic fields the mean of two readings be taken, the instrument being turned 180 deg. about the axis of rotation of the moving element for the second reading. In the case of unshielded moving-iron and electrodynamic instruments it is probably better to leave the instrument in one position and take the mean of two readings, the current through the

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[28] F. P. Cox states (*Elec. Review*, N. Y., 53, pp. 589-591, 1908) that, "Normal earth's field introduces an error of about 0.1 per cent in unshielded instruments, and such instruments should not be used nearer together than 18 inches. If placed side by side these instruments react on each other and an error of 1 per cent may very readily be introduced."

H. B. Taylor (*Elec. Jour.*, 2, pp. 474-481, 1905) says,

"A space of from two to three feet may be taken as a safe distance to allow between direct-current meters of the ordinary portable type. When space is very limited, two instruments can often be brought closer together without causing trouble, by placing one of them with its scale inverted with respect to the other, so that the neutral parts of the magnets are nearest to each other and the pole pieces are as far apart as possible".

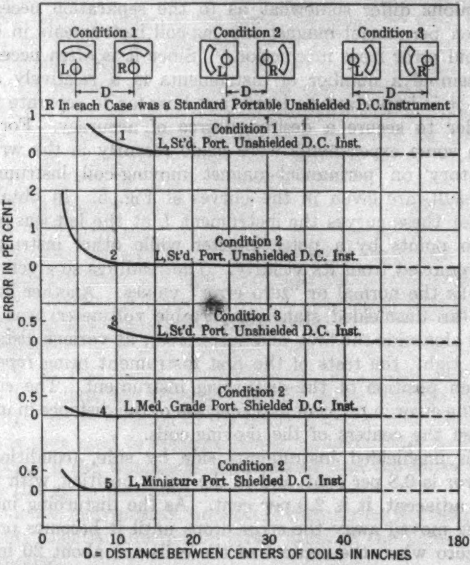

Fig. 5.—Errors Produced in D-C. Instruments by the Stray Field from an Unshielded Portable D-C. Instrument

instrument being reversed for the second reading. Instruments of these types should not be used for d-c. measurements if permanent-magnet moving-coil instruments are available.

To avoid the effects of stray fields, astatic instruments are made by combining two operating systems on a single axis. For example, two permanent-magnet moving-coil systems are used, with the moving elements coupled to add their torques and with the two magnets arranged to have opposite polarities. Another astatic arrangement consists of two moving-iron systems, the polarities of the two solenoids being opposite.[29] Astatic electrodynamic instruments have been made, but have not come into any extended use, probably because of the adoption of magnetic shielding by the leading American makers. It should be noted that "astatic" instruments are strictly astatic only for fields which are uniform over the space occupied by the coils, or for non-uniform fields which may happen to produce the same torque on each coil. The field about a conductor carrying a current is not uniform, but decreases inversely as the distance from the conductor. If an astatic instrument is placed near the conductor, one coil of the astatic system may be so much closer to the conductor than the other as to give an appreciably greater torque on the nearer coil, and hence an error will be caused by the stray field. In using astatic instruments the same precautions should be taken as for simple instruments, if the maximum possible accuracy is desired.

A special form of permanent-magnet moving-coil instrument[30] has been developed to reduce to a minimum the disturbing effect produced by the stray field from a vertical bus bar, since such vertical bars are the most frequent source of trouble. This is accomplished by placing the magnet so that the lines of force of the stray field run through the steel from side to side of the bar, rather than along its length. It is stated that even very strong stray fields, which would demagnetize the magnets of instruments as ordinarily made, will produce little or no effect when they pass through the steel at right angles to the direction of the permanent magnet flux, even if the instrument is entirely unshielded. It is also stated, in the same connection, that even heavy shielding will not prevent demagnetization when short-circuits occur in nearby conductors on large power systems.

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[29] Edgcumbe, Industrial Electrical Measuring Instruments, 2d edition, p. 142.

[30] P. MacGahan, Nat. Elec. Lt. Assoc., 36th Convention, Hyd.-Elec. & Trans., Technical, pp. 602-604, 1913.

Connections to ammeters and the current terminals of wattmeters and watthour meters should be run as close together as practicable in order to avoid forming a loop. This precaution is not important for small currents, but for a given design (constant ampere-turns for all ranges) the effect of stray field at the operating system, caused by the current in a loop of given form and size, increases directly as the ampere range. This may be seen from the fact that the current coils of large range instruments have only a few turns, so that even one turn in a nearby loop will have an appreciable effect on the readings.

It seems to be the consensus of opinion of American makers that electrodynamic voltmeters, ammeters and wattmeters should be protected from stray field by a laminated iron shield. In one moderate priced type this shield is open at top and bottom, but in the highest grade instruments the shield completely surrounds the operating system except for a limited opening at the top. In one make the laminated shield is surrounded by a drawn steel shield, and in the polyphase wattmeter the shielding of the two systems from external disturbances is supplemented by a laminated iron shield between the two systems to prevent interaction. At least two companies supply shielded portable moving-iron ammeters. In checking shielded electrodynamic instruments on direct current it is necessary to take readings with the current first in one direction, then reversed, the mean being taken as the normal value. The necessity for this procedure in unshielded instruments is caused by the interaction of the earth's (or local) magnetic field with the current in the moving coil, and in the shielded instrument it is due to a slight residual polarity in the shield.

Stray magnetic fields may be detected and their intensity determined by using an unshielded electrodynamic wattmeter with only the voltage circuit energized, or an electrodynamic voltmeter with its fixed coils cut out of circuit. The excitation should be from a d-c. source for unidirectional field, and from an a-c. source of the proper frequency for alternating field. In either case the instrument must be oriented until the deflection is a maximum, and in the a-c. case, the phase of the exciting current must be adjusted for maximum deflection. To calibrate the apparatus, the experiment may be repeated with the moving coil at the center of a coil of few turns of large diameter (say two or three feet radius.) The field intensity over the space occupied by the moving coil may be taken as constant, and is given with sufficient accuracy by the formula

$$H = \frac{0.2\,\pi NI}{r} = \frac{0.63\,NI}{r} \qquad \textbf{(1)}$$

where N is the number of turns in the coil, I the current in amperes, r the radius of the coil in centimeters. If the radius is expressed in inches, the constant becomes 0.25 instead of 0.63. The formula assumes that the cross section of the coil is small compared with the radius. In the a-c. case, H and I are effective values. The voltage used in calibrating the instrument should be the same as that used in measuring the stray field, or if not, an allowance must be made.

(h) *Electrostatic Effects.* Electrostatic attraction or repulsion of the moving system of an instrument may seriously affect its accuracy. This effect is most pronounced in instruments of relatively low torque, such as electrodynamic instruments, portable and some forms of switchboard moving-iron instruments, and in permanent-magnet moving-coil instruments, especially those having long pointers. It is possible with many of these instruments to cause the index to move one-fourth of the way up scale by rubbing the cover glass with a dry cloth. This should be avoided when readings are about to be taken, or, if necessary to clean the glass, the charge on the glass should be dissipated by breathing on it.

In the use of an electrodynamic wattmeter, it is very important to make the connections so that the fixed coil and the moving coil are at nearly the same potentials. With improper connections, errors may arise from electrostatic action between the coils, and there is also danger of breakdown of the insulation. This danger is especially great when a multiplier is improperly used with a wattmeter to extend the voltage range. It is suggested that the manufacturers might well agree on a standard method of marking the terminals of wattmeters and that more stress should be laid by the maker on the matter of proper connections, which are in nearly all cases discussed in instructions separate from the instruments. The fundamental principle of correct connection as regards the two coils might well be briefly set forth in the calibration card attached to the instrument.

Another condition which may occasion error from electrostatic action is the testing of a wattmeter on separate sources of current and voltage, or in its use with current and voltage transformers. In the latter case, the grounded side of the secondary circuit of the voltage transformer should run directly to the moving coil, in order that the latter may be at the same potential as the current coil. In testing the wattmeter on separate d-c. sources, some writers recommend joining the two coils at the instrument terminals. As this may cause trouble if the circuits are accidentally connected elsewhere, the writer prefers to join the two points to a d-c. voltmeter whose maximum reading is not less than the voltage used in testing the wattmeter. The index of this voltmeter should remain at zero. If a deflection occurs, leakages or accidental contacts are present in the test circuits or the sources. Another method suggested is to make the connection between the coils through a piece of fine fuse wire.

Another class of electrostatic trouble has been noted in voltmeters of relatively high range with metal covers, namely, electrostatic action between the pointer and the insulated cover.[31] This action has been noted in voltmeters of 750-volts range, and is greater in instruments with long pointers. The pointer is at the potential of the moving coil, which in railway systems with grounded negative would be 500 volts or more above earth, since the moving coil (in one well-known make at least) is connected to the + terminal of the instrument. The potential of the metal cover is not definite, but depends on circumstances as to leakage from the instrument circuits to the cover and the insulation of the latter from ground. It is evident that in general the potential will be lower than that of the pointer, and the cover may even be at earth potential, thus placing full line potential between cover and pointer, and making very appreciable errors. This difficulty has been noted in testing such voltmeters at the Bureau of Standards, and the method of avoiding it is to connect the cover to the + binding post during the test. Since the makers have not yet adopted means for avoiding the difficulty, so far as the writer is aware, and since the user is very liable to encounter it, attention is here called to it. A remedy would be to connect the cover to the moving coil, but this would be objectionable because of the possibility of accidental contact between the cover and other conductors or the operator. One possible remedy would consist in lining the cover with insulating material within which would be a lining of thin foil or sheet metal which would be connected to the moving coil and to the magnet system and scale plate. Where the voltmeter is to be connected to grounded circuits, the same result could be attained by connecting the cover to the moving coil and to the grounded side of the circuit, using a built-in reversing switch with the moving coil to bring the deflection up scale regardless of which pole of the circuit is grounded. The latter method requires circumspection on the part of the user, and from this standpoint is not as good as the former.

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[31] F. M. Farmer, Electrical Measurements in Practise, p. 54.

Use has been made of a conducting lining in wattmeters and other instruments having hard rubber tops, but so far as the writer is aware no such precaution has been adopted for direct-current instruments.

Electrostatic fields from high-voltage conductors in the vicinity of instruments are theoretically capable of affecting the indications of the instruments unless the latter are either in metal cases or in glass cases lined with a grid of conducting material suitably connected to the movement. This effect seems unlikely to occur in practise because of the necessity of keeping high-voltage conductors at a distance from observers. It will be greatest in electrostatic instruments [32] because of their low torque and the fact that the operation of the instrument depends on electrostatic forces. The trouble can be avoided by suitable screening.

Electrostatic difficulties may occur in the use of reflecting galvanometers, due to differences of potential between the moving coil and the magnet system and the case. Both the latter should be connected to one terminal of the coil and well insulated from the support on which the galvanometer is placed.

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[32] K. Edgcumbe, Industrial Electrical Measuring Instruments, 2d. ed., p. 186.

(*i*) *Variation of Frequency, Wave Form and Power Factor.* The effect of ordinary variations of frequency from the normal value is negligible in moving-iron, electrodynamic, and electrothermic instruments. The effect on induction instruments, unless carefully designed and compensated, is much greater. Even the most carefully compensated induction instruments must be used within a rather narrow range of frequency, especially wattmeters. Induction wattmeters may be compensated for moderate variations of frequency at unity power factor, or for large variations of power factor at normal frequency, but will show appreciable errors if both frequency and power factor differ from normal and from unity respectively.

Variations of wave form from the sine shape, when not excessive, produce very small effects on electrodynamic instruments and no effect on electrothermic instruments. The effect on moving-iron ammeters depends on the magnetic quality of the iron and the flux density at which it is worked. If the permeability is practically constant throughout the working range, the torque will always be proportional to the square of the current, which is the condition to be fulfilled if the instrument is to indicate the effective value of the current. As the iron approaches saturation the torque tends to vary as the first power of the current, which makes the instrument tend to indicate the average value of the current rather than the effective value. It will be seen that the quality of the iron and the correctness of the design have much to do with the proper functioning of moving-iron instruments, and it is doubtless due to the lack of these essential conditions that early forms of moving-iron instruments functioned badly on change of wave form and in other respects, and thus created a prejudice against the type which perhaps is not yet entirely dispelled.

Induction instruments, from the principle of their operation, are evidently subject to error due to change of wave form. There seems to be very little published data on this matter.

(*j*) *Mechanical Balance of Moving System.* Unless the axis of rotation of the moving system of an instrument is exactly vertical, any lack of mechanical balance will introduce a gravitational torque which will affect the accuracy of the instrument. It is partly for this reason that laboratory instruments are made with vertical axis, and this feature is one of the strong points of the horizontal edgewise instrument. One of the first things to examine in an instrument about to be used on important work is the condition of the moving system as to balance. Rough usage, or severe overloads, or excessive heating from any cause, is very apt to unbalance the moving system.

Switchboard instruments may be checked for balance by turning them into different positions about an axis parallel to the axis of rotation of the moving system. The same test is sometimes applied to portable instruments, and is satisfactory if approximate results are desired. For the most accurate balancing of portable instruments it is better to place the instrument in various positions on a plane support which is inclined say 10 deg. to 20 deg. to the horizontal. The difference between the two procedures lies in the fact that in the latter method the lower jewel and pivot come into contact at more nearly the working point of the pivot. When the instrument is checked for balance with axis horizontal, the point of contact of pivot and jewel is different from the working point, thus changing the axis of rotation slightly and giving an inaccurate indication of balance.

In American instruments of the better class the balancing is effected by varying the position of small weights which are adjustable in two directions at right angles. In balancing such an instrument, place it on a sloping support, first in the position which makes the pointer horizontal, then in another which is at right angles to the first. In the first position, any unbalance which appears can be removed by moving the weight which is adjustable along the direction of the pointer, and in the second position any unbalance in the line at right angles to the pointer may be corrected. If these two adjustments are carefully made, the instrument should be in balance when turned into any position on the sloping support, and should therefore be in the best possible balance for use on a nominally horizontal support.

Maintenance of balance in service requires that the moving system be as rigid as is consistent with lightness, and that the temperature shall not attain to values at which windings soften and alter their shape. It is evident that a given small weight placed on a moving system originally balanced will be able to produce less error as the operating torque of the instrument is greater. This constitutes an additional reason for the use of the highest practicable value of the ratio of torque to weight.

6. Features of Design Affecting Accuracy

In the following section will be discussed certain features of instrument design which have a more or less direct influence on the accuracy of the results, especially those features which can be checked by inspection.[33]

The designer of electrical instruments must fulfill commercial conditions as well as mechanical and electrical requirements. Cheapness and ease of manufacture, assembly and repair, and the ability to withstand a reasonable amount of handling in shipment, are matters which can not be disregarded if the product is to meet competition. The writer has endeavored to keep this fact in mind in the present discussion.

(*a*) *Shunts and Resistors.* These are always sources of heat when in use, and provision should be made for getting rid of the heat without raising the temperature of the operating system undesirably. It is possible to have shunts and resistors built in with the moving system with no ventilation, provided the power lost in the instrument is small, say only a few watts.

(*b*) *Permanent Magnets.* A general principle applying to the permanent magnets used in instruments and meters is that the magnetic circuit should be as nearly closed as possible. Since a gap must exist in order that the magnet may be of use, the practical question is to set a limit which shall give a reasonable degree of permanency, and endeavor to design the instrument within this limit. The following empirical relation was first given by Hookham.[34] If A is the area of cross section of the air gap, L the distance between the pole pieces, a the cross section of the magnet, and l the length of the magnet (mean length of the lines of force in the steel) then A/L should equal about 70 times a/l. This ratio of A/L to a/l may be called the permanency factor of the magnetic circuit. Heinrich and Bercovitz[35] restate Hookham's formula, but say that the permanency factor should not be less than 100. In the usual construction of permanent-magnet moving-coil instruments, L is equal to the sum of the lengths of the two air gaps, or in general, it is the total length of air gap.

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[33] There is relatively little in print on the detailed design of electrical instruments. Edgcumbe's "Industrial Electrical Measuring Instruments," 2d edition, contains much information on the design of permanent magnets, springs, moving coils, fixed coils, current transformers, etc. See also a paper on "Calculation of Moving-Coil Measuring Instruments", by F. Janus, *E. T. Z.* 28, pp. 560-3, 1905.

The Hookham permanency factor of a magnet is a purely geometrical relation, and does not take into account other matters which affect permanency, such as composition of steel, heat treatment, aging treatment, and flux density in the steel. It is, however, a useful guide in design and in comparing types of instrument. Values of this factor for switchboard voltmeters, as given by Heinrich and Bercovitz, range from 35 in poorly designed instruments to 500 in the best ones. In addition to the great advantage of a higher permanency, the use of a high value of permanency factor also results in less stray flux from the magnet, more of it being utilized, and the instrument is also less affected by the presence of adjacent masses of iron or by stray fields.

(c) *Springs*. The quality of the springs used in an electrical instrument has an important bearing on the accuracy of the measurements. Tempered steel springs as used in watches would be the most desirable if they were not magnetic. It is necessary to use non-magnetic material, and the one which has been most generally used is phosphor-bronze. On account of the high resistivity of the latter, it is necessary to employ other bronzes in millivoltmeters where the resistance of the instrument must be kept low. Since in general the elastic properties are less desirable when a large percentage of copper is used, the requirements as to low resistivity and high elasticity are conflicting, and a compromise must be made.

The elastic performance of the spring will depend upon its design, for any given material. The thicker the spring, for a given length and angular deflection, the greater the stresses in the inner and outer surfaces of the material. It is stated[36] that the stress should not exceed 600 kg. per cm.^{2} for phosphor-bronze, and that for a deflection of 90 deg. this is equivalent to saying that the length of the spring must be not less than 1500 times its thickness. If this condition is not complied with, or if the spring material is of inferior quality for the purpose, the instrument will show an undue amount of zero shift, which will reduce the accuracy of the measurements.

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[34] On Permanent Magnets, by George Hookham, *Phil. Mag.*, 5th Series, vol. 27, p. 186; 1889.

[35] *Handbuch der Elektrotechnik*, vol. 2, part 5, p. 29.

Another kind of zero shift is caused by the gradual uncoiling of the springs over a period of years. This effect is noticeable in instruments employing but one spring, such as the moving-iron type. In moving-coil instruments it is customary to use two spiral springs wound in opposite directions, so that the tendency in one spring to unwind would offset a similar tendency in the other spring.

Zero adjusters are becoming more generally used, and are very convenient. As ordinarily made, however, the range of adjustment is far greater than that necessary to correct for the slight amount of zero shift likely to occur in the reasonable use of the instrument. This excess of range of adjustment introduces the liability of error, since the zero adjuster may be used to bring a bent pointer back to zero, thus introducing errors over the whole scale which in some types may be objectionably large. While it might be some additional trouble for the maker, the service accuracy and reliability would be improved by limiting the range of zero adjustment to a small amount, say not over 1 deg. or 2 deg. of arc.

A permanent shift of zero in an instrument with a uniform scale can be corrected for, but transient zero shift occurring during the use or test of an instrument can not be accurately allowed for because it is not possible to know the effective zero position corresponding to given readings.

(*d*) *Ratio of Torque to Weight of Moving Element.* In order that the frictional error of an instrument shall be kept below the limit of reading, it is necessary to have good workmanship in the pivots and jewels, and also a value of the ratio of torque to weight which is not less than a specified amount for the given kind of instrument. Janus[37] gives the rule that the quotient of the torque in centimeter-grams divided by the weight in grams should not be less than 0.17; Heinrich and Bercovitz[38] give the value 0.05. Both of these cases refer to a full-scale deflection of 90 deg., and do not specify any particular class of service. A more definite statement[39] is that of Edgcumbe, namely, that the ratio of torque in centimeter-grams to weight in grams, for 90 deg. deflection, should not be less than 0.05 for portable instruments or 0.10 for switchboard instruments, the higher value for the latter being necessary because the axis is horizontal, the pivots usually are coarser, and the service is more severe. For switchboard service MacGahan[40] considers that 0.15 is the minimum satisfactory value.

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[36] Edgcumbe, Industrial Electrical Measuring Instruments, 2d ed., p. 31.

[37] Janus, *E. T. Z.* vol .28, p. 560; 1905.

The presence of frictional error or "sticking" may be observed by testing the instrument in comparison with a suitable standard, bringing the instrument under test carefully to a given reading without overshooting, first from lower, then from higher values. The difference between the means of sets of readings approached from the two directions will be twice the frictional error.

In this country it is more usual to express the torque of electrical instruments and meters in millimeter-grams.[41] The corresponding values of ratio of torque to weight then become ten times the value above given.

It should be noted that an instrument may have a high ratio of torque to weight and still be objectionable because the weight is excessive. In fact, it appears that as the weight on the pivots is reduced the value of torque-weight ratio may be lower for a given limit of frictional error.

The importance of high torque-weight ratio and light weight of moving element in watthour meters is now generally recognized, and a statement of these values is customary in announcing new designs. The corresponding practise in indicating instruments seems to be only beginning.[42]

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[38] Heinrich and Bercovitz, *Handbuch der Elektrotechnik*, vol. 2, part 5, p. 14.

[39] Edgcumbe, Industrial Electrical Measuring Instruments, 2d ed., p. 32.

[40] *Elec. Jour.* vol. 8, p. 1100; 1911.

[41] The use of the millimeter-gram is required by the A. I. E. E. Standardization Rules (No. 236).

[42] A recent catalog issued by an American maker gives values of torque and weight for direct-current and moving-iron instruments.

(*e*) *Damping Devices.* The usual methods of damping involve either eddy currents or fluid friction. Eddy-current damping is the ideal method in the permanent-magnet moving-coil instrument, and is realized by winding the moving coil on a conducting frame (usually of aluminum) in which eddy currents are set up whenever the coil moves. Eddy-current damping is also used in two of the leading American makes of electrodynamic instruments.

The most commonly used fluid-friction damper is the air damper, which is adapted to both switchboard and portable instruments. Recording instruments with moving systems of large inertia require more damping than can be supplied by an air damper, and oil damping is used. Another typical case requiring oil damping is that of the electrostatic voltmeter with its large moment of inertia and small operating forces. The inconveniences attending the use of oil make it undesirable where one of the other methods can be used.

Damping devices, if free from static friction, as they should be, can affect the accuracy of measurements only indirectly. If the damping is too small or too large to give critical motion of the moving system, larger errors may be made in attempting to read the momentary value of a fluctuating load. For loads which fluctuate very greatly, overdamping is sometimes used in order to smooth out the fluctuations and give an average value.

The one disadvantage of air damping is that it requires very small clearances between the vane or piston and the chamber in which it moves. Compared with the eddy-current damper consisting of an aluminum vane moving between the jaws of a permanent magnet, the air damper adds less weight and less moment of inertia to the moving system. The magnet of the eddy-current damper introduces a stray field, and is in turn exposed to the demagnetizing action of the alternating current in electrodynamic instruments, and for this reason it could hardly be used but for the fact that such instruments of the better class are now generally provided with magnetic shields around the operating system.

(*f*) *Instrument Losses.* The torque and torque-weight ratio of some forms of instrument may be increased by designing the windings for greater current densities and consequently greater internal losses. This procedure is open to several objections. The increased losses cause greater temperature differences within the instrument, and thus tend to produce larger errors due to self-heating. They also affect the conditions in the circuit. Increased losses imply greater resistance in series windings and lower resistance in voltage windings. When instrument transformers are used, their accuracy may not be so good when the instruments have higher losses. Finally, the margin of permissible overload is cut down as the losses are increased. In general, instruments should be designed with the smallest losses compatible with adequate torque.

When using instruments under conditions such that the power lost in the instruments needs to be taken into account, the connections should in general be made so that the error would be a minimum if not allowed for.[43] For example, if the power taken by a 100-volt 1-ampere lamp is to be measured, using an ammeter of 0.1 ohm resistance and a voltmeter of 10,000 ohms resistance, we may connect the voltmeter either around the lamp or around both the lamp and the ammeter. In the former case the error is 1 per cent, since the ammeter reads the sum of the lamp current and the voltmeter current. In the second case the voltmeter reads the lamp voltage plus the drop through the ammeter, and the error is only 0.1 per cent. If the voltage across the load is small compared with the rated voltage of the instruments, it may be better to put the shunt instruments across the load only. For example, if the above ammeter and voltmeter are used to measure the power taken by a 10-ohm resistor on a 10-volt circuit, the error would be 0.1 per cent with the voltmeter connected across the load, or 1 per cent if across both load and ammeter. Similar considerations hold for the case of measurement of small values of power with an uncompensated wattmeter.

7. Effect of Details of Construction on Accuracy

(*a*) *Good Connections.* It is of importance in all instruments, and especially in millivoltmeters and in instruments operated from current transformers, that the electrical connections shall be well made and capable of remaining so in use. Binding posts should be pinned or otherwise secured so that they will not turn, and the electrical connection should be made independent of the attachment of the post to its support, so that the loosening of the post will not introduce a bad contact. Neglect of this principle has given rise to inaccuracy and uncertain readings, not only in instruments, but also in current transformers, where a defective contact affects the ratio and phase angle.

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[43] H. B. Taylor, *Elec. Jour.*, vol. 2, pp. 474-481; 1905.

One of the best methods of checking the connections of an instrument is to measure its resistance with a Wheatstone bridge, using one cell (or at most only a few cells) to supply the current. Such a measurement of the voltage circuit of a wattmeter, for example, will often reveal the existence of a bad contact at the key which would not be detected in the ordinary use of the instrument, but which might become bad enough to introduce unsuspected errors later.

(*b*) *Contact Keys*. These should be made so as to close with a slight wiping motion to keep the surfaces clean. The contact points should be of platinum or some other metal or alloy suitable for the purpose. Built-up keys having the parts separated by thin sheets or washers of insulating material are more liable to give trouble by leakage or insulation breakdown between the parts than keys in which the metal parts are independently carried by the hard rubber or other insulating support.

(*c*) *Spring Abutments*. Instruments occasionally develop during shipment apparent zero errors and friction, which are found to be caused by the catching of one or more turns of the spring on some projecting part of the spring abutment. The latter should be made in such a way that this cannot occur.

(*d*) *Scales*. Cardboard scales often give trouble by loosening, especially in moist climates, and they should preferably be held by screws or rivets in addition to the usual cement. Buckling of the scale may bring it near enough to the needle to introduce friction.

8. The Measurement of Voltage

(*a*) *Direct-Current Circuits*. While some other types can be used for this service, the permanent-magnet moving-coil voltmeter is the acknowledged standard except for certain low-priced instruments used for testing dry cells, for which the moving-iron type is used.

While there is no theoretical limit to the maximum range for which direct-current voltmeters may be made, in practise the limit is set by the high resistance of the multiplier and the consequent large shunting effect of leakage paths. For example, an ordinary portable d-c. voltmeter to measure up to 15,000 volts would require a multiplier of about 1.5 megohms resistance, and if a leakage path occurs, with a resistance of 150 megohms, an error of 1 per cent will result. The same leakage path across the series resistor of a 150-volt instrument would produce an error of only 0.01 per cent. By subdividing the multiplier and applying the guard-wire principle to protect each subdivision from leakage, the error in measuring the high voltage can be reduced as much as is desired, but at the cost of complication, extra cost, and added power loss. For switchboard use there are on the market d-c. voltmeters with oil-insulated series resistors up to 15,000 volts.

In using high-range voltmeters, or in checking them by the potentiometer method, considerable care is necessary to avoid errors due to electrostatic action on the index of the voltmeter and to leakage currents through the galvanometer used with the potentiometer. It is probable that many unsuspected errors occur in the use of potentiometers having delicate reflecting galvanometers. This is especially liable to occur in humid weather. As an illustration, the results of an experiment carried out some years ago at the Bureau of Standards may be cited. Two five-dial potentiometers of the high-resistance type, with reflecting galvanometers, were used with volt boxes to measure the voltage of a storage battery, which was about 500 volts. One of the potentiometer outfits was carefully shielded by the guard-wire method, the other was used in the ordinary manner. The shielded one gave very steady readings, as would be expected, while the other showed variations up to 1.5 per cent, depending upon the condition of the battery as to insulation from ground. The temperature during this experiment was 26 deg. cent. and the relative humidity 90 per cent. By placing the unshielded potentiometer and all its accessories, including the galvanometer, on hard rubber blocks, the maximum error was reduced from 1.5 per cent to 0.3 per cent. While this experiment was carried out with potentiometers, it is evident that similar difficulties may arise with high range voltmeters.

The lower limit of measurement with portable d-c. instruments of regular construction is about 20 millivolts (for full scale deflection) if temperature corrections are applied, or 200 millivolts for instruments requiring no temperature correction. This applies to instruments compensated by the addition of resistance in series with the moving coil. By the use of suitable resistances in parallel and in series, compensated instruments are made for as low as 45 millivolts for full scale deflection.

When a d-c. voltmeter is connected to a battery or other source having an appreciable internal resistance, the reading of the voltmeter is lower than the e.m.f. of the source by an amount which depends on the relative magnitudes of the resistance of the source and of the voltmeter. If the resistance of the source is unknown, the magnitude of the error is unknown. The following method will show whether such an error exists and will make it possible to compute from two readings the true e.m.f. of the source.

Connect the voltmeter to the two points between which the measurement is to be made; let its reading be denoted by V_1. Then shunt the voltmeter by a resistance equal to its own resistance; let the corresponding reading be V_2. If the internal resistance of the source be negligible, the two readings will not differ. If they differ, the e.m.f. E of the source may be computed by the formula

$$E = \frac{V_1 V_2}{2\,V_2 - V_1} \tag{2}$$

This method is not limited to the simple case of a source with an unknown resistance in series, but is quite general, and may be stated as follows. If A and B are any two points on a network of conductors containing unknown but constant direct electromotive forces and resistances, and if E is the value of the potential difference between A and B, then the use of a d-c. voltmeter in the manner above described will give the value of E. In other words, a voltmeter used in this way will give the same result (within the limits of accuracy of the method and the instrument) as a null measurement with a potentiometer. This is true, whether the voltmeter be connected directly to the points A and B, or through "pressure wires" of appreciable resistance.

The accuracy of this method is greatest for values of V_2 nearly equal to V_1, and decreases as the ratio V_2/V_1 becomes less. For the case of internal resistance of the source equal to that of the voltmeter, $V_2 = 2V_1/3$, and an error of 0.1 per cent in V_1 and V_2 will cause errors of $+0.4$ per cent and -0.3 per cent respectively in the calculated value of E.

In using this method, if V_2 differs from V_1 by only a small amount ΔV, say a few per cent, the value of E is given well enough by the approximate formula

$$E = V_1 + \Delta V \tag{3}$$

A modification[44] of the above method consists in getting the reading V_1 as above, and then inserting a resistance in *series* with the voltmeter, equal to the resistance of the voltmeter. The unknown e.m.f. E may then be found by the formula

$$E = \frac{V_1 V_2}{V_1 - V_2} \tag{4}$$

A possible application of the above method would be to avoid error in the use of thermocouple pyrometers due to resistance in the leads. The temperature is measured by the reading of a millivoltmeter connected to the couple, and the reading for a given temperature will vary with the lead resistance. By providing such millivoltmeters with a resistance which can be connected in parallel by a key, the two readings may be used to determine the true electromotive force of the couple. Another obvious use is for the measurement of voltage at feeder ends by means of pressure wires of appreciable resistance.

With the increasing need for accuracy in temperature measurement and control, methods have been devised to eliminate the effects of lead resistance. In the instrument known as the pyrovolter the moving coil of the millivoltmeter is first used as a galvanometer to indicate when the IR drop through a resistance coil is equal to the value of the electromotive force E to be measured. By pressing a key, the circuits are then changed so that the moving coil now carries the current I. The scale is marked to read the IR drop, and hence the quantity to be measured. The instrument thus draws no current from the source, and within limits, is equivalent to a millivoltmeter of infinite resistance; since it therefore has something in common with the potentiometer, it might be called a "potentiometric millivoltmeter." There is a limit to the amount of resistance permissible in the leads, beyond which the sensitivity of setting the IR drop to equality with E is impaired, and with it the accuracy of the results. It must also be kept in mind that the result depends directly upon the constancy of magnet and springs, effect of local disturbing fields, etc.

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[44] Suggested by Dr. P. G. Agnew.

A more recent device for the same purpose is called the "compensated indicator." It consists of a millivoltmeter having a variable rheostat in series, also a key by which a shunt circuit may be connected across the moving coil and a part of the series resistance cut out. In operation, the variable rheostat is adjusted until no change in reading occurs when the key is depressed. The reading then gives the value of the electromotive force of the couple. The compensated indicator and the pyrovolter have other applications besides in pyrometry. They may be used to measure the potential difference between two points of a network, or the electromotive force of a battery, without the necessity of allowing for the current taken by the instrument, as is necessary with ordinary indicating instruments.

(*b*) *Alternating-Current Circuits.* For this work the type of voltmeter to be used depends on the nature of the service, as previously discussed. Voltmeters are made for direct connection to a-c. circuits up to 750 volts, and in some forms up to 2300 volts, but the latter practise is not recommended.

The measurement of alternating voltages less than one volt is difficult. A high-grade electrodynamic voltmeter of 1-volt range has a resistance of 2 ohms, so that it draws 0.5 ampere from the source. If such an instrument were rewound for a maximum reading of 0.25 volt, it would require a current of two amperes, which would be too large to pass through the springs. For such a range, an unshunted hot-wire ammeter of say three to five amperes range may be used, but the current taken from the source is excessive. When it is desired to measure the current taken by a load at low voltages of the order of 0.25 volt, the current required by a hot-wire ammeter used as a voltmeter may introduce a serious error, which may be difficult to allow for when the phase angles of the load and the voltmeter are different. To overcome this, the following method may be used.

Fig. 6 shows a load *C* for which it is desired to find the current for a given applied voltage. *A* is a two-range ammeter having the terminals of the two sections of the winding accessible.[45] The total current taken by the load and the voltmeter is passed through one section, and the voltmeter current is passed through the other section in reverse direction. The voltmeter current thus cancels itself so far as the reading of the ammeter is concerned. The voltmeter corrections should be determined with the ammeter coil and the leads up to the points *B* and *D* included as a part of the voltmeter.

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[45] Two-range ammeters with links for changing the range, and hence accessible terminals of the two parts of the winding, are available.

(*c*) *Voltage Transformers.* These are practically identical with distributing transformers of small capacity. For moderate voltages, no special difficulties are present in their design, such as are encountered in the current transformer. The rated output of the voltage transformer is made smaller than the permissible output as limited by heating considerations, in order to limit the change of voltage with change of load to a sufficiently low value. It is good practise to choose the winding ratio so that the "voltage ratio" (ratio of primary impressed voltage to secondary terminal voltage) will be slightly below its nominal value at no load and above it at full load, so that the ratio will be correct at some intermediate load approximating average conditions of use.

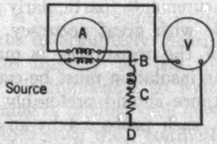

Fig. 6

The secondary terminal voltage is in general not in exact opposition to the primary voltage, but is displaced from it by a small angle which for the lack of a better term[46] is called the "phase angle." This quantity however, does not have to be considered in connection with the measurement of voltage, and will be discussed under "Measurement of Power."

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[46] Strictly speaking, the phase angle of a voltage transformer would be the angle 180 deg. $\pm\gamma$, where γ is the angle as defined above. In the same way, the phase angle of a condenser should be defined as the angle between the impressed sinusoidal voltage and the current, and is nearly 90 deg. In practise, however, the "phase angle" of a condenser is taken as the small angle by which the above differs from 90 deg. Various names have been suggested for the small angle by which a vector quantity departs from its ideal position; for example, "angle of defect." This is objectionable because it implies either that the apparatus has a defect, or that the angle may be too *small*, whereas it may be too large. Since the word "bias" may mean a swerving to one side or another of a desirable or normal direction, its use in such expressions as "phase bias," "bias angle," or "angle of bias" would be etymologically correct.

The most accurate method of determining the ratio and phase angle of voltage transformers is one which is practically an a-c. potentiometer arrangement. This method has been devised independently by several workers in this field.[47] It consists in applying the primary voltage to a high non-reactive resistance and in balancing the secondary voltage against the drop over a portion of this resistance. The ratio of the total resistance to the portion is equal to the "voltage ratio." It is obvious that the method is theoretically an accurate one, since resistance measurements (particularly resistance comparisons) may be made with great accuracy. However, in the present case one of the resistances is quite large for high voltages, so that the insulation must be carefully maintained, and the high resistance should preferably be divided into a number of sections, each protected by an electrostatic screen to prevent capacity currents from flowing between the sections themselves and between the sections and surrounding objects. At the Bureau of Standards, the testing of voltage transformers by this method has so far been restricted to an upper limit of 30 kilovolts. The ratio of voltage transformers for voltages considerably above this value cannot be measured as accurately as is desirable, and it seems probable that the ratio as fixed by the ratio of turns may depart considerably from the values which would be given by computation based on the performance of the same iron core wound for lower voltages. The reason is that at high voltages there are capacity currents between the primary sections themselves and between sections and other parts, such as core and case. These currents are superposed on the exciting current in the primary winding, and alter both ratio and phase angle in a way which does not at present admit of calculation. The attempt has been made to measure the high secondary voltage of step-up transformers by a voltmeter connected to a special low-voltage winding (the so-called "test coil," "voltmeter coil," or "tertiary coil") located at the center of the high-voltage winding, and evidence is adduced to show that in certain cases good agreement was obtained between two transformers equipped with voltmeter coils. The use of such a coil will give accurate results, in general, only if the coil is in every way a representative sample of the high-voltage winding. It may be possible to locate the voltmeter coil so that the flux through it is the average flux through the high-voltage winding, but the effect of capacity between coils can not be readily cared for. Differences in ratio of 0.6 per cent have been observed in 100 kv. transformers due to these capacity effects and it seems probable that they may often be larger. It seems evident that there is a great need for means of accurately measuring high voltages, which shall operate directly on the high voltage. Such apparatus, even if not applicable to routine measurements, would be of great value in settling such questions as the accuracy of results obtained with the "voltmeter coil."

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[47] Agnew and Fitch, *Bull. Bur. of Standards*, vol. 6, p. 281, 1909; *Elec. World*, vol. 54, p. 1042, 1909.
L. T. Robinson, TRANS. A. I. E. E., vol. 28, p. 1005, 1909.
F. A. Laws, *Elec. World*, vol. 55, p. 223, 1910.
Sharp and Crawford, TRANS. A. I. E. E., vol. 29, p. 1517, 1910.
Agnew and Silsbee, TRANS. A. I. E. E., vol. 31, p. 1635, 1912.

(*d*) *Crest Voltmeters.* With the constantly rising values of transmission voltages, the insulation of transformers and other high-voltage apparatus becomes more and more important. Since the evidence indicates that the breakdown of insulation is determined by the crest value of a voltage wave, rather than the effective value, it is necessary to have means for measuring crest voltages. This is the more necessary because distortion of the voltage wave is liable to occur in the testing of insulation, on account of the electrostatic capacity of the load.

The needle gap has been used for years for measuring crest voltage, and the sphere gap is a more recent apparatus which is more reliable and better adapted to high voltages. The needle gap is very much affected by distortion of the electrostatic field caused by surrounding objects, and the sphere gap is similarly affected, though to a smaller extent.

Direct-reading methods of measuring crest voltage have been devised, and instruments based on some of them are on the market. One of these[48] uses hot-cathode valves to rectify the charging current of a condenser, with a d-c. instrument to measure the current. This method is correct only when the voltage wave has but one maximum and one minimum per cycle. Another form[49] is essentially an oscillograph element which throws upon a screen a band of light the length of which is proportional to the peak voltage. Of these two instruments, the former is used directly on the high-voltage, while the latter is necessarily connected to the low-voltage secondary (or tertiary) winding of a transformer, and hence any uncertainty as to the ratio of transformation affects the accuracy of the result.

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[48] L. W. Chubb, Trans. A. I. E. E., vol. 35, p. 109; 1916.

There is reason to believe that the corona voltmeter[50] is capable of giving accurate measurements of crest voltage, and that its calibration may be computed from measured dimensions and the electric strength of air under given conditions of temperature and pressure. Its establishment as a standard instrument by which other crest voltmeters may be checked seems to depend only upon further work in which high voltages shall be measured and their wave forms determined by methods which are free from the limitations and uncertainties of those now available.

9. The Measurement of Current

(*a*) *Direct-Current Circuits.* The permanent-magnet moving-coil ammeter is the standard for all but small low-priced instruments used for dry-cell testing and for automobiles. For the latter the polarized vane[51] type is largely used.

The practise of American makers seems to be to supply medium size switchboard ammeters with self-contained shunts up to about 75 amperes. Above this, (and also below, on order) instruments with separate shunts are supplied. One maker lists seven-inch instruments self-contained up to 200 amperes. Small instruments, such as three-inch round and the smallest fan-shaped instruments, are self-contained up to 15 to 30 amperes. Large illuminated-dial, also some fan-shaped instruments, are made only for external shunts. The widest variation of practise is found in standard portable ammeters, which are supplied self-contained up to 500 amperes by one maker, while two others limit internal shunts to 30 amperes in order to avoid errors due to heating. This is a matter which affects the accuracy of electrical measurements, and it is therefore advisable to compare briefly the self-contained ammeter with the separate-shunt ammeter.

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[49] Middleton & Dawes, Trans. A. I. E. E., vol. 33, p. 1204; 1914.

[50] Whitehead and Pullen, Trans. A. I. E. E., vol. 35, pp. 809-833, 1916.

[51] This instrument, brought out by Deprez in 1880, contains a pivoted soft-iron vane in the field of a permanent magnet. A coil carrying the current to be measured sets up a field at right angles to the field of the permanent magnet, and the direction of the resultant field thus varies with the current. The vane assumes the position of the resultant field. This construction was used in the Thomson-Rice ammeter and voltmeter in the early days of electric lighting. In recent years the need for a simple and very cheap instrument for automobile use, capable of showing reversal of current flow, has caused this type to be made in large numbers.

In the self-contained ammeter the shunt is not open to the air[52], and hence its rise of temperature under load is greater than it would be if separate from the instrument. While the effect of this heating on the shunt itself may be made small by the use of suitable alloy, the heat liberated raises the temperature of the moving coil, springs, and other parts, thus introducing opportunity for error, since even compensated millivoltmeters may have errors if their parts are at different temperatures. Another objection to internal shunts for large currents is the magnetic effect of the current in the loop formed by the conductors which connect the instrument into the circuit. Large conductors are difficult to handle, and cannot be readily twisted together to make them non-inductive, as can be done with smaller wires. A further advantage of the separate-shunt ammeter is that its range can easily be altered by the use of other shunts.

On the other hand, the separate-shunt ammeter has the disadvantage that its moving-coil circuit is completed through four binding-post contacts, which may become dirty or corroded, and two flexible leads, which may be inadvertently exchanged for others[53] of similar appearance, or which may have their resistance affected by the breaking of strands.[54]

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52. This does not apply to the self-contained portable ammeter of 500 amperes capacity above referred to. This line of ammeters has provision for ventilation of the shunt, in some of the higher ranges.

53. Instrument makers are not always sufficiently careful in this matter. In a shipment of instruments sent to the Bureau of Standards for test were a portable millivoltmeter and a labratory standard millivoltmeter, with leads which were of different length and resistance and which bore no numbers or other marks to show with which instrument they were to be used.

54. Durand, Congress of the Applications of Electricity, Marseilles, 1908, p. 620, states that he has often found d-c. ammeters reading 10 to 20 per cent low from this cause. This evidently occurred with ammeters having a low resistance in the instrument, with low shunt drop. He gives this difficulty as one reason for avoiding low values of shunt drop, and he prefers 100 millivolts.

One American manufacturer has for several years supplied small portable ammeters with three self-contained ranges on the plan of the Ayrton-Mather galvanometer shunt. This is an arrangement which deserves to be more widely known and used. It has the advantages of avoiding flexible leads, as well as binding-post and switch contacts in the millivoltmeter circuit. There is one common + terminal, with a – terminal for each range, and the change from one range to another is made by shifting the – lead from one – terminal to another. Thus the only contact which is shifted is in the line circuit and has no effect on the accuracy of the instrument. It would be an advantage to the users of standard portable self-contained ammeters if all makers would furnish them with multiple ranges on this plan.

It was once common practise to make direct-current ammeters with non-interchangeable shunts and leads. It is now the prevailing custom of American makers to adjust all shunts of a given class to a uniform drop, which varies from 50 to 100 millivolts. The Meter Committee of the National Electric Light Association investigated the matter of standardization of shunts by sending a circular letter to ten instrument makers. This letter contained sixteen recommendations, some of which related to mechanical details such as width of slots and number of bolt holes, while others specified the use of a standard millivolt drop, the accuracy of adjustment (1 per cent), and the electrical properties of the shunt metal, namely, a practically zero temperature coefficient and no thermoelectric effect against copper. The Committee suggested the tentative adoption of the following values of millivolt drop; switchboard type instrument shunts, 50 and 60 millivolts; portable instrument shunts, 100 and 200 millivolts. The Committee concluded from its analysis of the replies that a large percentage of shunts was being manufactured in accordance with its recommendations, and that further conference with the makers might lead to further progress.

The matter of standardizing all shunts of a given maker has a bearing on instrument accuracy. If shunts of a given kind and make are only roughly adjusted, say within 10 per cent of a nominal value, the adjustment of the instrument resistance must be individually made to suit a particular shunt. In service, such instruments may be connected to the wrong shunts through oversight. If two shunts differing in drop by 10 per cent, which were correctly adjusted to their respective millivoltmeters, are interchanged, the instruments will differ by over 20 per cent when connected to measure the same current. A case of this nature is reported in which the error resulted in the overloading of one generator and the underloading of another, with generally unsatisfactory results, for more than a year before the trouble was discovered and rectified.[55]

The range of measurement by means of d-c. ammeters is very wide. Excluding the very sensitive unipivot instruments and considering only regular two-pivot instruments of good torque-weight ratio, capable of operation in any position, the lower limit is perhaps illustrated by a standard portable instrument giving a deflection of 150 divisions for 300 microamperes. No upper limit of current exists for the shunted ammeter, except that fixed by the requirements of the user. Shunts of 20,000 amperes rating are regularly listed by several American makers, and larger ones may be had on special order.

The construction and accurate adjustment of very large shunts presents some difficulties. The usual practise in making shunts of small and medium capacities is to solder all of the sheets of alloy into two terminal blocks. This requires that all of the sheets shall be soldered into a given block at one heat. This becomes very difficult in large shunts, and the accurate adjustment is not easily accomplished because of the lack of resistance standards of such very low values and the impossibility of getting enough current to give a sufficient drop for an accurate test. It is possible to avoid this by making up the shunt of sections in parallel, each section being individually adjusted and provided with potential wires which have resistances proportional to the resistance of the shunt and large in comparison with the latter. The sections may then be bolted or soldered together at each end to form a shunt of large capacity. The free ends of the potential wires leading from each terminal are joined together to form the two potential points of the large shunt. With this arrangement, the separate sections are readily adjusted, and small irregularities in the division of current among the

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55. McTighe, *Elec. Jour.* vol. 3, pp. 358-9, 1906.

different sections will not materially affect the resistance of the shunt as observed from these potential points.[56]

The method of connecting the copper bars to the ends of large shunts has an appreciable effect on the accuracy. Changes amounting to as much as 10 per cent have been reported.[57]

When a d-c. ammeter is connected into a circuit to measure a current, the reading of the ammeter, corrected for any error it may have, does not give the value of the current which was flowing in the circuit before the ammeter was inserted. This is because the addition of the ammeter increases the resistance and therefore diminishes the current. While this effect is small in many or most practical cases, there are occasions where the current is flowing in a low-voltage low-resistance circuit, and the use of a high-resistance ammeter, such as one of the hot-wire type, may introduce an appreciable error. The following method makes it possible to determine the value of the current I which was flowing before the ammeter was inserted:

Let the reading of the ammeter be I_1. Then insert in series with it a resistance equal to its own resistance; let the reading now be I_2. Then the current originally flowing is

$$I = \frac{I_1 I_2}{2\,I_2 - I_1} \tag{5}$$

This result is of the same form as that for the similar case of the voltmeter. It is possible to derive a slightly simpler formula by shunting the ammeter with an equal resistance, but this procedure is not to be recommended, especially for low-resistance ammeters.

(*b*) *Alternating-Current Circuits.* For this work, moving-iron ammeters are probably the most generally used, because of their simplicity, relative independence of frequency, and the fact that if necessary they can be checked (at least approximately) on direct current.

In the laboratory the moving-iron ammeter is very convenient for measurements not of the highest accuracy. For more accurate measurements on ordinary commercial frequencies, the electrodynamic ammeter has a number of advantages, including the possibility of accurate test on reversed direct current. For higher frequencies it is necessary to use hot-wire or thermocouple instruments. The writer has been informed by some users of thermocouple instruments that they are more robust and give much better satisfaction than the hot-wire instruments. They are displacing hot-wire instruments in the most important commercial application of the latter, namely, in radio work.[58]

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[56] The principle involved in this method is discussed by Wenner from a different point of view; *Bulletin of Bur. of Standards*, vol. 8, p. 575, 1912, Scientific Paper No. 181.

[57] Rosa, Trans. A. I. E. E., vol. 24, p. 223, 1905; Durand, Congress of the Applications of Electricity, Marseilles, 1908, pp. 625-6.

Moving-iron ammeters are made in ranges from about 0.1 ampere up to 600 amperes. Above the latter value it is necessary to use current transformers. This is frequently done for much lower ranges, and is always done when the circuit is of high voltage, in order to insulate the instrument from the circuit. The accuracy of the measurement is then a composite of the individual accuracies of the ammeter and the transformer. The ratio of transformation (primary current divided by secondary current) is in general not quite equal to the nominal value as marked on the name plate, and it varies with the frequency, primary current, and secondary burden. This matter is discussed in more detail in the following paragraphs.

(*c*) *Current Transformers.* An ideal current transformer would give a secondary current equal to $1/n$ of the primary current (where $n/1$ is the marked ratio) for all values of primary current, frequency, wave form, and secondary burden. Furthermore, the phase displacement between the primary current and the secondary current would be 180 deg. under all conditions.

Because a component of the primary current is required to magnetize the core, the secondary ampere-turns are in general smaller than the primary ampere-turns, and the angle between them differs from 180 deg. by a small angle. By using a slightly smaller number of secondary turns than the number required by an ideal transformer for a given ratio, the secondary current may be brought near to the desired value for a given set of conditions. The phase angle cannot be so readily controlled, and since it depends largely on the vector relation between the secondary current and the component which magnetizes the core, the usual method of keeping the phase angle down is by the use of proper material and good design.[59]

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[58] G. Y. Allen, *Elec. Jour.* vol. 16, pp. 494-500, 1919.

For a given current transformer the ratio will vary with the primary current and frequency, and with the secondary burden.[60] There is no method in use at present by which the performance of a current transformer under one set of conditions can be readily computed from its performance under another set. It is therefore necessary, in accurate work, to test the transformer under the conditions of use. In general, an increase in the secondary burden will increase the values of the ratio and the phaseangle for all values of secondary current and will also increase the rate of change of ratio and of phase angle with changing secondary current. The secondary burden should therefore be reduced to a minimum. A decrease in frequency acts in much the same way as an increase in the secondary burden.

The requirements which current transformers must meet differ widely. If only an ammeter of low impedance is to be operated at one stated frequency, the current ratio only is of importance. If the transformer and ammeter may be calibrated and used together, a relatively large variation in ratio with change of secondary current is not objectionable, as it can be taken care of in the marking of the ammeter scale. However, if a current transformer is to be capable of satisfactorily operating one or a number of ammeters, the latter being calibrated independently of the transformer, it is necessary to use what might be called a higher grade of transformer. To produce it, in general one must use more iron in the core, or more copper in the windings, or both. If, further, it is necessary to operate a wattmeter or a watthour meter, and the power factor of the system is appreciably below unity, a still better grade of transformer must be used, since in addition to good ratio performance a small phase angle is required.

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[59] For a clear and practical discussion of instrument transformer design see Edgcumbe, Industrial Electrical Measuring Instruments, 2d ed., pp. 312-338.

[60] The term "secondary burden" is recommended by the Institute as a substitute for the somewhat ambiguous terms load, secondary load, or secondary connected load. It is further recommended that secondary burden be expressed quantitatively as the resistance in ohms and inductance in henrys of the external circuit connected to the secondary winding of the transformer.

In the above statements, *grade* is intended to apply to the materials and the electrical and magnetic design, not to the mechanical features of construction, nor to the ruggedness of the windings, breakdown strength of insulation, etc. These latter features, while not directly related to the inherent accuracy as determined by design and materials, are of great importance because of their relation to service accuracy and reliability. Current transformers are often subjected in service to severe electric stresses due to high-frequency line surges, and to mechanical stresses caused by electrodynamic action between the windings when short circuits occur. Their construction therefore requires careful attention to details.

Using the word *grade* in the above qualified sense, it would seem desirable to have a set of performance specifications for various standard grades of current transformers.[61] These specifications would include the permissible deviation of the current ratio from the marked value over prescribed ranges of secondary current, secondary burden, and frequency, and would set limits for the phase angle under the same conditions. This would remove the uncertainty that now exists in comparing or selecting current transformers, since the present name-plate rating in watts or volt-amperes gives but little definite information which may be used as a basis of comparison, especially between the transformers of different makers.

Some general statements may be made in regard to the suitability of current transformers for accurate measurements. It is stated that,[62]

> A well-designed current transformer will have from 600 to 1200 ampere-turns in the primary at its rated full-load current. There is no objection to increasing the upper limit to almost any extent when the current to

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[61] The "Code for Electricity Meters" of the Joint Meter Committee of the A. E. I. C. and the N. E. L. A. contains specifications for the acceptance of types of current and voltage transformers. While this is limited to the use of such transformers as auxiliaries to watthour meters, and does not consider other grades, it would serve as a beginning for more inclusive specifications. The advance in the art of making transformers is such that the limits of tolerance for ratio and phase angle given in the Code are too broad for present practise.

Edgcumbe, in Industrial Electrical Measuring Instruments, 2d ed., pp. 337-8, gives concise specifications for the accuracy of current and voltage transformers, with tolerances considerably smaller than those of the Code for Electricity Meters.

[62] Edgcumbe, Industrial Electrical Measuring Instruments, 2d ed., p. 321.

be measured exceeds 1200 amperes, but there is no advantage in doing so for smaller currents, and the cost of construction is increased. These figures refer to transformers intended for a secondary load of from 15 to 40 volt-amperes at normal full-load current and at a frequency of 50. For other frequencies the permissible volt-ampere load varies in direct proportion with it, and for heavier loads an increase in the ampere-turns is desirable.

Another general statement is that current transformers having hinged or split cores should not be used with wattmeters or watthour meters, on account of the large phase angle which results from the large reluctance of the air gap in the core, and to the small value of full-load ampere-turns (125 to 250). These transformers are intended only for the approximate measurement of current in station or outside lines where it is necessary to link the transformer with the cable without cutting the latter. Because of the low value of ampere-turns, such transformers are used with ammeters specially calibrated with them, and they should be calibrated for the frequency on which they are to be used. Similar statements apply to the bushing type of current transformer of low full-load ampere-turns and excessive length of magnetic circuit.

Low-range instruments should not be substituted for five-ampere instruments with the expectation of better accuracy at low loads, unless the transformer has been tested with such instruments. The impedance of instruments of a given type increases approximately as the inverse square of the ampere rating.

The secondary circuit should not be opened while current is flowing, because the voltage at the secondary terminals may become dangerously high. The high value of flux may cause excessive heating of the core with liability of damage to the insulation. Furthermore, the magnetic condition of the core will be affected, with a resulting change in the ratio and phase angle, especially at small values of secondary current. The same abnormal condition of the core will occur if a direct current of sufficient strength flows through either of the windings. If magnetization occurs, the transformer may be brought back to normal condition (demagnetized) by operating at a flux density above the knee of the magnetization curve and gradually reducing the flux to a very low value. A method usually given for accomplishing this result consists in passing at least half the rated primary (alternating) current through the primary winding with a resistance of 10 to 20 ohms or more connected to the secondary in series with the secondary instruments. This resistance should then be gradually reduced to zero by steps of one ohm or less. Loose or corroded contacts, or excessive resistance in the secondary should be carefully avoided, and the resistance of the current windings of secondary instruments should be measured by a bridge to detect possible variable resistance due to loose or dirty connections.

When it is necessary to secure the best possible accuracy from wattmeters or watthour meters operated from current transformers, it is not advisable to connect relays or circuit-breaker trip coils in the same secondary circuit. In case it should be necessary to use the same transformer for instruments, relays, and trip coils, a high-grade transformer of liberal secondary rating should be used, and a reduced degree of accuracy should be expected. For important metering installations the current transformers operating the meters should have no other secondary burden.

When the secondaries of current transformers in a polyphase line are interconnected, it is difficult to calculate the effective secondary burden on the individual transformers in order to apply corrections for ratio and phase angle. For this reason, interconnection of secondaries is not advisable where the highest accuracy is desired.

The question of permanency of the constants of instrument transformers is one that has been up for a number of years. No systematic study of the question appears to have been published. The following statement has been made:[63]

> The permanency of commercial (instrument) transformers is such that their ratios may be relied upon to remain unchanged for a period of five years (but not longer without calibration) provided that the transformers have not been subjected to overloads, primary short circuits, or open-circuiting of the secondary of the current transformer under load.

Assuming that none of these abnormal conditions had been present, and that no contacts had occurred inside the windings, the only thing that could cause changes in ratio or phase angle is a change of the magnetic properties of the core. The kinds of transformer steel in use prior to the advent of silicon steel showed changes of this kind which in some cases were very marked. It is generally understood that modern high grade silicon steels are practically non-aging. In view of the fact that methods for testing current transformers are being devised[64] which are adapted to use by central stations, it is to be hoped that data will be kept for a sufficient time to enable the question of instrument transformer permanency to be settled.

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[63] Electrical Meterman's Handbook, 2d ed., p. 327; taken from "Code for Electricity Meters," 1912 ed., 123.

Multiple-range current transformers in which the same primary windings are grouped in various connections by means of links or plugs usually have practically identical phase angles and proportional current ratios on the different connections. Current transformers in which the primary winding is put in place by the user[65] sometimes show slightly different values of ratio for different locations of the primary windings. If the primary leads are kept well centered in the hole and the return portions several inches away from the transformer these variations will usually be less than 0.1 per cent.

10. THE MEASUREMENT OF POWER

Direct-current power may be measured by voltmeter and ammeter, or with an electrodynamic wattmeter. The latter requires that the mean of reversed readings be taken in order to eliminate the effect of local fields on unshielded wattmeters, or of residual magnetic polarity in shielded ones. For the measurement of a-c. power the wattmeter is the most suitable instrument.

(*a*) *The Electrodynamic Wattmeter.* For the accurate measurement of power at usual commercial frequencies this instrument is practically alone in the field.[66] This refers more particularly to portable or laboratory instruments which must be capable of use on various frequencies, power factors, and wave forms. These instruments are of great commercial importance because they are used to check watthour meters, either directly or through portable watthour meters ("rotating standards"). Their accuracy limitations therefore directly affect the customer's bill and the company's income, and they are well worth careful study.

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[64] P. G. Agnew, *Bull. of the Bureau of Standards*, vol. 11, p. 347, 1914 (Scientific Paper No. 233); *Electrical World*, vol. 62, p. 898, 1913; O. A. Knopp, *Electrical World*, vol. 67, p. 92, 1916; F. B. Silsbee, *Bull. of the Bur. of Standards*, vol. 14, p. 317, 1917 (Scientific Paper No. 309.)

[65] This kind of current transformer needs a standard name. It is called by the following names at present: through-type, inserted primary type, hole type, form R, and doughnut type.

[66] The induction wattmeter was formerly made as a portable instrument, but it was subject to the limitation of a fixed frequency, and had a rather large temperature coefficient. At present, all portable wattmeters made in this country are of the electrodynamic type.

In principle, an electrodynamic wattmeter consists of a fixed coil, through which the load current flows, and a moving coil through which flows a current which is proportional to the voltage across the load. The moving coil is usually pivoted within the fixed coil, and its motion is opposed by a spiral spring. The fixed coil is wound with comparatively few turns of thick wire, or in some instruments with copper strip or bar, while the moving coil is wound with many turns of fine wire, and in series with it is a relatively large noninductive resistance. When currents flow through both coils, the moving coil tends to move so as to make the mutual inductance of the two coils a maximum. In the torsion-head form of wattmeter the moving coil is deflected by the action of the currents, but is brought back to the zero position by turning a head which winds up a spring attached to the moving coil. The zero position in such wattmeters is chosen as the position of zero mutual inductance. In the ordinary deflection wattmeter the mutual inductance is zero at about half scale deflection, and changes sign as it goes through the zero position.[67]

The torque of a wattmeter is at every instant proportional to the product of the currents in the two coils. If the moving coil had no inertia and no damping, its deflection at any moment would thus be proportional to the instantaneous power. Because of its inertia its deflection indicates the average power, which is the quantity usually sought.

(*b*) *Losses in Wattmeter Windings; Compensating Coils.* In using a simple wattmeter as above described, the voltage circuit (moving coil plus added resistance) may be connected either directly across the load, or so that the drop through the series coil of the wattmeter is also included. In the first case, the wattmeter reading includes the power loss in the voltage circuit, and in the second case it includes the loss in the current coil. The choice of one or the other of these connections depends upon the circumstances in the same way as in the use of a voltmeter and ammeter to measure power. The use of a compensating winding on the fixed coil makes it possible to connect the voltage circuit of the wattmeter directly across the load terminals and still get a reading of power which does not include the loss in the voltage circuit. The additional complication is justified by the convenience of not having to apply corrections, but it introduces some risk of error due to wrong connections. To avoid this, it is well to check the correctness of connections experimentally. When using a compensated wattmeter to measure the power taken by a load, break the load circuit by removing the wire from one terminal of the load, while leaving the voltage circuit excited. If the connections are correct, the wattmeter will show zero deflection, unless it is not entirely compensated. If the connection of the voltage circuit is incorrectly made, so that the compensating coil is not in circuit, the wattmeter will read up scale by an amount equal to the watts loss in its voltage circuit.

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[67] The Thomson inclined-coil wattmeter is an exception. In it the mutual inductance is positive over the whole scale, increasing toward the full scale point. The point of zero mutual inductance is at a considerable angular distance below the zero of the scale, and is a point of unstable equilibrium when currents are flowing through the two coils. The mutual inductance increases in both directions from its zero position.

In testing a compensated wattmeter on separate sources of current and voltage, the compensating winding should be out of circuit. To test this point, apply an alternating voltage to the voltage circuit, with no current flowing in the current circuit. If the connections are correctly made, there will be no deflection; otherwise, a deflection down scale will result. This test can not be made so conveniently on direct current, on account of the effect of the local magnetic field.

Serious errors may result in the measurement of small values of power with compensated wattmeters of low range, unless the compensating winding has been very carefully located so as to have the same effect upon the moving coil as that of the fixed coil, for all positions of the moving coil.[68] A case came under observation some years ago at the Bureau of Standards which illustrates this point. A wattmeter of 150 watts rating was checked for compensation in the manner above described, and was found to be very closely compensated. It was then tested by the method of separate sources, and found to be reasonably accurate. In service, however, using the compensating winding, the results of measurements on standard lamps showed very large inconsistencies, and the wattmeter was again investigated for compensation, not only at the zero point, but at points up scale by shifting the spring abutment. It was found that the compensation held good only at the zero point, and was very far from correct at other points. Investigation showed that the compensating coil had been wound up separately as a coil of short axial length compared to that of the current coil, and had been located at one end of the current coil, inside of the latter. The instrument was later rewound, using a cable of insulated strands, the center one being selected as the compensating winding. This method gave practically exact compensation for all positions of the moving coil. It may be pointed out that one of the ways of improving compensation for such losses is to have the losses as small as possible. For this and other reasons, the losses in both coils of a wattmeter ought to be kept as low as possible, while maintaining sufficient torque.

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[68] A. L. Ellis, Compensating Wattmeters, TRANS. A. I. E. E., vol. 31, pp. 1579-1590; 1912.

(c) *Effect of Self Inductance in the Voltage Circuit of a Wattmeter.* Self inductance in the voltage circuit of a wattmeter is unavoidable, as the torque depends upon the mutual inductance of the moving coil and the fixed coil. The self inductance of the moving coil affects the accuracy of a measurement of a-c. power by reducing the value of the current in the voltage circuit and by making it lag behind the applied voltage by a small angle. Since the magnitude of these effects does not depend simply on the inductance but on the ratio of inductance to resistance, it is evident that the resistance which is placed in series with the moving coil should be non-inductive, and as large as possible consistent with other requirements. By winding resistance wire in a single layer on mica or other insulating cards, resistors are made which have extremely small inductance and capacity, and this construction is much used.

(d) *Correction Factor for Self Inductance of the Voltage Circuit.* If a wattmeter is used to measure the power of a sinusoidal alternating current I which lags by an angle θ behind the e.m.f. E, and if the inductance and resistance of the voltage circuit of the wattmeter are L and R, so that $\tan \alpha = \omega L/R$ is the tangent of the angle of lag of the current in the voltage circuit, then the correction factor C by which the reading of the wattmeter must be multiplied to correct it for the effect of inductance in the voltage circuit is

$$C = \frac{1 + \tan^2 \alpha}{1 + \tan \alpha \tan \theta} \tag{6}$$

In a wattmeter which is fit to use, $\tan \alpha$ will be quite small at ordinary frequencies, so that its square can be neglected, and we have the simpler form,

$$C = \frac{1}{1 + \tan \alpha \tan \theta} \tag{7}$$

(*e*) *Additive Correction for Self Inductance of Voltage Circuit.* While this expression is usable for moderate values of $\tan \theta$, it becomes less applicable as θ approaches 90 deg., near which value the correction factor becomes infinite. It is therefore preferable to make use of an additive correction, as first proposed by Drysdale.[69] To make use of this correction it is necessary to know the volt-amperes EI and the sine of the load phase angle, with L and R as before. Then the correction C', which must be added to the wattmeter reading, is

$$C' = -\frac{\omega L}{R} . EI \sin \theta = -EI \sin \theta \tan \alpha \tag{8}$$

In both of these correction formulas, $\tan \alpha$ is positive for inductance in the voltage circuit, and θ is counted positive when I lags behind E. Cases have arisen in which the series resistor in the voltage circuit, being wound as a bifilar coil, had enough electrostatic capacity to more than neutralize the effect of inductance, so that the current in the voltage circuit led the impressed voltage. In such cases $\tan \alpha$ is taken as negative.

The additive correction may be written

$$C' = -(EI \tan \alpha) \sin \theta$$

which shows that for a given wattmeter and a given value of volt-amperes the correction *in watts* is zero at unity power factor ($\theta = 0$) and increases as a sine function up to a maximum of $-EI \tan \alpha$ at $\theta = 90$ deg. At 50 per cent power factor, with a given value of volt-amperes, the error due to inductance in the potential circuit, expressed in watts, has thus already reached 87 per cent of its maximum value. Since the error in a good wattmeter, as expressed in displacement of the needle from its true position at a given power factor for the value of volt-amperes giving full scale deflection at unity power factor, is at most only a few tenths of a division, it may be seen that this error is not serious in most cases, and that the error in divisions for all points below 50 per cent power factor may be considered approximately equal to the error at 50 per cent power factor.

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[69] C. V. Drysdale, *Electrician* (London) vol. 46, pp. 774-8, 1901; vol. 76, pp. 523-5, 1916.

In the formulas for the additive correction and the correction factor the angle θ appears, and while it might be measured by a power factor meter, it is usually not known except from the values of volts and amperes, and the watts as read from the wattmeter. The latter being subject to error due to the various phase angles, the value of θ so found is only an approximate one. It is possible to use this value to get a first approximation to the true power, from which a more accurate value of θ can be found and used to get a second approximation. At this point we may note a further advantage of the additive correction as compared with the correction factor. The latter contains the term tan θ, which increases very rapidly as θ approaches 90 deg. Hence any error in determining θ will affect the accuracy of the correction factor very much. In contrast to this, the additive correction contains the term sin θ, which has reached 87 per cent of its maximum value when θ is 60 deg., and which increases to its full value more and more slowly as θ approaches 90 deg. It may be said that for a wattmeter which is fit to use, the apparent power may be used to determine θ for substitution in the additive correction formula, without introducing appreciable error.

As an illustration, assume $L = 4$ millihenrys $= 0.004$ henry, $R = 4000$ ohms, frequency 60 cycles ($\omega \doteq 377$), and E volts and I amperes at unity power factor give full scale deflection of 150 divisions. Then the maximum error, which occurs at zero power factor, will be

$$\begin{aligned} E\,I \tan\alpha &= E\,I\,\omega\,L/R \\ &= E\,I\,\frac{377 \times 0.004}{4000} \\ &= 0.000377\,E\,I \\ &= 0.06 \text{ division} \end{aligned}$$

For power factors of 50 per cent and 86.7 per cent the error will be 0.05 and 0.03 division respectively.

The preceding example refers to a modern high-grade portable wattmeter with a maximum voltage rating of 125 volts. At low power factors, use may be made of the overload capacity of the windings. When these are pushed to their limits the maximum self-inductance error will be 0.2 division. Wattmeters of similar type made specially for use on very low power factor will have a maximum self-inductance error of about 0.4 division. The wattmeters above considered all have 150-division scales.

As another example, consider a wattmeter having an inductance of 9 millihenrys and $R = 1000$ ohms, full scale reading of 250 divisions for $E\,I$ volt-amperes at unity power factor. The maximum error will be

$$\begin{aligned} E\,I \tan \alpha &= E\,I\,\omega L/R \\ &= E\,I\,\frac{377 \times 0.009}{1000} \\ &= 0.00339\,E\,I \\ &= 0.82 \text{ division} \end{aligned}$$

In all of these statements on the error due to inductance in the potential circuits, it has been assumed that this cause only is acting. Other causes will be separately considered, and finally the manner in which these errors combine to affect the result.

(*f*) *Compensation of Self Inductance of Voltage Circuit.* The effect of inductance in the voltage circuit may be compensated over quite a range of frequency by shunting a portion R/n of the series resistor by a condenser of capacity C. The inductance of the voltage circuit being L and its resistance R, the following relation[70] should be satisfied

$$C = \frac{L}{R^2/n^2} \tag{9}$$

While this expression is not rigorously exact, it is sufficiently exact for all practical purposes in wattmeter compensation.

The formula shows that an infinite number of combinations of C and R/n may be used. It is therefore important to see whether there is any advantage in choosing some particular value. Since for a given wattmeter and given voltage range L

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[70] Sumpner, *Jour. Soc. Teleg. Eng.*, vol. 16, p. 334, 1887; Drysdale, (London) *Elec.* vol. 46, p. 774, 1901; Rosa, *Bull. of Bur. of Standards*, vol. 3, pp. 48-50, 1906.

and R are determined, we have

$$C/n^2 = L/R^2 = \text{constant.} \tag{10}$$

Thus if we make n large, that is, place the condenser around only a small fraction $1/n$ of R, the capacity of the condenser must be large, and conversely, if the condenser is shunted around a large part of R (n small) the capacity required decreases as n^2. In addition to bringing the current in the voltage circuit into phase with the applied e.m.f., as a secondary effect the resistance R is decreased by an amount ΔR and the current I for a given voltage increased by an amount ΔI, thus requiring a correction to the wattmeter reading[71]. It may be shown that for any value of $\omega^2 C^2 R^2/n^2$ which is small compared with unity we may write

$$\frac{\Delta R}{R} = -\ \omega^2 C^2 R^2/n^3 \tag{11}$$

$$\frac{\Delta I}{I} = -\ \frac{\Delta R}{R} = \omega^2 C^2 R^2/n^3 \tag{12}$$

The change in current being inversely proportional to n^3, it is advisable, from this viewpoint, to shunt a large portion of the resistance with a relatively small value of capacity. A practical disadvantage is that if the insulation of the condenser should puncture in service, the unshunted part of the voltage circuit, including the moving coil, would be liable to burn out. It is therefore preferable to compromise by shunting say a third of R with capacity. In any given case, the limitations should be examined.

(*g*) *Correction Factor for Self and Mutual Inductance of Wattmeter.* The effect of the mutual inductance between the fixed coil and the moving coil of a wattmeter is usually stated to be negligible. On the other hand, makers of wattmeters in which the moving coil is always in the position of zero mutual inductance when readings are taken have sometimes put this feature forward as an advantage because it eliminates errors due to mutual inductance. The writer does not know of any definite published data as to the magnitude of this effect. It seemed desirable to have a formula for determining the error, since in special cases of high frequency or low voltage ranges it might be appreciable. The following discussion is based on an equation derived by Laws,[72] which, slightly rearranged, is as follows:

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[71] This matter was brought to the writer's attention some years ago by Dr. P. G. Agnew.

$$P = P_w \left(1 + \frac{\omega^2 (L_p \pm M)^2}{R_p^2} \right) - \frac{I_l^2 \, \omega^2 (L_l \pm M)(L_p \pm M)}{R_p} \tag{13}$$

in which P = actual power taken by the load, the loss in current coil of wattmeter being included.
P_w = reading of wattmeter (corrected for scale error).
ω = 2π times the frequency.
L_p = inductance of voltage circuit of wattmeter.
R_p = resistance of voltage circuit of wattmeter.
M = mutual inductance between current circuit and voltage circuit of wattmeter.
I_l = current taken by load.
L_l = inductance of load, including current coil of wattmeter.

Also, let
R_l = resistance of load, including current coil.
θ = lag angle of load.

The connections are understood to be made so that the voltage applied to the voltage coil includes the drop in the series coil, which thus virtually becomes part of the load.

The above equation may be put into the form

$$P = P_w \left(1 + \frac{\omega^2 L_p^2}{R_p^2} + \frac{\omega^2 M^2}{R_p^2} - 2 \frac{\omega L_p}{R_p} \cdot \frac{\omega M}{R_p} \right)$$

$$- I_l^2 \, \omega \, \frac{L_l \, \omega L_p - \omega L_p M - L_l \, \omega M + \omega M^2}{R_p} \tag{14}$$

It will be noted that in the equation (**14**) we have put the − sign before quantities containing M, instead of the ± sign given by Laws. The reason for this will now be given.

In most wattmeters of the deflection type the mutual inductance changes sign at approximately half-scale deflection. In the full-scale position the fluxes of the two coils are in the same general direction. If we consider the analogous case of an electrodynamic ammeter having the two coils in series, we have the known condition that the torque is in such a direction as to increase the total self inductance L, which is

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[72] F. A. Laws, *Electrical Measurements*, pp. 315-16. A similar equation can be derived for the case where the voltage circuit of the wattmeter is connected in parallel with the load only.

$$L = L_1 + L_2 + 2\,M_{12} \tag{15}$$

where L_1 and L_2 are the self inductances of the fixed coil and the moving coil, and M_{12} is the mutual inductance between them. On this basis, we ascribe the + sign to M when the coil is above the position of zero mutual inductance, in which position the reactance voltage in each circuit due to its self inductance is in the same direction as the voltage induced by the current in the other circuit. If this convention as to signs be adopted, equation (**14**) will have a + sign before the terms containing M when the moving coil is below the point of zero mutual inductance, and a − sign when the coil is above this position. We may therefore write equation (**14**) in general with a − sign as an operator before the terms containing M, with the understanding that M is a quantity which is inherently + or − according as the moving coil is above or below the point of zero mutual inductance.

For $\dfrac{\omega\,L_p}{R_p}$ in equation (**14**) we may write $\tan\alpha_L$, where α_L is the angle of lag in the voltage circuit, the mutual inductance being assumed zero. Also, for $\dfrac{\omega\,M}{R_p}$ we may write $\tan\alpha_M$, since $\omega\,M$, when mutiplied by the load current I_l gives a quantity $\omega\,M\,I_l$ which may be called the mutual reactance voltage in the voltage circuit, and which is analogous to $\omega\,L\,I_p$, the reactance voltage due to the self inductance of the voltage circuit. Also, for $\omega\,L_l$ we may substitute $R_l\tan\theta$. Making these changes

$$\begin{aligned} P = {} & P_w\,(1 + \tan^2\alpha_L + \tan^2\alpha_M - 2\tan\alpha_L\tan\alpha_M) \\ & - I_l^2\,R_l\tan\theta\,.\,(\tan\alpha_L - \tan\alpha_M)\,(1 - M/L_l) \end{aligned} \tag{16}$$

Since $I_l^2\,R_l = P$,

$$\begin{aligned} & P\,[1 + \tan\theta\,(\tan\alpha_L - \tan\alpha_M)\,(1 - M/L_l)] \\ & \qquad = P_w\,(1 + \tan^2\alpha_L + \tan^2\alpha_M - 2\tan\alpha_L\tan\alpha_M) \end{aligned}$$

from which

$$P = P_w\,\frac{1 + \tan^2\alpha_L + \tan^2\alpha_M - 2\tan\alpha_L\tan\alpha_M}{1 + \tan\theta\,(\tan\alpha_L - \tan\alpha_M)\,(1 - M/L_l)} \tag{17}$$

If M be assumed = 0, $\tan\alpha_M$ = 0, and this equation becomes

$$P = P_w \frac{1 + \tan^2 \alpha_L}{1 + \tan\theta \tan\alpha_L} \tag{18}$$

which is the common form of the expression for correcting for the error due to self inductance. If self inductance be assumed = 0, or compensated by a condenser shunting a resistance, equation (**17**) becomes

$$P = P_w \frac{1 + \tan^2 \alpha_M}{1 - (1 - M/L_l) \tan\theta \tan\alpha_M} \tag{19}$$

This equation is seen to be similar in form to equation (**18**), but contains the term $1 - M/L_l$ in the denominator.

In all wattmeters fit to be used, $\tan^2 \alpha_L$, $2 \tan \alpha_L \tan \alpha_M$, and $\tan^2 \alpha_M$ should be absolutely negligible, so that the equations may be used in the following simplified forms:

Self and mutual inductance present:

$$P = P_w \frac{1}{1 + (1 - M/L_l) \tan\theta\, (\tan\alpha_L - \tan\alpha_M)} \tag{20}$$

Self inductance present alone:

$$P = P_w \frac{1}{1 + \tan\theta \tan\alpha_L} \tag{21}$$

Mutual inductance present alone:

$$P = P_w \frac{1}{1 - (1 - M/L_l) \tan\theta \tan\alpha_M} \tag{22}$$

As L_l approaches zero, the term $(1 - M/L_l) \tan\theta$ in the preceding equations approaches the indeterminate form $\infty \cdot 0$. This may be avoided by putting equation (**17**) into either of the following forms:

$$P = P_w \frac{1 + \tan^2\alpha_L + \tan^2\alpha_M - 2\tan\alpha_L \tan\alpha_M}{1 + (\tan\theta - \frac{R_p}{R_l} \tan\alpha_M)(\tan\alpha_L - \tan\alpha_M)} \tag{23}$$

$$P = P_w \frac{1 + \tan^2\alpha_L + \tan^2\alpha_M - 2\tan\alpha_L \tan\alpha_M}{1 + \left(\tan\theta - \frac{M\omega}{R_l}\right)(\tan\alpha_L - \tan\alpha_M)} \tag{24}$$

These equations do not require the value of L_l. The terms in the numerator after unity may be dropped in practical use of the equations.

In order to show the order of magnitude of the mutual-inductance and self-inductance effects, we may take a modern high grade switchboard wattmeter of 5 amperes and 110 volts rating, capable of 100 per cent overload in current, used on a 60-cycle load of 0.50 power factor, 550 watts, 1100 volt-amperes, current lagging. For this wattmeter, $R_p = 3400$ ohms, $L_p = 3.4$ millihenrys, $M = +94$ microhenrys at full scale, and $R_l = 5.5$ ohms. We then have

$$\tan \alpha_L = \frac{2\pi \times 60 \times 0.0034}{3400} = 0.000377$$

$$\tan \alpha_M = \frac{2\pi \times 60 \times 0.000094}{3400} = 0.000010$$

$$\tan \theta = \tan 60 \text{ deg.} = 1.732$$

Using equation (**21**),

$$P = P_w \frac{1}{1 + (1.732 \times 0.000377)}$$

$$= P_w \frac{1}{1.00065} = P_w (1 - 0.065 \text{ per cent.})$$

The error due to self inductance alone is thus 0.065 per cent, which is below the limit of reading for a switchboard instrument, but is just barely appreciable on a portable instrument.

Using equation (**24**), neglecting the second order terms in the numerator, we have for mutual inductance alone

$$P = P_w \frac{1}{1 + \left(1.732 - \dfrac{0.000094 \times 377}{5.5}\right)(-0.00001)}$$

$$= P_w \frac{1}{1 - 0.000017} = P_w (1 - 0.002 \text{ per cent}).$$

This is far below the limit of reading, even for the best portable instruments. With high frequencies, low current ranges, or low volt ranges it would be advisable to check the effect of both self and mutual inductance.

(*h*) *Effect of Eddy Currents in Fixed Parts of Wattmeter.* If conducting masses are near the current coil of a wattmeter, eddy currents will be induced in them when current flows through the current coil. For sinusoidal currents, the e.m.f. induced will be 90 deg. behind the flux due to the fixed coil.

The eddy current will lag behind the induced e.m.f. by an angle which at ordinary frequencies is probably small for high resistance alloys, but which may be relatively large for copper. The moving coil is deflected by the resultant flux, namely, the vector sum of the main flux and the eddy current flux. Since the latter lags behind the main flux by more than 90 deg., the resultant flux is of somewhat smaller magnitude than the main flux and lags behind the latter. The effect upon the reading of the wattmeter is thus two-fold; (a) the decrease in magnitude tends to reduce the wattmeter reading; (b) the lagging of the resultant flux may tend to increase or decrease the reading, depending upon the vector relations of the voltage and current in the load. The effect of lagging the resultant flux by an angle α_E is evidently the same as would be produced by a cause (such as excess capacity effect in the voltage circuit) which advances the current in the voltage circuit by the angle α_E with respect to the applied voltage. Thus one part of the effect of eddy currents is of opposite direction to the effect of inductance in the voltage circuit. If the latter effect is to cause the current in the voltage circuit to lag by an angle α_L, then both effects may be taken care of by replacing α and α_L in the preceding formulas by $\alpha_L - \alpha_E$. This "equivalent phase angle"[73] of the wattmeter may also be extended to include the phase angles of current and voltage transformers used with the wattmeter.

The above discussion does not take into account the eddy currents set up in fixed conducting masses by the current in the moving coil. Aside from the fact that the motion of the latter would complicate the matter, the ampere-turns of the moving coil at rated voltage are only a few per cent of those of the fixed coil at rated current, and hence the eddy currents set up by the former may be neglected.

A few instances of appreciable error at ordinary frequencies resulting from eddy currents may be instructive. A new wattmeter in which the two current coils were supported by brass castings was found to have errors indicating eddy currents. On investigation it was found that in assembling the instrument certain insulating washers had been omitted, thus forming a closed conducting path around these castings. The error was 1 per cent at power factor 0.75, 60 cycles. Another instance, in which the eddy current error was 1.5 per cent at 87 per cent power factor, 60 cycles, was traced to short-circuited turns in the compensating winding. A similar result would be given by a short circuit in the current coil which would cut out some of the turns from the path of the load current and give a path for eddy currents. Wattmeters as received from the maker, or which are about to be used for important tests, should be checked on low power factor, in addition to the test with reversed direct current.

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[73] L. T. Robinson, in Pender's Handbook for Elec. Eng., 1st edition, p. 1819.

(i) *Effect of Eddy Currents in Moving Parts of Wattmeter.* Eddy currents in the moving coil or its metal fittings will introduce errors of the same sort as those due to mutual inductance, but they may be of appreciable magnitude. Eddy currents in the moving coil can occur only when one or more turns have been accidentally short-circuited. An idea of the magnitude of the effect may be gained by connecting the terminals of the moving coil together, excluding the series resistor, and passing rated current at a commercial frequency through the current coil. The interaction between this current and the current induced in the moving coil will deflect the coil, sometimes by several scale divisions, toward the point of zero mutual inductance. This effect decreases as the point of zero mutual inductance is approached, and for a given coil position it varies as the square of the current in the current coil. The observed effect is caused by eddy currents in *all* the turns of the moving coil, usually several hundred, and the deflection for a part of the coil short-circuited will be in proportion to the number of turns short-circuited. No case of this kind has come under the writer's notice, but as the matter can be readily checked, it should be kept in mind as a possible source of error in power measurements. In the manufacture of wattmeters and a-c. voltmeters all coils should be tested before assembly to detect possible short-circuited turns.

(j) *Methods of Testing Wattmeters at the Bureau of Standards.* Electrodynamic wattmeters are tested by the Bureau of Standards on direct current, using the mean of reversed readings in order to eliminate error due to local magnetic field or residual polarity in iron shields. A further test which is usually made is the determination of the "a-c—d-c. difference," which is made as follows: The current coil of the wattmeter is connected in series with the current coil of a standard reflecting electrodynamometer, the voltage coil of the wattmeter in parallel with that of the electrodynamometer. Alternating current is passed through the current coils and alternating voltage of the same frequency, adjustable in phase, is applied to the voltage coils. The current and voltage are regulated to give a stated deflection of the wattmeter, and the electrodynamometer is read. Using direct current and voltage, the wattmeter is then brought to the same deflection as before, first with current and voltage of a given direction, then with both reversed. As a check, the a-c. reading is then repeated. The mean of the two a-c. readings, minus the mean of the two d-c. readings, is then divided by the mean of the d-c. readings in order to obtain the relative difference (usually expressed as a percentage) in the reading of the wattmeter on d-c. and on a-c. This difference gives an index of the quality of the wattmeter as to freedom from errors due to self inductance and eddy currents. High grade portable wattmeters, tested on commercial frequencies at the volt-amperes giving full scale deflection for unity power factor, give a-c.-d-c. differences of not over a few tenths of a division. Wattmeters which have been carefully compensated for inductance give practically no difference.

(*k*) *Multiple Ranges in Wattmeters.* Multiple ranges are secured in wattmeters by making the current winding in parts (usually two) which can be joined in series or in parallel, and by bringing out taps from the series resistor in the voltage circuit. It is important that the separate parts of the current winding should be alike in resistance and in their magnetic effect upon the moving coil. This can be checked, in instruments having accessible links for changing the range, by putting the two parts of the current winding in series opposing, and exciting the voltage circuit from a source approximately in phase with the current. A deflection indicates a difference in the coils. Strictly, this test should be carried out throughout the scale, by shifting the spring abutments so as to bring the coil into various initial positions over the scale.

(*l*) *Polyphase Wattmeters.* A polyphase wattmeter consists of two single-phase wattmeter systems with their moving elements on a common spindle. Because of the limited separation between the elements, some provision for preventing interaction between the fixed coil of each element and the moving coil of the other element is necessary if the instrument is to give accurate readings. In certain instruments, for example, some graphic wattmeters, no such provision is made, but in high-grade portable polyphase wattmeters a magnetic screen is interposed between the elements, or each element is enclosed in a magnetic shield. The latter arrangement not only prevents interaction but shields each element from outside magnetic disturbing forces.

Polyphase wattmeters can hardly be given as high a degree of accuracy as the single-phase wattmeter of the same type and grade, because it is practically impossible to adjust the two elements to follow the same scale law. By careful workmanship, the two elements may be brought very nearly into agreement as to scale law and sensitivity, as is shown by the fact that one well-known maker guarantees portable single-phase wattmeters to 0.25 per cent and the corresponding polyphase wattmeters to 0.5 per cent. The necessity for having the two elements as nearly identical as possible arises from the fact that in measuring polyphase power the parts of the torque contributed by the two elements become more and more unequal as the power factor of the load is reduced. At power factor 0.50 all the torque is furnished by one element, and in the ordinary use of the wattmeter it might be either element indifferently. It is thus essential that the deflection corresponding to a given power be the same for each element, and that this equality of elements should hold for all parts of the scale. A check of this feature may be made on a single-phase circuit by putting the current coils of the two elements in series so as to oppose their torques and the voltage coils in parallel. A deflection shows an inequality of the torques of the two elements, and such deflection should be only a small fraction of 1 per cent. This test should be repeated for a number of points over the scale, because the equality of elements may be exact at the zero position and not at other points. The spring abutments should be shifted in order to bring the coil successively into the various positions. It is best to precede this test by a test for interaction between elements, which is carried out by exciting the voltage coil of one element from a single-phase supply and passing rated current, approximately in phase with the voltage, through the current coil of the other element. The resulting deflection, if any, ought to be very small, say not over 0.1 division.

(*m*) *Measurement of Polyphase Power*. The measurement of polyphase power may be made in various ways, which are described in text-books [74] and will not be considered in detail here. The two-wattmeter method seems to have the preference. This presents no difficulties on a two-phase circuit, since each element measures the power in one phase. On a three-phase circuit, however, the two elements measure the power of three phases, and care is necessary to have the proper connections and to interpret the readings correctly, because at power factors below 0.50 the reading of one wattmeter must be subtracted from that of the other. The power factor under which each element is working is not the same as that of the load, and hence with unbalanced load corrections for inductance of the voltage circuit of the wattmeter and for the phase angles of instrument transformers are more difficult to apply. The use of a separate wattmeter in each phase avoids these difficulties, but requires one more instrument and access to a neutral point, which may however be an artificial one.

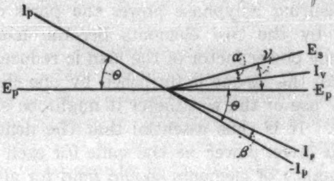

FIG. 7

(*n*) *Use of Instrument Transformers with Wattmeter.* The use of instrument transformers with a wattmeter introduces the necessity for taking account, not only of the departure of the current ratio and voltage ratio from their nominal values, but also of the phase angles. If the latter were zero or negligible, the power under measurement would be the reading of the wattmeter, corrected for scale error, multiplied by the product of the actual values of current ratio and voltage ratio. If the phase angles are not negligible, the manner in which they affect the result may be seen from Fig. 7. In this, I_p and E_p represent the primary current and primary voltage respectively, and the power to be measured is their product multiplied by cos θ. In the actual measurement, however, we must use the secondary voltage and current, E_s and I_s, in place of the primary quantities. The current in the voltage circuit of the wattmeter, I_v, lags behind E_s by the angle α. The reading of the wattmeter is thus proportional to

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[74] Standard Handbook for Electrical Engineers, 4th ed., pp. 156-8; Pender's Handbook for Electrical Engineers, 1st ed., pp. 1823-1826; Laws, Electrical Measurements, pp. 330-340; Farmer, Electrical Measurements in Practise, pp. 160-4; Edgcumbe, Industrial Electrical Measuring Instruments, 2nd ed., pp. 201-8, 222-4.

$$E_s I_s \cos \left[\theta - (\alpha + \beta - \gamma)\right]$$

instead of to $E_s I_s \cos (\theta - \alpha)$ as in the case of a wattmeter used without transformers and having an angle of lag α in the voltage circuit. It may be seen that the phase angles of the voltage circuit and of the two transformers may be lumped into one equivalent phase angle. The angular relations shown in Fig. 7, while exaggerated as to magnitude, are in the directions usually found, as follows: The secondary voltage leads the reversed primary voltage; the current in the voltage circuit of the wattmeter lags behind the secondary voltage at its terminals; the secondary current leads the reversed primary current. It is usual to give the + sign to the angle α when the current in the voltage circuit lags behind the voltage applied to it. There is a lack of uniformity of practise, however, as to the sign which should be prefixed to the transformer phase angles. The convention used by the Bureau of Standards is to measure from the reversed primary quantity to the secondary quantity, and to apply the sign which would be given in trigonometry, namely, that angles described by a vector moving in a counterclockwise direction are +. The usual case, namely, secondary current or voltage leading the reversed primary current or voltage, would be given a + sign. The contrary convention has also been advocated.[75]. Using the convention of the Bureau of Standards, and referring to Fig. 7, it will be seen that the angle β of the current transformer is to be added to the angle of lag α in the wattmeter, and from this sum is to be subtracted the angle γ of the voltage transformer. The remaining angle may be used as the equivalent angle of the whole combination, in applying the correction formulas.

The preceding discussion of power measurement with wattmeter and instrument transformers applies to the simple case of a single-phase circuit, or in the measurement of three-phase power by a wattmeter in each phase. When a three-phase balanced load at unity power factor is measured with two wattmeters, one wattmeter measures half the load under conditions as in a single-phase circuit of 0.866 power factor, current leading, while the other operates under the same power factor, current lagging. If the equivalent angles α are alike for the two wattmeters with their instrument transformers, the errors introduced will be equal and opposite, and (neglecting a term $\cos\alpha$ which differs from unity by only a negligible amount) the sum of the readings of the two wattmeters will give the actual power. In general, for a balanced polyphase load in which the phase angle is θ and the equivalent angle of each wattmeter and its transformers is α, the total correction to be added to the wattmeter reading is

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[75] L. T. Robinson, TRANS. Am. Inst. Elec. Eng., vol. 28, pp. 1005-1052; Pender's Handbook for Electrical Engineers, 1st ed., p. 1640.

$$C_1' + C_2' = \sqrt{3}\,E_s I_s \tan\alpha \sin\theta \tag{25}$$

If three wattmeters were used, each would measure the power $\frac{\sqrt{3}}{3} E_s I_s \cos\theta$, and the correction to each would be $\frac{\sqrt{3}}{3} E_s I_s \tan\alpha \sin\theta$. The sum of the three corrections is thus equal to the correction above given for the measurement of the same total power by the two-wattmeter method. Nothing is gained in the way of increased accuracy by the use of three wattmeters.

For example, if the power factor of the load is reduced to 0.50, the power should all be indicated by one wattmeter, while the other should indicate zero. For this case we must use the wattmeter correction in the additive form, since the correction factor becomes useless at zero power factor. The first wattmeter operates under the condition of 0.866 power factor, current lagging by 30 deg. If the current and voltage in each case are I_s and E_s, the correction to the first wattmeter will be, by equation (8)

$$\begin{aligned} C_1' &= -\,E_s I_s \tan\alpha \sin 30 \text{ deg.} \\ &= -\,1/2\, E_s I_s \tan\alpha. \end{aligned}$$

Similarly for the second wattmeter,

$$\begin{aligned} C_2' &= -\,E_s I_s \tan\alpha \sin 90 \text{ deg.} \\ &= -\,E_s I_s \tan\alpha \end{aligned}$$

The sum of these gives the total correction, namely,

$$C_1' + C_2' = -\frac{3}{2} E_s I_s \tan \alpha.$$

$$= \sqrt{3}\, E_s I_s \left(-\frac{\sqrt{3}}{2} \tan \alpha \right)$$

$$= -\sqrt{3}\, E_s I_s \tan \alpha \sin 60 \text{ deg.}$$

This is the result that would be obtained by applying the additive correction formula to the total volt-amperes $\sqrt{3}\, E_s I_s$, and shows that the correction to the polyphase measurement is the same as would apply to a single-phase measurement of the entire amount of power at 0.50 power factor, using a wattmeter and transformers having the equivalent angle α.

The magnitude of the phase angle in commercial voltage transformers may be estimated from the following data of tests made by the Bureau of Standards on 55 transformers during the period 1908–1918: Phase angle at no load, average + 9 min., maximum + 69 min., minimum + 2 min. The value of 69 min. is exceptional, the maximum, if this were omitted, being 29 min. The phase angles at full non-inductive load were: algebraic average, − 21 min; extreme values, − 38 min. and + 62 min.; omitting the last-named the next largest positive value was + 26 min.

While it is difficult to give any useful general statement regarding the magnitude of the phase angle of current transformers, since it depends on so many factors, it may be said that good current transformers, used within their rated limits of frequency and secondary burden, will have a phase angle of not over 1 deg. to 1 deg. 30 min. at 60 cycles and not over 1 deg. 30 min. to 3 deg. at 25 cycles, both at 0.5 ampere secondary. At 5 amperes secondary, the phase angle will be about one-fourth to one-third of these values. In the preceding statements, the reversed secondary current is understood to lead the primary current. In some cases, especially when the secondary burden is highly inductive, the reversed secondary current may lag behind the primary current for a part of the curve including the five-ampere point. Good transformers with small secondary burden, say a wattmeter or a watthour meter and connecting leads having a resistance of a few tenths of an ohm, will average about one-half of the above values.

(*o*) *Use of Iron in Electrodynamic Wattmeters.* On account of the relatively low torque of the usual electrodynamic wattmeters, the attempt has been made to use iron in the coils. This construction has been investigated by Drysdale, Sumpner and others,[76] but does not appear to have gained any footing in this country. It seems evident that such constructions would be unsuitable for precise testing, especially where more than one frequency is necessary, and that they would be limited to use on switchboards at a given frequency. One advantage of some of these constructions is that they are but little affected by stray magnetic field.

(*p*) *Induction Wattmeters.* These wattmeters are similar in principle to the induction watthour meter. They are inherently less accurate than electrodynamic wattmeters, but for switchboard use they have the advantages of simple construction, ruggedness, high torque, no moving wire, and long scale. They have the disadvantages of being useful on only one frequency, and a much larger temperature coefficient than electrodynamic wattmeters. Their accuracy is also affected by voltage and wave form variations, which do not appreciably affect electrodynamic wattmeters. While they are entirely suitable for the *operation* of a plant, they should not be used in such work as acceptance tests, or the accurate testing of watthour meters.

(*q*) *Electrostatic Wattmeters.* These wattmeters have found very limited application in this country, being confined to such special laboratory work as the determination of small amounts of power at very low power factors, such as the dielectric losses of insulating materials. They are not nearly as convenient to use as electrodynamic wattmeters, and their use would seem to be justified only for very special work for which ordinary methods are inapplicable.

(*r*) *Hot-Wire Wattmeters.* This form of wattmeter contains two hot wires, one carrying a current proportional to the sum of the line voltage and the drop at the terminals of a shunt carrying the load current, and the other a current proportional to the difference of the line voltage and the shunt drop. This instrument seems to have little to recommend it, as it is stated[77] to have rather more pronounced faults than the usual hot-wire ammeters and voltmeters. It seems to be a possibility, however, for the approximate measurement of power at high frequencies.

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[76] Edgcumbe, Industrial Electrical Measuring Instruments, 2nd edition, pp. 219-222.

[77] Edgcumbe, Industrial Electrical Measuring Instruments, 2nd edition, pp. 237-8.

11. THE MEASUREMENT OF ELECTRICAL ENERGY

(*a*) *General Discussion.* The measurement of electrical energy is of great commercial importance. It is not possible in this paper to do more than briefly refer to some of the principal factors which affect the accuracy of energy measurements. The many questions involved in the maintenance of accuracy of electric energy meters are being covered by the large organizations of central station companies, and the results appear in the *Meterman's Handbook* and the reports of the N. E. L. A. Committee on Meters.

In contrast to the large variety of types of indicating instruments, the direct-current energy meters in this country are limited to the commutator motor meter and the mercury motor meter, while the induction meter is the only one in use for a-c. work. In spite of this limited number of types, the problems involved in maintaining meter accuracy are very numerous, and it seems doubtful whether any other measuring device has had so much labor spent upon it in a given time. Too much can not be said in favor of the policy of co-operation which prevails among the large central stations, through their national association, in the way of pooling information and experience with meters for the common good.

What has been said of indicating instruments as to good design, workmanship, proper installation, maintenance, and testing, applies with even greater force to meters. In the following discussion it is assumed that the reader is familiar with the details of meter construction and operation.

(*b*) *Accuracy Curves of Watthour Meters.* It is desirable that the curve showing the "per cent registration"[78] as a function of the load should be a straight line from a very few per cent of full load to very heavy overloads. This can be realized only imperfectly in the meters now available, with the possible exception of the Aron pendulum meter, which has apparently not found any application in this country. The departures from the ideal curve are generally as follows: Zero registration for very small loads, a per cent registration in excess of 100 per cent at from say 25 to 75 per cent load, and a continually decreasing per cent registration as the load increases beyond 100 per cent of rating. The shape of the accuracy curve in the light-load region (say 5 to 10 per cent of rated load) is different in different types and makes of meters. In some cases, as in portable watthour meters, it may be necessary or desirable to adjust the meter so that the curve from 25 per cent to 100 per cent of rated load will be as flat as possible, even if this does cause the meter to be several per cent fast in the region of 5 to 10 per cent load. This is because a portable watthour meter ought not to be used below say 25 per cent of rated load, but should have such ampere ranges that commercial ranges of meters can be tested at light and at full load without using the standard at less than 25 per cent load. This is important in both a-c. and d-c. meters to avoid frictional errors in the standard, especially in the d-c. meter, and for the further reason that the d-c. portable watthour meter is especially subject to errors from local magnetic fields, which errors become relatively greater as the load decreases.

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[78] No single satisfactory word seems to have been proposed for this quantity, which may be defined as the percentage of the energy actually passed through the meter which is registered by it. Considered as an instantaneous quantity, it may be defined as the ratio (expressed as a percentage) of the actual speed of the meter disk at any moment to the speed of a correct meter of the same range and constants, measuring the same load.

From the central station standpoint, the under-registration at light loads and at overloads is important, because it is necessary in most cases to use a meter of much smaller rating than the connected load, in order to get approximate accuracy on very light but long-hour loads. When occasion arises for operating a large part of the connected load, the meter is slow.

(*c*) *Effect of Room Temperature on Meter Accuracy.* In a properly made d-c. commutator meter, and also in mercury meters, a rise of temperature increases the resistance of the voltage circuit at practically the same rate as the resistance of the drag disk, so that the only remaining variable is the strength of the magnetic field cut by the disk. The change in the accuracy of a good meter of this type should not exceed

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79. Values of temperature coefficient for five American makes of d-c. watthour meter are given by Fitch & Huber in *Bulletin of the Bureau of Standards*, vol. 10, pp. 166-8, 1913; Scientific Paper No. 207.

0.1 per cent per degree cent.[79] Cases where a much higher value has been found are usually traceable to the use of resistance alloy instead of a pure metal for the series resistance in the voltage circuit.

In the induction meter the conditions are inherently favorable for low temperature coefficient, because the same disk forms a part of the motor device and the drag device, so that any variations in its temperature have no effect of the first order. The other sources of temperature coefficient are the drag magnets and the electromagnets. Change of temperature of the voltage winding and the lag compensating circuit also have a slight effect. The actual temperature coefficient of induction meters is small. No recent determinations have been made by the Bureau of Standards, but data from tests made a number of years ago shows that the temperature coefficient is under 0.1 per cent per degree cent. for all but very abnormal conditions of power factor.

(*d*) *Effect of Self Heating on Meter Accuracy.* In the d-c. commutator meter the effect of self heating is quite marked. The armature and its series resistor have a high resistance-temperature coefficient, and in consequence will increase in resistance after the line voltage is applied, until a steady state is reached. Because of this fact, it is necessary to have the meter at operating temperature before determining its accuracy, and the same precaution is necessary with the portable watthour meters used to check service meters.[80] A second effect of self heating in the d-c. meter is the increase of resistance of the armature due to the heating of the series coil by the load current. This heating increases as the square of the load current, and its tendency is to reduce the per cent registration of the meter more and more as the load increases.[81] The use of the full-load adjustment to make the meter correct at full load in spite of this effect gives the accuracy curve the same shape as in the induction meter, namely, over-registration from say 25 to 75 per cent of rated load, and an increasing under-registration beyond full load.

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[80] An exception to this statement is the General Electric type CB-5 portable watthour meter, in which the usual pure metal drag disk is replaced by an alloy disk of lower temperature coefficient. This calls for a correspondingly low temperature coefficient in the voltage circuit, with resulting smaller change in accuracy as the meter warms up after being put in circuit.

[81] Fitch & Huber, *Bulletin of the Bureau of Stanards*, vol. 10, pp 162-6, 1910; Scientific Paper No. 207.

In the d-c. mercury motor meter the power taken by the current element is so low (0.2 watt at full load, in the meters tested by Fitch and Huber) that the effect of self heating from this source is small, and of opposite sign from that in the commutator meter.

In the induction meter the losses are relatively much smaller than in the d-c. meter, and the combined motor and generator action of the disk would make the meter practically unaffected by heating of the disk from the windings even if it were much greater. The effect of self heating may be said to be negligible in induction meters.

(*e*) *Effect of Stray Magnetic Fields on Meter Accuracy.* The d-c. commutator meter of the usual form is seriously affected by even moderate values of stray field, because of the relatively low flux density of the working field set up by the series coils (less than 100 gausses at full load). The effect of a given value of stray field upon the accuracy increases as the load on the meter decreases. It is therefore standard practise to adjust such meters in position after installation, and it is necessary to avoid locations where the magnetic field is liable to change. The meter should not be placed near structural iron which may become permanently magnetized by a short circuit through the meter. The mercury motor meter is only slightly affected by stray field. The induction meter is not affected by the earth's or other unidirectional magnetic field, but only by alternating fields of the same frequency as that of the current in the meter. It is relatively much less susceptible to error from stray field than the d-c. commutator meter.

In commutator meters of special construction the effects of stray field are reduced. The following devices are used: four-pole arrangement of the series coils and armature windings; the use of two bipolar armatures of opposite polarity; and the use of two sets of bipolar field coils in connection with a special form of armature having radiating strips of silicon steel to intensify and direct the flux set up by the armature coils. In some large capacity switchboard meters the drag magnets are enclosed in an iron box to shield them from stray field.

(*f*) *Effect of Change of Voltage on Meter Accuracy.* In d-c. meters, an increase of voltage raises the temperature of the voltage circuit without a corresponding change in the temperature of the drag disk. In consequence, such meters run more slowly at higher voltages, and vice versa.[82] The effect is more pronounced in the mercury motor meter than in the commutator meter. No simple method of obviating this effect seems to be available. The use of a voltage circuit and drag disk of lower temperature coefficient would not be feasible for service meters, as the torque developed by such a disk at a given speed in a given magnetic field is lower in proportion to its lower conductivity.

The current taken by the voltage circuit of an induction watthour meter on a given voltage is determined very largely by its reactance. The increase of resistance of the winding has only a small effect. On account of the presence of iron in the magnetic circuit, the reactance will change somewhat as the voltage changes. The data available indicate that induction meters are considerably less affected by change of voltage than direct-current meters, at least for commercial ranges of voltage variation.

(*g*) *Effect of Change of Frequency, Power Factor, and Wave Form on Meter Accuracy.* By proper design, induction watthour meters may be made insensitive to small changes of frequency on either side of the normal, provided the power factor is near unity.[83] Departures from normal frequency at low power factors introduce larger errors for which no simple method of compensation seems to be at hand. This is an argument for the practise of holding the frequency constant, as by the method using a standard clock. With frequency variation eliminated, errors at low power factor will not occur if the meters are properly lagged.

In some tests of American a-c. watthour meters at the

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[82] Fitch & Huber, *Bulletin of the Bureau of Standards*, vol. 10, pp. 166-8, 1910; Scientific Paper No. 207.

[83] In some tests made by the Bureau of Standards on American induction watthour meters a number of years ago, a variation of ± 10 per cent in frequency at unity power factor affected the accuracy by from 0.1 per cent to 0.7 per cent in different makes. The same variation in frequency, at power factors of 0.75, 0.50, and 0.25, changed the accuracy by about 1.5 per cent, 3 per cent, and 5 per cent respectively.

Bureau of Standards a number of years ago, the change from a sine wave form to one containing 30 per cent of third harmonic, at normal voltage and frequency and unity power factor, affected the accuracy by amounts varying from a few tenths of a per cent up to nearly 3 per cent. Durand[84] cites a case coming under his notice in a French installation where the watthour meters were correct at a certain hour of the day, while at other hours the errors attained to 15 per cent. In the latter case the network was supplied from another alternator having considerable difference of wave form from the one operating in the first case. It is a fortunate circumstance, from the standpoint of meter accuracy, that the tendency of recent years toward better control of frequency has been accompanied by the effort to design commercial alternators to give a good approximation to the sine wave form.

Another cause for the variation of wave form is the varying load on a line supplied from a given generator. When the load is heavy the wave form of the line voltage may approximate closely to that of the generator, but when the load is light the charging current of the line may considerably distort the wave form of the voltage along the line. It seems reasonable therefore to conclude that in designing induction meters the factors which tend to accuracy on varying wave form should be given due weight.[85]

(*h*) *Use of Instrument Transformers with Watthour Meters.* The effects of the phase angle and ratio errors of instrument transformers used with watthour meters are identical with the corresponding effects on wattmeters. A few points of difference in details may be noted. The watthour meter, unlike the wattmeter, has adjustments which permit the regulation by the user of the accuracy and of the phase angle. Since the ratio of the transformer, the accuracy of the meter, and the phase angle all vary with change of circuit conditions and of secondary current, it is obvious that the adjustment of the whole can be made only for some average load and conditions. The most convenient method of adjusting a meter with its transformers, when the primary voltage and current are not too high, is obviously by the use of a precision wattmeter connected directly in the primary circuit. This method ceases to be useful, even for primary voltages now considered relatively low, on account of such disturbing elements as leakage across insulating surfaces, electrostatic action on pointers, and electrostatic capacity between sections of the series resistance and their surroundings,[86] all of which are difficult to eliminate or allow for. There is also the added hazard to those making the test. It is therefore necessary in practise to use wattmeters with high grade instrument transformers, having small errors which are determined by laboratory methods.

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[84] A. Durand, Le Compteur Électrique, International Congress of the Applications of Electricity, Turin, Vol. 2, pp. 743-776, 1912.

[85] W. H. Pratt, *General Electric Review*, Vol. 18, pp. 277-281, 1915.

(*i*) *Portable Watthour Meters ("Rotating Standards")*. These meters are substantially equivalent to service meters except as to the register, connections, and case. In using them, it should be kept in mind that they are subject to all the limitations of the service meter, and should be carefully handled and frequently checked. Portable d-c. watthour meters in particular must be used with great care if results of reasonable accuracy are to be expected. If line conditions are fairly steady, more reliable results can probably be obtained with the ammeter, voltmeter, stop watch method. If the current and voltage are very unsteady, the portable watthour meter must be used. Care should be taken to avoid error due to the local magnetic field. A magnetic compass placed at the point where the portable watthour meter is to be used will show the direction of this local field, and the portable watthour meter should be placed with the planes of its series coils parallel to the direction previously indicated by the needle. Care should be taken to have the portable watthour meter at least two feet away from the compass, in order that the stray field from the drag magnets may not produce an error in the determination of the direction of the local field. Direct-current instruments should be at least five feet away from the compass for the same reason.

Portable a-c. watthour meters give much better results than the d-c. meters because of the absence of commutator, lighter weight of moving element, larger ratio of torque to weight, and greater independence of stray fields. They are subject to the same general limitations as the corresponding a-c. service meters. There are two ways of looking at this fact. According to one, it is an advantage to have the portable meter affected by abnormal frequency, low power factor, etc., in the same way as the service meter, because when this is the case the test of service meters under conditions which happen at the time to be abnormal will give the same result as a test with the conditions normal. The weak point in this argument is that if conditions are more often abnormal than normal, the test fails to show the deviation of the service meter from normal accuracy. It also frequently happens that the service meters are of more than one type, and are variously affected by a given departure from normal conditions. On the whole, it seems better to make the accuracy of the portable watthour meter as independent of voltage and other conditions as possible, so that every test of a service meter shall show its accurracy at the time.

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[86] Edgcumbe, Industrial Electrical Measuring Instruments, 2nd edition, pp. 216-7.

The current range of many portable watthour meters is varied by changing the grouping of a sectional current winding. When bad contacts occur at the links, plugs, or contact surfaces of a drum controller, this introduces little error in the results when the meter is used in the ordinary way. However, when the meter is used on the secondary of a current transformer, such bad contacts may introduce serious errors by putting more than the normal resistance in the secondary circuit, with resulting change of ratio and phase angle. Some metals which have been used in drum controllers for this purpose have shown a tendency to oxidize and introduce high resistance into the circuit. Meters having such means for changing the range should be checked by measuring the resistance of the current coils with a bridge, before using them with current transformers.

(*j*) *Permanency of Meter Drag Magnets.* Since a change of 1 per cent in the strength of field of the drag magnets affects the accuracy by 2 per cent, it is evident that permanency of the magnets is a very important matter. These magnets, especially in direct-current meters, are liable to be seriously altered by the magnetic field from the series coils when short circuits occur. This effect may be minimized by arranging the magnets with their long dimension parallel to the plane of the field coils and keeping them as far from the field coils as the design will permit.

The shortage of tungsten during the war compelled makers to use steels which depend upon other materials, such as chromium, for their coercive force. While there is reason to believe that with careful control of all processes of manufacture it is possible to make chrome steel magnets of high quality, it will probably be advisable to keep a closer watch than usual on the accuracy of meters manufactured during the war, since the permanency of magnets is better established by actual use than by laboratory tests.

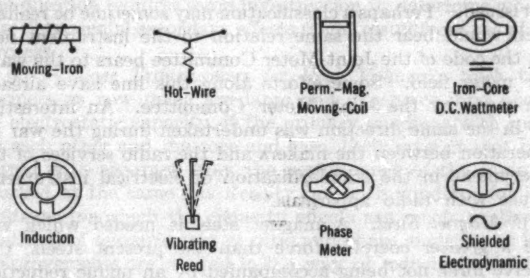

Fig. 8.—Symbols Placed on Instruments by a Foreign Maker to Denote the Operating Principle

12. FURTHER DEVELOPMENTS DESIRED

(a) *Better Understanding of Accuracy Requirements by the User*. This would benefit both user and manufacturer, and tend to simplify the purchase and sale of apparatus. It is suggested that the makers can aid in this matter by devoting more space in their bulletins to the question of the accuracy needed for different classes of work, the importance of accuracy in certain classes of measurement, and the uselessness and expense of needless accuracy. More definite statements of accuracy obtainable with the instruments should be given. Some makers publish definite accuracy guarantees, while others do not. This condition, coupled with competition, is a hardship to the user and to the maker who gives guarantees.

It is suggested also that every instrument should bear an inscription or symbol which will clearly show the operating principle.[87] This has been done for switchboard instruments by a foreign maker, as shown in Fig. 8.

(*b*) *Standardization.* The art of metering energy has been greatly advanced by the preparation and use of the "Code for Electricity Meters" prepared by the Electrical Testing Laboratories under the auspices of the Joint Meter Committee of the National Electric Light Association and the Association of Edison Illuminating Companies. The British Engineering Standards Association has prepared similar specifications for indicating instruments, a revised edition of which has recently been issued. Perhaps a classification may sometime be realized which would bear the same relation to the instrument field that the code of the Joint Meter Committee bears to the watthour meter field. Some efforts along this line have already been made by the Joint Meter Committee. An interesting step in the same direction was undertaken during the war by cooperation between the makers and the radio services of the Government in the standardization of electrical instruments for use with radio apparatus.[88]

(*c*) *Magnet Steel.* A magnet steel is needed which will have a greater coercive force than the present steels, this coercive force not being accompanied by an undue reduction of permeability or residual induction. Such a steel would be valuable for the drag magnets of watthour meters, especially for direct-current meters. It would also be useful for indicating instruments, especially for the small sizes which are coming into extensive use. The improvement in the steel could be used to get increased strength and permanency with the same amount of steel, or the same strength without sacrificing permanency with a smaller amount of steel, as might be desired in the case of instruments and magnetos for airplane use, where weight must be minimized. Other applications would include polarized relays, and similar apparatus employing permanent magnets.

(*d*) *Spring Materials.* It would be desirable to have non-magnetic alloys suitable for springs for electrical instruments, which would show less elastic fatigue than those now available. For some applications, a low specific resistance is also very desirable. On account of the thinness of instrument springs (only a few thousandths of an inch in some cases) and the fact that the strength of the spring varies as the cube of the thickness, it is important that the alloys used should be very resistant to oxidation or attack by sulphur or other impurities in the atmosphere. It would seem that gold plating would protect springs from this deterioration, but the question requires some investigation to determine whether any detrimental secondary effects might result from the plating process.

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[87] As an illustration may be mentioned the case of a high-grade portable shielded moving-iron ammeter which is put out in a style identical with a number of electrodynamic instruments of the same maker, but with nothing to show the operating principle. Such an instrument was used for a series of tests by a trained laboratory man, on the assumption that it was an electrodynamic instrument.

[88] G. Y. Allen, *Elec. Jour.* vol. 16, pp. 494-500, Nov. 1919.

(*e*) *Voltage Transformers for High Voltages.* Such transformers as now made are large and expensive, and because of electrostatic capacity of the primary windings with respect to each other and the core and case, their ratio cannot be inferred with accuracy from the performance of the same core worked at the same flux density but with windings for lower voltages for which the capacity effects are much smaller and methods of measurement of ratio and phase angle are at hand. Improvements are desired in the way of reducing weight and cost, and in correcting for electrostatic effects or determining and allowing for such effects on the ratio and phase angle.

(*f*) *Current Transformers.* The use of current transformers with a watthour meter implies heavy current or high voltage or both, and hence a commercially important amount of power. In all such cases, and especially in the larger installations, we ought to have current transformers whose phase angle is much smaller and more nearly constant over a wide range of operating conditions than at present. The ratio should also be very constant, and within a small fraction of 1 per cent of the marked value. With such transformers, and with good voltage transformers, meters could be lagged and adjusted on the secondary side, thus greatly reducing the labor and skill required.

Improvements are needed in current transformers which of necessity have a low value of primary ampere-turns, such as cable-testing and bushing transformers; also, in those which must be made with an abnormally long magnetic circuit, namely, air-insulated high-voltage current transformers for outdoor use.

(*g*) *Instruments for Plant Acceptance Tests.* For such tests instruments are needed which will give much greater accuracy and reliability under service conditions than can be had with present instruments. Such instruments could be larger and more expensive than the present portable instruments, because on some contracts a fraction of 1 per cent bonus or penalty would pay for them. The deflection potentiometer, with volt box and current shunts, is an example of the kind of instrument required, but is available only for direct current and voltage. Instruments of similar capabilities ought to be available for a-c. power at least.

(*h*) *D-C. Watthour Meters.* Further improvements are needed in the commutator meter which will decrease the attendance required, give reasonable immunity from stray fields, and permit operation from shunts without introducing errors from temperature differences. Some method should apparently be devised to shield the motor element from stray field and at the same time protect the drag magnets from the effect of short circuits through the meter.

(*i*) *A-C. Watthour Meters.* No figures are available on the various operating characteristics of the meters now being manufactured. They seem to have approached pretty close to practical lower limits of size, weight and cost, though from the station standpoint no doubt still further improvements in the latter would be welcomed.

For important installations it is desirable to have induction meters in which first cost[89] has been subordinated to greater accuracy, especially on overloads. Such meters could be larger and heavier, and could require more power in their windings, provided they gave improved results. It does not seem logical to use for such work a type of meter designed to be used by the million on small installations.

Some evident methods of improving the present watthour meter for this purpose may be mentioned. The braking flux due to permanent magnets should be largely increased, thus reducing the relative braking effect of the current-coil flux, which latter causes the present accuracy curve to droop heavily on overloads, and also reducing the relative braking effect of the voltage-coil flux which is one of the causes of change of accuracy with change of voltage. To improve the accuracy on varying frequency at low power factors, the use of iron of much better magnetic quality suggests itself. Possibly the use of silicon steel melted in vacuum and properly heat treated would give the desired result. In any event, enough iron should be used in the voltage magnet to enable it to be worked at the most favorable induction.

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[89] See discussion of paper on "Guarantees of Accuracy of Instruments," by G. Campos, *L'Elettrotechnica*, vol. 4, p. 123, 1917.

The use of a much greater braking flux would have the further advantage of giving a lower full-load speed, with corresponding reduction in wear of pivots and jewels.

DISCUSSION ON "THE ACCURACY OF COMMERCIAL ELECTRICAL MEASUREMENTS" (BROOKS), NEW YORK, N. Y., FEBRUARY 20, 1920.

W. H. Pratt: The subject of the paper is "The Accuracy of Commercial Electrical Measurements". That is a very broad topic, and it suggests that we ought to consider the meaning of "commercial". As I interpret the paper, it is intended to include the measurements which are just below the accuracy of refined laboratory work, but certainly acceptance tests scale very high and other lines of measurement which are evidently included do not rank at a lower stage.

There is one feature of this paper which I think should be particularly noted, and that is, due to its very nature and the repeated warnings that occur from page to page, of how to avoid errors, it is possible that it may give the reader the impression that instruments are not very good things, whereas I know that the author meant no such thing, and I know that his purpose in writing the paper was to show how good the available instruments are, and to point out how by discriminating use of our available devices even more accuracy than is customarily obtained, can be achieved.

On page 501 there is a discussion of scale law. I think that this must be viewed in a very broad way. If an instrument is to be used in a certain manner, and can be given a certain type of supervision, undoubtedly one form of scale distribution might be preferred to another, which could be better suited, were the facilities for following the performance of the instrument through its life different.

On page 529 there is mention made of the static protection in certain a-c. instruments, to prevent the effect of electrical charges disturbing the indications of the needle, and somewhere in the paper, near that point, it is stated that certain d-c. instruments are not so protected. I fail to remember cases where that is not effectively taken care of.

On page 534 the suggestion is made that if zero adjusting devices are arranged so as to give a very limited range of adjustment, this adjustment could be better effected. It seems to me that it would be very difficult, almost impossible, to do what is here suggested. One or two degrees, truly, is a very considerable displacement of zero, and would not occur in any ordinary instrument, except as a result of accident, nevertheless, other considerations, such as general repairs to the instrument, might easily dictate that considerably more than one or two degrees of adjustment should be provided.

On page 571 there is a reference to the use of polyphase wattmeters, and it is pointed out that the general accuracy is not as great as with single-phase instruments. Abstractly that is true, but we must remember that there are a great many types of measurements where the load is somewhat unsteady, where the result will be much more accurate if absolutely simultaneous addition is effected than if addition were effected by observing the reading of two separate instruments. On the whole, though, it is simply a general statement that the measurement of the more complex polyphase circuit is more difficult than that of a simpler single-phase circuit.

At another point in the paper it is stated that the moving iron instruments are not adapted to d-c. measurements where the moving coil instrument is available. There are some special types of measurements where this general statement fails to apply, for instance, in circuits carrying rectified direct currents, on which perhaps arc lamps and incandescent lamps are simultaneously connected. It is very important, if very high efficiency incandescent lamps are used to measure the current on the basis of the r. m. s. effect rather than the average basis. In such cases, even if current would ordinarily be described as direct current, the moving-iron instrument might be the necessary instrument to employ.

Under "Measurement of Energy", I think it is well to point out that there are available in this country a number of watthour meters which are of very high accuracy, considering the extreme range of conditions that must be met. Furthermore, this device was developed, with no limit to the expense on investigation, the results are such that added expense put into the structure in all probability would bring about entirely incommensurate returns in the way of added accuracy.

The accuracy is, of course, not 100 per cent, in fact, one limitation of this whole subject, and one that gives it its particular aspect, is that with measuring devices, we attempt to obtain the absolute truth, which we never can obtain, that is, we are always a little wrong.

On page 584 a criticism is made of the use of chromium steels for damping magnets and magnets in general, and it points out that the devices made during the war should be very carefully watched. It is only fair to state that manufacturers of devices using permanent magnets were fully acquainted with the chromium steels for many years before they made the change, but there were certain manufacturing advantages connected with the use of tungsten steel. During the war it was necessary to go to the chromium steels, and once the change has been made, there is no apparent reason for changing back. Almost the only shortcoming of chromium steel is that you cannot get quite as high a strength with a given amount of material. On the other hand, magnetic problems in general are such that by using a trifle more material you can get back to where you were.

On the last pages of the paper there are a number of suggestions made as to topics which should be taken up for improvement. I think it proper to say that these and many other investigations have been undertaken, and that we will all welcome the finding of new materials which would permit the general improvement of measuring devices. There are few devices that are in use that have been given a more careful scrutiny with regard to the details than measuring devices. It is mentioned on the last page that there should be improvement in d-c. watthour meters. I think that it is again only fair to say that this topic has been most extensively investigated, and if you should refer to the records of the large users of this device, you will find that there is not a great deal to be gained, even with perfect accuracy of the meter.

The suggestion was made that springs might be advantageously gold-plated. We must remember that part of the spring which is most important in the indications of an instrument is its outer skin, and that the suggestion of gold-plating does not seem to suggest an improvement in this respect.

Again the possibility of making use of more expensive meters for the measurement of large blocks of a-c. power is mentioned. Here again it must be pointed out that the devices which are in use are the result of extremely expensive investigation, and there have been very few sacrifices in obtaining accuracy to what might be called commercial considerations.

A fact that electrical measuring devices are small, and that the work which is put into them is a very considerable part of the whole cost, carries with it the meaning that there is very little reason for scrimping materials in order to save costs, and thereby reduce the character of the performance.

J. R. Craighead: In discussing terminology on page 543, Mr. Brooks objects to the use of the term "phase angle" in reference to instrument transformers because this "phase angle" really differs by 180 deg. from the real phase angle between primary voltage or current and secondary voltage or current. This has always seemed to me an unnecessary refinement. The potential or current at the secondary terminals of a transformer has either 180 deg. relation or zero phase relation, approximately, to that at the primary terminals provided you do not pay too much attention to what is inside the box. If you take simply a case of primary terminals, and secondary terminals, the phase relation may be taken as either reversed or not reversed. The method of handling connections in meter installations, pays no attention to reversal in the transformer but goes by the marked terminals. For practical purposes, therefore, the term "phase angle" may be properly used as an abbreviation of "phase angle between primary and reversed secondary" voltage or current.

Mr. Brooks discusses the limitations in the building and also in the testing of potential transformers and compares them to small power transformers. Now in the first place the potential transformer is only superficially like a small power transformer; in practically every design feature it has an individuality of its own; its exciting current is likely to be extremely different from the exciting current of the power transformer of the same size, its rating, as Mr. Brooks states, is very much below that which would be determined by its heating, its regulation is based on different requirements. The extension of the design to the higher voltages, consequently, does not necessarily offer exactly the same difficulties although they may be no less than those that are found in power transformers.

In respect to the testing, the author is right in stating that the higher you go in voltage the more difficult it becomes to test with accuracy for either ratio or phase angle, but it is possible to reach test values, somewhat beyond the 30,000 volts mentioned and still obtain results which are apparently consistent. The results also show that, at least, for a moderate range of voltages, the performance of a potential transformer can be fairly definitely related to its core and winding, without excessive variation due to the capacity effect.

We have conducted tests up to 45,000 volts for about ten years, with seemingly accurate results on both ratio and phase angles. We are at present building an extension of the outfit to enable us to make tests up to 70,000 volts, and while I have no tests as yet in the higher range, I believe it is quite certain we shall get reasonably accurate results up to that point.

With respect to the crest voltmeter, I believe Mr. Brooks does not mention the use of the kenotron with electrostatic voltmeter for the measurement of high potentials. This is a very acceptable and valuable means of measurement, for potentials as high as we can provide electrostatic voltmeters. On account of the leakages and corona effect in most electrostatic voltmeters, that limit is not yet very high. We have used them up to voltages of 25,000 crest value with comparatively small errors, but above this point the errors increase rather rapidly.

The kenotron voltmeter is also usable on the tertiary coil of test transformers, to which reference is made in another part of the paper. Oscillograph tests have shown that with a suitably designed tertiary coil, suitably placed, and with normal conditions of operation of the transformer, the accuracy is entirely satisfactory for the ordinary limits of high potential testing. I do not believe the tertiary coil is, or ever will be, the full equivalent of the potential transformer because the variation of load in the power transformer must of necessity cause a wider variation under testing conditions that would be obtained with the potential transformer itself, but it is quite a practicable instrument for the ordinary grades of accuracy.

On page 553 Mr. Brooks makes the statement that the ruggedness of windings, etc., are not directly related to the inherent accuracy. I am afraid I shall have to disagree with that. One of the most difficult questions that we have to meet is the design of current transformers for carrying very large overload without injury; that is, a transformer using normal full-load current of perhaps 100 amperes may have to be so constructed that it will carry 200 times normal load for two seconds or something of that kind. This has an important and direct effect on the accuracy of the transformer. It is possible to build such a transformer to standard accuracy, only by very greatly exaggerating the size of all the parts. It is possible to build a transformer within standard size only by considerably diminishing the accuracy and ability to carry secondary burdens.

The same is true regarding excessive requirements for insulation. As soon as the necessary dimensions of the opening of the core are increased by the special requirements, a change becomes necessary in the design of the apparatus, if the same phase angle and ratio are to be maintained.

Mr. Brooks states certain limits of design for satisfactory current transformers. While suggestions of this kind are of value to describe apparatus, they should not be used as arbitrary restrictions on the designer. A statement is quoted in the paper which limits current transformers of standard accuracy to a range of ampere turns from 600 to 1200 except for single-turn transformers. It is possible to build a good transformer for certain purposes below the minimum figure. It is possible to build a very excellent transformer for other purposes above the maximum figure, and most current transformers built at the present time to meet standard accuracy and standard requirements are above the maximum figure. This is especially noticeable in the case of the bus bar type of current transformers, where they run to 20,000 amperes, which would mean that a single turn corresponds to that many ampere turns.

In reference to the precautions to be used in operating current transformers, the following statement is made on page 554: "The secondary circuit should not be opened while current is flowing, because the voltage at the secondary terminals may become dangerously high. The high value of flux may cause excessive heating of the core with liability of damage to the insulation." Some little time ago a section was inserted in the Standardization Rules of the A. I. E. E. requiring that all current transformers should be so built that they can operate continuously with the secondaries open, without permanent damage. Practically all American transformers at the present time are now being built that way—the insulation is sufficient so that it is not damaged by the application of the voltage developed at the secondary of the transformer by normal primary current.

Magnetization of the core of a current transformer by the opening of the secondary with current in the primary is still a trouble which must be taken care of. The transformer must be demagnetized after the occurrence. Further, the voltage developed on the secondaries is always high enough to be unpleasant, and in some cases dangerous to the operator, if he happens to get across it.

Mr. Brooks discusses phase-angle errors of wattmeters, as affected by transformers and recommends Formula 6 on page 560, the so-called tangent formula, for correction of phase-angle errors. Another formula based on the cosines is widely used, and has some advantages over the tangent formula.

If $\cos\theta$ is the true power factor.

$\cos\theta_2$ is the apparent power factor shown by voltmeter, ammeter and wattmeter.

α is the equivalent phase angle of the wattmeter

β and γ are respectively the phase angles of the current and potential transformers.

Thus $\theta = \theta_2 + \alpha + \beta + \gamma$

$$\text{True watts} = \text{apparent watts} \times \frac{\cos\theta}{\cos\theta_2}$$

The convention as to positive and negative angles differs from that described by Mr. Brooks. θ_2 is a positive angle if the current is lagging. α, β and γ are positive if under lagging current conditions they increase the reading of the wattmeter, negative if they decrease it. Thus for current transformers a secondary current leading the reversed primary current gives a positive phase angle (β). In potential transformers a secondary voltage leading the reversed primary voltage gives a negative angle (γ). If the current in the wattmeter potential circuit leads the voltage applied, the angle (α) is negative.

The reason for preferring the cosine formula is that it is scientifically correct to start with, and most direct. You simply take a thing that has wrong power factor in it, and you divide it by that power factor, which is in the denominator, and multiply it by the true power factor, which is in the numerator, and that gives the correct value. In practise the correction factors $\frac{\cos\theta}{\cos\theta_2}$ are tabulated for various apparent power factors and values of $\alpha + \beta + \gamma$, rendering their application extremely simple.

There is also the advantage that this formula works with entirely satisfactory accuracy at extremely low power factors, where the tendency of the tangent formula, on account of the exaggeration of the tangent, as it approaches 90 deg., is to become incorrect. The same advantage which Mr. Brooks notes for the additive formula on this page, is obtained directly by the multiplying formula using the cosines.

On page 562 Mr. Brooks speaks of compensating for phase angle in the wattmeter. I note, in passing, that compensation can also be obtained with multiple circuits taken from the secondary of the current transformer or, better by a tertiary winding placed in the current transformer, to be loaded with compensating loads. These means while practicable, are rarely adopted, except in experimental work of a high class, because they are unnecessary.

Mr. Brooks also issues a caution on page 584 regarding the shifting of portable watthour meters from one current range to another, by means of series multiple

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(See paper by L. T. Robinson, TRANS. A. I. E. E., Vol. 28 page 1011-1012.)

connections, and calls attention to the fact that a bad connection affects the secondary burden of the current transformer and is a sufficient cause of error, to be worthy of consideration. There are a number of current transformers, particularly those used for testing, in which there is a series or multiple connection on the primary, allowing the current to have several ratios. If these connections are bad, so that the current flows through completely to one side of the transformer, while another coil receives little or no current, there will be a certain amount of variation of the ratio and phase angle of the transformer. The exact amount depends on the design. This is nearly always under one per cent, but with a very poor design, might be more than that.

This is also true in potential transformers. Where it is necessary to change the double-ratio potential transformer from one ratio to another, by connections on the high-tension side, it is necessary to see that a good connection is made. We have found one or two cases in which an error of as high as one or one and one-half per cent may be caused if failure of the multiple connection makes practically only one half of the transformer operative.

Now, just a word in regard to page 575, where Mr. Brooks mentions the results actually obtained in a large number of tests. He states that the phase angle at no-load of a certain number of commercial voltage transformers ranged from a maximum of 69 minutes to a minimum of two minutes with an average of 9 minutes. Modern design tends towards a small angle under that condition, and it is practicable to obtain transformers for either 25 or 60 cycles, which will not exceed ten minutes of phase angle at that point. Probably the transformer which showed 69 minutes was made some years ago, because up to a few years ago there was a large number of transformers being made using such high flux density that their no-load phase angles were extremely large.

The phase angles mentioned at full non-inductive load represent ordinary practise at the present time; that is, about a degree maximum negative phase angle, and a much smaller phase angle in the opposite direction.

E. P. Peck: Instruments, I have found, are generally considered troublesome things, and they do not receive very much attention, because they are not understood. They are quite often overlooked and neglected, not properly cared for.

As to some of the experiences I have had with testing instruments, a case very much in point was an acceptance test on a plant for five 12,500-kv-a. generators. The plant was designed and built. I got word that an acceptance test was to be made on the plant in two days, and I was to furnish the instruments for the test. I knew it was to be an important test from the size of the plant, and took all the precautions that could be taken, in a commercial laboratory to take care of such a test.

We calibrated two complete sets of instruments, and took them ninety seven miles up in the country for the test. When we got there, we found the contract was worded in such a way that an increase of efficiency in the units of one per cent meant $1000 bonus, and a decrease of efficiency of one per cent meant $1000 deducted from the contract price, so that one-tenth of one per cent error in the test meant $100. and that looked big to me;– particularly big, in those days when it took me three months to get $100 to put in laboratory equipment. There was $1000 one way or the other, on one small division of the indicating wattmeter.

The plant had been built, it cost several million dollars, three specialists had been obtained to measure the water end of the plant, and the electrical end was considered of such little importance, that only two days' notice was given, and no special instruments were purchased or thought of. They used merely the instruments on hand in a central station laboratory.

In this test we used every precaution we could think of, particularly after the contract had been read with the test in mind.

The indicating wattmeters that we used in the test were checked before they were taken up to the point where the test was made, and checked after they were taken back to the laboratory. We found there was a difference of 0.4 of a division in the reading of the meters, on the instrument test made before the acceptance test, and on the instrument test made after the acceptance test. It amounted to $400 one way or the other, depending on whether the meter changed on the way to the plant where the test was made, or changed on the way back from the plant. We averaged the thing, and gave either the other company $200 or took $200 ourselves.

The correction necessary on account of errors due to current and potential transformers, was over 2.5 per cent. It might have been easy to have overlooked current and potential transformer errors on that test, in which case $2500 would have been taken by one party or the other.

F. V. Magalhaes: Mr. Brooks has analyzed very thoroughly the magnitude of the errors that creep in due to bent pointers. Bent pointers can occur easily even with rather careful use of instruments, but I do not believe that Mr. Brooks has pointed out the guard for the error, namely, a periodic checking of instruments for scale check in addition to their electrical accuracy. These scale checks should be made periodically irrespective of the previous known history of the instrument. Conditions that may produce bent pointers are not always reported, and it is safer to keep a continuing record of the scale of the instrument by periodic checks. The point may be obvious, but has not been pointed out in the paper.

On page 555 Mr. Brooks comments on the question of permanency of the constants of instrument transformers. He says the question has been up for a number of years, and quotes from the Electrical Meterman's Handbook. I should like to state that while at the time the Handbook was compiled the recommendation for periodic interval of ratio test was five years, a sub-committee of the Meter Committee of the National Electric Light Association has been working practically continuously since that time, collecting data from the various utilities represented in that association relative to instrument transformers. The last two reports of the Meter Committee of the National Electric Light Association presented some of these results although no final conclusions were drawn. In general, a period of ten years looks to be a fairer period than five years for periodic ratio check. Some State Commissions have already changed their rule from five years to ten years for the periodic tests.

Mr. Brooks on page 588 in enumerating various things that he feels should be brought to the attention of the manufacturers for development, makes a comment as follows: "The use of a much greater braking flux would have the further advantage of giving a lower full-load speed, with corresponding reduction in wear of pivots and jewels."

With meters having heavy moving elements, it is entirely possible to use diamond jewels with a good grade of steel pivot, and obtain satisfactory service. It is a question whether the lowering of the speed would not increase errors at light load.

One other point—I feel it is a matter of regret that the title of the paper "Commercial Electrical Measurements" could not have been stretched to include stop watches. A stop watch is used on a great many of the field tests involving electrical instruments, and personally I would have been very grateful if Mr. Brooks had analyzed the weaknesses and faults of stop watches as well as the errors that might creep into their use. In the field at the present time it is about all that can be used. There are some improved methods of timing used in the laboratory, but personally I would have been glad to have had Mr. Brooks include an analysis of the stop watch and its proper use with the other instruments that are used with it.

F. H. Bowman: Mr. Brooks uses the term "permanent-magnet moving-coil" type of instrument. That is distinctly descriptive. That is nothing more or less than what is known as the d'Arsonval instrument. It strikes me that "d'Arsonval instrument" is entirely descriptive. We have analogy for that in the other terms, such as the watt, the farad, and others, and "d'Arsonval" instrument is a shorter term than the one used.

Mr. Brooks speaks also of an electro-dynamic instrument, or electro-dynamometer instrument. That can quite well be shortened to dynamometer instrument, which is a reminiscence of the old Siemens dynamometer, the origin of the instruments of the type described.

Furthermore, we have the phrase "moving iron instrument", which to me is one which has a core of iron. There might be recognized the repulsion type of iron instrument, where there is a repulsion type of moving iron. We could have a moving type of iron instrument and the repulsion type of iron instrument, which, although they both use iron in their structure are, to some extent, different.

The next point I wish to raise is with reference to the question of the uniform versus the non-uniform type of scale. Mr. Brooks' conclusion is that in future designs engineers should lead toward the one that, with a wattmeter gives a perfectly uniform scale. I think for a voltmeter it will give a scale tending more to the quadratic or square scale. That, with certain types of instruments, lends itself to a uniform scale. There will be no difficulty in making the d'Arsonval galvanometer, or anything else, substantially uniform under those circumstances. Call it either dynamometer wattmeter, or, as Mr. Brooks says, electrodynamic type of instrument. That gives all of our scale, and you would have to go to expedients to get it other than uniform.

On the other hand certain types of instruments will not lend themselves to the quadratic scale, but are a sort of mongrel.

When I am buying an instrument, rather than have the manufacturer attempt to sell me one which he has pulled and hauled and twisted, so that he gets one not actually uniform, but pulled into a semblance of uniformity, at the sacrifice of torque or efficiency, I would prefer the natural scale, and, in fact, I think it is an axiom that when you try to digress from a natural law you are apt to get into difficulties, which leads me to believe if you have a certain type of instrument, it is well to stick to the natural type scale for that instrument, especially for that particular kind of instrument you are working with. The scale which is pointed in just the portion that you wish to use it, for example, in the voltmeter around the 100 volt point. You do not care about the zero part of the scale and care little or nothing about the divisions from 130 degrees up. You are interested in the voltages from 100 to 115 and naturally prefer to have that rectified as far as possible.

As regards shifters, I am not sure whether Mr. Brooks favors them, and would like him to enlighten us. In any event, we can see the trend of instrument design is toward the use of zero shifters, for both a-c. and d-c. instruments. I would prefer an instrument that would give a slightly greater adjustment than the one or two angular degrees specified in the paper. I would prefer if the zero shifter itself was made mechanically strong, particularly that it was constructed so that it would have a uniform rotation, that is, it would not be a zero shifter to go a little one way and go against a stop and a little in the other way and go against a stop. The zero shifter is naturally weak, and when you put it against an obstruction it sometimes fails.

As a conclusion to the paper, there is a paragraph headed "Further developments desired." Now, if these further developments, which Mr. Brooks points out, are the goal to which the instrument designers should strive, then expressing that idea, as Mr. Brooks has, advances the art. If, however, a reading of that gives one the impression that there are radical defects in instruments and meters as now put out, then I believe that harm has been done to the art.

For example, on the subject of magnets. Tungsten has in the past been frequently not obtainable. Chromium is obtainable, and chromium makes a good magnet; there is no doubt that a chromium magnet is as permanent as a tungsten magnet. The question could be brought up—why was not chromium used in the beginning? That could be answered, no doubt, but the fact remains that chromium magnets are as permanent as tungsten magnets, other things being equal.

As regards spring material, I believe there is little trouble experienced with the fatigue of springs. I would consider it poor practise to gold-plate a spring, because the spring is naturally thin, and as Mr. Brooks points out, the torque depends on the cube of the thickness, and if on the outside of the spring we add a little soft electro deposit of gold, we are doing what we should not do, making the outer surface soft where it ought to be very hard.

Alexander Maxwell: In reading Mr. Brooks' paper, and others which have been presented before dealing with commercial electrical measurements, one notices almost invariably that the electrical measurements discussed are the comparatively refined measurements made in commercial laboratories; the accuracy of field measurements is not often discussed. Certainly it is difficult to discuss field measurements, as it becomes mainly a discussion of personal errors, but nevertheless it is important to keep in mind the lower accuracy attainable in the field. In the field you have great extremes and sudden changes of temperature, poor light, observers in uncomfortable postures, and many other factors all tending to reduce the attainable accuracy. I think that it should go into the record in any discussion of commercial measurements that field measurements are always appreciably less accurate than measurement made with the same instruments in the laboratory.

Regarding the use of symbols to indicate the operating principle for commercial instruments, it occurs to me that the direct and simple thing to do would be to mark the type of movement on the scale or on the case in plain English. The idea is good, and perhaps symbols might be used, so long as they were better than the ones shown in the paper.

H. H. Sticht: Referring to Mr. Brooks' classification of instruments, I am sure, speaking as a manufacturers' agent, that most manufacturers would be glad to see such classification made, because in a great many cases instruments are sold under false pretense.

Reference is made to the periodical checking of instruments. The use of electrical measuring instruments in industrial plants is increasing very rapidly, but in this class of users it is not generally recognized that instruments should be tested. Although the Meterman's Handbook mentions this, I think Mr.

Brooks has done well to bring it to the attention of engineers and electricians who are not connected with central stations.

Mr. Brooks' suggestion for the zero adjuster is very good, particularly for the industrial man, as distinguished from the central station man, who as a rule does not have a standard instrument for checking purposes. If such a man keeps on bringing the pointer back to zero, by means of the adjuster, whenever he bends the pointer, he will never find out that his instrument is incorrect and therefore continue to use it as an accurate instrument. On the other hand, in the instrument business as in everything else you get what you want; in many cases the consumer wants as large a zero adjustment as possible, and the manufacturer has to make it. During the war we had a number of instruments rejected, because they had but two per cent zero adjustment. I think that this is the reason why most manufacturers put in more than the amount specified in the paper.

As far as symbols are concerned, I think something should be placed on the scale, as suggested by Mr. Brooks. I do not care whether it is a symbol, or whether the name is there, but to the large number of users in industrial work a voltmeter is a voltmeter. It is for this reason that I believe a symbol or name on the scale would have a good effect, since a man would then know if he were working with a d'Arsonval, soft iron, or dynamometer instrument.

B. W. St. Clair: I want to say a good word for the present product of the American instrument makers. There are plenty of test instruments in the market today that are thoroughly reliable, and accurate. If rightly handled and carefully treated they can be depended upon for very consistent and accurate results. I have watched the performance of some five or six thousand of them in use every day on all kinds of testing, and, except in those cases where they are misused or abused, their performance is perfectly satisfactory. This applies to the better grade of instruments, for it must be acknowledged that there are some types much advertised that are subject to many troubles.

The first point I wish to mention has to do with instrument scales and zero adjustments. Zero adjustments were never put on instruments, and especially a-c. instruments, to correct for bending of the needles. The proper cure, and in fact the only cure for a bent needle is to straighten it. If an attempt is made to bring the needle back to zero with the zero shifter the needle will not line up with the divisions.

Under these conditions it is, of course, possible to make large errors in readings. Zero adjustments are designed to take care of changes in the spring regulators or to correct for internal changes in the spring that affect its zero position.

As regards scales of a-c. instruments it seems to me that there are some advantages to logarithmic scales. It makes possible the taking of readings at most any part of the scale with equal accuracy. Alternating-current voltmeters are used generally for voltages between 100-120, 200-240, etc., and it would seem a desirable practise to make the readability over this range as great as it can be by closing up the scale beyond these points. Voltmeters are hence an exception to this logarithmic scale. In wattmeters where the total scale range will be needed, the logarithmic scale enables a given amount of testing to be gotten out with a minimum number of instruments. In Fig. 2 of the paper are shown two wattmeter scales, one evenly divided and one logarithmic. For ordinary testing it would be possible to use the logarithmic scale from 100 to 500 watts or a range of values of 5 : 1. On the other hand it would only be possible to use the evenly divided scale from 50 to 150 watts or a range of 3 : 1. In the first case the precision of reading in per cent would be approximately constant while in the latter it would increase from a relatively low value to a maximum at full scale.

I cannot agree with the author that it is easier to mark uniform scales than logarithmic. In the manufacturing of high grade instruments, each instrument has its own individually calibrated scale, whether uniform or logarithmic. The actual dividing of the scale into the proper divisions between the cardinal points is done by jigs, and it is just as easy to divide a non-uniform scale as a uniform one.

I also want to say a good word for the compensating wattmeter. I am afraid you may have received a wrong impression as to its ability to hold its calibration. It is true that the compensating feature is one that requires very careful adjustment, but once that adjustment is made it is very permanent. It is something that depends only upon the geometry of the instrument, and the constancy of a resistance. Both these factors are highly permanent ones. This compensating feature is independent of springs, magnets or other things whose permanency might be doubted.

The question of pivots and jewels is one that is discussed whenever meter and instrument men get together. However, it is one of the least sources of trouble in either a meter or an instrument if they are treated with a certain degree of attention and respect. Instrument bearings never wear out. Meter bearings very seldom do. I have seen diamond meter bearings carrying 150 grams load at 180 rev. per min. that have made 600,000,000 revolutions with no sign of wear. While instrument bearings do not wear out they are easily damaged. To reduce the friction torque to a minimum it is, of course, necessary to reduce the radius of the area of contact between the jewel and the pivot as near to zero as possible. In well made bearings this may be so small that even with a moving element weight of from 5 to 10 grams the pressure per square centimeter is of from 100,000 to 250,000 kilograms. Because of the small area of contact and the enormous pressure that the contact is under, rough treatment imposes enormous crushing forces upon the pivot even with very light moving elements. A very little jar is sufficient to upset the end of the pivot, *i. e.*, "mushroom" it. This results in an enormous increase of the area of contact and a big increase in the friction torque. The ability of an instrument to withstand jars, depends upon the quality of engineering and the manufacturing skill that goes into the bearing. For this reason the oft repeated phrase,—"the ratio of torque to weight" is a very fictitious criterion of instrument quality. I am afraid we have been too prone to judge an instrument by this quantity without paying due regard to other factors of more importance. It is possible to secure high torque at the sacrifice of other desirable qualities. Besides torque such things as freedom from temperature errors, frequency errors, low power factor errors, and the like must be considered. In a-c. instruments (and it is only there that there is any excuse for relatively heavy moving elements) the uncertainty of reading on account of the bearing friction is much reduced by the slight vibration given the entire moving system.

Then a word about magnets. There are other things in magnet steel that should be considered in the selection of materials for instrument magnets, besides coercive force and permanence. High permanence is desirable, for high permanence means high torque. Large coercive force is also desirable for that means large permanence under demagnetization. But more important especially than coercive force, is the manner in which the $B-H.$ curve crosses the B axis. Other things being normal, that steel which approaches nearest to a right angle intersection of this axis has the highest permanence when made up into an instrument. The demagnetizing forces that an instrument meets, are in general not 40 or 60 *H*, but are relatively weak forces, and it is therefore, more necessary to have the steel show small changes under these forces than it is to have it show a high absolute coercive force. Another point that needs consideration is the ease of working and the necessity of working to close limits of temperature, and other factors. Steels which show high breakage losses during hardening and grinding are not desirable manufacturing materials.

Our present steels are very permanent. d'Arsonval types of instruments, when well made, and when not abused, show inappreciable yearly changes of calibration. Watthour meter magnets, whose variation in strength affects the accuracy of the meter as the square of flux changes cause no apparent speeding up from year to year. The point is this: Our present steels are very trustworthy when properly made and treated. The measurement art is not suffering because of low grade permanent magnets. Better materials are always acceptable, but there is no cause for a general worry on the part of meter and instrument users that their measuring equipment is in unstable calibration. There has been much agitation over the general substitution of chrome steel for tungsten steel during the war. Chrome steel gives magnets that are just as permanent as tungsten steel. There is little choice between them from a magnetic standpoint. In general, the average permanence of chrome is slightly lower than tungsten steel, and the coercive force slightly higher. It is very doubtful if there will ever be a general return to the use of tungsten steel.

A. L. Ellis: About the matter of bent pointers and the zero adjuster for correcting the trouble, it should be borne in mind that a good instrument does not have a pointer that is easily bent. The thin tip on the pointer is necessary in order to be able to read the scale closely, but the remainder of the pointer should be strong enough, and is strong enough, in practically all instruments, to maintain its form. Applying zero adjusters to instruments is a matter of choice. If the zero adjuster is put on an a-c. instrument it should only be used, as Mr. St.Clair has pointed out, for correcting a shift of the control spring, and not a shift due to a bent pointer. The zero adjuster would be useful, however, in certain types of a-c. instruments, notably switchboard instruments, where it could be used as an adjustment for correcting a scale error at a particular point when the instrument is being checked, but in this case it would probably be useful only for voltmeters.

With reference to the frequency with which instruments should be checked, much depends upon their use. Frequently a high grade instrument will be kept in a laboratory and referred to only for rough measurements, in which case it would not be necessary to check the calibration in many years simply because a high degree of accuracy is not required. In cases where instruments are used to turn out quantity production in a manufacturing plant, frequent checking is required to insure uniformity of product and guard against an error due to accidental damage of the instrument.

With regard to the scale divisions—the character of the scale on an instrument can be varied somewhat at will, depending upon the position of the moving and stationary parts with respect to one another at the starting point. This matter has been fairly well discussed.

With reference to magnet steel—chromium magnet steel, as a matter of fact, gives just what Mr. Brooks asks for as compared with tungsten, that is, greater coercivity. Having that, you naturally expect a more permanent magnet.

As to the electrostatic field from the case, the metallic cases in most instruments are grounded to the moving system so that there should not be any trouble from this source.

With reference to the interference of unshielded d-c. instruments with one another, it should be borne in mind that one can easily get into trouble where only one instrument is used. The unshielded d'Arsonval instrument is in error 0.3 of a division at full scale deflection, (1/5 of 1 per cent) depending upon whether the instrument faces east or west, due simply to the normal earth's field. It may be much greater in New York City.

One word about springs. Phosphor bronze control springs give no trouble whatever as regards oxidation if the instrument designer and instrument user keep sulphur away from them.

Harvey P. Sleeper: I think there are perhaps one or two points to be considered in the use of a polyphase meter to measure the power in a three-phase circuit; for example, if we use the two wattmeter methods, you then have two meters in place of one to read. Of course, that is a difficult operation for one person, particularly when the pointers may land at different points on the scale.

The matter of the sectional shunt is interesting to me. It is stated in the paper, and very correctly, that the trouble is confined in the manufacture of heavy capacity shunts. This is, of course, true, but it occurs to me that the trouble which is encountered in balancing the potential between the sections of the shunt by use of the potential leads could be offset by putting the additional effort into the manufacture of the shunt as a single unit, that is, not in sections.

With regard to the matter of a compensating wattmeter, it might be of interest to the operating man to suggest that it is only on the smaller capacity meters that the compensated winding really comes into any practical effect, and that the power consumption on the heavy capacity windings is really negligible.

In regard to the symbol suggested for the marking of instruments to indicate their type, I think that is an excellent idea. A meter should not be a mystery to a man, and he should be able by inspection to tell what type of movement it is, and I think this system of marking offers a very simple solution of it.

H. B. Brooks: Taking up Mr. Pratt's remarks, I wish to say that I did not intend to give a bad impression by saying so much about the *errors* of instruments, in fact, I strove to be diplomatic, and I said "factors affecting the *accuracy* of the instrument." I do not wish to be an alarmist, but I feel that if we understand our enemy, we have him half conquered, and if the user understands just where he may get into trouble with an instrument, he will know what to do to avoid that trouble.

Mr. Pratt and several others have said that instruments are protected electrostatically if the case is connected to the moving system. Direct-current voltmeters in which the brass case is set on a mahogany base, do not have this connection, at least those I have knowledge of have no connection between the circuits and case. The reason is obvious. The brass case is where it may be touched by the operator and the leads may drop on it. The suggestion in my paper would obviate the difficulty—it is to put an insulating lining in the case and inside of that a metal lining which is connected to the circuit, and in this way you can be free from electrostatic trouble, and free from trouble from accidental contact.

A number of speakers have touched on the matter of zero-adjusting devices, and it has been stated that my suggestion is difficult to carry out. By having a double adjustment, so that within the instrument you could make a relatively large shift—which would be used by the makers,—and then another secondary adjustment of one or two degrees, which the user can effect, the maker can use his discretion in setting his part of the double adjustment, and that will give him all the latitude he needs for manufacturing conditions. Under these circumstances the user would have only a small margin. If the pointer should be bent it should be necessary to open the case and straigthen it out, and it should not be possible to pull a bent pointer back on to zero with the external zero adjuster.

Several speakers, including Mr. Pratt, have mentioned that polyphase wattmeters, in practise, often give more accurate results than two wattmeters. I agree with that. I do not wish to say a word against the polyphase wattmeter, but only to point out that it is not commercially possible for the maker to get as good an accuracy with it as with the single-phase wattmeter of similar type.

Mr. Pratt speaks of the use of a moving-iron instrument for certain circuits, because we are more concerned with the root-mean-square value, and that the iron instrument is better. I agree with him there. Ordinarily, however, when we make measurements of current in a direct-current circuit, we are interested in the arithmetic mean, as in charging a storage battery from a pulsating circuit.

Mr. Pratt states that watthour meters have been developed with no limits to the expense. He referred particularly to direct-current watthour meters, and it was to those that I referred when I suggested that they looked too much like Dr. Thompson's device. My answer would be, evidently we have worked that principle to the limit, and it seems we need some entirely new development in the direct-current metering art. I might take this illustration from Dr. Whitney. Originally we had speaking tubes, and we might have conducted a research into the various factors on speaking tubes, to find out what was the best type of tube, and best material, but someone came along and said—"Let us shrink the tube to a wire," and Graham Bell reduced the tube to a wire. Somebody else got interested in the matter—"Let us do away with the wire entirely and talk through space." Apparently we need to do something of that kind with the direct-current metering art and get some radically new proposition.

Mr. Pratt and others spoke of the chromium steel magnets. I regret that my statements were not sufficiently clear on this point. I did not intend to reflect on chromium steel magnets, because I have been informed that about the only limitations are that the manufacturing operations must be kept within sharper limits. You have not the same wide latitude as to heat treatment and temperatures as with tungsten steel, and the feeling is, apparently, that if you will take pains in the manufacture, you can make good chrome steel magnets. I simply wanted to caution users of meters manufactured in war time to keep the point in mind, and to see what chromium magnets would do in service.

In regard to the goldplating of springs, I would say that the thickness of gold which would be necessary to protect against corrosion is so microscopically small that its effect upon the elastic properties would probably be negligible. The bronze will give the necessary elasticity, and a very thin film of gold will protect it. Mr. Ellis referred to the fact that phosphor bronze springs would not corrode if we would keep sulphur away. Some makers, at least, have put sulphur in instruments by using hard rubber parts, and at one time soft rubber was used, which is also capable of giving out sulphur. I have seen cases in which the sulphur from rubber tubing had blackened the springs. Instruments are frequently used in laboratories where the products of combustion of illuminating gas, which contain sulphur, are allowed to diffuse into the atmosphere. Therefore, I think we should have some protection for the springs. Mr. Pratt mentioned the fact that watthour meters have been developed after much investigation. I do not wish to champion the "super-meter" unduly, but I think this suggestion of the Italian engineer is a novel one, and we should consider the advisability of spending $30 or $40 or $50 for a meter to measure large quantities of energy which involve the accounting for thousands of dollars per month. The matter might well be investigated to see how much better meters could be made if we increased the braking flux and used a superior grade of steel in the electromagnet.

Mr. Craighead spoke of the use of the term "phase-angle" in connection with instrument transformers. I agree that if you get away from the physics of the transformer, and consider it as an iron box, with primary and secondary leads, you can think of your secondary voltage as in phase with the primary voltage. That is easy, but this matter comes up in other ways. For example, we have condensers which, on a sinusoidal voltage, should take a current leading by 90 deg., but actually the current does not lead exactly 90 deg., and the small angle by which it departs from 90 deg., we have been calling the phase-angle. We need an expression which shall signify the little angle by which a quantity departs from some ideal angle. At the Bureau of Standards we have talked that over a number of times, and have suggested such terms as "angle of defect," which sounds bad, and this suggestion of angle of bias" is only thrown out to see what you think of it.

Mr. Craighead spoke of the limitations on building and testing high-voltage potential transformers and I am interested to know that he has gone to 45 kilovolts and expects to go to 70 kilovolts. The figure of 30 kilovolts which I mentioned was what we have been able to do, and where we felt we ought to stop for the present. I do not doubt that we might go further than that, and with transmission voltages going up faster than we can keep up with them, it seems evident that we must consider radical improvements in methods of measuring high voltage.

Mr. Craighead spoke of the use of the kenotron and electrostatic voltmeter to get the crest value of voltage. Limitations on the space available for my paper prevented me from saying much about the crest voltmeter, though it is an important subject in which I am much interested. Dr. Silsbee of the Bureau of Standards is using that method for measuring the peak of the voltage induced in magneto armatures and seems to find it a good method.

Mr. Craighead takes me to task for saying that we should have a small phase angle in current transformers. I agree with him that a zero phase angle is desirable, but I did not want to ask for too much, and asked for a small constant angle. The value zero is certainly small, and I will be glad to accept that in any transformer which Mr. Craighead turns out for me.

Mr. Craighead said that the ruggedness of the windings is not related to the inherent accuracy. Looking at it from the understanding he got from my statement, I agree with him, but I did not mean it in the way in which he took it. Suppose I put it in this way. Let two iron cores identical in form and material be submitted to two persons, one a manufacturer who has had experience and will put in the right kind of insulation in the right way to make it a good rugged transformer, and let the other be wound with the same number of turns of the same size of wire, by an amateur. Then what I have called the inherent accuracy, which depends on iron performance and turns, etc., will be the same for both transformers. The service accuracy after a short circuit hits the transformers will be very different, and doubtless the accuracy of the well-made transformer will be greater, but I am speaking only of the inherent performance which will be shown on a laboratory test.

Mr. Craighead spoke of my quotation from Mr. Edgcumbe in regard to the number of ampere turns necessary for a good current transformer. I am aware that is a sort of empirical way of stating the case. But the user has few criteria for selecting transformers, and if he can put in his specification that the ampere-turns must not be less than 1000 or 1200, he is, at least, protected from a manufacturer who might put out a 500-ampere-turn transformer. That is merely a start, in the way of telling the user what he ought to have, in order to get good results. The value 1200 ampere-turns, which I quoted, is expressly stated not to be an upper limit.

I am glad Mr. Craighead corrected me in regard to the statement with reference to opening the secondary circuit. That caution has been common from the time I can remember working with current transformers, and I am glad the rules now require the transfomer to be so built that it can have the secondary open. Nevertheless, we should strive to educate the user to keep the secondary circuit closed.

In regard to the matter of correction formula for wattmeters, I think Mr. Craighead's formula has perhaps some points in which it does not quite come up to the Drysdale additive correction. The great advantage of the Drysdale formula is that it gives you so much at a glance. If you will put the term cos α equal to unity, the remaining formula tells at a glance the magnitude of the error in watts, and you can readily express it in divisions at full volt-amperes.

The compensation of phase angle in the current transformer, of which Mr. Craighead spoke, using a tertiary winding on the transformer, is a device that is well-known; at least it is on record in the patent literature. It requires to be adjusted by some means which would enable you to know when you had zero phase angle. This limitation would tend to prevent its general application.

In regard to the matter of multiple ranges in current transformers, we have found at the Bureau of Standards that in such transformers from good makers, the relative ratios are very accurately what they should be when the links are changed to give different connections. If the sections of the primary winding are properly distributed over the core, you might as an extreme case have three sections open out of four, and all the current going through the remaining section, with practically no error, and the only disadvantage would be the excessive heating of that one section.

The excessive value of 69 minutes was for a 25-cycle potential transformer and I think Mr. Craighead has suggested the reason, namely, that the transformer was designed with too high a value of the flux density.

In regard to the matter of conventions as to algebraic sign to be used in connection with wattmeter measurements, the important thing is to agree on some standard convention and then to have everybody use it.

Mr. Peck gave an interesting illustration of an acceptance test in which a wattmeter differed by 0.4 per cent, before and after the test. This tends to confirm the statement that we need a radically better type of alternating-current instrument for such work.

Mr. Magalhaes states that bent pointers can occur easily and that scale checks should be frequently made. I agree with that, and would suggest that we use the type which will give the least error if the pointer does get bent, and also that we should in addition make the scale checks.

Mr. Bowman apparently does not like the term "permanent-magnet moving-coil." I do not like the length of it myself, but have not been able to think of any other term that properly describes the type. The term "moving coil" would also apply to electrodynamic wattmeters and voltmeter, and is therefore ambiguous unless further qualified. I do not think the term d'Arsonval should be used for several reasons. In the first place, the d'Arsonval galvanometer was anticipated many years by Lord Kelvin's siphon recorder, which is simply the d'Arsonval galvanometer, plus a little glass tube carrying ink. Personally, if I were to give the name of any man to that instrument, it would be the name of that great pioneer who thirty years ago showed us how to make servicable instruments—Dr. Weston. Instead of applying the names of individuals to types of instruments, however, I prefer descriptive terms, such as hot-wire, moving-iron, and the like.

I think the term "electrodynamic" is preferable to dynamometer because dynamometer is a noun, and when we have a good adjective, we are hardly justified in using a noun as an adjective. Furthermore, the derivation of the word "dynamometer" suggests the measurement of force and does not bring in the idea of electric action. The term "electrodynamic" is not long and includes everything.

Mr. Bowman spoke of the fact that we need another term rather than moving-iron for the repulsion type of instrument. As I see it, both the so-called plunger type, in which a solenoid attracts a core, and the repulsion type in which there is one soft-iron piece within a coil, which is repelled by another, are simply two kinds of moving-iron instruments,—the essential thing is that each has a piece of iron which moves.

I agree with Mr. Bowman that the natural scale law of the instrument should not be unduly modified at the expense of efficiency. This has been shown by the experience of a maker of induction ammeters, who at one time forced these instruments to give an approximately uniform scale, but who has returned to the quadratic scale.

I heartily favor the use of zero adjusters. They are a great convenience. I only want the maker and user to bear in mind the difficulties that may arise, and guard against them.

Mr. Maxwell spoke of the fact that field measurements have not been touched on, and that there is an unknown extra amount of error in the use of an instrument in the field as compared with its use in the testing room. That is true, and I hope that those who have had extensive experience with field tests will collect and publish definite data on this subject in the near future.

I think that symbols to denote the operating principle of instruments are better than words, for this reason—we are going to manufacture things not for America only, but for the world, and our export trade in measuring instruments should greatly increase. I think, therefore, that some symbol that suggests to the user what the instrument is like is much better, and if properly chosen it can be understood by a man who cannot read the language of the instrument maker. I do not advocate the particular symbols shown in my paper, which are given merely to show what has been done.

I am glad to note that Mr. Sticht advocates the idea of a classification of instruments. Ten years ago, a prominent instrument engineer advised us that he wished we would lay out such a classification because, as he put it, it would relieve the manufacturer from the embarrassment of claiming everything for all types of instruments.

I am sorry to know that the government has rejected instruments because they had only two per cent zero adjustment. It is evident that the question needs to be investigated so that general agreement may be had.

Mr. St. Clair stated that the subdivision of the logarithmic scale between cardinal points is as easy as that of the uniform scale. The trouble with this procedure is that if the divisions between two given cardinal points are made of uniform length, some are too long, some too short. This may be discovered by noticing the abrupt change in the length of a division which occurs as each cardinal point is passed. These intermediate divisions are seldon if ever checked, so that if the cardinal points check correctly the instrument passes for an accurate one in spite of the errors which occur in actual use between cardinal points. This difficulty is absent in the uniform-scale instrument.

In regard to the usual idea concerning torque-weight ratio, I am convinced that it is not logical, and some other than a linear relation should exist. Experience seems to show that as we go to lower and lower values of weight, we can go to lower values of torque-weight ratio. I cannot make an exact statement of the matter, but believe that if the matter were properly studied we should find that some other than the simple torque-weight relation exists. It is the opinion of some of us who have considered the matter a little that a better rule than the present one is that for a given degree of excellence of pivots and jewels the ratio, torque divided by weight *squared*, should equal a constant.

Mr. Ellis stated that instrument pointers are strong and not liable to bend. That is true of certain modern types, but on the other hand, there are older forms in use, and many of them have pointers which are easily bent, and until they are eliminated we should keep the matter in mind.

Mr. Sleeper referred to the method of constructing large shunts, which I have given, and states that difficulty would be involved in balancing the parts of the shunt. This method has been used in the construction of a large shunt which had to be very precise, and no difficulty was found in carrying it out, and the mechanical difficulties which this method avoids are very great.

change in the length of a division when [illegible] cardinal point is passed. These [illegible] are scales, however, [illegible] so that if the cardinal points check correctly the instrument [illegible] for an accurate [illegible] of the [illegible] in a vertical [illegible] between cardinal points. This difficulty is inherent in the uniform scale instrument.

In regard to the usual idea concerning torque-weight ratio, I am convinced that it is not [illegible] and some other than a linear relation should exist. Experience seems to show that as we go to lower and lower values of weight, we must have lower values of the torque-weight ratio. I cannot make an exact statement of the matter, but believe that if the matter were properly studied we should find that some other than the simple torque-weight relation exists. It is the opinion of some of us who have considered the matter a little that a better rule than the present one is that, for a given degree of steadiness of pointer and [illegible], the torque divided by weight [illegible] should equal a constant.

Mr. Ellis stated that instrument pointers are at present not liable to break. That is true of certain modern types, but on the other hand, there are older forms in use and many of them have pointers [illegible] easily bent, and until they are eliminated we should keep the matter in mind.

Mr. [illegible] referred to the method of [illegible] which I have given and states that difficulty would be involved in balancing the parts of the [illegible]. This method has been used in the construction of a [illegible] which proved to be very [illegible], and no difficulty was found in carrying it out, and the practical difficulties with this method would not be very great.

Presented at the 358th meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., March 12, 1920.

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SHORT-CIRCUIT PROTECTION FOR DIRECT-CURRENT SUBSTATIONS

BY J. J. LINEBAUGH

Railway and Traction Engineering Dept., General Electric Co., Schenectady, N. Y.

The author includes an outline of the progress made in protection of direct-current machinery from short circuits since the publication of a paper at Atlantic City on this subject.

The improvements mentioned include the refinement and perfection in details of the flash barriers; a new design of high-speed circuit breaker for both direct-current substations and electric locomotives; a new high reluctanc ecommutating pole for 60-cycle synchronous converters and a new design of protected brush holder. An instructive analysis of conditions during direct-current short circuits is shown by several photographs, oscillographs and diagrams. Special reference is made to operating results on the electric zone of the Chicago, Milwaukee and St. Paul Railroad.

THE investigation of means to prevent flashing of direct-current machinery and development of suitable equipment has been continued since the presentation of the paper on "Protection from Flashing for Direct-Current Apparatus," by Mr. J. L. Burnham and the writer at the Atlantic City Convention in 1918.* This paper outlined in a general way the results obtained with quite a number of different methods of reducing or preventing flashing under extreme overload or short-circuit conditions.

This study indicated that a special form of flash barrier with arc coolers and a new form of high-speed breaker with current-limiting resistance had proved the most promising development. Tests showed that the two types of protection provided complete protection from a "dead" short circuit caused by short-circuiting the terminals of a machine without external resistance.

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*Trans. XXXVII Part II, 1919, pages 1341-1365.

These two types of protection have been further perfected and are now in regular commercial use. They are used either separately or together and in many instances considered standard railway practise.

The improved type of high-speed breaker described has been perfected in all details as shown in Figs. 1 and 2.

Thirteen of these breakers were installed by the Chicago, Milwaukee & St. Paul Railroad as part of the electrification of their Coast and Cascade Divisions. Fig. 2 shows the breaker with the arc chute installed but with covers removed. This view shows the calibrating rheostat used to set tripping point of the breaker. Description of this breaker will not be repeated as general theory of operation and design is described in the paper referred to, and more detailed description of the perfected breaker will be described in article by Mr. J. F. Tritle in the April number of the *General Electric Review*.

This breaker was used instead of the first type of circuit breaker which has given successful operation during the past three years in the fourteen substations of the 440-mile original electrification of the Chicago, Milwaukee & St. Paul Railroad. This new breaker has the advantage of lower cost and greater simplicity.

One of these breakers is used with each of the 2000-kw., 3000-volt, synchronous motor-generator sets in the Tacoma, Renton, Cedar Falls, Hyak and Cle Elum substations and the remaining five breakers on each of the new gearless type passenger locomotives. On account of the lower cost of these breakers and advantages of using the "unit" system throughout, each of the sets is protected by its own high-speed breaker instead of one breaker per substation, the arrangement in the original installation.

The general connections, location of circuit breaker, etc., are shown in Fig. 3.

The circuit breakers for the substations and locomotives are exactly alike with exception of interlocking and calibration for tripping points.

The circuit breakers were given a very exhaustive test in connection with one of the 2000-kw. sets in test before shipment, with very successful operation in all details. It was found that the generators could be short-circuited with only sufficient cable in the circuit to connect the different meter shunts, short-circuiting contactor and high-speed breaker without damaging machines in any way and with practically no flashing at the brushes. Figs. 4 and 5 show photographs of two of these short circuits, one of which gave so slight a flash that it can hardly be seen while the other shows very slight flashing in the flash barriers. Fig. 6 shows oscillogram of one of these short circuits giving a very good idea of the high speed of the breaker and the protection afforded. It will be noted that the current was limited to about 7000 amperes and reduced to three times load in 0.016 seconds. The current starts to decrease in 0.0081 of a second and it will be noted that the area repre-

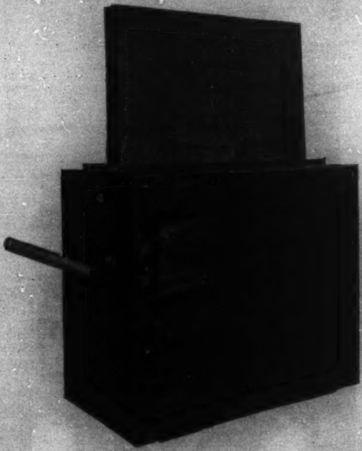

FIG. 1—1500-AMPERE, 3000-VOLT, D-C. HIGH-SPEED CIRCUIT BREAKER

senting the load which would be likely to cause flashover is very small and of such short duration that very little gas or arc could be formed. Fig. 7 has been prepared to show graphically the much greater

Fig. 2—1500-Ampere, 3000-Volt D-C. High-Speed Circuit Breaker With Asbestos Lumber Casing Removed Showing Rheostat Used to Obtain Different Tripping Points

protection afforded by circuit breaker of such high speed over that obtained with the usual type of breaker. This figure shows the current curve of a 2000-kw., 3000-volt machine on short circuit when protected by the high-speed breaker and by a standard 3000-volt breaker designed for higher speed than usually obtained with regular carbon-break 600-volt breakers. The area of each curve above the load which would cause flashing has been cross-hatched to show the ratio between the two areas which gives a very good idea of the value of the great speed and the reason there is so little flashing. During these tests about one photograph out of four was similar to Fig. 4, while the others were of the character shown in Fig. 5.

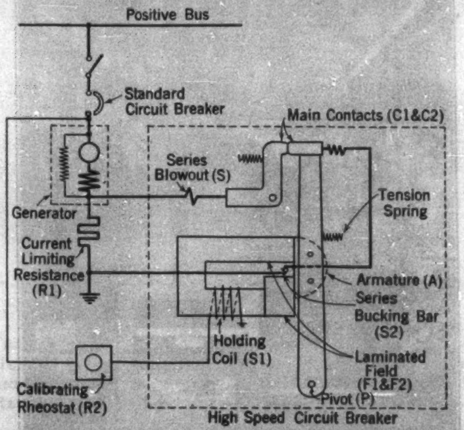
Positive Bus
Standard Circuit Breaker
Main Contacts (C1&C2)
Series Blowout (S)
Tension Spring
Generator
Current Limiting Resistance (R1)
Armature (A)
Series Bucking Bar (S2)
Holding Coil (S1)
Laminated Field (F1&F2)
Pivot (P)
Calibrating Rheostat (R2)
High Speed Circuit Breaker

Fig. 3—General Connections of High-Speed Circuit Breaker

A special reliability or endurance test was made as part of the acceptance tests of the breaker during which about 65 short circuits of different magnitude, fifteen of which were "dead" short circuits, were applied at intervals of about 2½ minutes without cleaning the commutators or giving them any attention whatever. At the conclusion of these tests, five "dead" short circuits were thrown on the set within ten minutes. At the end of these tests, commutators were in excellent condition without any need of cleaning or attention of any kind.

These circuit breakers are now in regular operation and reports received of their operation have been very gratifying.

The application of the high-speed circuit breaker to direct-current electric locomotives is another distinct advance as in addition to protecting the apparatus on the locomotive, it prevents the short circuits from affecting the substations.

FIG. 4—SHORT CIRCUIT ON 2000-KW., 3000-VOLT MOTOR-GENERATOR SET PROTECTED BY HIGH-SPEED CIRCUIT BREAKER AND FLASH BARRIERS. TRIPPING POINT 2550 AMPERES. LINE RESISTANCE 0. CURRENT LIMITING RESISTANCE 1.2 OHMS

If both the substations and locomotives are equipped with this type of high-speed circuit breaker, current under maximum conditions would never reach a value much greater than 7000 amperes. The value of the maximum short-circuit current decreases very rapidly with distance from substation as shown in Fig. 8. These records were taken with a 1500-kw., 3000-volt

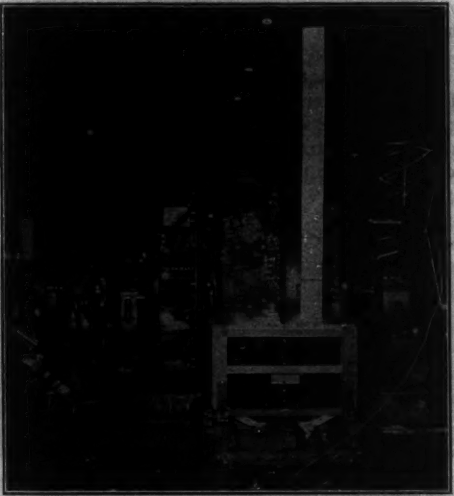

FIG. 5—SHORT CIRCUIT ON 2000-KW., 3000-VOLT MOTOR-GENERATOR SET PROTECTED BY HIGH-SPEED CIRCUIT BREAKER AND FLASH BARRIERS. TRIPPING POINT 2940 AMPERES. LINE RESISTANCE 0. CURRENT LIMITING RESISTANCE 1.2 OHMS

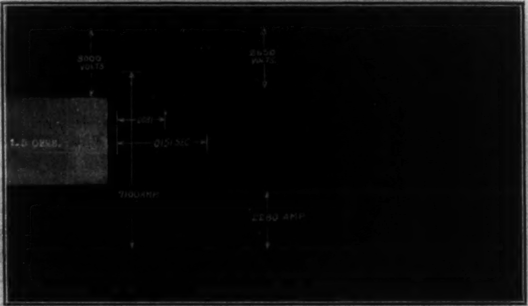

FIG. 6—SHORT CIRCUIT ON 2000-KW., 3000-VOLT MOTOR-GENERATOR SET PROTECTED BY HIGH-SPEED CIRCUIT BREAKER AND FLASH BARRIERS. TRIPPING POINT 2250 AMPERES

motor generator set connecting the overhead trolley consisting of two No. 4/0 wires directly to the 100-lb. track rails and closing the circuit at the substation. It will be noted that the maximum peak current is only 3600 amperes with short circuit 4800 ft. from the substation and 2860 amperes, 9600 ft. from the substation as compared with 7000 amperes "dead" short circuit across the terminals of the machine.

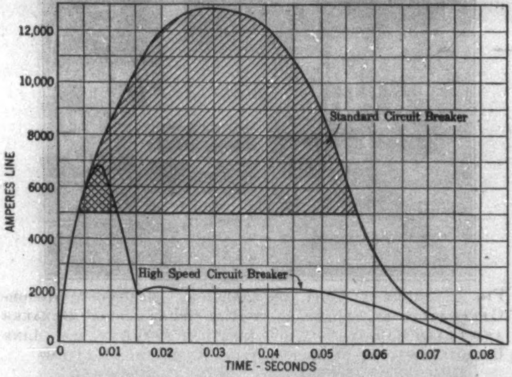

Fig. 7—Short Circuit on 2000-Kw., 3000-Volt Motor Generator Set Protected by High-Speed Circuit Breaker with Curve Showing Current Which Would be Obtained with Standard Circuit Breaker. Ratio of the Area of the Two Curves Above 5000 Amperes Gives Indication of the Protection Afforded

The value of this type of protection was proved very conclusively during these tests due to the accidental grounding of one of the switchboard busbar insulators. The high-speed breaker inserted the current limiting resistance so quickly that some time was required to locate the trouble as burning at the grounded point was so slight as to be hardly noticeable, indicating that even under such extreme conditions current is reduced so quickly that sufficient time is not allowed to cause current to generate sufficient heat to cause destruction of the current-carrying parts.

Another of the incidental advantages of this type of protection is the elimination of disturbances on the a-c. side of synchronous converters or motor-generator sets ordinarily caused by d-c. short circuits due to the fact that the load is decreased so quickly that momentum of the armatures supplies the energy and the load is not increased materially on the a-c. side. The overload relays are therefore not affected, increasing very greatly the general operating efficiency of a substation, eliminating time required to start up set from the a-c. side, etc. After the occurrence of a short circuit it is only necessary for the operator to close the high-speed breaker and then the main switchboard breaker which is interlocked with the high-speed breaker after which the main switch is thrown in following regular switching practise. If the short circuit still persists, the high-speed breaker will again open but with no flashing or damage to brushes or

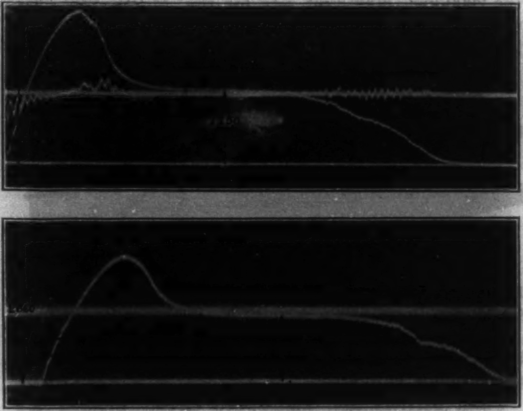

FIG. 8—SHORT CIRCUIT ON 1500-KW. 3000-VOLT MOTOR-GENERATOR SET AT DIFFERENT DISTANCES FROM SUBSTATION. GENERATORS PROTECTED BY HIGH-SPEED CIRCUIT BREAKER AND FLASH BARRIERS

commutator and greatly decreased duty on the regular breaker.

The flash barriers described in the original paper have not been changed in any essential details, improvements being along the line of simpler construction, ease of removal for inspection and improvement in appearance.

These barriers with iron wire arc coolers are standard equipment on all 3000-volt motor-generator sets as well as on all machines used in connection with automatic substation control. Fig. 9 shows one of the 2000-kw., 3000-volt sets for the Milwaukee electrification equipped with these barriers, while Fig. 10 shows barriers for 600-1200-volt, 60-cycle, 500-kw. synchronous converter.

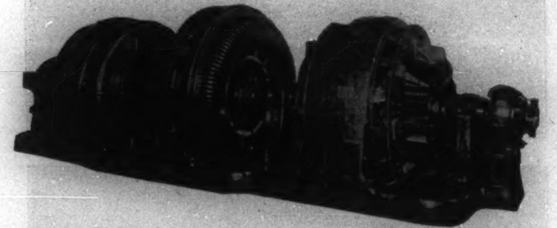

FIG. 9—2000-KW., 3000-VOLT D-C. SYNCHRONOUS MOTOR-GENERATOR SET EQUIPPED WITH FLASH BARRIERS

Flash barriers and high-speed circuit breaker are standard equipment on 1500 to 3000-volt substations and it is believed this practise will be extended to include 600 and 1200-volt apparatus.

Another advance in short-circuit protection is the protected type of brush holder recently perfected as shown in Fig. 11. This brush rigging is protected on all sides where flashing might occur by asbestos lumber so that an arc cannot readily hold between brush holders of opposite polarity and prevents the formation of iron or copper vapor which might cause a flash to the frame and cause damage to the brush rigging or commutator. A removable cover is provided for inspection and removal of brushes. It is made of an iron sheet for convenience as there has been no tendency for the arc to strike this part of the brush rigging during tests or in actual operation. It will be noted that this type of brush rigging lends itself very readily to the addition of flash barriers as shown in Fig. 10.

This type of brush rigging has been standardized for all 600-volt, 60-cycle synchronous converters.

The use of the high reluctance commutating pole is a very promising improvement which has just been made in 60-cycle, 600-volt synchronous converters and has been standardized for all 60-cycle machines. Tests indicate that the use of these poles raises the flashing point at least 50 per cent. In actual commercial use the improvement is greater than indicated by this figure as a very great proportion of short circuits which originally caused flash-over would not cause flashing on machines equipped with this new type of commutating pole winding.

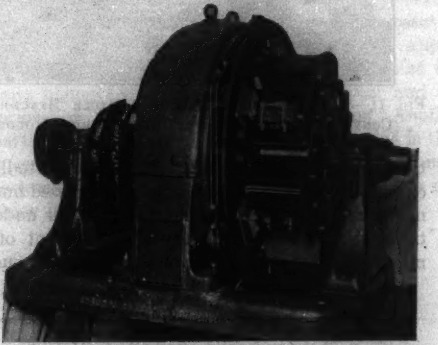

Fig. 10—500-Kw., 600/1200-Volt 60-Cycle Synchronous Converter Equipped with Flash Barriers

These improvements are of particular value for 60-cycle converters which are inherently more sensitive than 25-cycle converters.

Brief attention should be called to the great protection afforded by tapping the feeder at some dis-

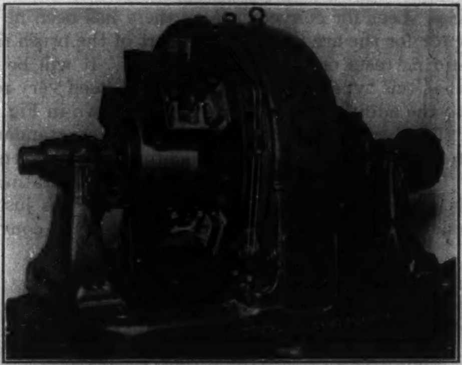

FIG. 11—500-KW., 600/1200-VOLT 60-CYCLE SYNCHRONOUS CONVERTER WITH PROTECTED TYPE OF BRUSH HOLDER

tance from the substation. This is undoubtedly the cheapest type of protection which can be used but cannot be relied upon to prevent flashing over under extreme short circuits. A very slight amount of permanent resistance such as would be given by such an

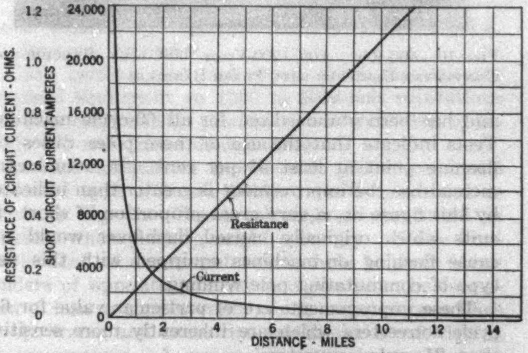

FIG. 12—CURVE SHOWING RATIO OF SHORT-CIRCUIT CURRENT TO DISTANCE FROM SUBSTATION

arrangement greatly reduces the number of flash-overs with very little loss of energy or voltage. Under ordinary conditions it is believed that the distance to the first tap need not be greater than 2000 ft. A greater distance than this causes an appreciable loss of energy and drop in voltage. Fig. 12 shows very clearly the great benefit of a small amount of resistance in reducing the maximum possible current on a short circuit.

If complete immunity is desired from short circuits, the high-speed circuit breaker and barriers undoubtedly offer the best known solution. With this protection, feeder taps can be connected to the overhead trolley directly at the substations, reducing losses to a minimum. Maintenance on the substation apparatus will also be decreased as burning from short circuits undoubtedly causes most of the wear and deterioration on brushes and commutator. Another particular advantage of this type of protection is that it can be applied to old generators or synchronous converters of any voltage without change in the machine itself.

Presented at the 358th meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., March 12, 1920.

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FLASHING OF 60-CYCLE SYNCHRONOUS CONVERTERS AND SOME SUGGESTED REMEDIES

BY MARVIN W. SMITH
Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

WITH the increasing application of 60-cycle synchronous converters to railway service, the question of flashing is receiving considerable attention. A series of tests has recently been made, and others are still in progress, with a view to determining the possibilities of protection from flashing by various methods, including the "quick-acting breaker," "flash suppressor," "flash guards" and various modifications and combinations of these. The converter that has been used for these tests is a standard 500-kw., 600-volt, six-phase, 60-cycle, 1200-rev. per min. machine. It was supplied with power from a 5000-kv-a. generator through three standard high-reactance transformers. These transformers had a reactance of approximately 17 per cent at normal load and 10 per cent on short circuit.

Reasons why 60-cycle converters are susceptible to flashing. The protection of the 60-cycle converter is more difficult than the protection of any other class of commutating machines due to the inherent limitations of distance between neutral points on the commutator. The limiting distance between neutral points is fixed by the frequency and peripheral speed of the commutator. For instance, considering the maximum allowable peripheral speed as 5500 feet per minute, the distance between neutral points in the case of a 60-cycle machine will be approximately 9.2 inches regardless of the speed or number of poles. This may be readily seen from the following simple relation:

$$\text{Dist. between neut. points} = \frac{\text{circum. com.}}{\text{poles}}$$

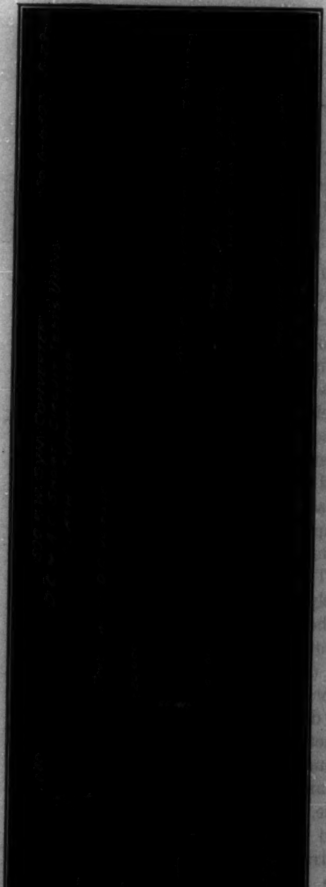

FIG. 1

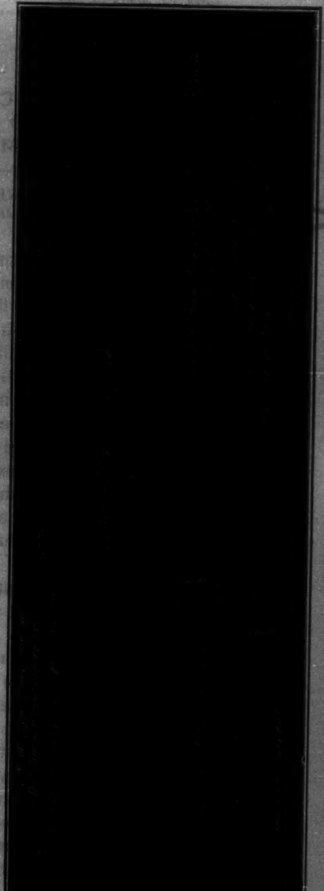

FIG. 2

$$= \frac{\text{periph. speed}}{\text{rev. per min.} \times \text{poles}} = \frac{\text{periph. speed} \times \text{poles}}{\text{poles} \times 120 \times \text{freq.}}$$

$$= \frac{\text{periph. speed}}{120 \times \text{freq.}}$$

An interesting and rather surprising feature shown by the tests is the extremely rapid rate of increase of the direct current on short circuit as compared with direct-current generators. For example, tests made on the generators of the 2000-kw. sets for the Chicago, Milwaukee and St. Paul Railroad, show the initial rate of increase in current on short circuit to be approximately 1,100,000 amperes, or 1650 times full load, per second. Also short-circuit tests on a 1000-kw. 600-volt direct-current generator show an initial rate of increase in current of approximately 2,700,000 amperes, or 1620 times full load, per second. On the 500-kw. converter, however, the average initial rate of increase is approximately 3,300,000 amperes, or 4000 times full-load current per second. This large difference between the rates of increase of short-circuit current for the d-c. generators and the converter cannot be accounted for by the difference in the inductances of the machines. In fact, the calculated inductances of the generators, which are of considerable lower frequency than the converter, are lower than the calculated inductance of the converter. The difference may be due to the fact that the alternating current which is a motor current building up in the opposite direction from the direct current, produces, or tends to produce flux linkages in the armature winding which oppose those of the direct current, with the result that the induced voltage opposing the rise of direct current is reduced. In other words, a higher rate of increase in current is necessary to produce the required counter voltage, which accounts for the higher rate of increase in current in a synchronous converter than in a direct-current generator. Reference to oscillogram Fig. 1 shows that the current reaches its maximum value in approximately 0.006 seconds. This rapid increase in short-circuit current together with the limited distances between brush-holder arms make the 60-cycle converter particularly susceptible to flashing trouble.

CONDITIONS CAUSING FLASHING

Flashing does not appear to be entirely dependent upon the point or value at which the current is arrested. Oscillograms in Figs. 2 and 3 show dead short circuits on the machine when protected by a modified high-speed railway-type breaker. The current was limited to approximately 14,500 amperes, 17½ times full load, the rate of current increase being limited to some extent by the use of a small reactor as a shunt for the trip circuit of the breaker. (The maximum short-circuit current value of this machine is approximately 19,000 amperes or about 23 times full load). However, at this speed the machine was completely protected, except for slight pitting of the brush holders, even at this high current value. The voltage records show clearly that the machine cleared itself immediately after the complete opening of the breaker, which took place in approximately 0.015 seconds. Fig. 4 shows an overload only one-fifth the above value in connection with the ordinary low-speed breaker, from which it is evident that the machine bucked over immediately after the breaker opened even at this lower value of current. When a machine is on short circuit a large percentage of the voltage is consumed internally. The heavier the short circuit the larger is the percentage of the voltage thus consumed. Then so long as the d-c. breaker is closed the voltage between brush arms and hence the tendency to flash is a minimum. The voltage between neutral points on extreme overload and short circuit with the d-c. breaker closed, is also dependent somewhat upon the brush pressure. The higher the brush pressure the lower the contact drop and hence the external voltage between neutral points. This contact drop may be quite appreciable if the brush pressure is low enough to allow the force of the arc to lift the brushes off the commutator, which means a lower percentage of the voltage will be consumed internally, with a result that the voltage on the commutator will be higher. Assuming a reasonable brush

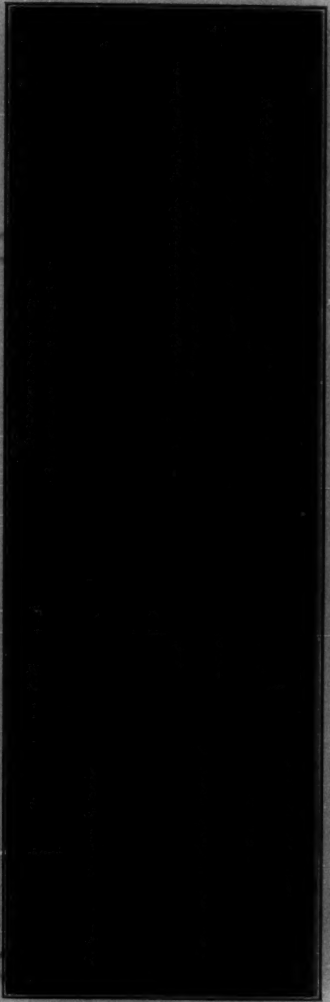

Fig. 3

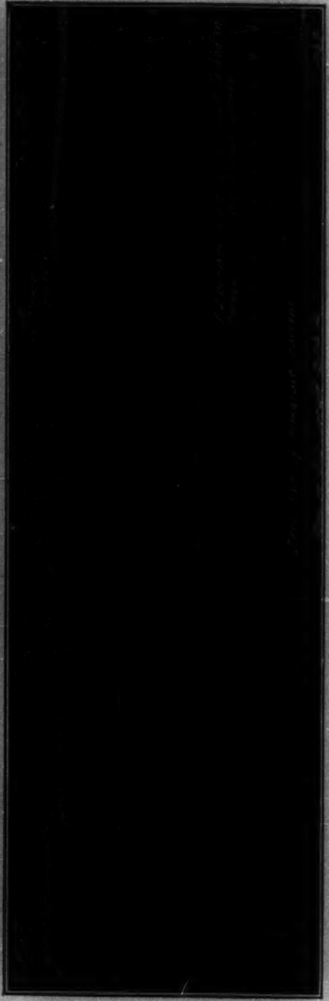

Fig. 4

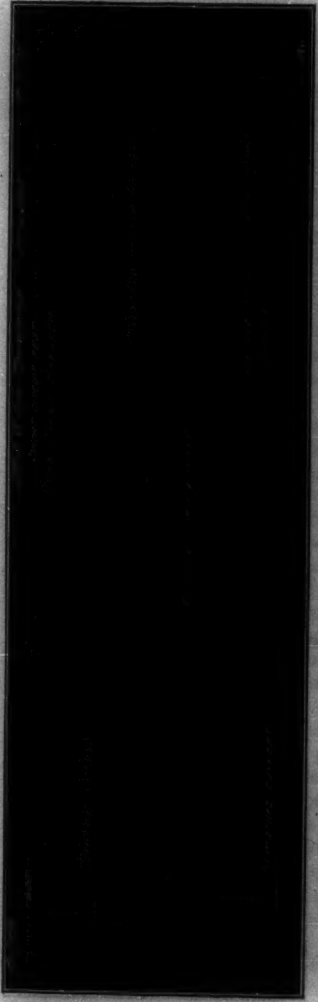

Fig. 5

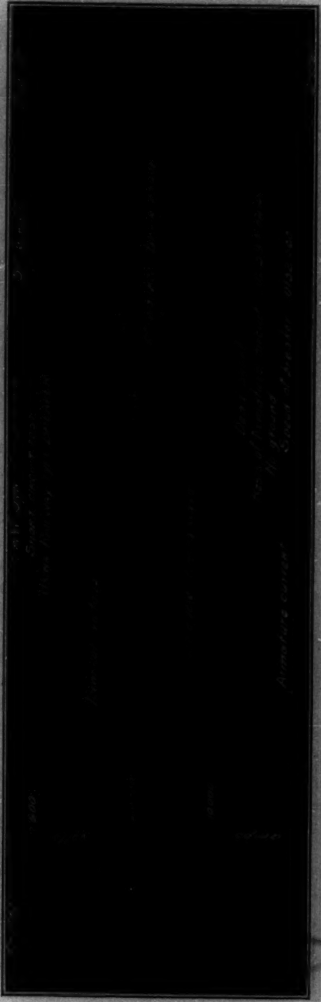

Fig. 6

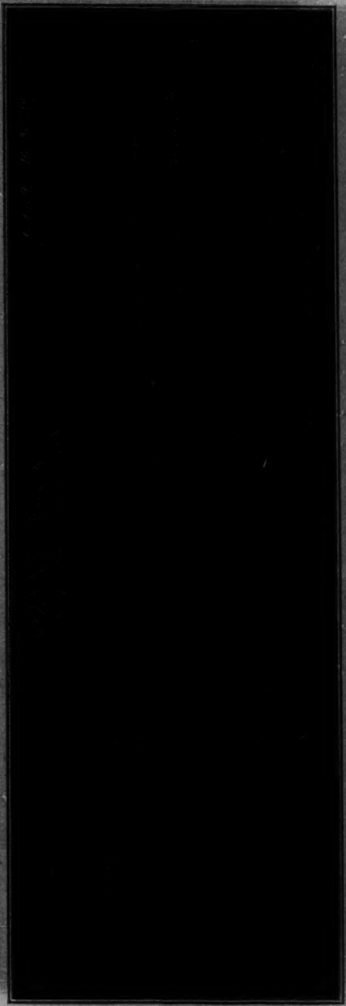

Fig. 7

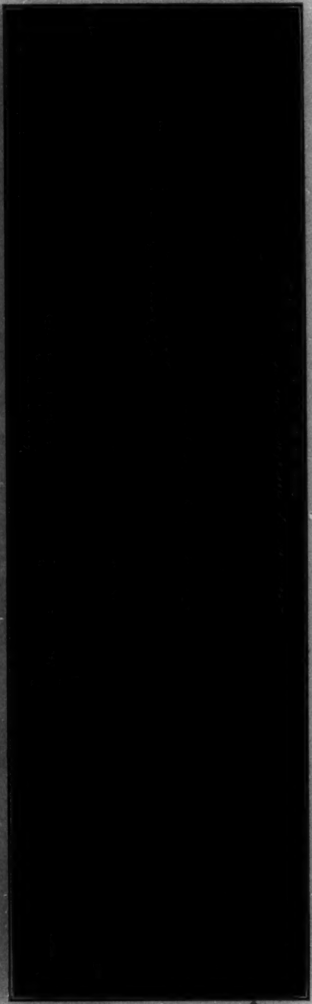

Fig. 8

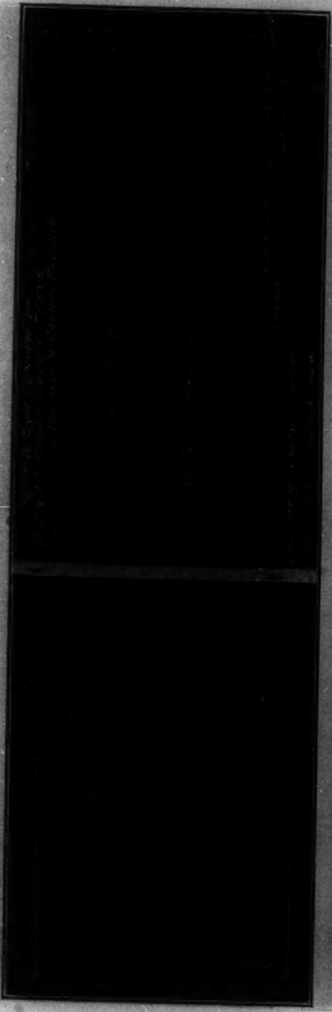

FIG. 9

FIG. 10

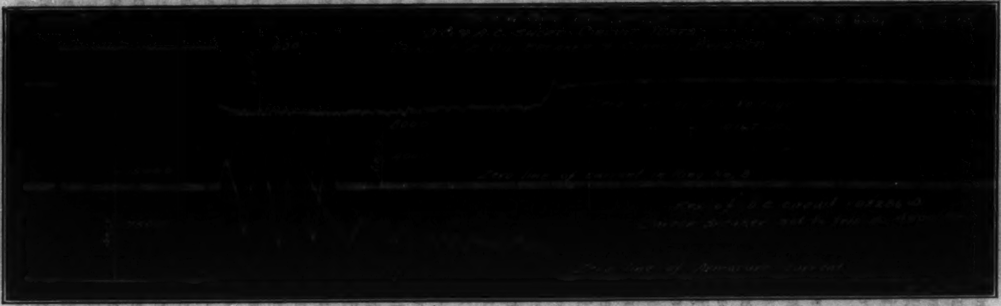

Fig. 11

pressure and contact drop, it is practically impossible for a machine to buck over and hang on between arms on dead short circuit so long as the d-c. breaker is closed, for the voltage on the commutator is practically killed just in the same manner as in the case of the flash suppressor referred to later. It is usually the opening of the d-c. breaker that does the damage. Tests in connection with both the low-speed and high-speed breakers and, in fact, ordinary operating experience show that a machine rarely bucks over until the d-c. breaker opens. See Figs. 4, 5 and 6. There may be considerable flame on the commutator due to the heavy current, but it does not have the tenacity nor power to cause any serious damage until the voltage is restored. Figs. 1 and 7 covering oscillograms taken in connection with the flash suppressor in which approximately one cycle elapsed between the time of the d-c. short circuit and the application of the a-c. short circuit, show clearly that the machine did not buck over before the a-c. short circuit was applied. During this time the commutator rotated through a distance corresponding to two neutral points. Figs. 8 and 9 show overloads of 3 and 6 times full load respectively when the d-c. breaker was tied in and did not open at all. However, the d-c. supply was tripped off about 0.06 seconds after the instant of short circuit as shown by the slight dip in the current and voltage wave. It is evident from the steady nature of the voltage wave that the machine did not buck over and hang on between arms, although there was considerable flash on the commutator.

Therefore, the ideal circuit breaker is not necessarily one which opens the circuit before the machine bucks over (for it usually does not buck over until the breaker opens) or before the current reaches a certain value, but one which opens the circuit before sufficient gas and volatile matter has been formed over the commutator to cause the machine to buck over when the voltage is restored by the opening of the breaker. It may be said that flashing is roughly a function of the voltage and distance between neutral points and the amount of gas or volatile matter over the commutator. This gas is again dependent upon several other variables, such as, the inherent commutating characteristics of the machine (which is a measure of the local short-circuit currents under the brush), the value of the load current and the time it has been flowing, the grade of brush, etc.

It is the writer's opinion in view of the above tests that a moderately high-speed breaker is no better than an ordinary slow-speed breaker. In the case of a moderately high-speed breaker probably as much gas as possible has been formed over the commutator by the time the breaker opens, *i. e.* after this point the gas is dissipated as fast as it is generated. Then a very slow-speed breaker may be even better than a moderately high-speed breaker due to the fact that the voltage will have time to die down appreciably before the breaker opens, especially, if the a-c. supply is opened in the meantime.

Opening the a-c. breaker is of course undesirable because the machine has to be synchronized again. However, this procedure minimizes the flashing considerably. Figs. 10 and 11 cover an overload of approximately seven times full load and a dead short circuit respectively taken under this condition. As shown by the a-c. wave, the a-c. supply was tripped off approximately 0.05 seconds after the instant of the d-c. short circuit. The opening of the d-c. breaker was purposely delayed as long as possible (by changing spring adjustments) so as to allow time for the current and voltage to die down due to the decreasing speed as well as the dying down of the field flux. In the case of the dead short circuit it may be seen that the current has dropped to almost one-fourth of the maximum short-circuit value and the voltage has dropped to less than one-half normal value when the d-c. breaker opens. The machine immediately clears itself upon the opening of the d-c. breaker as shown by the voltage record. During the period prior to the opening of the d-c. breaker there was of course considerable flame on the commutator but due to the limited voltage on the commutator (with the d-c. breaker closed) the arc did not have the tenacity to hang on, and by the time the d-c. breaker opened the current and voltage had reduced to such an extent that the arc could not establish itself.

INFLUENCE OF COMMUTATING CHARACTERISTICS ON FLASHING

Oscillograms covered by Figs. 12 and 13 which were taken at lighter loads and where the a-c. supply was cut off prior to the opening of the d-c. breakers, show clearly by the dip in the current and voltage wave that the machine bucked over slightly before the d-c. breaker opened, which appears contrary to previous conclusions. However, at this light load the voltage on the commutator is not appreciably reduced. Furthermore, with the a-c. supply cut off, the neutralizing effect of the a-c. armature reaction upon the d-c. armature reaction is absent. Therefore, the converter, operating as a straight d-c. generator with the increased effect of the armature reaction upon the commutating-pole circuit, is enormously under-compensated, with the result that the sparking under the brush is greatly increased which together with the high voltage between neutral points causes the machine to flash over even with the d-c. breaker closed. This again emphasizes the importance of the inherent commutating characteristics of the machine.

The relative strength of armature and commutating-pole fields has a considerable influence upon the commutation and hence upon the flashing of a synchronous converter on sudden changes of load and extreme overload or short circuit. Under normal load conditions the a-c. m. m. f. opposes the d-c. m. m. f. In the interpolar space the resultant armature reaction is only about 15 per cent of the d-c. armature reaction, and is in the same direction as the d-c. reaction. The commutating-pole field ampere-turns under this condition are just sufficient to buck down this resultant m. m. f. and in addition to force sufficient flux across the commutating-pole gap to generate the required counter voltage for commutation. However, at the instant of short circuit the converter acts largely as a d-c. generator delivering the first rush of current from its own inertia, with the result that the machine is enormously under-compensated. In fact, with the relatively low a-c. armature reaction at the instant of short-circuit, the resultant armature ampere-turns may be even greater than the commutating-pole ampere-turns, which means that the commutating-pole flux is actually reversed. This of course depends upon the relative amount of the commutating-pole m. m. f. expended in bucking down the armature m. m. f. and in forcing the flux across the gap. The larger the proportion of the ampere-turns expended in the commutating-pole air gap, or its equivalent, the less will be the effect of the armature reaction upon the commutating-pole flux and hence upon the commutation and flashing of the machine. Furthermore, on extreme overload or short circuit the machine falls back in phase similar to a synchronous motor carrying load with the result that the a-c. armature m. m. f. lags (in space position) with respect to the d-c. armature m. m. f. which is fixed in position by the position of the d-c. brushes. Therefore, even after the first instant of short circuit and after the a-c. m. m. f. has established itself the resultant armature reaction is still proportionately higher than the increase in load due to the relative shift in the position of the a-c. and d-c. armature reaction. This produces a result similar to the conditions in the first instant of short circuit, *i. e.* an under-compensated machine. Trouble from this source can also be minimized by increasing the strength of the commutating-pole field relative to the armature.

Sudden changes in the applied frequency to the converter is equivalent to the machine being out of phase and produces similar results from the standpoint of flashing. In fact, our attention was first called to the importance of the relative strength of the armature and commutating-pole fields by a case of trouble from this source in connection with flashing on a 1000-kw. 600-volt 25-cycle converter at the plant of the Philips Sheet & Tin Plate Co., at Wierton, West Virginia in the early part of 1917. This converter is connected to the secondary of an induction motor which drives a rolling mill and which is subjected to very sudden changes in load. These sudden changes in load produce sudden

Fig. 12

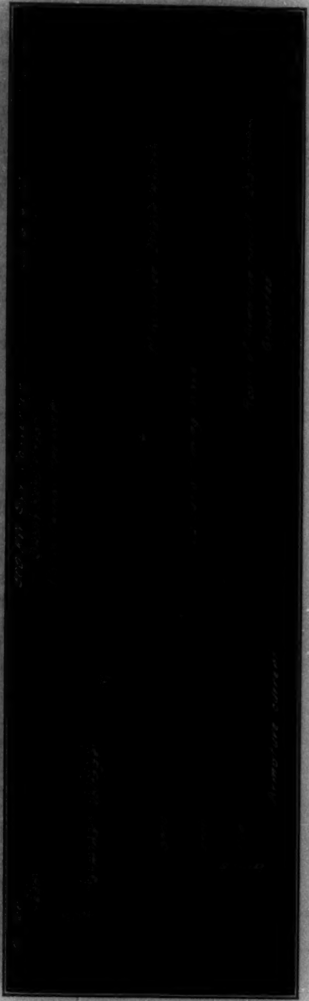

Fig. 13

changes in speed of the induction motor with a resulting sudden change in the frequency applied to the converter which causes flashing in a manner referred to above. A change was made in the commutating-pole circuit to strengthen the commutating-pole field in an effort to eliminate the flashing trouble. This change was to

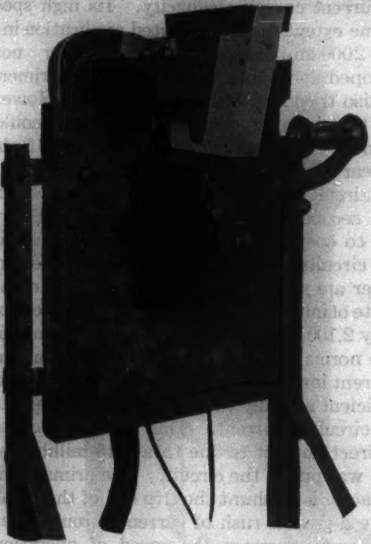

FIG. 14

place, approximately, three-quarters of an inch of brass liners at the back of the commutating pole. New commutating-pole windings to supply the additional ampere-turns required as well as new commutating poles to allow space for the brass liners, were also required. These brass liners being of conducting material, also have a tendency to damp out the pulsations of the commutating-pole flux caused by the pulsating armature reaction. This change practically eliminated the flashing trouble on this machine and showed a field for development along this line.

QUICK-ACTING CIRCUIT BREAKER

Tests made in connection with a modified high-speed railway-type breaker have been previously referred to see Figs. 2 and 3. This breaker has sufficient speed to protect the machine in most cases but is not suitable for this size machine on account of its limited continuous current carrying capacity. Its high speed is due to some extent to its over-rated application in this case.

A 2000-ampere high-speed breaker now being developed and which is still in the experimental stage was also tried out with some success. However, even with this breaker complete protection could not be obtained. Illustrations in Figs. 14 and 15 show front and rear views respectively of this breaker. On dead short circuit this breaker has a speed of, approximately, 0.014 second for complete opening. The arc tips begin to open in approximately 0.004 second. Dead short circuits on the machine when protected by this breaker are shown in Figs. 16 and 17. In these cases the rate of initial increase in current is limited to approximately 2,100,000 amperes per second, about 65 per cent of the normal rate of increase. This reduction in rate of current increase is such that the speed of the breaker is sufficient actually to limit the maximum value of the short-circuited current. This lower rate of increase in the direct current is due to a 0.15-millihenry reactor which was put in the circuit. The primary purpose of the reactor is to shunt the trip coil of the breaker so as to give a greater rush of current through the trip coil and thereby increase the speed of operation of the breaker. Although this reactor was effective in increasing the speed of the breaker as well as limiting the rate of increase in the direct current, a very undesirable and vicious voltage was induced in the coil upon the opening of the d-c. breaker, which in some cases appeared to re-establish the arc across the breaker.

This breaker was about as much underrated as the modified railway breaker was overrated on this application and it is believed that it would have a slightly higher speed on higher currents than dealt with in this case. A smaller breaker with lighter moving parts is now being considered on which it is expected to obtain higher operating speed.

Some very interesting high-speed photographs of the machine when flashing and when protected by this breaker are reproduced in Figs. 18 and 19. The camera used in taking these photographs has twenty-two stationary lenses with a rapidly revolving shutter in

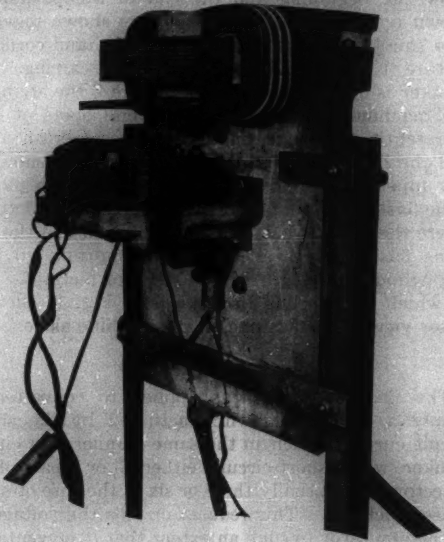

Fig. 15

the rear which opens them in rapid succession. The time interval between each exposure is of course dependent upon the speed of this shutter which is variable. The progression of the phenomena is in the order of the numerals on the plate. The pictures marked with the sub letter *S* were taken at the same instant (by two different lenses) as the picture marked with the corresponding number without the sub letter *S*. When properly mounted these pairs of pictures can be viewed to a great advantage through a steroscope which brings out the third dimension or depth of the picture.[1]

Fig. 18 which corresponds to oscillogram in Fig. 16 shows a picture of both the machine and breaker when flashing. There is approximately 0.0018 second interval between each picture. Pictures 1 and 17 which show the beginning and the end of the phenomena shown on this plate overlap and are shown together. The small light spot in the lower left hand corner of picture 1 shows the beginning of the arcing. The breaker is not yet visible. In picture 2 the arcing on the machine is increasing the several spots of light representing sparking under the several brush arms, and the beginning of the breaker opening is shown in the upper right-hand side of the picture. In picture 3 the flashing is still progressing, and so on to picture 7 where the breaker has completely opened and the machine flashed over. The machine continues to flash throughout the length of this series of pictures. Fig. 19 which corresponds to oscillogram Fig. 17 shows a closer view of the flashing on the machine alone.

FLASH SUPPRESSOR

The flash suppressor as applied to converters is simply a high-speed switch actuated by the short-circuit current, much in the same manner as a circuit breaker, which short-circuits either all or a part of the collector rings (usually three or six in the case of a six-phase machine.) This reduces or kills the voltage on the commutator to such an extent that it prevents the machine from flashing over. A detailed explanation of the flash suppressor is given in an article on this subject by Mr. F. T. Hague and Mr. N. W. Storer in the May 1918 issue of the *Electric Journal*; and its particular application to the generators of the Chicago, Milwaukee and St. Paul motor-generator sets is given in an article by Mr. Daivd Hall in the January 1920 issue of the *Electric Journal*.

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[1] A detailed explanation of the construction and operation of this camera is given in an article by Mr. J. W. Legg in the December 1919 issue of the *Electric Journal*.

Fig. 16

Fig. 17

The operation of the flash suppressor means practically a short circuit on the a-c. system through the converter transformers. The alternating current drawn on a dead d-c. short circuit where the high reactance transformers are used for compounding purposes, was only 15 to 20 per cent higher with the suppressor in operation than without. Thus on dead d-c. short circuit the suppressor did not add so much to the duty of the machine and a-c. system. However, each overload on the d-c. side above a predetermined value for the suppressor to operate meant a short circuit on the a-c. side. Another undesirable feature was the fact that the machine fell out of step and had to be synchronized again. In an effort to limit the heavy alternating current drawn by use of the flash suppressor and so eliminate this objection, tests were made with inductance in the local suppressor circuit for the purpose of finding out the maximum value of inductance which could be used to limit the alternating current and still reduce the d-c. voltage sufficiently to give protection on the d-c. side. For each setting of inductance the d-c. short-circuit load was increased to the point of prohibitive flashing. It was found that in order to give protection the value of this inductance had to be such that upon the opening of the d-c. breaker the d-c. voltage would not rise above the value it had dropped to due to the d-c. short circuit, *i. e.* there must be no appreciable rise in voltage when the d-c. breaker opens. Also the tests showed that reasonable protection could be had at about eight times full d-c. load without drawing over nine times full-load alternating current, or at about 12 times full direct current without drawing over 10 to 12 times full-load alternating current.

With the idea of eliminating the undesirable feature of having to re-synchronize the machine after the operation of the flash suppressor, a three-pole oil circuit breaker was put in the local leads of the suppressor circuit in connection with the above tests, so as to open the suppressor circuit and throw the machine back on the line. The trip circuit for the oil breaker was energized immediately after the opening of the d-c. breaker by means of an auxiliary contact on this breaker. Figs. 20 and 21 show tests in which the flash suppressor circuit was opened and the machine thrown back on the line. It was found that the suppressor circuit could be opened and the machine thrown back on the line in this manner without any particular disturbance up to the point where the inductance in the suppressor circuit must be decreased (for protection to the d-c. side) sufficiently to cause the machine to fall out of step. This point is at approximately ten times full load. Some evidence of the tendency of the machine to fall out of step is shown by the slight oscillation in the d-c. voltage wave in Fig. 21. The oscillogram in Fig. 22 shows a short circuit of approximately 13 times load and the pulsations in the d-c. voltage wave show clearly that this machine is out of step. The frequency of these pulsations corresponds to the slip frequency on the commutator and shows the extent to which the machine is out of step. When the suppressor circuit is opened and the machine thrown back on the line, when it is out of step, it flashes over due to phase displacement on the a-c. side regardless of the value of the d-c. load.

Fig. 18

All of the above tests show clearly that the flash suppressor will give ample protection from flashing to the converter as far as the a-c. side is concerned. However, there are several problems connected with the application of the flash suppressor to synchronous converters, such as, the increased duty on the collector ring, disturbance to the a-c. supply systems and the machine's falling out of step, which have not yet been worked out.

Fig. 19

Flash Guards or Barriers

Various forms of flash guards have so far proved to be unsuccessful. The general construction of these guards consisted of two continuous end rings, one at the back of commutator next to commutator necks and one at the front of commutator, with barriers between arms extending down to within approximately 1/32 in. of the commutator. These guards were made of asbestos lumber, a composition of asbestos and asphalt. Considerable trouble was experienced at first from the arc going under the front guard ring to the commutator *V* ring. This was remedied by making this front ring counterbored so that it extended over the face of the

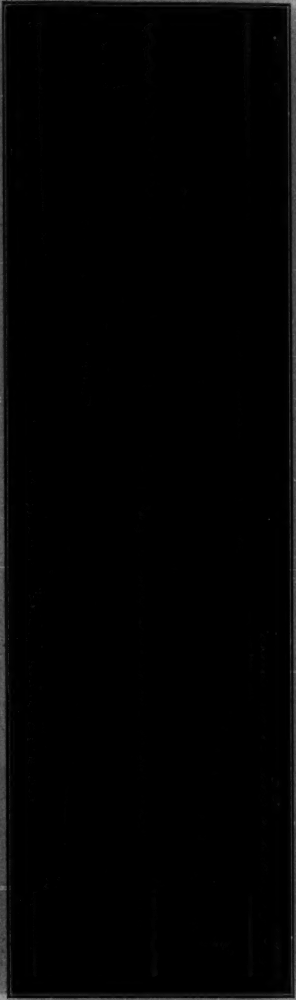

FIG. 20

FIG. 21

FIG. 22

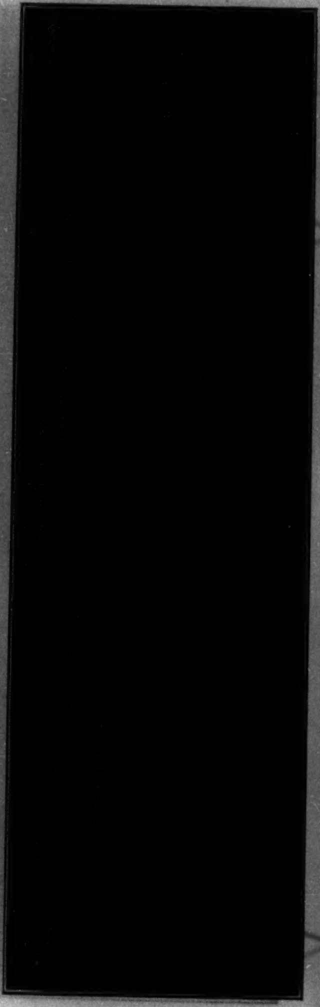

FIG. 23

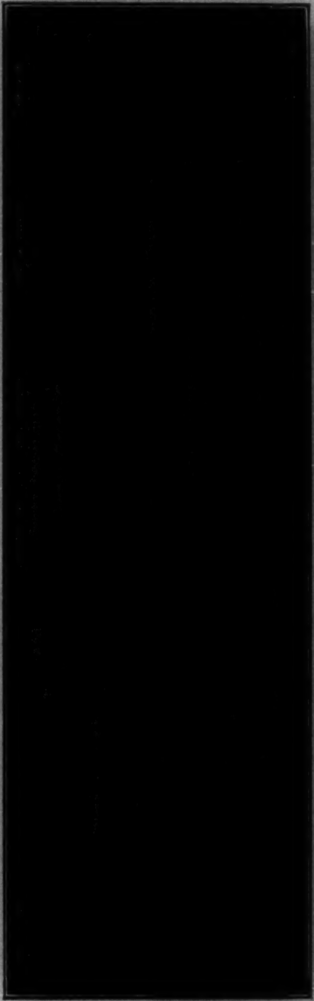

FIG. 24

FIG. 25

Fig. 26

commutator a small distance as well as below the front end of the commutator. However, even this did not prevent the machine from flashing; with the arcing space confined by these guards the arc seems to become explosive and force itself to parts that it otherwise has no tendency to go. Furthermore, after several flashes the surfaces of these guards become so carbonized and covered with metal particles that they actually aggravate the flashing rather than prevent it. In fact, in several instances the guards were so carbonized and metal-smeared after the flash-over, that streamers would run along the surface at no load and normal voltage. All tests indicated that these barriers should be of an open construction and as far away from the source of the arc as possible so as to give it sufficient room to expand and dissipate itself without becoming explosive. In other words the barriers should be next to the parts to be protected (as the pedestal and leading side of brush holders) and as far away from the arc as possible. This fact is borne out by oscillogram in Fig. 23. In oscillogram shown by Fig. 24 the machine flashed over to the front pedestal (no flash guards on machine). In an effort to keep the arc from jumping to the pedestal, the front ring, which had been slightly carbonized on a previous flash-over was then bolted back on front of the brush-holder brackets and the short circuit repeated as shown by Fig. 23. This time the arc jumped along the surface of the front ring between the two front brush holders on the two lower arms and practically demolished these two brush holders. Later this ring was fastened to the pedestal, away from the commutator and the source of the arc, and gave complete protection clear down to dead short circuit except for slight pitting of the brush holders.

This shows very clearly that the flash guards increase the tendency of the machine to flash over after they have become carbonized. No suitable material for this purpose which will not carbonize has been found so far. Best results have been obtained by protecting the pedestal with an arc shield and leaving as much space as possible on the commutator for the arc to expand. This also gives free access to the commutator, the lack of which is one great disadvantage and objection to any form of flash guard. This same protection can be offered more readily and easily by insulating both pedestals much in the same manner as for the elimination of bearing currents. This, however, has the disadvantage that a dangerous voltage may exist between the pedestals and ground in case the armature becomes grounded. With the pedestals insulated this machine can be dead short-circuited with the ordinary carbon breaker with no particular damage to the machine except a slight pitting and marking of the brush holders. Figs. 25 and 26 show dead short circuits with the machine grounded and ungrounded under the above condition. Evidence is again shown of the machine falling out of step as discussed under the heading of flash suppressors. The fact that the d-c. voltage is pulsating undoubtedly prevents the arc from hanging on. It goes out when the d-c. wave passes through zero. For this reason a dead short circuit on a converter in connection with a slow-speed breaker does not appear to be as vicious or tenacious as a short circuit at a point just before the machine falls out of step and the d-c. voltage is maintained in one direction. This point is about 10 to 12 times load in the case of this machine. This is also about the point of maximum EI output.

Presented at the 358th meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., March 12, 1920.

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AUTOMATIC RAILWAY SUBSTATIONS

BY FRANK W. PETERS

Railway & Traction Engineering Dept., General Electric Co.,
Schenectady, N. Y.

This paper reviews the broad range of conditions to which railway automatic substations have been applied and also discusses the economies and operating advantages effected by their use. A description is given of the modern equipment with details of its operation. Special reference is made to improvements in design of control apparatus, to the positive sequence of starting the machines and the protection afforded the apparatus against overloads or other irregularities either outside or internal to the stations.

PRESENT day railway conditions are such that any improvement leading to the reduction of invested capital or decreased operating charges is of vital importance. Automatic substations as shown by several years' operation have resulted in decided economies and improved operating conditions, and as the subject has not been presented before the A. I. E. E. since an account[1] of the original installation was prepared a review of the modern automatic equipment seems desirable.

The first automatically controlled railway substation was placed in service during December 1914, on the Elgin & Belvedere Electric Railway. The station equipment prior to being made automatic consisted of a single 300-kw., 600-volt, 25-cycle, three-phase synchronous converter with three single-phase 110-kv-a., 26,000-370-volt self-cooled transformers and standard manually-operated switchboard apparatus. The individual devices comprising the first equipments were with but few exceptions, those which had previously been developed by electrical manufacturers for use in other applications. This condition was a decided advantage to the operating companies since developmental charges were eliminated, rendering the equipments comparatively low in cost, which in addition to the successful operation of the early installations made possible the rapid growth of automatic application to electric railways.

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[1] Allen and Taylor Vol. XXXIV, Part 2, page 1801.

Shortly after the initial installation on the Elgin & Belvedere Electric Railway, the two remaining substations on that road were made automatic, followed by the Potomac Electric Power Co. which made automatic a 500-kw. substation. About that time (1916) the Des Moines City Railway and Interurban Railway Co., two adjacent roads, outlined and have practically completed a program involving automatic substations, which has resulted in the most notable example of their use up to the present time. These roads now have in operation a total of three 300-kw. and nine 500-kw., 600-volt automatic synchronous converter equipments. The confidence inspired in the minds of operating engineers and the rapid adoption of this comparatively new phase of electric railway operation was in no small measure accelerated by the successful performance of this rather broad application.

The range of requirement to which the new scheme could be successfully applied was demonstrated when in 1917 two 300-kw., 600-volt, 25-cycle synchronous converters operated in series on 1200 volts by the Milwaukee Railway & Light Co. were automatically equipped. Two automatic substations each containing one 600-kw. 1500-volt d-c. induction motor-generator set were also installed in 1919 by the Salt Lake, Garfield & Western Railway. The Rhode Island Co. have for some time been operating a station containing two 300-kw., 600-volt converters in parallel, while in other localities portable automatic substations have been functioning successfully. Railway converters now automatically equipped range in size from 200 to 1500 kw. with motor-generator sets from 300 to 2000 kw. The total capacity operating in this manner and including those in process of installation is estimated to be 45,500 kw. while the number of automatic equipments involved is approximately 79, 59 of which having been applied to 300- and 500-kw. sizes, and the remainder cover the range of various types and sizes of installations briefly referred to.

While it is the intention to confine the scope of this paper to railway activities, it is interesting to note that a hydroelectric station on the Iowa Railway & Light Company's system containing three 500-kv-a. synchronous generators has been operated automatically since 1917 while the Interstate Light & Power Co. in the same year placed in operation a 3000-kv-a. automatic synchronous condenser. The Union Electric Light & Power Co. is applying the scheme to a 2000-kw., 250-volt lighting synchronous converter. Railway operation, up to the present time, has presented apparently the most attractive field for the use of automatic features, but a well grounded start in other lines has been made and it is only reasonable to expect considerable activity in the lighting and hydroelectric branch of the industry.

Without question a direct reduction in operating expense has been the prime motive for the purchase by railway companies of automatic substations. The saving is effected in several ways, although it is a variable and depends on conditions under which a particular station is operated as well as the number and capacity of machines in the station.

Labor Saving. Considering the several items of saving more or less in their order of importance, the matter of eliminating operator's wages stands out most prominently. In those localities where three eight-hour shifts are in force in single-unit substations the labor saving alone from automatic operation will often wipe out the original cost of the equipment in two to three years. Where only one or two station attendants are employed, as generally is the case on the average interurban system, the net return is not so great, but usually is sufficient to represent a desirable return on the investment in extra equipment incident to automatic operation. When two or three machines are installed in a single station an equal number of complete automatic units are necessary, requiring two or three times the initial investment of a single-unit station. This fact makes it more difficult to justify automatic operation of such a station if only the saving of station attendant's wages is considered. A further study, however, of other savings and advantages inherent to automatic operation as applied to a specific case may show economy of sufficient magnitude to warrant two-unit automatic substations.

The procuring of competent labor constitutes one of the trying difficulties which operating companies have had to contend with during recent years. A natural result of such a condition as applied to manually operated substations is a poor handling of apparatus with a corresponding increase in maintenance costs and less efficient operation. The substitution of automatic stations for the manually operated types eliminates this difficulty in the proportion to the extent of substitution and also reduces the ill effects resulting from labor difficulties. While these features are assets, they are somewhat intangible and not easily capitalized.

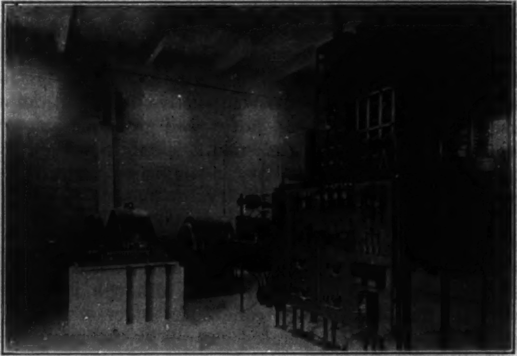


Fig. 2—Interior View of Automatic Substation Containing 600-Kw. 1500-Volt D-C. Induction Motor-Generator Set—Salt Lake, Garfield and Western Ry.

A complete elimination of all labor charges against these stations is not feasible since a regular system of inspection must be maintained. Some companies include this work as a portion of a patrolman's or other workman's duties, while other companies which operate several of these stations employ a man who devotes his entire time to this phase of the work. Those stations having high-tension aluminum-cell lightning arresters require daily visits by an inspector to charge the arresters, at which time a casual inspection of the automatic equipment may be performed so as to preclude any minor irregularities becoming serious. Where a reliable high-tension arrester is used which does not require charging such as the oxide film type, two or three hasty inspections per week have been found ample. At intervals of approximately two weeks a thorough inspection should be made and a detailed report prepared to indicate the condition of every device. The amount charged against each station for this service, including maintenance as reported by several companies, varies considerably but will average approximately $300.00 per year.

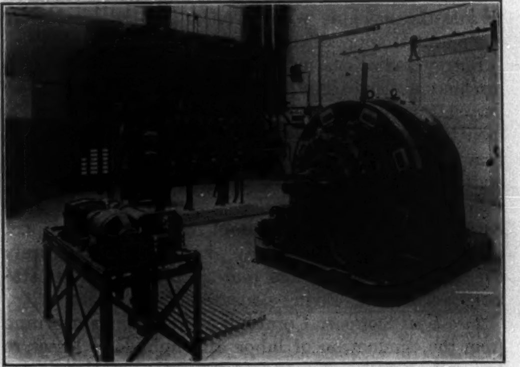


FIG. 3—INTERIOR VIEW OF 1000-KW. 600-VOLT SYNCHRONOUS CONVERTER AUTOMATIC SUBSTATION—PACIFIC ELECTRIC RAILWAY

Light Load Savings. Saving in light load losses, due to automatically shutting down a station when little or no power demand exists, is most noticeable on interurban roads maintaining an infrequent headway. Often a station in manual operation will run idle from a quarter to half the total operating time particularly so if the schedule results in train passing points in the immediate vicinity of the substations. In the event passing points are between stations the machine shutdown period will be somewhat reduced. The total energy saving from this source as applied to single-unit interurban stations depends upon the track layout, characteristics of the schedule and energy required to idle the machine which for a 500-kw. converter equipment is approximately 16 kw.

Energy saving in interurban stations having two machines automatically operated may work out to be very attractive especially if the second machine is used to help out on rush service or move occasional heavy express or freight trains. With automatic operation the second machine would cut in only when a demand for additional power existed and would drop out when not required. Prompt action in this respect is a decided factor in economy which while not entirely overlooked in the average manually operated substation, is a possible saving considerably neglected.

Two Unit Stations. Combining the savings accruing from the elimination of labor and light-load losses in two-unit parallel-operated automatic stations, together with other advantages, the arrangement is justified as evidenced by the operation of the two-unit station on the Rhode Island Company's system as well as a station now being installed by the Dayton & Troy Electric Railway Co. containing two 300-kw., 600-volt converters and a station on the Cleveland, Railway Co.'s system containing two 1500-kw., 600-volt converters.

Coal Saving. The elimination of coal or perhaps considerable energy used by electric radiators for heating the station or operator's booth is another direct saving attributed to automatic equipments. In the more northern localities this item is sufficient to warrant consideration especially in view of the scarcity and price of coal prevailing in recent years.

Building Design. Building design best suited to accommodate automatic substations as shown in Fig. 4 is often materially different than that common to manually operated stations. Simplicity, with resulting lowered initial cost, seems to predominate. The structure is arranged to house all apparatus indoors, which with locating the windows out of reach affords a protection from vandalism. Ample ventilation is provided by louvers located in the side walls near the floor level and ventilators in the roof.

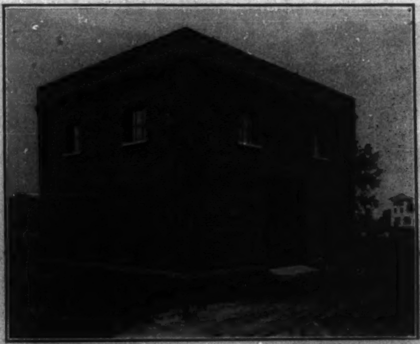


Fig. 4—Exterior View of Polk Boulevard Automatic Substation Containing 500-Kw. 600-volt Synchronous Converter—Des Moines City Railway

Load Factor Improvement. It is a well-known fact that the load factor on the average interurban substation is exceptionally low. Otherwise expressed, this means an unusually large investment in electrical apparatus compared to the average all day power delivered from the station. This condition arises because sufficient capacity must be installed to accommodate not only the regular service, but also to take care of excess demands usually of short duration such as results from accidental bunching of trains, snow plows or occasional heavy freight trains. These irregularities because of their rather brief demands on any one station do not, by heating limitations necessitate greatly increased substation capacity, but they do severely tax the station apparatus from the point of commutation and short period capacity. Since in the past there has been no alternative but to provide large capacity machines to protect against these contingencies, the condition of low load factor resulted in high investment charges for the average energy delivered.

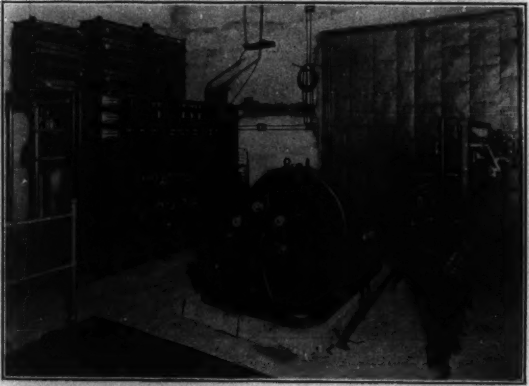


Fig. 5—Interior View of 200-Kw. 600-Volt Synchronous Converter Substation on Cincinnati Lawrensburg and Aurora Electric Street Railway

The automatic substation functions in a manner partially to remedy the situation so that the average load approaches the maximum load demand on the station, which condition permits increased traffic being served by a given substation or conversely permits a smaller substation capacity to serve a given traffic. This condition is brought about by inserting a high-capacity resistance in the direct-current side of the machine at times of severe overload. The direct purpose of the resistance is to limit the current delivered to a reasonable value, which prevents the abuse of apparatus arising from any load demand. The effect of this device is to cut off the high current peaks so frequent to substations serving rush service, locomotives, heavy freight trains or other requirements where the momentary demands tax the commutation capacity of the machines. With the peak loads limited to a safe value, it then becomes possible to permit the average load to approach somewhat nearly the continuous capacity of the machine which condition results in load factor betterment. Caution must be exercised, however, not to exceed the limit in this direction since the resistors may be in circuit sufficiently to interfere too greatly with the average trolley voltage, thereby decreasing train speeds as well as incurring an undesirable energy loss from heating in the resistors. Based on this principle as well as the fact that the average heating of transformers is low in the smaller interurban automatic substations due to frequent light-load shut-downs, it has become generally standard practise in these stations to provide transformers having a capacity equal to 80 per cent of the converter rating. In justification of the idea it may be noted that there are in operation at

Fig. 6—Portable Automatic Substation Containing 500-Kw. 600-volt Synchronous Converter—Kansas City Railways Co.

least two railway systems where full advantage of the principle is realized with exceedingly satisfactorily results.

Feeder Saving. Reduction in feeder copper is effected by the application of automatic substations based on the fact that elimination of labor cost in such stations permits a closer spacing and smaller capacity than would be the case with the manually operated type. For the same line characteristics then less feeder copper is required. The installation of automatic stations has in several instances resulted in the removal of sufficient copper to offset materially the cost of the station. Frequently, conditions arise on existing roads when it is necessary to bolster up some particular section of the system due to rapid growth in service requirements. An automatic station is often the cheapest means of producing the desired result since it is more economical to operate than a manual equipment.

Electrolysis. The effects of electrolysis may be to an extent eliminated by the use of automatic substations since their economic spacing is less than those manually operated. Such an arrangement shortens the negative return with a consequent reduction in potential difference between the rail and adjoining water mains. The National Bureau of Standards recognize this benefit in connection with the principle of automatic substation installations.

As previously stated the first automatic installation consisted of devices which with but few exceptions had already been developed by manufacturers for application to other service such as power station, car equipment, steel mill and similar industrial requirements. Experience soon taught, however, that successful automatic substations required an ultra-reliable class of apparatus since the failure of a single device means a shut-down of considerable duration because it necessitates sending a man from some distant point to investigate the trouble. This fact has been realized by the manufacturers and based on experience during recent years a class of apparatus has been developed for these stations which embodies a degree of reliability consistent with the exacting demands of present-day railway service.

Details of Operation. The functions of starting and connecting the machines to the line upon power demand and finally shutting them down after the demand for power has ceased are all carried on in their proper sequence without any assistance whatsoever from an operator. In present day practise the great majority of these stations are controlled entirely by the automatic equipment in accordance with the above statement, but a few are, for specific reasons, remotely controlled by dispatchers with the aid of a pilot wire. A remote controlled station as generally applied may be considered in the same class with a purely automatic station since the control current merely replaces the automatic devices which determine when the station is to start or stop. The remaining apparatus which performs the actual switching operations are identical in both cases.

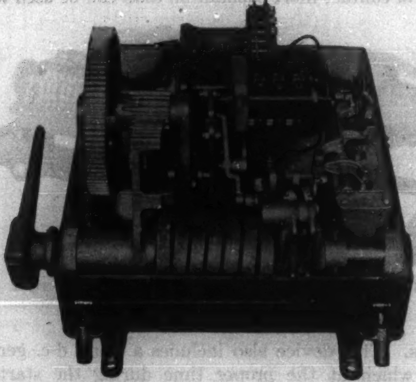


Fig. 7—Motor-Operated Oil Circuit Breaker Mechanism for Automatic Substations

The type of automatic equipments in the most extensive use consists of a group of relays, grid resistors and standard contactors, which together with a motor driven drum controller shown in Fig. 8 perform the usual function of starting, stopping and protecting the machines against irregularities without the aid of an attendant. In general, relays are used where the functions of starting, stopping and protecting the machines depend upon voltage, current or independent time values. During starting and stopping, however, numerous operations must be performed in a definite sequence, which if not strictly adhered to, is conducive to service interruptions. The motor-driven drum controller is used to obtain this fixed time relation of events and to substitute, wherever possible, a type of contact more substantial than can be used with relays. This device also includes a small d-c. generator which at the proper time during the starting operation separately excites the converter field, thereby definitely and immediately insuring the correct polarity.

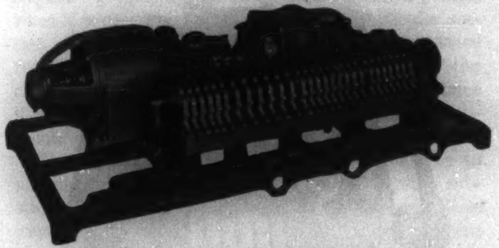


Fig. 8—Motor-Operated Controller and Exciter for Automatic Substations

Protective devices having the following duties are provided to perform the functions ordinarily left to the discretion of the operator.

1. To limit the overloads.
2. To limit the temperatures.
3. To shut down the machine.
    (a) When a-c. or continuous d-c. short circuits occur.
    (b) Upon failure of alternating current.
    (c) Upon failure of any device.
    (d) In case of excessive speed.
    (e) Upon reversal of direct current.

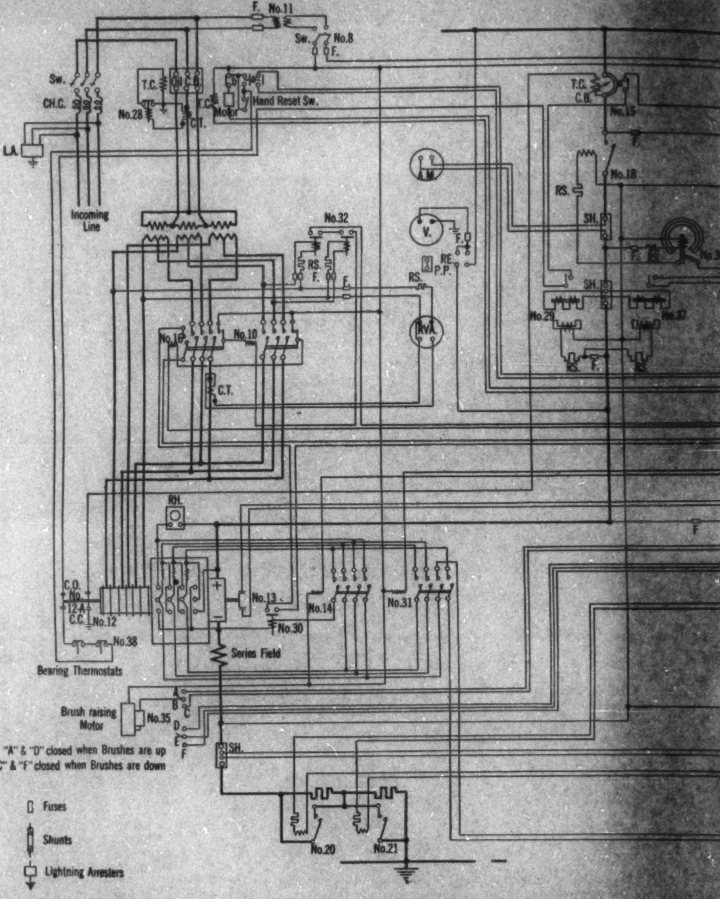
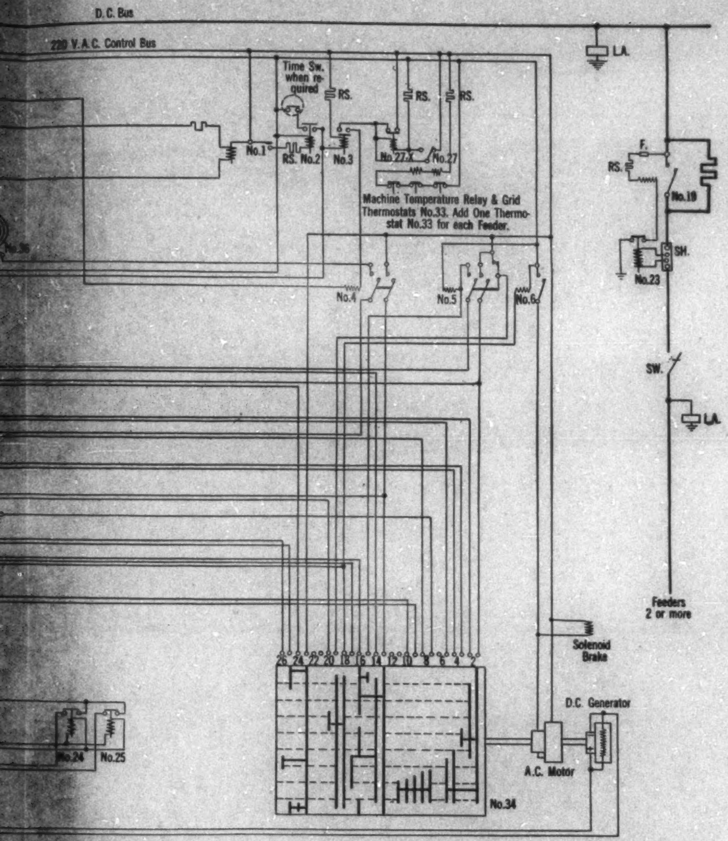

F. No.11
Sw. No.8
F.
Sw.
CH.C.
T.C.
O.C.B.
Hand Reset Sw.
No.28
C.T.
T.C.
Motor
L.A.
Incoming Line
T.C. C.B.
No.15
F.
A.M.
No.18
RS.
SH.
V.
F.
RE P.P.
RS.
No.32
RS.
F.
RVA
SH.
No.29
No.37
RS.
F.
RS.
No.16
No.10
C.T.
RH.
F
C.O.
No.
12-A
C.C.
No.12
No.13
No.30
No.14
No.31
No.38
Bearing Thermostats
Series Field
Brush raising Motor
No.35
A
B
C
D
E
F
"A" & "D" closed when Brushes are up
"C" & "F" closed when Brushes are down
SH.
No.20
No.21
Fuses
Shunts
Lightning Arresters

Fig. 1—Typical Wiring Diagram for

(1) Contact making voltmeter. (2) Time delay starting relay. (3) Time delay shutting down relay. (4) Contactor for making control circuit connections. (5) Contactor for making control circuit connections. (6) Contactor for making control circuit connections. (7) Three-phase oil circuit breaker. (8) 220-volt a-c. control switch. (10) a-c. starting contactor. (11) Control transformer, 220 volts. (12) Speed limit switch (over speed). (13) Speed control switch (under speed). (14) Converter field contactor shunt excitation. (16) A-c. running


D. C. Bus
220 V. A. C. Control Bus
L.A.
Time Sw. when required
RS.
RS.
RS.
No.1
RS. No.2
No.3
No.27-X
No.27
Machine Temperature Relay & Grid Thermostats No.33. Add One Thermostat No.33 for each Feeder.
No.36
No.4
No.5
No.6
F.
RS.
No.19
SH.
No.23
SW.
L.A.
Feeders 2 or more
Solenoid Brake
D.C. Generator
A.C. Motor
26 24 22 20 18 16 14 12 10 8 6 4 2
No.34
No.24
No.25

[PETERS]

OR AUTOMATIC RAILWAY SUBSTATION

contactor. (18) Main line a-c. contactor. (19) Feeder load limiting contactors. (20) Converter load limiting contactors. (21) Converter load limiting contactors. (23) Feeder load limiting relays. (24) Converter load limiting relay. (25) Converter load limiting relay. (27) A-c. low-voltage relay. (27x) Auxiliary relay for No. 27. (28) A-c. overload relay. (29) D-c. reverse current relay. (30) Converter field relay. (31) Converter field contactor separate excitation. (32) Single-phase protective relays. (33) Thermostat relays. (34) Motor operated drum controller. (36) Polarized relay. (37) Direct-current underload relay.

4. To prevent machine starting.
    (a) During low a-c. voltage.
    (b) During single-phase a-c. supply.

By referring to Fig. 1 which is a typical wiring diagram of an automatic 500-kw., 600-volt equipment, the sequence of operation may be followed. For convenience of reference the principal devices have been numbered or otherwise labeled. It will be noted that the 220-volt a-c. control bus is continuously excited from the control transformer No. 11 and the operating coil of contact-making voltmeter No. 1 is always connected between trolley and ground.

Assuming a particular station is shut down and a train is approaching. As it increases its distance from the next station on the line it will eventually cause the trolley voltage to drop and at a predetermined value, usually 450 volts, contact-making voltmeter No. 1 opens, de-energizing the operating coil of relay No. 2, which had been previously held open by excitation from the 220-volt a-c. control bus through relay No. 1. The closing of No. 2 closes relay No. 3 causing it to pick up and close contactor No. 4 provided the hand reset switch and contacts of a-c. low-voltage relays No. 27 are closed. Relays No. 2 has a dashpot to prevent momentary fluctuation of low voltage from producing false operations of the machine. With the drum controller No. 34 in the "off position" as would be the case before the machine starts, contactor No. 4 completes a circuit through segments No. 13 and No. 16 on the drum controller and the limit switch of the brush-raising device which closes contactor No. 6, thereby starting rotation of the motor-driven drum controller. Controller segment No. 15 soon closes contactor No. 5 which in turn energizes the motor-operated oil switch mechanism causing the main converter transformers to become energized by the closing of oil circuit breaker No. 7. The operating coil connection of contactor No. 5 is then transferred from segment No. 15 to No. 14. This circuit passes through an auxiliary switch on the oil circuit breaker to insure the return of all devices to their normal position should the breaker open for any reason. When segment No. 2 makes contact, starting contactor No. 10 is closed connecting the converter to the low-voltage taps provided the a-c. supply is delivering three-phase current as determined by relay No. 32. Shortly the drum controller stops rotating because of the gap in segment No. 16 and waits if necessary for the converter to come up to speed. At approximately synchronism, speed-control switch No. 13 closes, bridging by aid of segment No. 20 the gap in segment No. 16, causing the controller again to start rotating so as to complete the function of connecting the machine to the line.

Next segment No. 3 closes contactor No. 31 connecting to the converter fields a 250-volt supply obtained from the small generator on the drum controller, thereby immediately ensuring proper polarity. Contactor No. 31 is then opened by segment No. 3 and the self-exciting field contactor No. 14 closed by segment No. 4 and running contactor No. 16 closed by segment No. 5 connecting the converter to normal secondary a-c. voltage. Starting and running contactors No. 10 and No. 16 are both mechanically and electrically interlocked with respect to one another to insure against accidentally short-circuiting a portion of the transformer secondary winding. Segment No. 26 next starts the motor-operated brush rigging causing the converter brushes to be lowered which completes the operation of preparing the machine for connection to the d-c. bus. Segment No. 7 is next energized with 600 volts direct current and shortly thereafter segment No. 8 closes the d-c. line contactor No. 18 whose control circuit is in series with converter field relay No. 30, polarized relay No. 36 and auxiliary switches on running contactor No. 16 and control contactor No. 4, thereby ensuring before closing No. 18 that the converter has proper polarity, correct field and full voltage a-c. running connections.

As soon as the line contactor closes the machine delivers load to the bus through the load limiting resistors which, however, are soon short-circuited by contactors No. 20 and No. 21 operated by segments No. 9 and No. 10. The drum controller is then stopped by segment No. 17. When connection to the bus is made through No. 18 the flow of current closes relay No. 37, which will cause relay No. 3 to remain closed regardless of relay No. 1 whose function started the station. In other words the control of the station is now dependent on the contacts of No. 37 which will remain closed so long as a predetermined current is being delivered to the bus. Should the current fall below a set value, relay No. 37 will open and cause relay No. 3 to drop out after a certain period of time and shut down the station. Relay No. 3 has a dashpot and is timed so that momentary low values of current causing No. 37 to open will not shut down the equipment.

When the station does shut down relay No. 3 opens contactor No. 4 causing running contactor No. 16 and d-c. line contactor No. 18 to drop out and disconnect the machine. Contactor No. 5 opens after contactor No. 4 which operation establishes through an auxiliary contact a circuit to contactor No. 6, thereby starting the controller and running it to its "off position." While doing this, however, segment No. 24 trips out the oil circuit breaker and segment No. 25 causes the converter brushes to be raised in preparation for starting upon the next load demand.

The preceding covers briefly the necessary operation in starting and stopping the machines, but there-remain the equally important functions of protecting the equipment from irregularities caused by disturb ances on either the a-c. or d-c. side of the station or within the apparatus itself. Briefly these contingencies are taken care of as follows:

In the event a heavy d-c. overload occurs, relay No. 24 will pick up and open contactor No. 20, thereby inserting resistance in the circuit. Should the overload increase to a greater value, relay No. 25 will operate and insert more resistance, and in stations not provided with individual feeder protection a third step of resistance is provided to limit still greater overload demands. The value of resistance used is such as to permit short circuit in the immediate vicinity of the station without injuring the machine. The resistor capacity, which determines the length of time heavy overloads can be carried by the resistors without serious heating, is to a degree a function of the service requirements but the duration of those irregularities cannot be foretold with accuracy and the practise of providing liberal capacity in the resistors has not only proved desirable but very necessary. In some stations individual feeder protection which consists of an overload relay No. 23, a contactor No. 19 and a resistor in each feeder circuit is installed, thereby localizing to a degree the function of overload protection to each feeder. With such an arrangement only two sections of resistance are used in the machine circuit.

Protection from overheating the machine, its bearings and load limiting resistors is obtained by use of temperature relays No. 38 and No. 33 arranged to shut down the station immediately should such a condition arise.

A reversal of operation as a d.c. motor is prevented by relay No. 29 and over-speed by speed-limit switch No. 12-A. Both of these devices necessarily operate a control circuit which immediately opens contactor No. 4 and shuts down the station. A shunt-trip hand-operated d-c. circuit breaker No. 15 is in series with No. 18 and only used to protect against the possibility of the line contactor freezing closed. Should this condition occur the converter would motor from the d-c. end upon the a-c. end being disconnected and the excessive speed resulting would trip the circuit breaker No. 15 through the operation of speed switch No. 12.

In case a short circuit occurs on the a-c. side of the equipment, the definite time-limit overload relay No. 28 will trip out the main oil circuit breaker, shutting down the station and at the same time opening the hand reset switch which necessitates reclosing by hand before the station can be started again. This feature insures an inspector visiting the station to investigate the cause of the serious a-c. overload.

Low a-c. voltage relay No. 27 which is calibrated for a definite value is connected so as not to permit the station to start or to shut it down if running should the high-tension voltage become so low as to interfere with proper operation.

If for any reason a single-phase condition exists on the secondary side of the transformer during starting operations, relay No. 32 will lock out starting contactor No. 10 and prevent the converter from being connected to the transformer.

Polarized relay No. 36 protects against the possibility of the machine ever being connected to the line in the reverse direction. Unless proper polarity has been established before connecting the machine to the bus, line contactor No. 18 will not close.

In stations containing a motor-generator set instead of a synchronous converter, certain modifications to the equipment are necessary to accommodate the starting operations, but the scheme of operation with few exceptions is similar to the converter equipments. Oil-immersed starting and running contactors are used because of the higher transformer secondary voltage and a certain amount of overload protection is obtained by inserting one or two steps of resistance in the generator field circuit in addition to two steps of series resistance in the main d-c. circuit. This arrangement reduces initial cost since the field resistance and its contactors are of small capacity. An energy saving in resistor heat loss is also accomplished. The 250-volt generator on the drum controller becomes unnecessary in the case of a motor-generator automatic equipment.

The recent development of flash barriers for railway machines affords a device of much benefit to automatic substations and these devices are now included by one manufacturer as a regular part of the equipment. Where conditions are particularly severe and much trouble is experienced from flashing, barriers and a quick-acting circuit breaker will practically protect the machine.

The recent development in automatic equipments has largely consisted of perfecting and making more reliable the present type of station as well as arranging for and applying the principle in other applications such as lighting, mining and hydraulic generating stations. Among specific instances of improved design may be mentioned the elimination from the equipment of all disk-type interlocks and the substitution of substantial finger-type auxiliary contacts. Relay contact elements have been improved where necessary so as to provide a quick make and break action and on d-c. circuits blow-out coils have been added. An improved type of motor-operating mechanism as shown in Fig. 7 for oil circuit breakers is now in use in which compactness and reliability are the outstanding features. D-c. contactors having a rupturing capacity sufficient to handle any load conditions have been developed and are performing their function with complete success. Reverse current and under-load relays of substantial construction and capable of accurate calibration at low current values have been designed to accommodate the conditions of automatic substation operation.

The use of automatic control in railway substation has in a comparatively short time expanded to where it is firmly established in city and interurban railway operation. The successful experience of the past has resulted in larger capacity stations serving heavy traffic being made automatic. The adaptation of this type of control to electric trunk line service at 2400 or 3000 volts direct current as well as more extended use in the strictly industrial field is not far beyond the horizon.

Presented at the 358th meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., March 12, 1920.

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AUTOMATIC SUBSTATIONS FOR HEAVY CITY SERVICE

BY R. J. WENSLEY

Westinghouse Electric & Manufacturing Co.

WHEN automatically controlled substation equipment was in the development stage, the general consensus of opinion was that its greatest field lay in relieving the smaller interurban systems of a greater portion of their substation operating labor. While many such systems have installed one or more automatic substations, they have not displayed the interest in the matter that was expected. In many cases the officials have professed their total inability to finance the new apparatus even though the investment would show a very attractive saving.

On the other hand, it is very gratifying to those directly interested in the subject, that the larger city street railway systems are eagerly taking up the question of rebuilding their distribution systems by taking full advantage of the decided savings made possible by automatic operation of the converting equipment. Orders received from this source have more than made up for the failure of the smaller roads to equal expectations.

The earlier street railway systems made little or no attempt to lay out a distribution system with economy or efficiency. Any location available was seized on as a power house site. Cables were run the shortest possible distance to the trolley and from that point the trolley was usually the only means of carrying the current. As service demands increased due to larger cars and closer schedules, the voltage drop became too great for satisfactory service even by the rather lax standards of those days. To remedy this the trolley voltage was gradually increased from the early standard of 500 volts to 550 and then to 600 volts and the feeders were run parallel to the trolley so as to help the voltage at the ends of the lines.

As the traffic demands increased another factor entered into the distribution problem, that of electrolytic destruction of underground metallic structures such as water pipes, gas mains and so forth. This necessitated heavy copper feeders in parallel with the rail to prevent the returning current from seeking to travel over the underground piping system. The negative booster was also freely used to compensate for the drop in these return feeders, This of course was practically pure waste so far as efficient utilization of the power was concerned since the entire output of the boosters was expended in heating the cables. It was however a good preventive of electrolytic troubles and therefore widely used in spite of its wastefulness.

All this time the source of power supply was almost invariably a slow-speed d-c. machine operating at 550 to 600 volts, and direct-connected to a reciprocating engine. These units grew to unwieldy size in the attempt to keep up with the ever increasing demands of the public for more transportation facilities. The engine driven alternator and then the turbo alternator came to the front as the most economical method of power generation and the railway substation followed soon after. There were of course, railway substations in operation long before the turbine became popular but these were exceptions to the rule.

Some of the larger cities soon installed a distribution system composed of one or more central steam plants with a number of large substations located in various parts of the city. Owing to the cost of operating labor these were kept down to the least possible number even at the expense of considerable investment in feeder copper. There are many of the old systems still in operation with an unwieldy amount of feeder copper in the air and with I^2R losses amounting into the millions of kilowatt hours per year. Poor trolley voltage with consequent slowness of schedule speeds is a natural result of this. The possible savings to be had from reduced copper losses, reduction in running time and decreased amount of motor repairs by rebuilding the distribution system would in many cases enable a property to make a better showing of earnings than it is making at the present time.

The development of automatically controlled substation equipment has now placed in the hands of the distribution engineer a possible method of reconstruction that is little short of ideal. Substations can now be scattered about without the specter of heavy and ever rising operating labor expense to be watched. The increase in the number of substations enables the holding of good trolley voltage without the heavy expense in large feeder capacity as has previously been the case. Trouble due to electrolysis is greatly minimized and in many cases entirely eliminated by the increase in number of substations.

These substations are of course not one hundred per cent efficient so care must be used in applying this remedy that it not be overdone. Sizes of machines and stations must be carefully calculated so that the load factor will be as high as possible. In deciding on the best sizes of equipment to install, several factors must be considered. Interchangeability is always desirable and therefore there should be as few different sizes of machines as consistent with economical operation. Wherever possible the machines in any one station should be of the same size so that their places in the operating schedules may be interchanged. In most city applications two machines will prove to be the most desirable number for an automatic station with space provided for a third machine to take care of future growth. In most cases this problem of added capacity can be better solved by the installation of additional substations when the load grows too great for the original installation of converting equipment. This latter method keeps down the feeder copper requirements.

Where two machines are selected for a given location each one must be capable of carrying the normal off peak load without assistance. The combined capacity must be sufficient to take care of the peak load. In some cases this will result in more capacity than necessary during the off peak period, and in other cases just

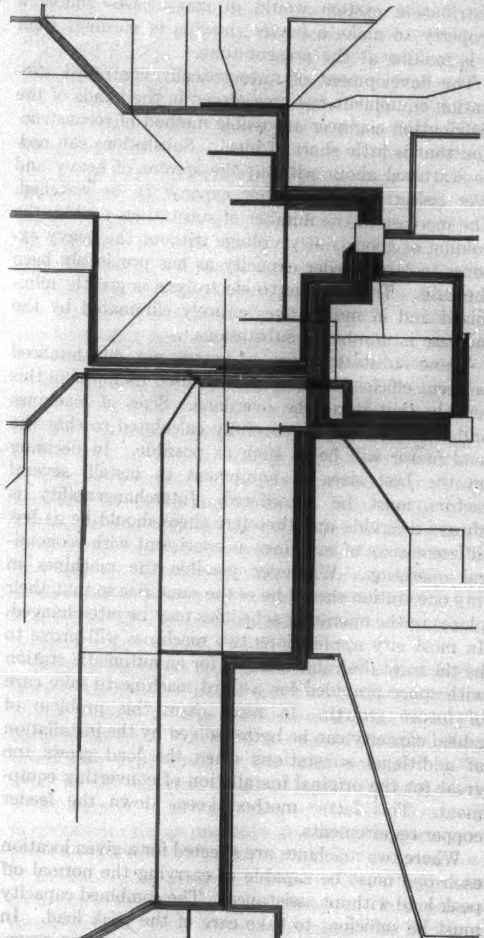

FIG. 1

the reverse, depending on the off peak load being less or greater than half the peak.

By selecting machines of combined continuous capacity equal to the r. m. s. value of the peak load, a reasonable reserve capacity is allowed since the two-hour rating will probably allow one machine to pull over the peak if the other is out of service. To help in such cases, a limited number of feeders should be run through from one station to another so that the feeder may be opened in the station in trouble thus transferring a portion of its load to an adjacent station. The automatic load limiting resistances supplied as part of a railway automatic substation will also help out in such cases at the expense of the trolley voltage.

To illustrate the way in which the problem of applying automatic substations to city service should be approached, a city of about 300,000 population has been selected to serve as an example. This city is now served by an excellent example of the old method of centralized distribution. Fig. 1 shows in a general way the stupendous amount of feeder copper that has been installed in the attempt to give good service. The two plants are about three-quarters of a mile apart on the same side of the city and about one mile from the loop district where lies the greatest load concentration.

The heaviest loaded lines are on the opposite side of the city from the power plants. The lower plant is the oldest one and is mainly equipped with reciprocating units. The total d-c. capacity of this plant is about 8000 kw. The upper plant is a modern turbine station which supplies power for several interurban systems. In this plant are located three 2000-kw. synchronous converters. This gives a total d-c. capacity of about 14,000 kw.

Most of the feeders shown in the cut are 1,000,000 cir. mils. There is a total of 25,800,000 cir. mils in feeders both positive and negative leaving the old plant, 23,000,000 cir. mils leaving the new plant and 8,000,000 cir. mils in tie lines between the plants. One of the heaviest loaded lines runs from the loop

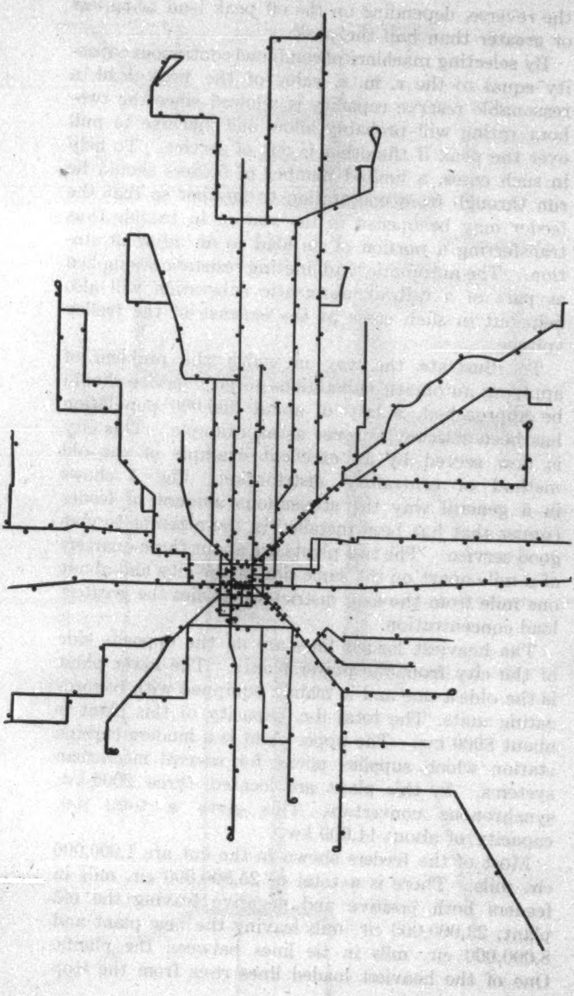

FIG. 2—SPOT MAP—AUTOMATIC SUBSTATIONS

directly away from the plants with the end of the line about seven miles distant measured along the feeder run.

Of this total feeder capacity about 40,000,000 cir. mils are in the positive feeders, the remaining being in the return feeders. The average length of feeder to the center of its load is about two miles. The peak load on the two plants is about 22,000 amperes. Assuming for estimating purposes that the rail and return feeder drop will be somewhere near equal, we find that at peak load the average voltage at the load centers will be 475 volts with 600 volts on the station bus bars. The I^2R losses reach the enormous value of 2750 kw. or 20 per cent of the total d-c. input, over the peak period. The average trolley voltage at the car will be somewhat less than that given for the load centers. In many cases the voltage on the car is so low that it is almost impossible to read news print during the evening rush hour.

This city operates about 300 cars during the periods of heavy traffic most of which are double truck, 20 tons in weight with two 60-h.p. motors geared for 27 miles per hour. The schedule speed is about six miles per hour during the evening rush hour and even this can hardly be held. Due to the low average voltage the cars cannot accelerate rapidly as they should and bunching soon occurs which only aggravates the trouble.

The problem is to lay out a new distribution system which will reduce the transmission losses to a minimum, raise the average trolley voltage to a reasonable value, keep the stray earth currents to a minimum, and at the same time avoid raising the expense for operating labor to an undue amount. All this must be accomplished at the lowest possible net cost.

As a preliminary to deciding on the size and location of substation equipments a spot map of the system should be prepared based on the rush hour schedules. A sample of this is shown in Fig. 2. A factor which upsets the spot map as shown is the fact that there are fourteen interurban lines entering this city all of which operate fairly heavy cars; 45 tons weight with

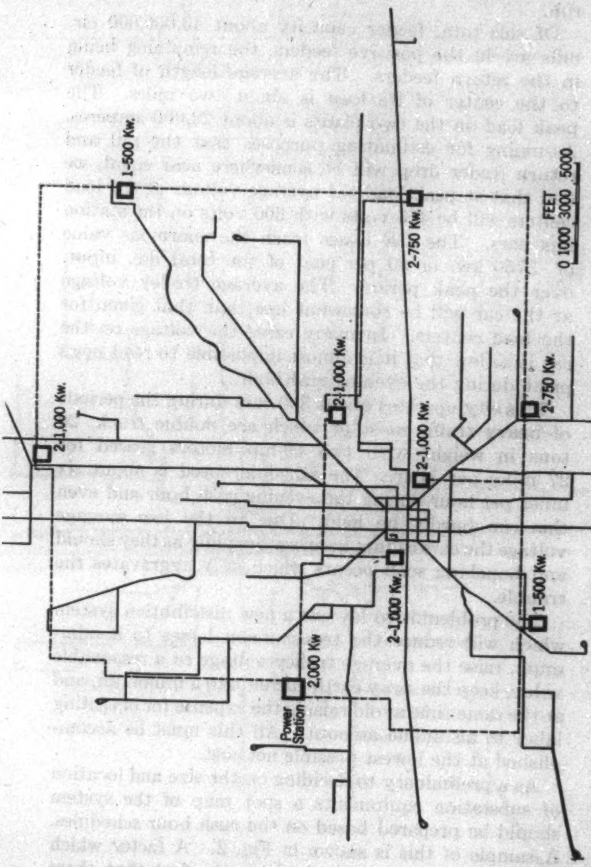
1-500 Kw.
2-750 Kw.
FEET
0 1000 3000 5000
2-1,000 Kw.
2-1,000 Kw.
2-1,000 Kw.
2-750 Kw.
2-1,000 Kw.
2-1,000 Kw.
1-500 Kw.
2,000 Kw
Power Station

FIG. 3—PROPOSED SUBSTATION LAYOUT—AUTOMATIC SUBSTATIONS

four 90-h.p. motors geared for sixty miles per hour would represent the average. In laying out the power supply it has been assumed that two of these cars are at the center of each line which they use for entering the city.

Certain other conditions such as heavy seasonal loads must be considered. There are in this city three parks that are heavily patronized and a fair ground that is a very heavy peak for one week in the year. Such considerations as these have been used to modify the strictly mathematical method of locating the new substations. This method is the one described in Richey's handbook as having been used to lay out the feeder system in Chicago, and consists of assuming that the distributed load on each line is concentrated at the center, then, treating the loads as actual weights, find the center of gravity. This locates the substation in the ideal position to feed its section. This ideal location is of course seldom practical and due consideration must be had for local conditions.

In figuring the spot map shown in Fig. 2 the average current per car has been taken as 53 amperes. The interurban cars have been figured as equal to two city cars. The city has been arbitrarily divided into nine districts for the purpose of apportioning the substation capacity. Care was taken in making this division to keep each section down to less than 2000 kw. in possible load as this seemed to be the largest size station that would be advisable on this size of system, having due regard for investment in feeder copper. The center of gravity was then found for each section and the substation located as near this point as a study of local conditions would warrant.

The two substations feeding the congested portion of the city were located on each side of the loop district just far enough away to be on the edge of the underground district. This is desirable from the standpoint of economy in installation, although the most desirable location from a standpoint of copper loss and feeder copper would be right in the center of the loop. This would however result in excessive investment charges for real estate and underground construction. Locations east and west of the center were chosen rather than north and south because of real estate values.

The final location decided for these substations is shown in Fig. 3. The high-tension line locations were picked out by the superintendent of distribution of the street railway company with a view to using as much of the present pole line as possible and of selecting streets for the necessary new lines where there would be the least opposition from the residents and the city authorities. The feeders form a ring bus around the city with a tie line across the center.

The intention is to sectionalize the feeder in each substation and equip the section breakers with reverse-power relays so that any section will be cut out in case of trouble, thus leaving the entire system in operation with the exception of the damaged portion of the line. Owing to the local situation with regard to the telephone companies and the city authorities the maximum voltage which it would be advisable to use above ground is 6600. While this is not the most economical voltage for the amount of power and the distance, it has been adopted because of the great expense of underground high-tension feeders at 11,000 or 13,200 volts.

At this voltage the feeder belt may be of 4/0 wire, and the cross tie line of 350,000-cir. mil cable which will give approximately 5 per cent power loss in the a-c. system at full load.

The substations as laid out have the following capacity machines installed: Three with two 1000-kw. converters each; Three with two 750-kw. converters each; Two with one 500-kw. converter each; One of the existing 2000-kw. machines to be retained at one of the steam plants. Total converting capacity installed 13,500 kw. Total peak load at present time 13,200 kw.

While this is apparently only sufficient nominal continuous capacity for the peak, it must be remembered that 20 per cent of this peak is lost with the present feeder arrangement so that if the feeder losses can be reduced to 5 per cent this will leave an actual surplus capacity of 15 per cent. This taken with the two-hour overload rating of the converters will give ample margin of reserve capacity.

The feeder calculations are based on maintaining an average voltage of 550 at the car during the evening rush hours. This property now has the equivalent of approximately 110 miles of 1,000,000-cir. mil copper in the distribution system. With the new substation locations we find that the rail resistance is within allowable limits in nearly all parts of the system. In only a few places will negative feeders be required and these will be quite short. With the widely distributed sources of d-c. energy, the carrying capacity of the trolley wires themselves become an important factor, in the distribution of the 600-volt current.

Of the present feeder system it will be possible to remove approximately seventy miles of 1,000,000-cir. mil cable or its equivalent. At 18 cents per pound this will amount to $205,380 which is the first important item of credit for the new installation.

The old d-c. plant should be scrapped entirely and the proceeds applied on the new equipment. It is estimated that the second hand and scrap value of this plant will be at least $100,000.

The I^2R losses on this system were calculated last year and the value of the current lost in the feeder system only, not including local track and trolley losses, was figured as being $100,000. These figures are not vouched for by the writer but as can be readily imagined from the description of the feeder system, this amount does not seem unreasonable. Since approximately one third of the feeder copper is to be left in service we will assume that the copper losses in the remaining feeders will be in proportion to the amount now in service. This will give us a saving in copper losses amounting to $66,000 annually.

If the rush hour schedules of each car are so speeded up as to save five minutes on the rush hour trip morning and evening, with 300 cars and with platform labor at 40 cents the annual possible saving will be $14,600.

The automatic substation buildings should be of the most simple type of construction possible with no heating apparatus and with no accommodations for operators. If extensive repair work becomes necessary in the winter time a portable electric heater can be installed. By constructing this type of building absolutely without ornamentation, the cost should not exceed $40 per kw. complete. Twenty-five miles of a-c. line will be required which can be partly installed on existing poles. The cost of this should not exceed $5000 per mile.

Gross cost of the new system;

11,500 kw. of substations at $40	$460,000
25 miles of line at $5,000	125,000
Reconstruction of feeder system	25,000
Power house equipment, transformers, etc	100,000
Total cost of automatic substations	$710,000
Credit power plant scrap	100,000
Credit copper removed	205,380
Net cost of automatic substations	$404,620
Fixed charges at 15 per cent	$75,693
Annual operating labor	7,000
Gross annual charges against automatics	$82,693
Savings due to the new equipment;	
Annual saving in copper loss	$ 66,000
Annual saving in platform labor	14,600
Annual saving in labor due to shutting down of old power plant	50,000
	$130,600
Annual cost of substations	$ 82,693
Net operating credit annually	$ 47,970

In addition to the credit shown above, the system would have an increased capacity of 15 per cent or 20 per cent over the present one and would be attracting more patronage by better schedule speeds and better lit cars. It would also be in ideal position to take care of expansion in its business which is almost certain to come as the city in question is growing rapidly and is attracting many new and large manufacturing establishments all of which are locating well toward the outskirts of the city thus requiring increased transportation facilities.

If manually operated substations were to be installed it would not be possible to show an operating credit since only three or four stations at the most would be installed on account of the cost of operating labor. More of the old feeder copper would be required and the saving in copper loss would not be as large. More negative copper would be required to keep the return drop within allowable limits.

The entire equipment should be automatic in operation except the one converter retained in the power plant. One of the converters in one of the loop stations would be set for continuous operation so that night operation would be taken care of. With the heavy amount of feeder copper still remaining, it is doubtful if the voltage drop caused by the night cars would start the outlying substations.

If in practise this assumption should prove not warranted, then it would be necessary to install time switches or pilot wires to prevent these outlying stations from operating at greatly reduced load with consequently poor efficiency during the night.

Many modifications of this layout are possible without materially altering the saving shown. The writer feels that many such interesting problems in automatic control will be met with in the next few years and that its use will spread even faster than the expectations of those who have been following its development. Automatically controlled substations have at this date been applied to railway work, both city and interurban, to Edison three-wire systems, to large industrial plants, to synchronous condenser equipments, and to coal mines.

Heavy steam railroad electrifications have not as yet been tackled although several such jobs have been estimated and it will only be a relatively short time until such an equipment will be installed on one or another of the heavy electrifications.

From an engineering standpoint there seems to be great possibilities in this field.

DISCUSSION ON "SHORT-CIRCUIT PROTECTION FOR D-C. SUBSTATIONS" (LINEBAUGH), "FLASHING OF 60-CYCLE SYNCHRONOUS CONVERTERS AND SOME SUGGESTED REMEDIES" (SMITH), "AUTOMATIC RAILWAY SUBSTATIONS" (PETERS) AND "AUTOMATIC SUBSTATIONS FOR HEAVY CITY SERVICE" (WENSLEY), Pittsburgh, Pa., March 12, 1920.

F. D. Newbury: I wish to discuss the two papers on flashing of converters. While the important results from this study of flashing will undoubtedly appear in improvements in the operation of the converters, one very interesting incidental benefit is the better appreciation of the phenomena of flashing.

Most of us, I am sure, in thinking of flashing, and the relation of speed circuit breakers to the problem, have had the idea that if a breaker could be produced that would open the circuit a little bit quicker than the ordinary types, the problem would be solved. Mr. Smith's oscillograms show conclusively that a moderately high speed breaker may increase the difficulty.

The circuit must be opened very quickly indeed or better not open it at all. Flashing, in a number of instances, in Mr. Smith's tests, did not occur until after the breaker opened even when the circuit remained closed for sufficient time for the current to reach its maximum value and remain there for an appreciable time. The real problem then is the development of very high speed breakers that will limit the current to a non-flashing value.

One reason why high speed is particularly important with the 60-cycle converter is the extreme rapidity with which the current increases on short circuit. If Figs. 7 and 8 of Mr. Linebaugh's paper showing the current increase when a d-c. generator is short-circuited be compared with the oscillograms in Mr. Smith's paper showing the current increase when a 60-cycle converter is short-circuited, the marked difference in rate of current increase will be observed.

If in the case of a 60-cycle converter the circuit increases to a flashing value before the high speed breaker opens sufficiently to limit the current and prevent the formation of conducting gases, then the circuit will be completed by the arc at the commutator and the opening of the breaker will not stop the flashing. This is clearly shown by the high speed camera pictures in Fig. 18 of Mr. Smith's paper.

There is one important point on which the two authors do not agree—this is the effectiveness and advantages of flash barriers. Even granting they are effective, we will all be better satisfied, I believe, if the answer to this flashing problem can be found in some other direction. Barriers that more or less completely box up the brushes greatly increase the difficulty of proper care and maintenance of brushes and commutator and for this reason they are undesirable, in my opinion, from an operating standpoint.

The improvement in performance that comes with the higher reluctance commutating pole has no accompanying disadvantage other than the slightly increased converter cost. It is easy to understand why the higher reluctance commutating pole does improve the converter performance on short circuit. One of the reasons for greater flashing in synchronous converters is the d-c. generator action at the instant of short circuit. Inasmuch as the converter normally requires only 15 to 20 per cent of the commutating-pole ampere turns, as the equivalent d-c. generator the converter is considerably under compensated when it functions as a generator. The use of larger gaps—or equivalent reluctance—and the accompanying additional winding tends to correct this condition.

I am glad to see the matter of feeder taps referred to by Mr. Linebaugh because, after all, that is the most satisfactory way of obtaining the necessary degree of protection in many cases. The loss involved in extended feeder taps is surprisingly small unless the length of tap is carried to an extreme. This of course is due to the small load factor of the ordinary small substation with which this question usually arises. A moderate amount of resistance between the converter and the trolley is good insurance against flashing trouble, that can be obtained at moderate cost.

S. Q. Hayes. I had a very interesting coincidence happen in connection with this meeting today regarding automatic substations. Yesterday I happened to receive from the Brown Boveri Company, in Switzerland, one of their publications for December,1919, and in it they stated that the Brown Boveri Company had been experimenting with a full automatic rotary converter substation at Riehen, near Basle, Switzerland. This supplies the Basle-Lorrach tramway. This is probably the first automatic substation in Europe, and has been working satisfactory, so it is stated.

This station starts automatically in the morning, shutting down at night, and opens and recloses its breakers temporarily in case of open circuit in the primary, or of overload or short circuit in the secondary. In case of a long period of open circuit, an alarm is given relative to the interruption of power.

It is stated that the station operates through a combination of relays and contactors which control various switches. The automatic starting in the morning and shutdown at night is accomplished by a clock of simple construction. They state that they are going to give a more complete description in the course of a couple of months.

C. H. Jones: I will relate a few of the experiences of the Chicago, North Shore & Milwaukee Railway, which is operating automatic substations, and has been operating them for about two years and a half now. We now have five of these stations. Three of them are 500-kw., one 300-kw. and one 1000-kw. stations. They are all in heavy interurban traction service. The car equipment is 47-ton, steel cars, 440-h.p. motors per car, operated in from one to four-car trains.

During the two and one-half years of experience with these stations we have had very fine success and we think very favorably of them, in fact, the operation has been considerably more satisfactory than manually-operated stations ever were. You can absolutely depend on them but there is this point—you cannot expect an automatic station to go on forever without getting some attention. It is useless to think you can put in an automatic station and lock the door and leave it forever and expect satisfactory service. Like every other piece of apparatus it needs careful operation and careful maintenance. The amount of maintenance which should be given varies considerably in the minds of many operators. Personally, I believe that it is absolutely necessary to inspect your station at least once a day. It is also very desirable to know what your station has been doing between the inspection period.

This matter last referred to is one concerning which the manufacturer has not yet seen fit to furnish the operator with means of ascertaining, that is, there is no recording device which indicates what has taken place between the inspection periods provided. I believe that to be absolutely necessary and in order to obtain some such record, we installed in our station quite a number of small mechanical counters connected to the various pieces of apparatus, such as contactors, or controllers, oil switch operating mechanism and things like that.

These counters register every time an operation of a particular piece of apparatus takes place, and by checking up one counter against another in the course of time you will get a very good indication of the general action of your station.

You will find at times that a certain piece of apparatus has failed. The real cause of it you do not know, and as long as it does not repeat itself, you are not particularly interested, there may be a failure at some time or other, but if you continue to get a failure on certain pieces of apparatus, it is time to make a closer inspection and observation of the station, in order to find out what may be causing these minor failures. That has been our case and the counters have paid for themselves many times over, but I believe that the manufacturers will do a great deal toward helping the operators, if they will provide some such device as I have referred to when the apparatus is first installed. The operator will eventually find out he ought to know these things, but it may take him some little time before he discovers it, and if these devices are provided in the first place, it will improve the general operation of the automatic substation equipment.

Up to this time, all of our five stations have been 25-cycle, but we now have on order a couple of 60-cycle equipments. One of these equipments is to change over a present 25-cycle station, and the other one is a new 1000-kw. 60-cycle equipment complete.

With reference to the papers on prevention of flashing in the converter we have had some little experience along that line, not so much in the automatic stations, as in the hand-operated stations. The older type of rotary converter, the non-inter pole type, was not subject to flashover; in fact, we have some old type 500-kw., which to my knowledge, and the knowledge of many of the men who have been with the company longer than I have been, have not been known to flash over. In the case of the new commutating pole rotaries, they are susceptible to the flashing. In the case of the later, 1,000-kw. equipments, we have had them provided with flashing barriers, and I will say that the converter has never flashed over. Many times we have heard them squealing, just as if they were going to flash over, but they never do go over. This occurs principally when the rotary is located right close to the starting point of heavy four-car trains that draw down something like 1000 amperes per car on starting. Rotary converters with barriers have never flashed over even under the above named conditions.

Sometime ago we installed two converters, before the flash barriers were furnished for them, and they were continually flashing over, when operating conditions such as I have stated, occurred. We put flash barriers on them and they have not flashed since, so personally I think a great deal of the flash barriers. Some of my friends do not think so much of them.

David C. Hershberger: In the application of automatic stations we find that the machines have a greater margin in capacity than with present manual control. Cases have come to the speaker's attention where machines are now loaded up to their limit, but by applying automatic control, have a greater leeway in capacity, because they are better able to carry over the peak loads. This is made possible by the use of a load-limiting resistance, whereby the trolley voltage is reduced at the substations which are carrying an excessive load. If adjacent stations are in operation, it results in shifting part of the load to these stations. However, if the adjacent stations are not operating, it merely means that the peak load demanded from the station which is operating is reduced. This is especially true on the interurban systems, where the continous capacity of the machine is not such an important factor as it would be on a city system, so that in that respect it does influence the application of machines, especially as regards sizes.

Another phase which needs consideration is that of inadequate d-c. distribution systems. One instance came to our attention where with manual substation control the system of distribution was inadequate. By re-locating some of the stations and re-distributing the units, it was found that the trolley voltage could be materially improved, thus avoiding the necessity of installing additional feeder. On some installations it is possible to take down copper and affect an economy, but where the operation of trains and rolling stock is so hampered by low voltage, that it would be necessary to put up additional feeder lines, the investment in automatic control is well worthy of investigation.

Still another requirement which has come to our attention is that of controlling auxiliary circuits. Many of the present manually operated stations have a number of auxiliary circuits, such as lighting circuits, circuits for supplying industrial loads and pump motor circuits, etc., that are hand controlled. The automatic control of such circuits has been solved, and they are handled very satisfactorily. For instance, take a circuit where the industrial load is variable, and where there are overloads which would trip the breakers, they can be arranged to close, say three times and then lock out until they have been reset. This practise is followed by some companies in manual operation, and it is just as feasible with automatic operation as it is with manual operation. A pump motor circuit, for instance, can be controlled by a float-type switch. Street and other lighting circuits are now successfully handled by time clocks.

The labor situation touched on by Mr. Peters demands careful consideration. When the system is shut down, due to substation labor trouble, it results in a financial loss. The liability to shut down with automatic stations is less than with manual stations, and therefore the saving which would result from the use of the automatic stations in this way should be credited to them.

C. A. Butcher: Very little has been said of high-tension switching. On the interurban roads there is much competition with the service furnished by steam roads; many of the interurban lines are supplied at the present time through long distance transmission lines, and in a great many cases steam plants are maintained. Often during stormy periods the long distance transmission lines have gone out of service, and it has been necessary to transfer to the steam generating plant and maintain service until the transmission line can be restored. That means that automatic substations operating from these long distance transmission lines are thrown out of service, and unless we have duplicate lines, it is necessary to wait until the steam plant has been put into operation to excite that line.

A number of the larger interurban roads throughout the country are operating on a duplicate system of transmission lines. In some cases they are supplied from separate sources, and in some cases from the same source. In order to take care of the automatic substations on transmission lines, which are paralleled over the right of way, it is necessary to provide some scheme of automatic transfer for the high-tension switch. This has been very effectively taken care of by the addition of a number of relays which adjust themselves to the conditions. If the transmission lines to which the station is normally connected go out of service, a set of relays falls out and tests the other circuit, and if that is in working order the station automatically is immediately transferred to that line, and when the high-tension long distance line is restored to service, it is ordinarily required that these stations be transferred to their normal connection. The same relays when the high-tension line is restored, automatically switch the substations immediately back to that connection.

The automatic service restoring relay system has been in use for a number of years, but it does not give the immediate operation the service companies would like to have. If, for example, a power feeder is knocked off the boards, usually the operator is instructed to wait one minute before he closes the circuit, and after he makes three attempts to fix the circuit and fails, he tries no longer, but reports the outage to the dispatcher and the difficulty is then turned over to the trouble gang, to clear up.

In order to give that same service automatically, there has been developed a motor-operated relay, which automatically closes the circuit breaker periodically. That is, it will close it in one minute, five minutes, or any predetermined length of time, depending upon how the relay is set, and will close it any number of times before the line is definitely locked out of service, and then by some means, either by pilot wires or otherwise, indication is given to the load dispatcher, who immediately gets into touch with the maintenance crew, and the circuit is taken care of. That is in addition to some of the problems already brought up, and must be taken care of in many existing stations, and will undoubtedly have to be taken care of in a great number of new installations.

Donald Bowman: Mr. Wensley in presenting his subject is very optimistic as to the future possibilities of the automatic substation, as applied to low-tension direct-current service in our big cities.

In the operation of the low-tension direct-current substations in our big cities, where everything is underground, and where the total capacity of the system may exceed 100,000 kw. the element of continuity of service is of equal importance with operating costs, because if continuity of service cannot be maintained, the central station will be given a very serious black eye.

In considering the installation of a number of direct-current substations, which will function at different times, according to load, I can readily see how such operation would be satisfactory if everything were carried out as per schedule, but at different times such inadvertent things occur as manhole fires and burning out of units in generating stations and substations, which seriously derange the normal sequence of operations. At those times it becomes necessary to have emergency switching equipment which will operate on both the high-tension and low-tension distribution systems. In the past, these emergency operations have been developed according to the needs of the moment, and have not been arranged as per schedule in advance.

I would like to ask Mr. Wensley to state how some of these unforseen contingencies such as the burning out of units in generating stations and in substations, burning out of transmission lines in manhole fires, and other failures of a similar nature can be successfully handled with the automatic low-tension direct-current substation.

J. F. Tritle: As there were no questions in regard to Mr. Linebaugh's paper, I would like to add a few points in regard to the high-speed circuit breaker, shown in Fig. 3. Mr. Smith in his paper called particular attention to the fact that a moderately high-speed circuit breaker may be even worse than a low-speed circuit breaker. We have found this to be exactly the case in our experiments, tests indicating that a circuit breaker to prevent flash-over, should operate, stop the current rise, and reduce it below the normal flashing value, in something less than the time required for a commutator bar to pass from one brush holder to the next. On 60-cycle machines, this means a speed of something like 0.008 second for the complete cycle.

Most of the oscillograms in Mr. Linebaugh's paper show the results of tests on 2000-kw., 3000-volt motor-generator sets for the Chicago, Milwaukee & St. Paul electrification, which were eight-pole, 514-revolution machines giving approximately 0.015 second for a commutator bar to pass from one brush holder to the next.

The results obtained were so encouraging that the investigation was extended to 600-volt machines. The latest test and experiences indicate that the form of breaker shown in Fig. 3 will quite positively protect up to date 600-volt, 60-cycle synchronous converters. On dead short circuits, the main contact tips, with the breaker adjusted to trip on three times normal load, started to break contact in from 0.002 to 0.003 seconds after the short occurs. The maximum current peak occurs somewhere around 0.006 to 0.007 seconds and we can get the current down to normal or zero in about 0.010 seconds. With the 300-kw. machines the maximum current peak is about 5000 amperes; in the 500-kw. machines around 8000 amperes, and in the 1000-kw. machines approximately 10,000 amperes.

We recently put five dead short circuits on a 1000-kw. 60-cycle converter in a period of approximately six seconds; in other words, only about 1.4 seconds between short circuits. A contactor was used in series with the high-speed circuit breaker for closing the circuit and the high-speed breaker automatically tripped and completely ruptured the circuit similarly to an ordinary circuit breaker, except for the much greater speed of operation.

Our experience also agrees with Mr. Smith's in regard to the tendency towards flashing after a moderately high-speed circuit breaker opens, rather than while the short is on the machine. I remember a series of tests on a 60-cycle machine in which simultaneous oscillograph records were taken of the d-c. and the a-c. current which showed this tendency very clearly. If the short circuit was cleared by the high-speed circuit breaker in less than 0.01 seconds, there was practically no flashing on the commutator and no pulsation on the a-c. end. However, if the short or heavy overload was left on the machine longer than 0.01 seconds, the machine got out of phase a maximum angle in about six or seven cycles after the short circuit occurred and back in phase again in about 17 or 18 cycles. The flashing was materially worse if the circuit was opened by a moderately high-speed breaker while the a-c. current was near its maximum value, in greatest out-of-phase position, than if opened by a relatively low-speed breaker while the a-c. current was near its minimum value for the in-phase position. When in phase no appreciable synchronizing current is required but if out of phase when the load is tripped the synchronizing current is still required. This current excites the commutating pole sufficiently to give high voltage under the brushes which causes severe sparking and flashing.

A complete line of 60-cycle machines has now been developed having high reluctance poles and radial type protective brush rigging. Complete protection against d-c. flash-overs is apparently afforded these machines by the high-speed circuit breaker and as far as we can forsee at the present time, the flash barriers will not be necessary on them.

One of the principal points which I desire to leave with you is that operating companies need have little concern in the future about flashovers from direct-current short circuits on 60-cycle machines, it being principally a question of the kind of protection provided. Mr. Jones has described the results obtained with flash barriers when properly maintained. There seems to be general agreement about the effectiveness of high reluctance commutating poles. Our tests and experience indicate that practical immunity from flashover can be obtained with the high-speed circuit breaker. The breaker is simple, reliable and strong and has been practically standardized for use in connection with our 1500- and 3000-volt d-c. machines. A line of 600-volt breakers is also being developed as rapidly as possible.

L. D. Bale: The steady growth of Cleveland has made it necessary to provide additional cars to take care of the demand for increased transportation facilities.

With the placing of these new cars in service, together with the fact that at present there exists several locations where insufficient trolley voltage is maintained during the critical peak hours to handle the present schedule makes it necessary either to provide additional feeders or substations. To attempt to rectify this low voltage condition and provide for additional cars by means of feeders would involve a heavy expenditure to say nothing of accompanying losses, therefore, additional substations were decided upon, these stations having an aggregate capacity of 12,000 kw. made up of eight 1500-kw. 60-cycle rotary converters.

Taking advantage of the advance in the art, making it possible to operate substation equipment automatically, enables the location of a greater number of smaller stations having smaller distributing areas than would be economically possible if it were necessary to operate such stations manually, therefore, four automatic substations are to be built each containing two 1500-kw. converters respectively. Each of these stations having practically a capacity load within one mile radius.

Aside from the fact that this particular type of 1500-kw. 60-cycle rotary converter has been standardized by The Cleveland Railway Company, two units are to be used in each of these stations rather than one large unit because of the higher efficiency procurable by maintaining the rotary capacity in service at any time, as near as possible to that of the existing load, also with two units in the station instead of one, greater reliability of service may be expected.

The fact that with each of these stations, practically a capacity load originates within its individual distributing zone, taken with the savings made possible through automatic operation, in addition to a much more economical distributing system and the elimination of possible labor disturbances, the elimination of considerable existing feeder copper and its accompanying losses, the better return-circuit conditions obtainable and the determination of the company to eliminate wherever possible the use of manual labor, has been the deciding elements in the adoption of this type of substation of comparatively large size.

As to definite figures in connection with these stations, I should like to wait until such time as at least one or two of the automatic stations are completed and in operation, that we may have an opportunity to secure reliable costs. All data of this nature at the present time are in the form of estimates which under existing conditions should not be quoted.

E. W. Cook: Referring to section of paper under Details of Operation. No provision has been made to prevent the immediate restarting of the machine following shut down by reverse d-c. current even if the trolley voltage is normal and graphic meter records taken on the 1000-kw. substation shown in Fig. 3 of

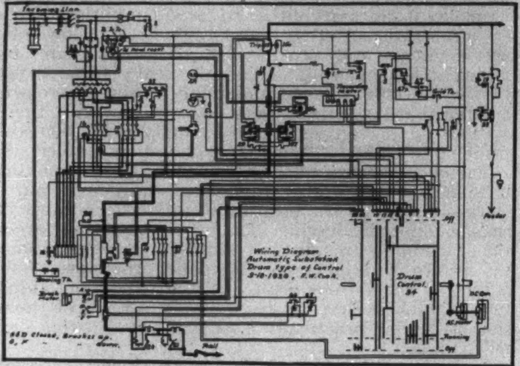

FIG. 1—WIRING DIAGRAM FOR AUTOMATIC RAILWAY SUBSTATION

1. Contact making voltmeter, contact closed on normal trolley voltage.
2. Time-delay starting relay, contact closed when coil is de-energized.
3. Time-delay shutting-down relay, contact opened when coil is energized.
5. Contactor for making control circuit connections.
6. Contactor controlling drum control motor.
7. Three-phase oil circuit breaker.
7a. Interlock closed when oil breaker is closed.
7b. " " " " " " open.
7c. " " " " " " closed.
7d. Hand reset switch in parallel with 7a.
8. 220-volt control bus knife switch.
10. A-C. starting contactor.
11. Control transformer 220-volt bus.
12. Over-speed limit switch.
13. Under-speed control switch.
14. Converter field contactor self-excitation.
15. Main d-c. breaker.
16. A-C. running contactor.
18. Main line d-c. contactor.
19. Feeder load limiting contactors.
20. Converter " " "
21. " " " "
23. Feeder " " relays.
24-25. Converter load limiting relays.
27-27x. A-C. low-voltage relay.
28. A-C. abnormal overload relay (hand reset).
29. D-C. reverse-current relay.
30. Converter shunt field relay.
31. Converter shunt field contactor separate excitation.
32. Single-phase protective relays.
33. Grid resistor thermostats.
34. Motor operated drum controller.
36. Polarized relay.
37. D-C. underload relay.
38. Bearing thermostats.

the paper indicate that under certain load conditions the machine could be "motored out" and restarted on an average of once every two minutes for more than an hour at a time.

This restarting is caused by the time delay, about five minutes, of relay No. 3 which allows the machine to be motored out four or five times before No. 3 can open and it is only necessary to have a momentary load of a few hundred d-c. amperes to reset No. 3 and start the cycle all over again.

In attempting to overcome this feature it was noticed that No 4 control circuit contactor is unnecessary and could be eliminated by transferring its work to No. 5.

Referring to Fig. 1, the sequence of operation would be as follows:

Low d-c. voltage opens No. 1 and after a definite time delay No. 2 closes, energizing drum fingers 11 and 12 and the coil of No. 5 through the contacts of No. 27x, 15a, 29, 38 and 7d; No. 5 closes and through drum segments 1-16 and interlock on brush operating device closes No. 6 which starts drum control motor.

Shortly after the drum starts to revolve segments 18-19 make contact ready to turn drum to "off position" if No. 5 should open, a moment later segment 13 makes contact and 11-12 break contact leaving No. 5 locked closed through its main a-c. contact and contacts of No. 27x, 15a, 29, 38 and 7d; any one of these relays or interlocks could open No. 5 and it would not reclose until the drum is turned to the off position and then only by the closing of No. 2 which normally occurs when No. 1 is opened by low d-c. voltage.

This arrangement prevents restarting after being "motored out" unless normally started by low d-c. voltage.

As the drum continues to revolve segment 15 closes the oil circuit breaker and a little later the holding circuit of No. 5 is transferred from segment 13 to segment 14 through interlock 7c on oil breaker, if 7c fails to close No. 5 is opened, drum is set in "off" position and if d-c. voltage remains low this cycle would continue to be repeated. Assuming normal starting, segment 2 through interlocks of No. 32 and 16 closes No. 10 and starts the converter, a few seconds later the holding circuit of No. 6 is transferred from segment 16 to 20 through underspeed switch No. 13 and the drum motor stopped until converter reaches normal speed when drum motor starts and soon returns the holding circuit of No. 6 to segment 16.

Segment 3 closes No. 31 and fixes the polarity of the converter, segment 4 closes No. 14 making the converter self-excited; No. 31 and 14 are mechanically interlocked.

Segment 5 through interlock on No. 10 closes No. 16 main running contactors, segment 26 lowers the brushes and this transfers the holding circuit of No. 6 from segment 16 to 17 and would stop the drum if brushes are not lowered. Segments 7, 8, 9 and 10 are energized from the positive lead of converter, segment 8 closes No. 18 through interlocks on No. 30, 16, 5 and 36; segments 9 and 10 close resistor shunt contactors; No. 21-20 and segment 17 breaks contact stopping the drum in the running position.

Shutting down by low d-c. load: No. 37 closes and energizes No. 3 from one phase of the converter, after a definite time delay No. 3 opens the holding circuit of No. 27x which opens No. 5 which in turn opens No. 16, 18 and as drum turns to the off position No. 7 is opened, brushes raised and the converter is shut down; No. 3 closes shortly after No. 16 is opened.

By reverse d-c. current: No. 29 opens the holding circuit of No. 5 which shuts down the converter as described under low d-c. load.

By low a-c. voltage: No. 27 shunts the holding coil of No. 27x which opens and in turn shuts the converter down by opening No. 5.

Abnormal a-c. overload: No. 28 trips No. 7 and opens hand reset switch 7d, leaving converter shut down until manually restarted.

Hot resistor grids: Thermostats open holding circuit of No. 27 which opens No. 27x and No. 5 shutting down the converter until thermostats cool and reset.

Hot bearing: Thermostats open No. 5 and converter is shut down until manually restarted.

Additional modifications and equipment: Polarized relay No. 36 should stop the drum until the d-c. voltage is equal to or more than the line voltage; this is very desirable where the control is applied to motor generator sets.

No. 1 and 2 might be combined to reduce the number of relays required and similarly No. 27-27x.

A suitable recording graphic meter is a very necessary part of the equipment; this meter should record on a single strip chart the d-c. amperes and voltage and have pens to indicate when No. 10, 18 and 37 operate.

J. L. Burnham: This paper presents data very similar to that given in a paper on Protection from Flashing for Direct-Current Apparatus presented at the June 1918 annual convention. In that paper it was stated that complete protection from flashing could be secured by the use of the high-speed circuit breaker and flash barriers of a peculiar construction with cooling screens to condense the vapor formed by the arc. It was pointed out at that time that a flash barrier without means for disposing of the conducting vapors formed by an arc would be worse than no barriers. This seems to be completely confirmed in this paper.

Mr. Smith has stated the advantages that were secured by the use of high reluctance commutating poles in a special installation. He also states that the high reluctance was also accompanied by extra damping which may have had something to do with improving the performance under the special conditions described. No mention is made of having tried high reluctance poles on 60-cycle converters for railway service. The writer first pointed to their advantages in a discussion on commutating pole converters in 1910 (TRANS. A. I. E. E. 1910, Vol. XXIX, Part 2, pp. 1663-4). A number of tests were also made in 1918 which showed the great advantage of high reluctance in the commutating pole, particularly when carried to a point where the excitation required is in excess of the direct-current armature reaction. In Fig. 2 an oscillographic record is given showing very plainly what takes place when about six times full load is thrown on and tripped off with a breaker of ordinary speed. From the values of current on this oscillogram the excitation on the commutating pole, resulting from its field windings and armature reactions are plotted, Fig. 3, for com-

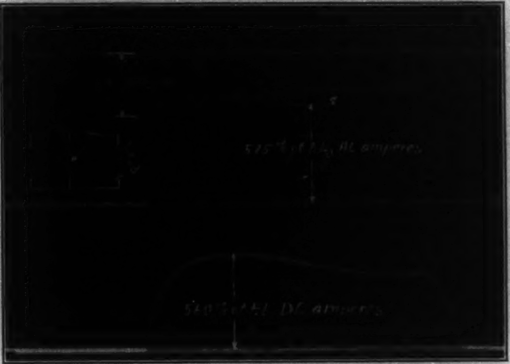

FIG. 2

parison to what is actually required for best commutation. The percentage departure from the correct value, with the usual all steel commutating pole is about 70 per cent as compared to about 20 per cent for the high reluctance commutating pole with a large portion of non-magnetic material next to the magnet frame. As usually short circuits may be several times greater than this load it will be seen that the commutating field might actually be reversed with the all magnetic pole, whereas it would be changed only in strength with the high reluctance pole.

A comparison by test of the loads to cause flashing under similar conditions with the all magnetic and

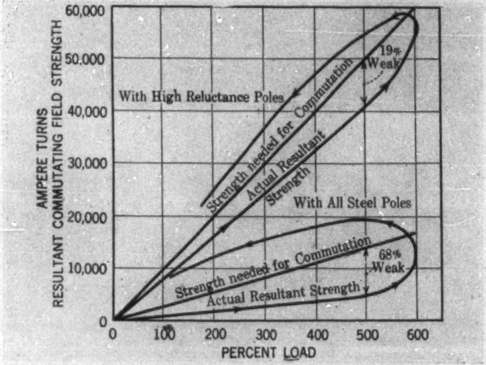

Fig. 3—Transient Effect of Armature Reaction in a Synchronous Convertor During Short Circuit

with high reluctance poles show that the latter will increase the amount of short-circuited load about 50 per cent before flashing occurs.

Tests that have been made with high-speed breaker and high reluctance commutating pole on 600-volt, 60-cycle converters show that complete protection against any amount of d-c. short circuit is obtained.

From Mr. Smith's paper I understand that he has not been able to obtain complete protection with the high-speed breaker. From the figures he gives on speed I believe his results are due to the breaker not being fast enough and also to the fact that all steel commutating poles are used. Mr. Smith has given the factors which cause flashing as "The inherent commutating characteristics of the machine, the value of the load current and the time it has been flowing, the grade of brush, etc." With the use of high reluctance commutating poles I have found that the inherent commutating characteristics of the machine under short circuit are decidedly improved for reasons given above and also that the effect of saturation is reduced. With the high-speed breaker the time the excessive load is on the machine should be sufficiently short to avoid an appreciable formation of gas or volatile matter between neutral points on the commutator and to cause a minimum pulsation. These factors have been kept in mind in the development of a line of 60-cycle railway converters designed by the writer in the last two years. The complete line is now being built with high reluctance poles and several sizes are in operation. For complete protection from flashing of this line of machines high-speed breakers which will completely open the circuit in 0.01 second from the beginning of short circuit are available. Still higher speed (0.006 to 0.007 of a second) is obtained by connecting across the high-speed breaker a resistance which will limit the current to a value easily handled by the converter.

The action of the so-called suppressor described in the paper is interesting in that protection under certain conditions may be obtained without seriously increasing the short-circuit current on the a-c. side. However, the same a-c. short circuit is applied to the machine by the suppressor regardless of the value of the d-c. short circuit which may be only a moderate overload. Mr. Smith states that this scheme is good for only a few limited combinations of load and is, therefore, not applicable for protection in commercial work.

Marvin W. Smith: I agree with Mr. Trittle that the best solution for flashing seems to be in the direction of the high-speed breaker, but we have found, in order to give complete protection, that the breaker must be of very high speed. We found, however, several cases where flashing did not occur before the opening of the breaker, even though the commutator had moved a sufficient distance for the bar to pass from one brush arm to the other. The fact is mentioned in connection with Fig. 1, where reference is made to the flash suppressor, in which the short circuit on the a-c. side was delayed even greater than one cycle. During this time the commutator bar passed a distance of two brush arms, and the machine did not flash over during that time, but does flash over at the opening of the d-c. breaker. There is no question but what, as Mr. Trittle has mentioned, there is considerable advantage in the higher reluctance commutating pole, and we had rather see the problem solved in that way, than by flash barriers or some other means which obstruct the commutator and prevent accessibility.

F. W. Peters: The economies made possible by the use of automatic substations in concentrated service have been presented very effectively by Mr. Wensley. In this connection he spoke of the recent Cleveland installation, or rather the decision to install this large concentrated converter capacity, automatically operated, in the heart of the city. In the past, the majority of automatic applications have been in conjunction with interurban substations where the capacities are relatively small and a correspondingly low cost of automatic devices could return, by labor saving, an appreciable amount on the capital invested. When we deal, however, with city service involving several large units in a station, the initial automatic equipment cost is increased at a rate much greater than the corresponding saving in labor and, therefore, it becomes difficult to determine the economy of such an installation. In view of this fact, I would request Mr. Wensley to enlarge upon this point and bring out just what is attracting the managers of concentrated city service to adopt automatic operation in large stations, and I have in mind particularly the Cleveland Railway Substation involving 3—1500-kw. automatically operated converters.

The question of using automatic equipment in industrial applications, particularly of direct current, is a point well taken and you gentlemen no doubt are familiar with cases where large factories have their generating units in one particular portion of the plant from which power is distributed in a very uneconomical and cumbersome way, by the use of heavy bus copper. It appears that by distributing the conversion units around a factory, direct improvement may be brought about from an efficiency standpoint and also by the elimination of extensive bus layout, and disadvantage encountered due to concentration of power at one point where a short circuit tends to pull out everything.

It is very gratifying to hear what Mr. Jones, Engineer of the Chicago, North Shore and Milwaukee Railway has to say about the operation of automatic substations. He has had considerable experience with this type of station and I am sure we could report still more progress were all engineers equally interested and painstaking. The exhaustive records of automatic substation operation as obtained by Mr. Jones are very valuable and have at all times been open to review by manufacturers for the purpose of making modifications and improvements wherever possible.

Mr. Jones' comments on flash barriers are very well taken and it brings this to my mind—flash barriers, from their very construction, show that they depend largely for their success upon careful adjustment, that is, protection from flashing is obtained by diverting the arc in such a way that it will be extinguished without damaging the equipment. Naturally care must be taken to properly guide the arc in a safe path. In certain cases, flash barriers have been installed and even automatic stations have been started where the idea seemed to be "now I have it, lets lock it up and let it run." That fortunately is not always the case and where a reasonable amount of maintenance has been given to the apparatus, it has, of course, resulted in a much more satisfactory operation than in those cases where the equipments have been neglected. Sometimes operators have not taken the trouble to thoroughly understand the principle on which the automatic equipments function.

Mr. Hays spoke of an automatic equipment brought out by the Brown, Boveri Company, and I am sure it will be interesting to read the details of their arrangement at such time as we have that privilege. Europe is a little slow in so far as automatics are concerned as evidenced by the fact that Australia has had several equipments in successful operation for sometime and a complete automatic equipment is now in transit to New Zealand.

Mr. Cook in his discussion presents a condition of false "motoring out" which the more modern equipments have been modified to protect against. In so far as combining in one device the functions as now performed by two or more devices, we encounter the limitations as determined by manufacturing conditions where it is often much cheaper to use a combination of devices already developed, than to design a new piece of apparatus whose limited production would necessitate higher costs.

The flexibility of automatic substations has been emphasized by Mr. Butcher, and in no case has it been impossible to meet practical operating conditions. Frequently local arrangement of the distribution or operation necessitates treatment as a special case, and progress, if compared to the complications of other automatic applications such as steel mill and automatic operation of car equipments, should not be considered as having reached its limits, in spite of the many accomplishments of the last few years.

R. J. Wensley: Taking up Mr. Bowman's questions first, regarding continuity of service in large Edison 3-wire systems, and heavy city service, this is a real problem and Mr. Bowman has touched on one of the biggest problems that the automatic substation has to meet. We have, at the present time, as part of standard production, a well-developed line of selective power relays. We can do almost anything in the way of sectionalizing a system by eliminating bad feeders, thus stopping back flow of current, and clearing out from any tie bus any set of feeders that may be giving operating trouble. We can feed into heavy city substations through several different tie buses, from several different directions. We can by the use of selective reverse-power relays, definitely clear off of these buses, feeders which may be giving trouble, so as to leave uninterrupted the power supply on these buses.

This, of course, does not cover all possible cases. There are operating contingencies that come up that cannot be foreseen.

The system with which Mr. Bowman is connected, the Commonwealth Edison Company of Chicago, probably has the most complicated Edison network in the world. I do not think there is any other system that has the same kilowatts in converting and battery capacity on one bus. Such a system could not be made fully automatic in the true sense of the word. They have a load dispatcher. There is no particular reason why the principle of the automatic telephone cannot be applied to such an emergency, so that the load dispatcher, instead of having a telephone report made to him of the condition of a circuit breaker, can have full control over the circuit breaker from his own office. He can press a button and close the circuit better than an operator half a mile down the street. There are special means in automatic substations to enable large Edison systems to go back into service after total outage which are much superior to manual operation. Of course, these same features can be applied to manual stations, but they are inherently a part of the automatic station. I refer to the load limiting resistances and the resistance schemes of getting back and establishing service.

Suppose they have an extensive and disastrous feeder burnout, and the entire Edison network is put out of commission. It is almost impossible for any one station, or any one group of stations to take that load up. It practically becomes necessary to get out the entire operating force and send them out through the town to pull the main switches of all big buildings, before service can be restored.

With the automatic station, we have load limiting resistances, and cannot take more than load and one-half on the converter. After an outage the load dispatcher can restore service by starting all stations as nearly as possible at the same time. The load limiting resistance prevents overloading of the first converters thrown on the bus.

The automatic control is much quicker and more positive than any operator. The automatic control can supply power while the operator is deciding which switch to throw. This is not merely talk. It has been done time and again. I have seen the railway type of automatic control on 300-kw. non-interpole converter put on the line in 12 seconds. The operator cannot think that quick, or cannot throw switches that quick. I do not mean to say that we can take a 2000-kw. booster type of converter and do that. We can take a system of the type Mr. Bowman represents and install automatic control, and if the load dispatcher attacks the problem intelligently, I will guarantee that three minutes after the power supply is resumed the Edison system will be in full operation. This is in sharp contrast to a period of three hours as certain instances have shown are required to put the system back where they do not have automatic operation.

Just a word regarding flash-over protection and automatic substations. So far in the railway substations we have not gone into the high-speed breaker as part of our automatic control, to any great extent. We have talked about it, and in a few cases tried them out, but the average automatic substation today is not operating with high-speed breakers. We are operating with what is decidedly a low-speed breaker. It is only a contactor having no accelerating spring and no triggers or tripping devices. It falls open by gravity, assisted by the pressure of the contact. The overload relay is of the low-speed sort, and no attempt is made to get high-speed operation.

We limit the short-circuit current by running feeder taps out to the furthest convenient distance from the station. The short-circuit current dies down to some extent, before the first contactor opens. This contactor does not open until a good many hundredths of a second after the short circuit first strikes the machine. Then, instead of suddenly opening the circuit thus allowing abnormal rise in voltage between commutator bars we insert a single section of the load limiting resistance and limit the short-circuit to four or five times the machine rating. The next step then cuts in, and then a third step cuts in, thus limiting the current to about load and one-half. The statement has been made by both of the authors of the papers on flashing that the flash-over in most cases occurs after the breaker opens. Since we do not open any breaker, but instead reduce the current gradually we do not expect much trouble from flashing.

In this way we hope in automatic stations to get around the necessity for employing the complicated high-speed breakers, and have confined ourselves to the most simple form of contactor.

*Presented at the 358th meeting of the American
Institute of Electrical Engineers,
Pittsburgh, Pa., March 12, 1920.*

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THE BALDWIN-WESTINGHOUSE CHICAGO, MILWAUKEE & ST. PAUL ELECTRIC LOCOMOTIVES

BY N. W. STORER
General Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

REQUIREMENTS

IN the summer of 1917, the Chicago, Milwaukee & St. Paul Railway Company issued specifications covering the apparatus necessary for the electrification of their line from Othello to Seattle and Tacoma. The following were the principal requirements for passenger locomotives as finally determined by the Railway Company:

Power Supply. To be 3000 volts direct-current.

Route. Either the line between Harlowton and Avery, or the newly electrified line from Othello to Tacoma and Seattle.

Grades. The limiting grades are 2 per cent compensated for a distance of 20 miles from Piedmont to Donald on the east slope of the Rocky Mountains, and 2.2 per cent compensated for 17.8 miles from Beverly Junction to Boylston on the western division.

Alignment. The sharpest curve on the main line is 10 deg., but the locomotive must negotiate a 16 deg. curve in the yards satisfactorily.

Load. Twelve steel coaches weighing 950 tons.

Speed. The locomotive to be designed for a speed of approximately 25 mi. per hr. up a 2 per cent grade, about 35 mi. per hr. on 1 per cent grade, and to have a maximum speed of 65 mi. per hr.

Mechanical Design. The locomotive to have a four-wheel guiding truck at each end, of the "Woodard" type.

The driving wheels to be not less than 60 in. in diameter.

Regeneration. Trains to be held on down grades by regenerative braking.

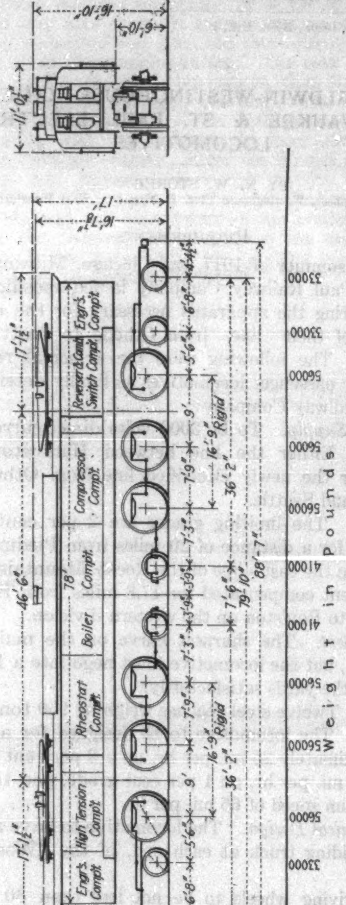
16'-10"
9'-10"
11'-0½"
17'
16'-7⅛"
17'-1½"
46'-6"
78'
17'-1½"
14'-6"
15'-9"
Engr's Compt
Reverser & Comb. Switch Compt
Compressor Compt
Compt
Boiler
Rheostat Compt
High Tension Compt
Engr's Compt
4'-4½"
6'-8"
5'-6"
9'
7'-9"
7'-3"
3'-9"
3'-9"
7'-3"
7'-9"
9'
5'-6"
6'-8"
4'-4½"
16'-9" Rigid
36'-2"
7'-6"
16'-9" Rigid
36'-2"
79'-10"
88'-7"
33000
33000
56000
56000
56000
41000
41000
56000
56000
56000
33000
33000
Weight in Pounds

FIG. 2—OUTLINE SHOWING EQUALIZATION SCHEME, WEIGHT DISTRIBUTION AND WHEEL BASE

Train Lighting. Current to be supplied from the locomotive for lighting the train and charging the train storage batteries at a voltage of 60 to 85.

Train Heating. The locomotive to be equipped with an oil-fired boiler, to be supplied by the Railway Company, to furnish steam for heating the train.

Storage capacity to be provided for 30,000 lb. of water and 750 gallons of fuel oil.

Thermostats. Thermostats to be provided that will automatically start the blowers for cooling the main motors when the motor temperature reaches a predetermined value, in order to limit their operation to that time when they are actually needed.

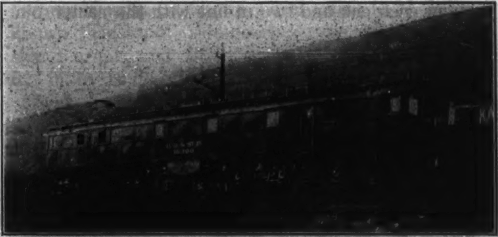

Fig. 1—Baldwin-Westinghouse 3000-Volt D-C. 275-Ton Passenger Locomotive for Chicago, Milwaukee & St. Paul Railway Co.

DESCRIPTION

A portion of the order for locomotives was secured by the Westinghouse Electric & Manufacturing Company, and this paper covers the brief description of the Baldwin-Westinghouse locomotive as furnished.

General. A view of the complete locomotive is shown in Fig. 1, and a diagram showing the wheel arrangement, axle loading and equalization, in Fig. 2.

The locomotive is built in a single unit, having one long cab, carried on running gear of the 4-6-2 – 2-6-4 type.

The locomotive weighs 275 tons.

Main Running Gear. The running gear consists essentially of two Pacific type running gears, coupled back to back. One-half of the running gear is shown in Fig. 3. This shows the running gear complete with the motors mounted and the air conduit carried on top of the motors.

The side frames are steel castings, joined over the four-wheel trucks by a heavy "A" frame casting; also by heavy cross-ties between the drivers which also support the motors, carry the center pin and carry the coupling between the two running gears.

Each half running gear has six spring-supported plungers on which the cab rests. There are two supports at each end and two in line with the center pin. By the use of shims, the distribution of weight between the two ends of each running gear may be adjusted as desired.

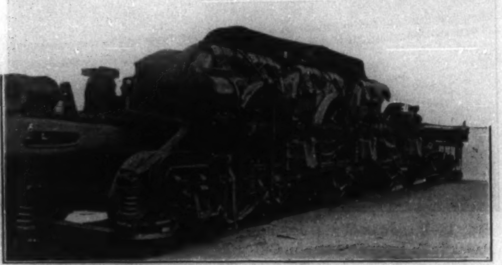

FIG. 3—ONE-HALF LOCOMOTIVE RUNNING GEAR, SHOWING DRIVING MOTORS, COUPLING BAR AND AIR CONDUIT

The equalization, as shown in Fig. 2, is of the standard three-point type; the leading bogie being cross equalized with the leading pair of drivers and the pony axle being equalized with the two adjacent driving axles on the sides.

Extra points are provided in the equalizing levers so that practically any distribution of weight that is desired can be attained.

The driving wheels are 68 in. in diameter; the journals 8½ in. by 14½ in., located outside of the wheels.

The drawbars, with Minor fraction draft gears, are carried in the "A" frame casting, previously mentioned.

The coupling between halves of the running gear consists of a long bar of a box section. This is shown in Fig. 3. The coupling pins are 10 in. in diameter and are located well inside the pony axles. The pins are hollow, filled with oil-soaked waste and have oil holes provided that keep the pins well lubricated.

Bogie Trucks. The four-wheel trucks are of the "Woodard" type with outside journals; 36-in. rolled steel wheels and cast steel side frames. The journals are 6½ in. by 14 in.

Pony Trucks. The two-wheel truck is of the well known "Rushton" side-bearing type, also with outside journals and 36-in. wheels. The journals are 6½ in. by 14 in.

Brakes. Brake shoes are provided on all drivers. A modified form of the 14-EL Westinghouse Air-Brake Company equipment is used.

Cab. The cab is 78 ft. 0 in. long, 10 ft. 2 in. wide; is strong and rigid so that it can be lifted at the ends. The main strength lies in the two bridge girders extending from end to end. The heavy cross-braces and the side members and top of the raised deck down the middle of the cab, form a construction that is light, but stiff. The cab is divided by cross partitions into compartments, one at each end for the engineer, and the others for the various parts of the cab equipment.

Locomotive Capacity. The total motor rating of the locomotive is 4200 h. p. on the one-hour basis. The continuous rating is 3400 h. p. The tractive effort and the speed are given on the nameplate of the locomotive as follows:

Rating	Tractive effort		Speed	
	Full field	Short field	Full field	Short field
1-hour rating............	66,000	57,000	23.8	27.2
Continuous rating.......	49,000	40,800	26.0	30.4
Weight on drivers	168 tons			

ELECTRICAL EQUIPMENT

Driving Motors. The six driving motors are of the twin armature type. See Fig. 4. Both armatures are contained in a single frame, arranged to secure the maximum economy of weight. The fields are of the standard four-pole type with four salient poles and four inter-poles for each armature. There are brush-arms on each commutator which are easily accessible.

Each armature is wound for 750 volts, but the two armatures and the two sets of field windings are connected permanently in series so that the rating of the complete motor is based on 1500 volts.

FIG. 4—700-H. P. 1500/3000-VOLT D-C. TWIN ARMATURE MOTOR

The motor is designed for field control by means of inductive shunts.

The one hour rating is 700 h. p. The continuous rating is 567 h. p. with forced ventilation and 400 h. p. without blowers.

The characteristic curves, when running as a series motor, are shown in Fig. 5. Curve sheet Fig. 6 shows the performance when regenerating.

Quill Drive. The motors are mounted rigidly on the cross ties of the running gear, one directly above each driving axle. Each motor is geared to a quill centered in bearings in the motor frame and surrounding the driving axle with a clearance all around when axle and quill are concentric of 1¾ in. The quill is connected to the drive wheels by long helical springs which are clamped rigidly at the ends in castings which are bolted one to the quill flange and the other to the drive wheel. There are seven springs at each end, worked in compression in one wheel, while those in the other are in tension. All springs with clamps are interchangeable. They are easily accessible for inspection, and any spring may be removed without disturbing any other part of the running gear.

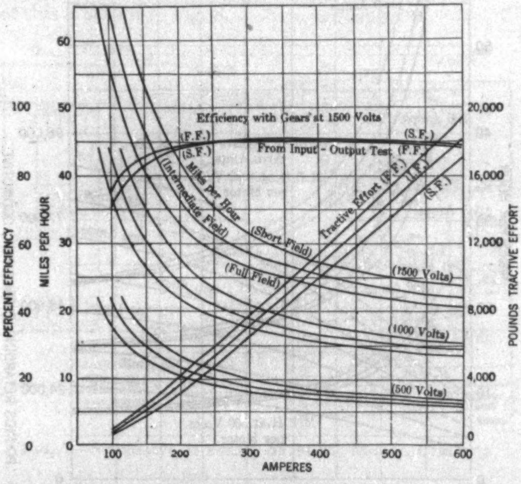

FIG. 5—MOTORING PERFORMANCE CURVE OF DRIVING MOTOR AT 1500, 1000 AND 500 VOLTS

This drive is similar to the well-known quill drive of the geared locomotives of the New York, New Haven & Hartford Railway.

Main Motor Control. A schematic diagram of the main motor circuits with starting resistances and all stabilizing resistance and exciters for regenerative braking is shown in Fig. 7. The six motors are arranged to be connected in three combinations:

1. All in series, giving one-third speed.

2. Three in series, two in parallel, giving two-thirds speed.

3. Two in series, three in parallel, giving full speed.

Inductive shunts are applied to the fields on all three of these positions.

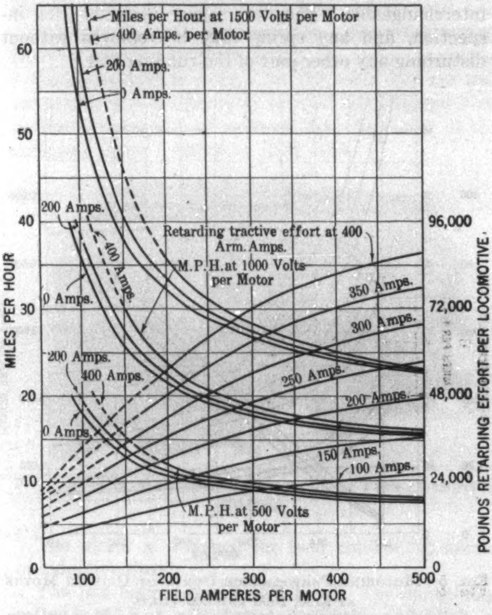

Fig. 6—Regenerating Performance Curves of Locomotive with 3000 Volts on Line. Speeds with First, Second and Third Motor Combinations. Current Shown is Current per Motor

Shunt transition is used in passing from one combination to the next.

Regenerative Braking. When regenerating, the motors are separately excited from two axle-driven generators which are carried on the inside axles of the two four-wheel trucks and geared to them like ordinary interurban railway motors. These generators are separately excited and the field strength of the main motors is controlled by varying the fields of the exciters.

The scheme that is used for regeneration includes the use of stabilizing resistance, which is connected in series with the exciter armature, main motor field circuit, and also with the main motor armature circuit, so that the field excitation is dependent, to a certain extent, on the armature current. A schematic diagram of this is shown in Fig. 8.

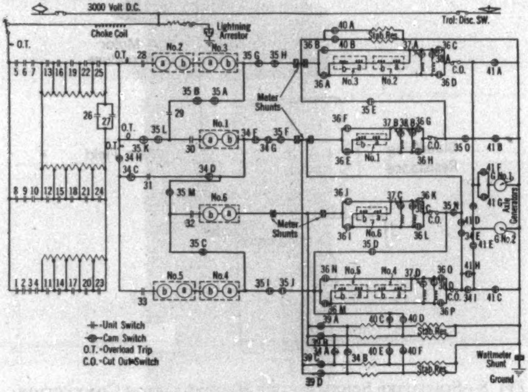
3000 Volt D.C.
Choke Coil
Lightning Arrestor
Trol: Disc. SW.
O.T.
5 6 7 13 16 19 22 25
26 27
8 9 10 12 15 18 21 24
1 2 3 4 11 14 17 20 23
O.T. 28
No.2
No.3
a b
35 G 35 H
35 B 35 A
29
Meter Shunts
O.T. 35 L 35 K
No.1
30
34 F 35 F 34 G
34 H
34 C 31 34 D
35 M
No.6
32
35 C
No.5 No.4
33
35 I 35 J
40 A 40 B Stab.Res.
36 B 37 A 36 C 38 A
36 A 36 D
C.O.
41 A
35 E
36 F 37 B 38 B 36 G
36 E 36 H
35 O 41 B
41 F G No.1 41 G
Axle Generators
G No.2
36 J 37 C 36 K 38 C
36 I 36 L
35 N 41 D 34 E 41 E
35 D
36 N 37 D 36 O 38 D
36 M 36 P
41 H 34 I 41 C
39 A 40 C 40 D
39 B 34 A 34 B 40 E 40 F
39 C 39 D
Wattmeter Shunt
Ground
-Unit Switch
-Cam Switch
O.T.-Overload Trip
C.O.-Cut Out Switch

FIG. 7—SCHEMATIC DIAGRAM OF MAIN MOTOR CIRCUITS

Master Controller. The master controller is shown in Figs. 9 and 10. It has four control drums and four operating handles.

1. The speed drum, which controls the resistance switches and line switches; field shunts during motoring and the exciter voltage during regeneration.
2. The reverser drum, which performs the usual function.
3. The motor combination drum, which has three positions, each corresponding to one of the three motor combinations.
4. The regenerative drum, which changes the connections from motoring to regenerating and vice-versa.

The master controller is arranged so that the controller may be thrown from the "off" position to the second or third speed combination, if desired, without passing through the lower combinations.

The method of operation is as follows:

To Start the Train. Motor combination lever in first speed position; regenerative lever in motoring position; the reverse lever in forward position.

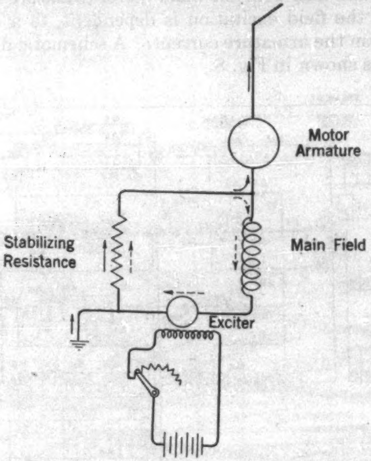

FIG 8—SIMPLIFIED SCHEMATIC OF REGENERATION CONNECTION

Move the speed lever to the first notch and gradually notch up until the train starts and then continue until the resistance is all cut out and the train is running with the motor field shunted in the first motor combination.

Then throw the motor combination lever to the second position; back off the speed handle to the first position (at the same time pressing a button in the end of the lever so that it cannot pass the first position). At that point, with the resistance all in circuit and the three legs of resistance in parallel, the change-over takes place; one-half of the motors being shunted momentarily and then thrown in parallel with the other half. The resistance is then notched out as before and the motor-fields shunted.

Then the motor combination handle is thrown to the third position and the speed handle backed to the fourth notch when the change is made from the second to the third combination, and the resistance speed handle again notched forward. With the resistance

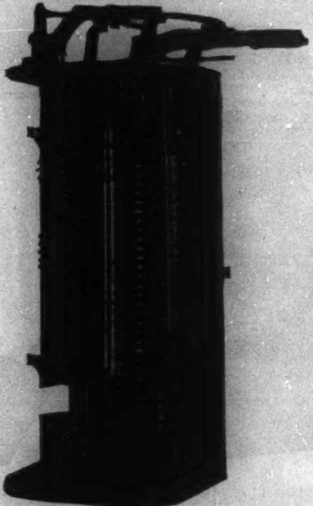

Fig. 9—Master Controller with Cover Removed, Showing Drums for Speed Control, Reversing, Motor Combinations and Regeneration

all cut out, the motors may be run with full field or first or second shunt.

The engine-man is prevented from accidentally starting the locomotive from rest, in the second or third combination, by an interlock in the controller which prevents the line switches from closing when the motor combination lever is in the second or third position, unless he pushes a button in the top of the master controller.

It is, therefore, necessary when applying current with the locomotive at speed, to press the button when it is desired to go immediately into the second or third combination.

There are thirteen resistance steps, all of which are available in both first and second speed combinations. On the third combination, there are ten steps that are useful; making a total of thirty-six resistance steps and two field shunting notches in each combination, or a total of forty-two steps in the master controller. The ease of manipulation of this arrangement of master controller has already been established in service.

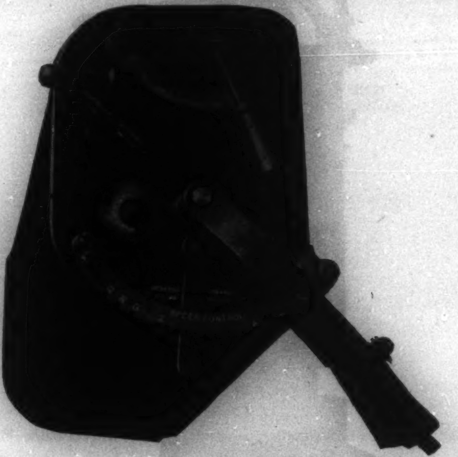

Fig. 10—Top View of Master Controller

Main Resistance. The main resistance is arranged in three groups, each of which has five switches, each short-circuiting a section of resistance. These three groups are connected all in series for the first combination, and three in parallel for the second and third combinations. The sequence of closing the resistance switches is the same on all combinations. The resistances are always connected in series before the master controller reaches the "off" position.

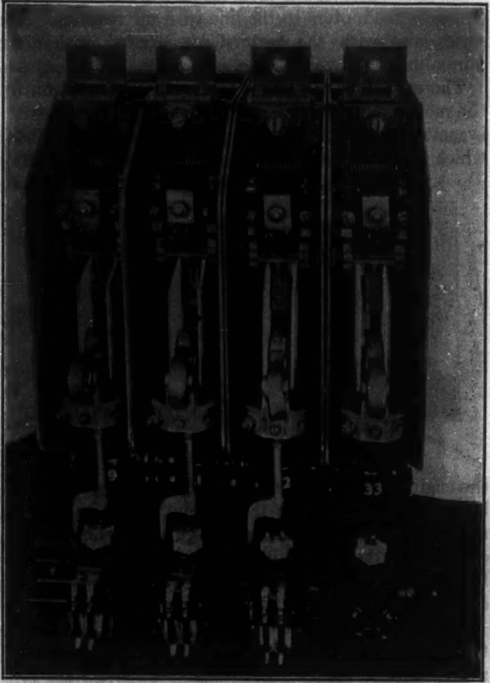

Fig. 11—Rear View of Group of Four High-Voltage Unit Switches

Overload Trips. The overload trips are arranged to open the resistance switches and insert the entire resistance in series before the line switches are opened.

Regeneration. The operation to begin regeneration is as follows:

1. Throw the regenerative handle to the regenerative position on the master controller.

2. Place the motor combination handle in position corresponding to the speed of the locomotive.

3. Move the speed lever to the first step. This connects the motors to the line with all resistance in series and with the motor-fields excited with the minimum exciter voltage.

The exciter voltage is automatically increased until the motor voltage equals the line when the speed lever is moved to the sixteenth notch of the controller, after which the exciter voltage and speed can be changed by the engine-man by moving the speed handle alternately back and forth between the sixteenth and the seventeenth notches, if the exciter voltage is to be decreased; or between the sixteenth and the fifteenth, if it is to be increased. The motor-operated rheostat will move one step for each time the controller is moved.

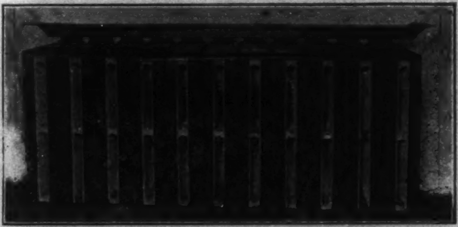

FIG. 12—FRONT VIEW OF UNIT SWITCH GROUP

To change the motor combination during regeneration, throw the speed handle to the "off" position; move motor lever to the new position and start regeneration as before.

Unit Switches. These are shown in Figs. 11 and 12, and are indicated on the diagram by two vertical dashes. The switches are numbered from "1" to "33". The switch is provided with a very powerful magnetic blowout with arc chutes of arc-resisting material, and an arc splitter. It is electro-pneumatically operated and is designed so that it is very thoroughly protected from insulation troubles and for ease of inspection and over-hauling.

Cam Switches. The cam switch groups are shown in Figs. 13 and 14, and indicated on the diagram by two vertical dashes enclosed in a circle. The switches are designated by numbers and letters. The number corresponds to the group in which the switch is placed and the letter corresponds to the particular switch in the group.

Those of Nos. 37 and 38, which are the groups operating the field shunts, also Nos. 39 and 40 which control the stabilizing resistance during regeneration, are provided with magnetic blowout; but the switches for the reverser, which never close or open the circuit with current on, have no arc chutes or blow-outs. Groups Nos. 34 and 35, which make the different motor combinations, have barriers placed between adjacent switches which have a large difference in potential, but no magnetic blow-out, since these switches are never used for opening the circuit under load. The groups are operated electro-pneumatically.

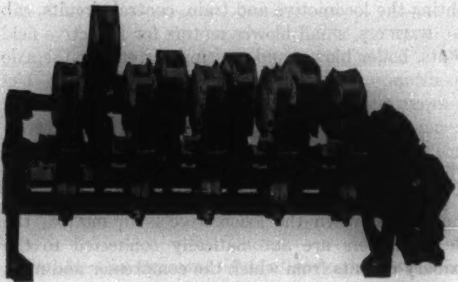

FIG. 13—CAM OPERATED SWITCH GROUP WITH MAGNETIC BLOWOUTS

Auxiliaries. Power for the auxiliary motors, control circuits, train lighting, motor excitation, etc., is derived from three sources:

1. *Motor-Generator.* (See Fig. 16). The motor receives current from the line; the generator delivers

current at a constant voltage of 85. This is required primarily for train lighting.

2. *Storage Battery.* Of the MV-25 ironclad Exide type containing 38 cells. This battery has a capacity of 300 amperes for approximately one hour.

3. *Two Axle-driven Generators.* Which are designed primarily to furnish current for exciting the main motors during regeneration.

The storage battery is always available to supply current for locomotive lights, control circuits, and the air-compressor motor for short periods. When the locomotive is in service, the generator of the motor-generator set is always in parallel with the battery, and from this dual source is always taken the power for lighting the locomotive and train, control circuits, cab floor-warmers, small blower motors for inductive field shunts, boiler blower and exciting current for the axle generators. In addition, the current for the air-compressor motor is supplied from this dual service whenever the locomotive is standing still or regenerating.

The axle generators are, as stated, used primarily for exciting the main motors during regeneration. At other times, when the locomotive is in motion, the axle generators are automatically connected to the auxiliary circuits from which the compressor and main blower motors take power. At this time the voltage is automatically maintained at 90.

Air-Compressor. A two-stage compressor with inter-cooler, having a displacement of 150 cubic feet per minute, is provided. It is a double-acting, upright type, driven by an industrial type motor.

Blowers. Two blowers are provided to supply forced ventilation to the driving motors. They are driven by motors that are duplicates of the compressor motor.

Auxiliary Control. The axle generator circuits are shifted from one circuit to another by a group of the same type of cam operated switches as are used in the main motor circuits.

The field rheostats for the generator of the motor-generator set and the axle generators are operated by small motors. As the fields of the two axle generators are connected in series, only one rheostat is required for them.

The switches for controlling the blower and compressor motors and the high-voltage motor are magnetically operated.

The motor of the motor-generator set is protected from overload or short circuit by a small permanent resistance in series and a set of three expulsion type fuses which blow in series, the first two inserting additional resistance in the circuit, and the last opening only after a relatively high resistance has been inserted and the current consequently limited to a low value.

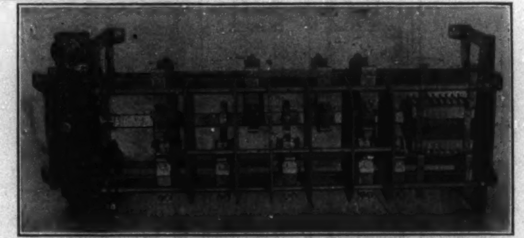

FIG. 14—CAM OPERATED SWITCH GROUP WITH BARRIERS

Meters. A Sangamo wattmeter is provided which has separate dials for integrating the motoring watts and regenerating watts. A full set of motor ammeters is provided, and also a voltmeter reading line volts. An ampere-hour meter is provided for the storage battery.

Train-Heating Plant. A boiler with a capacity for evaporating 4000 lb. of water per hour is provided for heating the train and engine-man's cab. The boiler and water storage tanks are located in the middle of the cab and occupy a relatively large proportion of the space. The tank for fuel oil is located directly beneath the boiler, and the water tanks, two in number, just fore and aft of the boiler.

Pantagraphs. Two pantagraphs of the double slid-

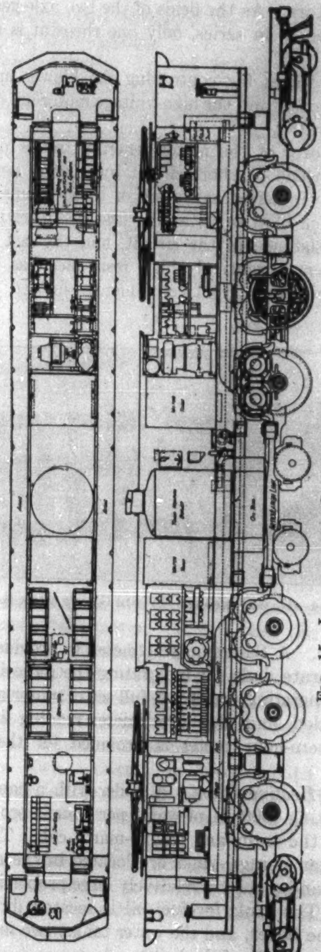

FIG. 15—LONGITUDINAL SECTION OF LOCOMOTIVE

ing shoe type are provided. These are raised by air and lowered by gravity.

Wiring. All cable is in steel conduit, and all exposed connections are solid copper bar or strap.

Mechanical Design. This locomotive has been designed to possess certain mechanical features which have developed through years of experience with steam locomotives. These characteristics are believed to be advantageous when applied to electric locomotive design as well. They may be summarized about as follows:

1. Distribution of weight, fore and aft, with respect to wheel base.
2. Height of center of gravity.
3. Length of rigid wheel base.
4. Guiding trucks.
5. Equalization.
6. Coupling between truck frames.
7. Diameter of driving wheels.
8. Non-spring-borne weight on driving axles, etc.

The principal means that contribute to good operation are:

1. The location of the mass of the locomotive as close to the center as possible.
2. High center of gravity of that portion of the locomotive carried on the running gear.
3. A long rigid wheel base to insure stability and also to decrease the transfer of weight due to drawbar pull.
4. Guiding trucks, located well outside the mass of the running gear, to contribute to stability, as well as leading the locomotive into curves.
5. Proper equalization to secure stability and weight distribution.
6. The minimum permissible restraint to free movement of each axle and portion of the running gear.
7. Large diameter of drivers and low "dead" weight on axles.

In designing a locomotive cab, it is very advantageous to concentrate the heavy equipment between the center pins. A good rule is to have the radius of gyration with the cab swinging from one center pin, shorter than the distance between center pins.

In the running gear, it is desirable to have the radius of gyration measured from the center pin, less than half the wheel base. The closer the weight is concentrated to the center pin, the less tendency there is to nose and the easier the duty on the wheel flanges and track. The steam locomotive seldom has a good fore and aft distribution of weight, but its high center of gravity compensates for it.

As concerns the size of drivers and dead weights, it may in general be said that, other things being equal, the effect on track is inversely proportional to the square of the wheel diameter, and directly proportional to the dead weight.

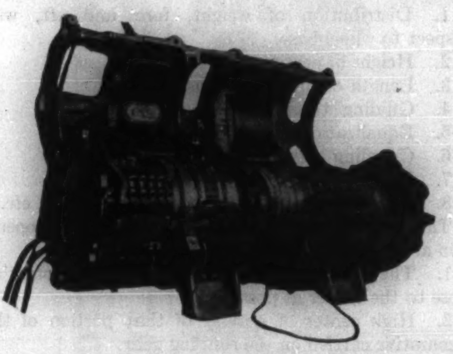

FIG. 16—3000-VOLT-85-VOLT MOTOR-GENERATOR SET—TOP HALF OPENED

This locomotive embodies all the foregoing features to a remarkable degree. Particular attention was given to weight distribution. The cab has the boiler, water and oil tanks, storage batteries, air-compressor, resistors, motor-generator set, and the heavier parts of the control equipment, concentrated between the center pins. The driving motors are mounted above the axles on the running gear, thus getting the weight well inside the wheel base, but placing it relatively high. The height of the center of gravity of the complete locomotive is 68 in., a value that corresponds well with that of a steam locomotive.

The Pacific type of running gear with its long rigid wheel base and the guiding trucks is a particularly stable design and is especially good with the weight distribution that obtains on this locomotive. The height of the center of gravity of trucks with motors mounted is 43¾ in.

The quill drive, which is a further development of the one used on the New Haven locomotives, gives each driving axle perfect freedom to move vertically the full distance permitted by pedestal jaws without affecting the motors or frames, except through springs. The only "dead" weight carried is the weight of wheels, axles, journal boxes and spring clamps; a total of 7032 pounds for the wheels, axles and spring clamps which are rigidly fastened together, and 770 pounds for the journal boxes. This weight with the larger diameter of drivers (68 in.), and the total weight of 56,000 lb. per axle, gives a combination that is much better than has been considered very good practise on steam locomotives.

FIG. 17—TWO 700-H. P. MOTORS COUPLED TOGETHER FOR TEST

Great effort has been made to cushion the locomotive against shock, either while running or from bumping. The apparatus in the cab is especially well protected since the cab rests on spring-supported plungers which are in series with the main semi-elliptic springs. The cab is protected against bumping strains by floating center pins, which, while held rigidly against lateral motion, are cushioned against longitudinal motion by heavy springs. It has been the practise heretofore with this general type of locomotive to have one center pin rigid in the running gear frame and allow the other one to move freely in a longitudinal direction. It is felt that it is much better to allow a slight relative motion between the two center pins, but to prevent the bumping shocks by spring cushioning.

ELECTRICAL DESIGN

Main Motor. This was constructed of the twin armature type for several very good reasons:

1. The two armatures can both drive the same gear. A single armature motor would require twin gears, each of the same gear face as the present single gear in order to keep the tooth pressure down to the same value.

2. The space between wheels available for motors is limited and must be divided between gears and the motor itself.

The use of a second gear would take just that much space out of the armature core, and thus make a very large diameter necessary to get the output required.

3. A large diameter motor and double gears would increase the weight and cost of the locomotive.

4. A large diameter would make the motor extend farther up into the cab, thus necessitating raising the floor above the motors and cutting down the space available for other apparatus.

5. The use of two armatures makes it easy to design the motors so as to secure the advantages of low-voltage armatures for commutation.

6. Small armatures are easier to handle and cheaper to maintain than the large ones would be.

The capacity of the motor was fixed by the number of driving axles. Six driving axles were used as this seemed to be the smallest number that could be used with the limiting weight on the drivers. It was especially fortunate, as it led to the adoption of the double Pacific type of running gear with its excellent riding characteristics.

The characteristic curves of the motor, shown in Fig. 5 are taken from test; the efficiency being measured by the "input-output" method with all losses included. In making this test, two motors were connected together through gears, quills and quill springs; one motor being run as a motor, driving the other as a generator. See Fig. 17. The current from the generator was fed back through the motor by boosting its voltage, and the additional current required was supplied from an outside source. The measurements of the power supplied by the booster and from this outside source constitute the total losses in the two machines. The test was made on motors which had been operated only a few hours, consequently, the friction losses are probably higher than they would be after being in service for awhile; but the electrical and mechanical losses are thus included in the total, and the efficiency thus derived is the same as if the motors were on the locomotive. It is worthy of note that the efficiency thus measured is materially higher than that calculated by the A. I. E. E. Rules from "no-load" losses and the fixed percentage of losses for gears and other load losses. These tests show plainly that those losses assumed for the Institute Rules are too high for large motors.

Fig. 4 shows the pinion end of the motor, and on the motor frame are shown the shields which protect the motor from melting snow falling into the outlet openings in the motor frame.

The motor armatures have very effective fans mounted on them which give the motors quite a high continuous rating without the use of any forced ventilation. It is expected that the blowers will not be used at all with normal train weight, except on heaviest grades.

Control. 1. The first thing necessary in designing the control for an electric locomotive is to decide on the combination of motors that is to be used.

2. To decide on the schematic arrangement of switches.

3. To decide on the necessary sequence of switches and arrange the master controller and suitable interlocks in the control circuit to insure following that sequence.

4. Calculate resistance values and the number of steps necessary to secure smooth control.

5. To design master controller, switches and other apparatus to meet these requirements.

6. To arrange overload trips so as to afford adequate protection from overloads and grounds in any part of the circuits, without producing dangerous voltage surges or serious arcing.

Where regeneration is to be provided for on a d-c. locomotive, it is essential,

1. To decide on a scheme that is inherently stable.

2. To arrange a schematic diagram of switches corresponding to the scheme, and to utilize as far as possible the same switches and sequence of switches as when motoring, so as to keep the total number of switches as low as possible.

3. To provide for beginning regeneration at any speed within the normal running range without having a heavy current in either direction that would produce a surge in the train.

4. To arrange the control so that the engine-man will have control of the regenerated power and speed at all times.

With six motors to drive the locomotive, some new problems were introduced.

The universal practise with d-c. motors heretofore has been to use the series-parallel arrangement, or at most, series, series-parallel and parallel, giving one-fourth, one-half and full speed. This scheme takes motors in multiples of two, or when two motors are connected permanently in series, in multiples of four.

With six motors and twelve armatures on 3000 volts, it would be possible to obtain the regular series-parallel control in several ways, had it been desirable.

1. A twin motor could have been wound for 3000 volts, and six motors then divided in three sets of two motors; each set being arranged for series-parallel.

With this arrangement, the motors would at least be less rugged and more liable to flashing than where they are wound for 1500 volts, and the control would be much more complicated.

2. The motors could be wound for 1000 volts each and connected permanently three in series.

This would give the simplest control; but connecting three motors on independent axles in series, would lead to dangerous over-speeding and two to three times the normal voltage on the motors in case of wheel slippage. It is common practise on high-voltage d-c. cars and locomotives to run with two motors permanently in series. The maximum voltage one can get in case of wheel slippage is less than double the normal voltage. This method has given very good results, but it is as high as it is safe to go with geared motors.

3. The motors could be wound for 2000 volts, that is, 1000 volts per armature, and three armatures connected permanently in series.

This would be open to the same objection from over-speeding and over-voltage on the motors whose armatures are in different circuits, and the motors would not be interchangeable, for some would have their armatures and fields connected in series inside the motors, and would thus have only four leads, while the others would have to have a separate set of leads for each armature and field winding.

In none of these three cases (except the second), would the control be materially simpler than the scheme that was finally adopted; but the best reason for the adoption of the one-third, two-thirds and full speed arrangement lies, not so much in any reasons that have been mentioned heretofore, as in the great advantage of the three-speed locomotive for handling the service. The one-third and the two-third speed arrangements cover the range so much better than the one-half speed, or than the one-quarter and one-half speeds, that there is scarcely any comparison possible. The half speed is too low for a running speed and too high for switching. The one-third speed is an excellent switching speed, and the two-thirds speed is a good running speed and is especially good for regenerating on heavy grades.

Another considerable advantage lies in the decreased rheostatic losses on this locomotive compared with one having the ordinary series-parallel control. The three speeds alone make a very considerable reduction in rheostatic losses; but when combined with the field shunting positions on each combination, reduce the rheostatic losses to less than one-half of what would be obtained with the ordinary series-parallel control. This, of course, means a very decided reduction in the weight of resistance necessary, as well as an increase in efficiency.

The combination of unit switches and cam switches gives a perfectly successful, and at the same time, the lightest combination of switches possible. The unit switches are used in the high-voltage circuit wherever they are required to open the circuit with any current; thus the line switches, resistance switches and safety switches in the motor combination circuits are of the unit type and are interchangeable, except for interlocks which are easily transferred. The reverser switches, field shunt switches and those for regenerative change-over, exciter and auxiliary change-over are of the cam type, and operated by compressed air. These switches are assembled in groups of from four to sixteen switches; the sixteen-switch group being the reverser.

Shunt transition is used in passing from one motor combination to another as it is simple and gives sufficiently smooth transition to be very satisfactory. In changing from the first to the second combination, one-half of the motors are shunted momentarily; but in going from the second to the third speed, only one-third of the motors are shunted.

The sequence of switches is so simple that relatively few of the unit switches have interlocks on them, but the cam groups are all interlocked so that it is impossible to get false operation. The switches in any given cam group have the mechanical interlock provided by the cam shaft.

One of the fundamental principles on which this locomotive was designed was that the motor circuit should not be opened, either in normal operation or under emergency, until the current had been cut down by the introduction of resistance.

The specifications describe how this is done on this locomotive. It is considered to be a very valuable, if not an absolutely necessary, feature. It not only divides the arcs among a large number of switches and, therefore, promotes safety, but it prevents the generation of high voltages from suddenly rupturing a heavy current.

The line switches are located in a separate compartment from the resistance switches so that the four switches in series are always kept free for the final break.

Another fundamental principle in the regeneration was that the motors should never be connected to the line without the maximum starting resistance in series between motors and the line. This is a valuable provision in motoring, but is necessary in regeneration to prevent undesirable surges in the train.

This locomotive has approximately 17 ohms of resistance on the first step, consequently, the maximum current that could flow from the line would be less than 200 amperes, if there were no voltage generated in the motors at all. If this current is divided between two or three circuits, as would be the case on the second or the third combinations, the torque resulting, especially with the very small current in the motor field, would be negligible. The exciter rheostat is so arranged as to increase the voltage of the exciter immediately on closing the line switch until the flow of current through the resistance is practically zero, which means that the voltage of the motors equals that of the line. This method gives perfect safety in connecting the motor to the line and the stabilizing resistance and the axle-driven exciters add still more to the safety.

The stabilizing resistance protects the motors against sudden changes in line voltage, while the axle-generators protect against variations in grade, which would otherwise tend to vary the speed.

In case of a sudden reduction in line voltage, which would be followed by a heavy increase in the regenerated current, the increased voltage drop in the stabilizing resistance resulting from this, immediately cuts down the field current and generated voltage of the main motor sufficiently to limit the increase in regenerated current to a safe value.

In the case of the passage of a train from a sharp curve on a compensated grade to a tangent, there would be a tendency for a sudden heavy increase in speed, because of the steeper grade and the decrease in train resistance. As the speed increases, the voltage of the axle-generators automatically increases and, therefore, the field current of the main motors, which holds the train with a very slight increase in speed.

This scheme of regeneration offers a high efficiency as the only extra losses are those in the exciter and in the stabilizing resistance, both of which are small.

Auxiliaries. It has been the experience with high-voltage d-c. equipments that the auxiliary motors are very much more difficult to design than the main motors. More trouble is ordinarily experienced with auxiliaries than with the main current apparatus and in order to make such apparatus reliable, a great deal more expense is involved than would normally be warranted. It was the policy on this locomotive to limit the high voltage to the minimum possible number of circuits and the minimum amount of apparatus. It was impossible to avoid it altogether on the moving apparatus, due to the fact that the train lighting must be supplied for hours at a time in case of any emergency when the train is held up by a wreck or because of storms. The motor-generator set, which is designed primarily for this purpose, is the only piece of moving apparatus among the auxiliaries which has the high voltage applied to it. The voltage selected for the auxiliary circuits was primarily decided by the voltage required for train lighting.

While 85 volts is lower than would otherwise be selected for the auxiliary motors, it is perfectly safe and is a voltage that is so low that the motors can be forgotten, as far as the questions of insulation and commutation are concerned. The auxiliary motors are, of course, very much smaller than would be the case, if the high voltage were applied to them.

A storage battery, from the standpoint of operation, is always a desirable thing on the locomotive as it offers an opportunity for making a complete inspection of control apparatus by furnishing current for the lights and control, and for keeping up the air pressure without having the high-voltage current on the locomotive at all. Having a source of power to furnish light on the locomotive is a very desirable feature.

The combination of the storage battery and the axle-generators will furnish ample auxiliary power to take a locomotive to the end of a run, if the motor-generator set fails. The same combination will take a train safely down the longest and steepest grade with the air-brakes, if power is for any reason cut off the line. If the train is going up a grade and power is cut off, the battery will furnish air to hold the train with the brakes for a considerable time, so that it will not be necessary to block the wheels as is the ordinary practise, unless power is off for a considerable time.

Low-voltage motors are so small, comparatively speaking, that although there is a great difference in the power required for the compressor and the main blower motors, it was felt that the advantage of having the motors interchangeable was worthy of the increase in cost of the blower motors. The design of the compressor adopted is favorable to this, so that the motor is practically a standard industrial type of motor.

Thermostat. The thermostat, which is used for starting the blower motors, in accordance with the requirements, consists simply of a cast-iron resistance made up in tubular form, which is in series with the main motor armature. This resistance is enclosed by several micarta tubes so that it has substantially the same thermal characteristic as the main motor. There is a Sylphon tube contained within the resistance tube, which expands when a predetermined temperature is reached and closes the magnet circuit of the blower motor switch, thus starting the blower when that temperature is reached. The apparatus is very simple and rugged; and while, of course, it cannot follow the temperature of the motor absolutely, it is sufficiently accurate for the purpose.

CONCLUSION

The initial operation of these locomotives has given every reason to anticipate that good riding qualities, the advantage due to low-voltage auxiliaries, the flexible speed control, stability of regeneration, safeguards against short circuits, and the simplicity and ruggedness of the individual parts of the locomotive, form a combination that will be a great success in the service for which it was designed.

Presented at the 358th meeting of the American Institute of Electrical Engineers, Pittsburgh, Pa., March 12, 1920.

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PASSENGER LOCOMOTIVES FOR CHICAGO, MILWAUKEE & ST. PAUL RAILWAY

BY A. F. BATCHELDER
Ry. Locomotive Dept., General Electric Co., Schenectady, N. Y.

AND

S. T. DODD
Ry. & Traction Engineering Dept., General Electric Co., Schenectady, N. Y.

DECEMBER 9th, 1915, may be considered the date of the initial electrical operation over the electrified lines of the Chicago, Milwaukee & St. Paul Raliroad. During the following winter the electrification was extended over 440 miles of route from Harlowton, Montana, to Avery, Idaho, a section which crossed the Belt Mts., the Rocky Mts., and the Bitter Roots. The locomotives for this initial electrification were of the geared type, designed and built especially with a view to the most economical operation of the freight service. The locomotives for passenger service differed from the freight locomotives only in the details where it was absolutely necessary to meet the operating requirements, such as changing the gear ratio to increase the speed and providing car heating and lighting equipment.

Three years later, in 1918, the successful operation of the original equipment had convinced the railroad company of the economical advantages of electric operation, and they decided to equip an additional section extending over the Cascade Mts. between Othello, Washington, and Tacoma, Washington, a distance of 212 miles. In choosing the equipment for the new extension, it was decided to give special emphasis to the requirements of passenger service and to purchase locomotives which were primarily designed with this in view, taking advantage of any details which would assist in the proper and economical operation of passenger trains. For the freight service it was decided to retain the geared locomotives that were in use on the

Harlowton-Avery Division, changing the gear ratio where necessary to meet freight conditions, and using only locomotives of the new design for passenger service.

To meet the specification for passenger locomotives, the General Electric Company has designed, completed, and tested, a locomotive which appears to embody the necessary qualifications and to successfully fulfill the requirements, both from electrical and mechanical standpoints. In designing the locomotive, particular attention has been given to the features affecting safety, reliability, efficiency, convenience of operation, effect on track, and cost of maintenance. The locomotive has especially good riding qualities; it has no apparent effect on the alignment of the track, and to a marked degree, it is free from transverse movements or oscillation which would tend to create lateral pressures against the rails.

It is the intention of this paper to give a description of this locomotive which differs in many ways from the locomotives which are now in operation on the Harlowton-Avery Division, to indicate the reason for choosing this design, and to call attention to some of the principal features which differ from usual practise. Briefly stated, the service requirements for the locomotive are to operate a 950-ton passenger train over the mountain divisions of the Chicago, Milwaukee & St. Paul Railway at speeds of 25 mi. per hr. up 2 per cent grades with maximum operating speeds of 60 mi. per hr. on the level, and to provide regenerative braking on the down grades at speeds consistent with safe operation. Fig. 1 is a photograph of the locomotive in three-quarter view. Fig. 2 is an outline drawing of the side elevation, giving the general dimensions. Fig. 3 is a section through the apparatus cab, showing the location and arrangement of the principal pieces of apparatus.

It will be seen that the running gear is composed of four individual trucks; two end trucks having three axles each, and two center trucks having four axles each. These trucks are connected together by special articulation joints. The motor armatures are mounted on the axles and the motor fields are carried on the truck frames.

The superstructure is made in two sections of similar design with a third section between them. The third or central section contains the train heating equipment, which consists of an oil fired steam generator, together with water and oil tanks. This unit is complete in itself, and is carried over supports attached to the two middle trucks. It can be readily removed for repairs without interfering with any other part of the locomotive. It is placed between the two operating cabs in order to be easy of access to the engineer's helper or fireman, from either location.

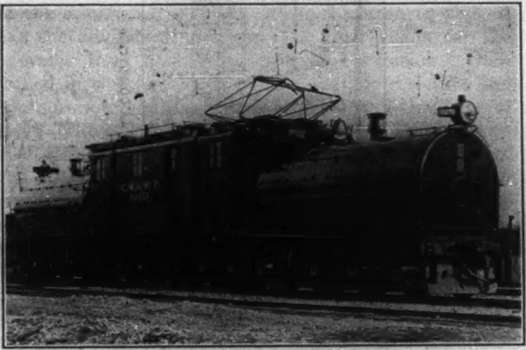


FIG. 1—THREE-QUARTER VIEW OF LOCOMOTIVE

The two end sections are similar to each other in appearance. The operator's cab in either section is on the inner end next to the heater cab above described, in order that the operator can be convenient to the heater and in order to allow a maximum space for apparatus in the apparatus cab or outer end section. Another advantage of this arrangement of cabs is that the operator can have access to any section of the locomotive requiring his presence without passing through a section containing high-tension apparatus. The engineer's or operating cab contains a main or master con-

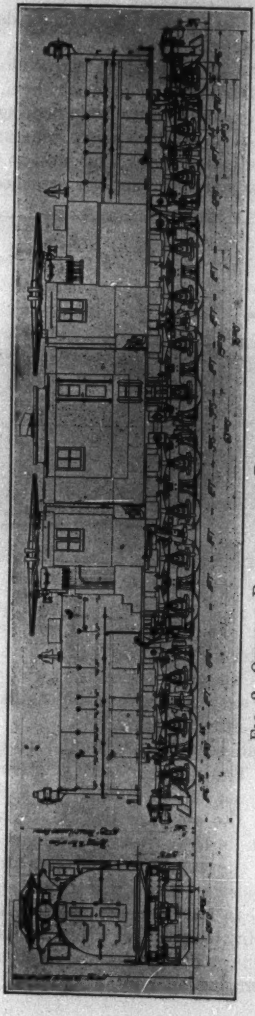


Fig. 2—Outline Drawing and Dimensions of Locomotive

troller, the air brake valves and handles, and an instrument panel, containing air gages, ammeters, and speed indicator. The engineer uses either of the two operating cabs according to the direction in which he is running.

A door gives access from the operating cab to the apparatus section which extends with a cylindrical top to the extreme end of the locomotive. The cylindrical construction naturally adapts itself to the protection of the apparatus included and in addition to this, it has the advantage of allowing a clear vision for the operator from his normal operating position. Contained in this apparatus section are the resistors and contactors to control the power circuits of the locomotive. The starting resistors are placed in two rows on each side of the central passage just above the floor of the superstructure and are covered at the sides by removable covers which when opened will allow the separate resistor boxes to be slid out upon the longitudinal running board outside of the apparatus cab. The air compressor for the air brakes, the motor-generator set for train lighting, and the storage battery for marker lights and emergency control, stand upon the same level as the resistors, and can be removed or replaced in a similar manner. Above the resistors are located the contactors with their arc chutes facing a central aisle two feet wide. This allows ample arcing space and room for inspection of contactors. Above the contactors is the cylindrical roof of the locomotive with trap doors for inspection of the back connections and insulation and for removing the contactors in case replacement is necessary. The whole design and arrangement of this apparatus cab lends itself to a maximum economy of cost and material, as well as to convenience of inspection and repair of apparatus.

MOTORS

The motors are of the well known bi-polar gearless design which were adopted by the New York Central Railroad 14 years ago, and which have proved by fourteen years' service, operating heavy passenger trains between Grand Central Station and Harmon, to be well suited for the service. This motor has demonstrated its remarkable reliability and low cost of maintenance.

To insure light weight per axle, flexibility in control. good truck arrangement for curving as well as for high-speed running, twelve motors are chosen, each of relatively small capacity. They are especially designed to withstand high temperature, being insulated with mica and asbestos.

Fig. 4 shows the motor armature complete, built directly on the axle with the wheels pressed and keyed in place. The continuous rating of each motor at 1000 volts and with 120 degrees rise by resistance is 266 h.p., corresponding to 3500 lb. tractive effort at the rim of the drivers at a speed of 28.4 mi. per hr. Forced ventilation is employed for cooling. The armature core is provided with holes for the passage of ventilating air. Ventilating blowers are located above each motor armature and deliver air at the commutator end of the motor where it divides, a part passing through the armature and a part back through and around the field coils where it escapes upwards and is afterwards used for ventilating the starting resistors.

This type of motor gives very high power efficiency in average operation, it having no journal bearings or gearing. It lends itself nicely to simple and compact locomotive design as the frame is made use of to furnish the entire path for the magnetic flux. The pole pieces and field coils are fastened to the cross transoms of the trucks and the magnetic flux passes horizontally in series through all twelve motors, finding a return path through the locomotive frame. The articulation joints between the trucks are made in such a manner that large surfaces are in contact to provide an easy path for the flux. The pole pieces are made flat in order to prevent the pole pieces from coming in contact with the armature during the vertical movement of the truck frame on its springs or when removing or assembling the armatures. A minimum clearance of 1/8 in. on each side is allowed between the armature and the pole piece tips. The brushholders are bolted to the transom allowing the brushes to move up and down with the fields as the frame rides on the truck springs.

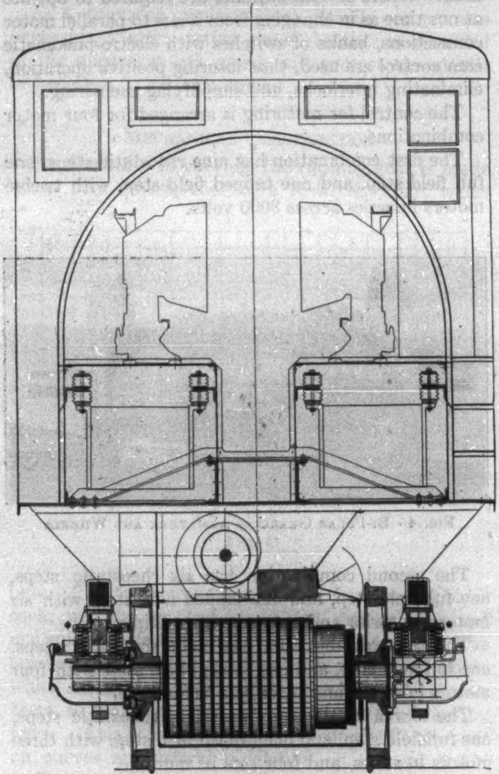


FIG. 3—CROSS-SECTION OF APPARATUS CAB

CONTROL

In choosing the control apparatus special care has been taken to use individual pieces of apparatus best suited to the particular requirements. Where single independently operating switches are necessary as on the resistance notches, electro-magnetic control is used. Where several switches are required to operate at one time as in changing from series to parallel motor connections, banks of switches with electro-pneumatic cam control are used, thus insuring positive operation, eliminating interlocks, and simplifying the wiring.

The control for motoring is arranged for four motor combinations.

The first combination has nine rheostatic steps, one full field step, and one tapped field step, with twelve motors in series across 3000 volts.

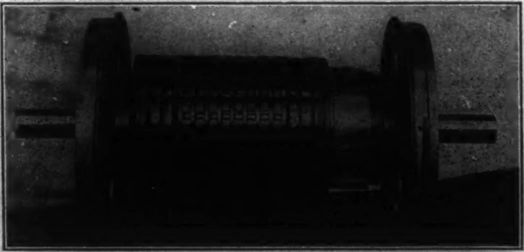


FIG. 4—BI-POLAR GEARLESS ARMATURE AND WHEELS

The second combination has six rheostatic steps, one full field step, and one tapped field step, with six motors in series and two sets in multiple.

The third combination has eight rheostatic steps, one full field step, and one tapped field step, with four motors in series, and three sets in multiple.

The fourth combination has eight rheostatic steps, one full field step, and one tapped field step, with three motors in series, and four sets in multiple.

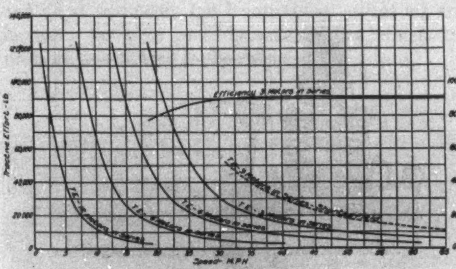
This results in a total of 39 control steps with a choice of eight operating speeds, exclusive of the resistance steps. The locomotive characteristics on the various steps are clearly shown in Fig. 5.

The regeneration of power for braking is accomplished in a simple manner by using some of the motors for exciting the fields of the others, which in turn are used as generators to return power to the line.

As a provision against short circuits, or extreme overloads, a quick acting circuit breaker is provided in the apparatus cab which will protect the circuit in less than 1/100 of a second.

MECHANICAL CONSTRUCTION

For flexibility in curving, the running gear is made up of four trucks, each of a relatively short wheel base. The two middle trucks have four driving axles each; and the two end trucks, two driving axles and one guiding axle each, making a total of 14 axles. The trucks are connected together with articulated joints which allow of no relative lateral movement between them, so that each truck positively leads the following truck. This is for the purpose of reducing flange wear on curves and lateral oscillation on tangent track.



Characteristic Curves
3000 Volt DC Gearless-Motor Locomotive
For C M & St P RR

FIG. 5

The most important problem that has to be faced in the design of a locomotive for high-speed passenger service is the problem of limiting as far as possible the lateral oscillations of the locomotive structure which tend to distort the track, and to minimize the effect on the track of such oscillations as occur. If a locomotive were built with a rigid wheel base as long as the total wheel base of the present locomotive (67 ft.), the lateral oscillations could not reach any large angular value. However, on account of the long wheel base, such a locomotive would be incapable of taking curves. By articulating the wheel base, as we have done, the locomotive is capable of accommodating itself to track curvature, but at the same time, on account of the articulation between trucks, and the consequent guiding effect of one truck on another, the lateral oscillations on tangent track are minimized in the same manner as would be done by the use of a long rigid wheel base.

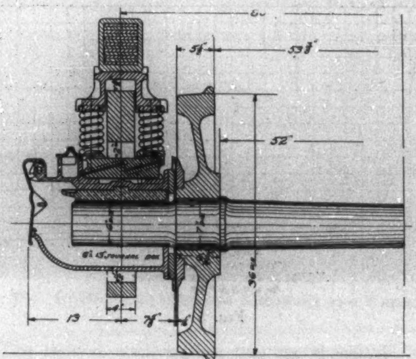


FIG. 6

To soften any lateral blow that may be given against the rail, the leading and trailing axles are allowed a movement of one-half inch relative to the truck frame, either way from their central position. This movement takes place against a resistance introduced by wedges above the journal boxes which tend to hold the box in its central position and to give a dead beat action opposing the motion. This wedge construction is illustrated in Fig. 6. To further protect the track from lateral displacement on the ties, the outer end of the superstructure is carried on rollers, bearing on inclined planes upon the truck frames, while the inner end of the superstructure is rigidly fastened to one of the middle trucks. This construction tends to hold the leading and trailing trucks in their central position. When a blow is delivered by the leading or trailing truck against the rail head, the superstructure, is displaced laterally across the outer truck. In such a sideways displacement, the weight of the superstructure rolls up on the inclined plane on that side, and thus transfers weight to the rail that is affected and increases the adhesion of the rail to the tie. This action really has two results. It not only increases the holding power between rail and tie at that point, but it introduces a time lag and increases the time and distance during which the pressure is delivered to the rail head.

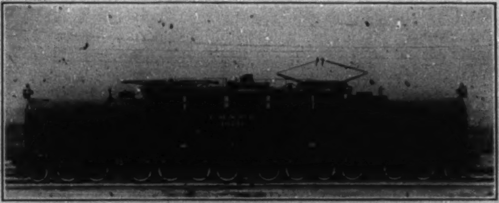


FIG. 7—SIDE VIEW OF COMPLETE LOCOMOTIVE

As a matter of record, it should be said that the first of these new locomotives was delivered to the railway company at Deer Lodge, Montana, on December 14th, 1919, and was put in operation handling passenger trains between Deer Lodge and Avery.

For convenience of reference a table is attached giving a summary of the principal dimensions and characteristics of this locomotive.

Total weight		521,200	lb.
Total weight on drivers		457,800	
Weight per driving axle		38,150	
Dead weight per driving axle		9,500	
Weight per idle axle		31,700	
Dead weight per idle axle		3,560	
Length overall	76 ft.	0	in.
Width overall	10 ft.	0	in.
Height over cabs	14 ft.	11⅝	in.

Height over pantograph, locked down	16 ft. 8 in.
Total wheel base	67 ft. 0 in.
Max. rigid wheel base	13 ft. 9 in.
Diameter of driving wheels	44 in.
Diameter of idle wheels	36 in.
Size of journals	6 in. by 13 in.
Dimensions of operator's cab	5 ft. by 10 ft.
Dimension of heater cab	14 ft. 11 in. by 10 ft.
Heater capacity	4,000 lb. steam per hr.
Water capacity	30,000 lb.
Oil capacity	6,000 lb.
Compressor capacity	150 cu. ft. per min.
Number of motors	12
Type of motor	(bi-polar) GE—100
Diameter of armature	29 in.
Clearance between bottom plate and top of rail	5¼ in.
Working range of pantograph	9 ft. 0 in.

Locomotive Rating	*Tapped field*	*Full field*
Total horsepower, 1 hr. motor rating	3,600	3,500
Total tractive effort 1 hr. motor rating	37,500	48,500
Speed mi. per hr	35.6	27.1
Total horsepower continuous	3,200	3,200
Total tractive effort continuous	32,000	42,000
Speed, mi. per hr	37.8	28.4

DISCUSSION ON "PASSENGER LOCOMOTIVES FOR CHICAGO, MILWAUKEE AND ST. PAUL RAILWAY" (BATCHELDER AND DODD) AND "THE BALDWIN-WESTINGHOUSE CHICAGO, MILWAUKEE AND ST. PAUL ELECTRIC LOCOMOTIVES" (STORER), PITTSBURGH, PA., MARCH 12, 1920.

S. Q. Hayes: There is one point I would like to ask Mr. Dodd. One of the illustrations of the G. E. locomotive—the one looking through the apparatus cab—apparently showed all the control leads lying loosely ontop of the contactors off on the left hand side. I was wondering whether that were temporary, or whether that was their permanent position in place.

R. L. Wilson: In the early days of the application of electric traction to railroads, I had some experience with it as a practical man. In those days the electric locomotive was so complicated—or it seemed that way to me, at any rate—that there not did seem to be any hope of anybody but an engineering apprentice understanding it. Since that time, I am informed that a man with ordinary intelligence can handle it. From what I have seen tonight it still looks complicated. However, the reliability of all these devices has been proved.

Now, to me, the main point that has been brought out tonight is the absolute divergence of opinion, if you like to so term it, as to how to meet a given problem, particularly mechanically. In looking at the different illustrations, we see that the locomotive described by Mr. Storer has a running gear very similar to the ordinary Pacific type of steam locomotives, which everybody is familiar with, and which apparently has proved satisfactory in a mechanical way, so far as running and the effect on the track is concerned. I imagine that fact is what lead to the adoption of that particular type of running gear, by the engineers who designed the locomotive in question. What led to the other design I do not know, unless it would be to gain electrical efficiency at high speeds. It seems to me that inherently that design would be expected to react on the track more than the other, and that certain devices would have to be introduced in the design and construction to overcome that inherent tendency.

I was much interested in Mr. Dodd's description of what the devices were, the wedges which are on the leading truck and the rollers rolling on an inclined plane elsewhere. That idea of a roller was used,—or something corresponding to it—in the first engines which were installed on the New York, New Haven & Hartford system, which had a tendency to oscillate. In that case it was not a roller rolling an inclined plane, but rather a cam effect at each corner, so that when the cab tended to oscillate, it raised the weight on this cam, which tended to bring the cab back into position and stop the oscillation. It seems to me that the fact that these rollers come into play, and act only on one side, and leave the other loose would not be a good thing, although it may work out otherwise.

Now, as Mr. Storer said, the low speed motor is inherently subject to higher losses, copper losses particularly, and I noticed in the paper here that the tractive effort of the locomotive that Mr. Dodd described is 46,000 lb., and that somewhere else it says something about the heating limit being 120 deg., corresponding to 3500 lb. tractive effort. As I understand, the specifications call for 25 miles an hour up a two per cent grade or thereabout, which corresponds to a tractive effort of about 50,000 lb., which would bring an overload on a motor designed for 46,000 lb. tractive effort.

I was wondering, in going up a long grade, which would take an hour to ascend, what temperature the motors would likely attain.

E. H. Martindale: In the paper presented by Mr. Dodd the statement is made: "The continuous rating of each motor at 1000 volts and with 120 deg. rise by resistance is 266 h. p., corresponding to 3500 lb. tractive effort at the rim of the drivers at a speed of 28.4 mi. per hr." I ask Mr. Dodd what the maximum operating temperature is which is allowed on the motor.

R. J. Wensley: I would like to ask Mr. Dodd a a question. A couple of years back, or possibly more, there were one or two disastrous wrecks on the New York Central with the low gravity motors, and there were some comments made at the time as to the effect of the low gravity motors on the track, and some claimed that the low gravity motor was to blame for the wreck, claiming that it caused the locomotive to climb off the track. I never heard that satisfactorily explained. Possibly Mr. Dodd can do that.

F. D. Hall: Considerable stress has been laid on the superior efficiency of the gearless motor at high speed. What is the average speed of the locomotive in its day's run?

R. E. Ferris: I would like to ask Mr. Dodd if he anticipates any trouble from snow when the motor is mounted on the axle, and in close to the truck, as the snow conditions are very severe on the Milwaukee Road.

A. M. Candy: Mr. R. L. Wilson brought out a question which I think should prove quite interesting namely, the reference to the complexity of the modern electric locomotive. About ten years ago I had the pleasure of being numbered amongst a group of some 25 or 30 young men who were assigned to work on the Pennsylvania terminal electrification in New York City. When the locomotives for that service were placed in operation a few weeks previous to the official opening of the terminal, there was some question as to whether or not the average railroad man(the locomotive engineer and fireman) could handle these locomotives satisfactorily.. As a result, one of us was assigned to each locomotive to be on board at all times when it was in service, so that if anything went wrong of a minor nature the train service would be delayed as little as possible. Each of us prepared himself with a small notebook, in which we had all the schematic diagrams and main wiring diagrams of locomotive, together with the names and numbers of all the wires, control fingers, interlock contacts, etc. We had the sequence of the various switches and the names and numbers of practically all of the control wiring committed to memory, so that at a moment's notice we could tell exactly where to look for trouble in the event of failure of any particular switch or any of the auxiliary equipment. We did an unlimited amount of tutoring with the engineers to teach them this information, and it was quite customary for us to quizz one another as to the method of procedure assuming certain failures should take place. In fact we vied with each other to devise imaginary troubles and the simplest means of correcting or eliminating the failures. The most of us were located on this job and worked faithfully for approximately one year. During all that time, however if my memory serves me correctly, there was not one of us who ever got an opportunity to display the knowledge which he had, because the apparatus, even in the minutest detail, never failed to function properly. Of course, the locomotives were inspected at regular intervals, but the strength and simplicity of the equipment was such that even though the locomotives were operated a great many more miles per inspection than is customary with steam locomotives, they did not cause any train detentions.

It seems to me that in so far as the subject of general electrification of steam railroads is concerned at least, it would be better for us to consider the electric locomotive from the standpoint of being a perfected complex mechanism rather than an experimental complicated mechanism, and that it would be better to point out the advantages of the electric locomotive versus the steam locomotive, rather than to argue extensively among ourselves as to which particular type or design of locomotive is most satisfactory.

There is one other point which I desire to mention, and that is the one brought out by Mr. Dodd in reference to his locomotive, namely, the very free and unobstructed view presented to the engineer through the cab window, looking out along the arch shaped dome over the switching and resistance units. Judging from the illustration thrown upon the screen, it strikes me that the view is certainly not any more obstructed than that obtaining for the average steam locomotive. I do not believe that Mr. Storer mentioned this point in connection with his locomotive, and therefore, I think it would be in order if he would make a few statements relative to this feature.

L. J. Hibbard: Mr. Dodd, in describing his locomotive, mentioned the fact that each main motor had a separate blower motor. I would like to have him tell me what type of motor they use for running each separate blower, and if they expect to have any trouble on these separate blower motors.

Calvert Townley: If I may, I would like to emphasize the remarks brought out by the next to the last speaker. It is well known, that every electrification of steam railroads has been successful. We have now been electrifying roads for a number of years with different types of locomotives. Some of them are more efficient than others, some have lower maintenance cost than others, they vary in degrees of success, but everyone of them has been successful, and as a matter of fact, notwithstanding the large number of types which have been put into use, there are not anything like as many electric locomotive types as there are standard steam locomotive types, and the railroads, accustomed as they are to dealing with steam locomotives, do not think anything of having a large number of different types of steam locomotive on their lines for use for different purposes.

In much of our discussion of this subject, we argue about the value of respective locomotives, and we are in danger of giving our steam railroad friends the idea that the electric locomotive development is doubtful, that we are not sure of ourselves, and that perhaps they had better wait a while and see what happens—

that is the attitude they are likely to take under such circumstances.

Of the two locomotives described tonight, one will perhaps show a better efficiency than the other, one may show a lower cost of maintenance than the other, but I feel sure that each of them will do the work of the Chicago, Milwaukee and St. Paul Railway, to the satisfaction of the railroad company. My reason for speaking in this way is not to discourage discussion, I hope the discussion will bring out every feature of possible advantage—but I think we should emphasize the uniform success which has attended all electrification. Do not let the steam railroad people think we are not sure of ourselves.

If the electrification of steam railroads, ever gets started, it will call for so great a number of locomotives, and so rapidly that if all the existing facilities of the different companies, be devoted to that purpose, they will be nowhere near sufficient. Therefore, it is perhaps well that they do not go too fast. No steam railroad man with an electrified section, would now consider going back to steam, even though from our view point he may have been most reactionary, and looked askance at the electrified part of his lines. After he has become familiar with the electrified system, he recognizes its many advantages, tremendously increased traction, greater speed improved traffic possibilities and other incidental advantages which the electric locomotive affords to an extent not practical with the steam locomotive. The poorest electric locomotive compared with the best steam locomotive easily exhibits a decided margin in its favor.

C. M. Davis: I ask Mr. Storer, to explain the detail of the exciting motor on the leading truck. As I understand it, this motor is geared to the truck axle in much the same way that the ordinary railway motor is geared—this being the case, the gearing, of course, works just the reverse to what it does in the case of the railway motor, when the motor is used to drive the car, that is, a large gear in this case is driven by small pinions.

I should like to have brought out the method of accomplishing this gearing, whether springs are used to soften blows on the teeth when running at high speeds, 60 miles an hour, or what provision is made for taking care of that particular feature.

S. T. Dodd: Mr. Hayes asked about some leads shown in one of the illustrations. The leads were temporary only. The picture was taken before completion of the cab and was used by me to show the location of apparatus before installation of the outside part of the cab.

Another question was asked as to service capacity. I can only say that the motors have ample capacity to perform the service specified by the railroad company and outlined in the paper.

Mr. Ferris asked about the possibility of snow troubles. There will be no trouble on this score as we know from our experience with the New York Central gearless locomotives. If I had been prepared for such a question, I could have shown pictures of electric locomotives running through snow piled up to the windows. Possibly he misunderstands the construction of the gearless motors and thinks they are entirely unprotected. As a matter of fact, there are protecting sheets under the armatures and across the ends so that the armatures are practically as well protected from weather as in the ordinary railway motor. You know that with modern insulation we have very little trouble in that respect from railway motors.

Mr. Candy has referred to the complexity of the electric locomotives. The question of complexity of any mechanism depends largely on the amount of analysis to which you subject it. A steam locomotive is a complex piece of apparatus if you go into a careful analysis of all its features from the chemical composition of the coal to the output at the drivers. I am afraid that electrical engineers too often look at problems from the electrical standpoint and do not pay sufficient attention to the mechanical standpoint. We build locomotives to do service and that is what we should keep in mind rather than the details that go to make up the final combination. Maintenance of equipment tells the whole story, and in that respect the electric locomotive has a record of which to be proud.

The question has been raised as to the center of gravity of these locomotives. The center of gravity of the locomotive as a whole is 57 in. above the rail head. In the first place, I wish to point out that the location of the motor is not the only thing that determines the height of center of gravity. The weight of drawbar and couplers, the end and side frame castings, the cab construction and other mechanical features fix the height within rather narrow limits and the location of the motors is of secondary importance. In the second place the location of the center of gravity has very little to do with the riding qualities of a double ended locomotive. With such a locomotive equipped with guiding trucks at each end, the locomotive is steered by these guiding trucks and the thrust against the rails is determined by the construction of those trucks rather than by the location of the center of gravity of the main body of the locomotive. This is the reason that in my paper I emphasized the construction of the leading and trailing ends of the locomotive. I wished to show the precautions we had taken to cushion the track thrust when running at high speeds.

Two very different types of locomotives have been presented tonight. It is a source of congratulation both for the users and manufacturers of electric locomotives that such should be the case. In the United States at the present time there are only about 350 electric locomotives in service on steam roads. If we include the trolley roads there may be 700 electric locomotives in service. At the same time there are 63,000 steam locomotives in service. We are just beginning to touch the job of the construction and application of electric equipment to the transportation problem. The sooner we learn how to build locomotives which will meet all the conditions of service, the better it will be for all of us, and I agree with Mr. Storer that it is a splendid thing for all of us to obtain this experience and find out the lines along which we must progress because the big developments are still ahead of us.

N. W. Storer: While appreciating very much the discussions we have had tonight, I regret very much that some of the steam railway engineers who are present did not take part in it. It is all very well for electrical engineers to discuss electric locomotives and to extol their virtues, but we need the side lights that the railway engineers can throw on the subject.

Mr. Townley made an excellent point in bringing to our attention the fact that the electric locomotives that have been built have all done their work better than any steam locomotives that have been put on the road. We should keep that fact in mind.

In the few remarks I made to draw out Mr. Dodd, I had no intention to disparage the gearless type of locomotive. I am more than happy to have that locomotive built, and also one of our own manufacture, which is so absolutely different, and to have them both tried out on the same road. It makes little difference which one does the work the best. It is going to be to the advantage of the electric railway industry to have all types of locomotives thoroughly tested out, in order that the best ones may finally be determined. We are very glad that the two types are to be operating side by side, and if the low friction losses in the gearless type of locomotive, and the absence of gears and other connections between the armatures and the axles, are sufficient to over-balance some of the other features which are not so desirable and which need not be mentioned by me, it may solve some of the difficulties in connection with electric locomotives, one of the greatest of which is the transmission of the torque of the motor to the driving wheels.

I agree very largely with what Mr. Dodd has said about the kick of the locomotive and about the rolling of the cab with the weight up high. There is no question about it, where the cab weight is up high, as it is in our locomotive, it steers a straight course regardless of variations in the road-bed. It is certainly easier on the road-bed to have the weight high than it is to have it down close to the track.

Mr. Davis has asked a question about the performance of the little axle-driven generator. As pointed out in the paper, this is geared directly to the axle like an ordinary interurban motor, and is about the same size. The output of the generator is never more than 25 to 30 kw., so that the duty on the gears is very limited and no special provision has been made to avoid vibration. We do not anticipate any trouble from it.

Presented at the 359th meeting of the American Institute of Electrical Engineers, Boston, Mass., April 9, 1920.

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THE FIXATION OF ATMOSPHERIC NITROGEN BY THE SILENT ELECTRIC DISCHARGE PROCESS-I

BY C. FRANCIS HARDING
Head, School of Electrical Engineering, Purdue University

AND

K. B. McEACHRON
Research Assistant, Engineering Experiment Station, Purdue University

THE very great increase in the demand for nitrates during the war for the manufacture of explosives and fertilizers, together with the shortage of imported Chile saltpeter, which was the principal source of nitrates before the war, has been emphasized sufficiently in previous papers. This condition of increased demand and curtailed supply led not only to the establishment of many manufacturing plants for the production of nitrates by well-known methods, but also to laboratory research and commercial development in connection with promising new processes.

Such a process, for obtaining nitric acid from the air by means of the silent electric or corona discharge at high voltage, has been investigated throughout a period of several years at Purdue University under the auspices of the Engineering Experiment Station of that institution. Improvements in the yield, resulting from a given input of electrical energy, are being made quite frequently as the investigation progresses and many of the peculiarities of both electrical and chemical reactions of the oxides of nitrogen, combined with ozone, are being disclosed as phenomena incidental to the process under consideration. It has, nevertheless, been considered advisable to present, as a progress report, the results obtained thus far in order that, in the spirit of co-operative research, discussions and suggestions of value will result which in turn will hasten the development of the process. With this object in view, the results of the investigation being carried on at Purdue University are presented in this paper.

It is well to point out at first the distinction between the process herein described and the arc method of nitrogen fixation. In the former, air is passed through an electro-static field of such intensity that marked corona and some static sparks are produced, but no power arc is established and the temperature is increased but slightly above that of the incoming air. While the arc process of nitrogen fixation is dependent principally upon the very high temperature of the power arc, the silent discharge reaction is the result of electrical ionization of the air at a relatively low temperature.

The fact that electro-static brush discharges and sparks will produce, in a confined volume of air, various oxides of nitrogen and ozone, has been known for many years. Oxidation of these lower oxides of nitrogen by an excess of ozone results in nitrogen pentoxide, N_2O_5. This gas, when absorbed in water, produces nitric acid from which the desired nitrates may readily be formed.

EARLIER INVESTIGATIONS

The investigation, to which this paper forms an introduction, was the result of the translation from the German, by Messrs. G. N. Unger, G. W. Payne and F. S. Weimer, of an article entitled, "The Formation of Nitric Oxides by the Silent Electric Discharge in a Siemen's Tube." The latter was submitted by Hugo Spiel for a doctor's degree at The Technical High School, Vienna, in 1909. Spiel gives credit for the first publication of the effect of this silent electric discharge upon air as follows:

> The first observations of the silent electric discharge in literature appeared about 1870, following the work of Andrew, Hauzau, Jean, Thenard and Boillit, with ozone and nitric acid. It concerned the decomposition of carbonic acid and turned the general interest to the chemical effect of the electric discharge in gases. In 1873 the French physicist, Moncel, published a work entitled, "Concerning the Condensed Discharge of the Induction Coil," in which he claimed to have discovered the above phenomena in 1853.

Following a review of the previous work done by others upon the problem, Spiel described a series of laboratory research investigations performed by him upon fixed volumes of air and various artificial nitrogen-oxygen mixtures. Spiel's apparatus consisted of a small glass or quartz Siemen's tube, using acidified water electrodes upon either side of an annular air space of only 3 mm. thickness. The electro-static discharge, within this space, was produced by an induction coil and condenser having a sparking distance of 40 cm., and a frequency of approximately 40 cycles per second.

As a result of the exposure of a confined volume of air to the electro-static discharge, at different pressures and voltages, Spiel noted during a single test; first, a decrease in pressure and later, at the minimum pressure, a reversal of the reaction. This reversal was indicated by the presence of brown fumes of NO_2. If the electric discharge was continued beyond this point, the pressure increased to the initial, or to a greater pressure, when equilibrium was established.

The conclusions reached by Spiel have been summarized in the appendix of this paper, because of the bearing they have upon this investigation and the inaccessibility of the original article.

EARLY INVESTIGATIONS AT PURDUE UNIVERSITY

In planning a further investigation of this process,[1] it was decided that although the work must necessarily be performed with laboratory apparatus upon a comparatively small scale, when contrasted with commercial arc plants, the equipment should be designed and constructed in such a way as to provide for a continuous flow of air through the electric field. There is no reason apparent, therefore, to prevent the construction and continuous operation of a commercial plant based upon the same principle.

To produce the so-called "silent electric discharge" in a column of air, it is, of course, necessary to apply an electric potential between conducting plates or concentric pipes, Fig. 1, sufficient in magnitude to exceed the dielectric strength of air under the particular pressure at which such air is being forced through the apparatus, *i. e.*, the potential gradient must be such that the molecules of the air are torn apart into ions and neutral atoms which may reunite in new combinations. Such ionized air is a comparatively good conductor of electricity and unless special precautions are taken in the design of the tube, a power arc will result at one point and the potential will be lowered at other points in the field, thus eliminating the reaction.

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[1] Thesis entitled "The Fixation of Atmospheric Nitrogen by the Silent Electric Discharge at High Voltage," 1917 by G.W. Payne, G. N. Unger and F. S. Weimer.

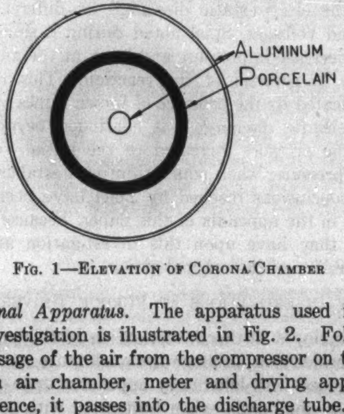

FIG. 1—ELEVATION OF CORONA CHAMBER

Original Apparatus. The apparatus used in this first investigation is illustrated in Fig. 2. Following the passage of the air from the compressor on the left through air chamber, meter and drying apparatus in sequence, it passes into the discharge tube. This consists of a vertical aluminum tube 37.8 in. (96 cm.) in length and 2.91 in. (7.4 cm.) internal diameter, inside of which was mounted concentrically a glass tube of slightly greater length, 2.5 in. (6.35 cm.) diameter and 0.125 in. (0.318 cm.) thickness. This glass tube provides the barrier of higher dielectric strength than air to prevent the arc from forming. The potential is applied between a coaxial aluminum rod of 0.625 in. (1.59 cm.) diameter and the outer aluminum tube. The air passes upward between the glass and aluminum tube, and thence the gases pass downward between the central rod and the glass tube, emerging from the bottom into the mixing chamber. Receptacles are provided at the right with glass beads and distilled water or a solution of *N*/10 potassium hydroxide (KOH)for the fine subdivision and absorption of the gases. Traps are placed upon either side of the drying bottles to prevent the sulphuric acid, (H_2SO_4) which is used as the drying agent, from being carried into the meter or discharge tube. A similar trap is located between the mixing chamber and absorption apparatus to prevent the absorption water from being forced back into the mixing chamber. Manometers and thermometers are installed, as indicated in the figure, for the purpose of determining pressures and temperatures of entering air and effluent gases, respectively.

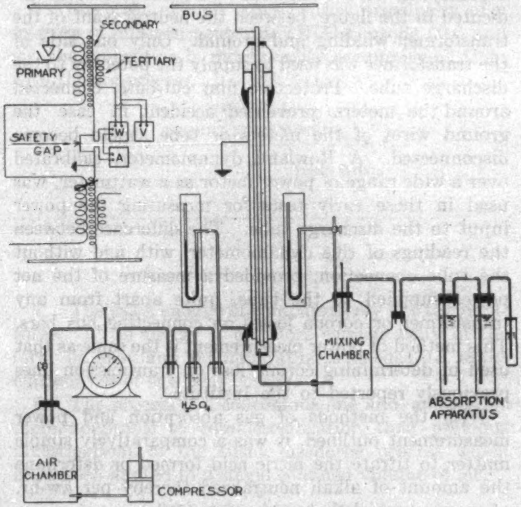
BUS
SECONDARY
PRIMARY
TERTIARY
W
V
SAFETY GAP
A
MIXING CHAMBER
H_2SO_4
ABSORPTION APPARATUS
AIR CHAMBER
COMPRESSOR

Fig. 2—Original Apparatus

Power Measurement. A potential in the neighborhood of 30 kilovolts, alternating at a frequency of 60 cycles, was furnished by a 200-kilovolt, 50-kv-a., air-cooled transformer. The tertiary coil of this transformer, whose potential was calibrated against the A. I. E. E. standard sphere gap in the secondary circuit, provided a convenient means of reading voltage. The potential terminals of the wattmeter were also connected to the tertiary coil. An ammeter and the current coil of a wattmeter were connected, as indicated in the figure, between the neutral point of the transformer winding and ground. Only one side of the transformer was used to supply the potential to the discharge tube. Protective film cut-outs, connected around the meters, prevented accident in case the ground wires of the meters or tube should become disconnected. A Rowland dynamometer, calibrated over a wide range of power factor as a wattmeter, was used in these early tests for measuring the power input to the discharge tube. The difference between the readings of this dynamometer, with and without the tube connection, provided a measure of the net power supplied to the tube, quite apart from any transformer or corona losses on connecting bus bars. This method of power measurement is the same as that used in determining corona loss on transmission lines previously reported to the Institute.[2]

With the methods of gas absorption and power measurement outlined, it was a comparatively simple matter to titrate the nitric acid formed or determine the amount of alkali neutralized thereby per kw-hr. of energy expended.

Conclusions of Original Investigation. The results of three (3) tests by Messrs. Unger, Payne, and Weimer are listed in Table I.

A study of Table I indicates that thus far the yield by the silent discharge method is very much smaller than those of the arc processes. Some of the conclusions derived from the investigation as indicated seemed however, to warrant further research along similar lines.

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[2] "Corona Losses between Wires at High Voltages," C. Francis Harding, Transactions A. I. E. E. Vol. XXXI (1912), page 1035.

1. The reaction is not necessarily a thermal one, but nitrogen pentoxide (N_2O_5) is formed at low temperatures by the action of the corona at high voltages.

2. Although this reaction is accompanied by much free ozone (O_3), the lower oxides of nitrogen, so objectionable in the arc process, are apparently not present to any great extent, except in case of reversal of the reaction.

3. Since the effluent gases consist principally of a mixture of nitrogen pentoxide (N_2O_5), ozone (O_3) and air, the absorption is more readily accomplished than with the arc process.

TABLE I.

NITRIC ACID YIELDS.

Test	Grams HNO_3 per kw-hr.
No. 1	4.17
2	2.55
3	5.55

4. The gases emerge at comparatively low temperatures. Little energy is therefore carried away in the form of heat and no apparatus is needed to make efficient use of this heat energy.

5. The process is a continuous one and, as such, is more adaptable to commercialization than previous investigations of the silent discharge process which have been limited to a single confined volume of air.

IMPROVED APPARATUS

Encouraged by the possibilities of this process, in spite of the relatively low yields available thus far, the work was taken over upon a larger scale by the Engineering Experiment Station with the assistance of Messrs. H. W. Asire, S. S. Green and H. C. Thuerk.[3]

Apparatus operating upon the same principle, but much larger and more commercial in design, was constructed as indicated in Fig. 3.

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[3] Thesis entitled "The Fixation of Atmospheric Nitrogen by the Silent Electric Discharge Method," 1918 by H. C. Thuerk and S. S. Green.

The corona discharge takes place inside an aluminum tube five feet (1.52 m.) long and six inches (15.2 cm.) in diameter, mounted in a vertical position, the lower end being about 18 inches (45.7 cm.) from the floor. A 5/8-inch (1.59-cm.) aluminum rod is placed on the axis of the tube and extends 18 inches (45.7 cm.) above the end of the tube. The lower end of the rod is screwed into the end of a ½-inch (1.27-cm.) bakelite tube 15 inches (38.1 cm.) in length. A perforated bakelite disk, about four inches (10.2 cm.) in diameter, is arranged so that it may be screwed onto the outside of the bakelite tube. This disk furnishes the support for a glazed porcelain tube of three inches (7.62 cm.)

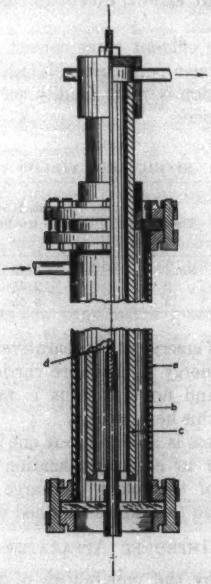

FIG. 3—DISCHARGE TUBE

a.—Wrought iron casing.
b.—Aluminum lining.
c.—Porcelain dielectric.
d.—Aluminum electrode.

inside diameter and ½ inch (1.27 cm.) thickness, the upper end of which projects beyond the end of the aluminum tube, but not as high as the aluminum rod. The top of the tube is closed with a bakelite cap placed over the end of the porcelain tube. A collar, also of bakelite, is inserted between the aluminum and the porcelain tubes. The joints between the porcelain and the bakelite are sealed with a rosin, beeswax and sealing wax compound, which is not appreciably affected by the gases. The bottom of the tube is sealed with a piece of thick plate glass, through the middle of which the bakelite tube is passed. By the use of a mirror, placed on the floor at the proper angle, a view of the inside of the tube is afforded.

Surrounding the aluminum tube is a length of wrought iron pipe, having flanges screwed on either end in such a way that the bakelite collar at the top and the plate glass at the bottom may be clamped securely.

In order that the temperature of the tube may be raised at will, a heating element, consisting of several hundred feet of iron wire, is provided. The heating element, wrought iron pipe and flanges are covered with a magnesia covering which decreases the radiation.

Air enters the tube near the top, and passes downward between the aluminum and porcelain tubes, through the holes in the bakelite disk and up through the inside of the porcelain tube, exhausting through a bakelite tube screwed into the cap over the end of the porcelain. The path of the air is shown in Fig. 4.

A mercury manometer, connected to the inlet pipe, indicates the relative pressure in the discharge tube. The temperatures of the incoming and outgoing gases are measured by alcohol thermometers whose bulbs are in actual contact with the gases. Mercury thermometers were not used on account of the breaking up of the mercury column due to the static charges formed. The mercury is also a conductor which is, of course, a serious disadvantage, and makes their use unsatisfactory.

The aluminum rod is connected to the high-tension transformer, while the outer iron casing is grounded.

With from 50 to 60 kv. impressed on the discharge tube, the space both inside and outside of the porcelain tube may be filled with corona discharge.

Absorption Apparatus. The absorption apparatus consists of an absorbing chamber, separator and precipitator. This apparatus will be more easily understood by reference to Fig. 5.

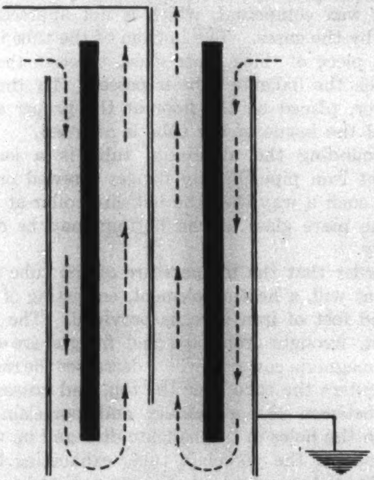

FIG. 4—PATH OF AIR IN DISCHARGE TUBE

The gases, leaving the discharge tube, bubble through a solution of sodium hydroxide ($NaOH$) of known strength, contained in the absorption chamber. The bubbles of gas are made to come into contact with quite a large wetted surface as the chamber is partially filled with very short lengths of small glass tubing. The use of the tubing makes possible relatively high air velocities with but little back pressure. The absorbing liquid is titrated both before and after the run, in order to ascertain the amount of HNO_3 absorbed.

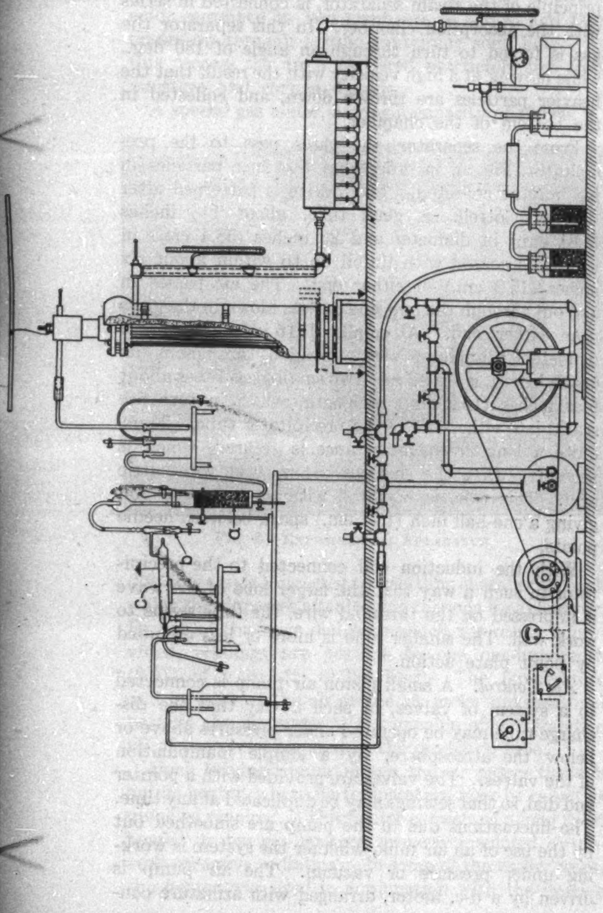

FIG. 5—IMPROVED APPARATUS

a.—Absorption chamber. b.—Separator. c—Precipitator.

At certain air velocities, a fog is formed over the absorbing liquid and, in order to recover the acid carried with it, a separator, Fig. 5, working on the principle of the steam separator, is connected in series with the absorption chamber. In this separator the gas is forced to turn through an angle of 180 deg., while moving at a high velocity with the result that the heavier particles are thrown down, and collected in the bottom of the chamber.

From the separator, the gases pass to the precipitator, Fig. 5, in order that the finer particles of fog may be collected. This device is patterned after that of Cotrell. A glass tube, about 1½ inches (3.81 cm.) in diameter and 23 inches (58.4 cm.) in length, is coated with tinfoil up to within about six inches (15.2 cm.) of either end. The air passes in and out through the two side outlets blown in the glass near either end. A small (1/16-inch (1.59-mm.) diameter) aluminum wire, threaded throughout its entire length, is sealed into two small glass tubes about four inches (10.2 cm.) in length, which in turn are sealed into either end of the precipitator tube. Thus, quite a long creepage distance is secured, which is necessary when the surfaces become wet with nitric acid. This tube is operated with an induction coil giving a one-half inch (1.27 cm.) spark between needle points.

With the induction coil connected to the precipitator in such a way that the larger lobe of the wave is impressed on the threaded wire, the tube seems to work well. The smaller lobe is more or less excluded by point plate action.

Air Control. A small piston air pump is connected to a system of valves, in such a way that the discharge tube may be operated under pressures above or below the atmosphere, by a simple manipulation of the valves. The valves are provided with a pointer and dial, so that settings may be duplicated at any time. The fluctuations due to the pump are smoothed out by the use of an air tank, whether the system is working under pressure or vacuum. The air pump is driven by a d-c. motor, arranged with armature control of the speed. A voltmeter is connected across the armature circuit to act as an indicator of the motor speed.

Air leaving the pump bubbles through sulphuric acid, contained in two chambers in series, and then passes on through a tube containing soda lime. By this process the air is rendered practically free from moisture and carbon dioxide.

A special gas meter, having a capacity of 100 cu. ft., (28.32 liters) is connected to a settling chamber through which the air passes after leaving the soda lime chamber. By use of the large dial with which the meter is provided, readings are possible to one one-hundredth (0.01) of a cu. ft. (0.283 liter). The temperature and pressure of the air at the air meter are measured because the air meter is not capable of withstanding all of the pressures used in the discharge tube.

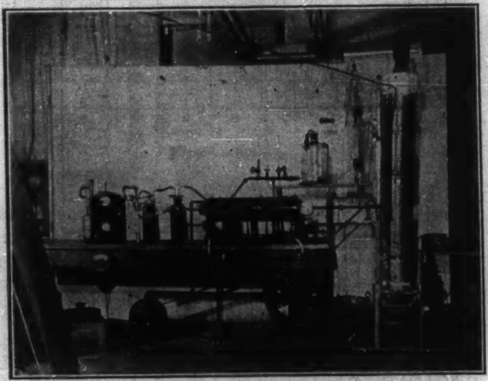

FIG. 6—EXPERIMENTAL APPARATUS

Upon leaving the air meter, the air passes through an air heater, where its temperature may be raised as much as required. This heater consists of a long tube, heated by ten gas burners, properly insulated to prevent excessive radiation. In some of the earlier work, the heater was used in conjunction with the heating element on the discharge tube to hold a constant temperature. It was found however, after several trials, that small temperature changes did not seem to affect the yields materially, and as a result all of the tests recorded in this paper were made at room temperature.

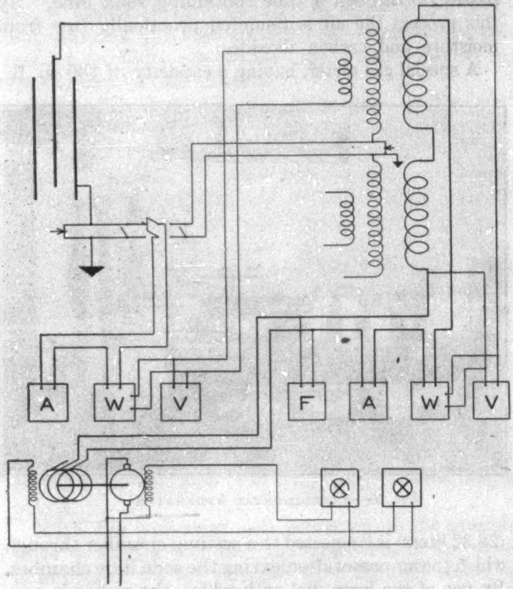

Fig. 7—Diagram of Connections

This was of course held as nearly constant as possible. It is planned to make a set of runs in which the temperature will be made the independent variable. Upon leaving the gas heater the air passes through a valve, used for regulating the tube pressure, and then enters the discharge tube.

Power Measurement. The high-tension winding of the transformer was grounded at its neutral point. The other terminals were connected to high-tension buses. Each half of the secondary may be operated up to 100 kv. The excitation for the transformer is furnished by an alternator which gives practically a sine wave.

A test table has been arranged, equipped with various meters and rheostats. On this table, switching arrangements, Fig. 7, are provided so that the current in either circuit, *i. e.* from transformer to ground or, from discharge tube to ground, may be read on the same milliammeter. These currents differ because of the capacity of the transformer coils to ground, and also because of the actual electrostatic leakage occurring in the transformer.

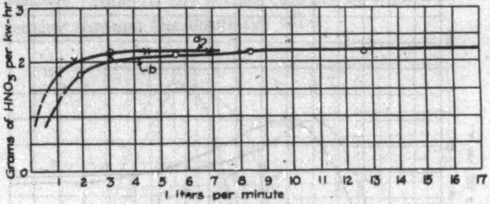

FIG. 8—460 MM.

a.—n. t. p. b.—actual.

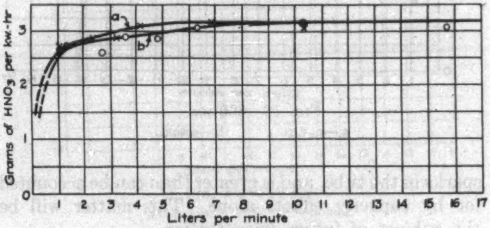

FIG. 9—560 MM.

a.—n. t. p. b.—actual.

A special low-reading wattmeter, whose potential coil is connected to the tertiary coil on the transformer, has its current coil in series with the milliammeter in order that readings of watts in tube circuit and transformer circuit may be taken independently. There is some considerable variation in the readings of the wattmeter in the two circuits with varying pressure in the discharge tube. This variation in wattmeter readings appears to be a function of the static sparks in the tube, and is greater than can be accounted for by capacity effects alone. This matter will be the subject of future investigation.

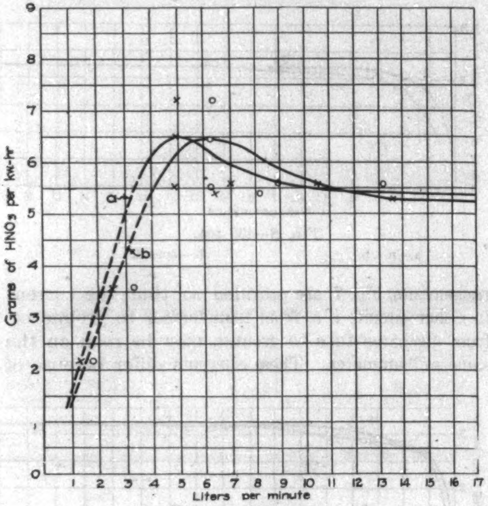

FIG. 10—660 MM.

a.—n. t. p. b.—actual.

Meters were connected in the primary circuit of the transformer for measuring the input power. The net power measured from the primary side has checked that observed on the secondary side within 10 per cent in practically every run made.

Results of Tests. A series of runs at pressures of 460, 560, 660, 710, and 780 mm. has been made. The power in the transformer circuit to ground was held constant during all of these runs. The frequency was 60 cycles. Each run was 45 minutes in length. For each pressure a series of runs was made with air velocity as the variable. Figs. 8, 9, 10, 11, and 12 show the yield of HNO_3 per kilowatt-hour of energy expended in the tube. The curves marked "actual velocity" refer to the actual velocity of a particle in passing through the discharge tube.

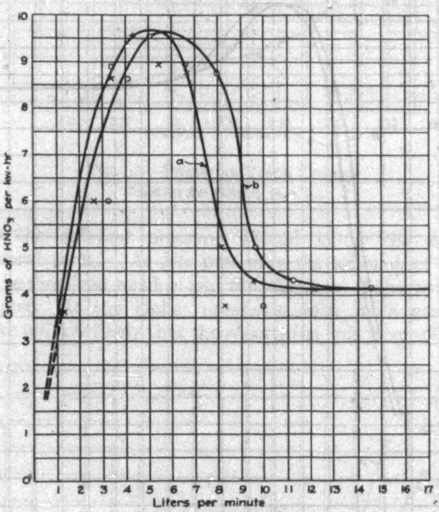

FIG. 11—710 MM.

a.—n. t. p. b.—actual.

Figs. 13 and 14 show the relation between pressure and yield with constant air velocity. From these curves it may be seen that the yield at the lower velocities is limited, but that at the higher velocities the limit has not yet been approached.

From the equations for the formation of nitric acid, it may be seen that the concentration of N_2O_5 will be half that of NO for any given value of nitric acid.

$$2NO + O_3 = N_2O_5$$
$$N_2O_5 + H_2O = 2HNO_3$$

The N O first formed is oxidized to N_2O_5 by the excess of ozone present so the N_2O_5 may be contrasted with 2 N O for the same yield of nitric acid.

The relation between per cent concentration of fixed nitrogen, calculated as N O, velocity and pressure is given in Figs. 15 and 16. These curves show that high concentration is to be secured at low velocity, but as seen in Figs. 13 and 14 at the expense of energy input. The concentration curves are therefore somewhat misleading from a commercial standpoint.

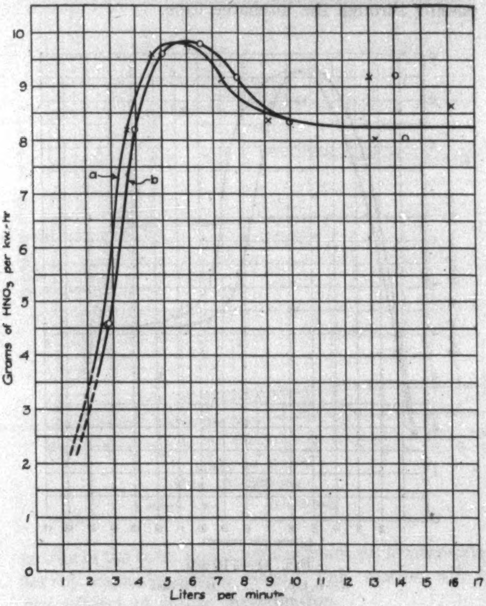

FIG. 12—780 MM.

a.—n. t. p. b.—actual.

It seems probable that all of the velocity-yield runs

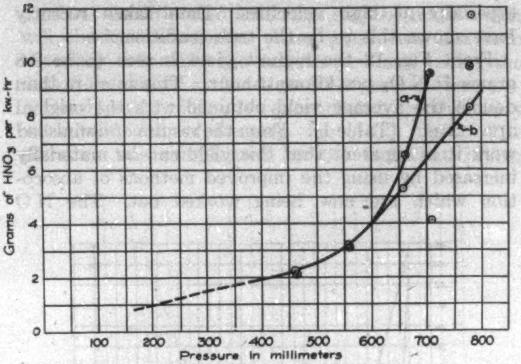

FIG. 13—PRESSURE YIELD CURVES
Constant Air Velocity (n. t. p.).

for the different pressures should follow the same general form. At low pressures the amount of gas to be dissolved was low and the efficiency of absorption higher. At the higher pressures, with more gas to take up, it is likely that the absorption was incomplete,

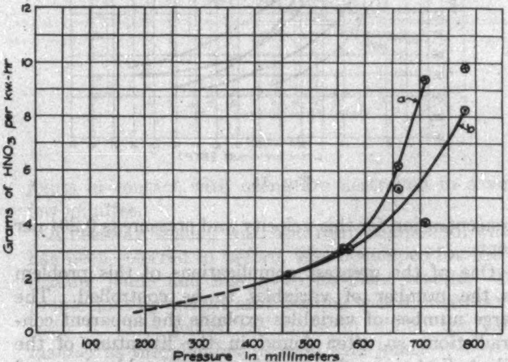

FIG. 14—PRESSURE YIELD CURVES
Constant Air Velocity (actual).

especially at high velocities. Data taken recently have shown this to be the true condition.

From Fig. 12 the highest yield is seen to be 9.8 grams HNO_3 per kilowatt-hour. This is more than double the average yield obtained with the original apparatus. (Table I). From the results of unfinished work it is apparent that this yield can be materially increased by using the improved methods of absorption which are now being worked out. The NO concentration for this velocity and pressure is 0.204 per cent.

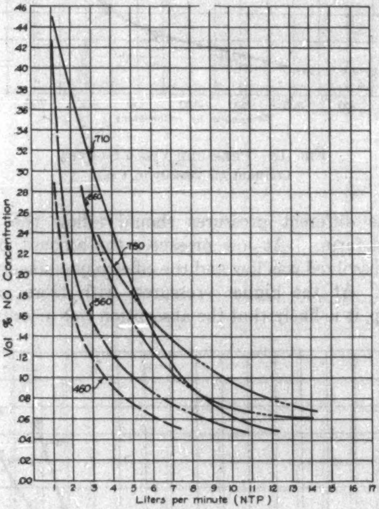

FIG. 15

One of the greatest complications of this problem is the number of variables to be controlled. The large number of variables explains the apparent contradictions so often found in the literature of the silent discharge. It has taken considerable time and patience to acquire the ability to secure sufficiently constant conditions so that runs may be made which will check satisfactorily. Very great care has been necessary in making the various runs and taking the data, to eliminate as far as possible personal errors.

Considerable difficulty encountered in the early work was that of keeping the joints tight. The situation was complicated by the fact that rubber joints in contact with ozone are destroyed in a very few minutes.

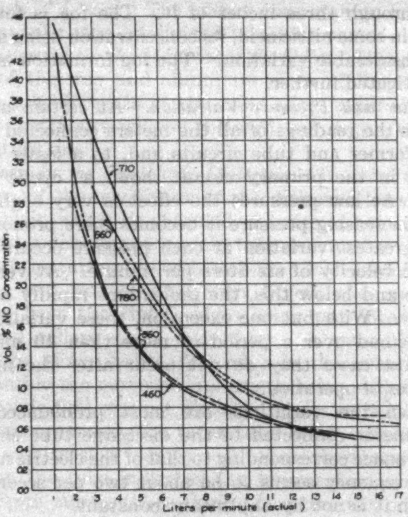

FIG. 16

A beeswax, sealing wax, and rosin compound, which has been referred to before, was developed for use in making joints and has given excellent results. The proportions by weight are: Sealing wax 3; beeswax 2; rosin 5. The joints are made in about the same manner as the plumber wipes a lead joint.

Incidental Phenomena.—Fog. At certain air pressures and velocities a white fog is formed over the surface of the absorption liquid. In the low pressure tests slight fog is formed, while at the higher velocities no fog appears. In many cases the fog is an initial condition disappearing after the first ten or fifteen minutes of operation. As the pressure is increased the fog becomes stronger and in general remains longer, until at 780 mm. pressure the fog, at a velocity of six liters per minute, is so dense that it is not possible to see through three inches of it. The fog is found to contain some nitric acid, its concentration being subject to considerable variation. The fog formation is being investigated further.

Meter and Pressure Variation.—At certain air velocities the readings of all the meters connected in the transformer and tube circuits and, to a lesser degree those in the primary circuit show a cyclic variation. At low pressures the effect is very slight, but with increasing pressure it becomes more pronounced. The greatest variation at each pressure occurs at an actual velocity of six liters per minute. At velocities above and below this, the indications rapidly become steady. With but one exception, these variations do not extend over a period of more than 40 minutes. In most cases they do not occur after the first 20 minutes of operation.

When the variations are most pronounced, the manometer connected to the discharge tube shows a disturbance corresponding to that of the electric meters. The frequency seems to be about two per second, although it is not by any means constant.

From the velocity yield curves given in Figs. 8, 9, 10, 11, and 12, it is to be noticed that the curves begin to flatten out at about the critical velocity of six liters per minute.

In the discharge tube, nitrogen pentoxide (N_2O_5) is being formed, while at the same time, if subjected to discharge for too long a period, it is again broken down. These two actions are therefore opposing one another and both are a function of time. This action may have some bearing on the observed variations.

Another effect which should be mentioned has been termed "Initial effect." During practically every run made, in order to hold the wattmeter deflection constant, the primary voltage had to be reduced as the run progressed. This average potential decrease during runs amounted to about 3 per cent. The tertiary coil voltage did not show as high a percentage variation and in some cases no change was noted. Most of the observed changes took place in the first fifteen minutes of operation.

Analysis of Losses. The energy consumption in the discharge tube may be divided into four parts.

1. Warming up the Material of the Tube. Using the weight of the iron and aluminum parts of the tube, and the specific heats of the metals, an expenditure of 14.14 watthours will be required per degree cent. temperature rise.

2. Loss in the Porcelain Dielectric. In order that some idea of the magnitude of the dielectric losses in the porcelain might be obtained, a series of heat runs on a regular discharge porcelain tube was made. The outside of the tube up to within about a foot (30.5 cm.) of either end was covered with tinfoil, the corona discharge from the tinfoil edges being prevented by pressboard collars sealed over them. A snug fitting metal tube three inches (7.6 cm.) in diameter was slipped inside the porcelain, its length being such that the dielectric field was not distorted by its ends. The tinfoil was grounded, the inner tube being connected to the high-tension supply. Alcohol thermometers placed on the tube indicated the temperature of the tube. From curves taken during this test the power loss in the porcelain dielectric will be of the order of 10 watts, its exact value depending upon the value of the current flowing, in any particular case. From the observed temperatures of the discharge tube and effluent gases the temperature rise of the porcelain is greater than that which would be caused by its own dielectric loss. On account of this the energy calculated as being used in warming up the porcelain will exceed the value obtained by test.

3. Heat Carried away by Discharge Gases. The amount of heat carried away by the air is very small, being of the order of 0.20 watthour per 1000 liters of air per degree cent.

4. *Energy used in the Chemical Reactions.* Using the figures given by Berthelot[4] the amount of energy required for the formation of enough N_2O_5 to produce one gram of nitric acid is 95.4 calories, or 0.11 watthour. For the ozone 667 calories or 0.78 watthours are required.

Assuming that the wrought iron tube and the aluminum tube are at the same temperature, and that the porcelain tube is at the temperature of the discharge gases the following energy relations are secured, based upon their respective weights, specific heats and assumed temperatures.

Metal parts average rise 5.6 deg. cent. loss = 79.2 watthours
Porcelain tube average rise 9.35 deg. cent. loss .... = 27.1 watthours
Effluent gases 9.35 deg. rise—478 liters loss = 0.9 watthour
Chemical reactions 2.13 grams HNO_3 (per hour) .... = 0.24 watthour
0.96 grams ozone = 0.78 watthour
108.2

The power read on the wattmeter when connected in the tube circuit was 134 watts, leaving a difference of 25.8 watthours to be accounted for. Some of this may be due to temperatures existing inside the discharge tube higher than those used in the calculations. This error is not likely to be very large, however.

Conclusions. This work is but well begun, and no prediction can be made with any degree of certainty, yet the results obtained thus far are quite promising. Improved absorption apparatus has already indicated that increased yields may be expected. The process is very simple, and the first cost and maintenance of a plant would not be high. The difficulties of operation associated with the use of high temperatures are obviated.

Referring to Anderegg's calculations,[5] based upon his theory of the reaction, the ideal yield from the silent discharge process, if all the electrical energy were available for the reaction, would be 250 grams of nitric acid per kilowatt-hour. The theoretical possibilities from the thermal reaction, according to thermodynamical calculations, represent a yield of only 134 grams of acid per kilowatt-hour at a temperature of 4200 deg. cent. If the former process can be developed to the present efficiency of the arc process, the yield will be greater and the first cost and operating expenses will probably be much less because of the simpler apparatus involved.

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[4] Smithsonian Physical Tables, 6th Edition.

[5] "The Calculation of the Efficiency of the Silent Discharge Process for Nitrogen Fixation," F. O. Anderegg, Science, 50, 49 (1919) and Chemical Abstracts *13*, 3090 (1919).

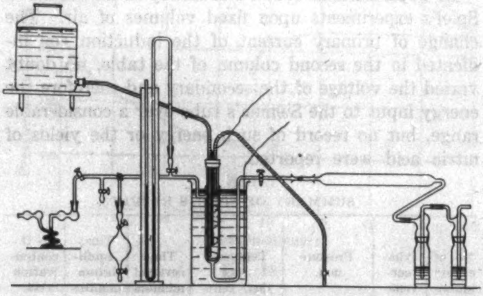

Fig. 17—Spiel's Apparatus

In general, those reactions approximating most closely natural phenomena are found ultimately to be most satisfactory and economical. Nature provides dilute nitric acid for the fertilization of the soil by the ionization of the air as a result of electric discharges in the atmosphere. The gases thus produced are absorbed by the moisture of the clouds or rain and are distributed over the land. Such a process is identical in principle with that under discussion in this paper.

This work has been accomplished only by the generous cooperation of many interested investigators. Dr. F. O. Anderegg in particular, Assistant Professor of Chemistry at Purdue University, has followed the work in detail and has made many suggestions of value. The hearty appreciation of the authors is extended to him, to other collaborators mentioned in the paper, to Professor H. C. Peffer, Head of the School of Chemical Engineering, and Mr. Emerson Pugh, instructor in the School of Electrical Engineering. The departments of Chemistry and Physics have also rendered services which are hereby recognized with appreciation.

The Engineering Experiment Station at Purdue University will continue the investigation of the "Fixation of Atmospheric Nitrogen by the Silent Discharge Process."

APPENDIX

In Table I will be found a summary of the results of Spiel's experiments upon fixed volumes of air. The change of primary current of the induction coil indicated in the second column of the table, no doubt varied the voltage of the secondary and therefore the energy input to the Siemen's tube over a considerable range, but no record of such energy or the yields of nitric acid were reported.

TABLE I.
SUMMARY OF SPIEL'S RESULTS

No. of experiment.	Prim. current.	Pressure mm.	Temperature deg. cent.	Time reversal minutes	Time equilibrium minutes	N O concentration vol. per cent
1	1.9	715 –744	12 –15.3	120	175	0.7
2	1.9	716 –748.5	14 –14.2	110	..	4.4
3	1.9	481.5–503.5	14.3	39	51	0.6
4	1.9	481.5–502	15	43	..	5.6
5	1.9	345.5–353.5	16 –16.1	14	85	0.5
6	1.9	344 –352	15.5–15.7	11	..	6.6
7	3.0	729.5–746	17 –17.1	30	50	0.4
8	3.0	721.5–746.5	16.1–16.2	31	..	4.2
9	5.0	731 –760	14 –14.2	11	80	0.1
10	5.0	731 –750	14	11.5	..	3.6

The analyses were made after the equilibrium was reached for the experiments of odd number. In the even numbered experiments, the analyses were made as near the reversal point as possible.

CONCLUSIONS

The conclusions derived by Spiel as the result of his work on air are quoted as follows:

1. "With the discharge in enclosed volumes of air the final equilibrium concentration gives nitric oxide below one volume per cent for these conditions."

2. "Decrease of the original pressure essentially alters the equilibrium concentration."

3. "Increase in the primary current decreases the equilibrium concentration."

4. "The pressure decrease, which the gas undergoes, as is to be seen from curve, Fig. 17, varies between 8 and 34 mm. This pressure decrease depends not only upon the formation of ozone, but also upon the formation of oxides of nitrogen for, as can be seen from the red color of the gas at the time of reversal, and as has been proven by the determinations of concentration, much more oxidized nitrogen is present before and at the time of reversal, than at equilibrium."

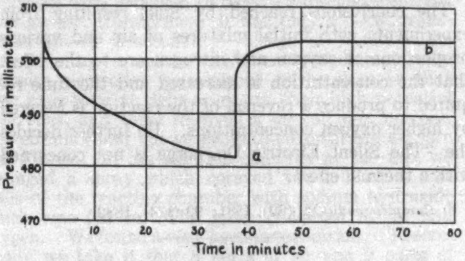

FIG. 18

a.—5.6 vol. per cent N O.
b.—0.6 vol. per cent N O.

This abrupt and strong reversal is a very peculiar phenomenon. It has been checked by Anderegg in the chemical laboratory of Purdue University. The reversal has been shown by Spiel to be affected by changes of temperature. An investigation is being undertaken by R. E. Nelson to see what effect the maintenance of a very constant temperature will have on the reversal.

5. "The concentration at the time of reversal in experiments at atmospheric pressure is about six

times the final concentration. For lower pressures this ratio is raised, the final concentration being lowered; the reversal concentration increases rapidly. With 500 mm. pressure at the time of reversal about 10 times, with 350 mm. about 12 times as much NO is present as in the final equilibrium."

6. "With increased current the reversal as well as the final concentration decreases considerably."

7. "The time after which reversal occurs varies between 12 and 110 minutes; the latter time, which was obtained with air at atmospheric pressure and 1.9 amperes, agrees with the results of Hantefeuille-Chappius."[6]

The conclusions reached by Spiel resulting from experiments with initial mixtures of air and various proportions of oxygen and nitrogen are to the effect that the concentration is increased and the time required to produce a reversal of the reaction is lowered by higher oxygen concentrations. He further decides the "The Silent Electric Discharge is not concerned with a thermal effect."

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[6] *Comptes rendus* 92 (80), 1881; 92 (134), 1882.

DISCUSSION ON "NITROGEN FIXATION BY THE SILENT DISCHARGE METHOD" (HARDING AND McEACHRON), BOSTON, MASS., APRIL 9, 1920.

Mr. Finch: I should like to ask Mr. Harding if he made any runs at a higher pressure, say 2 or 3 atmospheres, at the same time increasing the velocity?

Mr. Benjamin: I might state that in the demand for argon for filling electric light bulbs in 1916, I was called upon by my company to develop a method for the obtaining of argon. In connection with that I naturally turned to the Berklinite process to remove the nitrogen from the air and eventually developed an apparatus which is extremely simple, being a reaction chamber provided with 4 electrodes. These electrodes were nickel. Two diametrically opposite electrodes were arranged to receive a discharge current from an a-c. transformer; it was only a 5-kw. transformer giving a secondary voltage of 30,000 volts. This sets up a spark. Another two diametrically opposite electrodes were placed fairly close so that their axes were at right angles to their high potential electrodes, and these received a 30-volt d-c. current. The circuit could not be established until the a-c. arc was formed; in the reaction chamber at the top, we arranged a spray which sprayed the interior of the walls of the reacting chamber with sodium hydroxide which was fed in. The gases consisted of air and oxygen. We found a very peculiar condition. Theoretically we take it that 8 parts of air and 9 parts of oxygen would form a complete reaction, but we had very poor results, practically no results, until we supplied a surplus of oxygen, amounting to 11 parts of oxygen to 8 parts of air, and then the reaction became violent, the entire reaction chamber was filled with a yellow glow which you could not detect as caused by the d-c. arc or the a-c. arc, being a natural sodium flame effect. Our products were completely converted by circulating sodium hydroxide until the gases were completely consumed and formed sodium nitrate, the residual gas being the rarer elements of the air, such as argon, helium. The other remaining gases we did not attempt to analyze. This process was extremely simple, as it consisted of just one reaction chamber and air circulating system for the sodium hydroxide.

K. B. McEachron: The results, as you can see from the curves, indicate that the higher pressures and higher velocities give the best yields. We have not done any work at any higher pressures than those reported in the paper. In case the pressure was increased materially the voltage would also need to be increased to produce a satisfactory corona. For several reasons this is not feasible with the present design of tube, one of the most important limitations being that the creepage distance at the top of the tube is such that the voltage cannot be increased very materially without danger of flashing over.

We have considered the advisability of introducing certain absorbing liquids, as sodium hydroxide or water vapor, into the discharge chamber. We have found however that dry air is very much better than moist air, and all of the air we use has been dried. The trouble with moist air is that sparks form very readily which seem to hinder the reaction. If the air contains much moisture, a portion of the yield of nitric acid will adhere to the walls of the discharge chamber, thus diminishing the observed yield. The presence of nitric acid inside the discharge tube is also likely to cause a short circuit at the bottom of the tube, where the surface of the insulating materials becomes more or less conducting.

Mr. Tucker: The authors refer to a higher yield than that indicated in the paper; could they give us any figures on that?

K. B. McEachron: We are not quite ready to give any definite values for the increased yields referred to. I might say, however, that we have obtained yields more than twice those mentioned in the paper. The reason why we do not want to give this information is on account of the possible error in our power measurements. To measure 100 or 200 watts with an electric potential of 50,000 or 60,000 volts is a problem which offers considerable difficulty. Until we have more definite information regarding our power measurements, we do not wish to put these data on record. We hope to obtain a check on our measurements by the oscillographic method, which method we are using now.

Mr. Benjamin: I might state that I have experienced trouble with porcelain insulation in that respect and have resorted to lava, I believe made somewhere about Chattanooga, that gave perfect results and no crazing on the surface whatever such as we have experienced with porcelain, and gave perfect electrical insulation.

K. B. McEachron: The process described in the paper should prove to be a valuable one, if the yields can be made to equal or exceed those of the arc process. A large amount of work will necessarily have to be done in order that the conditions which produce the best yield may be determined. When new data of sufficient value has been obtained, further reports will be made to the Institute.

Presented at the 350th meeting of the American Institute of Electrical Engineers, Boston, Mass., April 9, 1920.

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MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-NICKEL ALLOYS

BY T. D. YENSEN

Research Metallurgist, Westinghouse Elec. & Mtg. Co., E. Pittsburgh, Pa.

Part 1. This investigation was undertaken to determine whether any iron-nickel alloys could be found having a higher saturation value than pure iron. Alloys were prepared containing 0-100 per cent Ni.

Pure Fe-Ni alloys do not forge readily, and to make them forgeable it is necessary to add alloying elements like Mn or Ti.

The results show that the saturation value decreases slowly with increase in Ni content up to 20 per cent Ni; then rapidly to 30 per cent; again rises rapidly to 50 per cent and falls off gradually toward 100 per cent Ni. At no point does it exceed that of pure iron.

For values of H between 100 and 400 the permeability is about 5 per cent higher for 6 to 8 per cent Ni than for pure iron, but this advantage is offset by the large increases in hysteresis loss.

Alloys containing 35 to 70 per cent Ni have high permeability at low and medium densities and low hysteresis loss, the highest permeability occurring for 50 per cent.

30 to 50 per cent alloys are characterized by a nearly straight line B-H curve from the origin to $B = 2000$ to 4000 gausses and also by low retentivity and coercive force, properties which are of value in connection with certain electromagnetic meters.

Part II. Previous investigations on commercial iron-nickel alloys have shown that 25 to 35 per cent alloys have irreversible magnetic and electrical transformation points occurring below ordinary temperatures. The present investigation confirms these results for pure alloys. A 30 per cent alloy, annealed and cooled to room temperature, had its saturation value, $4\pi I_s$ increased from 2500 to 17,800 gausses and its electrical resistance decreased from 81 to 32 microhms per cu. cm. after being cooled to liquid air temperature and reheated to room temperature. Alloys containing 15, 35 and 50 per cent nickel showed practically no change after the above treatment. After allowing all transformations from the austenitic state to the α state to take place the curves for $4\pi I_s$ and for electrical resistances both have definite cusps for 34.5 per cent nickel, corresponding to the compound Fe_2Ni, thus giving evidence of the existence of this compound. It is pointed out that the irreversible transformation causes an enormous increase in the hysteresis loss.

PART I

Introduction. The discovery of materials having higher magnetic saturation than pure iron is of great importance in connection with dynamo-electric machinery, because it is the saturation value of the material in the teeth of the armature that determines to a large extent the size of the machines. If the saturation value could be raised 10 per cent, this would mean that the length of the armature—and consequently the field and frame—could be decreased by approximately 10 per cent, with a resultant large saving in the cost of the machine. Cobalt has been known for some time to increase the saturation value; in the case of the alloy Fe_2 Co by as much as 10 to 13 per cent[1]. At high inductions the author has shown[2] that the permeability of Fe_2 Co may be as much as 25 per cent higher than that of pure iron. This alloy would therefore seem to be very suitable for the purpose mentioned were it not for its high cost. Efforts have consequently been made to discover less expensive materials with properties similar to those of Fe_2 Co.

Nickel has been suggested as a possible substitute for Co, in spite of the fact that according to Weiss[3] nickel lowers the saturation value of iron. Burgess and Aston[4] prepared a large number of alloys ranging from 0–100 per cent Ni and among these there were one or two alloys having a slightly higher permeability at high inductions than pure iron. The composition of these alloys was in the neighborhood of 5 per cent Ni. These results and certain others obtained with low nickel steels prepared commercially made it desirable to investigate carefully the possibilities in the direction indicated. Alloys were consequently prepared from the purest materials obtainable and by the most approved method known to yield pure alloys.

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[1] Weiss: Electromagnet with Ferro-cobalt pole pieces. *Comptes Rendus* 156, p. 1970, June 30, 1913. *Trans. Faraday Soc.* 8, p. 149, 1911-12.

Yensen: Magnetic Properties of the Iron-cobalt alloy Fe_2 Co, *Gen. Elec. Rev.*, 18, p. 881 Sept. 1915. PROC. A. I. E. E., 34, p. 2463.

Kalmus and Blake: Magnetic Properties of Cobalt & Iron-cobalt alloys, *Can. Dept. of Mines, Mines Branch, Report* 413, 1916.

Becket: Alloys of Iron and cobalt, U. S. Patent No. 1, 247,-206, Nov. 1917.

[2] Loc. Cit.

[3] Loc. Cit.

[4] Magnetic and Electrical Properties of Iron-nickel alloys, *Met. & Chem. Engr.* 8, p. 23, Jan. 1910.

Preparation. The alloys were made by melting the desired proportions of electrolytic iron and electrolytic nickel placed in magnesia crucibles in an Arsem type vacuum furnace under pressure of 1 to 2 mm. mercury. Chemical analysis of the above materials gave the following results:

Element	Electrolytic Iron	Electrolytic Nickel
S	0.003 per cent	0.006 per cent
P	Trace	0.001
C	0.014	0.030
Mn	Trace	Trace
Si	Trace	0.040
Cu	Trace	.....
Al	0.005	.....
Fe	99.978 (by diff.)	0.250
Ni	.....	99.673 (by diff.)

The first attempts that were made to forge the ingots revealed the fact that pure iron-nickel alloys do not forge very well at ordinary forging temperatures. Up to 8 per cent Ni, and in rare cases with higher nickel contents, fairly good forgings were obtained by forging at a dull red heat, but not by forging at a bright red heat. In other cases the alloys would forge at a black heat, or even cold, while some alloys could not be forged under any conditions. In order to obtain pure iron-nickel alloys in the form of test pieces it was, therefore, necessary in some cases to machine rings directly from the ingots. Forgeable alloys were obtained by means of certain addition agents. Ti and Mn worked successfully for this purpose, while Si, Al and Mg had no effect. The amounts of Ti and Mn required varied from 0.2 per cent for 10 per cent Ni to 1.0 per cent for 50 per cent.

Three kinds of test pieces were prepared:

1. Rings from ingots, 2.46 cm. (31/32 in.) outside

diam., 1.83 cm. (23/32 in.) inside diam. and 0.95 cm. (3/8 in.) long.

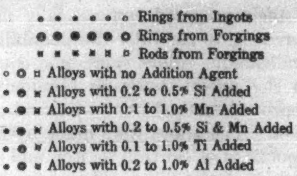
Rings from Ingots
Rings from Forgings
Rods from Forgings
Alloys with no Addition Agent
Alloys with 0.2 to 0.5% Si Added
Alloys with 0.1 to 1.0% Mn Added
Alloys with 0.2 to 0.5% Si & Mn Added
Alloys with 0.1 to 1.0% Ti Added
Alloys with 0.2 to 1.0% Al Added

Fig. 1—Key to Point-Notation of Iron-Nickel Alloys

2. Rings from forgings, 2.46 cm. (31/32 in.) outside diameter, 1.83 cm. (23/32 in.) inside diameter and 0.95 cm. (3/8 in.) long.

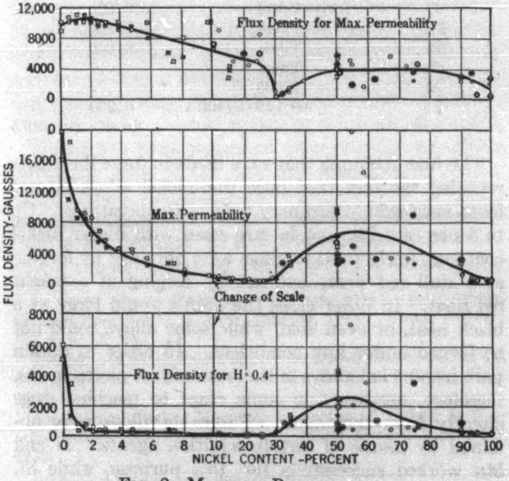

Fig. 2—Maximum Permeability

3. Rods from forgings 1.27 cm. (1/2 in.) diam. and 30.5 cm. (12 in.) to 35.5 cm. (14 in.) long.

The annealing was done at 900 to 950 deg. cent, in rare cases at higher temperatures, in a vacuum furnace, under a pressure of less than 0.1 mm. of mercury, the test pieces being buried in granulated MgO, and the temperature measured by means of a Pt-Pt Rh thermo couple. The cooling was extended over a period of 24 hours and was done at a rate of approximately 30 deg. per hour.

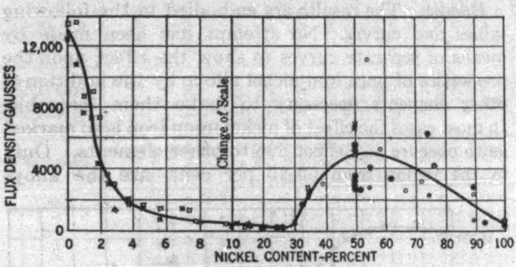

Fig. 3—Flux Density for $H = 1$

Testing. The magnetic testing of the rods was done by the Burrows compensated double bar and yoke method,[5] being the most approved method thus far

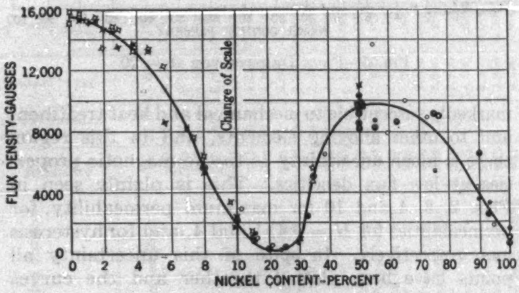

Fig. 4—Flux Density for $H = 4$

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[5] Bull. *Bur. of Stand.* Vol. 6, No. 1, Rep. No. 117. Bull. Eng. Exp. Sta. Univ. of Ill. No. 72 (1914), No. 83 (1915), Trans. A. I. E. E., 33, I. 451 (1914), 34, II., 2601 (1915).

developed for rods. The rings were tested by means of the ordinary two-winding method, no other method being either simpler or more accurate.

Results. The results are embodied in the following tables and curves. No attempt has been made by means of separate curves to show the effect upon the properties of pure iron-nickel alloys by the addition of other elements necessary to make them forgeable. In most cases the effect of nickel upon iron is so marked as to obscure any effect due to other elements. Only in the region from 50–70 per cent are the alloys markedly susceptible to mechanical and heat treatment and to other alloying elements, and in this region there is much uncertainty as to the magnetic properties at low flux densities. This is plainly seen in Figs. 2, 3, 4 and 10 for maximum permeability, for permeabilities for $H = 0.4$, 1 and 4, and for hysteresis loss, respectively. In spite of this uncertainty all points have been plotted together and the curves drawn through the centers of density of the points. The curves thus obtained for $H = 0.4$, 1, 4, 20, 100, 400 and for the saturation intensity, $4\pi I_s$, have all been replotted in Fig. 9 showing at a glance the variation in permeability for various magnetizing forces with variation in nickel content. These curves might be

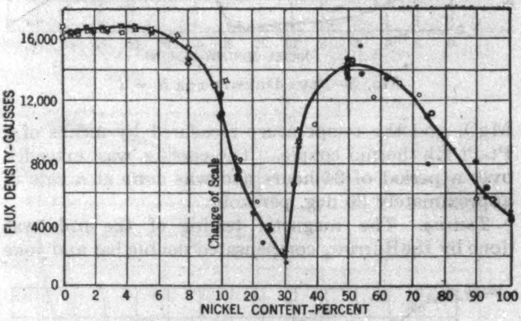

FIG. 5—FLUX DENSITY FOR $H = 20$

TABLE I—MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-NICKEL ALLOYS. ELECTROLYTIC IRON AND NICKEL, MELTED AND ANNEALED IN VACUO

Specimen No.	Ni-content per cent	Kind testpiece	Annealing Temp. deg. cent.	Electrical resist. microhms per cu. cm.	Max. permeability	B for μ_{max}, gausses	Density, B, in Kilogausses for H =							Satur. value, $4\pi I_s$	Hysteresis loss for B = 10,000, ergs/cu. cm./cycle	Retentivity for B = 10,000, gausses	Coercive force for B = 10,000 gilberts/cm.
							0.2	0.4	1	4	20	100	400				
2 Ni 201	0.50	Forged Rods	920	11.5	18,200	10,000	0.6	3.4	13.6	15.7	16.5	18.3	21.2	22,700	1,175	9,200	0.39
203	1.01[1]	" "	920	12.7	9,350	11,000	0.2	0.9	8.7	15.2	16.2	18.4	21.4	22,500	..	..	..
205	1.48	" "	900	13.8	9,100	10,000	0.2	0.8	9.0	15.4	16.7	18.6	21.5	22,700	..	..	..
207	1.96	" "	940	14.9	8,200	10,000	0.2	0.6	7.2	13.4	16.7	18.6	21.6	22,600	1,900	8,700	0.65
209	2.44	" "	940	16.1	6,680	10,000	0.15	0.4	3.6	14.9	16.7	18.75	21.65	22,600	..	..	..
211	2.84[1]	" "	920	17.1	5,650	10,000	0.15	0.4	3.0	14.6	16.8	18.6	21.65	22,700	..	..	..
213	3.85	" "	902	19.4	4,350	10,000	0.1	0.2	1.6	13.2	16.4	18.6	21.8	22,360	3,300	8,300	1.10
215	4.62[1]	" "	902	20.3	4,100	9,000	..	0.3	1.6	13.3	16.7	18.7	21.2	22,300	..	..	..
294	5.68[1]	Ring from ingot[2]	923	..	4,500	3,220	0.2	0.6	4.5	11.0	14.5	16.7	19.1	20,000	2,440	6,600	0.80
217	5.67	Forged rods	920	22.0	3,300	10,000	..	0.1	1.3	12.2	16.7	19.0	21.7	22,400	..	..	..
219	7.33[1]	" "	1,064	24.8	2,130	5,000	0.1	0.3	1.2	7.6	16.2	19.1	21.6	22,600	4,225	5,700	1.45
221	8.00	" "	920	25.2	1,710	8,000	..	0.1	0.5	6.4	15.4	19.1	21.6	22,100	..	..	..
237	9.61[1]	" "	920	27.1	810	10,000	..	..	0.3	3.4	13.0	18.7	21.0	21,200	9,270	5,600	3.3
295	11.10[1]	Ring from ingot	923	..	855	8,900	..	0.1	0.3	2.9	13.3	19.1	21.5	22,000	9,450	5,800	3.70
225	14.92[1]	Forged rods	960	29.8	460	3,000	..	..	0.2	1.5	8.2	17.3	21.0	21,600	12,500	4,200	5.00
296	15.87[1]	Ring from ingot	923	..	475	5,000	..	..	0.2	1.1	8.1	15.2	19.7	22,300	13.080	4,250	5.10
297	21.03[1]	" " "	923	..	276	4,500	..	..	0.1	0.5	5.4	15.2	20.0	22,200	30,300	5,200	13.00
233	34.81[1]	Forged rods	950	80.4	3,600	1,000	0.8	1.4	2.8	5.7	10.0	12.0	12.2	..	910	2,400	0.40
298	38.92[1]	Ring from ingot	923	..	3,060	2,300	0.3	1.0	3.0	6.4	10.4	12.4	12.2	11,900	1,030	2,400	0.45
2,102	54.08[1]	" " "	923	..	19,600	5,200	3.6	7.3	11.1	13.7	15.6	15.6	16.0	15,600	580	7,600	0.18
2,101	57:99[1]	" " "	923	..	14,500	1,888	2.6	3.6	5.2	7.9	12.2	13.8	14.2	13,800	194	3,400	0.14
2,100	66.96[1]	" " "	923	..	3,200	5,500	0.2	0.4	2.6	10.0	13.1	13.6	13.9	13,500	1,330	3,400	0.56
2,104	72.63[1]	" " "	923	..	3,230	4,200	0.2	0.6	3.1	8.8	12.4	12.6	12.9	12,500	922	2,500	0.40
285	75.80[1]	Ring from forging	931	..	4,230	4,400	0.1	0.7	4.2	9.0	11.1	11.4	12.0	12,000	627	2,350	0.34
															(B =4000)	(B =4000)	(B =4000)
2,103	89.83[1]	Ring from ingot	923	..	2,310	2,200	0.2	0.5	2.2	4.3	5.7	6.2	6.2	6,000	592	2,150	0.53
287	95.05[1]	Ring from forging	931	..	500	560	..	0.1	0.5	1.9	5.6	7.0	7.6	7,410	3,015	2,450	2.85
288	99.88[1]	" " "	931	..	400	400	..	0.1	0.4	1.0	4.4	6.0	6.8	6,550	5,650	2,450	5.20

[1] As per Chem. Anal. other alloys as per amount Ni added. [2] Low sp. gr.—not plotted.

termed the magnetic iso-gilberts for the iron-nickel alloys. To this set of curves the following general statements can be applied:

1. The saturation intensity, I_s decreases slightly—not more than 1 or 2 per cent—from pure iron to 20 per cent nickel. Between 20 and 30 per cent nickel I_s decreases to one-tenth of its previous value, increases again rapidly beyond 30 per cent, reaches a maximum for 50 per cent nickel and finally decreases

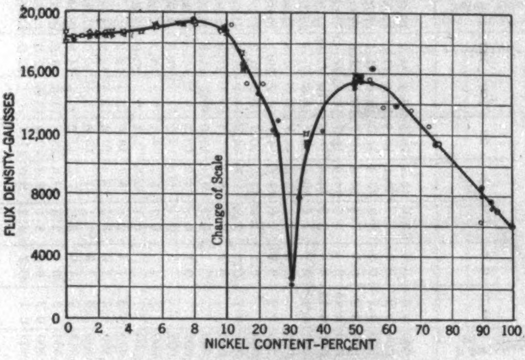

FIG. 6—FLUX DENSITY FOR $H = 100$

gradually toward 100 per cent nickel, the value for pure nickel being between ⅓ and ¼ that for pure iron.

2. The iso-gilberts for high values of H are of the same general shape as that for I_s, except that there is a slight increase—about 5 per cent—in permeability between 0 and 8 per cent nickel.

3. For low values of H the iso-gilberts drop rapidly from pure iron and remain very low until the 30 per cent point has been passed; reaching a maximum between 50 and 60 per cent nickel, for which in some cases a permeability has been reached approaching that for pure iron; and finally decreases to very low values for 100 per cent nickel.

The low saturation value occurring in the region of

TABLE II

MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-NICKEL ALLOYS. ELECTROLYTIC IRON AND NICKEL, MELTED AND ANNEALED IN VACUO. EFFECT OF SI AND MN AS ADDITION AGENTS

Specimen No.	Ni-content per cent	Si-added per cent	Mn-added per cent	Kind Testpiece	Annealing temp. deg. cent.	Electrical resist. microhms per cu. cm.	Max. permeab. μ_m	B for μ_m, gausses	Density, B, in kilogausses for H =							Satur. value, $4\pi I$	Hysteresis loss for B = 10,000, ergs/cu. cm./cycle	Retentivity for B = 10,000, gausses	Coercive force for B = 10,000, gilberts/cm.
									0.2	0.4	1	4	20	100	400				
2 Ni 202	0.50	0.2	..	Forged rods	920	12.7	11,000	10,000	0.2	1.4	10.8	15.3	16.4	18.2	21.1	22,500	1,570	9,100	0.52
204	1.00	0.2	..	" "	900	14.3	8,330	10,000	0.15	0.5	7.6	15.2	16.6	18.4	21.4	22,800	..	..	..
206	1.48	0.2	..	" "	900	15.1	8,530	11,000	0.2	0.7	7.2	15.3	16.8	18.3	21.6	22,500	..	..	..
208	1.96	0.2	..	" "	940	16.5	6,060	10,000	0.15	0.4	4.2	14.7	16.6	18.5	21.5	22,500	2,420	8,700	0.80
210	2.44	0.2	..	" "	920	18.0	5,650	10,000	0.15	0.4	3.0	14.6	16.8	18.4	21.5	22,200	..	..	..
212	2.90	0.2	..	" "	920	19.1	4,550	10,000	0.1	0.2	1.2	13.4	16.5	18.5	21.4	22,500	..	..	..
214	3.85	0.2	..	" "	902	20.7	4,500	9,000	0.1	0.3	2.0	13.6	16.9	18.6	21.5	22,500	3,030	8,100	1.02
216	4.76	0.2	..	" "	920	21.4	4,000	9,000	..	0.2	1.4	13.2	16.9	18.8	21.75	22,800	..	..	..
218	5.67	0.2	..	" "	920	25.8	2,130	8,000	..	0.1	0.6	8.5	16.4	19.1	21.7	22,240	..	..	..
220	7.00	0.2	..	" "	1,064	25.7	2,400	6,000	0.2	0.5	1.3	8.1	16.0	19.2	22.0	22,600	3,760	5,400	1.30
273	8.00	..	0.1	Forged rod	900	26.2	1,300	5,200	..	0.1	0.5	5.2	14.4	19.2	21.9	21,800	5,990	5,900	2.36
				" ring	900	..	1,350	5,400	..	0.1	0.5	5.5	14.6	19.4	22.2	22,700	6,340	5,800	2.20
272	10.00	0.2	0.1	Forged rod	900	28.9	640	6,400	..	0.07	0.2	2.2	12.4	18.5	21.5	21,850	11,200	5,300	4.10
				" ring	900	..	700	7,000	..	0.05	0.2	2.1	11.1	18.8	21.9	22,200	11,100	5,400	4.30
238	9.52[1]	0.2[2]	..	Forged rods	920	29.6	556	10,000	..	0.1	0.3	2.7	10.6	17.0	19.9	20,400	10,150	5,400	3.90
226	15.0	0.2	..	" "	960	31.0	450	4,000	..	..	0.2	1.4	7.5	16.5	20.9	21,300	12,300	4,200	5.00
232	30.0	0.2	..	Forged rods	950	82.0	500	250	0.1	0.2	0.4	1.0	2.1	2.6	3.1	2,700	169	200	0.40
				Ring from ingot	1,100	..	570	250	0.1	0.2	0.5	1.0	1.9	2.6	..	..	220[3]	200[3]	0.60[3]

TABLE II—*Continued*

Specimen No.	Ni-content per cent	Si-added per cent	Mn-added per cent	Kind Testpiece	Annealing temp. deg. cent.	Electrical resist. microhms per cu. cm.	Max. permeab. μ_m	B for μ_m, gausses	Density, B in kilogausses for H =							Satur. value, $4\pi I$	Hysteresis loss for B = 10,000 ergs/cu. cm./cycle	Retentivity for B = 10,000, gausses	Coercive force for B = 10,000, gilberts/cm.
									0.2	0.4	1	4	20	100	400				
283	32.6[1]	..	1.0	Ring from forging	931	..	2,320	720	0.5	0.6	1.7	3.4	6.5	7.9	8.7	8,570	334[5]	950[5]	0.38[5]
234	34.5	0.2	..	Forged rods	950	81.3	2,780	1,500	0.3	1.1	2.5	4.9	9.2	11.4	11.8	11,410	1,040	2,000	0.40
				Ring from ingot	1,100	..	2,210	1	0.2	0.8	2.1	5.0	9.0	11.2	..	..	1,800	1,500	0.80
261	50.0	..	0.75	Forged rod	900	..	9,600	2,870	0.2	4.0	6.8	10.9	14.5	15.7	15.9	15,540	810	4,800	0.29
				" ring	917	6	3,200	4,500	0.9	0.5	2.9	9.1	14.0	15.2	15.7	15,350	1,810	3,600	0.75
262	50.0	..	1.0	Forged rod	900	45.	9,600	2,000	0.2	4.0	6.8	10.9	14.1	15.1	15.5	14,900	810	4,800	0.29
				" ring	917	..	6,060	4,870	0.9	2.4	5.5	10.2	14.5	15.4	16.0	15,450	724	2,300	0.30
263	50.0	0.5	0.5	Forged ring	931	..	3,570	4,000	1.0	0.8	3.4	9.3	13.8	15.5	16.3	15,900	2,315	5,300	0.89
264	50.0	0.25	0.5	" "	900	..	2,940	4,000	0.4	0.8	2.8	8.5	14.2	15.5	15.7	15,800	1,390	2,500	0.60
281	51.4[1]	..	1.0	" "	931	..	3,110	5,600	0.2	0.6	2.7	9.8	14.7	15.8	16.3	15,960	2,155	4,400	0.80
289	54.9[1]	..	0.5	Ring from ingot	931	..	2,860	4,300	0.1	0.3	2.7	8.7	13.7	14.3	14.8	14,500	1,177	2,550	0.68
				Forged ring	923	..	5,000	2,000	0.8	2.0	4.0	8.2	13.7	15.2	15.4	15,100	579	2,300	0.32
279	62.2[1]	..	1.0	Forged ring	931	..	3,300	2,800	0.3	1.0	3.3	8.5	13.4	13.9	14.5	14,160	1,581	3,700	0.55
290	74.8[1]	..	0.5	Ring from ingot	931	..	2,200	2,200	0.1	0.4	2.2	5.3	7.9	8.8	9.2	8,840	706[4]	2,550[4]	0.68[4]
				Forged ring	923	..	9,000	3,600	1.4	3.5	6.2	9.0	11.0	11.4	11.7	11,300	541	3,650	0.24
291	89.9	..	0.5	Ring from ingot	931	..	1,040	2,400	..	0.1	0.6	3.6	6.7	8.0	8.5	8,210	1,453[4]	2,400[4]	1.42[4]
				Forged ring	923	..	2,660	2,800	0.3	0.9	2.7	6.5	8.0	8.5	8.7	8,300	1,222[6]	2,700[6]	0.60[6]
292	93.3[1]	..	0.5	Ring from ingot	931	..	800	2,400	..	0.1	0.3	3.1	5.8	7.2	7.6	7,230	2,055[4]	2,700[4]	1.8[4]
				Forged ring	922	..	630	3,400	..	0.1	0.3	2.4	6.3	7.6	7.8	7,450	4,480[5]	3,400[5]	2.7[5]
293	99.9[1]	..	0.5	Ring from ingot	931	..	500	2,000	..	..	0.4	1.5	4.7	6.2	6.8	6,490	3,640[4]	2,600[4]	3.2[4]
				Forged ring	923	..	316	2,750	..	0.1	0.4	0.8	4.3	6.0	6.3	6,000	5,870[4]	2,400[4]	5.6[4]

1. As per Chem. Anal. other alloys as per amount added.
2. C—0.061 per cent not plotted.
3. For $B = 2{,}000$
4. For $B = 4{,}000$
5. For $B = 6{,}000$
6. For $B = 8{,}000$

30 per cent nickel has been shown to be due to the fact that the magnetic transformation point of these alloys lies below ordinary temperatures. K. Honda in a recently published paper[6] has shown that cooling the alloys in liquid air and then heating them to room temperature raises I_s from the minimum value of about 200 to nearly 1000. At the same time the minimum point has been shifted from 28 to 30 per cent to about 34.5 per cent, the latter corresponding to the compound Fe_2Ni.[7]

Fig. 10 shows the variation of hysteresis loss for $B = 10{,}000$ gausses with nickel content. As might be expected from the iso-gilberts of the previous curve, the loss increases rapidly even with small additions of nickel, and the loss becomes very high for alloys containing 15 to 25 per cent, reaching a maximum of nearly 50,000 ergs per cu. cm. per cycle for the 25 per cent alloy. Between 30 and 70 per cent the loss is as low or even lower than for pure iron, but above 70 per cent the loss again increases rapidly reaching 20,000 for pure nickel.[8]

Fig. 10 shows the retentivity to decrease with increasing nickel content reaching a minimum of only 200 for the 30 per cent alloy. With further increase in nickel content the retentivity again in-

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[6] The Thermal and Electrical Properties of Nickel Steels, *Sci. Repts.*, Tohoku Imp. Iniv. 7, pp. 59-66. (1918).

[7] By the above treatment the iso-gilberts would, of course, also be changed, and the entire set of curves would appear very differently. This matter is discussed in Part II of this paper.

[8] All losses are based on a flux density of 10,000 gausses. In case of the 30 and 80 to 100 per cent alloys having saturation values lower than 10,000 the losses were obtained for as high densities as possible, and the hypothetical loss for 10,000 calculated by means of the formula

$$W_{10} = W_B\left(\frac{10{,}000}{B}\right)^{1.6}$$

where W_B is the loss actually obtained by test for the flux density B. The 1.6 law may not be strictly applicable in the case of the alloys mentioned, but is believed to be sufficiently accurate for the purpose of comparison.

TABLE III

MAGNETIC AND ELECTRICAL PROPERTIES OF IRON-NICKEL ALLOYS. ELECTROLYTIC IRON AND NICKEL, MELTED AND ANNEALED IN VACUO. EFFECT OF Ti AND Al AS ADDITION AGENTS

Specimen No.	Ni-content, per cent	Ti-added per cent.	Al-added per cent	Kind testpiece	Annealing temp. deg. cent.	Electrical resist. microhms per cu. cm.	Max. permeab. μ_m	B fo μ_m, gausses	Density, B, in kilogausses for H =							Satur. valve, $4\pi I_s$	Hysteresis loss for $B = 10{,}000$, ergs/cu. cm./cycle	Retentivity for $B = 10{,}000$, gausses	Coercive force for $B = 10{,}000$, gilberts/cm.
									0.2	0.4	1	4	20	100	300				
2 Ni 236	9.9	0.20	..	Forged rods	920	27.5	605	10,000	..	..	0.2	1.7	11.0	18.9	21.5	22,600	10,700	5,300	4.10
247	15.0[1]	0.20	..	Forged rods	900	30.8	405	4,050	..	..	0.1	1.3	7.3	16.5	20.5	21,600	15,150	4,400	5.70
				" ring	917	..	382	4,000	..	..	0.05	0.9	6.9	16.2	20.2	21,900	13,900	3,900	5.90
248	20.0[1]	0.20	..	Forged ring	917	..	240	6,000	..	..	0.05	0.4	4.8	14.7	19.4	22,200	28,200	4,900	12.00
249	25.0[1]	0.20	..	" "	917	..	187	6,000	..	..	0.05	0.3	3.6	12.0	15.9	18,400	36,900	5,000	15.00
251	50.0[1]	0.50	..	" "	917	..	5,700	4,000	0.6	2.0	5.2	9.8	14.5	15.7	16.0	15,600	852	3,000	0.40
252	50.0[1]	1.00	..	Forged rod	900	47.5	4,960	4,960	0.4	1.3	5.0	9.5	13.9	15.2	15.6	15,200	1,385	4,200	0.62
				" ring	917	..	3,200	3,200	0.4	1.0	3.2	8.2	13.6	15.0	15.6	15,300	1,270	2,600	0.60
2 Ni 240	23.8	..	0.20	Ring from ingot	950	..	200	5,000	..	..	0.03	0.2	2.8	12.1	16.3	19,200	10,860[2]	2,300[2]	12.80[2]
241	25.1	..	0.20	" " "	950	..	172	8,600	..	..	0.05	0.3	3.8	12.9	17.0	19,700	8,950[2]	2,400[2]	10.20[2]
242	29.8	..	0.20	" " "	950	..	400	400	..	..	0.4	0.8	1.5	2.1	2.8	2,860	278[3]	150[3]	0.50[3]

1. Per cent Ni added, alloy not analysed chemically.
2. For $B = 4{,}000$
3. For $B = 1{,}500$

creases, reaching a maximum between 50 and 60 per cent nickel.[9]

Fig. 12 shows the variation in electrical resistance checking very closely the results obtained by Burgess and by Honda.[10] The curve has been drawn through the points for pure iron-nickel alloys for all cases in which alloys were available in the form of rods. In every case it will be noticed that the alloys containing a small amount of silicon have a decidedly higher resistance than the alloys without silicon, as might be expected. The curve reaches a maximum for 30 per cent nickel corresponding to the composition of the alloy having the lowest magnetic saturation value. Honda has obtained a decrease in electrical resistance corresponding to the increase in the

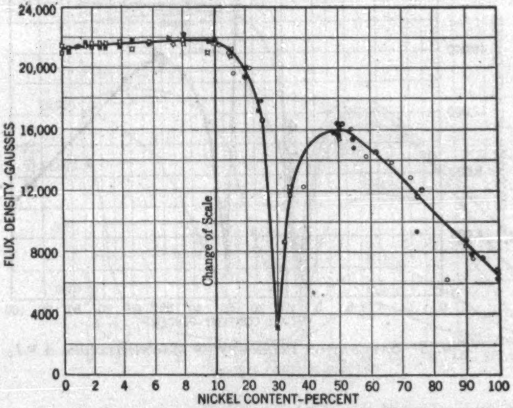

FIG. 7—FLUX DENSITY FOR $H = 400$

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[9] No correction has been made in case the alloy was tested at a lower density than 10,000, but the error is supposed to be small in every case. Examples: 2 Ni 285, containing 75 per cent nickel was tested at 10,000 gausses, showing a retentivity of 2350, while 2 Ni 290, having the same nickel content, tested at B = 4000 gave a retentivity of 2550.

[10] Loc. Cit.

saturation value above referred to, by cooling the alloys in liquid air and then heating to room temperature. For the 30 per cent alloy the resistance decreased from a value of approximately 90 to 30 microhms, whereas no change is shown for the 34.5 per cent alloy corresponding to Fe_2Ni, making this the alloy having the maximum resistance.[11]

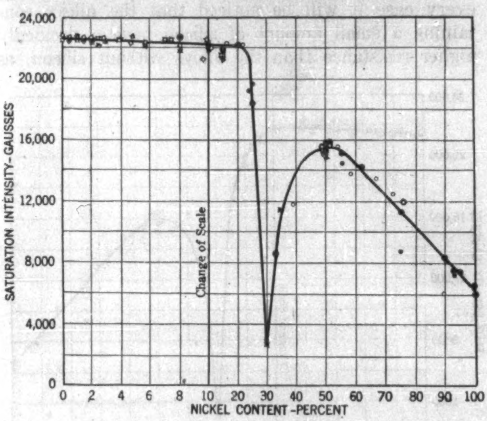

FIG. 8—SATURATION INTENSITY OF MAGNETIZATION $4\pi I_s$

The specific gravity shown in Fig. 13 was obtained incidentally as a means of obtaining the true cross section of the ring test pieces. The values increase uniformly from 7.9 for pure iron to 8.9 for pure nickel.

PART II

REVERSIBLE AND IRREVERSIBLE TRANSFORMATIONS

Introduction. In Part I of this paper it was pointed out that the properties of alloys in the neighborhood of 30 per cent nickel would be very different after being cooled to liquid air temperature from after being cooled merely to room temperature. It was further pointed out that this difference is due to the fact that these alloys—in addition to the reversible transformation point occurring above room temperature—have an irreversible transformation point occurring below room temperature. These transformations have been studied a great deal in recent years, the pioneers in the field being Hopkinson (1), LeChatelier (2), Osmond (3), Guillaume (4) and Dumas (5).[12] The alloys investigated by these scientists all contained foreign elements to some extent, particularly carbon and manganese. As a matter of fact manganese was considered an essential constituent, because without it, the alloys could not be

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[11] Further data on this subject is contained in Part II.

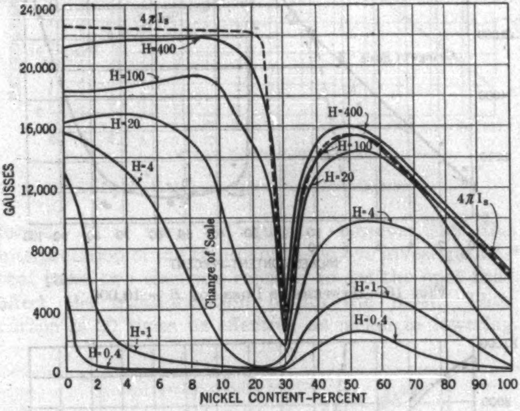

Fig. 9—Saturation Intensity $4\pi I_s$ and Flux Density for Various Magnetizing Forces

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[12] (1) *Proc. Royal Soc.*, 46 and 47, p. 23, 1889, 48 p. 1., 1890.

(2) *Comptes Rendus*, 90 p. 285, 1890.

(3) *Comptes Rendus*, 118, p. 532, 1894; 121, p. 684, 1895; 128, pp. 306, 1395, 1513, 1899.

(4) *Comptes Rendus*, Jan. 25, April 5, June 18, July 26, 1897.

(5) *Comptes Rendus*, 139, p. 42, 1899, *Jour. Iron & Steel Inst.*, II, p. 225.

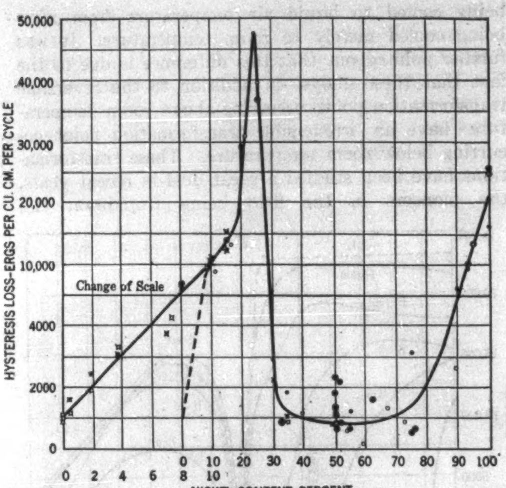

FIG. 10—HYSTERESIS LOSS FOR $B = 10,000$

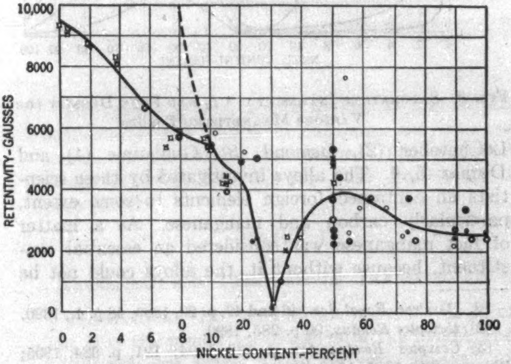

FIG. 11—RETENTIVITY FOR $B = 10,000$ GAUSSES

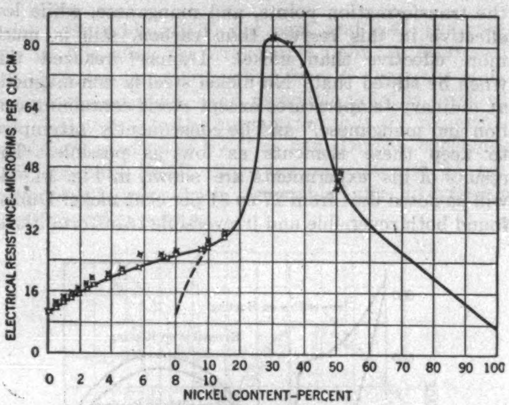

FIG. 12—ELECTRICAL RESISTANCE

forged. It is very unfortunate, however, for the interpretation of the results of the above investigations that these two elements referred to are the ones that affect the transformation points the most. Thus carbon is 20 times as effective as nickel in lowering

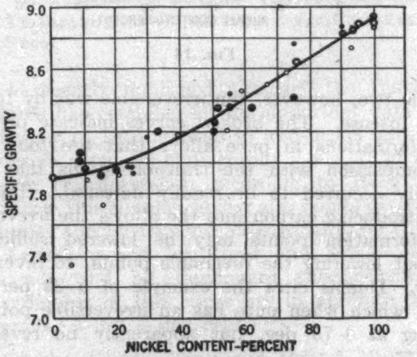

FIG. 13—SPECIFIC GRAVITY

the transformation points, and manganese, while less effective in this respect than carbon, still is much more effective than nickel. Dumas[13] realized this when he stated that "No nickel steel is non-magnetic at ordinary temperatures except steels containing carbon and manganese," and he consequently attempted to keep these elements as low as possible. The results of his experiments are shown in Fig. 14. It will be noted that from 27 to 31 per cent nickel Dumas found both reversible and irreversible transformations.

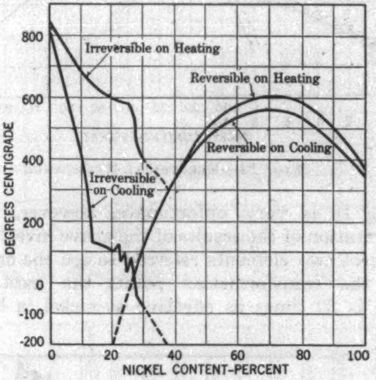

FIG. 14

Of the two, however, the irreversible was by far the more intense. The broken curves indicate probable transformations in pure alloys that are too feeble in comparison with the transformations that have already occurred to be readily detected. However, by introducing carbon into the alloys, the irreversible transformation points may be lowered sufficiently without lowering the reversible points, to reveal the latter. Dumas cites the example of a 23 per cent alloy, which when pure has an irreversible point on cooling at + 75 deg. but apparently no reversible point. By adding 0.85 per cent and 1.41 per cent Mn to this alloy the irreversible point is decreased to −188 deg., and on cooling from 0 deg.—the alloy is now non-magnetic—the reversible point is revealed at —150 deg. On the assumption that the reversible points are unaffected by C or Mn, Dumas concludes that the pure Fe-Ni alloys between 20 and 37 per cent nickel have both reversible and irreversible transformation points.

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[13] Loc. Cit.

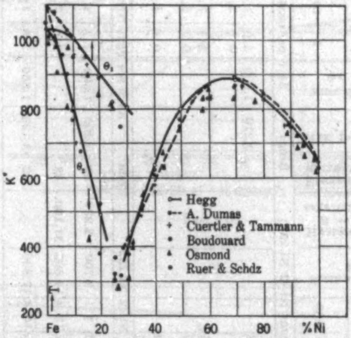

FIG. 15—ABSOLUTE CENTIGRADE TEMPERATURES OF MAGNETIC TRANSFORMATION (CURIE POINTS) FOR PURE FERRONICKELS (HEGG)

In a recently published circular[14] the Bureau of Standards has compiled all the important data published prior to 1916 relating to iron-nickel alloys and includes an extensive bibliography. Fig. 15 gives the magnetic transformation according to Hegg reproduced from this circular, giving also points determined by Dumas and by Osmond magnetically, by Guertler and Tammann and by Rurer and Schuz metallic graphically and by Boudouard thermoelectrically. The ordinates of this curve are in absolute degrees centigrade (= 273 + deg. cent.)

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[14] Circular No. 58. Invar and Related Nickel Steels April 4, 1916.

TABLE IV

MAGNETIC AND ELECTRICAL PROPERTIES OF Fe-Ni ALLOYS. REVERSIBLE AND IRREVERSIBLE TRANSFORMATIONS

Condition	Max. permeability, μ_m	Flux density for μ_m, gausses	Flux density (kilogausses) for H (gilberts/cm.) of				Saturation value, $4\pi I_s$	Hysteresis loss for $B = 10{,}000$—ergs/cu. cm./cycle	Electrical resist.—microhms/cu. cm.	Max. permeability, μ_m	Flux density for μ_m, gausses	Flux density (kilogausses) for H (gilberts/cm.) of				Saturation valve, $4\pi I_s$	Hysteresis loss for $B = 10{,}000$ ergs per cu. cm. per cycle	Electrial resistance. microhms per cu. cm
			1	10	100	400						1	10	100	400			
	30 per cent Ni (2 Ni 232)								Rings	34.5 per cent Ni (2 Ni 234)								
Annealed at 1,100 deg. c. in Vacuo..	560	220	0.4	1.5	2.8					2,210	1,500	2.0	7.2	11.2				
Cooled to —70 deg.	..	..	..	..	..							0.5	5.3	13.1				
Cooled to —70 deg. and heated to + 20 deg.	232	4,650	0.1	2.0	10.3							2.1	7.0	11.1				
Cooled to —185 deg.	279	5,580	0.1	2.3	13.0							0.3	5.3	13.4				
Cooled to —185 deg. and heated to + 20 deg.	244	4,880	0.1	1.9	12.0							0.6	4.7	11.2				
	30 per cent Ni (2 Ni 232)								Rods	34.5 per cent Ni (2 Ni 233 and 234)								
Annealed at 900 deg. c. in Vacuo....	500	250	0.4	1.5	2.6	3.1	2,700	2,230[1]	82	3,200	1,300	2.7	8.0	11.7	12.0	11,500	1,000	81
Cooled to —70 deg. and heated to + 20 deg.	213	4,550	0.1	1.8	10.2	13.4	14,600	31,100	38	..	..	2.6	7.7	11.6	12.0	..	..	80
Cooled to —185 deg. and heated to + 20 deg.	250	5,750	0.1	1.8	12.5	16.4	17,800	..	32	3,120	1,560	2.6	7.6	11.7	12.1	11,710	..	78
	15 per cent Ni (2 Ni 225)								Rods	50 per cent Ni (2 Ni 261 and 262)								
Annealed at 900 deg cent. in vacuo.	460	3,000	0.2	4.9	17.3	21.0	21,600	12,500	30	9,600	2,870	6.8	13.0	15.4	15.6	15,200	810	44
Cooled to —70 deg. and heated to + 20 deg.	494	3,600	0.2	4.8	17.2	21.0	..	..	30	..	..	6.8	12.9	15.2	15.5	..	..	44

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[1] Calculated from the loss for $B = 2{,}000$ by means of the 1.6 law $= W_{16} = W_2\,(10/2)^{16}$.*

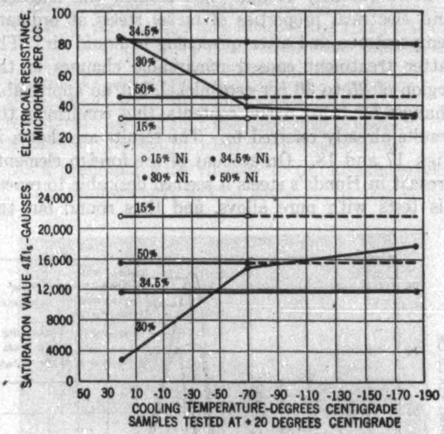
ELECTRICAL RESISTANCE, MICROHMS PER CC.
100
80
60
40
20
0
34.5%
30%
50%
15%
○ 15% Ni
● 34.5% Ni
● 30% Ni
● 50% Ni
SATURATION VALUE $4\pi I_s$ -GAUSSES
24,000
20,000
16,000
12,000
8000
4000
0
15%
50%
34.5%
30%
50
30
10
-10
-30
-50
-70
-90
-110
-130
-150
-170
-190
COOLING TEMPERATURE-DEGREES CENTIGRADE
SAMPLES TESTED AT +20 DEGREES CENTIGRADE

FIG. 16

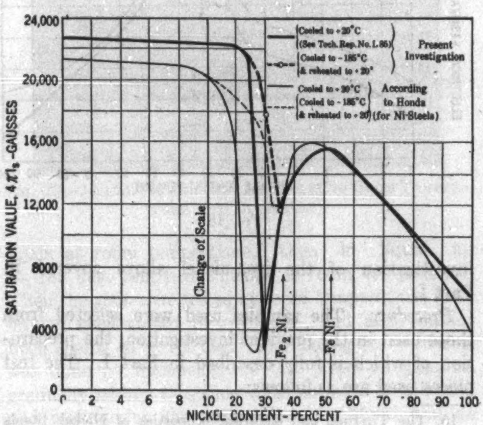
24,000
20,000
16,000
12,000
8000
4000
0
SATURATION VALUE, $4\pi I_s$ -GAUSSES
Cooled to +20°C (See Tech. Rep. No. L 85)
Cooled to -185°C & reheated to +20°
Present Investigation
Cooled to +20°C
Cooled to -185°C & reheated to +20°
According to Honda (for Ni-Steels)
Change of Scale
Fe_2Ni
FeNi
0
2
4
6
8
10
20
30
40
50
60
70
80
90
100
NICKEL CONTENT-PERCENT

FIG. 17

More recently Honda[15] has studied the magnetic and electrical properties of nickel steels at ordinary temperatures and after quenching in liquid air. The latter treatment caused remarkable changes in the region of 25 to 30 per cent nickel, but no appreciable changes for other nickel contents, thus confirming the results already referred to. The results are shown in Figs. 17 and 18. On account of the foreign elements present in Honda's steels it seemed desirable to repeat his tests with pure alloys, and thus round out the investigation of the iron-nickel alloys covered in Part I.

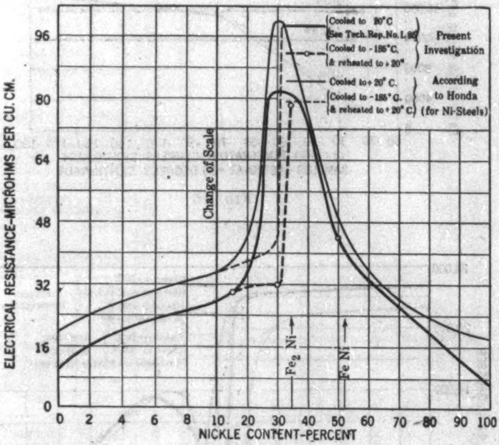

FIG. 18

Procedure. The samples used were selected from those used in the previous investigation, the preparation of which is fully described in Part I. The test pieces used are as follows:

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[15] The Thermal and electrical properties of Nickel Steels *Sc. Reports* Tohoku Imp. Univ. 7, p. 59, 1918.

2 Ni 225—15 per cent Ni, 2 Rods 1.27 cm. (½ in.) diameter.
2 Ni 232—30 per cent Ni, 2 Rods as above and 1 ring 3.83 cm. (1½ in.) inside diam. and 4.47 cm. (1¾ in.) outside diam.
2 Ni 233—34.5 per cent Ni, 1 Rod and 1 Ring as above.
2 Ni 234—34.5 per cent Ni, 1 Rod and 1 Ring as above.
2 Ni 261—50 per cent Ni, 1 Rod and 1 Ring as above.
2 Ni 262—50 per cent Ni, 1 Rod and 1 Ring as above.

As a preliminary the rings were tested successively at room temperature, in CO_2 snow (– 70 deg. cent.), again at room temperature, then in liquid air (– 185 deg. cent.) and finally at room temperature. Then the rods were tested at room temperature, after having been cooled to – 70 deg., and after having been cooled to – 185 deg. The cooling to – 70 deg. was done by immersing the test pieces in gasoline, gradually adding CO_2 snow until no more would melt and maintaining the temperature for about half an hour by adding more snow from time to time.

FIG. 18—2 NI 232, 30 PER CENT NI COOLED TO ROOM TEMPERATURE. (100 DIAMETERS)

Results. The results are tabulated in Table IV and shown graphically in Figs. 16, 17 and 18. It will be noted that while there is practically no change in the properties of the 15, 34.5 and 50 per cent alloys the 30 per cent alloy has undergone a marked change. Its saturation value, $4 \pi I_s$, was increased from 2700 to 14,600 by cooling to – 70 deg. and to 17,800 by cooling to — 185 deg. Similarly, the electrical resistance was decreased from 82 to 38 and to 32 by the above treatments. It is probable that the change would have gone to completion at – 70 deg. if the testpiece had been held there sufficiently long and that the irreversible transformation point therefore lies between 0 and — 70 deg. thus confirming Dumas' results.

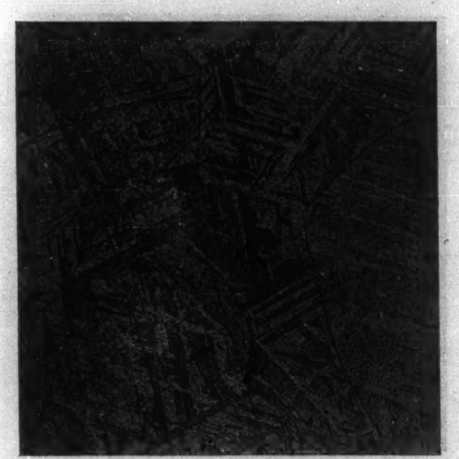

Fig. 20—2 Ni 232 30 per cent Ni Cooled to —70 deg. cent. and Reheated to Room Temperature. (100 Diameters)

While only a few points were determined, they are sufficient to demonstrate that the cusp of both curves, after the completion of the transformations, occurs for approximately 34.5 per cent nickel instead of for 30 per cent as the previous investigation would lead one to believe. This points strongly towards the existance of the compound Fe_2Ni, corresponding to 34.6 per cent nickel, suggested by earlier investigators. The existence of the compound FeNi corresponding to 51.5 per cent nickel, is more doubtful, because there is no definite indication of it in the electrical resistance curve.

It is interesting to note that while the saturation value of the 30 per cent alloy was increased to a remarkable extent by being cooled to below the irreversible transformation point, the permeability at low flux densities was decreased and the hysteresis loss enormously increased, the latter 15 times. In other words, the reversible transformation produces a material with low hysteresis loss and low magnetic saturation, while the irreversible transformation produces a material with 6 times the saturation value, but with 15 times the hysteresis loss.

It is also interesting to note that there apparently is no change in the microstructure of the 30 per cent alloy accompanying the marked magnetic and electrical transformation. Fig. 19 shows the microstructure of the alloy cooled to room temperature, and Fig. 20 after cooled to −70 deg. cent. and reheated to room temperature. The latter corresponds to magnetic and electrical properties widely different from those of the former, and yet the microstructures at 100 diameters are identical as far as can be seen. It is therefore apparent that the magnetic and electrical transformations are atomic or ionic in their nature.

DISCUSSION ON "ELECTRIC AND MAGNETIC PROPERTIES OF NICKEL-IRON ALLOYS" (YENSEN), BOSTON, MASS., APRIL 9, 1920.

S. L. Gokhale (by letter): In this discussion I intend to limit my remarks to two points only, *viz.*, saturation values and methods of measuring them.

For measurement of magnetization, with magnetizing force up to 400 gilberts, Mr. Yensen has used the Bureau of Standards method, popularly known as Burrows method. He has not specified how the saturation values have been measured, and extrapolation according to Kennelly's law, does not in all cases fit the figures given by him. According to Mr. Yensen, the saturation value of iron is not improved by the presence of nickel. In this respect his findings are opposed to those of Burgess and Aston, who seem to have produced three samples having saturation values higher than pure electrolytic iron, by introducing 7 to 10 per cent of nickel, as has been observed by Mr. Yensen on page 792.

Burgess and Aston have used for their tests, methods which are now superseded by the Burrows method. Their tests stop at $H = 100$, and it would be scarcely fair to place their findings along side those of Mr. Yensen which are made with a better method and extend over a much greater range.

Computing the saturation values for different alloys by the extrapolation method, using data in Mr. Yensen's paper, I find that the quality of iron improves by the introduction of nickel up to about 7 per cent of nickel, in the presence of 0.2 per cent of silicon and with only about 4 per cent of nickel when there is no silicon. For complete computation giving the saturation values, by extrapolation from Mr. Yensen's data, see Tables I and II attached.

This raises the question of the validity of the extrapolation formula, for the determination of saturation values. The importance of the saturation values of materials both for theory and for practical engineering application is too obvious to need further comments, and it is scarcely necessary to emphasize the need of some direct and reliable method for obtaining the necessary data. To this task I have devoted a considerable part of my time during the last six years.

The experimental work has just been completed (that is, in the only sense in which an experimental work can ever be completed). A method for measurement of saturation values, has been developed giving results correct to 0.3 per cent. A paper for publication explaining the method is in progress and will

TABLE I

SATURATION VALUES FOR IRON—NICKEL ALLOYS

With pure Iron and Nickel
By extrapolation method based on Kennelly's Law.
From data by T. D. Yensen, (page 797).

Specimen	Nickel per cent	B for $H = 400$	$H = 100$	$B - H$ for $H = 100$	$H = 400$	ρ for $H = 100$	$H = 400$	$\Delta \rho$	Sat. $\frac{\Delta H}{\Delta \rho}$	Sat. (Yensen)
2 Ni.										
201	.50	18.3	21.2	18.2	20.8	.00550	.01923	.01373	21830	22700
203	1.01′	18.4	21.4	18.3	21.0	546	1903	1357	22100	22500
205	1.48	18.6	21.5	18.5	21.1	540	1895	1355	22150	22700
207	1.96	18.6	21.6	18.5	21.2	540	1885	1345	22300	22600
209	2.44	18.75	21.65	18.65	21.25	536	1880	1344	22330	22600
211	2.84′	18.6	21.65	18.5	21.25	540	1880	1330	22550	22700
213	3.85	18.6	21.8	18.5	21.4	540	1870	1330	22550	22360
215	4.62′	18.7	21.2	18.6	20.8	537	1923	1386	21650	22300
294	5.68	16.7	19.1	16.6	18.7	601	2140	1539	19500	20000
217	5.67	19.0	21.7	18.9	21.3	529	1880	1351	22200	22400
219	7.33′	19.1	21.6	19.0	21.2	526	1885	1359	22090	22600
221	8.00	19.1	21.6	19.0	21.2	526	1885	1359	22090	22100
237	9.61′	18.7	21.0	18.6	20.6	537	1940	1403	21350	21200
295	11.10′	19.1	21.5	19.0	21.1	526	1895	1369	22000	22000
225	14.92′	17.3	21.0	17.2	20.6	581	1940	1359	22100	21600
296	15.87′	15.2	19.7	15.1	19.3	661	2070	1409	21300	22300
297	21.03′	15.2	20.0	15.1	19.6	661	2040	1379	21800	22200

TABLE II
SATURATION VALUES FOR IRON—NICKEL ALLOYS

Iron and Nickel with 0.2 per cent silicon
By extrapolation method based on Kennellys' Law
From data by T. D. Yensen pages 779 and 800.

Specimen	Nickel per cent	B for $H = 100$	$H = 400$	$B - H$ for $H = 100$	$H = 400$	ρ for $H = 100$	$H = 400$	$\Delta \rho$	Sat. $\frac{\Delta H}{\Delta \rho}$	Sat. (Yensen)
Ni										
202	0.50	18.2	21.1	18.1	20.7	.00553	.01933	.01380	21750	22500
204	1.00	18.4	21.4	18.3	21.0	546	1905	1359	22100	22800
206	1.48	18.3	21.6	18.2	21.2	550	1885	1335	22400	22500
208	1.96	18.5	21.5	18.4	21.1	543	1895	1350	22200	22500
210	2.44	18.4	21.5	18.3	21.1	546	1895	1340	22300	22200
212	2.90	18.5	21.4	18.4	21.0	543	1905	1362	22000	22500
214	3.85	18.6	21.5	18.5	21.1	540	1895	1355	22150	22500
216	4.76	18.8	21.75	18.7	21.35	535	1872	1337	22490	22800
218	5.67	19.1	21.7	19.0	21.3	526	1878	1354	22150	22240
220	7.00	19.2	22.0	19.1	21.6	524	1850	1326	22600	22600
273	8.00	19.2	21.9	19.1	21.5	524	1860	1336	22500	21800
272	10.00	18.5	21.5	18.4	21.1	543	1895	1352	22200	22700

NOTE: Sample No. 273 contain 0.1 per cent manganese but no silicon content.
Sample No. 272 contain 0.1 per cent manganese and 0.2 per cent silicon
The remaining samples contain 0.2 per cent silicon.

be published at the earliest possible date. This method briefly stated is as follows: The sample under test, generally a rod ½ in. diameter and about 12 in. long, or an equal sized bundle of strips is magnetized by a coil capable of producing 3000 gilberts. The magnetic circuit is completed by a heavy yoke of soft iron. The magnetizing coil is connected in series with the primary coil of a variable mutual inductor. The potential coil surrounding the sample near its middle is connected in series with the secondary coil of this variable inductor, and also in series with a galvanometer of high sensitivity. See Fig. 1.

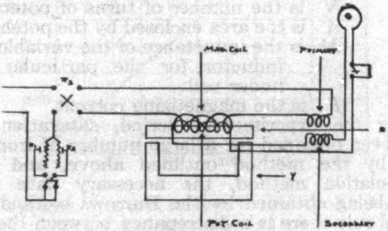

FIG. 1

S = Sample Under Test. Y = Yoke. R_S = Reversing Switch. R_1 and R_2 = Rheostats for Controlling Independently, Two Points on the Saturation Curve. Other Details Omited in Diagram.

The magnetizing current is first adjusted to give a force of about 2200 gilberts, exact adjustment or measurement not being necessary; this adjustment is made by the rheostat R_1. The magnetizing force is now raised to about 3000 gilberts, the necessary adjustment being made by the rheostat R_2. If at $H = 2200$ the sample is saturated, it will remain saturated at $H = 3000$; the increase of flux in this case will be due to increase of H only. If also the design of the permeameter is such as to make H, proportional to the magnetizing current within the limits of H, 2200—3000, this increment in H can be neutralized by a proper setting of the mutual inductor, and conversely if the mutual inductor can be so arranged as to give a zero deflection for all upward changes in the magnetizing current, the sample should be regarded as saturated.

A reversal of the magnetizing current will under these circumstances give an effective reversal of the intrinsic flux *B-H* only; or in other words the ballistic deflection of the galvanometer under these conditions will give directly the saturation value, without any intermediate determination of *H* or without any computations or corrections dependent on *H*. In this method it is therefore, not necessary to measure the value of *H*, either accurately or approximately, but if desired, it may be computed by the formula

$$H\,N\,A = \frac{M\,I}{10}$$

Where H is the magnetizing force producing saturation.
N is the number of turns of potential coil.
A is the area enclosed by the potential coil.
M is the inductance of the variable mutual inductor for the particular setting under test.
I is the magnetizing current.

During the experimental period, saturation values have been obtained for a large number of iron alloys, both by the method outlined above, and by the extrapolation method, the necessary data for the latter being obtained by the Burrows method. As a general rule there is a discrepancy between the results obtained by the two methods; in extreme cases this discrepancy has reached 10 per cent, but as a general rule it does not exceed 3 per cent.

Such a discrepancy may be due partly to errors inherent in Burrows method and partly in the extrapolation formula. The possibility of error in the data obtained directly by the method outlined above, is not to be ignored, but evidence of such error has not so far appeared. Incidentally I may mention at this point, that the high saturation values $4\pi J$ shown by Mr. Yensen are very unusual.

Prof. B. O. Pierce has given results of test on four samples of American Ingot iron; the highest saturation value obtained was 21780. (*Proceedings of the American Academy of Arts and Sciences; June, 1913,* p. 139). An analysis of his data indicates an error in the measurement of *H*, which when corrected for, reduces the saturation value to 21475, which is in close agreement with the result of my tests on similar material. That conclusion is also confirmed by the researches of Hadfield and Hopkinson (*Journal I. E. E.* Dec. 1910—page 251), who obtained for the purest available sample, a saturation value of 21100.

In view of these facts I think we ought to make sure of our data, and therefore, give more attention to the methods of test before drawing any important conclusions.

T. D. Yensen: Replying to the discussion contributed by Mr. Gokhale in regard to the saturation values of Iron-Nickel alloys and methods of measuring them I very much regret that I did not mention in my paper the method that was actually used by us, as this might have saved Mr. Gokhale's time in preparing his tables. We actually did use the extrapolation method based on Kennelly's law. The reason that Mr. Gokhale has obtained values that differ from those given in my paper is that he makes use of the inductions for $H = 100$ and $H = 400$ in order to compute the slope of the $\rho - H$ curve, whereas $H = 100$ lies below the bend in the $\rho - H$ curve in every case that has come to my notice. This is illustrated in the accompanying table and Fig. 2 for Alloy 2Ni 204. Here it will be noticed that

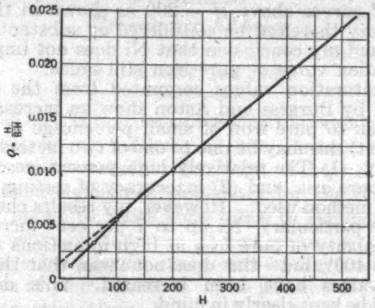

FIG. 2—COMPUTING SATURATION VALUE FOR SPECIMEN 2 Ni 204 BY KENNELLY'S LAW

Using Values for $H = 100$ and $H = 400$

$$\left.\text{Sat.} = \frac{\Delta H}{\Delta \rho} = \frac{300}{0.0135} = 22200 \right\} \text{Incorrect}$$

Using Curve

$$\text{Sat.} = \text{Slope of Curve} = \frac{500}{0.02195} = 22800 \text{ Correct.}$$

H	*B*	*B-H*	ρ
20	16600	16600	0.0012
100	18430	18330	0.0055
200	18870	18670	0.0102
300	20730	20430	0.0147
400	21430	21030	0.0190
500	21810	21310	0.0235

the $\rho - H$ curve is a straight line down to $H = 200$ but that for lower values of H, the points lie on another straight line having a greater slope than the former. It is, therefore, evident that, by making use of the values for $H = 100$—as Mr. Gokhale has done—saturation values are obtained that in every case are too low. All of my values were computed from the slopes of the $\rho - H$ curves above $H = 200$ as shown in the figure, and may therefore be considered as substantially correct, and my conclusion that Ni does not improve the saturation value of pure iron still holds.

If saturation values computed from the data obtained by Burgess and Aston show an increase by the addition to pure iron of small percentage of Ni (7-10 per cent) this may be due to one of two factors, or both, namely; (1) The relatively high percentage carbon in their pure iron, and (2) inaccuracy of testing, inherent in the method used. However, my results check theirs in one particular;—Ni up to 7 per cent increases the permeability of pure iron at high inductions (for $H = 100$ to 400); but—this does not mean that the saturation values have been increased. This distinction should be kept clearly in mind.

In regard to the saturation values of American Ingot iron, to which reference is made by Mr. Gokhale, I may say that we have tested many samples of this iron and of Armco iron (which is practically the same) and have obtained saturation values of 22,000 for the material as received. When this material is vacuum treated however, the saturation values can be raised to 22,700 or practically the same as that obtained for pure vacuum fused electrolytic iron. The conclusion seems therefore natural, that the reason for the lower values obtained by Professor Pierce for Ingot and by Hadfield and Hopkinson for the purest available iron (which I believe was Swedish charcoal iron) is impurities in the samples, either gaseous or solid.

The method for measuring the saturation value directly as proposed by Mr. Gokhale—although the principle may not be new— is undoubtedly valuable in so far as it may be used as a check on the extrapolation method, but otherwise I believe the latter is sufficiently accurate and simple for most purposes.

I hope I have succeeded in removing any further skepticism that may exist in regard to the reliability of the results.

In conclusion I wish to acknowledge my indebtedness to Mr. Thomas Spooner and his assistants, for the magnetic testing.

Presented at the 30th Annual Convention of the American Institute of Electrical Engineers White Sulphur Springs, W. Va., June 29, 1920.

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AN ENGINEERING ANALYSIS OF THE LABOR PROBLEM

Presidents Address

BY CALVERT TOWNLEY

THE Constitution prescribes that the President must deliver an address at the Annual Convenvention,—whether you want to hear it or not. It has been sometimes customary for the Executive who is about to retire from office to summarize the achievements of his term. It has seemed to me that these have been better chronicled as they occurred in the pages of your JOURNAL. The pastime of prophecy has some attractions, but is of doubtful value, and somewhat dangerous as well. The advances in the art are well covered by many valuable technical papers of the year including those which are to be submitted at this Convention. There remain broad public questions regarding which engineers are peculiarly well qualified to have opinions and to influence the opinions of others and it has therefore seemed to me that I might perhaps come nearer justifying the constitutional provision by dealing with one of these.

One of the most important problems which the American people now have to solve is that of the relations between the employer and the employee classes, generally referred to as capital and labor. A large measure of our national success will depend upon this solution, and as engineers are even more directly affected by and therefore more intimately connected with industrial prosperity than are the other professions, for example, of law, medicine and theology, it is more obviously their duty to assist in the solution of the problem as much as they can. Although this subject has been voluminously discussed by numberless people for a long time, it has usually been treated from the standpoint of what ought to be— according to the views of one or the other of the interested parties. Questions of justice and injustice, legality, the so-called "rights" of labor and the protection of capital, of expediency, are brought in. There is one angle from which, so far as I know, it has not been treated, namely from the standpoint of what is, instead of what ought to be; that is to say, from the engineering standpoint,—one of fact, human nature and of economic laws as contrasted with the laws enacted by man. It therefore may be now opportune, especially to this audience, to offer a contribution from this view point, with the belief that the direction of the attention of the members of our organization to the fundamentals of so important a question may serve to clear away some of the fog which always seems to becloud the view.

To begin with, the first law of nature is self preservation. It is manifested in its highest form as patriotism, in its lowest as sordid greed. Between these two extremes appear all forms of selfishness, many of them by no means blameworthy, others under control, or not infrequently dormant, but nevertheless certain to manifest themselves when circumstances compel. For example, selfishness is the actuating motive in barter. It leads the seller to try to sell at the highest price and the buyer to try to buy at the lowest. Barter and trade are at the foundation of industry and having been always recognized as essential to prosperity have been encouraged and protected. This form of selfishness has acquired great respectability—but while the practise of matching wits in trade is supported by the people of all lands, the sanctity of contract is also fundamental, *i e.*, that form of selfishness which leads to the violation of an agreement once made is everywhere decried and condemned. It has been demonstrated that selfish human nature will assert itself and that good faith cannot be relied upon, therefore recourse has been had to another powerful trait, fear. The exercise of the police power, with its penalties for transgression, is based on fear of consequences.

The ideal condition for barter is where both parties are absolutely free agents and neither therefore under duress. Although such a condition is seldom realized, when the duress becomes so extreme as to constitute oppression, the desperation of the oppressed may be counted upon to devise resistance. Just as surely as political oppression has throughout all history led to political revolution, so has commercial oppression caused commercial revolution. Too great power, if continued long enough, will always be abused and just as such power in government becomes tyranny, similarly too great advantage in barter begets commercial robbery. These statements sound like platitudes and I should hesitate to repeat them were it not for the fact that they not only bear directly on the present situation between labor, capital and the public but they seem to be so frequently forgotten or ignored. Please remember that I am not blaming or praising any one. I am trying to recite cold facts and to call things by their right names.

In the time-honored barter between employer and employee, the employer formerly held the strategic position. If he refused to pay the wages demanded by any individual there was usually some one else willing to accept a lower rate and the unemployed one, not being mobile and usually having to work to live, was soon forced to yield. As business concerns grew larger this advantage naturally increased and just as naturally the employers' power was abused. By and by the resulting oppression became sufficiently acute to provoke general resistance and the labor union came into being. Workmen banded together to eliminate competition among themselves. It should be noted that the establishment of unions did not suspend the operation of the law of supply and demand; rather did it allow that law freer play byc ounterbalancing the power of the union against the previous strategic advantage of the employer. The effect was generally beneficial because it corrected an evil, and the general public while not particularly interested looked on approvingly. But early in their history the unions made a serious although not a surprising blunder. They set up a system of contracts between employer and employee and, because they could escape doing so, provided no means by which the workmen could be penalized for violation, except by the union itself. This flagrant flouting of a fundamental did not for a time have any serious effect, but the blunder once made was never thereafter corrected and it is probable that even today it would be difficult to find a labor leader who could be made to see the seriousness of this mistake, much less to endeavor to rectify it. I pass quickly over a long period of years after the first organization of unions during which unionism had its ups and downs and came to be regarded, if not with equanimity, at least with tolerance by the employer class, and come to the recent period of the World War. Here an entirely new condition was created—an imperative demand for men came up almost over night. It simply had to be satisfied at no matter what price. At the same time not only was the supply diminished by those required for the armed forces but immigration stopped as well. The effect on the price of labor was axiomatic—it rose. The unions being the vehicles through which the demands of many classes of workmen could most readily be expressed at once became active. The law of supply and demand was fighting on their side and they made the most of it. Wages and prices rose to hitherto unheard-of figures. Labor had too much power; its demands had to be met whether or no, and of course it abused its power. Then the unions made another mistake. They credited too much of their successes to their organization and failed to appreciate the part played by the law of supply and demand, also they underestimated the power of resistance which could be engendered by oppression. In their turn they became the oppressors—insistent, arrogant. Where their principal weapon, the strike, was not effective as against the employer class in one industry, they enlisted the workmen in other industries, and who had no grievance, in sympathetic strikes. They asserted their power to dominate the public convenience, safety and health in order to coerce their opponents into submission to their will. Were it necessary I could recite many specific instances to prove this fact but I assume it to be so generally known and accepted that a recital would be superfluous. Now this is the condition to day and it is one which has caused much grave concern and has produced many strange ideas. We hear that a new order of things has come to pass; that the "rights" of labor must now be respected; that the workman will hereafter have a greater share of the products of his toil; that he must share in the management of industry and have a recognized place in government, and the like. To my mind the facts warrant none of these assumptions. There is no new order,—economic laws are the same as they always have been. They are as ruthless and as inexorable as are the laws of physics. Neither has human nature changed. The unions are in the saddle, but, to use a hunting expression, they are riding for a fall. Please let me remind you again that I am not blaming the unions, or any one else. I am trying to confine myself to statements of facts and to logical opinions based thereon according to the precept and training of the engineer. Suppose we analyze the fundamentals.

In our system of govermnment where every man can vote and have his vote counted, and with our multitude of newspapers informing everybody of what is going on, there is little chance that a condition can arise such as that of China or of Russia or even of Germany. Further while we talk a great deal of politicians and parties, we know that on any really vital question the people are going to make up their minds themselves and will elect men who will carry out the wishes of the majority. In other words the great majority rules.

The number of workmen in labor unions is variously estimated to be from ten to forty per cent of the men employed in trades where there are unions. These figures represent the extreme claims of the contending parties. We shall probably not be far wrong if we take a compromise figure of say 20 per cent. In the largest class of all, the farmer there are no unions and similarly none in many other avocations, so if we consider the total voting strength of the country, the percentage of men in unions is certainly not over 10 per cent and probably not over 5 per cent. Of this small proportion a few are the leaders, a larger number enthusiastic followers and the great majority largely governed by conditions. By that I mean they are loyal unionists when the union is succeeding but desert on small provocation when it is defeated. This is almost obviously true because the workmen are banded together to accomplish selfish purpose and no other. The individual workman cares no more for his fellow as a class than does the employer. It follows therefore that a union must continue to succeed or it will disintegrate and disappear. The history of the last thirty years has recorded many once powerful unions even the names of which are now almost forgotten. The "Knights of Labor" and "The Amalgamated Association of Iron and Steel Workers" are two examples which illustrate this point. The strength of organization contending against the mob is very great and it is probably true that so small a proportion as ten per cent or even as five, would prevail under these conditions. That in fact is the situation confronting the nation today. This small organized percentage supported by the operation of the law of supply and demand has prevailed, but looking into the future, it seems clear that there is only needed a sufficient incentive to induce the unorganized 95 per cent, or even a small part of them, to get together in opposition to the five per cent in order that the latter may be overwhelmed. This incentive has been or will be furnished by union oppression. I say "has been or will be" because the abuse of too great power always grows. Therefore if the unions shall still continue to gain their ends, even in the face of their present insolence and ruthless disregard of others, their arrogance and self confidence born of such successes, will inevitably lead them on to acts of greater and yet greater oppression until they force their opponents to combine against them for defense or even for self preservation. The moneyed class never has had, and probably never will have much sympathy or cooperation from the general public for the very obvious reason that the general public is jealous of it. Sympathy goes naturally to those who are worse off. The moneyed class being better off than the great majority therefore comes in for sympathy with a minus sign, that is, jealousy. An incentive which will induce the public to oppose the unions can therefore be sufficient only when the reason is strong enough to overcome this jealousy. That reason becomes sufficient when the need for self preservation is made clearly apparent. Already there are indications that the people will not much longer submit to domination by the unions. The Governor of Massachusetts defied them, was shortly thereafter reelected by an abnormally large majority and has now been nominated on the Republican National ticket for Vice-President which latter distinction it is safe to say he owes almost entirely to this incident. The State of Kansas has enacted a law to curb union interference with the peace and comfort of her citizens and her Governor has become a popular National figure in consequence. The temper of much of the daily press has changed, and where formerly there was much said about the so-called "living wage" and criticism of "capitalistic greed" "there now appear articles about the vicious cycle of mounting wages and costs", "loafing on the job" and the "need to teach the people thrift and economy". These and other signs are merely symptoms of the incentive to resistance that may be expected to be superinduced by oppression. The aroused public would not be just to the unions, and an attack once thoroughly launched might be expected to go further than it should. However I am not here discussing justice or how either party should behave but rather what they have done in the past and under pressure of human nature and economic laws most certainly will do in the future.

Whether or not an aroused public does curb union domination, the present high wage era will not be radically affected. That is a condition controlled, not by the unions at all, but by the law of supply and demand. Even the complete abolition of the unions while it might check further increases and perhaps bring about some recessions, and while it would reduce living costs some what by cutting out interruptions, and tend to increase production, would not itself either increase the supply of workmen available nor decrease the demand for their services. Indeed while I have discussed the fundamentals of barter between the employer and the employee and have endeavored to show the natural reactions resulting from barter with either party under duress, it is not at all certain that conditions will become so extreme as to bring about the reaction described. That is to say, instead of the abuse of power by the unions being curbed by the organized opposition of other classes of our people it may be reduced or even thwarted by the operation of the law of supply and demand. Lest I may be misunderstood I might here state that because I have been confining myself to a discussion of the relations between the employer and the employee and have had much to say about the employee's abnormally high wages, I do not want to be understood as giving that condition as the only cause of the prevailing high prices. Advantage has very generally been taken of the opportunity to increase profits by those who had wares to market, which is only another way of saying again that selfishness is a universal trait of which no class or classes has a monopoly. A bettering of conditions may be affected by a decrease in the demand for or an increase in the supply of labor. Already there is a marked reduction in the sales of certain products resulting from an unwillingness or the inability of many people to pay the exorbitant prices asked. Government reports of falling exports indicate that the expected lessening of the foreign market for our high priced products is approaching and it is not at all unlikely that this may be followed by heavy increases in our imports which will displace American made goods. This result may come about both because foreign nations are getting into their stride of production again and because the high prices prevailing here have naturally created an attractive market. A combination of these and other conditions will reduce the demand for our products and consequently for the workmen to produce them. The immigration authorities of the port of New York report their facilities overtaxed to handle the large numbers coming to our shores, and say that even those numbers would be far larger if there were only more ships to carry the people who want to come. It is natural to suppose that as the heavy burdens of after-war taxation are brought home to the foreign people an increasing percentage will seek to escape them by coming here to live, and also that the gradual restoration of trans-Atlantic shipping to its normal schedules will afford the greater facilities thus demanded. This augmentation of population—largely workmen of course, will swell our supply and likewise tend to correct any shortage that may exist.

If my analysis of the facts is correct, it is clear that the present domination by organized labor is temporary and also that the era of high prices will pass. Therefore, no material permanent change in either our social order or in our industrial structure is to be anticipated. In the contest between brains and brawn, waged since the world began, brains have always won and brains always will. Free play for the natural forces of trade may be counted upon to exercise a beneficent influence and it should be hampered and interfered with by government restrictions as little as possible. We cannot of course determine from history or from any facts at hand, how long a time it will take for conditions to become normal again but what we need now is clear thinking, courage and patience. I know that we can rely on the engineer for clear thinking and I haven't the slightest doubt about his courage, but I am not so sure about his patience. You should be leaders of thought in your several circles. You can help to allay much of the present anxiety about the so-called unrest and the apprehension as to the future; therefore I feel warranted in asking you to give your close consideration to the subject matter of what I have said this morning.

Presented at the 30th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29, 1920.

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ANNUAL REPORT OF THE ELECTROCHEMISTRY AND ELECTROMETALLURGY COMMITTEE

To the Board of Directors.

THE chairman of the Electrochemistry and Electrometallurgy Committee for the year 1917–1918 suggested to the Board of Directors of the American Electrochemical Society that it should approach the A. I. E. E. with the idea of forming a joint committee to consider matters that were of common interest to the two societies. Such a committee could consider papers which might be of importance to both societies, could bring to each society's attention matters of common interest and make arrangements for occasional joint meetings. The result was that the Board of the American Electrochemical Society appointed Messrs. Fink, Parmelee and Schluederberg "to cooperate with" this Committee.

Nothing was done in this matter in the year 1918-1919; but this year it was taken up again with the result that a joint meeting of the A. I. E. E. and the A. E. S. was held in Boston April 9th, 1920. The meeting was successful, the combined registration amounting to about 425, with the result that the seating space of the rooms used for meeting was somewhat overtaxed.

It is believed that the occasional holding of joint meetings with the A. E. S. similar to that held this year is desirable and it is suggested that the plan be followed up next year when an even more successful joint meeting might be held. The weak point of the April meeting was the failure to print in advance some of the papers presented at the meeting by members of the A. I. E. E. The blame for this rests with the chairman of your committee who had not provided for his unforseen detention in Europe during the first part of the year, with the result that sufficient time was not given to authors to prepare their papers before the meeting. The printing of the papers in advance of the meeting is important so as to stimulate discussion and the exchange of views between the members of the two societies which have so much ground of common interest.

FRANCIS A. J. FITZGERALD, *Chairman.*

ANNUAL REPORT OF THE COMMITTEE ON ELECTRICAL MACHINERY

To the Board of Directors.

THE Committee on Electrical Machinery placed before itself the aim to cover as nearly as possible the range of work in which there had been changes of importance which had not yet become a matter of record in the TRANSACTIONS of the Institute.

The Committee joined with other committees in the presentation of papers at the Midwinter Convention on the general subject of super power stations for the eastern coast states. This subject will receive renewed attention from time to time.

The polyphase alternating-current commutator motor for variable speed operation received comprehensive treatment in two papers, one by B. G. Lamme, presented before the Schenectady Section, and the other by John I. Hull, to be presented at the Annual Convention.

The subject of temperature measurements in large alternating-current generators and allied questions received a complete and exhaustive treatment in four papers to be presented at the Annual Convention. Rationalization of temperature measurements and uniformity of policy have been established as a result of these papers and of the investigations which led up to them.

A mathematical paper by R. E. Gilman discusses the losses due to eddy currents in stranded conductors embedded in slots, thus carrying on further the fundamental work begun by A. B. Field.

A symposium of papers on the design and the operating characteristics of welding machinery has been arranged for the Annual Convention so as to make a record in the TRANSACTIONS of the progress made in this important direction.

In presenting this brief report, the Chairman desires to point out that he has considered it the essential function of his Committee to trace, through the papers solicited and the meetings arranged, the progress of the art which it was desired to record. Thus it is unnecessary to deal again here with the subject matters comprehensively presented in the papers.

In conclusion, the Chairman wishes to thank the members of his Committee for their support and co-operation.

B. A. BEHREND, *Chairman*

ANNUAL REPORT OF THE PROTECTIVE DEVICES COMMITTEE

To the Board of Directors:

EARLY in the year the Protective Devices Committee appointed a number of sub-committees to investigate, study and report on various subjects and these are discussed in some detail in the following summary.

SUBCOMMITTEE ON TERM, "INSTANTANEOUS RELAY"

E. E. F. CREIGHTON, *chairman*

This Subcommittee resulted from the criticism of the term "instantaneous" as applied to a relay, it being contended that the term did not properly describe the device. A number of other terms were suggested of which the term "quick acting" appeared to have the preference. On account of the wide diversity of opinions among the members of the committee on the subject, no definite recommendation has been made, and it was agreed that the Chairman of the Subcommittee would summarize the various opinions on the subject and submit them for publication in the JOURNAL.

SUBCOMMITTEE ON SCHEMES OF RELAY PROTECTION

P. H. CHASE, *chairman*

Following the recommendation in the Report of the Committee last year, a number of letters were written to the executives on a selected list of operating companies, and about thirty-five replies have been received from companies who were willing to cooperate in the work assigned to this subcommittee, and one of their engineers delegated as the correspondent on the subject. Questionnaires regarding schemes of relay protection have been forwarded to these correspondents. The United States and Canada has been divided into six geographical sections and each section has been assigned to a member of the subcommittee who is with an operating company in that district. These members are charged with examination and classification of replies received and with distribution of the information to the other members of the subcommittee. It is felt that the work of the subcommittee to date has been very largely of a preliminary nature and that its value will be more apparent later in the year.

SUBCOMMITTEE ON OIL CIRCUIT BREAKERS AND SWITCHES

H. R. WOODROW, *chairman*

This Subcommittee is cooperating with similar committees of the National Electric Light Association and the Power Club, and is securing information on the subject from a limited number of the larger operating companies. There has not been sufficient time since the request for information was issued, to secure answers from the various companies and to summarize the results. The subject of high-tension fuse protection has also been referred to this Subcommittee.

SUBCOMMITTEE ON SYSTEM DISTURBANCES

D. W. ROPER, *chairman*

It was the opinion of a number of members of the Committee that the serious system disturbances experienced by the operating companies were due in a large measure to the defective action, or perhaps the absence, of suitable protective devices, and that a study of such disturbances might indicate the lines along which the future work and study of the Protective Devices Committee should properly be directed. The members of the Subcommittee are interchanging information on this subject and in recent months have been somewhat embarrassed by the amount of data available. An unusually large percentage of the larger operating companies have experienced severe system disturbances and in some cases the behavior of the protective devices was an important factor in the disturbances. The Subcommittee is now endeavoring to formulate some standard method of reporting on such matters, so that the reports can be properly condensed and summarized and be made available for the purposes of the Protective Devices Committee.

SUBCOMMITTEE ON REACTORS

F. E. RICKETTS, *chairman*

In the last few years there has been a marked improvement in the design of reactors such as, methods of winding to reduce eddy currents, improvements in structure to increase the insulation strength, and to work the copper to the best advantage. In some cases this improvement in design has been taken advantage of to reduce the size of the reactor so that the coils of today are small when compared with those of equal rating built a few years ago. This, however, is not a feature that should be carried to extremes as the current limiting reactor is, fundamentally a protective device and this function should be fully recognized in its development. Every effort should be toward the perfection of the design to avoid failure of the device itself and at the same time avoid the reduction of any of the protective features.

The application of reactors has been reviewed at different times in papers presented before the Institute and various schemes of connection have been proposed; but there is still some difference of opinion as to the proper place to insert reactors to obtain the best results, and there are many factors entering into this question. Each application requires a separate study but in general the best protection for the service is obtained by installing the reactors in individual feeders, and this is the practise followed by most companies-provided the necessary space is available. However, some engineers prefer to sectionalize the system by inserting reactors in the bus, but when this is done, consideration should be given to the possbility of increasing the chance of the different sections getting out of synchronism which becomes very much of an operating menace where the amount of reactance in the circuits of the generators is much greater than that necessary to give the greatest synchronizing power.

SUBCOMMITTEE ON RESISTORS FOR USE WITH POTENTIAL TRANSFORMER FUSES

R. N. CONWELL, *chairman*

One of the larger companies in New York has made some elaborate and complete short-circuit tests on potential transformer fuses installed with and without protective resistors in series. Following these tests a number of companies in and around New York have adopted the standard practise of installing protective resistors in series with the potential transformer fuses which are connected to their high-tension buses for the purpose of limiting the current, and in this manner preventing damage to apparatus or interruption to service which had previously occurred. A summary of the results of the investigations, with the practise of the several companies is being prepared for submission to the Publication Committee for use in the JOURNAL. A canvass by the Chairman brought forth the information that certain other companies are using a different type of fuse which has given excellent results in service, and which does not require the use of a protective resistor in series.

PRIMARY CUTOUTS FOR LINE TRANSFORMERS

This subject was discussed at some length in the meetings of the Committee. It was the opinion of a number of the members of the committee that are connected with operating companies that the development of primary cutouts for line transformers had not kept pace with the development of the transformers themselves. One company reported they had considerably more trouble originating in the primary cutouts than in the transformers which these cutouts were intended to protect. Several of the larger companies install the larger sizes of overhead line transformers and all sizes of subway types of transformers in manholes without any primary cutout as their experience indicates that they have fewer interruptions in this manner than they would if they attempted to use cutouts with these transformers. It is the opinion of the members of the Committee that if the larger manufacturing companies would devote as much engineering skill to improvements in primary cutouts as they do to improving the design of line transformers, this difficulty which is one of long standing, would very quickly be eliminated.

During the discussion of the subject of primary cutouts it was brought out that one of the largest companies had, some years ago, abandoned the use of all types of cutouts or lightning arresters which were enclosed in an iron case. This was found necessary on account of the troubles with these devices during rain storms and wet snow storms, and that the difficulty had increased with the size of the generating or transforming units which supplied the current to the lines on which these devices were installed. During the past year another large operating company has taken exactly the same action and for the same reason. The Committee therefore, recommends that operating companies take note of this situation as with the increase in the size of their generating and transforming units they will ultimately be called upon to take similar action and they should, therefore, investigate types of such devices without the metal enclosing case with a view to its adoption for their future work.

During the year the Committee has undertaken to make investigations which will not be completed during the fiscal year. The Committee members who are taking an active part in this work all feel that the work is not only of particular interest to the members of the subcommittees, but should ultimately result in something of value to the Institute. It is recommended that the investigations of these subjects be continued next year, and that the members of the subcommittees who have been actively engaged in these several investigations be continued on the Protective Devices Committee during the ensuing year.

D. W. Roper, *Chairman.*

ANNUAL REPORT OF COMMITTEE ON INSTRUMENTS AND MEASUREMENTS

To the Board of Directors:

THE Instruments and Measurements Committee submits the following report covering its own activities during the past year and the developments and progress in that part of the electrical field covered by its title.

A considerable number of papers was arranged for during the year and at the Mid-Winter Convention in February, one morning session, one afternoon session and part of a second afternoon session were assigned to this Committee. A most interesting and valuable group of papers was presented. The sessions were very well attended and the discussion was limited only by the available time.

The papers presented at the sessions referred to above are as listed below:

1. "Measurements of Projectile Velocities" by Dr. P. E. Klopsteg and Major A. L. Loomis.
2. "A New Form of Vibration Galvanometer" by Dr. P. G. Agnew.
3. "Precision Galvanometer for Measuring Thermo E. M. Fs." by Messrs. T. R. Harrison and P. D. Foote.
4. "Notes on Synchronous Commutators" by Professor J. B. Whitehead and Mr. T. Isshiki.
5. "Oscillographs and Their Tests" by Professor A. E. Kennelly and Mr. A. A. Prior.
6. "The Accuracy of Commercial Electrical Measurements" by Mr. H. B. Brooks.

Without entering into a complete review of all the papers or the discussion, brief mention should be made of the paper on the Check and Calibration of Oscillographs and the paper on the Accuracy of Commercial Electrical Measurements. The oscillograph is rather generally used as a means of practically ultimate analysis of electrical functions and phenomena, and the paper provides the means of checking the functions and characteristics of the oscillograph itself. The contribution to the papers of the Institute entitled "The Accuracy of Commercial Electrical Measurements" is a most thorough and comprehensive analysis, review and statement of the characteristics and limitations of the many commonly used electrical instruments. This paper will, without doubt, serve as a ready reference for many years and should be of value to many engineers in other than actual testing work. It covers in a thorough and intelligent manner the errors that may be expected in the use of various instruments under various conditions.

The definition of power factor in polyphase circuits is a matter that has been active and of interest from many angles throughout the past year. While it has been the direct work of a Special Joint Committee of the N. E. L. A. and the A. I. E. E., its work falls within the general purview of this committee and a brief mention in this report seems entirely proper The Special Joint Committee canvassed the manufacturing, operating, scholastic and purely engineering branches of the electrical industry and prepared a report of its findings. It was intended to have this report presented at the Mid-Winter Convention last February. It was impossible, however, to analyze and intelligently condense in the form of a report, the volume of material received. It was decided, therefore, to have the report ready for presentation at the Summer Convention of the A. I. E. E. and to have a session assigned to the general subject. Arrangements will be made for discussion by various interested individuals to present the point of view of various interests affected by the settlement of the question. It was hoped that this method of procedure would provide an adequate form for the use of the many interested individuals and permit the ultimate acceptance of a satisfactory solution of the question.

Your committee has surveyed the field of manufacture of instruments and measuring devices and finds in general a few reports of new developments, some reports on standardization and adaptation to commercial usage of the designs dictated by the war and practically unanimous reports from all of the manufacturers of extreme difficulty in contending with the shortage of material and labor turnover in the production of the standard and accepted lines of apparatus.

The various companies manufacturing watthour meters and demand devices, practically all report the completion of experimental models, finished designs or actual production of devices for measuring kv-a-hr. and demand devices for measuring the demands of kv-a. and kw. and power factor. These various devices may make the measurements separately or simultaneously on a single record. Further there is a steady continuation in the development and standardization of maximum-demand devices to complete the series of devices suitable for easy adaptation to d-c. watthour meters, a-c. watthour meters and various frequencies.

Aside from the adaptation to various types of meters there is the continuation of the development providing devices actuated on the time basis, either on spring driven clocks, spring driven clocks electrically wound or motor driven clocks.

The Warren motor referred to in last year's report of this committee continues to be developed in a most satisfactory manner. It is being used for the control of frequency on generating systems through the master clock. It is also being installed in many forms of graphic and recording instruments used for various purposes throughout the stations of generating and distributing systems.

In the field of indicating instruments, both for direct- and alternating-current use, it is found that the designs which were developed particularly for war purposes, have been modified sufficiently to meet the regular demand and there are now available lines of both switchboard and portable types of instruments that were not available two or three years ago. These instruments are in general of a very satisfactory type and serve a very useful purpose in providing other sources of supply of instruments which did not exist some years ago.

The greatly increased activity in the measurement of high frequencies used in wireless work caused the development of vacuum thermal converters. These devices serve very satisfactorily for other purposes than for wireless work inasmuch as they permit the precise measurement of extremely small values of alternating current, a field of measurement which has not until recent times been covered by other than laboratory instruments.

In canvassing the field of telephone and telegraph engineering for instruments and measurements, the committee finds in answer to inquiry practically nothing to report, but mainly the continuation of developments which were started during the war period and previously reported.

Reports from those companies specializing in the manufacture of laboratory and precision instruments indicate a very valuable development of this type of instrument for certain special as well as general purposes. The line of development has been to produce on a commercial scale automatic precision electrical instruments, either of the indicating or the graphic type. Wheatstone bridges and potentiometers are normally considered as indicating instruments or devices manually operated by means of dial switches or plug contacts. One of the companies manufacturing this type of apparatus reports the development of an automatic potentiometer and an automatic Wheatstone bridge, both used for recording or controlling processes or a combination of the two. These automatic equipments are supplied for the control of chemical and electrochemical processes and some installations of these are in actual use. For chemical and electrochemical work the automatic Wheatstone bridge can be operated on a commercial 60-cycle a-c. circuit and a special a-c. galvanometer has been especially developed for this equipment.

Two of the important applications of the a-c. Wheatstone bridge equipment, are the recording of surface condenser leakage in power plant operation and the controlling of the finish point during the evaporation process in plants making condensed milk. The potentiometer equipment is also used for accurately recording and even controlling the acidity and alkalinity of solutions through the measure of the potential between two electrodes immersed in the liquid.

The recording or indicating potentiometer when used in connection with thermocouples produces a very satisfactory and accurate record of temperature. The measurement of temperatures is, of course, directly related to design and operating requirements of electrical apparatus and these devices serve very satisfactorily indeed to analyze the daily or weekly cycles of thermal performance of cables, transformers, turbine generators or in fact any of the various commonly used types of apparatus in generating and distribution systems.

S. G. RHODES, *Chairman.*

ANNUAL REPORT OF THE TELEGRAPHY AND TELEPHONY COMMITTEE

To the Board of Directors:

FOLLOWING out the plan adopted last year as to the form of annual report to be submitted by the committee the following report includes references to technical progress only. The report is made up of contributions forwarded by each member of the committee.

STANDARDIZATION

During the past year considerable progress has been made in standardizing telegraph and telephone practises of the railroads of this country and Canada. Committees composed of representatives of the railroads, the commercial telegraph and telephone companies, and various manufacturers of apparatus, have prepared specifications for materials, apparatus and methods which, while conforming to the generally accepted standards in each case, include special features adapting them to railroad requirements. It is expected that these specifications will be formally adopted by the American Railroad Association in the near future, and that the desired approach to uniformity of practise will soon follow.

TOLL LINE CONSTRUCTION

Toll circuits may be composed of either open wire on pole lines, aerial cable, or underground cable. Any one of these will give high grade transmission when improved equipment and methods of operation are used. Therefore, the tendency in toll line construction is to build plant which will be most economical and at the same time reduce the cases of interference to service from storm breaks and various other troubles to a minimum.

Underground cables properly installed and maintained are open to the least interference to service.

However, it is not economical to place underground construction unless more than two full-size cables are required. Generally, underground cables cannot be justified except between the larger centers and their surburban sections.

Aerial cables are liable to less interference than open wires, but economy restricts their use largely to cases where the existing number of circuits and the rate of increase require extensive rebuilding, or additional pole lines to continue on an open wire line basis. Where the number of circuits will not justify aerial cable, open wire circuits must be constructed and maintained.

UNDERGROUND CABLE CONSTRUCTION

No radical changes have recently been made in engineering or construction methods involved in connection with laying of underground cables. The maximum size of cable used contains 1200 pairs of No. 24 B. & S. gage wires. Experimental work is being carried on for the purpose of increasing the number of wires without increasing the diameter of the cable sheath.

Troubles in underground cables have been due in part to disorderly arrangement of cables in manholes and the manner in which the splices were made. Great stress is now being laid upon the importance of having the manholes in good condition, supporting the cables in an orderly manner, and making splices with extreme care. The cost of labor required for installing underground cables is relatively small compared with the total cost of the cable in place, and therefore it is felt that all reasonable efforts should be made to install and splice the cables in such manner as to reduce the cable troubles to a minimum.

INDUCTIVE INTERFERENCE

The Railroad Commission of the State of California has recently published a comprehensive work entitled, "Inductive Interference," in which is given a large amount of technical information gotten together by the Joint Committee on Inductive Interference, which investigated this subject in California over a period of about five years. This work makes available to interested parties a great deal of information of fundamental importance.

The activity in both the power and the communication fields since the war, has demanded renewed close attention to the study of inductive interference.

The growing use of electric arc furnaces which usually produce great distortion of wave form has given rise to serious new problems in noise interference prevention.

The methods of overcoming interference with communication circuits from abnormal conditions on adjacent power circuits have not been so well developed as the methods of overcoming disturbances from the normal operation of power circuits. Constant increase in the amounts of energy carried by power circuits has also made these problems more difficult. Especial attention must be given, therefore, to this phase of interference, particularly along the lines of limiting the number of abnormal conditions, reducing the length of time in which the circuits are abnormal and in limiting the magnitude of unbalanced power currents during such abnormal periods.

Information on the subject of interference from electric railways has been increased by two or three important investigations, but is still far from complete.

PRINTING TELEGRAPH TENDENCIES

While the invention of systems of printing telegraphs has been active substantially since the introduction of Morse's telegraph in the year 1844, the use of printing telegraphs on a commercial scale in America—aside from stock and news ticker systems—has been extensive only since about the year 1900. During the past twenty years improved printing telegraph systems have been extensively applied in moving traffic over trunk circuits where the volume of business is heavy and continuous. On the lines of the Western Union Telegraph Company practically 80 per cent of the trunk line traffic is so despatched. The Postal Telegraph-Cable Company, on the other hand, makes no use of printing telegraph systems at the present time.

On railroad telegraph lines printing telegraphs are used also to a large extent.

The next stage of development—now very likely close at hand—is the application of printing telegraphs to wider uses and to circuits having a much lighter than trunk line traffic load. Extending to light-load circuits the use of printer systems makes simplicity of operation, low first cost, and low maintenance cost of first importance.

With the object of meeting the requirements for simplification, work is now under way with tape printing devices. Also, it is probable that two or three other systems of simplified telegraph printing will be given trials in service before long.

RADIO SIGNALING

The year 1919 has seen the application of discoveries and inventions which were necessarily veiled in secrecy during the years of war. Under stimulus of war necessity remarkable progress was made in the application and extension of the known principles of the art, standardization of radio apparatus and of its quantity production and use. These advances were made possible by the cooperation of industrial companies, their engineers, and engineers and scientists with academic connections. The efforts of these individuals or groups were ably coordinated and stimulated by the military and naval branches of the Government.

Progress during the past year has been more in the nature of a consolidation of the scientific advances of the war time than in the nature of an entry into new fields. The entire art of radio communication now stands on a firmer basis, with a greatly enlarged personnel of trained engineers and research investigators.

Radio telephone communication was maintained between the seat of Government and the steamship George Washington, on which President Wilson made his European voyages. In these demonstrations the radio station at New Brunswick, N. J., was used.

The guidance by radio of airplanes engaged in peace-time activities is well illustrated in the directing towards landing places of the planes of the Post Office Department. Communication with planes on long voyages was accomplished in the memorable voyage of the N C-4.

High-speed radio telegraphy has been greatly accelerated by the use of the Hoxie recorder and the Alexanderson magnetic amplifier. Direction-finding by radio compasses and its corollary of directive-receiving has been placed upon a firm scientific basis.

The possibility of selecting signals with reference to their direction has been successfully applied by Weagant, Wood, Taylor, Alexanderson, and others, to the selection of desired signals and the discrimination against atmospheric disturbances and signals of foreign stations. These principles have also permitted engineering advances in duplex, or two-way operation of pairs of stations, and in increased possibilities of multiplex transmission and reception. Knowledge and experience along the lines of duplex and multiplex operation has given added interest to the questions of regulation of wave length, range, and of the effect upon radio development of legislative action.

In the manufacturing field there has been progress not only in standardization of apparatus and in better performance, but also in the production of equipment for high-power continuous-wave stations; compact and simple apparatus for medium-power stations and short-range sets suitable for amateur use.

The withdrawal of military restrictions has resulted in a stimulus to amateur operation. By the encouragement of amateurs and of the research departments of the universities and engineering schools, the art should attract additional workers and provide for its future the necessary increase in an enthusiastic and well-trained personnel.

AUTOMATIC TELEPHONY

During the past year there was begun a movement which may become general looking to the converting of manual telephone exchanges to automatic operation. The same movement is said to be under way in foreign countries. From the mechanical standpoint there were no outstanding improvements in automatic telephony brought into public use. Progress took the form of a general refinement of details necessary to meet operating conditions.

DONALD MCNICOL, *Chairman.*

ANNUAL REPORT OF THE COMMITTEE ON TRANSMISSION AND DISTRIBUTION

To the Board of Directors,

THE Committee on Transmission and Distribution submits the following report for the year 1919-1920:

GENERAL

Early in the year the Board's Committee on Technical Activities suggested that it might be advantageous to change somewhat the character of the Annual Reports of the Technical Committee so that each of these reports would become an authoritative source from which could be obtained a complete resumé of developments in its particular field.

This matter was thoroughly discussed at a meeting of the Technical Committee Chairmen, who were of the opinion that it would be inadvisable to take such a step

The chief objections to the plan may be summed up as follows:

1. The preparation of a really complete and authoritative annual resume by each Technical Committee would require a great deal more time than could possibly be devoted to it by the Committee members.

2. Several publications prepare annual digests of developments and an attempt by the Institute to prepare similar statements would be more or less in the nature of duplication.

3. The preparation of a summary by the Committee instead of the present method of reviewing papers presented, would eliminate the personal element. This might tend to take away the incentive which now exists for the preparation of papers, as under present conditions, the preparation of papers is solicited, and when abstracted for the Annual report, full credit is given to the author.

It therefore seems that the advance in the art is best recorded by the present methods, and that those who desire a summary of developments, obtain this summary in a more accurate form than would be possible if the Committee's Annual Report were to deal in generalities.

During the year your committee in conjunction with the National Electric Light Association Underground Systems Committee, formed a joint Subcommittee on Cable Specifications.

This Subcommittee, after a number of conferences in which practically all the cable manufacturing companies were represented, completed its standard specification for paper-insulated lead-covered cable. This specification, which is accompanied by an appendix giving very complete notes explaining the various clauses, is printed in full as part of the report of the Committee on Underground Systems of the National Electric Light Association. The specification is also printed in pamphlet form for general distribution among cable engineers.

As a result of this standardization, cable manufactured under these specifications will be of the first quality and in accordance with the latest and best practise.

In addition to the work on preparation of cable specifications, this Joint Subcommittee is also preparing data and carrying on investigations of the dielectric losses in cables, relation of heating of cables to their load-carrying capacity, and such other information as is essential to the proper operation of underground cable systems.

In this connection, the Subcommittee has plans under way for the construction of an experimental conduit line about 200 feet in length. With the facilities offered by this experimental line the following phases of the problem will be investigated:

1. Relative temperature drops taken up by the cable insulation, air surrounding the cable, duct walls and surrounding soil.

2. Determination of watts lost as related to the temperature curve under various conditions.

3. Best arrangement of ducts to reduce heating to a minimum.

4. Determination of a reliable method of rating cables.

PAPERS ON TRANSMISSION AND DISTRIBUTION

There has been considerable discussion during the past year of the desirability of erecting high-voltage tie lines to connect existing or proposed, large power developments.

A paper presented at the Pacific Coast Convention in Los Angeles, September, 18th, 1919, by Messrs. R. W. Sorensen, H. H. Cox and G. E. Armstrong, deals with the high-tension tie line in so far as the state of California is concerned.

The authors summarize the power resources of the state, estimate the probable 1926 demand, and discuss the advantages of an all state tie line. The paper points out that the fundamental problems in 220,000 volt transmission are well understood, and that as the need of such a line is imperative, arrangements should be made without delay for the working out of details, and the construction of the line.

The super-power generation and transmission problem for industries and railroads on the North Atlantic Seaboard has been very thoroughly covered in a paper begininng on page 10, by Mr. W. S. Murray.

The author's plan, which has the endorsement of the important engineering societies, provides a means whereby the present estimated machine capacity of 17,000,000 horse power, divided 10,000,000 for industrial and 7,000,000 for railroad purposes, in the region between Boston and Washington, can be lifted from a present load factor of 15 per cent to a load factor of 50 per cent or more.

By utilizing the more economical generating apparatus at such a high load factor, it is expected that present fuel consumption can be cut in half and that the railroads can be relieved of carrying a considerable amount of the coal which is annually transported to the seaboard.

In conjunction with Mr. Murray's paper there is also a symposium giving the views of a number of eminent engineers on the subject of super-power generation and transmission.

In the April, 1920 JOURNAL, Mr. H. H. Plumb has contributed an extremely interesting paper on a rational method of determining the economic voltage for a long transmission line. The paper presents a rapid solution for the size of copper conductor based on Kelvin's law, and with this as a foundation, develops the solution of the economic voltage for copper, copper-clad steel, aluminum and other conductor materials.

Mr. W. A. Del Mar in the January, 1920 issue of the JOURNAL, has worked out a method by which different insulating materials may be compared even when the tests are made at different voltages, and on cables of different sizes. This method consists in expressing the specific quality of the insulation, in terms of a quantity which may be termed the coefficient of dielectric loss, and which is the product of the power factor and the specific inductive capacity.

TENDENCIES IN HIGH-VOLTAGE TRANSMISSION

The probable magnitude of the power requirements of the near future, together with a quickened appreciation of need for maximum economy in its production has led to widespread and serious consideration of projects for bulk transmission of power to load centers from distant energy sources. It is of interest to note that similar conditions exist and similar projects are being considered in many foreign countries. The large amounts of power and the long distances involved in such projects are beyond the economic range of transmission voltages thus far in use. In consequence there has been, during the past year, considerable study and discussion of the feasibility, design, cost and characteristics of transmission at a materially high voltage.

The study and discussion have centered largely around 220 kilovolts which has appeared to represent a sufficient increase in transmission capacity to meet the requirements of projects thus far under discussion, and at the same time involve no such large step beyond present practise as to introduce problems not susceptible of present commercial solution.

Development of trunk transmission at such a voltage as 220 kv. would involve new problems arising from the high voltage, the large amounts of power involved, and the high service standards which the importance of the transmission would demand. The various studies which have been made indicate that these problems differ in degree, but not in character, from those satisfactorily handled in present practise. The production of equipment for such a voltage has been the subject of thorough study and investigation by the manufacturers, and definite assurance is given that equipment entirely adequate for the service can be supplied. For transformers in suitably large units for voltages of the order of 220 kv. it is stated that no radical departures are involved in either design or construction. As regards oil circuit breakers, where the problem is one of high capacity rather than of high voltage, it is claimed that satisfactory designs have been worked out. In general, for extra high voltage installations the conclusions from various studies have been that design policy should tend to extreme simplicity of layout and staunchness of equipment. One feature of this policy would probably be omission of arresters or similar protective equipment. While present type of arresters could presumably be adapted for such a voltage, it appears to the opinion of those who have thus far studied the problem that they would be neither needed nor efficient. The idea appears to be becoming quite general among operating engineers that arresters of the usual type, with a series gap, afford little protection against dangerous low-voltage high-frequency surges, while the need for protection against excess potentials becomes progressively less for the higher operating voltages. Both theoretically and in the light of such experience as is available, circuits operating near the corona limit will tend to be self protecting against excess potentials as a result of corona dissipation, while atmospheric lightning disturbances in general appear to have relatively less destructive effects upon lines and apparatus for the very high voltages.

It appears probable that the one problem of 220-kv. transmission to which present practise cannot satisfactorily be extended without considerable modification is that of line insulators. While it is believed that present commercial types of suspension disk insulators could be made to give acceptable service, they are not regarded as likely to prove wholly satisfactory. Even if it could be assumed that the deterioration which has constituted such a serious defect with the earlier disks has in recent types been adequately overcome, it would still appear that both the electrical and mechanical characteristics of these insulators are not well adapted to the conditions of extra high voltage service. With long strings there is an excessive concentration of potential on the end disks and very little would be added to the total flashover of the string by increasing the number of disks. For 220 kv. this condition appears to be so pronounced that probably some remedy would have to be devised. It has been suggested that grading by using several types of disks with different electrostatic capacities would effect an improvement in the potential distribution over the string, although maintenance would be somewhat complicated, while another suggestion is for the installation of suitably shaped electrostatic shields at the ends of the strings. A difficulty considered even more serious is that mechanically these disks are not strong enough, in a single string, to support with adequate margins of safety the extreme loads which have been assumed as called for by 220 kv. service, due to the large size of conductor, the long spans probably demanded by considerations of economy and the heavy design loadings which the high service standards would dictate. The complication attending the use of two-string or three-string assemblies for suspension units is considered highly undesirable. In view of these limitations of the present commercial type of high-voltage insulator, the need for an improved or radically different design for higher voltages has been strongly emphasized. Some of the insulator manufacturers appear to be making extensive investigations and experiments in this field.

The long lines and high load capacities per circuit, conditions which call for extra high voltage service, have led to the recommendation that voltage be controlled mainly at the receiving end, either by synchronous condenser installations operated as an integral part of the circuit or, where practicable, by the use of excess generator capacity in the receiving system. For a given amount of power to be transmitted, ample condenser capacity, by improving voltage regulation, would reduce the number of circuits otherwise needed and by reducing the values of line current would permit of economical use of smaller conductors.

For 220 kv. transmission the expensive character of substation apparatus and the greater insurance of immunity from operating troubles which is afforded by simplicity of system would both favor keeping to a minimum the number of connections. It may be presumed that the tendency in developing such a system will be to use the high voltage for trunk transmission only, existing lower voltage transmission systems or extensions thereof being employed in general to reach customers and minor or secondary load centers.

The practise of interconnecting power supply systems in adjacent territories, which, during the war, gained new impetus as a result of the need for obtaining the maximum effective output from existing generating capacity, has been further extended, and numerous plans are in progress or under discussion for interconnections of considerable magnitude. The great interconnected system in the Southeastern states has been extended and the interconnections reinforced.

This movement toward interconnection is hastening the tendency, already evident, toward uniformity and simplicity in transmission standards and practises. Standardization of voltages in multiples of 11 kv. is well established, accepted steps being 22 kv., 33 kv., 44 kv., 66 kv., 88 kv., 110 kv., 132 kv., and 154 kv., with 220 kv. projected as the next step. Certain earlier standard steps, such as 60 kv. and 100 kv., are no longer used except in connection with existing systems. The advantage of keeping these standard voltage steps to a minimum are so great, even more from the point of view of future interconnection than of economy in manufacture, that certain of the steps in the above list may fall into disuse. Eighty-eight kv. for instance, does not seem likely to be continued as a standard, while possibly either 33 kv. or 44 kv. and either 132 kv. or 154 kv., may tend to drop out. Sixty cycles, likewise, is becoming firmly established as the standard frequency for the United States, and its virtually exclusive use for general power supply can be predicted for the near future. The practise of using a grounded neutral connection for the higher voltages is considered to have clearly demonstrated its superiority and is rapidly supplanting the isolated delta connection, while for larger systems at least the preference for the grounded neutral connection is evident also in case of the lower voltages.

In general it is coming to be clearly recognized that interconnection of power supply systems is not to be merely an occasional expedient but the normal condition, and that it will tend to become more and more extensive, adjacent systems being connected into groups, and these groups being joined by trunk transmission lines into systems covering a number of states, probably leading eventually to an interconnected system of nation-wide proportions. With this tendency toward widespread interconnection so unmistakeable, an important obligation rests upon the engineering profession to so plan present work as to enable it to fit into such development with a minimum of waste, particularly in such matters as standardization of voltages and frequencies in adjacent territories. The establishment of a comprehensive power supply system, district or regional, will mean much to the future of the electrical power industry and to the industrial future of the country, in which the question of power supply is clearly destined to become one of the determining factors.

TENDENCIES IN UNDERGROUND CABLE PRACTISE

The lack of accurate information on certain elements influencing the operation of underground cables, has in the past caused this branch of the transmission and distribution to be neglected to some extent, and some of the operating companies have been put to a great deal of trouble and expense on this account. At present, however, there is a growing interest in this subject which has already resulted in a marked improvement in the manufacture of cables, so that the carrying capacity of cables manufactured today is materially greater than that of cables manufactured a few years ago.

The principal improvement is the reduction of dielectric loss at relatively high temperature. This has been accomplished principally as the result of a thorough study of the properties of impregnating compounds. It is now possible to manufacture cable the rating of which will be decreased only a few per cent under temperature as high as 100 deg. cent.

These refinements in manufacture make the three-phase underground cable fairly satisfactory for voltages up to 25,000, and for loads per circuit up to 10,000 kw., but if these limits are to be exceeded it will probably be necessary to go to single-conductor construction.

During the past year there have been a great many serious burnouts on underground systems where trouble was communicated from one cable to another until sometimes a whole duct line was involved, and at present there is a great deal of attention being given to methods for the prevention of the spreading of such troubles.

Results have been improved by changes in the design of duct lines, such as the wider separation of ducts especially where they enter the manhole. This wider separation tends to prevent the communication of trouble from one cable to another which is very dangerous where there are a great number of important cables in one line. For the same reason it is now the prevailing practise to use some form of conduit construction that gives an individual wall of concrete around each duct, as concrete is a very good arc-resisting material.

Protection of cables from fire in manholes by cement covering is still the prevailing practise. Methods of applying the cement varies to some extent, but the results are uniformly satisfactory.

There is also a recognized advantage in installing separate duct lines for cables operating at widely different voltages in order to avoid the possibility of limiting the load on some of the cables on account of the temperature limits of cables working at another voltage. This temperature influence is very important and should always be considered when there is a possibility of extreme limitations being placed on some cables by the influence of others.

As regards sizes and types of underground cable, manufacturers report a steadily growing demand for cables of 350,000 to 500,000 cir. mils. There is also a tendency on the part of customers to specify sector type cable on account of the smaller overall diameter for a given copper cross section.

The installation of these large size cables makes it more difficult to bend and train the cable in manholes and in some cases requires a modification in the design of manholes to avoid sharp bends.

The rectangular manhole with duct openings in the center of, and flush with one wall, compels the bending of the cable at too short a radius. Oval or rectangular manholes can be used, but as these shapes are not entirely satisfactory, it has been proposed that the rectangular manhole be set at 45 degrees with the center line of the conduit, and the duct entrance be recessed so as to permit of training through the manhole with long radius bends.

The Hochstadter type of cable in which the individual conductors are covered with a thin metal foil, and the assembled conductors sheathed without belt insulation, is now being supplied commercially.

Users of split-conductor cable report satisfactory operating results and state that the advantages of the selective protection obtained by the use of this type of cable more than offset the disadvantage of the smaller current-carrying capacity for a given duct size.

E. B. MEYER, *Chairman.*

ANNUAL REPORT OF THE IRON AND STEEL INDUSTRY COMMITTEE

To the Board of Directors,

DURING the Convention held at Lake Placid in June, 1919, Dr. C. P. Steinmetz in discussing the work of the technical committees of the Institute expressed the belief that the annual reports of such committees should be planned to provide for the use of the entire membership of the Institute, a comprehensive record of the present status and recent progress of electrical engineering in the particular industry or branch of the art to which each committee had been organized to apply itself.

Accordingly, Mr. Wilfred Sykes, the Chairman of the Board's Committee on Technical Activities, suggested to the various 1920 Technical Committees that their reports "should cover notable installations or developments during the year and in view of the fact that, in the past, little has been done in this respect, that the reports for 1920 might be a review of the state of the art up to date."

Guided by these suggestions it has been the aim of the Iron & Steel Industry Committee, to at least prepare as a foundation, the present status of electrical application to the iron and steel industry, upon which subsequent committees might base annual reports showing recent progress. The field is an extremely broad one dealing as it must, with practically every process in the transformation of the raw material into the many forms of finished product. Electricity is utilized in the handling of the ore and provides the means of handling the finished products as well as the final heat treatment of the more highly developed products. From the mechanical point of view, it has been stated that from the ore to the finished product the hand of man has no occasion to touch the materials and it can be stated further that throughout this same transformation, practically no step is taken in which electricity is not playing an increasingly important part.

While the iron and steel industry in America, dates back into the country's earliest history, electricity only comparatively recently began to play its important part, to be more exact, about 1891. At present the total generating capacity required by the industry is approximately one million kilowatts and each new development in the art of steel making opens up new fields for electrical application so that in many important respects, the making of steel is becoming an electrical industry.

Supplement A has been prepared to indicate the major applications of heat and power entering into the making of iron and steel products from the raw materials, and to show the extent to which electrical heat and power have been or may be applied. As to the auxiliary applications of power in the handling, manipulating, and finishing of the materials and products, electricity is almost universally used as supplementary data will show.

It was decided to consider the mining of the raw materials as outside the scope of the Iron & Steel Industry Committee and it has dealt only with the material after arrival at the blast furnace. Reference again to Supplement A will indicate the processes to which the Committee has applied itself. Furthermore, it has not been intended to prepare a handbook for electrical engineers engaged in the iron and steel industry, but rather provide a source of general information for the large proportion of the Institute membership engaged in the many other lines of electrical engineering.

Using Supplement A as a basis, memoranda have been or are being prepared along the following general lines, believing that brief consideration of each subject separately, could be used to better advantage and more readily added to than to attempt a consecutive description.

Blast Furnace Group

Memorandum A 1—The status of iron ore smelting in the electric furnace.

Memorandum A 2—Synthetic electric furnace pig iron.

" A 3—The electrical precipitation of dust of blast furnace gases.

Electric Melting Furnace Group

*Memorandum B 1—Statistical data—The growth of the application of electric furnaces.

* " B 2—Types and electrical characteristics.

Electric Heat Treatment Furnace Group

Memorandum C 1—Electrical heated soaking pits.

" C 2—Carbon resistor type furnaces.

" C 3—Typical heat treating plants (Carbon resistor).

* " C 4—Metallic resistor type furnaces.

Rolling Mill Group

*Memorandum D 1—Statistical data.

* " D 2—Types of Drives—Non Reversing — Adjustable, Speed — Reversing.

Miscellaneous Group

*Memorandum E 1—The electric hydraulic system as proposed for large forge presses.

The Committee has prepared and sent to all the principal Iron and Steel Companies, the following questionnaire and the results will be tabulated as a general indication of the extent and characteristics of both electric power supply and application. This data will appear in the form of supplement B at an early date.

ELECTRICAL STATISTICS

Name and location of plant...
POWER SUPPLY (period January to December, 1919, inclusive)
Purchased from central stations.......................... kw-hours
Generated by own steam units from coal-fired boilers....... kw- "
" " " " " from gas or waste heat boilers kw- "
(including exhaust pressure turbines)
" " " gas engines using waste gases........... kw- "
MOTOR EQUIPMENT a-c....phase...cycle...volts d-c....volts
Number of motors............. ....
Total horse power............. ....
GENERATING CAPACITY a-c.....volts. d-c.......volts.
Average kw. capacity in use—kw..... kw....... { Do not include sub-station d-c. apparatus }
Average load factor......... %.... %.....

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*In preparation for issue at an early date.

MEETINGS AND PAPERS

No attempt has been made to secure papers or arrange Institute meetings for the consideration of iron and steel Industry subjects, owing to the extensive interlocking of the memberships of the A. I. E. E. and the Association of Iron & Steel Electrical Engineers which holds monthly meetings of high grade in Pittsburgh, Cleveland, Chicago, Birmingham and Philadelphia. In some cases joint meetings between the local sections of these organizations have been successfully held and an extension of this practise is recommended. The nature of the papers presented at the A. I. & S. E. E. meetings is made clear by the list supplementing this report (Supplement C) and it is recommended that closer relations be established between the two organizations through the publication, in the JOURNAL of the A. I. E. E., of notices before and after the A. I. & S. E. E. meetings, together with other items of mutual interest.

It is further recommended that the Board of Directors consider the scope of the work of the Association of Iron and Steel Electrical Engineers and with the appointment of the iron and steel industry committee for the coming year, formulate a definite policy as to the nature of the Committee's activities in order to avoid duplication of effort on the part of the two organizations, the memberships of which are closely interlocked.

The Board of Directors of the A. I. E. E. has authorized this Committee to discuss with the Board of the

A. I. & S. E. E. the question of co-operation in the matter of Standardization Rules and it is regretted that no opportunity for such conference has presented itself. Such action is strongly recommended at an early date.

Many valuable papers relative to electric furnaces in the iron and steel making processes are presented in the meetings of the American Electrochemical Society and are recommended for notice and possibly publication in the A. I. E. E. JOURNAL.

The Committee wishes to acknowledge with thanks, the hearty co-operation it has received from the individuals and companies appealed to in the gathering of the data presented herewith.

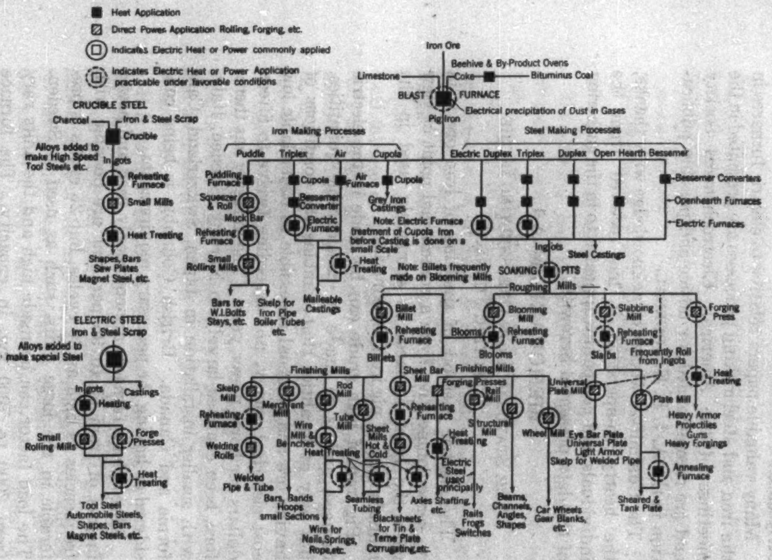
MEMORANDUM A 1

THE STATUS OF IRON ORE SMELTING IN THE ELECTRIC FURNACE

The electric method of smelting of iron ore is in no sense competitive from a commercial point of view with the blast furnace, as commonly used in the manufacture of pig iron, in America.

Pig iron of high grade can be made in the electric furnace, and is so made in considerable quantities, but only at localities that combine the conditions of having ore and charcoal accessible to dependable low cost electrical energy, not exceeding twelve to fifteen dollars per horse power year. The industry has therefore, been confined largely to Norway, Sweden, Italy, with smaller installation in Japan, Switzerland, California and Canada. British Columbia is being considered as having the necessary requirements.

Three-phase furnaces are commonly used and power factors of from 65 to 95 per cent are possible with frequencies as high as 60 cycles, but 25 cycles is preferable in the large units, such as used in Sweden. The load factor can be made high and the units vary from 2000 to 7000 kw. in capacity. The furnace voltage is low, being below 100 volts, and voltage regulation is accomplished by means of taps in the high-tension side which limits the use of very high trans-


■ Heat Application
▨ Direct Power Application Rolling, Forging, etc.
Indicates Electric Heat or Power commonly applied
Indicates Electric Heat or Power Application practicable under favorable conditions
CRUCIBLE STEEL
Charcoal
Iron & Steel Scrap
Crucible
Alloys added to make High Speed Tool Steels etc.
Ingots
Reheating Furnace
Small Mills
Heat Treating
Shapes, Bars Saw Plates Magnet Steel, etc.
ELECTRIC STEEL
Iron & Steel Scrap
Alloys added to make special Steel
Ingots
Castings
Heating
Small Rolling Mills
Forge Presses
Heat Treating
Tool Steel Automobile Steels, Shapes, Bars Magnet Steels, etc.
Iron Ore
Limestone
Beehive & By-Product Ovens
Coke
Bituminus Coal
BLAST FURNACE
Electrical precipitation of Dust in Gases
Pig Iron
Iron Making Processes
Puddle
Triplex
Air
Cupola
Steel Making Processes
Electric
Duplex
Triplex
Duplex
Open Hearth
Bessemer
Puddling Furnace
Squeezer & Roll
Muck Bar
Reheating Furnace
Small Rolling Mills
Bars for W.I.Bolts Stays, etc.
Skelp for Iron Pipe Boiler Tubes etc.
Cupola
Bessemer Converter
Electric Furnace
Air Furnace
Cupola
Grey Iron Castings
Note: Electric Furnace treatment of Cupola Iron before Casting is done on a small Scale
Heat Treating
Malleable Castings
Bessemer Converters
Openhearth Furnaces
Electric Furnaces
Ingots
Steel Castings
SOAKING PITS
Note: Billets frequently made on Blooming Mills
Roughing Mills
Billet Mill
Reheating Furnace
Billets
Blooming Mill
Reheating Furnace
Blooms
Slabbing Mill
Reheating Furnace
Slabs
Frequently Roll from Ingots
Forging Press
Heat Treating
Finishing Mills
Skelp Mill
Reheating Furnace
Welding Rolls
Welded Pipe & Tube
Merchant Mill
Bars, Bands Hoops small Sections
Rod Mill
Wire Mill & Benches
Heat Treating
Wire for Nails, Springs, Rope, etc.
Tube Mill
Seamless Tubing
Sheet Mills Hot & Cold
Blacksheets for Tin & Terne Plate Corrugating, etc.
Sheet Bar Mill
Reheating Furnace
Finishing Mills
Forging Presses
Heat Treating
Electric Steel used principally
Axles Shafting, etc.
Rail Mill
Rails Frogs Switches
Structural Mill
Beams, Channels, Angles, Shapes
Wheel Mill
Car Wheels Gear Blanks, etc.
Universal Plate Mill
Eye Bar Plate Universal Plate Light Armor Skelp for Welded Pipe
Plate Mill
Heavy Armor Projectiles Guns Heavy Forgings
Annealing Furnace
Sheared & Tank Plate

ELECTRICITY IN THE MAKING OF IRON AND STEEL PRODUCTS

NOTE—Practically all auxiliary applications of power for the transportation, manipulation, hoisting, pouring, shearing, finishing, magnetic lifting, etc. of the steel and iron materials, raw materials and machinery or tools used, are electrical.

mission voltages without intermediate stepdown transformers.

The furnaces used in Sweden where the greatest amount of electric pig iron is produced (approximately 70,000 tons in 1918) consist of a furnace shaft quite similar to a small blast furnace independently supported above a relatively shallow crucible of larger diameter which is covered by a circular arch through which the carbon electrodes project into the charge which is fed to the top of the furnace as in a blast furnace.

MEMORANDUM A 2

SYNTHETIC ELECTRIC FURNACE PIG IRON

Conditions of shortage of pig iron and the unusual availability of steel turnings incident to the war period, developed the manufacture of synthetic pig iron in electric furnaces by re-carburizing fusion of steel turning or scrap. This process was used principally in Canada and France, the latter country having made at Levet, Nanterre and Limoges over 150,000 tons during the war period. The future of this industry depends upon cheap water power, availability of materials and the relative cost of coal.

Typical Furnaces in Canada. A small 250-kw., single-phase furnace at Orillia, Ontario, produced six to seven tons of low phosphorus pig iron per twenty-four hours. A six-ton, three-phase furnace at St. Catherines, Ontario, with 1200-kw. transformers produced approximately 20 tons per day, with a 700-kw. input and with full 1200-kw. input—35 tons per day. For each hundred weight of turnings charged, there was added 5 per cent in the form of carbon and silicon and the yield was approximately 95 lb. of pig iron. The furnaces were of the enclosed type as usual to the making of steel and the current was of standard frequency. The load factor and power factor can be made high.

The French Furnaces. Open type furnaces of greater capacity, are used and are principally of single-phase construction with one moveable electrode and the other imbedded from below in the hearth. Standard frequencies with high load factor and power factor prevail. A 2500-kw. furnace produces 80 to 100 tons per day with a power consumption of 675 kw-hr. per ton. Complete foundries for the casting of shells were in some cases located adjacent to the pig iron furnace and by a further treatment in single-phase electric mixing furnaces of closed type, with an expenditure of approximately 50 to 100 kw-hr. per ton, the metal was made to meet the specifications for such castings (carbon 2.90, silicon 1.75, manganese 0.50, sulphur-trace, phosphorus 0.05).

These single-phase mixer furnaces are operated in groups of three and have one adjustable electrode above with the other electrode imbedded in the hearth from below.

MEMORANDUM A 3

THE ELECTRICAL PRECIPITATION OF DUST OF BLAST FURNACE GASES

The Cottrell system of electrical precipitation has been applied in two instances to the cleaning of blast furnaces gas. The advantages are such that such equipment is likely to become common among the blast furnaces.

In applying this system the gas piping from the blast furnace is arranged so that the gas is conducted through a group of vertical pipes from bottom to top and the high-tension electrode in the form of chains or wires held taut by weights, are supported in the center of the pipes from insulators at the top. The dust and fume particles are repelled by the charged electrode and are deposited on the sides of the pipes from which it is jarred by vibration of the pipes at suitable intervals (once each hour or more). This jarring can be accomplished automatically by relays adjusted to voltage drops and amperage increases which occur when the accumulation of dust becomes excessive. During the shaking down of the dust the precipitating apparatus is shut down for the necessary two or three minutes required.

A typical installation, put into service during 1919, takes three-phase, 60-cycle, 220-volt current from the power system, two 25-kv-a. transformers in open delta being ample for the entire power requirement which is 15 to 20 kv-a. to clean 45,000 to 50,000 cubic feet of gas per minute. The 220-volt current is applied to a small synchronous motor operating at 1800 rev. per min. and driving a mechanical rectifier, and is also fed through transformers which step the voltage up to 35,000 to 50,000 volts, adjustable by taps. The rectifier converts this high-tension current to the unidirectional current required by the precipitating electrode. The equipment can be made to suit any standard frequency.

A second installation has been made more recently in connection with a blast furnace and is of about one-half the capacity of the first equipment.

MEMORANDUM C 2

CARBON RESISTOR TYPE FURNACE FOR STEEL TREATING

Furnaces of this type are usually rectangular in shape with doors at each end for the receipt and delivery of the materials to be treated. Parallel with the length of the furnace, one on each side, are located two resistor troughs. The resistance elements composed of broken carbon are thrown loosely into these carborundum fires and troughs which are supported on brick pillars along the side wall and protected by being recessed back from the space into which the steel materials are introduced. The heat from the resistance element is radiated to the walls and top of the furnace which is lined with refractory materials which become heated and in turn heat the charge. Carbon or graphite electrodes at each end of the troughs provide the means of introducing the current and the control of the current, hence the heat is effected by varying the voltage impressed at the electrodes, this variation being obtained by means of numerous taps on the secondaries of the special transformers.

The materials to be treated are in some cases put into the furnace on cars for a period treatment and in other cases the operation is continuous, the materials being slowly pushed or conveyed through the furnace.

Advantage is invariably taken of the possibility of making these furnaces automatic, both as regards the cycle of temperature treatment and the charging and discharging.

The following list of furnaces, together with the dates of installation, will indicate the development and application in steel treating:

Plant	Date	Application	No.	Electrical capacity each	Tons of steel heated per 24 hours
No. 1	1915	Annealing castings	1	150 kw.	12
2	1916	Annealing castings	1	300 kw.	24
3	1916	Treating draw-bar knuckles	2	1–600 kw. 1–300 kw. Tandem operation	144 total
4	1916	Annealing steel castings	1	600 kw.	12
5	1917	Treating small steel parts for aeroplanes and tractors	1	150 kw.	24
6	1917	Treating auto gears	1	40 kw.	4
7	1918	Duplicate of plant	3		
8	1918	Duplicate of plant	5		
9	1918	Annealing R. R. axles	1	150 kw.	12
10	1918	Treating cast steel anchor chain	2	1–600 kw. 1–300 kw. Tandem operation	150 total
11	1918	Treating crank shafts for liberty motors		1–600 1–300 Tandem operation	144 total
12	1918	Annealing steel	1	600 kw.	75
13	1919	Roller bearing and ball race treatment	1	150 kw.	12
14	1919	Annealing steel sheets	1	600 kw.	150
15	1919	(2) Duplicate of plant	10		
16	1920	Treating bolts and nuts	1	40 kw.	4
17	1920	Roller hardening	1	75 kw.	5

MEMORANDUM C 3

TYPICAL HEAT TREATING PLANTS

(Carbon Resistor Type)

Plant A. Automatic furnace handling roller bearing and ball races.

Materials to be treated are placed in metal baskets holding approximately 125 lb. each and fed into one end of rectangular carbon resistor type of furnace maintained at approximately 1525 deg. fahr. by an input of 150 kw. A clock contactor regulates, over a variable range, the rate of flow of material through the furnace and ordinarily 1000 to 1200 pounds of material is handled per hour.

The discharge of the heated material is into a drawing or quenching tank, the oil of which is maintained at a temperature of approximately 350 deg. fahr. by means of immersed nichrome resistors requiring about 40 kw.

The temperature control of the furnace and drawing tank is by pyrometers and the adjustable clock contactor devices make the operation entirely automatic as to the temperature, the time in the furnace and the time in the tank. Motor-driven conveyers, door operating mechanisms and oil pumps are all under the control of the time and temperature measuring devices.

Plant B. Annealing furnace for treating low carbon cold rolled strip.

The annealing furnace is approximately 225 ft. long and 22 ft. wide with the heated chamber in the middle. Two parallel tracks pass through the furnace and heated chamber, and cars of steel strip each holding about 20 tons of material are fed on to tracks from opposite ends, the purpose being to absorb in the cold carload on one track the heat from the heated carloads on the other track on the way to the discharge point.

The temperature attained in the electrically heated chamber is approximately 1200 deg. fahr. A movement of the cars takes place every six hours, discharging one twenty-ton car of material at each end or approximately 150 tons of annealed material per 24 hours with an electrical input of approximately 120 kw-hr. per ton.

Plant C. Annealing furnace for treating alloy steel.

Arrangement is similar to Plant B except furnace is shorter and the nature of the steel requiring a 40-hour treatment at about 1400 deg. fahr. The ten cars in the furnace at one time each hold 30 tons of steel or a total of 500 tons, 120 tons of which is in the middle electrically heated chamber. The delivery is approximately 72 tons per day with an electrical input of approximately 250 kw-hr. per ton of metal annealed.

SUPPLEMENT C

REPRESENTATIVE PAPERS RECENTLY READ BEFORE THE ASSOCIATION OF IRON & STEEL ELECTRICAL ENGINEERS MAIN AND SECTION MEETINGS.

- Organization of Electrical Department in the Iron & Steel Industry.
  - Inspection and Operation.
  - Educational Training for Employees.
  - Electrical Repair Shop.
  - Storeroom and Spare Parts.
- Records and Tests.
- A-C. *vs.* D-C. Motors for Rolling Mill Table Drives.
- Recent Improvements in Industrial Control.
- Present Status of Electric Furnaces in the Steel Industry
- Electric Heat Treatment.
- Overload Protection for Motors.
- Overload Protection on Cranes.
- General Specifications for A-C. Motors for Main Roll Drive.
- Present Status of Arc Welding in the Iron & Steel Industry.
- Safety Rules for Government of Employees Working on Electrical Equipment.
- Babbitt and Babbitting.
- Grounded Neutral.
- Current-Limit Reactance.
- Welding, Electric *vs.* Gas.
- Electric Rolling Mill Drives.
- Steel Plant Power Generation from Waste Heat and Coal.
- Electrical Equipment of the Largest Plate Mill in the World
- Electrical Installations at Trumbull Steel Co.
- Steam *vs.* Electric Driven Mills.
- Electrical Features of a Modern Steel Plant.
- Steel Mill Electrical Repair Shop Practise.
- Influence of Gear Ratio on Speed of Operation, Motor Heating and Contactor Wear in Auxiliary Steel Mill Drives.
- Automatic Electrolytic Oxy-Hydrogen Plant.
- Heating of Underground Cables.

ANNUAL REPORT OF THE INDUSTRIAL AND DOMESTIC POWER COMMITTEE

To the Board of Directors,

THE Industrial and Domestic Power Committee has continued its earlier plan of work without change or modification. We are convinced that it is sound. We know the results now appearing will justify the study. We strongly urge its continuance.

Eleven subcommittees are analyzing electrical power applications in eleven different branches of industrial activity. The analysis consists of a study of the various processes involved, the movements in the processes and the electrical application to the movements. No standardization studies are considered. This is a province of other committees. The work is entirely ethical. In no way is our work overlapping studies by other Institute committees, or by committees external to the Institute.

A similar study of the electrical power applications in the use of prominent industrial tools is being conducted.

The work is being reported by monographs. Each monograph comprises a section of subcommittee study, complete in itself and yet forming a part of the whole. Each subcommittee is planning its own work to its own ideas, but all in conformity to the general plan. In the final execution of the plan, a progression of monographs carefully edited and approved by the Industrial and Domestic Power Committee will reach headquarters and will be available for presentation to Section or Institute meetings; for publication in the JOURNAL or for sale by headquarters at a nominal fee. With the interest being generated in the plan, we look confidently for extremely valuable returns; for treatises on industry of practical value. They in no sense can replace the work of the consulting engineer.

They should be a guide in the study and planning of industrial applications.

In our work we have been helped by men not of the main committee, but regularly appointed by the President to our several subcommittees. These appointments have been cheerfully accepted, and such results as are being obtained are because of their service, and we may suggest that this plan has brought into active Institute service men whose abilities for such service were not otherwise being reached.

The personnel of the several subcommittees follows:

1. Subcommittee on Motors with Particular Reference to Speed-Torque Characteristics.

A. M. Dudley, *Chairman.*
R. H. Tillman, A. C. Lanier,
Wilfred Sykes, John C. Parker.

2. Subcommittee on Domestic Power Application.

H. Weichsel, *Chairman.*
James Dixon, C. L. Kennedy,
A. F. Welch, L. L. Keilholtz.
Bernard Lester, Edgar D. Doyle.

3. Subcommittee on Applications in Printing Industry.

W. C. Kalb, *Chairman.*
W. E. Date, John D. Nies,
J. C. Lincoln, Carl F. Scott.

4. Subcommittee on Applications to Cranes and Hoists.

R. H. McLain, *Chairman.*
H. W. Eastwood, E. Friedlaender,
James A. Shepard.

5. Subcommittee on Applications to Machine Tools.

H. D. James, *Chairman.*
T. E. Barnum, W. C. Yates,
W. T. Snyder, R. H. Goodwillie.

6. Subcommittee on Applications to Passenger and Freight Elevators.

R. H. Goodwillie, *Chairman.*
H. D. James, H. P. Reed,
David L. Lindquist, Charles H. Roth.

7. Subcommittee on Application in Textile Industry.

H. W. Cope, *Chairman.*
C. T. Guildford, S. B. Paine,
J. C. Ramsay, D. H. Sadler.

8. Subcommittee on Applications in Cement Industry.

R. B. Williamson, *Chairman.*
H. Weichsel, C. A. Kelsey,
J. F. Siegfried, Arthur Simon,
S. A. Staege, M. R. Woodward.

9. SUBCOMMITTEE ON APPLICATION IN WOODWORKING INDUSTRY.
L. E. Underwood, *Chairman*
Truman Hibbard, Robert L. Smith
S. A. Staege.

10. SUBCOMMITTEE ON APPLICATION OF ELECTRIC ENERGY IN INDUSTRIAL HEATING.
E. V. Buchanan, *Chairman*.
T. E. Penard, H. O. Swoboda,
W. S. Scott, H. A. Winne.

11. SUBCOMMITTEE ON APPLICATIONS IN RUBBER INDUSTRY.
W. E. Date, *Chairman*.
M. Berthold, A. C. Bunker,
B. T. Mottinger, C. A. Rice,
H. F. Schippel.

Most of the subcommittees have made reports of their activities for the term. It is impossible to give these complete in this report without making it unwieldy. It is impractical to condense them. They should have individual consideration, and their reading amplifies this picture of our work. For this reason they are forwarded herewith for the information and consideration of the Board of Directors. The subcommittees in the main have worked hard and conscientiously to develop the plan, the main committee acting as a clearing house, and in an advisory capacity. All subcommittee chairmen are members of the parent committee. But the burden of the work has fallen to the subcommittees, and their work merits every appreciation.

The outstanding feature in electrical industrial development during the present term has been magnitude of production. Demand has continued greater than supply taxing combined production to the greatest limit. Probably the development and increase in use of fractional horse power motors for domestic and small industrial use has been the greatest contributing factor in this total. In this fractional field, quantity of production especially is to be emphasized. In general, quantity demand has been the real problem, the design and application of new productions being forced to second place.

In larger horse power fields, the increasing use of the synchronous motor is to be noted. As a corrective of the power factor of industrial loads it continues not only desirable but a growing necessity to meet a continual problem. In this connection, the use of the static condenser has continued as a power factor corrective.

The use of the direct-current motor in its several types continues accurately as does the use of the induction motor. In general there is continued progress in the standards of application of the several types of motors and in the recognition of these standards and away from misapplication, and great credit must be accorded to the discerning intelligence of the engineers connected both with the production and use of motors that has cooperatively tended to bring this about.

The Industrial and Domestic Power Committee particularly appreciates the advice of your committee on technical activities, together with the help constantly accorded from our Secretary's office at headquarters. As a committee, we have tried to recognize fully the trust imposed and bespeak for our successors your full cooperation in the work we are undertaking.

A. G. PIERCE, *Chairman.*

ANNUAL REPORT OF THE MARINE COMMITTEE

To the Board of Directors,

IT will no doubt be recalled the inception of the Marine Committee was in 1913. Little actual work was accomplished by this Committee until last year, when the compilation of a set of rules for electrical installation on shipboard was started, together with dissemination of knowledge relative to that subject to the electrical engineers of the newly created ship yards.

The immensity of the work, that of preparing the Marine Rules started, was the more fully appreciated as the work progressed, and while the work of last year's committee was very commendable, the volume of the work completed was performed by this year's committee. The number of members constituting the committee this year was greatly reduced to facilitate the work and curtail expenses as much as possible.

Eight meetings of the committee were held. The first meeting was given over to the appointment of the various Committees and the outlining of the work of each. The Ship Installation, Marine Propulsion and Historical Committees constituted the main divisions. The first two of these committees were supplemented by subcommittees to look after the vast amount of detail work involved in the activities of the main committee.

In last year's Annual Report of the Marine Committee it was suggested that the unfinished work of that Committee be continued this year. This unfinished work consisted of the completion of the rules for governing the installation of electrical apparatus on shipboard. The work on these rules that was not completed consisted of the following:

Revision of Rules prepared last year.
Control Equipments.

Alternating-Current Apparatus Control.
Radio Installations.
Running Lights, etc.
Fire Alarm Systems.
(not completed by subcommittees.)
Tabulation of Generating Sets.
Storage Battery Installation.
Gyro-Compass Installation.
Gyro Stabilizing Installation.
Tabulation of Wires and Conductors.
Appliances in vicinity of Compass.
Inspection Report to Classification Societies.

The Rules for Electrical Installations, as stated above, have now been completed and are in the hands of the Secretary to be passed on by the Board of Directors. These Rules will be published in pamphlet form and it is the intention that they be sold at a nominal figure.

Too much credit cannot be given to the various members of the Ship Installation Committee and the Chairman, Mr. G. A. Pierce, in particular for the work accomplished, as we believe these rules to be the most complete and comprehensive ever formulated by any society, covering as they do practically every phase of the electrical field as applied to ship work and extending the field of the Institute to practical installations. This being an innovation on the part of the Institute, the attitude of the ship owners, builders and classification societies relative to their adoption is eagerly awaited. Several new features are also introduced, which will probably extend the field of this work to a marked degree.

The rules as now completed have been referred to the various classification societies and the emergency Fleet Corporation, all of which were given opportunity to comment. This method of action has gone far to put in composite form the work as viewed from the different angles. When published, copies of the rules are to be forwarded to the Institute of Electrical Engineers of England, France and Italy.

As the work of the Ship Installation Subcommittee neared completion, it was found that, due to the magnitude of this work, it would be necessary to have someone care for the preparation of it, in order that it could be put in final form for publication. It was therefore decided to appoint an Editing Committee, whose duty it would be to correlate the vast amount of information contained in these rules. The work of this committee was no easy task and we therefore take this opportunity of complimenting it on its labors in connection with this work.

The Marine Propulsion Committee has prepared its historical and data work and it is suggested that this be put in form to be published in the JOURNAL of the Institute, as a means of drawing out discussion and additional data as regards this subject. It was decided to await experience on electrically propelled ships, soon to be put in service, rather than introduce data based on theory only. It is suggested that this feature be given careful thought and consideration during the coming year, as many new ideas will probably result from the experience gained by that time.

HISTORICAL COMMITTEE

The Historical Subcommittee has prepared a Historical Review of the Use of Electricity on Shipboard. This review accompanies and forms a part of this report. A more complete account will be prepared for publication in the JOURNAL of the Institute. This is considered a very important work, as the evidences of the use of electricity in the marine field are only fragmentary and should be placed in a form of record, as a matter of history. When this is accomplished, it will then be a very simple matter to add, from time to time, the outstanding events in their chronological order. This historical feature is one that should not be lost sight of in the work of future committees.

In line with the requirements of the rules as now completed, governing the installation of electrical apparatus on merchant ships, it is suggested that next year's committee take up the matter of approval of various fixtures, fittings, etc., with the end in view of having these meet the requirements of the new rules. It is believed that the preparation of General

Construction Specifications, covering various appliances should be performed by the Marine Committee.

One of the ideas brought forth during the year, was the consideration of terminal facilities at marine piers. This is more or less a new activity, as very little has been done thus far to facilitate the handling of cargoes at these terminals. The field is large and is therefore due a considerable amount of thought.

In conclusion, I feel that particular comment should be given to large attendance of the members of this committee at the regular meetings and the splendid spirit of co-operation existing between these various members. It is only in such harmony that the greater results, for which we all hope, are to be attained.

ARTHUR PARKER, *Chairman.*

HISTORICAL REVIEW OF THE USE OF ELECTRICITY ON SHIPBOARD

The marine field has probably been one of the most backward in the adoption and development of electrical apparatus which in the main, we believe has been due to the traditional conservatism of the seafaring man. While electricity was used to a very limited extent as far back as 1882 for interior communication, watertight bells, buzzers, annunciators, etc., having been installed by the Chas. Cory & Son Co. on the *Santa Rosa* about this time, and in the Navy electrical instruments were installed on the *Trenton* in 1883, apparently one of the first, if not the very first, electric light installations was also made on the *Trenton* about this time, at New York Navy Yard. All early lighting installations were Edison bipolar generators belt driven to an Armington-Sims or other reciprocating type engine. Voltmeters were not used until after the early '80s, and field regulation was not indulged in until about the same time, all of the early installations simply having a pilot light on the generator for gauging the voltage and candle power of the lamps.

Early wiring was installed in wood mouldings, and we believe that Habirshaw wire was considered the standard in the early installations, which was rubber covered with tape outside. Branches were in most cases spliced and the first junction branch boxes were used about 1885-1886. Fuses were used about the same time, usually using an ordinary single fuse placed in the branch circuit on a small insulating base which was inserted in the moulding.

The first marine telephones, we believe, were installed on the U. S. Cruiser *New York* (the first *New York*) about the year 1890. They were single direct circuits, *i. e.*, two telephones used in place of voice tubes. The instruments were of the "Bell" type and installed by Chas. Cory and Sons. The first "loud speaking" marine telephones were installed on the *Korea* and *Siberia* and the U. S. S. *Charlestown*, about 1895.

The first complete central energy (operator's) switchboard telephone systems were installed on the S. S. *Le Grande Duchesse* built at Newport News about the years 1897-1898. Pneumatic electric bells and annunciators were used quite extensively on ships during the period 1878-1890.

While the records are not clear we have every reason to believe that electric bells were installed in the Navy on ships at least ten years previous to the first electric light installation on the *Trenton*.

The most conspicuous example of the use and rapid development of electricity on shipboard has been in the American Navy, which has not only been for years a larger user of electrical machinery than the merchant marine, but also has led the navies of the world in this respect.

Lighting equipments were installed in the Navy on several of the early vessels, beginning with the *Trenton*, and certain power applications installed, such as the ammunition hoists on the Cruiser *New York* and the Battleships *Massachusetts*, *Indiana* and *Oregon*, turret turning equipment on the Cruiser *Brooklyn*, and certain minor auxiliaries such as ammunition hoists on the gun boats *Wilmington* and *Helena*. The extensive use of electrical apparatus for lighting and power purposes was not made however, until the Battleships *Kearsage* and *Kentucky* were constructed in 1898-1900, these vessels having seven 50-kw. engine-driven, 80-volts generating sets, which were operated in series on a 3-

wire system to give 80 volts for lighting, and two voltages 80 and 160, for power motors. The major portion of the auxiliaries on these battleships were electrically operated, including the winches, boat cranes, ventilation fans, broadside ammunition hoists and turret auxiliaries. The anchor windlass and steering gear however, were steam driven.

After the *Kearsage* and *Kentucky* some succeeding battleships continued in many of their auxiliaries the use of steam drive, and the *Kearsage* and *Kentucky* are the only instances in the Navy of the use of the 3-wire control.

About 1902 the Navy Department devised an electric steering gear operated on the follow-up system and purchased apparatus of different manufacturers to effect installation on one of the small monitors which had then been recently constructed. The results obtained from this equipment were not satisfactory however. This question remained under discussion for a number of years until about 1909, when a contract was placed for supplying electric steering gear for the Cruiser *Des Moines* with the Cutler-Hammer Mfg. Co. For this purpose the use of an electric motor taking its power direct from the dynamo mains and operated by an automatic controller was advocated. The results of this trial equipment were so satisfactory that it eventually resulted in the use of electricity for the steering of all capital vessels in the Navy, the later installations, beginning with the Battleship *New Mexico*, employing the use of the electro hydraulic gear instead of the direct motor drive, and to the exclusion of the steam drive.

Beginning with the Battleships *Nevada* and *Oklahoma*, electrical equipments were provided for the anchor hoist gear, which proved so generally satisfactory that all subsequent capital ships have specified electrical drive for this auxiliary, thus practically completing the application of electricity to all auxiliaries on capital naval vessels.

The use of electricity for lighting and auxiliary power purposes in the Navy having proved so generally successful, the General Electric Co. was granted contract for the installation of electric drive on the Collier *Jupiter* in 1911 which proving satisfactory, a contract was given to the same company for the installation of electric drive on the Battleship *New Mexico*, which continuing the satisfactory results obtained, all battleships and battle cruisers authorized since the *New Mexico* have included the electric drive, and undoubtedly it will remain the means of propelling capital ships of the Navy.

In the submarine branch of the Navy various means of supplying the necessary power for propulsion on early boats when submerged were used, such as hand power, compressed air, and steam. With the development however, of the storage battery, the application of electrical apparatus to the submarine made rapid advancement and made the modern type of submarine possible. In the modern submarine, besides using motors and storage batteries for propulsion purposes, all auxiliary machinery such as hydroplane, steering gear, pumps, windlasses, machine tools, periscope hoists, etc., are electrically operated, electricity also being used for lighting, sound detecting devices and interior communication devices of various kinds. The growth of electricity in the case of submarines may be noted from the fact that the early successful types were equipped only with electric lighting, bell signals and motors for propulsion under 50-h.p., whereas in the most recent electric propulsion, motor power has reached about 3000 h.p. As a matter of historical interest, the first application of the use of propulsion motors for submarines arranged with two armatures on the same shaft was made by the Lake Torpedo Boat Co. on submarines built in Russia for that Government and designed in 1906 and furnished with contactor type of controllers arranged for series parallel operation of the armatures as supplied by the Cutler-Hammer Mfg. Co., this type of motor and system of control being now extensively used in the later submarines of the American Navy.

The American Navy therefore, at the present time, is a user of electricity for practically all purposes for lighting and power and extensively for interior communication work.

While the American Navy has led all other navies of the world in the use of electricity, Europe seems to have led this country in the trial of electrical auxiliaries for use on merchant vessels, and while electricity has been used generally for lighting and some interior communication purposes on merchant vessels for years, it has been employed but to a very limited extent for power purposes until the advent of the oil-engine-driven ship, the most notable instances of this kind being the fleet of vessels constructed by Burmeister and Wain of Copenhagen, the *Christian X* being one of the first vessels so constructed by this company, and its first voyage to this country created marked attention. Since that time it is understood that some 70 or more vessels of this type have been constructed and projected by that company.

In 1916 and '17 a number of merchant vessels were projected in this country, including tankers and cargo vessels employing electrical auxiliaries, among others being the Tanker *La Brea* with electric operated pumps and the Tanker *Solitaire* just completed by the Texas Co., having oil-engine drive and electrical auxiliaries throughout, including electric heating. Other instances are six tank vessels constructed by the Pennsylvania Shipbuilding Co. propelled by geared steam turbines and having electrical auxiliaries.

At the present time there are a number of oil-engine-driven cargo vessels with electric auxiliaries and others with electric drive and electric auxiliaries being projected in this country, and it is believed from this time the field will rapidly broaden until a great many of the merchant vessels will be so propelled and operated.

Extensive progress has been made recently in the merchant field in the use of the electric drive, the first installation on record in this country being two fire boats constructed in 1908 on the Great Lakes. Since these first installations in 1908, a total of 28 electrically propelled vessels have been constructed or projected, all but three of these being built or building in this country. Among these may be mentioned

ELECTRIC PROPULSION OF SHIPS (Not Including Naval)

Date	Name of ship	Type of ship	Type of propulsion	Size in tons	S H P	Propeller R P M	A. C. or D. C.	Voltage	Gen. units	Motor units	Remarks
1910	Jos. Medill	Fireboat	Steam electric	....	500	190	D C	250	2	2	Twin screw
1910	Graeme Stewart	Fireboat	Steam electric	....	500	190	D C	250	2	2	Twin screw
1913	Tynemouth	Cargo	Diesel electric	3400	500	78	A C	500	2	1	Method abandoned as Diesel engine misfit
1916	Mjolner	Cargo	Steam electric	800	850	90	A C	440	2	2	Geared single screw
1918	Wulsty Castle	Cargo	Steam electric	6400	1500	76	A C	650	2	2	Geared single screw
1918	Panoil	Tanker	Steam electric	1400	620	150–180	A C	440	1	2	Twin screw
1918	Mexoil	Tanker	Steam electric	1400	620	150–180	A C	440	1	2	Twin screw
1918	Fuel Oil	Tanker	Steam electric	1400	620	150–180	A C	440	1	2	Twin screw
1919	Mariner	Trawler	Diesel electric	500	400	200	D C	250	2	1	2–125 V. gen. Single screw 1–250 V. motor
1920	Crudoil	Tanker	Steam electric	1400	620	150–180	A C	440	1	2	Twin screw
1920	Elfay	Schooner yacht	Diesel electric	313	90	360	D C	125	1	1	Single screw
Building	Guinivere	Yacht	Diesel electric	642	550	220	D C	250	2	1	Single screw
Building	Powhaton	Passenger ship	Turbo Elec.	3440	3000	100	A C	1150	1	1	Synchronous motor
Building	4 ships	Coast guard cutters	Turbo electric	1600	2600	120	A C	1150	1	1	Synchronous motor
Building	United States Shipping Bd. 12 ships	Cargo	Turbo electric	9600	3000	100	A C	2300	1	1	Induction motor
Building	United Fruit Company	Cargo & Pass.	Turbo electric	6550	3000	100	A C	1150	1	1	Synchronous Motor

the four tankers constructed by the Pan American Petroleum Transport Co., the trawler *Mariner* equipped at the New London Ship and Engine Bldg. Co.'s works, the passenger ship *Cuba*, the 12 ships projected by the Shipping Board, and the cargo vessel *Wulsty Castle* constructed in England. This growing use of electric drive within the last two or three years would indicate a still further and more rapid adoption.

While the above gives a preliminary and general outline of the history and development of electricity on shipboard, a more complete and detailed historical review of this subject will be compiled during the coming year for submission to, and publication by, the Institute.

H. L. HIBBARD

ANNUAL REPORT OF THE LIGHTING AND ILLUMINATION COMMITTEE

To the Board of Directors,

I BEG to submit on behalf of the Lighting and Illumination Committee the following report for the year 1919-20.

ACTIVITIES OF THE COMMITTEE

Your Lighting and Illumination Committee, at its meeting on October 10th., 1919, decided to conduct a symposium on "Distribution Systems for Street Lighting" at one of the Institute meetings of the year, and to schedule, if possible, a paper on "An Analysis of Daylight Saving" at the Midwinter Convention. These plans resulted in holding the national meeting in Chicago on January 9th and 10th under the auspices of this Committee, at which three papers were read dealing with street lighting distribution systems. One of these papers on series systems was presented by W. P. Hurley, another, on multiple systems, by Ward Harrison, and a third, on constant potential series systems, by Charles P. Steinmetz.

The Chicago meeting was very successful and the discussions at both sessions were as complete as time permitted. At the afternoon session, prepared discussion was presented by C. H. Shepard of the Lincoln Park Board, Chicago; by N. B. Hinson, of the Southern California Edison Co.; by F. F. Fowle and W. F. Parker. In the evening, following Dr. Steinmetz's paper, illustrated discussion was presented by F. A. Vaughn together with a description of the new group lighting system now used in Chicago for underground distribution for residence street lighting, by Deputy Commissioner Nixon. The newest things brought out in this discussion were the Chicago group lighting system and the conduit return system used in Southern California.

At the Midwinter convention in New York in February 1920, a paper on "Daylight Saving" was presented by Preston S. Millar under the auspices of the Lighting and Illumination Committee. This paper contained a comprehensive analysis of the subject and the general purpose kept in view by the author in the preparation of the paper is indicated by the following quotation from the opening paragraph of his remarks:

> In accepting the invitation of the Lighting and Illumination Committee to present a paper on "Daylight Saving," the author stipulated that in order to consider the subject comprehensively, a considerable part of the paper would have to be devoted to matters remote from electrical engineering. The economic and sociological aspects of daylight saving surpass in importance the effect upon use of artificial light. Any treatment which should ignore these important features would lack perspective and would be likely still further to increase confusion on a subject which is greatly in need of clarification. Accordingly this paper includes a brief survey of daylight saving in its several aspects.

Your Lighting and Illumination Committee decided this year, as last year, to send a circular letter to the Chairmen of all of the Institute Sections, suggesting that one meeting of the year in each Section might profitably be devoted to an illumination topic. The following quotations from this year's letter will indicate the point of view taken by the Committee:

> At the last meeting of the Lighting and Illumination Committee of the American Institute of Electrical Engineers held on October 10, 1919, in Philadelphia, it was decided to suggest to the Chairmen of the various Institute Sections that it might be desirable to include on the program for the current year one meeting devoted to some illumination topic.
>
> Where the Local Section of the Institute is in a territory having a Local Section of the Illuminating Engineering Society it would be desirable in connection with the meeting devoted to illumination to have said meeting as a joint session between the two organizations.

The responses to this letter were widespread and a number of excellent papers dealing with various aspects of lighting and illumination were presented before some of the Local Sections during the course of the year.

DEVELOPMENTS IN THE LIGHTING FIELD

Under date of January 16, 1920, the Chairman of the Board's Committee on Technical Activities sent a letter to the Chairmen of the Technical Committees, which contained, among other items, the following:

> It was brought out very strongly at the last annual convention in the discussion of the Development Committee's report that the reports of Technical Committees should cover the activity in the particular line covered, so that anyone reading these reports from year to year would obtain a review of the progress of the art.
>
> These reports should cover notable installations or developments during the year and it is suggested in view of the fact that in the past little has been done in this respect, that the reports might be a review of the state of the art up to date. One plan that was suggested which seemed to have considerable merit, was that the Chairman of the Committee might apportion to different members of the Committee certain parts of the field covered by the Committee's activities, and these should be edited and combined to be the report of the Committee.

Acting on this suggestion, the Chairman of your Lighting and Illumination Committee has addressed the members of the Committee in an effort to gather the important developments in the lighting field. These have been edited and combined so as to be the main portion of this report. It should be stated in advance, however, that the developments in the field of street lighting distribution, which were covered in the papers read before the Chicago meeting in January, will not be discussed in this report, since they will be found completely treated in the TRANSACTIONS of the Institute. The same statement applies to the important question of "Daylight Saving" which was covered in the paper before the Midwinter convention, and which will be found in the Institute JOURNAL for February, 1920.

GENERAL NOTES

As one of the results of the war, in its effects on the lighting field, a number of interesting papers have been presented during the past year. The titles of several of these papers will indicate the rather unusual character of the material which has been gathered under war conditions. Among these may be mentioned the following: "The Science of Marine Camouflage Design," "Painting Battleships for Low Visibility," "Camouflage," "The Principles of Camouflage" and "Industrial Lighting and its Relation to the War."

Following the war, the past year has witnessed an unusual increase in the interest taken in lighting in general. It has been felt that the past twelve months have, in fact, marked an awakening among the users to the possibilities of artificial light. This has been evidenced by the efforts made by various societies, organizations and associations, to have placed before them the facts as to what light can do in the way of increasing production, decreasing accidents and of improving the morale of employees in industrial plants. It would appear that a transition period is in progress in the lighting field, if one can judge by the trend of the developments during the year. Among these developments the following may be mentioned;

1. A widespread recognition of good lighting as an important aid to manufacturing.

2. The increasing use of the foot-candle meter as a means of checking illumination intensity in various parts of a lighted room.

3. Progress of industrial lighting codes.

4. Progress in automobile lighting regulations in several states.

5. Development of the bowl enameled Mazda C lamp to give better diffusion. The latest lamps of this type have a feathered edge of the enameling to prevent a sharp "cut off" line.

6. The proposal of a plug outlet for permitting the convenient change of ceiling and wall fixtures.

7. The increasing tendency on the part of fixture manufacturers to make and advertise ready-to-hang types of fixtures. These fixtures are principally for commercial and industrial lighting but there is also some tendency to invade the residence lighting field, which seems to be an improvement both from the scientific and the commercial standpoints.

8. The formation of the fixture manufacturers association, with an attempt (as mentioned in item 6 above) to perfect a method of fixture hanging which will permit electric light fixtures to be hung as easily as a picture and moved, by the renter, as easily as a portable lamp can be moved, thus encouraging the renter to install improved fixtures as a substitute

for bad designs often found to be in use when a house or an apartment is rented.

9. Progress in the enforcement of the industrial and automobile headlight codes. New York, Connecticut, California, Wisconsin and Pennsylvania have adopted the I. E. S. S. A. E. automobile headlight specifications, and it is reported that in all of these states, except New York, there are practical attempts at enforcement, which are gradually producing results that are favorable.

10. An important development in street lighting units is the "Duoflux" standard. This unit contains two lamps (in one type, one 1000 candle-power and one 250 candle-power) and by means of a relay located in the casing of the fixture, the 1000 candle-power lamp will be extinguished at midnight, and the 250 candle-power lamp lighted.

DEVELOPMENTS IN LIGHTING UNITS

The RLM standard dome reflectors have during the year become more widely adopted and nearly all the leading steel reflector manufacturers recommend these standards for industrial lighting.

Among the new fixtures and auxiliaries developed during the year, there may be mentioned the enclosing type of interior units which consist of a combination reflector, a diffuser and an enclosing globe in one piece. Another interesting development has been that of simplified units of the semi-indirect type, and the increased application of the incandescent lamp for moving picture projection. In street lighting, the single unit of high candle power has quite largely replaced the cluster form of post, the former being considered superior both from the standpoint of efficiency as well as of appearance.

Luminous arc lamp efficiencies have been increased by the compounding under high pressure of the ingredients of the electrodes. It is reported that by this means, 30 to 40 per cent more light is produced than with the previous standard electrodes, with at least equal life. For the same light as formerly, increased life of the electrodes is expected.

IMPORTANT LIGHTING INSTALLATIONS

The unusual lighting effects secured at the Chicago and the Buffalo electrical shows in 1919 may be mentioned. At Chicago, the so called "Palace of Aladdin" was housed in a structure 50 feet high, and interesting lighting effects secured with glass jewels, painted mirrored glass and flood lights. At Buffalo, 5000 *white* Mazda lamps were employed along with nearly 5000 illuminated disks distributed among the roof girders. The application of spectacular lighting of this general nature, was seen at its best, perhaps, in the "Jewel Portal for the Victorious Army" in New York and the "Altar of Victory" in Chicago. The use of 30,000 so called jewels in each of these remarkable lighting displays, contributed largely to the spectacular effects secured.

The effect of the war on industrial lighting has been to stimulate the interest taken in better factory lighting the past year. The tendency to employ illuminating engineers for the handling of industrial lighting schemes, rather than to entrust this work to the electrical department as heretofore, has been noticed. In one large industry a research is being conducted to ascertain the effects of lighting on production. The large increase in wages in the industries, with very little increase in the cost of factory lighting, has reduced the cost of good lighting in terms of wage equivalents. This same fact has also made lighting the more important in terms of its effects on increased production and reduced spoilage.

A notable example of modern factory lighting installations is contained in a booklet recently issued by one of the lamp manufacturers. It would have been very difficult a short time ago to have published a book of this kind without relying upon the artist to touch out undesirable features and to paint in desirable ones. This entire booklet contains unretouched photographs of industrial lighting installations and is a commentary upon the improvements which have been made in this part of the illumination field.

In street lighting a number of important installations have been made. Among these may be mentioned the main street lighting in Salt Lake City consisting of 70 standards, each equipped with three 6.6-ampere ornamental luminous arc lamps. These units are spaced about 100 feet and the overall height is 29 feet. Although this particular system was put into operation prior to the current year, it represents an example of intensive street lighting which is coming more and more into prominence. More recent systems of intensive street lighting are the Triangle Lighting in San Francisco, first put into operation in January 1919, and the Broadway system in Los Angeles, put into operation in January 1920. The latter installation consists of 134 two-light ornamental luminous arc standards 106 feet apart and 27 feet high. The installation cost was about $6.50 per front foot of property, and the annual operating cost is about $1.00 per front foot.

The Broadway system at Saratoga Springs is of interest in that about a mile of street is to be lighted by 69 of the new "Duoflux" units mentioned above. Each of these units contains two lamps, one of 1000 candle power and one of 250 candle power. This installation, which is to be put into operation about June 1920, will cost about $32,000 for its installation and for operation, about $10,350 per annum.

CONTRIBUTIONS TO THE ART

Important papers contributed during the year include a discussion entitled "Coefficients of Utilization" by Ward Harrison and Earl A. Anderson, published in the March 20, 1920 issue of the *Transactions* of the Illuminating Engineering Society. This paper presents a method for the direct determination of coefficients of utilization applying to installations of all ordinary types of lighting units in rooms of varied proportions and different ceiling and wall colors. Actual colors for ceiling and wall surfaces covering 32 different shades with the corresponding reflection factors are included in the printed paper.

A paper on "Opportunities of Extending Lighting through New Applications" was read by R. M. Searle before the annual convention of the Illuminating Engineering Society in October 1919, which contains references to the following divisions of the lighting field: Street Lighting, Display Lighting, Flood Lighting, Lighting of Freight Terminals, Industrial Lighting, Stairway Lighting and Lighting for Community Affairs, this list giving an idea of some of the fields in which developments may be expected.

The following quotation from an address of President S. E. Doane before the Thirteenth annual convention of the Illuminating Engineering Society is of interest:

> The war has taught us that we have been regarding electric lighting, to use a phrase that one of my associates has given me, as a janitor service, whereas we should have looked at lighting as an item in the cost of production or in the cost of sales, and regarded it as any other factor in the cost. Then we would have examined, as have men within the last year or two, under what light we could get the maximum of visual acuity or speed of visual reaction. During the war time we have had some demonstrations. Mr. Durgin has given the most and the best of the practical demonstrations of how to apply this knowledge that visual acuity increases with the intensity of light and the speed of reaction increases production.
>
> Its application to production has been spectacular, and the results have been remarkable. As a matter of interest and as a measure of our opportunities I would like to use some figures from the lamp industry. Sixty-five per cent of the output of the lamp manufacturers, according to our estimates, is used in that portion of our business which would be affected by this knowledge. In other words, more than half of the electric lighting of this country can be affected and will be affected by the better knowledge we have of the production or increase in visual acuity and speed of visual reaction under intensive lighting.

For those who may be especially interested in the progress of the lighting and illumination art, attention is directed to the annual report of the Committee on Progress published each year by the Illuminating Engineering Society. The last report of this kind, covering 80 pages, will be found in the *Transactions* of the Illuminating Engineering Society for November 20, 1919.

C. E. Clewell, *Chairman.*

ANNUAL REPORT OF THE POWER STATION COMMITTEE

To the Board of Directors,

DURING the year 1919-1920 the Committee held two meetings, mainly devoted to discussion of papers presented, in person by the writers for comments and suggestions. The contributions to the activities of the Institute under the auspices of this Committee were as follows:

A symposium on steam turbine design, with the following papers:

Present Limits of Speed and Power of Single-Shaft Curtis Steam Turbines, by Eskil Berg.

Present Limits of Speed and Power of Single-Shaft Steam Turbines, by J. F. Johnson.

Present Limits of Speed and Output of Single-Shaft Turbo Generators, by F. D. Newbury.

A plea for standardization in statistical records of state and governmental bodies reporting efficiencies of power plants, with the presentation of the following paper:

Essential Statistics for General Comparison of Steam Power Plant Performance, by W. S. Gorsuch.

A symposium on excitation, with the papers to be presented at the Annual Convention as follows:

Considerations which Determine the Selection and General Design of an Exciter System, by J. T. Barron and A. E. Bauhan.

Factors in Excitation Systems of Large Central Station Steam Plants, by A. A. Meyer and J. W. Parker.

Exciters and Systems of Excitation, by H. R. Summerhayes.

Application of D-C. Generators to Exciter Service, by C. A. Boddie and F. L. Moon.

Exciter Practise in the Northwest, by J. D. Ross.
Generator Excitation Practise in the Hydroelectric Plants of the Southern California Edison Company, by H. H. Cox and H. Michener.

The committee also considered the question of making suggestions for the activities of the incoming committee for the ensuing year, and recommended for consideration the following:

Safe Maximum Limit of Operating Capacity for Each Section of Bus.
Reactive Component Dispatching.
Auxiliaries in Steam and Hydroelectric Plants.
Fighting Generator Fires.
Trend of Modern Power Station Design.

PHILIP TORCHIO, *Chairman.*

ANNUAL REPORT OF THE ELECTROPHYSICS COMMITTEE

To the Board of Directors,

THE Electrophysics Committee desires to receive suggestions for future work. It seems well, therefore, for this purpose to re-state from the report of last year the policy that this Committee has endeavored to carry out. It is as follows:

1. To encourage original papers of high technical standard, marking advances in electrophysics.

2. To have each year a broad, interesting, general lecture, free from mathematics, dealing with modern physics.

3. To promote a more complete cooperation and mutual understanding between the engineer and the physicist.

It is our object to keep open the "line of communication" between the pure physicist and the strictly applied physicist or engineer.

A joint meeting was held with the American Physical Society in Philadelphia, October 10-11th. The Physical Society session gave a notable discussion on the present status of theories of atomic structure. A review of this subject designed for engineers has been printed in the JOURNAL. The A. I. E. E. papers showed how these theories had been practically applied in the study of crystal structure and in the solution of vacuum tube problems. Other papers covered a variety of subjects marking advances in electrophysics.

Talks were given in an evening session by past presidents of the two Societies. It was the object of these talks to promote cooperation. The titles are significant—"The Indispensability to Each Other of Pure and Applied Science" by H. A. Bumstead; "Pure Science and Industrial Research" by J. J. Carty.

Additional technical papers have been given at other sessions.

F. W. PEEK, JR., *Chairman.*

ANNUAL REPORT OF THE TRACTION AND TRANSPORTATION COMMITTEE

To the Board of Directors,

TWO meetings were held under the auspices of the Traction and Transportation Committee; one at the mid-winter convention in New York in February, and the other at a regular meeting of the Institute in Pittsburg, March, 1920.

At the New York meeting there was presented and discussed a symposium paper entitled: "The Economic Supply of Power for the North East Atlantic Seaboard." This paper made reference to an item before Congress calling for a survey of the situation between Boston and Washington looking toward the establishment of a national policy in the matter of the generation and distribution of power through the means of which large economies could be effected as applying to labor, fuel and other materials.

The meeting passed a unanimous vote recommending that the Board of Directors of the Institute appoint a special committee to be known as the "Committee on Super Power System." This committee has since been appointed.

Since the above meeting, Congress has approved the proposed survey and appropriated $125,000 for the purpose.

The Pittsburg meeting included the presentation and discussion of papers relating to the design of the Minneapolis and St. Paul Electric locomotives by the General Electric Co. and by the Westinghouse Electric and Mfg. Co.; also papers concerning automatic substations and protection to substations.

Both of these meetings were largely attended and much valuable information and data were presented, all of which has appeared or will appear in the TRANSACTIONS.

There seems to be no disagreement on the part of any members of the committee in the conclusion (1) that super power systems stand as future guarantors for economical generation and distribution of power and (2) that the electric locomotive has demonstrated its ability as applying to passenger, freight or switching movement, to outclass the steam locomotive in capacity and control.

WM. S. MURRAY, *Chairman.*

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29–July 2, 1920.

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CLASSIFICATION OF LARGE TURBO GENERATOR FAILURES

BY PHILIP TORCHIO

Chief Electrical Engineer, New York Edison Co.

THE experience with this class of electrical apparatus is in a measure less satisfactory than could be desired. The design difficulties are undoubtedly greater than encountered in the design of low-speed generators as used in hydroelectric plants, but the exacting service requirements of large steam power undertakings demand the maximum of reliability from its generators. To meet these requirements, designing engineers must ultimately produce apparatus more dependable. At the same time users must install and operate their generators under conditions that will be most favorable to their maintenance.

For the only object of reviewing past failures to aid us to avoid repetitions in the future, I have accepted the invitation of the Machinery Committee to give a summary of typical generator failures of which I have a record or report from different installations in this country.

The size of generators investigated ranged from 5000 kw. to 30,000 kw., the larger units being of more recent manufacture, while the smaller date back from twelve to sixteen years.

The total failures, several occurring on the same unit, amounted to fifty-five, of which thirty-three occurred in armatures, sixteen in fields, four in armatures and fields, and two in terminals.

The classification of all these generator troubles is as follows:

Armature failures due to:		
Mechanical damages	3	
Heating of windings	17	
Heating of iron	2	
Loose laminations	1	
Moisture in cooling air	3	
Corona shield breaking	2	
Heating at end turn clamping	1	
Overload damaging end turns	2	
Causes undiscovered	2	33
Field failures due to:		
Open-circuited connections	3	
Grounding	7	
Grounding caused by bus short-circuit	4	
Loosening of damper windings	2	16
Armature and field failures due to:		
Moisture	1	
Undiscovered	3	4
Terminal failures due to:		
Moisture	2	2

From the above analysis, it appears that at least 50 per cent of armature failures are due to heating, damaging the insulation. To avoid similar troubles, designing engineers should use insulating materials of proved dependability, maintain low copper temperatures by proper subdivision of copper and transposition of strands in windings and provide liberal ventilation throughout the machine. The operating copper temperatures should be limited at or closely to 100 deg. cent. Too great range of operating temperatures is bound to cause generator failures.

It may be appropriate here to emphasize a broad general principle to which perhaps we have not given full consideration in the past, and that is that insulated electrical apparatus subjected to too wide operating temperature ranges may be seriously impaired by mechanical bulging and damages to the insulation and the incasing structure, without inherent deterioration of the insulation itself. We found that the insulation, while excellent to withstand the maximum temperatures, was mechanically damaged by the expansion and contraction of the copper which it surrounded. For instance, mica insulation installed in hydroelectric generator armatures, which have only a small fraction of length of a turbo generator armature and are operated at a practically steady continuous load, last indefinitely at high temperatures, while mica insulation, under same or lower temperature conditions, in steam turbo generators operated at partial loads for several hours, then loaded to a maximum and later shut down every twenty-four hours, will fail.

To weed out cracks or latent defects due to defective workmanship at factory or in installation, the test voltage for windings should be raised to three times full voltage plus 1000.

The large number of field failures on generators of older design using fibrous insulation were practically eliminated in more recent designs by the use of mica and asbestos tape insulation.

Experience seems to indicate that a solid forged field has greater resistance to field stresses under bus short circuits than laminated field structures or even structures made up of end plates and several central plates a few inches in thickness.

The loosening of damper windings seems to have been satisfactorily overcome, from the experience of the last few years.

From the standpoint of the users, it appears important that great care be exercised in the supply of cooling air to prevent moisture or condensation depositing on the windings.

DISCUSSION ON "CLASSIFICATION OF LARGE STEAM TURBO GENERATOR FAILURES" (TORCHIO), WHITE SULPHUR, W. VA., JUNE 30, 1920.

W. J. Foster: Mr. Torchio speaks of limiting the temperature in the copper to 100 deg. cent. That is an extremely difficult thing to obtain. At the present time it is regarded by most designers as impossible, with the specifications as made out, calling for certain outputs at certain power factors.

I simply wish to point out that there is hardly a user of turbo generators who cannot radically improve his temperatures by operating his machines at better power factor. I am not going to argue as to whether that is practicable or not, but I wish to point out the desirability of it.

Most 60-cycle, large turbo generators are 1800 revolutions and 80 power factor and will operate at temperature rises of only about one-half, somewhere between 50 and 60 per cent of the present temperature rises in the rotors, if operated at unity power factor with the same energy output. That is extremely desirable when we consider the length of life. I think that statement is a safe one to make—regardless of construction—even with the very best possible construction—longer life may be expected at such low temperatures as Mr. Torchio recommends.

There is one point in the paper in which I would like to take issue with Mr. Torchio—or rather I would insist upon a limitation—and that is, where he asks for an insulation test of three times normal plus 1000 volts. That is very well and practicable for generators of 6600 volts and thereabouts, since it is evident that all you need to do is to take your 11,000 or 13,200-volt design and wind it for 6600 volts. That is entirely practicable in those cases, but when it comes to the higher voltage machines, it is a very serious matter, and I regard it as practically impossible at the present time in the larger turbo generators.

In that connection, I will take advantage of the situation here and refer Mr. Torchio and other interested members to the little table on page 961 of my paper for this session, and ask them to compare columns 2 and 4, in which I regard the design represented in column 2 as having about as large a slot as should be incorporated in a machine, due to magnetic limitations and eddy current limitations. If you use that slot, and put in a coil with an insulation for three times normal, plus 1000 volts, you get the temperature rise shown in column 4 as compared with column 2.

F. D. Newbury: Mr. Torchio presents a list of turbo-generator failures from which it would appear that armature winding heating is the major cause of failure.

My data and experience do not show that armature heating is the chief cause of failure, although it should not, by any means, be overlooked. This history of Westinghouse turbo generators impresses one with the importance of operating hazards such as fires of external origin, abnormal voltage surges, and water carried into the generator.

The investigation I have made covers every Westinghouse turbo generator from 15,000 kv-a. rating upwards, that has been placed in operation up to January, 1920. It happens that these generators, without exception are mica insulated, with mica insulation between strands and between conductors as well as between coil and core. This should be remembered in considering the record of these generators in regard to failures caused by armature heating.

The first Westinghouse generators larger than 10,000 kv-a. were placed in operation in 1913. These were generators roughly 20,000 kv-a. in size, two pole in the case of 25 cycles and four pole in the case of 60 cycles. The design of such large two and four pole units in 1911 and 1912 was highly experimental and while these units met the standard temperature rise of 50 deg. by thermometer, we have since found that the true copper temperature of the strands nearest the air gap gave at least 200 deg. rise in some cases. Experience has shown how to avoid these high local temperatures mainly by reduction in eddy current losses.

I also wish to draw your attention to the inclusive and representative nature of the record in connection with heating troubles. It covers all generators of large size of a given type and it is safe to draw general conclusions, bearing in mind the type and detailed design of the armature insulation involved.

During these seven years' experience there have been 22 cases of major armature trouble, involving 12 different installations or designs and 16 individual units. By major armature trouble is meant trouble serious enough to result in the failure of one or more armature coils.

The classification of armature failures as to cause is in some cases a difficult matter. The events leading up to the trouble may be unknown and the evidence is very often burnt up. It has been thought best, therefore, to separate these cases into three classes as follows:

1. Where the manufacturer has accepted responsibility.
2. Where the operating company has accepted responsibility.
3. Where the cause is unknown or in dispute.

1. Eight cases were chargeable to design, workmanship or other cause for which the manufacturer accepted responsibility.

(a) Five cases (of these eight) were caused by defects in manufacture that developed very shortly after installation.

(b) Three cases (of these eight) concerned details of design that led to failure after four or five years operation. Two of these cases (in one design) involved high local temperatures caused by eddy currents in the top strands of the coil.

2. Eleven cases were chargeable to operating hazards for which the operating company accepted the responsibility.

(a) Four cases (of these 11) were caused by fires originating outside of the armature winding. In three of these cases the fire started in the cables just outside of the generator; in the fourth case the fire started in a series transformer accidentally open-circuited.

(b) Four cases (of these 11) occurred in one installation and were caused by abnormal voltage surges that caused the outside surfaces of the insulation to catch fire. In addition to these four cases that resulted in coil failures as many more fires started that were put out before such damage was done. This trouble disappeared after the generator neutral was grounded.

(c) One case (of these 11) was caused by ice or water (it occurred on Feb. 28) carried into the generator from the air washer.

(d) Two cases (of these 11) were caused by enforced operation under conditions that were known to be unsafe. One case involved unsafe overloads and the other case involved continued operation after it was known the armature needed minor repairs.

3. Three cases involved unknown causes or the cause was in dispute.

In two of these cases the operating company believed armature coil heating to be responsible.

This classification of armature breakdowns shows that out of 19 cases, where causes were agreed to, 11 cases were caused by operating hazards for which the generator can in no way be held accountable. The majority of these cases were caused by fires of external origin.

If all the cases in which armature heating was involved (even by suspicion) are grouped, there are only five cases out of 22 breakdowns. Two cases (involving one of the first designs) were caused by design proportions; in two others temperature was not the primary cause of breakdown, but the operating companies believed it to be a contributory cause; and in the fifth case enforced overloading was the primary cause.

R. B. Williamson: I would like to endorse Mr. Foster's remarks about the effect of power factor on turbo generators. The rotor of a turbo generator is relatively small, and the surface available for getting rid of heat is also small. With inductive load, the lagging current that the machine is compelled to furnish calls for increased exciting current in the rotor and very materially increases the heating. Thus the output which can be obtained from the machines at unity power factor, with a given rotor heating, is much greater than it would be, at, say, 80 power factor.

The question of voltage test of three-times normal plus 1000, as Mr. Foster pointed out, would entail no hardship with machines for 6600 volts, or possibly 8000 or 9000 volts, but with generators for 11,000 or 13,200 volts, it is a difficult matter to design machines for such high-voltage test. As Mr. Foster pointed out in his paper, a machine insulated for such test would involve thicker insulation and much higher internal temperatures. The latter might lead to trouble due to creepage of the coils or some effect of that kind, even if the higher temperature were not sufficient to damage the insulation.

W. F. Dawson: I do not want you to get the idea that it is only the mica insulation that will stand the gaff. I have been responsible for something like 2000 turbo alternators, and probably 90 per cent were not insulated with mica in the armature winding, and I can say very truthfully that there have been practically no failures in those generators from any causes of workmanship, material or manufacture. In the beginning we had some trouble due to the fact that the sharp corners of the armature slots bore against the insulation, but with the eased off slots, used by most of the American manufacturers, and the insulation properly applied to the coils, we have had almost unbroken success.

The great difficulty that was noted in Europe some ten or fifteen years ago, and particularly pointed out by Mr. Highfield was due in a large measure, as

Mr. Newbury has just mentioned, to the so-called pulled in windings. I believe even today some manufacturers in Europe adopt that method, and it is very handy from the manufacturing standpoint, but it is just about as futile in respect to keeping up insulation as anything can be. These windings were filled with air voids, and the potential gradient was steepest on the untreated cotton of the individual wires, and failure was bound to result, but with our American practise, we are in a very much better position. Of course, these machines I mentioned, are not high-voltage machines. We have adopted mica insulation wherever the voltage exceeds 4000, but I have put out enormous quantities of machines for industrial work at voltages of 4000 and under, with all treated fabric insulation, and I should say that the success has been, at least, 99 per cent.

Robert Treat: At the bottom of page 904 Mr. Torchio makes this statement:

"For instance, mica insulation installed in hydroelectric generator armatures, which have only a small fraction of length of a turbo-generator armature and are operated at a practically steady continuous load last indefinitely at high temperatures . . . . ."

I would like to inquire whether Mr. Torchio has in mind any definite limiting temperature which in his mind constitutes a "high temperature" as here used.

The writers opinion, based on a careful study of operating experience and based on a detailed knowledge of internal temperatures and coil and insulation design, is that breakdowns caused by armature heating are in reality due to abnormal local temperatures that have values of the order of several hundred degrees; and that temperatures of 100 deg. or 150 deg. ordinarily discussed in connection with guarantees have very little to do with the problem. This statement of course, applies only to windings completely insulated with mica within the slots.

This record of breakdowns brings to the surface another fact that is reassuring to the companies operating high-voltage units. If these generators be classified according to voltage it is found there are twice as many generators wound for 11,000 volts, or higher voltage, as there are generators wound for lower voltages. But these 22 cases of armature breakdown are equally divided between these two voltage classes. This means that the percentage of armature breakdowns in the high-voltage generators under discussion is only one-half that of the lower voltage generators. This record would be very different with treated cloth insulation (in large high-voltage generators) that is subject to cumulative heating on account of increasing dielectric losses with temperature; and with partially closed armature slots, still used by some European designers.

This record does not prove that high-voltage generators are necessarily safe or less subject to breakdown except in so far as breakdowns may be caused by voltage surges that are independent of generator voltage. Obviously a 10,000-volt surge would be dangerous in a 2400-volt generator but would be harmless in a 13,000-volt winding. The record does prove that 11,000 and 13,000-volt windings are just as reliable as low-voltage windings and, further, that breakdowns are generally caused by trouble unrelated to line voltage.

The facts brought out by this record may be summarized as follows:

1. The majority of armature breakdowns is caused by operating hazards originating outside the generator.
2. Armature heating in armatures completely insulated with mica is a minor cause of breakdown.
3. Large high-voltage generators, with mica insulation and open armature slots, are as reliable and probably are more reliable than large low-voltage generators.

James Lyman: We appreciate the almost insurmountable difficulties in obtaining all that we would like in the matter of insulation tests, and in the matter of temperatures, but what were apparently insurmountable difficulties five or ten years ago, the designing engineers of our large manufacturing companies have overcome, and it is to be hoped that they may, in the line of evolution, gradually work toward the limits that Mr. Torchio has given.

Mr. Newbury has stated that one-half as many burnouts occurred in the case of large generators wound for 11,000 volts and above, as have occurred in the same number of similar generators wound for lower voltages. Cannot we, therefore, assume that the insulation was quite equal to the service, but that the weakness in these machines was the high temperatures they attained?

Undoubtedly, mica insulation, when not subjected to any mechanical strain, is quite equal to the extreme temperatures that have been mentioned, but in turbo generators, especially in the very large units, the windings are subject to great mechanical strains, and I suspect that where very high temperatures are allowed, the burn-outs are due more to expansion and contraction, the movement of the mica insulation, which mechanically is not very strong, than to the insulating qualities of the mica.

Philip Torchio: Practically all the limitations which have been pointed out hinge on temperature limits and voltage tests, both of which are closely interrelated. In fact, Mr. Foster has no hesitation in applying the three-times normal voltage test if the machine, instead of being rated 100 per cent, is rated at only 75 per cent, because then he can put on the thicker insulation to stand the higher voltage test and still dissipate the heat and maintain a low temperature.

That prompts me to comment that the users are not requesting the large ratings and the last drop of capacity from the machines—they want safety and continuity of service. I do not propose to stand before you and say I would want to retard progress. We must make progress, and we share with the designers and the manufacturers the responsibilities and the burden of any experiment, but we must emphasize one point; that these large units are of such vital importance that the question of getting out the last 10 per cent or 15 per cent of capacity is of no importance whatsoever to the users in general, and by users in general I do not mean the operating companies alone, but I mean the customers who use the power. They want continuity of service and reliability, and in the end the apparatus with the longest life, the most reliable apparatus, is the apparatus which will give the cheapest service.

Along the same line of reasoning is my reply to Mr. Foster's reference to power factor. The operating companies are making a strenuous effort to improve power factor, and I want to take this opportunity of saying that it is not for sentimental or selfish reasons that we do try to get the power factor improved, but it is really because the low power factor is an obstacle to the progress of power generation, as Mr. Foster has forcibly demonstrated.

In answer to Mr. Treat's question as to what I mean by high temperature, I would state that as far as it refers to pure mica insulation, not mica built up with a binder, I mean temperatures of, say, 185 deg. cent., as prevailing on the 2200-volt generators at Niagara, which have been in satisfactory service for over 20 years. I had in mind such temperature as far as the limitation of pure mica itself to stand the voltage goes, provided it is not disturbed by local stresses, by periodic and frequent expansion and contraction, and by vibration and pounding due to the periodic beating of the strands in the windings of the coils. In the latter case and with built up mica, I would limit the temperature to about 100 deg. cent. to preserve the integrity of the binding material.

I would like to make some qualifying comments on Mr. Newbury's very interesting presentation, as my analysis of troubles also covers many of Mr. Newbury's troubles, but to enter into these details it would take up too much time. The important point is that the art is advanced by the lessons of past errors.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29–July 2, 1920

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VENTILATION AND TEMPERATURE IN LARGE TURBO GENERATORS

BY B. G. LAMME

Chief Engineer, Westinghouse Elec. & Mfg. Co.

IN the turbo alternator, the problems of air circulation, or ventilation, and of insulation have been carried further than in any other type of large apparatus. There are fundamental reasons for this. In the first place it may be said that there are four effective materials concerned in the turbo-generator operation, namely the iron for magnetic purposes, the copper in the windings, the insulation encasing the winding, and the ventilating air. The iron and copper have long been carried up to their practical limits, so that the major part of recent developments has been in the insulation and in the ventilation. In the second place the physical limitations of the materials having been reached, in many ways, it is no longer possible to increase the physical dimensions, as has been the case in lower speed machines.

As present insulations have been known and utilized fairly well up to their limits, for some years, it follows that the more recent growth has been largely along lines of improved methods of heat dissipation, which, in the final consideration, is largely the problem of air circulation.

Therefore, beginning with the problem of ventilation in general, it may be said that, broadly, there is but one method in use today, namely forcing a large quantity of air through the machine, at a high velocity due to unavoidable restriction of the air paths. The air-forcing means in all cases consists of powerful fans or blowers driven either by separate motors, or by the rotor shaft of the turbo alternator itself, There are advantages in favor of either arrangement, but these do not concern the problem broadly. The real problem is to force, through effective cooling channels or paths, sufficient air to take up the generator heat, without unsafe temperature limits being reached. It is the design of these cooling channels or paths that much of the modern turbo-alternator engineering has been expended. In the following treatment of the general problem, these various methods will be considered.

The types of ventilating channels may be classified broadly as *radial* and *axial*. Very few machines use either one exclusively. In axial ventilation the air travels in the general direction of the shaft, whereas in the radial arrangement the movement is in a right angle direction. The air-gap ventilation in the turbo generator is axial in most cases, while the internal ventilation of the rotor body or core is usually the axial and radial combined. In the same way the ventilation of the turbo stator core is usually a combination of the axial and the radial; for, even in the so-called radial types, the cooling air through the gap passes axially over the armature bore, before passing out radially. In the same way, in the so-called axial types the air through the axial core channels passes radially out at the core center.

In addition to the purely axial and radial arrangements, may be mentioned a third one, namely, circumferential ventilation, or air circulation around the machine circumferentially. This has been used to a certain extent in certain European types of machines, and in combination with axial and radial arrangements, it appears in certain American types. In fact, in the axial air gap ventilation, the circumferential action also occurs naturally, due to the speed of the rotor, the air traveling in a spiral path from the ends toward the center of the machine.

RADIAL VENTILATION

In the usual radial ventilation in the modern high-speed turbo alternator, the air enters the air-gap at each side, or end, of the core, and as it travels toward the center part, it is deflected outward at intervals, through the numerous radial ducts. In most cases the entire cooling air enters the gap at the two ends, and, therefore, its highest velocity is at the entrance.

As part of the air breaks away to pass out the first of the radial ducts, the volume of air in the gap decreases, thus successively lessening the pressure required to overcome the gap resistance. There is thus fairly high pressure at *a* in Fig. 1, due to the great restriction of the air path section at this point, and this tends to shunt part of the air out through the radial ducts over *a*.

At the center of the machine, due to the velocity of the ingoing air, there is "banking up" of the air, with corresponding pressure tending to drive it out radially. Here the velocity is thus converted into static pressure. Obviously, therefore, the air flow in the different ducts along the core, being due to different conditions of pressure, may be quite unequal. The real problem, therefore, is to obtain equal, or proper, air flow through the various ducts.

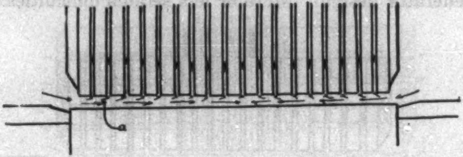

FIG. 1

An extension of this problem is found where the width of the core is greater than the permissible air flow can cool. Assume, for instance, with the largest permissible gap the design will allow, and the largest allowable diameter of rotor, that a core width of 80 in. can be ventilated by air from the two ends. Now what can be done with a core width of 100 in. or 120 in., which would be required for a 25 or 50 per cent larger output? One answer would be that it is impossible to use cores of such width. But that would put a stop to a certain trend of development and, therefore, some solution involving an important modification must be adopted. One of these is to put air in, radially, at the middle part of the stator core, and to allow it to flow axially *toward the ends,* in the air gap, and then out radially, as shown in Fig. 2. Here is a new problem, in that the entering air must be forced in against the rotation of the rotor core, and possibly against the velocity effects of the air from the end gaps. In consequence the driving pressure may have to be considerably different from that for forcing the air in at the end gaps. This thus further complicates the problem of the proper distribution of the air in the various ducts. Nevertheless this general arrangement has been used with considerable success.

The above description of the difficulties of the problem has been given, not as a criticism of this particular arrangement or method, but simply to illustrate some of the very real difficulties encountered in large turbogenerator design. More or less serious difficulties are found with *all* known methods, judging from the variable results obtained in different designs, even where the method itself is unchanged. The exact calculation of air pressures, and air flow in complex channels or passages, especially where a high-speed rotating element is present, is beyond the capabilities of the most experienced calculator, and, therefore, the experimental factor enters to a certain extent in all new designs. Good approximations can be made, in many cases, but no good designer will "fool himself" into thinking he can calculate the ventilation on a new construction with any great exactness. Even if he gets the right quantity of air through his machine, he cannot be sure it will distribute properly through the many parallel air paths, such as represented by the radial ducts, in

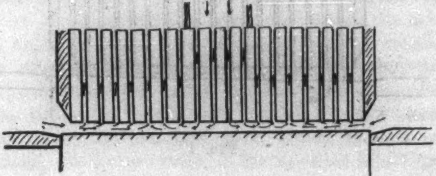

FIG. 2

the radial scheme, or the many axial ducts in the axial method.

Axial Ventilation

As stated before, axial ventilation is usually partly radial. In this method, as usually applied, air not only passes axially along the air gap and out at radial ducts at the center of the core, but axial ducts, or channels, parallel to the shaft or air gap, are in the stator core, back of the armature slots, for allowing air-flow in parallel with the gap. The intention is to force sufficient air through the gap to cool the rotor, and the stator tooth tips, while air through the stator ducts will further cool the stator teeth and armature core. In the early designs one large central duct was used, this being changed later to two or three narrow ones, while in the latest types, there may be a dozen or more radial ducts, taking up a considerable portion of the central zone of the stator. With this latter arrangement, therefore, the central part might be considered as radially ventilated, while the end parts are axially ventilated. The real merits and demerits of this later construction, compared with the so-called radial type can only be judged properly when the quantitative conditions of the problem are taken into account. This will be considered, later in the paper, under the treatment of losses, heat flow, radiating surfaces, etc. Without knowing these quantitative conditions it is impossible for any one to say, in any particular case, whether any one method or arrangement is materially better than another.

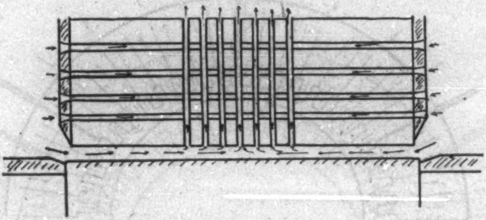

Fig. 3

A variation of the above axial scheme consists in supplying air, under pressure, to the central radiating ducts which then passes through the stator channels and the air gap, to the two ends. Or, again, the ventilating fan may operate reversed so that it *pulls* the air through from the central ducts. One advantage of the latter arrangement is that the friction loss due to the fan heats the outgoing instead of the ingoing air. This method will be referred to again under heating conditions.

It will be seen that with the axial arrangement, like the radial, the problem is to obtain the best air distribution with a large number of parallel paths and with different resistances of the various paths. One advantage of the axial arrangement is that part of the parallel paths,—those in the stator core,—are under possible control at the inlet.

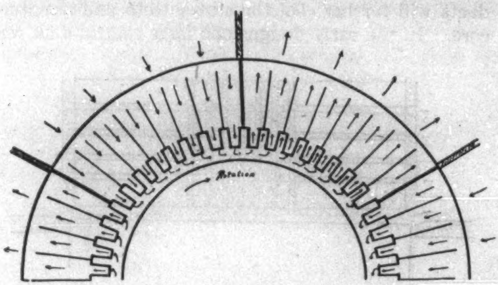

FIG. 4

CIRCUMFERENTIAL VENTILATION

This method should be considered, as it has been used commercially, to a considerable extent. In practise it is in reality a combination of radial and circumferential methods. As usually applied, there are radial passages through the stator core, but divided, circumferentially, into groups, so that the air passes inward to the air gap, in certain axial zones, and then circumferentially around the gap to the next zone where it passes out radially, as shown in Fig. 4.

At first glance this looks like an ideal arrangement, but a little study will show that it also has its difficulties, just like the other methods, for there is no way to insure proper division of the air among all the parallel paths. For instance, if one zone covers eight teeth, then due to crowding of the air in the gap, due to the rotor velocity, the pressures might be balanced in some of the tooth passages so that there is practically no air flow in those particular elements. This would mean very unequal temperatures in the tooth belt as a whole, and so the problem thus narrows down to one of proper air distribution, just as in all other arrangements.

A modification of the above, which has been used, combines radial and circumferential methods, as shown

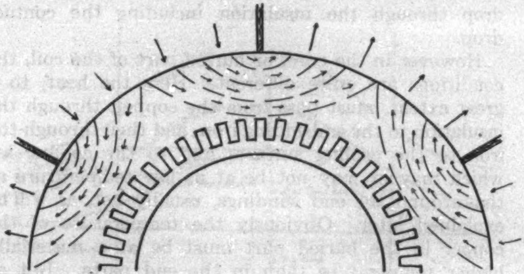

FIG. 5

in Fig. 5. In this the air from the outer core surface is simply deflected through the stator core, and cools the tooth belt mainly by conduction through a relatively long path. The stator gap surface and rotor must be cooled by the usual methods.

In addition to the above there are various other combinations of radial, axial and circumferential methods, each of which possess certain merits, but a description of which is unnecessary here.

HEAT FLOW FROM THE ARMATURE COPPER

There are three principal sources of heat in such apparatus, namely, the losses in the copper windings, the iron losses, and the friction of the air at the high velocities used. Considering first the armature windings, all the heat generated must flow through the insulation, the relative rate of flow at any part being dependent principally upon the temperature differences. In addition to the transverse heat flow, through the insulation, there will be flow along the copper itself from hotter parts toward cooler places, if such exist, which is almost always the case. In the end windings, for instance, the heat can flow directly from the copper, through the insulation, to the cooling air. Moreover, local or eddy currents in the copper are liable to be least in this part, due to the absence of the iron. Consequently, in the end windings the actual copper temperatures are higher than the surrounding air temperatures only by the amount of temperature drop through the insulation including the contact drop.

However in the core, or buried part of the coil, the conditions are quite different. Here the heat, to a great extent, must pass from the copper, through the insulation to the embedding iron, and then through the iron to the cooling surfaces and to the cooling air which may or may not be at as low temperature as that around the end windings, usually not, as will be explained later. Obviously the temperature of the copper in the buried part must be at a materially higher temperature than in the end parts. But as copper is a pretty good heat conductor, this difference in the copper temperature tends to equalize itself by heat flow along the coil itself, thus tending to lower the higher temperature and increase the lower. This action can be quite effective in equalizing the temperatures in the various parts in those machines where the heat paths are not too long compared with the section of the conducting path and the amount of heat to be conducted. In narrow core machines, with short end windings, the equalization may be within a few degrees, but in the usual wide turbo-generator cores, with the very long span end windings, this equalization only shows prominently in the dividing zone between ends and core. In other words there is a very considerable longitudinal heat flow at this part, but it is relatively small far in the core or far out on the end windings. This may be illustrated diagrammatically by Fig. 6. Here the temperature of the core with respect to the copper is dependent upon the load conditions. With heavy load this flow may be from the buried copper to the iron at all parts of the core width, while at light load the flow may be from the iron to the copper over the whole core width. At intermediate loads there may be heat flow from the iron to the copper near the ends of the core, and from copper to iron in the midportions.

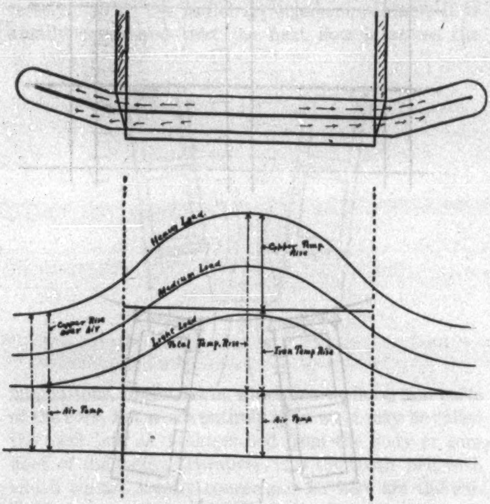
Heavy Load
Medium Load
Light Load
Copper Temp. Rise
Copper Rise over Air
Total Temp. Rise
Iron Temp Rise
Air Temp
Air Temp

Fig. 6

Considering the heavy load conditions it is obvious, in a wide core machine, that the center parts should show very materially higher temperatures than the end copper, such higher temperatures being due to two or more drops, (1) through the insulation, (2) through the iron and the surface contact, and (3) to the difference between the cooling air in the core and that around the end windings. This latter effect may be very considerable, as will be brought out later. It should be evident that if the temperature rise or insulation drop, as determined at the end windings, should be 40 deg. cent. for instance, then the temperature rise in certain of the embedded parts should be higher by the amount of the adjacent iron temperature, plus the rise in the cooling air at those parts. If the iron temperature should be 40 deg. cent. above the adjacent cooling air, and the latter should be only 10 deg. cent. above the outer or end air, then this represents 50 deg. cent. additional temperature to be added to the insulation drop from the copper to the iron, or a total of 40 deg. + 50 deg. = 90 deg. These should not be taken as representative figures, but are simply assumed, to bring out the general fact. The

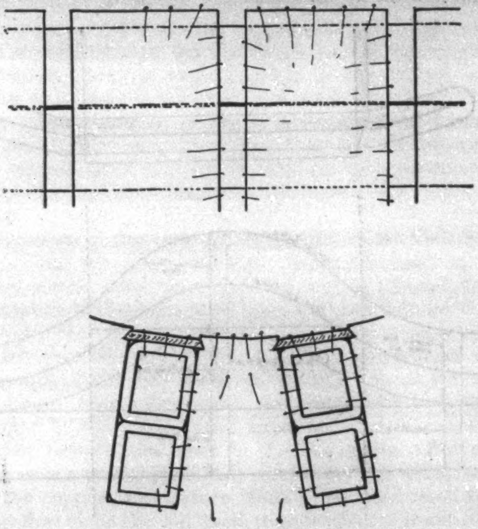

FIG. 7

object is to show that the central portion of the core will necessarily be considerably hotter than the ends, and that an end winding temperature measurement alone is of no value in such apparatus.

HEAT FLOW IN THE ARMATURE IRON

In practically all methods of turbo-generator ventilation there are two general paths of heat flow in the armature iron, namely, along the laminae, and transversely. With the radial arrangement of ducts it is usually considered that the heat flow is across the laminations, to the ducts, which is true in certain parts of the core, but is not entirely so in what may be called the *tooth belt*, as distinguished from the body or core back of the teeth. However, it is the tooth belt with which we are mostly concerned, for here are the embedded armature coils. Here the heat flows partly *across* the laminae to the radial duct surfaces, but in many cases quite a large part flows *along* the laminae to some cooler parts, such as the tooth tips, or the core behind the teeth. One reason for the longitudinal flow lies in the much better heat conducting path along the iron, compared with that across the thin laminae with their good coating of insulating (and, therefore, poor heat conducting) varnish.

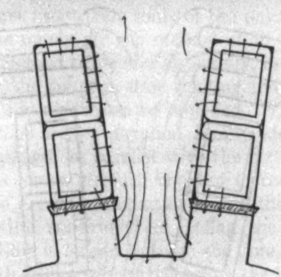
FIG. 8

The flow to the tooth tips is especially large in those cases where the armature coils are sunk some considerable distance below the gap surface, as shown in Fig. 8.

Here the tooth tip surface is very materially increased, and the heat dissipation is largely at the edges of the laminations. Experience has shown very considerable improvement in iron temperatures with this sunken coil arrangement.

In the body of the core, back of the slots, the heat flow, in the radial slot arrangement, will be mostly across the laminae. Due to this the ducts should be spaced at quite frequent intervals, there being, usually, one duct for at least each 2 in. of core width. Even with a 2-in. spacing, temperature differences of as much as 10 deg. cent. may be noted in cases, between the center of the pack and the end laminae.

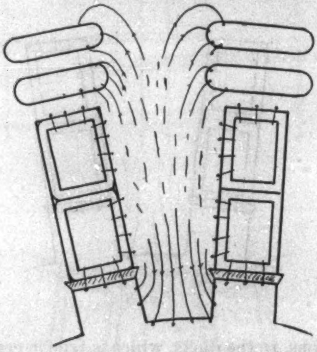
FIG. 9

In the axial arrangement of ducts, the heat flow is largely along the laminations, except at the central part where radial ducts are used.

At the tooth tips and air gap, obviously the heat dissipating conditions are very much as in the radial duct arrangement, especially where the coils are well below the surface. There is also a strong flow outward to the core and the axial ducts back of the coil slots, as indicated in Fig. 9.

From off-hand inspection, one would say that this method of heat dissipation is less effective than that with radial ducts, as the paths of heat flow are longer. But it is purely a quantitative problem, as the heat flow *across* the laminae is not nearly as effective as along them, possibly in the ratio of 1 to 10, so that the longer path may actually represent considerably less temperature drop than the shorter transverse one. On this basis, it then narrows down to the effective air surfaces for dissipating the heat, and upon the amount of air which can be utilized.

The primary reason for the axial method of ventilation is to get away from some of the physical restrictions of the purely air gap method of air supply. In the latter method the entire air supply, in most cases, is through the air gap, thus limiting the permissible volume of air which can be supplied. The axial arrangement, as usually supplied, simply adds core ventilating passages in parallel with the air gap. Sufficient air is forced through the gap to cool the rotor and also to exert a scouring action on the tooth tips, thus assisting materially in cooling the tooth belt. Additional air is forced through the core ducts, back of the tooth belt, to assist in cooling the teeth and to cool the core proper. As stated before, the effectiveness of the method depends largely upon the duct surface exposed and the quantity of air supplied, just as with other methods of ventilation. The real temperature drops are not so much in the iron itself, as from the exposed surfaces to the air, or what may be called the contact drop.

Effects of Heating of the Ventilating Air

As the ventilating air takes up the heat, it rises in temperature and becomes correspondingly less effective as a cooling element. In other words, any addition to the cooling air temperature means a corresponding addition, in degrees, to the temperature of the iron and copper. Therefore, the effectiveness of the air cooling depends to some extent upon the course of the air through the parts to be cooled. Theoretically, if any given part needs the most effective ventilation, then the coolest air should be fed directly to that part, with as little rise as possible. None of the present methods accomplish this completely, but some appear to be more effective than others.

Consider first, the radial arrangement, where the air enters at each end, through the air gap. The air is heated by friction as it passes along, as about half the total windage loss, in such machines, is in the fan and the other half is in the air passages. Also as the air passes over the rotor and stator gap surfaces, it takes up heat from these parts. In consequence, there is a gradual accession in air temperature, from the ends of the core, towards the center. This adds a corresponding increase to the temperatures of the iron and copper parts. The increase in the air temperature at the central part will be, presumably, about half the total air rise, or possibly more, as the loss due to the air friction in the gap, plus the rotor gap loss, plus the tooth loss dissipated at the air gap, may be

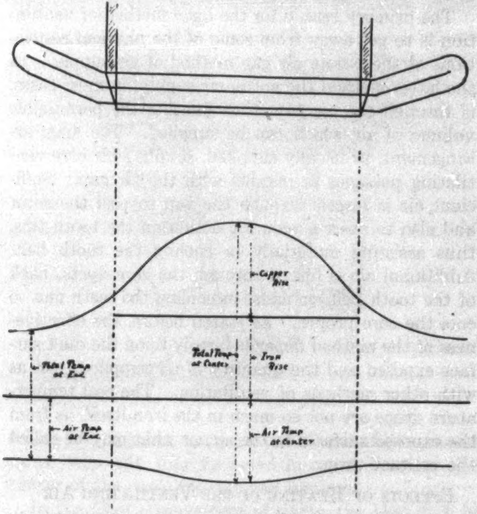

Fig. 10

taken as equal to, or greater than, the core and tooth losses dissipated directly into the radial ducts.

With auxiliary central duct ventilation, as described before in connection with Fig. 2, the rise in the air temperature at the tooth belt, is probably about midway between the temperatures of the entering and exit air.

In the axial method, of the more usual type, similar conditions hold, and the temperature rise of the air is somewhat greater at the central parts, than the usual radial arrangement, due to the fact that much the larger part of the heat dissipation to the air occurs by the time the air has reached the central portion. In consequence, the air temperature rise, at the center, is due to the core, as well as the tooth losses, considering only the core channel. In the air gap the temperature increase of the air is due to air friction and to the stator and rotor gap surface heat dissipation, just as in the radial arrangement. However, it must be borne in mind that the total volume of air through the gap is smaller than in the radial method, which should mean smaller friction loss. Taking into account both the decrease in loss, and the reduced volume of air, the increase in the air temperature in the gap, at the center, may or may not be larger than in the radial arrangement,—it is purely a question of proportion.

With the more recent types of axial construction, where the central portion of the stator embodies a large number of radial ducts, the end parts of the core being supplied with axial channels, it is a question whether there is much difference between the axial and radial arrangements, as far as air temperatures are concerned.

The above is on the basis of air flowing *towards* what are nominally the hotter parts of the machine. Let us now look at the matter on the basis of the air flow being in the opposite direction, with the axial method of ventilation. At first thought, this would not appear to change the conditions. However, there is one important difference. Here the temperature rise of the air is *least* at the central part of the core, and is higher near the ends, whereas the copper temperatures, as shown in Fig. 6, are naturally higher toward the center, and lower at the ends, due to longitudinal heat flow from the buried windings toward the ends. The resultant of the two effects is to tend to equalize the copper temperatures in the core, by bringing down the higher points and increasing the lower. There is, however, one disadvantage in this arrangement, in that there will be higher temperatures in the end windings, so much so that mica end insulation may be required, which usually is not so desirable on the curved end parts. Of course, there are possible remedies for this condition, such as an auxiliary cooler air supply for the ends, as from a separate fan, or fans.

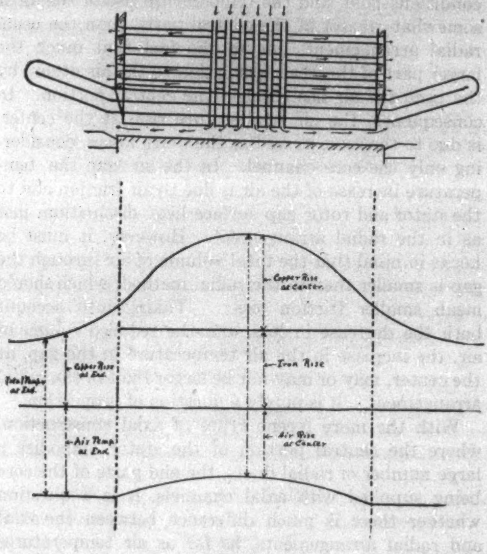

FIG. 11

By this reversed axial method, as it could be called, a reduction of the *central* copper temperatures of 10 deg. to 20 deg. might be possible, depending upon the individual designs. However, at some intermediate point *a* in the core there should be no gain, while at the ends there may be an increase of possibly 20 deg. If the hottest central part, with the usual arrangement, is 10 deg. hotter than the intermediate point *a* of no-temperature-change, then the actual peak or hottest spot temperature of the stator, as a whole, may be materially lowered by this arrangement. The rotor ventilation, however, presents a more difficult problem, as will be referred to later.

VENTILATION AND TEMPERATURE PROBLEMS OF THE ROTOR

In the foregoing, the problems of the stator have been considered, but the rotor must not be forgotten. The actual copper loss in the rotor of the modern huge capacity turbo-alternators may be as much as 80 to 100 kw., and most of the heat resulting from this must be radiated from the rotor air gap surface. In addition there are eddy current losses in the rotor iron, sometimes relatively large, due to the open stator slots, etc. And yet the excitation, or copper losses in turbo alternators are very small, in per cent, compared with other types of rotating electrical machines. In a 30,000-kw. turbo-alternator, which is by no means the largest size built, one quarter of one per cent, expended in the exciting windings, means 75 kw. loss in the rotor, which must be dissipated. The rate of heat dissipation from the gap surface of the rotor is often extremely high. Four to five *watts per square inch* of gap surface is not unusual. Comparing this with the 0.5 to 1.0 watt per sq. in., common in connection with dissipation of the excitation heat in more moderate speed machines, it may be seen that the above figures indicate a very abnormal rate of heat dissipation to air.

The writer believes he was one of the first to use purely air gap ventilation for the cooling of high-speed turbo rotors, and his early tests gave decided evidence of the great effectiveness of this method. However, there is naturally a limit to the effectiveness of such cooling, for there is some point where the improvement is quite small, with materially increased air velocities. Possibly air "cavitation" comes in, as one of the conditions which influence this result. The effectiveness of the air, in cooling the rotor, depends upon the scouring action of the air on the rotor surface, the hot layers in contact with the metal being continually torn off and replaced by cooler air. Experience shows that this action does not appear to be proportionally effective with air velocities above a certain point. Apparently the ventilating air along the air gap is crowded out against the stator surfaces, and thus possibly may become more effective in cooling the stator, with the higher velocities. Under such conditions, sinking the stator coils well below the gap surface should be especially beneficial.

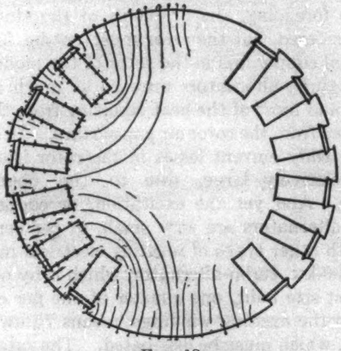

FIG. 12

It was stated that a large part of the rotor losses is dissipated as heat at the rotor gap surface. But much of this loss occurs at a considerable distance from the surface, and the resulting heat must be conducted through various paths to the cooling air. This constitutes quite a problem in itself.

Fig. 12 illustrates the arrangement of the rotor coils and core, in cross-section, in the usual radial slot type of rotor, with a two-pole machine. From this illustration it may be seen that there are two main paths of heat flow from the rotor coils, namely, outward directly toward the surface, and inward to the central body or core and then along the metal of the core to the pole surface and to the ends of the core. Obviously these latter paths are very much longer than the former, but to compensate for this, their sections are relatively larger, so that the density of heat flow is correspondingly lower. It is largely a question of relative proportions.

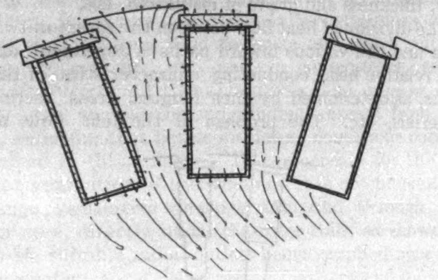

Fig. 13

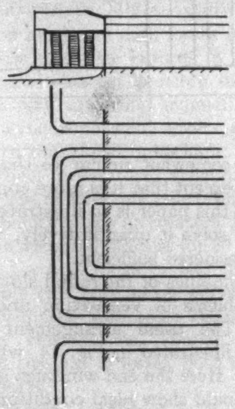

Fig. 14

Let us consider now, an individual rotor slot and its winding, as shown in Fig. 13. Here the first temperature drop of importance is from the copper to the surrounding iron, and the supporting wedges. The amount of drop is dependent upon the density of heat flow, thickness and grade of insulation, etc.

The division of heat flow between the direct outward path and the various inward paths is dependent upon the relative heat conducting characteristics of these paths, as determined by their lengths, cross sections, materials, etc. The problem of the heat flow and distribution is somewhat similar to that of the axial type of stator, except that it is more complicated. As the purpose of this paper is to illustrate the problem, rather than to solve it quantitatively, the above description is considered sufficient.

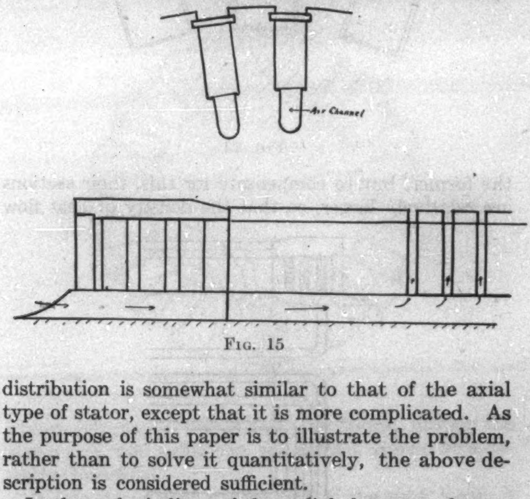

Fig. 15

In the end windings of the radial slot type of rotor still other problems of ventilation and heating are encountered. The usual arrangement of the end windings is as illustrated in Fig. 14, which shows six slots per pole. Here the end windings are exposed to the air, and should show ideal conditions for ventilation, provided good air circulating conditions are obtainable, which is not often the case. A heavy supporting end ring is required over the windings, and the coils must be well insulated from this. This enclosing ring is a direct hindrance to radial ventilation between the coils. Numerous ventilating holes through the supporting ring will aid, but in turn brings up problems of surface leakages and creepage distances, especially where the end windings have no outer wrappings, which is frequently the case. Outer wrappings over these end windings tend to bulge and crush out of shape, collect dirt, etc., and, in consequence, some manufacturers omit them altogether, and more or less completely enclose the end windings, except from below, believing that better insulating conditions can be obtained in this manner. To compensate for the reduced ventilation of the windings, it is not unusual to arrange ventilating channels into and through the rotor core, directly under the rotor coils, as shown in Fig. 15, suitable radial outlets being supplied near the core center.

With reversed ventilation as described in connection with Fig. 11, it is difficult to say just what course the rotor ventilating air would follow.

IRON LOSSES IN TURBO GENERATORS

The stator or armature core losses in large turbogenerators follow the same laws as in other types of rotating apparatus (except possibly in degree), due largely to the very abnormal magnetic proportions. In the largest synchronous converters, outputs of 300 kw. per pole are considered quite high; in large high-speed water-wheel generators, synchronous condensers, etc., 3000 kw. or kv-a. per pole is unusual; but, in the modern capacity turbo alternators, 8000 to 10,000 kv-a. per pole is accepted practise for 60 cycles, and up to 20,000 kv-a. per pole for 25 cycles. Such enormous outputs per pole can only mean abnormal magnetic proportions, compared with other classes of electrical apparatus. In consequence, special problems are encountered, in the way of losses, both in the iron and in the copper parts.

One of the special conditions is the relatively high voltage between laminations, compared with ordinary types of machines. For comparative purposes this may be expressed in terms of volts per inch of core. In ordinary types one volt per inch width of core is quite high, whereas in some of the modern turbo-alternators this figure is doubled, or even more in some instances. This, together with the enormous plate surface per pole, tends naturally to exaggerate eddy current losses in the iron. The laminae must be insulated better from each other, than in ordinary machines. The effects of the burred edges of the plates, due to punching, are more pronounced. For instance, a two-mil (0.002-in.) burr is considered quite small, in punching, yet with 17-mil thickness of plates, this burr represents about 12 per cent of the plate thickness, and several times the depth of the insulating coating on the plates. Such burrs, under great pressure, tend to cut through and make metallic contact between plates. Such minute contacts may seem to be of minor importance, and correctly so in machines with low volts per inch of iron. But in the huge turboalternators, with their higher voltages between plates, the effects of undue burr are quite considerable.

In the same way, extreme care must be taken to line up the stator plates or punchings, so that little or no filing is necessary in the coil slots. If the building is uneven, so that individual laminae project into the slots, these high points should be removed, otherwise the coils are liable to loosen in the slots, in time, through concentrated pressure, with consequent wear of the insulation at the projecting laminae. Fairly smooth slots are an essential condition, as shown by bitter experience. On the other hand, if one attempts to level down the high laminae by filing, as has been not uncommon practise in other types of machines, there is more or less danger, due to possible dragging over of adjacent laminae, with consequent metallic contact. It is almost unbelievable what high temperatures may be developed locally by such burring over at the edges of the laminae. In two instances directly within the writer's personal knowledge, such burring resulted in actual fusion of the metal at the filed or burred place. This fusion meant actual temperature of many hundreds of degrees centigrade, or far above any workable point. Yet, even under this condition, one of these turbo generators went through its complete shop test without evidence of trouble, and it was only by the merest accident that the damaged point was discovered. In the other case, three adjacent laminae had been burred together in some way, possibly due to filing, and had *welded* together at their edges, in the coil slot, and had so damaged the mica coil wrapper that a burn-out occurred in a few hours' time. Here the local temperature must have been far above 600 deg. cent., for incandescence, which begins between 550 deg. and 600 deg. is far too low to cause such welding. In both of the above cases the writer's inference is that the local temperatures attained approximated 1000 deg. cent. It was only due to the mica, that the breakdown did not occur immediately, in both cases.

Pressure apparently has a considerable effect on the losses in turbo-generator cores, possibly due to the establishment of better contact between adjacent plates, the burr cutting through the insulating coating on the plates. Quite often, in assembling the stator core, the resulting pressure is greatest next to the ends, and, in practise, greater losses and higher temperatures have resulted. To lessen the loss, by breaking up the contact between plates, etc., it is sometimes the practises to add separating sheets of suitable paper at frequent intervals throughout the core, the intervals being shortened near the core ends. This has proved quite effective, in the past, in equalizing the losses and temperatures due to core pressure. In one case where the core losses were unduly high, and there was excessive temperature in the end sections, the writer had the stator iron removed for about 6 in. at each end, and then replaced, adding sheets of fairly heavy "express" paper every 1/8 in. With the original amount of iron replaced in the core, thus representing greater compactness and higher pressure than before, the stator losses were down to normal, and the temperatures were quite well equalized. It might be mentioned that this particular stator used radial type ventilation with air ducts at quite frequent intervals. The above is simply indicative of a number of quite similar experiences. Some criticism has been raised regarding the durability of such paper, but long experience shows that where the stator iron has an ultimate temperature lower than that at which the paper deteriorates, apparently no trouble whatever results from its use. It should be borne in mind that this method of using the paper is not the equivalent of the tissue like paper coating on the individual lamina, sometimes used abroad in place of a varnish coating on the plates. This heavy paper at intervals is in addition to the coating on the plates, and is for the purpose of adding a large number of definite positive breaks in the metallic structure, these breaks being wider than the worst burrs which are liable to occur in the punchings.

While the effect of the punching of the plates is to produce burrs, which may be eliminated to a great extent by further treatment, it also has a harmful effect in increasing the magnetic losses in the iron, apparently due to the shearing effect. The action of punching or shearing seems to affect the material in proximity to the sheared edges. Consequently the harmful effect is largest, where the sheared edges bear a high ratio to the total iron, that is, at the teeth. Annealing after punching quite often shows a gain sufficient to warrant the additional expense. In some types of apparatus, however, it does not appear to be worth while.

ARMATURE COPPER LOSSES IN TURBO GENERATORS

Like the iron losses, the armature copper losses follow the laws of ordinary machines, except in degree. There are three principal losses to be considered,— (1) the usual I^2R due to the load current,— (2) eddy currents in the conductors due to the flux across the armature slots set up by the armature load current, and (3) the eddy currents in the coils set up by the main field, or gap, flux entering the top of the armature slots.

Loss (1) is taken care of by a suitable section of conductor. Loss (2) may be quite large in large turboalternators, due to the fact that the ampere-wires per slot are quite large in many cases, the number of slots being small due to the physical limitations of the dimensions of the machines. This means relatively deep slots and coils, and a tendency for excessive local currents, unless unusual subdivision or lamination of the conductors is resorted to, this being done quite completely in modern practise.

Loss (3) due to gap or field flux penetrating the slot and armature conductor is dependent upon the amount of such flux, which in turn depends upon the proportions of the slot with respect to the gap, etc. At first glance, one would be inclined to say that, with a small air gap, the gap flux would penetrate the armature slot more than with a wide gap, but, in fact, the opposite is the case. With the large gap, the relative lengths of the flux paths into the slots, are shorter, compared with the direct iron-to-iron paths, than with a short gap, other conditions being equal, and consequently the fringing into the slots will be greater, see Fig. 16. An obvious means for lessening this flux is to sink the armature conductors a considerable distance below the gap surface, and this is quite common practise in modern machines. Fairly large air gaps are necessary, from the design standpoint, on all large turbo generators; so that, without special precautions,

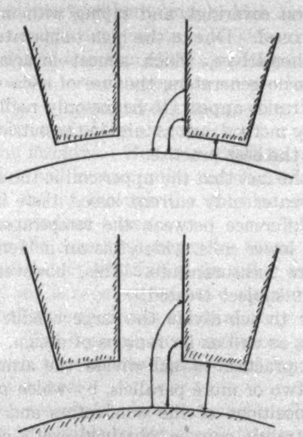

FIG. 16

such as sunken coils, well laminated conductors, etc., the losses from the gap flux would be much larger than in usual types of machines.

The lamination of the conductors of turbo generators has been quite a problem. Various methods of subdivision and of insulation of the individual strands, have been tried. Enamels and varnishes of various sorts, cotton coverings, and taping with mica paper, have been used. Due to the high temperatures inside the coils themselves, which almost invariably occur in large turbo generators, the use of mica on the individual strands appears to be the only really safe and satisfactory method just as mica in the outside wrapper represents the best practise.

Due to the fact that the upper coil in the slot naturally has greater eddy current losses, there is liable to be some difference between the temperatures of the upper and lower coils, which has an influence on the temperature measurements. This, however, belongs in the next subject treated.

In order to sub-divide the large conductors more completely, as well as for reasons of design, it is a not infrequent practise to sub-divide the armature circuits into two or more parallels, by which means certain transpositions of the conductors and their elementary strands can be obtained, with consequent reduction in the eddy current losses.

VOLUME OF COOLING AIR, AND WINDAGE LOSSES

Reference has been made, repeatedly, to the difficulties in obtaining the requisite cooling air. It is the enormous volume of air required that is back of much of the difficulty. As a rough approximation, let us assume the total generator losses, including windage, as 4 per cent of the rated capacity; or the loss in kilowatts is equal to 0.04 × kw. capacity. Also accepting the well-known relation that one kw. loss will raise the temperature of 100 cubic feet of air 18 deg. cent. in one minute, then four cubic feet of air per minute is required per kw. capacity of the machine for 18 deg. cent. rise of the air, or three cubic feet per kw. for 24 deg. cent. rise. This is only a crude quantitative statement of the problem, but it indicates that for a

30,000-kw. or kv-a. generator, for instance, a total of about 90,000 cubic feet of air per minute is required. This volume, in practise, will actually lie between 75,000 and 100,000 cubic feet, depending upon the total losses (including windage itself), and upon the air temperature rise allowed. In any case it is very large and the real problem is to obtain sufficient cross section of air inlet without unduly high air velocities. Assuming a maximum velocity of 9000 feet per minute, the minimum cross section of the air path must be about nine square feet, or equivalent to an opening three ft. by three ft.,—an apparently impossible figure from casual observation of the air gap and other air channels through the machine. But even 9000 ft. air velocity is very high and means quite considerable pressures for forcing the air through the machine, with consequent high windage losses.

Whether the core loss proper be taken care of by axial or radial duct ventilation, in either case a large volume of air is required to take up the total gap losses and this alone means comparatively large windage losses. This is an inherent condition, and the difference, with the various methods of ventilation, is only one of degree.

With any reasonably well designed method of ventilation, it is possible to get the requisite air through the machine with the desired cooling effect, provided suitable pressure is obtainable. However, the higher the pressure with a given volume of cooling air, the higher will be the losses, other conditions being approximately the same. In consequence, there is a limit to the permissible pressure which can be used.

The efficiency of the fan which provides the pressure, is an element of the problem, but when one considers that more than one-half of the total windage loss is inside the machine and independent of the fan itself, it is evident that the fan efficiency is not a dominating feature, although it is of importance.

The problem of artificial cooling is more complex in the turbo generator than in any other type of electric machine, and it may be said that more engineering effort has been, and is being, spent on this than any other problem in this class of apparatus. Each step upward, in capacity, or in speed, opens the problems anew, for in each of the preceding advances the designers have usually strained to the utmost to meet the necessities of the case.

To the uninitiated, the volume and the weight of air required to cool the modern turbo generator is always a matter of surprise. Without the figures before one, it is almost unbelievable that a modern large capacity turbo generator *puts through itself practically its own weigh tin cooling air in forty to sixty minutes.* Obviously, unless this air is kept very clean, the total amount of dirt and other foreign substance which pass through the machine, in even a few days' time, will be enormous, and a very small percentage of this, deposited inside the machine, may soon seriously clog the ventilating passages and thus interfere with the operation of the machine. This indicates why air washers and other devices for preventing, or lessening, the admission of dirt, have become almost a necessity in such machinery.

TEMPERATURE DETERMINATIONS

There are two special problems in the temperature determination of large turbo generators, namely, location of the hottest part, and measurement of the temperature in such a manner, or by such method, as will give a fairly close approximation to the actual results. The former problem is easier than the latter, as it simply involves a multiplicity of measurements on a given type or construction of machine, until a reasonably accurate location is determined, which may be applied on other machines of the same type. Uniformity of materials and construction is involved in this.

The second problem is difficult. If one could apply temperature indicators directly to the copper itself, of course the actual highest copper temperatures could be obtained directly. But this is not practicable except under what may be called laboratory conditions. The requisite continuity of the insulation over the copper makes difficult any measurement directly on the copper itself. We have, therefore, adopted certain approximate methods, which have proved fairly satisfactory, but which must be used with judgment, if a reasonably accurate approximation is to be obtained. The results are dependent upon the method of application, the type of ventilation used, and various other considerations.

The method now used most extensively depends upon the placing of the temperature indicator between upper and lower coils in the same slot, and, therefore, is predicated upon the two-coils-per-slot arrangement, which, however, is now the common practise in this country. The theory of the method is based upon the principles of heat flow from the two coils.

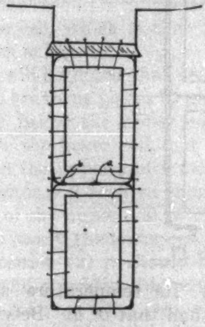
FIG. 17

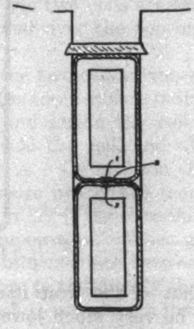
FIG. 18

Let Fig. 17 illustrate the usual two-coils-per-slot arrangement. The arrows shown indicate the general paths of heat flow from the two coils. Assuming the copper temperatures of the two coils as equal, and the width of the coils to be considerable, then at the mid-point *a* between the upper and lower coils the temperature should be practically midway between the upper and lower copper temperatures, and an indicator at this point should show about the same temperatures as the copper. This is not exactly true, for there is some heat flow along the insulation from the midpoint *a*, so that this point will probably be slightly lower than the true mid-temperature between the coils.

A point *b* between the coils, but at the corner next the iron, should have a temperature very nearly equal

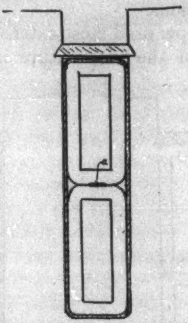

Fig. 19

to that of the iron itself. Its temperature should thus be very much lower than that of *a*. Between *b* and *a* there should be intermediate temperatures, depending upon the distance from the center point *a*.

It should be obvious, therefore, that a temperature indicator covering practically the full width of the slot,

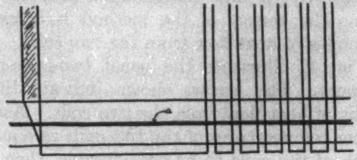

Fig. 20

will show some average value between *b* and *a*. Therefore, for greatest accuracy the indicator should be as narrow as possible, and be placed at *a*.

Another consideration involves the method of applying the usual slot cells. In common practise the turbo-armature coils are "pushed home" in hard fibre cells placed next to the iron. This serves as a protection for the insulation proper, in putting the coil in place, and also makes a tighter fit in the slot. There are two general methods of applying these cells. In one, the cell is placed over the lower coil, and then folded over its upper side, either during manufacture of the coil, or before the upper coil, with a corresponding cell, is put in place. This is shown in Fig. 18.

In the second arrangement, the two coils are set in a single cell, which is then folded over the top one, as shown in Fig. 19.

It will be noted that with the two-cell arrangement, there are three places to put the temperature indicator, —(1) below the upper coil and inside the cell,—(2) above the lower coil, but inside the cell, and (3) between the upper and lower cells. On position (1) the indicator will tend to show more nearly the temperature of the upper coil, while in (2) it tends to indicate more nearly the lower coil temperature. When placed in position (3) it should be intermediate between (1) and (2), provided this position is thoroughly shielded from external influences, which is not always the case, as will be explained further on.

In the single cell arrangement, there is but one location for the indicator between the coils, unless, as is found in some constructions, a separating or spacing strip of appreciable thickness is placed between the upper and lower coils. In such case, if placed above the strip, the indicator should show more nearly the upper copper temperature. Obviously, with this single cell, the indicator is well shielded from external influences, such as the ventilating air, that might come in contact with the measuring device at the radial air ducts.

Considering next, the application of such indicators to the axial and the radial types of stators, it will be noted that if the indicator be located in the axially ventilated portion of the core, as indicated in Fig. 20, at point *a*, for instance, there is no chance for the ventilating air to come in contact with the indicator, regardless of whether the single or double slot coil arrangement is used. In other words, the indicator is naturally well protected. However, if placed in the central radially ventilated section or zone, the conditions may be quite different, but not necessarily so, depending upon the construction and application of the indicator, etc. Here the coils are partly embedded in iron and lie partly in the radial ventilating ducts. The indicated result will depend upon whether the measurement is taken at *a* or *b*, for instance, in Fig. 21. This difference, however, may be quite small if the detector is thoroughly shielded from the ventilating air in the duct. It may be suggested that, as the coils are ventilated in the duct, the indicator should be ventilated also. But we are not after the temperature of the outside or exposed surface of the insulation,

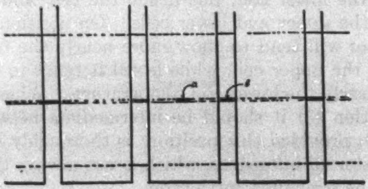

Fig. 21

but are attempting to approximate the internal copper temperatures. *The indicator should thus be as well shielded, if possible, as the copper inside the coils.* The single slot cell arrangement thus appears to be more accurate than the double cell, with the indicator outside the cells, when the indicator is placed close to, or bridges one or more ventilating ducts. Thus, in the axially ventilated type of stator, the ventilating device should not be used in the central radially ventilated zone, unless unusual precautions are taken to avoid the discrepancies which are liable to result. The same may be said of the radially ventilated stator as a whole.

It should be evident, from the above, that a certain amount of judgment and skill is necessary to obtain the closest possible approach to the true copper temperatures. Furthermore, the problem is complicated by the fact that the upper coil in the slot is liable to be hotter than the lower one, due to higher eddies in the upper copper. At best, therefore, the method as a whole is only an approximation, but it gives so much closer to the correct result that any of the measuring methods in use only a few years ago, that it marks a decided step in advance in generator temperature measurements.

As far as temperatures in the stator are concerned, they are not of serious importance except as they have an influence on the copper and insulation temperatures, and even the copper temperature, in itself, is of importance only as it affects the insulation. In fact, it is solely the insulation durability which we are after, in the last analysis. The improvement, and refinements in design, betterments in the grade of insulation, innovations in ventilation, etc., all are directed to the one end, not the temperature itself, but the factor of safety of the insulating materials.

DISCUSSION ON "VENTILATION AND TEMPERATURE PROBLEMS IN LARGE TURBO-GENERATORS" (LAMME), WHITE SULPHUR, W. VA., JUNE 30, 1920.

R. B. Williamson: On page 936, Mr. Lamme calls attention to a point in connection with large turbo generators that I think has not heretofore been brought out very forcibly; namely, the necessity of very thorough insulation of the laminations. This is a point that is important in all alternators but specially so in turbo generators.

Consider one section of the stator core. Each lamination is dovetailed into the stator frame and on each side of the section we have a ventilating spacer in the form of brass castings, or in the shape of steel strips welded to the laminations forming the ventilating duct walls. In a turbo-generator with a deep core behind the teeth and a relatively large flux in this part, there is considerable voltage between the two sides of a given core section. This may be of the order of 6 to 10 volts, and the vent spaces together with the yoke casting form a single-turn secondary in which the only thing that prevents current flowing is the varnish or other insulation between laminations.

With low-speed generators, such as water-wheel or engine-type machines, the core depths back of the teeth is small. Consequently the voltage between the two sides of a core section is proportionally smaller and there is not so much danger of circulating current as in turbo generators.

I recall one case of a turbo generator, where the insulating varnish broke down, and the current flowing across the laminations was such that it was possible by running up the magnetization to make some of the teeth red hot. If, therefore, great care is not taken in varnishing laminations and stacking the core, it is possible to lose all the advantage, so far as core loss is concerned, that may be gained by a careful selection of iron and in addition there will be the danger of local hot spots.

As Mr. Lamme points out, in some cases it is customary to put a thick layer of paper in the center of each core section, this breaks up the circuit between the two sides of the core section, and at the same time does not prevent the flow of heat towards the ducts on each side. In order to get the laminations in the best condition, it is desirable to anneal them, not so much to reduce the loss, but to remove the burrs, oil, etc., and leave the surface of the steel in good condition to receive the coating of insulating varnish.

W. J. Foster: Mr. Lamme states that the maximum flow of air is something like 80,000 to 100,000 cu. ft. per minute in a 30,000-kw. unit, which stated in terms we can understand, amounts to the weight of the generator itself passing through it every 40 to 60 minutes.

Recently we sent out a questionnaire to the engineers of different air conditioning companies with regard to the quantity of dirt that is to be found in the neighborhood of a modern steam turbine power house. The replies lead us to make this statement—that at the average power house, the quantity of dirt and impurities of all kinds is one hundred thousand millionths of the air itself. That is a pretty small fraction when you write it out in decimals, but when you multiply the quantity of air, it gives this answer—that every day where you pass 80,000 cubic feet of air through, you pass one and one quarter cu. ft. of dirt.

This dirt, fortunately, most of it passes right through, but if there is oily vapor or extreme moisture, or if the impurity itself is of the nature of carbon dust, which is apt to appear more than other forms of impurities, it is not long until the passageways are somewhat clogged up, and the surfaces are put in such condition that external fires that we are hearing so much about nowadays may start.

Mr. Lamme does not bring in one form of radial ventilation. On pages 959 and 960 of my paper you will find a modification of the radial ventilation which results in equalizing the temperature throughout the length of the machine.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29–July 2, 1920.

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TEMPERATURES IN LARGE ALTERNATING-CURRENT GENERATORS

BY W. J. FOSTER

Engineering Dept., General Electric Co., Schenectady, N. Y.

WHAT shall be considered a large generator—5000, 10,000, 15,000 or 20,000 kv-a? Shall it be the rating alone that is considered or shall we take such factors as speed into account?

Undoubtedly, a large proportion of the general public thinks of a large machine as one that occupies a large space compared with other machines used for the same purpose. They judge the largeness by physical dimensions alone. At the same time, it is safe to say the more intelligent of the general public think of size in terms of the work that can be done. Probably a machine of 10,000 kw. or higher, is regarded by them as a large machine. To the engineer, largeness involves the difficulties inherent in the design and construction. A 1000-kv-a., 10,000-cycle alternator is a large one; a 5000-kv-a. generator at 3600 revolutions per minute is large. Considered strictly with reference to the temperature problem, the engineer would hardly consider a 20,000-kv-a. generator a large one if the periodicity and potential were those in regular use in commercial work, and the speed 100 revolutions per minute, or thereabouts. However, for the purpose of this article we will consider 20,000 kv-a. as a large generator.

There are two principal factors in the temperature problem in every case; first, the total losses, or the amount of heat energy to be disposed of and its concentration, and, second, the means that can be provided for dissipating the heat in such manner as not to cause damage to any part of the machine. The problem may be attacked along the lines of reduction of losses or of devising such constructions that the heat may be more effectively dissipated.

In a rotating dynamo electric machine three sources of heat are always involved; first, hysteretic losses in the magnetic material; second, the resistance to flow of current losses in windings, and, third, the frictional losses in the bearings and of the windage. The first two are electrical in their nature; the last is mechanical.

Combined with hysteresis losses in the magnetic material are more or less eddy current losses. The total losses in the magnetic material are dependent upon such factors as the degree of lamination employed, the character of the insulation between laminations, the amount of pressure employed in clamping up the cores, as well as the character of the steel employed. In like manner, the resistance losses in the copper are frequently accompanied by eddy current losses, the amount of which is dependent upon such factors as the stranding of the conductor, the pitch of the winding and the arrangement of the turns. The windage losses, or the losses that result from either the fan action of rotating parts or from the disk action or rubbing of a revolving body on the surrounding air, are dependent upon the peripheral speed and the details of design of the parts that are producing fan action.

In considering the temperature problem, the first and most fundamental consideration is the relation of the space occupied by the object, to the total heat losses to be dissipated. A 20,000-kv-a. 100-rev. per min. machine compared with one of same output at 1800 rev. per min., has its losses generated in a space eight times as great in terms of cubical space occupied, or approximately two times in the projected area occupied. We may well think of the temperature problem in terms of heat losses to space occupied. Below a certain value of this constant it is absurd to use ventilation housings, no matter how great the rating of the machine, as such housings have the effect of preventing the natural means of heat dissipation, *viz.*. convection and radiation, and ventilation housings in such cases result in higher temperatures, unless forced draft is provided, which results in a decrease in the efficiency of the unit and can be justified only on the score of reduction of noise, or some similar reason.

Second in importance to the space factor comes the heat density factor. By this we mean the quantity of heat energy passing through a unit area of material.

The third factor is the thermal conductivity of the various materials,—a factor which depends not only upon the thermal properties per se of the materials but also upon the manner in which the materials are put together.

CLASSIFICATION OF MACHINES WITH RESPECT TO VENTILATION

Attempts have been made at standardization of the various classes, but I think it safe to say nothing yet has been suggested that appeals to engineers in general as entirely satisfactory. Possibly it is desirable to have a large number of classes of machines to fill in the gap between the extremes of the lowest speed small capacity machine that requires no special provision and may be said to depend upon *natural* ventilation alone, and the highest speed machine that requires the most careful artificial ventilation. It is difficult to classify the types that have already been developed to fill in this gap as they blend into one another.

The points to be kept in mind in the design and construction of machines in general, not particularly those standing at the extreme ends, as outlined above, are; first, obtaining supply of cooling air from a region well removed from the region into which the outlet air is dumped; second, the placing of barriers or bafflers to assist the flow of air and to prevent re-circulation; third, the providing of ample cross section in all parts of the paths of flow of cooling air and the avoidance of sharp contrasts in the cross-sectional area of the paths, especially the avoidance at any point of greatly reduced cross section that would introduce great resistance; fourth, the avoidance of "churning of air," or internal circulations, which are often hard to prevent by reason of the irregular shapes of the different parts of the machine.

CLOSED AND SEMI-CLOSED VENTILATED MACHINES

In almost all large generators—whether hydraulic or steam turbine, it is necessary to pipe air either to the machine or away from it, or both to and away from. Probably the most common practise is to pipe air to the machine, allowing it to escape through the stator frame into the dynamo room,—the escape often arranged so as to be upwards, which is preferable on account of the greater comfort to the operators, the reduction in noise and the slight reduction in temperature obtained by the lower temperature of the air immediately around the machine.

The author wishes to call attention to certain advantages that would result in reversing the common practise of the present time, in the ventilation of large generators in hydraulic units, and to take the air in directly from the room and pipe it away either to some point in the building removed from the machine or to out of doors. The advantages of this arrangement have already appealed strongly to the operators of some of the largest hydraulic generators, and such a system is now in use in a few plants. It is a much simpler matter to draw air into the rotor direct from the room at the two ends of the generator than to provide the necessary space for the air conduits and the housings required either at the one end or the two ends, which almost invariably involve greater distance between bearings and, consequently, an increase in both the diameter and length of the shaft and corresponding parts. A great advantage of the scheme of piping air away is the more comfortable temperature of the dynamo room in hot weather. It is never necessary to be in an atmosphere of higher temperature than that existing out of doors, whereas, in case of the more common practise, the air surrounding the machine has its temperature raised several degrees above that of out of doors, due to the heat that has been added to it by passing through the machine.

WATER COOLING

Water is an ideal agent for cooling purposes. At first thought it seems strange that it has not been made greater use of in removing heat from large machines. A small quantity of water, on account of its specific capacity, would suffice to remove heat from a large generator, but the difficulty is in arranging jackets that would prove safe and could be located in close enough proximity to the parts in which the losses are generated to remove such losses without a considerable drop in temperature through the intervening walls of material. It is apparent at once that water-cooling is much better adapted to the stationary than to the revolving parts. While it is possible to arrange for a flow of water through the revolving parts, it is probably not possible to so arrange the flow of water as to cool the surfaces where the most of the heat is generated. Hence, a system of water-cooling would be dependent upon the joint action of air-cooling and

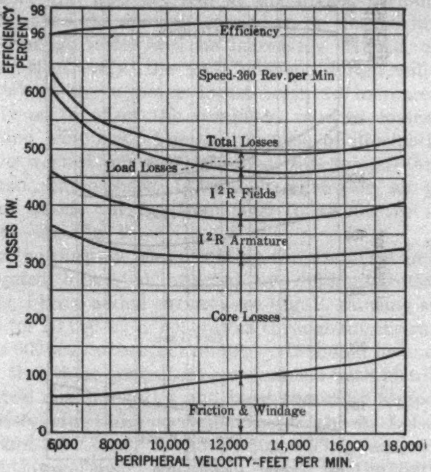
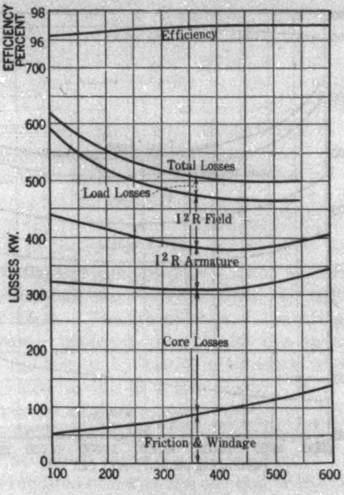


FIG. 1

would require a design that would re-circulate the air surrounding the rotor, in such manner as to most effectively carry the heat from the surfaces of the rotor to the surfaces of the water piping in the stator. Another objection is the danger of injuring a machine in case the water circulating system becomes leaky. Still another is the danger of too great an accumulation of dampness, due to condensation at certain times, of the moisture in the air on the water-cooled parts. It is doubtful whether water-cooling can ever be a competitor of air-cooling in dynamo electric machines.



Rev. per Min.

FIG. 2

PERIPHERAL SPEED OF ROTOR

It is possible that the advantage electrically of higher peripheral speed in almost all designs of large generators, is not fully appreciated by many designers themselves. The losses in both the iron and the copper are almost universally less at the higher peripheral speeds in any practical problem. Assuming the same characteristics electrically in all respects, such as saturation curve, armature reaction per unit pitch, etc., the losses in the armature teeth are less at the higher peripheral speed. They are exactly inversely as the peripheral speed, while the core losses proper remain practically constant. Copper losses in both armature and field are less at the higher peripheral speed, unless the speed is carried to an absurd limit. The windage losses will be increased. Hence, considered electrically, the most efficient design will be that where the windage losses begin to increase so rapidly as to offset the combined reduced losses of core and windings. For the purpose of illustration, I have worked out a 20,000-kv-a. 60-cycle, 360-rev. per min., three-phase, 11,000-volt generator at peripheral speeds varying from 6000 to 18,000 feet per minute—see Fig. 1.

For the purpose of illustrating the variation in the segregated losses, as affected by different rotative speeds, I have added curves—see Fig. 2, showing such losses for 20,000-kv-a. generators throughout the range 100 to 600 revolutions per minute. It should be understood that these generators are designed with identical electrical characteristics and have the same temperature rises, *viz.*, those corresponding to the A. I. E. E. Standard for Class "A" insulation. They are what the author considers normal in design for the output at the several speeds.

The 100-rev. per min. generator has a peripheral velocity of about 7500 feet per minute, the 600-rev. per min., of 15,000 feet per minute. The first has its losses generated in a space of approximately 2200 cubic feet, the last in approximately 550 cubic feet. The total losses of the first are 620 kw., or 270 watts per cubic foot of space occupied; of the last, 495 kw., or 900 watts per cubic foot of space occupied. Hence, the ventilation problem is quite different in the two cases. The first might be of the open type, drawing its ventilating air in from the room and returning it directly into the room; the last must be enclosed,—preferably totally enclosed.

VENTILATION DUCTS IN ARMATURE CORES

The common practise for ventilating armatures is to provide at short intervals in the laminated core, a narrow passage,—usually ⅜ in. or ½ in., for the air to be driven through radially by the fan action of the rotor, or in special cases by an external fan. The flow of air is in any special case dependent upon the details of construction, such as the character of the space blocks, how located with respect to the coils in the slot: the niceties introduced at the entrance from the airgap in the way of treatment of retaining wedges of the windings, the exact location of the end of the spacers, etc.

Good results are usually obtained by having a ventilating duct about every two inches,—the most efficient spacing being dependent upon such factors as the radial depth of the core, length of core, and the pressure of the cooling air. In long cores the spacing may be graded and the sections of core at middle made smaller than at the ends. This for two reasons,—first, some of the heat at the ends travels to the head of the core where the cooling conditions are usually good and, second, the ventilating air gathers up heat as it passes in from the head towards the middle and, hence, is not as good a cooling medium when it enters the ventilation duct as the air in the ducts nearer the head. When fans are mounted at the two heads of rotor, and end housings placed on the stator so as to establish a good air pressure, the pressure in the ventilating ducts increases from the head to the middle—hence, the quantity of air passing through is greatest in the duct at middle, decreasing toward the heads. The curve in Fig. 3, entitled "Velocity of Air at Entrance to Air Duct," was plotted from air pressure readings made on a large turbo-generator with equally spaced ventilating ducts. The other curves were determined from consideration of the heat dissipation problem. In like manner, Fig. 4 shows curves for a later turbo-generator with stator core sectionalized in such manner as to equalize temperatures throughout the length of core.

GIVING DIRECTION TO COOLING AIR

It is quite wonderful what improvements are sometimes accomplished in cooling machines by very simple expedients. It is sometimes advisable to arrange a machine so that it is obliged to take all of the cooling air in at one end and put it out at the other. This, as a rule, is especially helpful to the rotor.

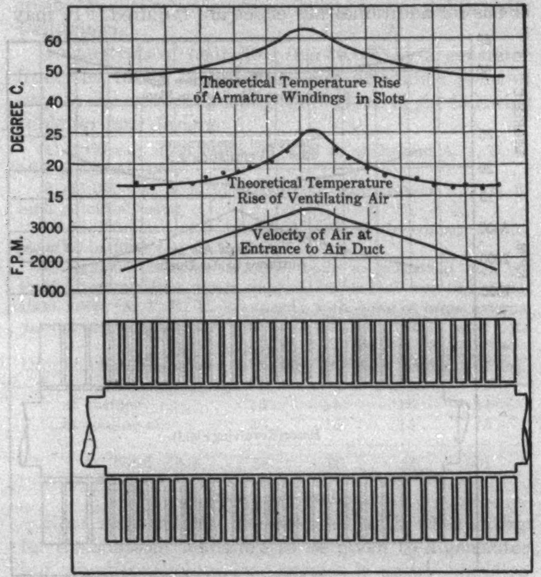


FIG. 3

But, in general, machines arranged with a radial system of air ducts through the core, should draw the air in equally from both ends. Often a machine that, seems on a casual glance to be symmetrical as to the two ends, proves a surprise in taking practically all its air from one end. In such case little, if any, air passes outward through the radial air vents,—in fact, sometimes the air will pass inwards in some of the vents. The remedy is usually very simple,—any little barrier interposed in the path of the axial flow will restore the desired circulation and often reduce the temperature several degrees.

The poles themselves act as the fan blades on the rotors of many salient pole generators, and no fans or fins for additional fan effect are required. It may not be generally known that even in cases where carefully designed fans similar to those used in large cylindrical rotor turbo generators are employed, the poles themselves contribute more to the blower action than the fans. The problem of ventilation in such machines is more complicated than in cylindrical rotor machines, where the blower action is more largely due to fans designed for the purpose.

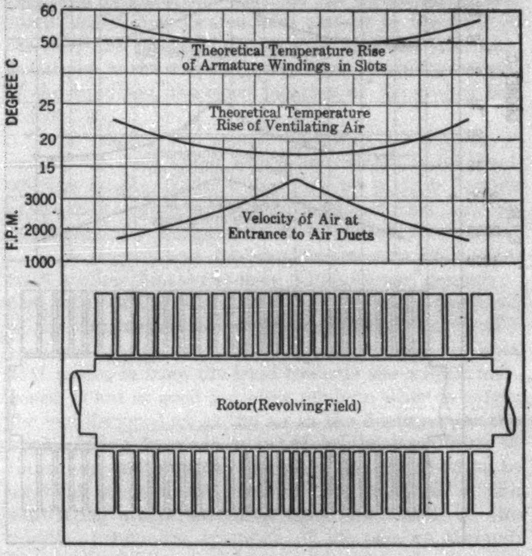


FIG. 4

HEAT FLOW

The most efficient ventilation of a large electric generator requires circulation of the cooling air in such manner as to bring it in contact with large surfaces of the solid materials in which heat is being generated and close to the sources of the heat generation.

The heat resistance of the various materials entering into the construction, such as copper, the magnetic steels, and the various insulating materials, is quite well-known.

An analysis of heat flow in a 30,000-kv-a. generator from the inside of armature coil at middle point of core to the ambient cooling air is given in the following table for four designs:

(1) 4000-volt mica insulated coils to withstand A. I. E. E. high potential tests;

(2) 11,000-volt mica insulated coils to withstand A. I. E. E. high potential tests;

(3) 11,000-volt mica insulated coils with copper density same as in 4000-volt design;

(4) 11,000-volt mica insulated coils to withstand high potential test of three times normal, instead of two times, plus 1000 volts (A. I. E. E. Standard) with coils of same external dimensions as those of (2), so as to be assembled in same slots.

	1	2	3	4
Drop through insulation...	21 deg.	30 deg.	48 deg.	67 deg.
" " core.......	6	4	6	5
" at surface...........	16	11	16	14
" in cooling air.......	15	15	15	15
Total drop.....	58	60	85	101

HEAT STORAGE

Heat storage must be reckoned with when ratings for intermittent loads are to be given to a generator, but when continuous load service is under consideration, as in nearly all large commercial generators, the heat capacity properties are chiefly of scientific interest, except when the duration of heat runs in acceptance tests is under consideration. Fig. 5 shows curves of time required to reach constant temperature, in the case of an 18,750-kv-a. turbo generator at overload corresponding to about 20,000-kv-a. which may be taken as typical of the modern large cylindrical rotor generator. This set of curves represents three runs under widely different conditions. The curves "Rise in Field Winding" and "Rise in Armature

Winding" were determined in the same run. The curve "Unexcited" shows the rate of temperature rise when the heat is generated by the windage alone, as measured by the detector embedded in armature slots. The curve "Excited to Normal Volts" shows the rate of rise measured in same manner, when the heat is that of core losses on open circuit in addition to the windage. Fig. 6 gives curves of temperature rise in a single run on a salient pole generator. Comparing the two sets of curves, it is interesting to note the quicker rise in the field winding of the salient pole machine, where nearly all the heat passes directly into the cooling air from the surface of the bare copper, over that of the cylindrical rotor machine where the field winding of each pole consists of several coils embedded in slots in the magnetic material.

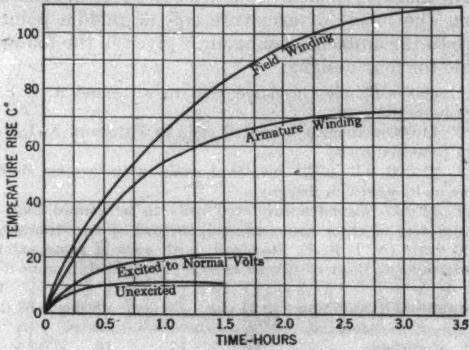


FIG. 5

HIGH VS. LOW TEMPERATURE GENERATORS

Undoubtedly any machine, electric generator or strictly mechanical machine, such as steam turbine, would be better off if it could always maintain the same temperature in all its parts. Change of temperature beyond certain limits repeated often enough, results in deterioration in most electric generators. This is due primarily to an effect of heat that is mechanical in its nature, *viz.*, a change in size. Much can be done to minimize the deleterious effects of change in size of the various parts, by introducing constructions in detail parts that automatically adjust for changing size. But it is extremely difficult in certain parts to protect materials of quite frail mechanical nature, like many insulations, from the effects of change in compression or what is more serious, slight movements of different degree in different places. Looked at in this way, it is desirable to have a generator of low temperature rise. But it is not always convenient or possible to build low temperature generators if machines are to be produced equal in capacity to prime movers. Furthermore, high temperature machines are justified in cases where increased efficiency or lower cost will more than offset the shorter life.

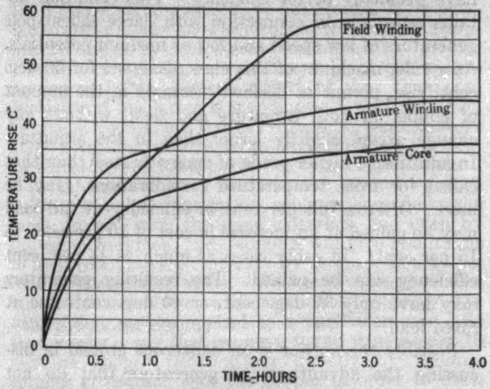


Fig. 6

With reference to relative efficiency and cost, it is quite apparent that the high temperature machine has the advantage in the case of a generator whose insulations are suitable for the higher temperature, and whose efficiency is still rising with increase of load, as in most alternating-current generators, and whose cost is only slightly increased by enlarging the shaft and other mechanical parts involved.

It may be laid down, as a rule, that the high temperature machine costs less, but it must not be taken for granted that its efficiency is better. In fact, most of the lower speed machines of low temperature, have better efficiency than corresponding machines of high temperature, unless the designer has been grossly careless in taking care of the ventilation.

It is possible to design most machines for low temperature rise without great additional cost, and have decidedly better efficiency. This condition obtains especially in connection with large salient-pole generators of low speed and low or medium potentials. As a rule, machines of this class, designed for 50 deg. cent. rise, permit of decided increases in the amount of copper in both armature and field, without any change except slightly larger slots in the armature. In addition, a higher grade of magnetic steel than that called for from temperature considerations, may be used. Often a full per cent in efficiency at full load may be gained at an increase in cost of 10 per cent to 15 per cent. In other cases as much as ½ per cent efficiency can be gained. The resulting generators may have only 35 deg. cent. or 40 deg. cent. rise at rated load.

Possibly the author is on dangerous ground in discussing the advantages of generators that do not conform to the Standardization Rules of the A. I. E. E. But it is not for a moment his intention to reflect in the slightest on the standards that have been set, but he wishes to point out gains to the user to be had by following along more conservative lines in certain cases. Again, he realizes the possibility of trouble to himself and his ilk from urgent requests that may be in store from buyers of generators, when generators are under consideration that cannot economically be built for temperatures below the Standards of the A. I. E. E. in the high and low temperature classes. It is well to add that designing certain sizes for low temperatures, where only slight gain or no gain whatever in efficiency results, often involves hardship and results in certain risks being taken that are wholly unjustified.

DISCUSSION ON "TEMPERATURE IN LARGE A-C. GENERATORS" (FOSTER), WHITE SULPHUR, W. VA., JUNE 30, 1920.

F. D. Newbury: I would like to ask Mr. Foster if these figures have been checked by direct measurement on the actual generator, and by direct measurement, I mean the measurement of the copper temperature by embedded detectors located within the insulations, and therefore in contact with the copper? In our experience and as brought out in detail in the paper of Mr. Fechheimer and myself, we have found it impossible to arrive at any knowlegde of the true copper temperature except by such direct measurement.

In this connection I would like to point out in connection with Mr. Foster's statement that "heat conductivity of insulation is well-known", that that may be true in a sense, but it is not true in the sense that we know a definite value that can be applied to a particular armature coil for a particular type of insulation.

In the case of tests made in the laboratory with built-up samples of insulation, supposedly made by the same process, with the same materials, and even by the same workmen, such samples will give results as to heat conductivity that vary by as much as fifty per cent of the smaller value. We know average values, but we do not know, with reasonable accuracy, the values that can be applied to a particular case. So that is why I say that we have found it necessary to measure copper temperatures directly by thermocouples in contact with the copper in order to draw conclusions that are of any value.

We have also found in the case of the detector method with the detectors located between coils, that that method is subject to wider variations, and in some instances is really less reliable than the resistance method. Of course, these new methods give entirely different results—one gives the local temperature, the other gives the average temperature of the complete winding, both inside and outside the core.

I would like to ask Mr. Foster if in a machine such as the one for which he has given us the temperature drop X, what the relative rise by resistance would be, because until we have some way of checking the probable error of the figure we cannot very well discuss them.

W. I. Slichter: The appearance of a heating unit which had been running for a year or so at temperatures of 500 deg. regularly, and occasionally temperatures of 600 deg. cent. and which consisted of ordinary resistance wire insulated by mica, struck me as so good that I wonder whether we might not very soon expect to increase the limit which we are now discussing of 185 to 200 deg. cent., for our mica insulation.

Philip Torchio: I would take issue with Prof. Slichter about suggesting temperatures like 185 degrees for mica insulation in turbo generators. I would go back to my limit of 100 degrees. For other apparatus, like heating units, I would go up to 500 or 600 degrees, or anything the Professor wishes.

I would like to make just a brief comment on the remark of Mr. Newbury on the heat conductivity of materials varying considerably, as much as fifty per cent., and also the statement by one of the writers that the heat conductivity varied considerably with and against the grain, varying as much as from one to 3.5.

I have had somewhat similar difficulties in trying to determine the heat conductivity of concrete due to the surface drop in making measurement. Furthermore, with windings insulated with mica, if the different layers of mica are not perfectly uniform, they create air spaces which will give all kinds of differences; in reality it is not the conductivity of the material that varies so much, but rather the physical condition of the layers, how they are put together, and also the differences in the measurements in determining the surface drop while the measurements are made.

A. S. Loizeaux: From what has been said this morning about temperatures in large turbo generators, it would seem that high temperature itself is probably responsible only for a rather small part of the failures, and that there is a considerable amount of experience of very recent origin that goes to add to that evidence. I think that the support of end windings, or the lack of rigid support, is probably responsible for failures that might ordinarily be laid to the door of high temperature. A very small movement, gradually resulting in breaking the insulation, is responsible, I think, for some failures.

The difficulty in supporting the end windings is a real one. I have found cases where they have been decidedly bent due to a heavy short circuit.

Another thing mentioned in the discussion of fires originating in current transformers. I think we should warn every operating company to take the current transformers out of the air ducts, if they are in there at present, because an explosion of a current transformer or the burning of it will easily set fire to the generator windings. Also the terminals and the main leads of the generator should be adequately spaced and insulated and supported. In order to prevent damage from heavy short circuits, the supports should be close together and extremely rigid.

Selby Haar: The objection to the high temperatures, as suggested by Prof. Slichter, was that it resulted in extremely low efficiencies, because of the large increase in copper loss at high temperatures. This hardly effects large turbo generators because they have high efficiencies under almost any circumstances, but the situation is far different for some other classes of machinery in which copper space is limited.

S. R. Bergman: The influence of the temperature on the efficiency of high-speed generators is small. In such machines the core loss, friction and windage losses are predominant, whereas, the copper loss is relatively small. An increase of 50 deg. to 100 deg. cent. in the temperature would, therefore, have but a small effect on the efficiency.

The reverse is true in low-speed machines in which the copper loss is predominant. In such machines an increase of 50 deg. to 100 deg. cent. in the temperature would have a marked effect on the efficiency.

W. J. Foster: It is all right, perhaps, in the sense in which Mr. Bergman means it, that the addition of some copper losses is of small consequence—that is true as far as efficiency is concerned, because the I^2R is ridiculously small, as pointed out by Mr. Lamme in his paper, but just the same the local heating is there, just as it is in the laminated core. If you happen to have laminations pressed together so they solidify along the slot for an inch or two, you may have the temperature go to red heat. That does not hurt the machine, as far as efficiency is concerned; it is only 0.001 of a per cent, perhaps, but it will destroy the machine, since it will cause a fire, so we must look out in the windings to keep temperatures down to a certain point that will be safe for their operation.

Right along that line Mr. Loizeaux brings up the point of the support. I do not know what Mr. Loizeaux has in mind when he speaks of rigid support, but I would like to point out the great difficulty in this whole matter of support of end windings is not to have it too rigid, but a little flexible, just to take the blows that Mr. Loizeaux points out occur, due to short circuits, improper phasing, and everything of that kind. I agree with Mr. Loizeaux that they should be very effectively supported, so that they can stand in the best manner the strains that they will meet in commercial service.

Another point brought out by Mr. Loizeaux was the different conditions that may prevail. That is a thing in regard to which the manufacturers are now having some contention with customers' engineers, some of whom are reaching the point of specifying, or asking the manufacturer to guarantee, that a generator if thrown in 180 deg. or 90 deg. out of phase, anywhere between proper phase-in and the worst phase-in, will be good for that kind of operation, under the conditions that arise in the power houses. Furthermore, they expect that guaranty to apply for a considerable period, possibly ten or twenty years.

We must remember that guaranties of that kind, to be reasonable, should be defined and limited, such, for instance, as that a generator will stand short circuits, at normal voltage, or maybe ten or twenty per cent above voltage. The manufacturer can agree to a reasonable guaranty of that kind, and have the test applied.

Now, with regard to the little table of temperature drop, about which Mr. Newbury asked, page 961, he has pointed out, there are probably two factors in there that involve uncertainties. He points out a statement which has been made by me, as to the fact that temperature drops in different types of material have been determined and are well-known, I would say that, in this particular case the temperature drop has been determined from the actual armature coils, but not in the machines.

F. D. Newbury: Do you excite the coils with direct current or alternating current?

W. J. Foster: Direct current. The only factor I am getting at is the temperature drop in the insulation. When it comes to the factor of how much heat there is there to get out, that is really where the uncertainty comes in, and in this case, the 20,000 kv-a. turbo generator of 11,000 volts, the allowance for eddy current losses was based on the famous classical paper of A. B. Field, with such modifications as were necessary. It is practically his formula but slightly modified, such modifications being based on experience.

The thickness of the insulation was another thing that Mr. Newbury asked for, and the character of the detector. The thickness of the insulation in that particular case is a trifle over 0.1 inch.

Regarding Mr. Newbury's further point as to whether the temperatures have been checked up by the drop through the windings as a whole. I wish to state that I have always regarded the determination of temperatures by that method as a good one. The Institute has never seen fit to standardize it, but I think it is a very resonable request for the buyer to make, that the temperature shall be determined by the drop in resistance at the end of heat run as well as by the temperature detectors. In the case of machines of the size for which these data are given, we have always found that that temperature rise was less than that revealed by the temperature coil, but I do not have the data on the largest generators, for the reason that we have never operated these in the testing department at home, and it is very difficult to obtain a machine outside for testing purposes.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29-July 2, 1920.

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SOME PRACTICAL EXPERIENCE WITH EMBEDDED TEMPERATURE DETECTORS IN LARGE GENERATORS

F. D. NEWBURY AND C. J. FECHHEIMER
Both of the Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

WE have now had some six years' experience with the built-in detector method of temperature measurement as applied to large a-c. generators and have accumulated experience that seems to warrant reconsideration of the Standards Rules relating to this subject.

When this method was first proposed and discussed in 1913 and 1914, it was believed that it provided a way by which the temperature of the copper—inside the insulation—could be very closely approximated by a device installed outside the insulation. When the detector—be it thermocouple or resistance coil—is properly installed, it does measure a temperature much nearer the inside copper temperature than does any other established method, but if the detector is not properly installed, even though located between coil sides, it will give considerably lower results.

Designing and operating engineers are interested in armature coil temperatures from two different standpoints; they are interested in the effects of heat on the insulation and from this standpoint are concerned that the temperature at no point in the entire winding shall exceed the safe operating temperature of the insulation located at that point of highest temperature; they are also interested in the linear expansion of the straight part of the armature coil due to temperature changes and, from this standpoint, they are interested in the *average* temperature of the entire length of the coil located in the armature core-slots.

The ideal measuring device from the standpoint of the chemical effect of heat on insulation is a detector, such as the thermocouple, that will measure local temperatures; although, obviously such a device, to be of service, must be located where the highest temperatures are to be found. The best device to gage the effect of heat in causing the winding to "creep" and chafe the insulation is a device such as a long resistance coil, that will average the temperature along the length of the armature coil.

In this paper some errors of measurement with the embedded temperature detector are discussed. Some pertain to details concerned with proper installation of the detectors, and while they, at first glance, may appear to be unimportant details, they are, according to our experience, of such importance as to warrant very careful study.

A discussion of errors of measurement requires that some standard of reference be established. In connection with the present subject the only satisfactory reference temperature is the temperature of the copper inside the insulation as obtained by direct measurement. In general, the statements made in this paper are based on tests in which such actual copper temperatures have been measured by thermocouples placed in contact with the copper, the leads of the thermocouples being carried through the armature coil insulation. These inside temperatures can be measured accurately and can be checked consistently without difficulty. Practically all the factors that lead to errors when temperature detectors are located on the outer surface of the insulation disappear when a flat ribbon thermocouple is placed in metal-to-metal contact with the copper conductor.

When a detector is placed between the two coil sides in a slot, the detector measures the actual temperature inside the insulation providing:

(1) that there is no flow of heat from the adjacent sides of the copper in the upper and lower coils to the slot sides; and

(2) that there is no difference in temperature between the upper and lower coil sides. In practically all machines both are present, to a greater or less extent. These two effects upon the reading of the detector are important, and therefore will be discussed.

FLOW OF HEAT TO THE SLOT SIDES

Recent measurements of thermal conductivities* show that the longitudinal conductivity of insulating material is considerably greater than the transverse, approximately 1.5 to 4 times. When built up in the usual manner around a coil the transverse conductivity is considerably decreased so that the longitudinal conductivity may be 3 to 10 times the transverse.

In Fig. 1 is shown a portion of a slot with the usual two coil sides and the approximate lines of heat flow as well as the isothermal (equipotential) lines. As drawn, the assumption was made that the insulating material was isotropic, not of greater conductivity in one direction than the other. It is evident that the path of heat flow from the portion of the copper under consideration is in part transverse to, and in part longitudinal with, the insulation. In accordance with this figure, there is heat flow from the adjacent sides of the copper in the two coils to the slot sides, which heat flow is, in the actual case, augmented by the relatively high longitudinal thermal conductivity. The particular isothermal line *abc* is at that temperature which is recorded by the detector at *b*, and the drop in temperature from the copper to the detector is approximately $\frac{(ga)}{(gf)} \times$ temperature difference between copper and iron. The actual case is too complicated to admit of accurate mathematical analysis, but an idea of the difference between the copper and detector temperatures can be gained from

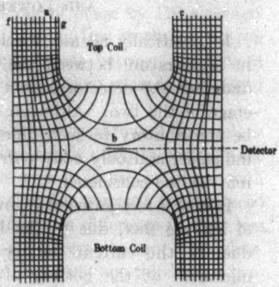

FIG. 1

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*"The Thermal Conductivity of Insulating and Other Materials" by T. S. Taylor, Electric Journal, Dec., 1919.

data on the 12,000-kv-a. alternator and small model contained in this paper.

The importance of this factor depends largely on the distance between copper conductors in the upper and lower coil sides. It may have little effect in low voltage windings but may be of considerable importance in 11,000- to 13,000-volt generators in which the copper of the two coil sides is separated by at least 0.5 inch of insulation.

DIFFERENCE IN TEMPERATURE BETWEEN THE UPPER AND LOWER COILS

In practically all a-c. machines there is a difference in temperature between the copper in the two coils, and then the detector cannot read higher than the average of the two. Although a number of factors may be contributory to such difference, most are of minor influence, and only eddy currents due to the load current will be considered.

It is well recognized that, owing to the rate of change of leakage flux, due to the load currents in the conductors, the current density in the conductors is a minimum at the bottom of the lower coil, and increases with increasing depth of conductor. Thus, the loss in the upper coil is greater than in the lower coil, and consequently the temperature of the upper coil is usually higher than the lower. A striking example of this was shown by the tests on the original Niagara generators described in a paper five years ago.* Those tests showed that with a load of 980 amperes, the maximum temperature of the top coil was 224 degrees, of the bottom coil 168 degrees (the average of which was 196 degrees), whereas the couple between coils showed 185 degrees. Thus, the difference between the top coil temperature and that given by the couple between coils was 39 degrees. While this is admittedly an extreme case due to the use of solid bars in a two-layer winding, eddy current conditions, even when all possible precautions are taken, are an important factor of design in the larger 60-cycle generators of the present day.

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*"Experimental Data Concerning the Safe Operating Temperature for Mica Armature—Coil Insulation." F. D. Newbury, A. I. E. E. Vol. XXXIV, p. 2747.

The eddy current loss in the top coil is influenced by the throw of the coils. Thus, if there are three phases with the usual six-phase grouping, and the throw of the coils is two-thirds of full pitch, the load currents in the upper and lower coils are displaced from each other by 60 electrical degrees. The density of the leakage flux interlinking the upper coil is reduced by the displacement and the eddy loss is lowered thereby. If the throw of the coils is one-third of full pitch with three-phase winding (six-phase grouping), the currents in the upper and lower coils are out of phase by 120 electrical degrees. The reduction in eddy loss in the upper coil is greater when the displacement between currents is changed from 60 to 120 degrees than when it is changed from 0 to 60 degrees.

If in a three-phase machine the throw of the coils is between full pitch and two-thirds pitch in some of the slots the currents in the two coils are in phase with each other, and in other slots the currents are displaced by 60 degrees. Hence, there is a difference in eddy loss in the top coil in some slots from that in other slots. Similarly, with a throw between one-third and two-thirds, the eddy loss differs in some slots from that which obtains in other slots.

In a two-phase machine with a throw of coils between one-half and full pitch, some slots have coils in which the currents are in phase and other slots have coils in which the currents are displaced by 90 degrees. If the throw is less than 90 degrees, in some slots the currents are in phase opposition, which gives the minimum loss.

In a three-phase, two-pole, 60-cycle, turbo generator that has been in service for about six years, with fairly deep conductors in the coils, there are 36 slots, and the throw of the coils is from slot No. 1 to slot No. 12. Thus, the throw is between one-third and two-thirds of full pitch. Tests made showed that with 60 electrical degrees phase displacement the temperature rise by thermocouple between coils was 63 degrees cent., whereas with 120 degrees phase difference, the temperature rise was 56 degrees; a difference of seven degrees.* It is therefore evident that the temperature detectors should be located in slots in which there is minimum phase difference between the currents in the upper and lower coils.

PROTECTION OF DETECTOR FROM THE COOLING AIR

With the usual two coil per slot arrangement there are two methods of wrapping the mechanically protecting slot cell around the coils, viz.:

(1) To place these cells on coils individually as shown in Fig. 2, or,

(2) To use a single slot cell to wrap both coils as indicated in Fig. 3.

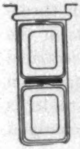
FIG. 2

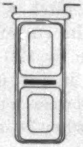
FIG. 3

With the usual radial scheme of ventilation (that is, with air ducts at intervals axially), it would seem that there is likelihood of the cool air passing between coils and cooling the detector below the temperature that would otherwise be read. This cooling air would not, however, have an appreciable effect upon the temperature of the coils, because the percentage increase in surface exposed to the cooling air is negligible. Tests made upon a 12,000-kv-a. alternator and more especially upon the model, results of which are given in this paper, bear out this supposition.

TESTS ON A 12,000-KV-A., 6600-VOLT, THREE-PHASE, 60-CYCLE, 150-REV. PER MIN., VERTICAL ALTERNATOR

The stator of the alternator had the following dimensions:

Internal diameter = 192 inches
External = 209 inches
Core width = 33 inches
Number and size radial vents = 13 − ½ inches
Number and size of slots = 324 − 0.79 in. x 3.45 in.
Number of conductors per slot = 4

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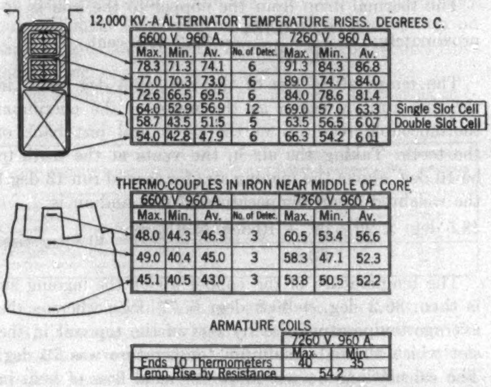
*The temperatures were measured by means of thermocouples between coil sides, not on the bare copper.

Size of conductor =4 (0.162 in.x0.25 in. bare)
4 (0.129 in.x0.21 in. asbestos covered)
Section of conductor =0.264 sq. in.
For arrangement of conductor see Fig. 4.
Connection, two-circuit star.
Throw of coils 1 and 7.

In order to make tests with thermocouples on the bare copper, these couples were installed in coils near the neutral point, and the neutral was grounded during the tests. In addition to these couples, the two-cell, and the one-cell arrangements were used—two cells from the center line of the machine to the right, and one cell to the left. The thermocouples were placed in the packages of iron adjacent to the central vent. Thermocouples were also embedded in the iron, as shown in Fig. 4, they being axially at the middle of the package to the right of the central vent.



12,000 KV.-A ALTERNATOR TEMPERATURE RISES. DEGREES C.

6600 V. 960 A.				7260 V. 960 A.			
Max.	Min.	Av.	No. of Detec.	Max.	Min.	Av.	
78.3	71.3	74.1	6	91.3	84.3	86.8	
77.0	70.3	73.0	6	89.0	74.7	84.0	
72.6	66.5	69.5	6	84.0	78.6	81.4	
64.0	52.9	56.9	12	69.0	57.0	63.3	Single Slot Cell
58.7	43.5	51.5	5	63.5	56.5	60.7	Double Slot Cell
54.0	42.8	47.9	6	66.3	54.2	60.1	

THERMO-COUPLES IN IRON NEAR MIDDLE OF CORE

6600 V. 960 A.				7260 V. 960 A.		
Max.	Min.	Av.	No. of Detec.	Max.	Min.	Av.
48.0	44.3	46.3	3	60.5	53.4	56.6
49.0	40.4	45.0	3	58.3	47.1	52.3
45.1	40.5	43.0	3	53.8	50.5	52.2

ARMATURE COILS

	7260 V. 960 A.	
	Max.	Min
Ends 10 Thermometers	40	35
Temp. Rise by Resistance	54.2	

FIG. 4

Two heat runs were made: the first at 6600 volts, 960 amperes per phase until constant; then, without shutting down, the voltage was raised to 7260, the current being kept at 960 amperes. Both runs were made at nearly zero power factor, lagging current.

The summary of these heat runs is given in Fig. 4. Before drawing conclusions it will be interesting to estimate the temperature drops through the insulation and to compare these calculated results with the test.

The ingoing air temperature was 31 deg., so that for the first heat run the total temperature of the top coil was about 107 deg. total. The calculated eddy current loss for the top coil is 48 per cent of I^2R loss.* The I^2R loss at 0 deg. cent. with 960 amperes per inch of length per coil is 1.09 watts. The average surface (taken as bare copper surface plus three times the distance from copper to iron) is 3.31 sq. in., the thickness of the single wall = distance copper to iron is 0.154 inch; the thermal conductivity of the insulation we shall take as 0.003 watts per inch cube per degree cent. Then the watts per square inch at 107 deg. =

$$\frac{(1 + 0.00427 \times 107)\ 1.48 \times 1.09}{3.31} = 0.707$$

The thermal drop from the copper to the iron is approximately: $\dfrac{0.707 \times 0.154}{0.003} = 36.2$ deg. cent.

The temperature rise of the iron 48.5 deg. cent. in the first run, is taken as the mean of the maximum thermocouple readings in the teeth and just back of the teeth. Taking the air in the vents at the teeth to be 10 deg. above the ingoing air (for second run 13 deg.) the weighted mean temperature of iron and air is:

$$\frac{48.5 \text{ deg.} \times 26\tfrac{1}{2} \text{ in.} + 10 \text{ deg.} \times 6\tfrac{1}{2} \text{ in.}}{33 \text{ in.}} = 40.8 \text{ deg. rise.}$$

The temperature of the copper above the ingoing air is then: 36.2 deg. + 40.8 deg. = 77 deg.; whereas the average temperature rise by test of the top coil in the slot which showed maximum temperature was 76 deg. The calculations do not allow for axial flow of heat in the copper to the ends, but the decrease in temperature at the middle due to longitudinal flow in this machine is not great.†

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*See paper by R. E. Gilman on Eddy Currents, page 977.

†This subject of relative longitudinal and transverse heat flow has been investigated mathematically by one of the authors and a paper on this subject will be submitted to the Institute in the near future.

Summarizing the calculations we obtain Table I.

TABLE I

	6600 Volts, 960 Amperes	7260 Volts, 960 Amperes
Weighted mean of tooth and air temp.........	40.8	50.4
Thermal drop through insulation............	36.2	37.1
Calculated copper temp. rise = sum of above..	77.	87.5
Average temp. rise of the coil in slot having max. temp. (test)......................	76.	88.1

This close agreement between the calculated and observed drop in temperature through the insulation under two different load conditions indicates the overall consistency and reasonableness of the test results and permits us to draw conclusions from them with added confidence. Attention is especially called to fact that the drop through the insulation cannot be altered (except for slight change in loss due to resistance being modified by change in total temperature) by change in ventilation. The only way the heat can be dissipated from the copper is by conduction through the insulation; some of the heat may flow longitudinally but still must be conducted through the insulation on the coil ends and then be liberated by convection from the surface.

The readings may be summarized still further as in

TABLE II

		6600 Volts, 960 Amperes Degrees	7260 Volts, 960 Amperes Degrees
Average temp. rise of top coil in slot having max. temp...............................		76.	88.1
Couples between coils	Max. temp. rise single slot cell..	64.	69.
	Max. temp. rise double slot cell.	58.7	63.5
Average temp. rise bot. of slot..............		47.9	60.1
Average temp. rise teeth....................		46.3	56.6

From Table II it will be seen that:

(1) There is a very material difference between the true copper temperature and the temperature as measured by means of detectors between coil sides; 12 deg. and 19 deg. for the 6600- and 7260-volt heat runs respectively. These differences are due to a combination of effects such as heat flow to the sides and difference in temperature between upper and lower coils. Expressed as a percentage of the total drop in temperature from the copper to the weighted mean of iron and air (76 deg.

– 40.8 deg. for first run), these differences are 34 per cent for the first and 51 per cent for the second heat run.

(2) The temperature reading is lower with the two-cell arrangement than with one cell. This amounts to 5.3 deg. for the first and 5.5 deg. for the second heat run. The differences between the maximum temperature on the copper and the maximum temperature by detector with two cells expressed as percentages of the total drop are 49 and 65 for the first and second runs respectively. The various figures are put in more convenient form in Table III.

TABLE III

	6600-Volt Run.		7260-Volt Run.	
	Degrees	Per Cent Total Drop in Insulation	Degrees	Per Cent Total Drop in Insulation
Difference between max. copper temp. and max. detector reading with one cell........	12.	34	19.1	51
Difference between max. copper temp. and max. detector reading with two cells.......	17.3	49	24.6	65

(3) There is a considerably greater difference between maximum and minimum readings with detectors exposed to cooling air as in the two-cell arrangement than with detectors in contact with copper or with detectors between coils but protected from air currents. This is shown by the following figures from Fig. 4.

	Max.	Min.	Ratio
Couples on copper......................	78.3	71.3	0.91
Couples bet. coils one-slot cell............	64.	52.9	0.83
Couples " " two-slot cell............	58.7	43.5	0.74

This is a matter of considerable importance as it is a measure of the reliability of the readings by the several methods. The greater variation in the case of the two-slot cell arrangement is possibly due to the greater effect of poor contact between the coil surfaces and the detector when the detector is exposed to air currents.

(4) A comparison of the average temperature rise by couple at the bottom of the slot and the average temperature rise of the teeth, shows that the former is not much higher than the latter. The thermocouple at the bottom of the slot measures the iron temperature at that spot and cannot read the copper temperature.

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The difference between the two readings is quite small, but that can be readily understood when one considers that there is considerable flow of heat from the coil, which must be accompanied by a thermal drop in the iron; the longitudinal thermal conductivity of the iron is only about one-ninth that of copper.

Further conclusions may be drawn by reference to Fig. 4.

(5) The temperatures by thermometers on the end windings give practically no information in regard to the maximum copper temperatures.

(6) The temperature rise by resistance is at best, the average resistance of the complete winding and therefore conveys no information in regard to maximum temperature, and little information as regards the average temperature in the slots.

TESTS WITH ARMATURE MODEL

In Fig. 5 are shown the details of a model designed to reproduce the temperature conditions in a radially ventilated armature core from which data of interest can be obtained more conveniently than in the actual generator. This model consists of sheet-steel laminations having two armature slots and built up into a core of four packages with ½-in. air ducts between. Two flat coils (connected in series) wound with small wire are located in the two slots. A second sheet steel core, one inch thick, is placed over the slots to complete the magnetic circuit. The coils were insulated with mica of a thickness sufficient for a 6600-volt generator.

In Fig. 6 are shown elevations of the model when placed in a wooden box with a small Sirocco fan used for driving air through the vent-ducts.

In Fig. 5, C_1, C_2, etc., I_1, I_2, etc., all refer to thermocouples, the "C" couples being placed between coils, thus measuring coil temperatures, and the "I" couples being imbedded in the iron. R_1, R_2, R_1', R_2', are resistance exploring coils. Couple C_2 is placed within R_1; C_2' is within R_1'; C_4 is within R_2; and C_4' is within R_2'. C_1, C_5, and C_5' are above the resistance exploring coils. As shown R_1 and R_1' are substantially the entire width of the slot; R_2 and R_2' are each about 9/16 in. wide. In

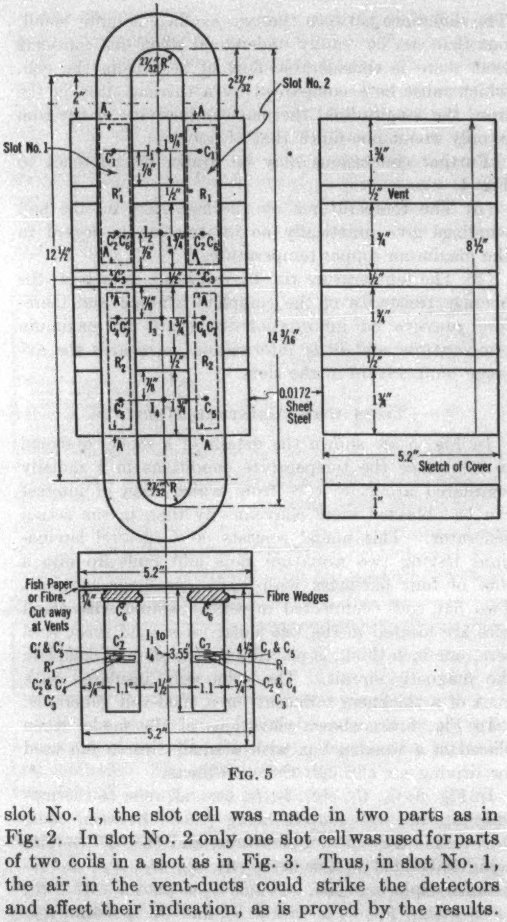
2 23/32" R
2 23/32"
Slot No. 2
2"
A
Slot No. 1
C'_1
I_1
1 3/4"
C_1
7/8"
R'_1
1/2"
R_1
C'_2 C'_6
I_2
C_2 C_6
12 1/2"
C'_3
C_3
C'_4 C'_7
I_3
C_4 C_7
R'_2
R_2
C'_5
I_4
C_5
14 3/16"
0.0172 Sheet Steel
2 23/32 R
1/2" Vent
8 1/2"
5.2"
Sketch of Cover
1"
Fish Paper or Fibre. Cut away at Vents
1/4"
Fibre Wedges
C'_6
C_6
C'_2
I_1 to I_4
C_2
3.55"
4 1/2
C'_1 & C'_5
C_1 & C_5
R'_1
R_1
C'_2 & C'_4
C_2 & C_4
C'_3
C_3
3/4"
1.1"
1 1/2"
1.1
3/4

FIG. 5

slot No. 1, the slot cell was made in two parts as in Fig. 2. In slot No. 2 only one slot cell was used for parts of two coils in a slot as in Fig. 3. Thus, in slot No. 1, the air in the vent-ducts could strike the detectors and affect their indication, as is proved by the results. In order to show that there was no appreciable difference between the two slots, data are also included taken with all space closed between coils (litharge and glycerine was used). In Table IV the data from heat runs with 20 amperes 60 cycles are given.

TABLE IV

TEMPERATURE RISES, DEGREES CENT.

Test with Alternating Current, 20 Amperes, 60 Cycles,
Approximately 374 Volts

Amperes per sq. inch = 1870.
Approx. tooth density = 92,000 lines per sq. inch.
" core " = 73,000 " " " "
" gap " = 51,200 " " " "
Eddy current loss top coil = 0.39 x I^2R.

Without Packing Between Coils		With Packing Between Coils	
Two-Slot Cells	One-Slot Cell	Two-Slot Cells	One-Slol Cell
Slot No. 1	Slot No. 2	Slot No. 1	Slot No. 2
C1′ = 30.6	C1 = 44.6	C1′ = 47.5	C1 = 43.5
C2′ = 34.5	C2 = 45.2	C2′ = 43.	C2 = 39.5
C3′ = 38.6	C3 = 48.2	C3′ = 43.	C3 = 41.0
C4′ = 34.6	C4 = 50.4	C4′ = 46.5	C4 = 44.0
C5′ = 32.	C5 = 48.5	C5′ = 45.5	C5 = 45.0
C6′ = 55.2	C6 = 54.5	C6′ = 54.5	C6 = 54.
C7′ = 58.	C7 = 59.6	C7′ = 58.5	C7 = 58.5
R1′ = 32.5	R1 = 45.	R1′ = 43.7	R1 = 41.
R2′ = 33.	R2 = 48.8	R2′ = 44.9	R2 = 42.
Entire winding by resis. = 54 deg.		53 deg.	
11 = 38.9	13 = 30.5	11 = 41.	13 = 30.
12 = 31.5	14 = 38.8	12 = 30.5	14 = 35.5

It will be noted that the temperatures in the slot with two cells is materially lower than with one cell, whether thermocouples or resistance coil method be used. Table V was made up in the same manner as Tables I and II; thermal conductivity was taken as 0.003 watts per inch^3 per deg. cent. The air tempera-

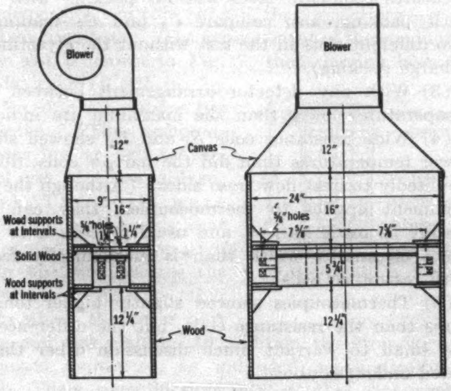
Blower
Blower
12"
Canvas
12"
9"
16"
24"
16"
Wood supports at Intervals
5/8" holes
1 1/4"
1 1/4"
5/8" holes
7 5/8"
7 5/8"
Solid Wood
5 3/4"
Wood supports at Intervals
12 1/4"
Wood
12 1/4"

Fig. 6

ture rise was taken as 3 deg. above ingoing air for figuring weighted mean of tooth and air temperatures.

TABLE V

Weighted mean of tooth and air temp		= 26
Thermal drop through insulation		= 29.5
Calculated copper temp. rise = sum of above =		= 55.5
Average temperature rise of copper top coil, test		= 56.6
Couples between coils	Max. temp. rise single slot cell	= 50.4
	Max. temp. rise double slot cell	= 38.6

Figuring in same way as for the 12,000-kv-a. alternator, and using 58.8 deg. for the internal temperatures (the average of the C_7 couples), Table VI is obtained:

TABLE VI

	Degrees	Per Cent of Total Drop in Insulation
Difference between copper temp. and max. detector reading with one cell	8.4	25.6
Difference between copper temp. and max. detector reading with two cells	20.2	61.5

From the tests on the model it will be noted that:

(1) The cooling effect of air which passes between coils with the two-cell arrangement very materially lowers the reading of the detector, whether thermocouples or resistance coils are used.

(2) The air which passes between coils with the two-cell arrangement has no influence upon the internal temperatures (compare tests without packing with those with packing; also compare C_6 and C_7 readings in two different slots in the test without the glycerine and litharge packing).

(3) With any detector arrangement between coils, temperatures lower than the maximum are indicated.

(4) Wide resistance coils R_1 and R_1' showed slightly lower temperatures than did the narrow coils, due undoubtedly to heat flow from sides. (Although the same argument applies to thermocouples, they can more readily be made narrow, and usual practise is to make them of smaller width than is mechanically feasible with resistance coils).

(5) Thermocouples showed slightly higher temperatures than the resistance coils, but the difference was too small to warrant much discussion other than as noted under (4) above.

The fact that the temperature rise by resistance agreed fairly well with the average of the thermocouples inside the top coil may be accounted for by:

(1) The ventilation of the ends was very poor (air was allowed to pass through ⅝-inch hole) and therefore its temperature was probably higher than the slot portion, thus raising the average temperature; and

(2) Very little time was lost between shut-down and taking of reading of resistance. Neither of these conditions can be obtained in a large alternator—the ends are very effectively cooled, and there is a considerable lapse of time after shutting down before readings can be taken. The latter implies that in a machine, the average temperature is higher than can be measured by resistance.

Errors in Determination of Temperatures by Placing Detectors Between Coil and Iron or Under Wedge

Previous reference to errors by this method has been made. A test on another machine will be given to assist in showing the futility of attempting to measure the copper temperature with detectors, say, between bottom of coil and slot, and between coil and wedge.

A 3750-kv-a., 480-volt, three-phase, 60-cycle, 3600-rev. per min. turbo generator was wound with one coil side per slot, one turn per coil, the connections having been two-circuit star. The arrangement of the conductor in the slot is shown in Fig. 7, there having been 72 strands, each (0.102 in. by 0.205 in. d. c. c.) ribbons per conductor. (The temperature was low enough to permit of use of Class A insulation.) The coils were turned over at the ends; that is, a strand at the top of a coil in one slot became the bottom strand in the other slot in which the other part of the coil was placed. In addition the strands were transposed depthwise, as indicated schematically in Fig. 8. There were 36 slots total, so that the number

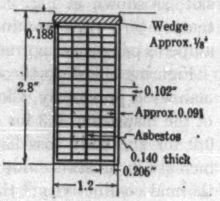

Fig. 7

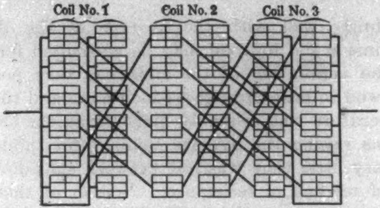

FIG. 8

of coil-sides per pole per phase per circuit was three. The three coils in such a phase group formed a group for transposition. Thus, starting at the beginning of a phase group, all the strands were joined together at one end of the coil, but at the other end the 72 strands were arranged in 12 groups, of six strands each. The top left group at the beginning of coil No. 1 becomes the bottom left group at the end of coil No. 1. To simplify the sketch in Fig. 8, the left sides of the 12 groups of coil No. 1 are shown joined in parallel; these are the beginnings of the coil group. At the other end of coil No. 1, the position of the groups are actually inverted, but they are not so shown. Then, the top left group which is actually the bottom left in coil No. 1 is joined to the third left group in coil No. 2; and the other end of that group is joined to the fifth left group in coil No. 3, etc.

Thermocouples were placed at the middle of the straight parts of the coil, before the coils were insulated, and couples were also placed at the bottom and top of slot, as shown in Fig. 9. In the same figure the final temperature rises are indicated for heat run at 4100 amperes per phase, normal voltage, zero power factor.

Figuring the drop through the insulation in the same manner as previously, allowing 0.091 in. for the thickness of the insulation; 0.3 for the watts per sq. in. at 0 deg.; 0.2 for the eddy loss as fraction of I^2R; 1.32 for the increase in resistance due to temperature; 0.0025 for the thermal conductivity;* the drop is:

$$\frac{1.2 \times 1.32 \times 0.091 \times 0.3}{0.0025} = 17.3 \text{ degrees.}$$

---

*The lower value of conductivity was chosen at the lower voltage because of the greater relative influence of air pockets with the thinner insulation.

The weighted mean of iron (34 deg.) and air (say 10 deg.) (for relative lengths of 3.33 to 1) is 28.4 deg. The mean of the three temperatures inside the coil is 47 deg., and the drop is (47 – 28.4 =) 18.6 deg. as compared with 17.3 deg. calculated.

It will be noted that:

(1) The detector at the bottom of the slot or under the wedge does not indicate the temperature of the copper.

(2) That the maximum temperature of the copper is probably not at either the top or at the bottom of the coil.

(3) The cool currents of air above the wedge may cause the detector below it to read lower temperature than one at the bottom of the slot, even though the copper at the top is at higher temperature.

As a matter of fact, as previously pointed out in commenting upon the tests on the 12,000-kv-a. alternator, the detector at the bottom of the slot reads the temperature of the iron with which it is in contact; the detector placed below the wedge reads the temperature of that portion of the wedge, and neither can take into account the drop through the insulation. The A. I. E. E. Standardization Rule number 356 allows for a correction factor of 10 deg. cent. plus 1 deg. cent. per 1000 volts above 5000. Such correction factor is necessarily more or less meaningless.

Thus, the tests and calculations on the 12,000-kv-a. alternator showed that the drop through the insulation was approximately 36 degrees with 91 per cent load or about 43 degrees with full load. With high voltages, especially in long-core 60-cycle machines of large output, the drop is necessarily greater, due to the heavier wall of insulation; in such machines the drop is of the order of 50 degrees cent.

In our opinion it is not possible to decide upon correction factors which are applicable, even for comparison, with the one coil per slot arrangement. In point of fact, machines built at the present time in large or moderate sizes (in which detectors are incorporated), have with very few exceptions, two coil-sides per slot. Therefore, it is our opinion that the method of measuring copper temperatures by means of detectors at the bottom of the slot or under the wedge, should not be recognized in the Standardization Rules.

LOCATION OF THE HIGHEST COPPER TEMPERATURE

It will be seen from an inspection of Fig. 4 that the temperatures at the top of the upper coil are slightly higher than at the middle of the upper coil, but that there is a larger difference in temperature between the middle and the bottom of the coil than between the top and the middle of the coil. In the model (see Table III) the temperatures at the bottom of the upper coil are higher than at the top. In the 3750-k.v.-a. generator (see Fig. 9) the temperature at the side was substantially the same as at the top (test showed 1 deg. higher at the side), but the temperature of the copper at the bottom of the coil was appreciably lower than at the side. In all three cases, the effect of eddy currents was to increase the loss toward the top of the coil; but the fact that the temperature at the top was lower than might have been anticipated was in every case because the thermal conductivity of the compact wedge was relatively high, and with the effective cooling of the outside of the wedge, the temperature at the inside was lowered. (See Fig. 9 and note the reduced temperature below wedge.)

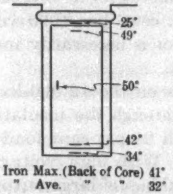

FIG. 9

The fact of the matter is that, if the temperature of the bare copper is to be measured, detectors should be located at various depths within the upper coil, and the maximum temperature taken to be the maximum copper temperature. If this is done in a number of coils in the machine and if the detectors are so located axially that their positions correspond to the axial position of maximum core temperature (to be determined by test on similar machines before manufacturing the coils) the maximum copper temperature will be measured with possibly a very small percentage error.

SUMMARY, CONCLUSIONS AND REMARKS

The points brought out in this paper may be summarized as follows:

(1) With detectors placed between coil sides, the difference in eddy currents in the upper and lower coils causes the detectors to read lower than the maximum.

(2) With detectors placed between coil sides, the flow of heat to the slot sides causes the detectors to read lower than the maximum.

(3) With detectors placed between coil sides, the phase difference in currents in the particular slot influences the reading, and the detectors should be located in slots in which there is minimum phase difference.

(4) With detectors placed between coil sides the possible circulation of air about the detector may have a material influence upon the reading of the detector. The slot cell should be so arranged that air currents cannot influence the reading of the detector. (See Fig. 3.) The single slot-cell provides excellent protection as the detector is then located in dead-air space.

(5) The air currents which lower the reading of the detector with the two-cell arrangement have an inappreciable influence upon the internal copper temperature.

(6) All heat from the copper must flow transversely through the insulation.

(7) The thermal drop through the insulation in large, long-core, high-voltage 60-cycle machines is of the order of 50 deg. cent.

(8) The thermal drop through the insulation cannot be appreciably lowered—at least in long-core machines—by improvement in ventilation.

(9) With any arrangement of detectors between coil sides, temperatures lower than the maximum are read, with the possible and rare exception of copper temperatures equal to or lower than the iron temperatures.

(10) With wide detectors between coil sides lower readings are obtained than with narrow detectors. The detectors should be made as narrow as is mechanically feasible.

(11) With detectors placed on the bare copper, the maximum temperature is not necessarily read at the top of the upper coil.

(12) A detector placed between coil side and iron or between coil side and wedge reads only the iron or wedge temperature at the particular spot; the reading gives no information in regard to the thermal drop through the insulation and the actual copper temperature.

(13) Thermometers placed on the end windings give no information in regard to the highest copper temperature in large high-voltage generators.

(14) Resistance measurements taken after completion of the heat run, convey no information in regard to maximum temperature, and but little information in regard to the average temperature in the slots.

(15) The machine usually cools too much after shutdown and before taking readings to obtain accurate data on the average temperature of the winding.

Whereas the only known way to measure the maximum copper temperature is by means of detectors on the bare copper, that method is not usually commercial, and therefore we must continue to employ detectors between coil sides. If a one-cell arrangement is used (Fig. 3), the detector if fairly narrow reads temperatures which, while they are slightly lower than the average of the top and bottom coil temperatures, afford a basis of comparison for the maximum temperature which is helpful to the purchaser and operator in judging on the one hand the value of the points of superiority, and on the other hand whether or not the machine is being operated at too high load. It is true that the method cannot cover all discrepancies, it cannot, with any simple correction factors, allow for the variation in eddy current loss in the various proportions of machines; it cannot cover the thermal drop due to the heat flow to the sides of the slot; it cannot allow for the uncertainty of locating the hottest spot over which there was lengthy discussion in the Standardization Committee in 1914, but it is a means of judging, and the only means that we now have that is entirely commercial. Inasmuch as any correction factor would be of comparatively little significance, it would, in our opinion, be well to omit such factor entirely, it being understood that in any case the reading by detector is probably somewhat lower than the maximum copper temperature.

The machines built at the present time in large or moderate sizes in which detectors are incorporated, have with very few exceptions, parts of two coils per slot. An occasional machine is built in which part of only one coil per slot is used, but such machines are usually wound for a lower voltage than is possible with the two-coil winding.* Therefore, it is our opinion that the method of measuring maximum copper temperatures by means of detectors at the bottom of the slot or under the wedge, should be omitted from the Standardization Rules.

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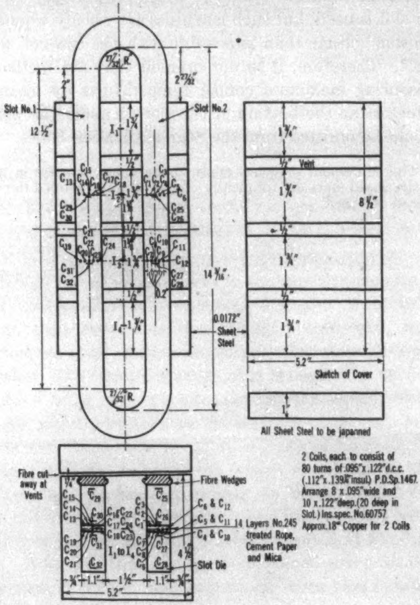
*This statement may not apply to European practise in which partly closed slots are frequently employed and "shoved-through" type of coil used.

DISCUSSION ON "SOME PRACTICAL EXPERIENCE WITH EMBEDDED TEMPERATURE DETECTORS IN LARGE GENERATORS" (FECHHEIMER AND NEWBURY), WHITE SULPHUR, W. VA., JUNE 30, 1920.

C. J. Fechheimer: In order that additional proofs might be secured of the influence upon the reading by detectors placed between coil sides of (a) heat flow to the slot sides, (b) width of detector, (c) difference in losses in the upper and lower coils due to eddy currents, an armature model similar to the one described in our paper was built, as shown in Fig. 1. The construction of this model differed from the previous one only in that additional insulation was placed between coil sides consisting of 14 layers, each layer 0.015 inch thick, of treated cement paper and mica, and in the arrangement of the thermocouples. Those indicated as C_3 to C_{24}

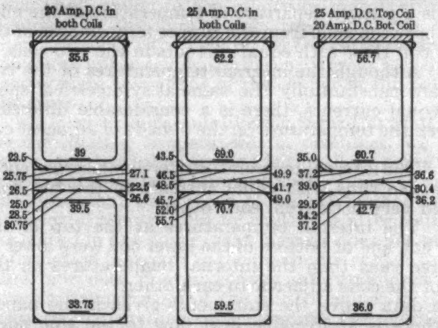


Slot No.1
Slot No.2
2"
2 27/32"
12 1/2"
1 3/4"
1/2"
Vent
8 1/2"
14 3/16"
0.2"
0.0172" Sheet Steel
5.2"
Sketch of Cover
1"
All Sheet Steel to be japanned
2 Coils, each to consist of 80 turns of .096"x .122" d.c.c. (.112"x .139 1/2" insul.) P.D.Sp.1467. Arrange 8 x .095" wide and 10 x .122" deep.(20 deep in Slot.) Ins.spec. No.60757 Approx.18# Copper for 2 Coils
Fibre cut-away at Vents
Fibre Wedges
C_6 & C_{12}
C_5 & C_{11}
C_4 & C_{10}
14 Layers No.245 treated Rope, Cement Paper and Mica
Slot Die
I_1 to I_4
4 1/2"
3/4"
1.1"
1 1/2"
1.1"
3/4"
5.2"

FIG. 1

inclusive were made of No. 27 B. & S. gage copper and advance wire; those inside the coils C_{25} to C_{32} inclusive were made of 0.005 in. by 0.25 in. copper and advance ribbons.

A series of tests is being conducted, and at present, we are prepared to give data on three. These tests were made with direct current, so that there was no eddy current loss; in order to initiate the greater loss in the upper coil which usually obtains when alternating current flows, different values of direct current were used in the two coils. It will be noted in Fig 1 that there are thermocouples in corresponding positions slots 1 and 2 and in Fig. 2 the averages of the temperature rises, for symmetrical points, above the ingoing air in degrees centigrade are recorded. In Fig 2 the temperatures are averaged from two thermocouple readings except those at the middle of the extra insulation between coils, the recorded temperature rises then being the average of four readings.


20 Amp.D.C. in both Coils
35.5
39
23.5
25.75
26.5
25.0
28.5
30.75
27.1
22.5
26.6
39.5
33.75
25 Amp.D.C. in both Coils
62.2
69.0
43.5
46.5
48.5
45.7
52.0
55.7
49.9
41.7
49.0
70.7
59.5
25 Amp.D.C. Top Coil
20 Amp.D.C. Bot. Coil
56.7
60.7
35.0
37.2
39.0
29.5
34.2
37.2
36.6
30.4
36.2
42.7
36.0

Rise by Resistance Top Coil 42.7° Bot. Coil 44.5° Rise of Iron End Packages 13° Middle Packages 10.5° Air Rise 2.5°.

Rise by Resistance Both Coils 79° Rise of Iron End Packages 24.5° Middle Packages 20° Air Rise 3.5°.

Rise by Resistance Top Coil 66.5° Bot. Coil 48.7° Rise of Iron End Packages 17° Middle Packages 13.7° Air Rise 2.5°.

Fig. 2

A study of the 20 ampere and 25 ampere heat runs will show that they are consistent with each other, if due allowance is made for the increase of the resistance of copper with temperature, and for the weighted mean of iron and air temperatures. The third heat run with different values of current in the two coils is also as consistent with the first two runs as may be expected with temperature tests, due allowance having been made for iron temperatures.

It will be noted that:

(1) There is a large difference in temperatuare between the inside and outside of the coil insulation.

(2) There is a comparatively small difference in temperature between the outside of the coil insulation at the center of the slot and the middle of the extra insulation.

(3) There is a marked temperature gradient from the center of the slot to the slot sides.

(4) With different values of current in the two coils there is a greater departure of temperature at the midpoint between coils from the copper temperature in the upper coil than with equal currents in the two coils.

(5) Although the internal temperatures of the two coils are substantially the same at symmetrical spots with equal currents, there is a considerable difference between the temperature on the outside of adjacent coil sides.

(6) The internal temperatures for symmetrical spots in the two runs with equal currents in the two coils differed very little from each other.

(7) The internal temperatures at the top of the upper coil and at bottom of the lower coil were lower in all three runs than the internal temperatures on the sides of the coils adjacent to each other.

The data prove the statements given in the paper, that there is appreciable heat flow to the slot sides; that the detector between coils does not read the true copper temperature; and that the width of the detector influences the reading thereof.

Conclusion (5) brings out the fact that there may be discrepancies of considerable magnitude as a result of unequal thermal conductivities in the insulating wall. Although there may have been minor factors which affected the temperatures at the outside of the coils, the difference in thermal conductivities, due perhaps to better surface contacts between layers of insulation on one coil side than the other, permitted greater rate of heat flow from the upper coil than from the lower. That the internal temperatures were only slightly influenced thereby, is probably because the rate of heat flow from the other sides were nearly equal in the two coils. The temperature rises of symmetrical points in the two slots also were different. Thus, in the 25-ampere run, the temperature rises at positions C_3 and C_{15} (Fig. 1) were 44 and 53 degrees respectively, an average of 48.5; and for C_9 and C_{21}, 57 and 54.5 degrees, an average of 55.7 degrees.

Although sufficient data are not at present available, if conclusion (6) were applicable generally, the temperatures on the copper could be taken in a smaller number of places in a machine than with the usual method with detectors between coil sides. Data in Fig. 4 of the paper also show that the percentage difference is smaller for internal differences than between coil sides.

Conclusion (7) should be considered in conjunction with that portion of the paper which relates to the "location of the highest copper temperature". It is again seen, that, owing to the relatively high thermal conductivity of the fibre wedge (the wedge was thinner than ordinarily) the rate of heat dissipation from the top of the upper coil was sufficiently great to reduce the temperature below that of the copper at the bottom of the same coil.

While we have various ideas in regard to what should be the limiting temperatures in machines, we cannot reach any decision as to the limit until we agree upon a method of measuring temperatures. Thus, it has been suggested that we go back to the old method of measurement of temperature by resistance after shut-down. As stated in the paper, that method does not give any information in regard to the maximum copper temperature, nor does it even convey information as to the average temperature of the buried copper in the slot; because that method only measures the average temperature of the entire winding after shut-down. This will be evident when we consider that the end windings are usually much cooler than the buried portions.

The time that elapses after shut-down, and before taking resistance readings, is usually sufficient to permit of a not inconsiderable reduction in average temperature.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 29–July 2, 1920.

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EDDY CURRENT LOSSES IN ARMATURE CONDUCTORS

By R. E. GILMAN
Westinghouse Electric Mfg. Co.

THE recent increase in the size of generators particularly of the turbo type has compelled a more complete investigation of the effect of stranded conductors in reducing eddy current losses. A reduction in losses has been necessary to secure safe temperatures inside the armature coil, since in larger units with few poles the ampere conductors per slot are necessarily large and the use of deep slots is forced upon the designer. Further the present practise of guaranteeing hot spot temperature or the safe heating temperature for the insulation, lends general interest to the subject and it seems advantageous for the engineering public to have more detailed information as to the magnitude of eddy current losses and as to some of the ways in which these losses can be controlled by the designer.

In looking up the literature on the subject of eddy currents in slot wound conductors, it appears that there is very little information of practical value in print.

One notable exception to this general statement is a paper, entitled "Eddy Currents in Large Slot Wound Conductors," by A. B. Field, presented at the 22nd Annual Convention of the A.I.E.E. in 1905. This paper covers only special applications of conductors. The limitations imposed in Mr. Field's paper are that the conductors are either solid or infinitely laminated and that the currents of all conductors in same slot are in phase.

In modern designs the stranding of conductors is limited to a finite number of strands depth-wise and there is often a large difference in loss between solid bars and infinitely standard bars of the same total cross section. So it becomes of interest to investigate the effect of, say, three or five strands. In the second place in large two-pole machines, for example, special strand groupings are sometimes desirable for mechanical reasons. And lately it is almost universal practise to use short-cord windings and therefore have conductors in the same slot carrying currents of different time phase. For these reasons the writer ventures to put forward the discussion which follows, as it may be of interest as additional data on a subject which has never been thoroughly discussed.

It is proposed to cover the following ground:

1. Develop formula for the calculation of the density in any point of a current-carrying wire (strand or conductor) provided the magnitude of the current in that wire and the total current in all the wires below it in same slot are known, together with the phase relation of these currents.

2. The calculation of this density will involve the evaluation of four constants of integration which are discussed in Part 2.

3. The development for the general case of the loss in a solid bar as compared to the same bar with uniformly distributed current.

4. The development of the loss ratio in an infinitely stranded conductor with various arrangement of groupings and the discussion of the effect of end windings in this case.

5. Approximation formula for the loss ratio of solid bars and the discussion of the limitation of its application.

6. Mathematical solution of current values in various strands for any number of strands and any arrangement of these strands. Development of formula for the equivalent loss ratio for a stranded conductor.

7. Examples of the application of the formulas to various grouping arrangements in the case of two strands.

8. Examples, same as in section 7, but using three strands.

9. Comparison of results in the case of solid bars,

with two strands, three strands and infinite strands, and the development of the law for any number of strands.

10. Test data covering the comparison of calculations and tests with various equivalent thicknesses of strands, various ratios of length of buried coil to free ends, and various number of strands per conductor from one to six.

In slot-wound conductors surrounded on three sides by iron there are two fluxes which produce eddy currents. The first flux is the one due to magnetomotive-force of the field and enters at the top of the slot adjacent to the gap. The strength of this field can be regulated by proper design proportions, such as overhanging tooth tips, sunk coils, or magnetic wedges and the use of low flux densities in the teeth. This source of eddy currents is not the subject of this paper and will not be further discussed. The second flux is that due to the m.m.f. of the currents in the coil itself. A little consideration will show that this second flux crosses the slot transversely from tooth to tooth through the body of the conductors and that it causes currents to flow along the top edge of the coil (air gap side) throughout the length of the iron and return along the bottom edge. The eddy currents themselves give an additional m.m.f. and react upon the system in such a way that the net result is a current density which is always of normal frequency, but varies in magnitude and phase as a function of the distance from the bottom of the conductor. In any plane that is parallel to the bottom of the conductor the magnitude and phase of the current are constant.

This is evident at once if we assume that the flux due to the armature current passes at right angles to the slot depth transversely across the slot. This assumption is made, also the further assumption that the reluctance in the iron circuit to this transverse flux is zero and that after a conductor reaches the end of the iron core that all induced fluxes link all the elements of any conductor equally and produce no e.m.fs. to cause eddy currents in the conductor. This latter condition is not strictly true.

1—CALCULATION OF DENSITY AT ANY POINT IN A CONDUCTOR

Let us consider a conductor carrying a current which may have any position in a slot with respect to other conductors which also carry currents.

Let Δ = The instantaneous current density at any distance x from the bottom of conductor

x = The distance from the bottom of conductor of Δ

f = The depth of conductor

a = The width of slot

r_1 = The ratio of copper width to the slot width

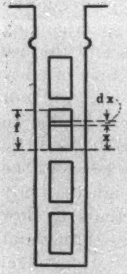
$a\,r_1$ = The copper width

ρ = The specific resistance of copper, which is taken as 0.625×10^{6} ohms per cu. inch at 0 deg. cent.

$\omega = 2\pi \sim$

$\sim$ = The frequency of current

V = The instantaneous value of the e.m.f. of self induction per unit of length produced at x by the transverse flux.



At the point x the e.m.f. of self induction is V; at a point $x + d\,x$ this e.m.f. is $V + dV$.

The current density at x is Δ, the current is $\Delta \times$ section, the resistance is $\frac{\rho}{\text{section}}$ per unit length. The resistance drop is $\Delta\,\rho$ at x and $(\Delta + d\,\Delta)\,\rho$ at $x + d\,x$. In the element of the conductor of width $a\,r_1$, depth $d\,x$ and length unity the four e.m.fs. given above form a closed circuit if the iron core is of unit length. If the core is of a length l then the factor l is multiplied into each value.

Since the sum of all e.m.fs. in a closed circuit must be zero we get at once

$$(V + d\,V) - V = (\Delta + d\,\Delta)\,\rho - \Delta\,\rho$$

or

$$\frac{d\,V}{d\,x} = \frac{d\,\Delta}{d\,x}\,\rho \qquad (1)$$

Let the total flux up to $x = \phi$
Then at $x + dx$ it is $\phi + d\phi$
If the induction per unit area at x is β then $d\phi = \beta\, dx$ and if t is time

Then $dV = \frac{d}{dt}\beta\, dx$ and $\frac{dV}{dx} = \frac{d\beta}{dt}$ and $\frac{d^2V}{dx^2} = \frac{d^2\beta}{dx\,dt}$

Further the current between x and $x + dx$ is $a\, r_1\, \Delta\, dx$ and produces an increment of flux density $d\beta$

Whence $d\beta = \frac{4\pi a r_1 \Delta\, dx}{10\, a}$ and $\frac{d\beta}{dx} = \frac{4\pi r_1 \Delta}{10}$

$\frac{d^2\beta}{dx\,dt} = \frac{d^2V}{dx^2} = \frac{4\pi r_1}{10}\ \frac{d\Delta}{dt}$ in absolute units

If we use volts, ohms, amperes, etc., and Δ is the density per unit area

Then $\frac{d^2V}{dx^2} = \frac{d^2\beta}{dx\,dt}10^{-8} = 4\pi r_1 \frac{d\Delta}{dt}10^{-9} = \frac{d^2\Delta}{dx^2}\rho$ **(2)**

Let $\alpha^2 = \frac{2\pi r_1}{\rho}\omega\, 10^{-9}$ and we get from **(2)**

$$\frac{\omega}{2}\frac{d^2\Delta}{dx^2} = \frac{\alpha^2\, d\Delta}{dt} \qquad \textbf{(3)}$$

This is the differential equation for Δ in terms of x and t as variables and the solution of this equation gives us Δ.

From the conditions of our problem we know that the frequency of Δ is the same as the frequency of the current in the conductor, also that Δ is of uniform value in any plane of the conductor which is parallel to the bottom of the slot.

Let Δ_m be the maximum value of the density at the point x under consideration, also let the instantaneous value of current in the conductor be $J \sin \omega t$ and let θ be the phase angle between J and Δ_m. Then $\Delta_m \sin(\omega t + \theta)$ is the instantaneous value of the density at our point x.

$\Delta_m \sin(\omega t + \theta) = \Delta_m \sin \omega t \cos \theta + \Delta_m \cos \omega t \sin \theta$ by expansion this can be rewritten $\Delta_m \sin(\omega t + \theta) =$
$A \sin \omega t + B \cos \omega t = \Delta$ **(4)**

A and B are functions of x and J and Δ is a function of x, J and t

From equations(**3**) and (**4**)

$$\left[\frac{d^2 A}{dx^2}\sin\omega t + \frac{d^2 B}{d x^2}\cos\omega t\right]\frac{\omega}{2} = \left[A\cos\omega t - B\sin\omega t\right]\alpha^2\omega$$

$$\left[\frac{d^2 A}{dx^2} + 2\alpha^2 B\right]\sin\omega t + \left[\frac{d^2 B}{dx^2} - 2\alpha^2 A\right]\cos\omega t = 0$$

and therefore $\frac{d^2 A}{dx^2} = -2\alpha^2 B$, $\frac{d^4 A}{dx^4} = -2\alpha^2\frac{d^2 B}{dx^2}$, $\frac{d^4 A}{dx^4} = -4\alpha^4 A$ $\frac{d^2 B}{dx^2} = 2\alpha^2 A$, $\frac{d^4 B}{dx^4} = 2\alpha^2\frac{d^2 A}{dx^2}$ and

$$\frac{d^4 B}{dx^4} = -4\alpha^4 B \qquad (5)$$

$\frac{d^4 A}{dx^4} = -4\alpha^4 A$ and $\frac{d^4 B}{dx^4} = -4\alpha^4 B$ are simultaneous equations of the fourth degree in x only and their solution involves four constants of integration for each equation. Since, however, the equations are simultaneous these constants for the two equations are related to each other so that it should be possible to get a general solution of the original equation (3) involving only four constants of integration. The two equations of (5) are linear in x with constant coefficients of the form

$$A_o\frac{d^n y}{d x^n} + A_1\frac{d^{-n1}y}{dx^{n-1}}\quad A_{n-1}\frac{d y}{d x} + A_n y = 0 \qquad (\mathbf{6})$$

Where A_o, A_1, etc., are constants. For y substitute ϵ^{mx}) where m is a constant to be evaluated. Since $\frac{d}{dx}\epsilon^{mx} = m\,\epsilon^{mx}$ and $\frac{d^2}{dx^2}\epsilon^{mx} = m^2\epsilon^{mx}$, etc., we get after rejecting the factor ϵ^{mx}.

$$\mathrm{A}_o m^n + A_1 m^{n-1} + - - - + A_{n-1} m + A_n = 0 \qquad (\mathbf{7})$$

and any value of m which satisfies(**7**) corresponds to some particular value of ϵ^{mx} which is an integral of (**6**). Therefore if m_1, m_2, m_3, etc., satisfy (**7**) then $y = c_1\epsilon^{m_1 x} + C_2\epsilon^{m_2 x} + etc.$, is the complete integral of (**6**).

From (**5**) we can write $D^4 + 4\alpha^4 A = 0$ if we put D for $\frac{d}{dx}$ and $A = C_1\epsilon^{m_1 x} + C_2\epsilon^{m_1 x} + etc.$, where m_1, m_2, m_3 and m_4 are the $\sqrt{2}\,\alpha$ multiplied by the four roots of

$\sqrt[4]{-1}$ or $\pm\left(\frac{1+j}{\sqrt{2}}\right)$ and $\pm\left(\frac{1-j}{\sqrt{2}}\right)$

$$A = \epsilon^{ax}\left[C_1\,\epsilon^{jx} + C_2\,\epsilon^{-jx}\right] + \epsilon^{-ax}\left[C_3\,\epsilon^{jx} + C_4\,\epsilon^{-jx}\right]$$

Let $C = a_1 + j\,a_2$ then $C_1 + C_2 = 2a$, $C_2 = a_1 - j\,a_2$ and $C_1 - C_2 = 2\,j\,a_2$. Hence we can write the solution of equations (5) as follows:

$$\left.\begin{aligned} A &= 2\,\epsilon^{ax}\left[a_1\cos\alpha\,x - a_2\sin\alpha\,x\right] + 2\,\epsilon^{-ax}\Big[a_3\cos \\ &\qquad \alpha\,x - a_4\sin\alpha\,x\Big] \\ B &= 2\,\epsilon^{ax}\left[b_1\cos\alpha\,x - b_2\sin\alpha\,x\right] + 2\,\epsilon^{-ax}\Big[b_3\cos \\ &\qquad \alpha\,x - b_4\sin\alpha\,x\Big] \end{aligned}\right\}\textbf{(9)}$$

But $\frac{d^2A}{dx^2} = -\,2\alpha^2 B$ and combining with equations (9)

$$\frac{dA}{dx} = \alpha\,\epsilon^{ax}\left[(2a_1 - 2a_2)\cos\alpha\,x - (2a_1 + 2a_2)\sin\alpha\,x\right]$$

$$-\,\alpha\,\epsilon^{-ax}\left[(2a_3 + 2a_4)\cos\alpha\,x + (2a_3 - 2a_4)\sin\alpha\,x\right]$$

$$\frac{d^2A}{dx^2} = 2\alpha^2\,\epsilon^{ax}\left[-\,2a_2\cos\alpha\,x - 2a_1\sin\alpha\,x\right] +$$

$$2\,\alpha^2\,\epsilon^{-ax}\left[2a_4\cos\alpha\,x + 2a_3\sin\alpha\,x\right] = -\,2\alpha^2 B \quad \text{———}$$

$$-2\alpha^2 B = -\,2\,\alpha^2\,\epsilon^{ax}\left[2\,b_1\cos\alpha\,x - 2\,b_2\sin\alpha\,x\right] -$$

$2\alpha^2\,\epsilon^{-ax}\left[\,2\,b_3\cos\alpha\,x - 2\,b_4\sin\alpha\,x\,\right]$ This reduces to $\epsilon^{ax}\left[-\,2\,a_2 + 2\,b_1\right]\cos\alpha\,x + \epsilon^{ax}\left[-\,2\,a_1 - 2\,b_2\right]\sin\alpha\,x$ $+\,\epsilon^{-ax}\left[2\,a_4 + 2\,b_3\right]\cos\alpha\,x + \epsilon^{-ax}\left[2\,a_3 - 2\,b_4\right]\sin\alpha\,x$ $= 0$ Separating terms in $\sin\alpha\,x$ and $\cos\alpha\,x$

$\epsilon^{ax}\left[-\,2\,a_2 + 2\,b_1\right] + e^{-ax}\left[2\,a_4 + 2\,b_3\right] = 0$ and

$$\epsilon^{ax}\left[-\,2\,a_1 - 2\,b_2\right] + \epsilon^{-ax}\left[2\,a_3 - 2\,b_4\right] = 0$$

Note: Through typographical error a has been used instead of α throughout paper in the exponent for ϵ.

And since these equations hold for all values of x

$$a_1 = -b_2 \quad a_2 - b_1 \qquad a_3 = b_4 \text{ and } a_4 = -b_3$$

Substitùting in equations for A B and

$\Delta = A \sin \omega t + B \cos \omega t$ the values of A and B from

$$(9)\ \Delta = \epsilon^{ax}\left[2 a_1 \cos \alpha x - 2 a_2 \sin \alpha x\right] \sin \omega t +$$

$$\epsilon^{-ax}\left[2 a_3 \cos\alpha x - 2 a_4 \sin\alpha x\right] \sin \omega t +$$

$$\epsilon^{ax}\left[2 a_2 \cos\alpha x + 2 a_1 \sin\alpha x\right] \cos \omega t +$$

$$\epsilon^{-ax}\left[-2 a_4 \cos \alpha x - 2 a_3 \sin\alpha x\right] \cos \omega t, \text{ which reduces to}$$

$$\Delta = 2\,\epsilon^{ax}\left[a_1 \sin(\alpha x + \omega t) + a_2 \cos(\alpha x + \omega t)\right]$$

$$-2\,\epsilon^{-ax}\left[a_3 \sin(\alpha x - \omega t) + a_4 \cos(\alpha x - \omega t)\right] \qquad (10)$$

This is the complete solution of Δ involving the four constants of integration a_1, a_2, a_3 and a_4. The evaluation of these constants is given in Part 2.

2—EVALUATION OF CONSTANTS IN THE SOLUTION OF Δ

The value of the constants a_1, a_2, a_3 and a_4 can be secured from the limiting conditions imposed upon the current in any strand or conductor as the use may be.

The most general condition is the one where the current J in the wire under consideration can have any value and any phase angle with respect to some reference current; and also the current I below the wire can have a value and phase in no way related to J.

Taking a wire which may be a strand of the conductor or a conductor itself: Let its depth be f, its current J, its phase displacement be θ. Let $\Delta_o \sin(\omega t + \theta)$ represent the mean density if the current is uniformly distributed over the wire. Let I be the current below the wire and let its phase angle be ϕ.

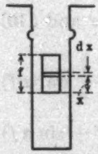
Then our limiting conditions are

$$\Delta_o \sin(\omega t + \theta) = \frac{1}{f}\int_o^f \Delta\, dx \qquad (11)$$

$$\left[\frac{d\,v}{d\,x}\right]_{x=0} = \rho\left[\frac{d\;\Delta}{d\,x}\right]_{x=0} = 4\,\frac{\pi\,\omega}{a}\,I\cos\,(\omega\,t+\phi)\;10^{-9} = \left[\frac{d\,B}{dt}\right]_{x=0} 10^{-8} \tag{12}$$

The development of equation (12) is as follows:

For any element of a conductor $d\,x$ the current above this element produces a flux which links this particular element and all elements below it, therefore producing no variation in these elements or $\frac{d\,v}{d\,x} = 0$ due to this flux.



We have then to consider only the current in the element and the current below it.

Let the depth of the wire be *f*.

Consider only the current up to the wire.

We have called this current I and its phase ϕ. We have then a flux density at X and all points above it of β_1

$\beta_1 = \frac{4\,\pi}{10a}\,I\sin(\omega\,t+\phi)$ and $\phi_1 = \frac{4\,\pi}{10a}\,I\sin\,(\omega\,t+\phi)(f-x)$ is the flux due to I which links the strand above the element $d\,x$.

The e.m.f. due to ϕ_1 in volts is V_1.

$V_1 = \frac{d\phi_1}{d\,t}\;10^{-8} = \frac{4\,\pi}{a}\;I\,\omega\cos(\omega\,t+\phi)\,(fx)\;10^{-9}$ and $\frac{d\,V_1}{d\,x} = -\left[-\;\frac{4\pi}{a}\;I\,\omega\cos\,(\omega\,t+\phi)\;10^{-9}\right]\frac{V_1}{d\;x}$ is negative as ϕ_1 decreases when X increases. It is readily seen that when X is zero that V_1 is the only e.m.f. producing eddy currents and we derive our equation (12) under the condition that x is zero. We have from equation (10)

$$\rho\,\frac{d\;\Delta}{d\,x} = 2\rho\,\alpha\left\{\Big[a_1-a_2-a_3-a_4\Big]\sin\,\omega\,t+\Big[\,a_1+a_2-a_3+a_4\Big]\cos\,\omega\,t\right\}_{x=0} \tag{13}$$

Combining (**12**) and (**13**) and remembering

$$\alpha^2 = \frac{2\,\pi\,\gamma_1}{\rho}\,\omega\,10^{-9}$$

we get

Note: On this page and in remainder of paper the letter r should be substituted for γ.

$$(a_1 - a_2 - a_3 - a_4)\sin\omega t + (a_1 + a_2 - a_3 + a_4)\cos\omega t$$

$$= \frac{\alpha}{\alpha\gamma_1} I\cos(\omega t + \phi) \qquad \textbf{(14)}$$

and therefore grouping all terms together which contain sin ωt or cos ωt and equating each group to zero we get

$$a_1 - a_2 = a_3 + a_4 - \frac{\alpha I}{\alpha\gamma_1}\sin\phi$$

$$a_1 + a_2 = a_3 - a_4 + \frac{\alpha I}{\alpha\gamma_1}\cos\phi \qquad \textbf{(15)}$$

From (**11**) $\Delta_o \sin(\omega t + \theta) = \frac{1}{f}\int_o^f \Delta\, d\, x$ and (**10**)

for value of Δ we get

$$\sin\omega t \Big[a_1\overset{\alpha f}{\epsilon} (\cos\alpha f + \sin\alpha f) + a_2\epsilon^{\alpha f}(\cos\alpha f - \sin\alpha f) \overset{c}{+} a_3\epsilon^{-\alpha f}(-\cos\alpha f + \sin\alpha f)$$

$$+ a_4\epsilon^{-\alpha f}(\sin\alpha f + \cos\alpha f) - \sin\omega t(a_1 + a_2 - a_3 + a_4)$$

$$+ \cos\omega t \Big[a_1\epsilon^{\alpha f}(\sin\alpha f - \cos\alpha f) + a_2\epsilon^{\alpha f}(\sin\alpha f$$

$$+ \cos\alpha f) + a_3\epsilon^{-\alpha f}(\cos\alpha f + \sin\alpha f) + a_4\epsilon^{-\alpha f}(\cos\alpha f$$

$$- \sin\alpha f) + \cos\omega t\,(a_1 - a_2 - a_3 - a_4) = \Delta_0\,\alpha f$$

$$\Big[\sin\omega t\cos\theta + \cos\omega t\sin\theta\Big] \qquad \textbf{(16)}$$

All terms in (16) contain sin ωt or cos ωt. Collecting each set separately and equating to zero and substituting the values of $-\alpha\frac{I}{a\gamma_1}\sin\phi$ from(**15**) and also $J = a\gamma_1\Delta_{of}$ and we find that(**16**) reduces to

$$\left.\begin{aligned}
&\frac{\alpha}{a\gamma_1}\Big[J\cos\theta + I\cos\phi\Big] = (a_1 + a_2)\,\epsilon^{\alpha f}\cos\alpha f\\
&+ (a_1 - a_2)\,\epsilon^{\alpha f}\sin\alpha f + (a_3 + a_4)\,\epsilon^{-\alpha f}\sin\alpha f\\
&- (a_3 - a_4)\,\epsilon^{-\alpha f}\cos\alpha f\,\frac{\alpha}{a\gamma_1}\Big[J\sin\theta + I\sin\phi\Big]\\
&= (a_1 + a_2)\,\epsilon^{\alpha f}\sin\alpha f - (a_1 - a_2)\,\epsilon^{\alpha f}\cos\alpha f + (a_3\\
&+ a_4)\ldots\epsilon^{-\alpha f}\cos\alpha f + (a_3 - a_4)\,\epsilon^{-\alpha f}\sin\alpha f.
\end{aligned}\right\} \textbf{(17)}$$

$$\frac{\epsilon^{\alpha f} + \epsilon^{-\alpha f}}{2} = \cosh\alpha f \quad 1\,\frac{\epsilon^{\alpha f} - \epsilon^{-\alpha f}}{2} = \sinh\alpha f.$$

If we combine (15) and (17) and if we let $\cosh\alpha f\sin\alpha f$

$= p$ and $\sinh \alpha f \cos \alpha f = q$, we can after rearranging obtain the following values for a_1, a_2, a_3, a_4

$$a_1 = K \Big[(p - q)\,(J \sin\theta + I \sin\phi) + (p + q)\,(J \cos\theta + I \cos\phi) + (p - q)\, I\, \epsilon^{-\alpha f} \sin(\alpha f - \phi) - (p + q)\, I\, \epsilon^{-\alpha f} \cos(\alpha f - \phi) \Big]$$

$$a_2 = K \Big[(p + q)\,(J \sin\theta + I \sin\phi) - (p - q)\,(J \cos\theta + I \cos\phi) + (p + q)\, I\, \epsilon^{-\alpha f} \sin(\alpha f - \phi) + (p - q)\, I\, \epsilon^{-\alpha f} \cos(\alpha f - \phi) \Big]$$

$$a_3 = K \Big[(p - q)\,(J \sin\theta + I \sin\phi) + (p + q)\,(J \cos\theta + I \cos\phi) - (p - q)\, I\, \epsilon^{\alpha f} \sin(\alpha f + \phi) - (p - q)\, I\, \epsilon^{\alpha f} \cos(\alpha f + \phi \Big]$$

$$a_4 = K \Big[- (p + q)\,(J \sin\theta + I \sin\phi) + (p - q)\,(J \cos\theta + I \cos\phi) + (p + q)\, I\, \epsilon^{\alpha f} \sin(\alpha f + \phi) - (p - q)\, I\, \epsilon^{\alpha f} \cos(\alpha f + \phi) \Big]$$

$$\text{Where } K = \frac{1}{4}\,\frac{\alpha}{a\,\gamma_1}\,\frac{1}{p^2 + q^2}$$

3—EXCESS LOSS IN CONDUCTOR DUE TO EDDY CURRENTS

Considering the case covered in Part 2 of a wire carrying a current J with a displacement θ and located in a slot over the current I with a displacement ϕ we can determine the copper loss in the wire. To obtain this loss we require the average value of Δ^2 with respect to x and the mean value with respect to time. Let this value be known as $\Delta^2{}_1$.

$\frac{1}{f}\int_0^f \Delta^2\, d\,x$ is the square root of the mean squared value of Δ with respect to x. The time of one cycle is from zero to $\frac{1}{\sim}$ where $\sim$ is the frequency. $\frac{1}{\sim} = \frac{2\,\pi}{\omega}$

The complete operation is indicated as

$$\frac{\omega}{2\pi}\,\frac{1}{f}\int_{o}^{f} dx \int_{o}^{\frac{\pi\,2}{\omega}} \Delta^2\, dt = \Delta^{2}_{1}$$

From 10

$$\Delta = 2\,\epsilon^{ax}\Big[a_1 \sin(\alpha\, x + \omega\, t) + a_2 \cos(\alpha\, x + \omega\, t)\Big]$$

$$-\,2\,\epsilon^{-ax}\Big[a_3 \sin(\alpha\, x - \omega\, t) + a_4 \cos(\alpha\, x - \omega\, t)\Big]$$

For convenience let $\alpha\, x + \omega\, t = A \quad \alpha\, x + \omega\, t = B$ when

$$\Delta = 2\,\epsilon^{ax}\Big[a_1 \sin A + a_2 \cos A\Big] - 2\,\epsilon^{-ax}\Big[a_3 \sin \beta$$

$$+\, a_4 \cos \beta\Big]$$

$$\Delta^2 = 4\,\epsilon^{2ax}\Big[a^2_1 \sin^2_1 A + 2\,a_1\,a_2 \sin A \cos A + a^2_2 \cos^2 A\Big]$$

$$-\,8\,\Big[a_1\,a_3 \sin A \sin B + a_1\,a_4 \sin A \cos B\Big]$$

$$+\Big[a_2\,a_3 \cos A \sin B + a_2\,a_4 \cos A \cos B\Big]$$

$$+\,4\,\epsilon^{-2ax}\Big[a^2_3 \sin^2 B + 2a_3\,a_4 \sin B \cos B + a^2_4$$

$$\cos^2 B\Big]$$

$$\text{Sin}^2 A = \tfrac{1}{2} - \cos\frac{2A}{2}$$

$$= \tfrac{1}{2}\Big[1 - \cos(2\alpha\, x + 2\,\omega\, t)\Big]$$

$$= (a)\,\frac{\omega}{2\pi}\int_{o}^{\frac{2\,\pi}{\omega}} (a)\, dt = \tfrac{1}{2}$$

$$2 \sin A \text{ ocs } A = \sin(2\alpha\, x + 2\,\omega\, t)$$

$$= (b)\,\frac{\omega}{2\pi}\int_{o}^{\frac{2\pi}{\omega}} (b)\, dt = 0$$

$$\text{Cos}^2 A = \tfrac{1}{2}\Big[1 + \cos(2\alpha\, x + 2\,\omega\, t)\Big]$$

$$= (c) \quad \text{etc.} \quad = \tfrac{1}{2}$$

$\text{Sin } A \sin B \quad = \tfrac{1}{2}\left[\cos 2\,\omega t - \cos 2\,\alpha x\right]$

$\qquad = (d) \qquad \text{etc.} \qquad = -\tfrac{1}{2}\cos 2\alpha x$

$\text{Sin } A \cos B \quad = \tfrac{1}{2}\left[\sin 2\,\alpha x + \sin 2\,\omega t\right]$

$\qquad = (e) \qquad \text{etc.} \qquad = \tfrac{1}{2}\sin 2\alpha x$

$\text{Cos } A \sin B \quad = \tfrac{1}{2}]\left[\sin 2\,\alpha x - \sin 2\,\omega t\right]$

$\qquad = (f) \qquad \text{etc.} \qquad = \tfrac{1}{2}\sin 2\alpha x$

$\text{Cos } A \cos \quad = B\ \tfrac{1}{2}\left[\cos 2\,\alpha x + \cos 2\,\omega t\right]$

$\qquad = \ 8 \qquad \text{etc.} \qquad = \tfrac{1}{2}\cos 2\alpha x$

$\text{Sin}^2 B \quad = \tfrac{1}{2}\left[1 - \cos(2\alpha x - 2\,\omega t)\right]$

$\qquad = (h \qquad \text{etc.} \qquad = \tfrac{1}{2}$

$2\sin B \cos B = \sin(2\alpha x - 2\,\omega t)$

$\qquad = (i) \qquad \text{etc.} \qquad = 0$

$\text{Cos}^2 B \quad = \tfrac{1}{2}\left[1 + \cos(2\alpha x - 2\,\omega t)\right]$

$\qquad = (j) \qquad \text{etc.} \qquad = \tfrac{1}{2}$

The average integrated value of Δ^2 between $t = O$ and $t = \frac{2\pi}{\omega}$ or $\frac{\omega}{2\pi}\int^{\frac{2\pi}{\omega}} \Delta^2\, dt = 2\,\epsilon^{2ax}\,(a_1^2 + a_2^2) + 4\,a_1\,a_3\cos 2\,\alpha x - 4\,a_1\,a_4\sin 2\,\alpha x - 4\,a_2\,a_3\sin 2\,\alpha x - 4\,a_2\,a_4\cos 2\,\alpha x + 2\,\epsilon^{2ax}\,(a_3^2 + a_4^2)$ and this expression can be called Δ^2_t

$$\frac{1}{f}\int_o^f \Delta^2_t\, d\,x = \Delta_1^2$$

This can be reduced to the following

$$\alpha f \Delta_1^2 = \epsilon^{2af}(a_1^2 + a_2^2 - \epsilon_1^{2af}(a_3^2 + a_4^2) + 2\sin 2\,\alpha f\,(a_1\,a_3 - a_2\,a_4) + 2\cos 2\,\alpha f\,(a_1\,a_4 + a_2\,a_3) - (a_1^2 + a_2^2 - a_3^2 - a_4^2) - 2\,(a_1\,a_4 + a_2\,a_3) \qquad (19)$$

If in formula (19) we substitute the values of a_1, a_2, a_3 and a_4 from (18) we can reduce to the following

$$\alpha\, f\, \Delta_1^2 = 2\,K^2\,(p^2 + q^2)\left[\,8\left\{J\cos\theta\, I\cos\phi + J\sin\theta\, I\sin\phi + I^2(\sin^2\phi + \cos_2\phi)\right\}\left\{\sinh\alpha f\cosh\alpha f\right.\right.$$

$$- \sinh\alpha f \cos\alpha f - \cosh\alpha f \sin\alpha f + \sinh\alpha f \cos\alpha f \Big\}$$

$$4 + \Big\{ J^2 (\sin^2\theta + \cos^2\theta) \Big\} \quad \Big\{ \sinh\alpha f \cosh\alpha f$$

$$+ \sin\alpha f \cos\alpha f \Big\} \Big] \tag{20}$$

$$K = \tfrac{1}{4} \frac{\alpha}{a\gamma_1} \frac{1}{(p^2+q^2)} (p^2+q^2) = \frac{\cosh 2\alpha f - \cos 2\alpha f}{2}$$

also since Δ_1 is the square root of the mean squared value of Δ and Δ_0 is the maximum value of the average density in the wire therefore the ratio of loss is

$$R = 2\,\frac{\Delta_1^2}{\Delta^2} \quad \text{whence } (\mathbf{20}) \text{ is}$$

$$R = \frac{\alpha f}{a^2\gamma_1^2 f^2 \Delta_0^2} \left[4\, \frac{\Big\{ J\, I \cos(\theta - \phi) + I^2 \Big\}}{\operatorname{Cosh} 2\alpha f - \operatorname{Cos} 2\alpha f} \right.$$

$$\frac{\Big\{ \sinh\alpha f - \sin\alpha f \Big\}}{\operatorname{Cosh} 2\alpha f - \operatorname{Cos} 2\alpha f} \; \frac{\Big\{ \cosh\alpha f - \cos\alpha f \Big\}}{\operatorname{Cosh} 2\alpha f - \operatorname{Cos} 2\alpha f}$$

$$\left. \int + \frac{\Big\{ \sinh 2\alpha f + \sin 2\alpha f \Big\} J^2}{\operatorname{Cosh} 2\alpha f - \operatorname{Cos} 2\alpha f} \right] \tag{21}$$

$a r_1 f \Delta_0 = J$ and if we consider the case where the current J and I are in phase then $\cos(\theta - \phi)$ is unity. Let $\left[\frac{I}{J} + 1\right] = m$. Then for this special case we have conditions given in Mr. A. B. Field's paper and obtain the same formula.

$$\frac{R}{\alpha f} = \frac{4\,(m^2 - m)\;(\sinh\alpha f - \sin\alpha f)\;(\cosh\alpha f - \cos\alpha f) + (\sinh 2\alpha f + \sin 2\alpha f)}{\operatorname{Cosh} 2\alpha f - \operatorname{Cos} 2\alpha f} \tag{22}$$

m in this case is the number of conductors in a slot counting from the bottom of the slot up to and including the conductor under consideration.

4—INFINITELY STRANDED CONDUCTORS

The most general type of conductor is a stranded one. The limits of stranding extend from the solid bar or one strand to infinite strands. We have developed the general case of the solid bar. Let us consider the other limiting case or an infinitely stranded bar. The one self evident fact is that with zero depth of strand there is no circulating current within the strand, consequently any circulating current within the conductor must travel the full length of the conductor rather than just the length of the core as in the case of a solid bar. If L_t is the length of the conductor and L_t the length of the iron we can let $\gamma = \frac{L_t}{L_1}$. Then equation (1) becomes in the case of an infinitely stranded bar

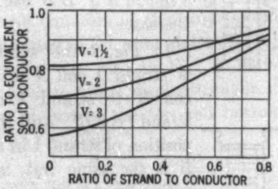
$$L_1 \frac{d\,v}{d\,x} = L_t \frac{d\,\Delta}{d\,x}\rho \quad \text{or} \quad \frac{d\,v}{d\,x} = \gamma \frac{d\,\Delta}{d\,x}\rho \tag{23}$$

Our general equation (2) can now be written

$$\frac{d^2V}{d\,x^2} = \frac{d^2\beta}{dxdt}10^{-8} = 4\pi\frac{d\,\Delta}{dt}10^{-9} = \frac{d^2\,\Delta}{dx^2}\gamma\rho \tag{24}$$

A little consideration will show at once that

$$\frac{d^2V}{dx^2} = \frac{d^2\,\Delta}{dx^2}\gamma\rho \text{ from (23) and } \frac{d^2V}{dx^2} = 4\pi\frac{d\,\Delta}{dt}10^{-9}$$



independently of the value of I below the conductor under consideration. See equation (12) and note that

$$\frac{d^2{}_1V}{dx^2} = \text{zero}$$

With our general equation for Δ as given in (24)

that is $\frac{d^2\,\Delta}{d\,x^2}\gamma\,\rho = 4\,\pi\frac{d\,\Delta}{dt}10^{-9}$ or $\frac{\omega\,d^2\,\Delta}{2\,dx^2} = \frac{x^2\,d\,\Delta}{r\,dt}$ (24)

our limiting conditions are the same as for equation (2) namely equation (11) and (12) and the only difference in the solution of the heating equivalent is the substitution in (24) of $\frac{\alpha^2}{\gamma}$ for α^2 in equation (3).

For an infinitely laminated conductor of the depth f we can take our losses from the solid bar of a depth of $\frac{f}{\sqrt{\gamma}}$. In the above we have taken the case where all strands are short-circuited at each end of one conductor or half a turn. In stranded conductors there are in general use two kinds of coils which differ from this construction. In these coils the strands are short-circuited at the start and finish of the completed coil. The coil itself may consist of one or more complete turns. The two types of coil differ in the relative location of the strands in successive conductors. In this paper the types are designated as "straight up" shown in Fig. 1 and "turned over" Fig. 2.

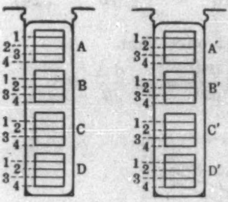


Fig. 1

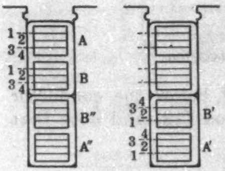


Fig. 2

In Fig. 1 the winding order is strand 1 coil A to strand 1 coil A' to strand 1 coil B to strand 1 coil B' and so forth.

The relative location of strand 1 in coil A A' B B' etc. is always the same.

In Fig. 2 the winding order is strand 1 coil A to strand 1 coil B' to strand 1 coil B to strand 1 coil B'. And the relative position of strand 1 in coil A and B is the same but is reversed with respect to the bottom of the slot in A' B'.

In the case shown in Fig. 1 we see that V varies in value depending upon whether we are considering conductor A B C, etc., but we find that $\frac{d}{dx}\left[\Sigma V - \gamma n \rho \Delta\right] = O$ where n is the number of conductors per slot depthwise. $\frac{d^2V}{dx^2} = 4\pi\gamma\frac{d\Delta}{dt}10^{-9}$ for any value of V with the same value of X no matter which conductor is considered.

Our fundamental equation for Δ is the same as in the previous case, that is $\frac{d^2V}{d\,x^2} = 4\,\pi\,\frac{d\,\Delta}{dt}\,10^{-9} = \frac{d^2\,\Delta}{dx^2}\,\gamma\,\rho$ **(24)**

and our limiting conditions are $\Delta_o \sin \omega\, t = \frac{1}{f}\int_o^f \Delta\, dx$

and when $x = o$

$$r\, n\, \rho \left(\frac{d\,\Delta}{d\,x}\right)_{x=o} = 10^{-9}\, 4\,\frac{\pi}{\alpha}\,\omega\, J \cos \omega\, t \Big[1+2+3+\quad +n-1\Big]$$

$$\text{or}\quad \gamma\,\rho\,\frac{d\,\Delta}{d\,x} = 10^{-9}\, 4\,\frac{\pi\,\omega}{\alpha}\left(\frac{n-1}{2}\right) J \cos \omega\, t \qquad \textbf{(25)}$$

These limits result in a solution for R the same as (22) except that $\frac{n-1}{2}\,J$ in (25) replaces I in equation (12). J is the current in the conductor. Therefore in place of (22) we get an equation in all respects similar except that $\frac{n-1}{2}$ replaces $m-1$ or $\frac{n+1}{2}$ replaces m

$$R = \alpha f\, \frac{4\left(\frac{n^2-1}{4}\right)\begin{pmatrix}\sin h\,\alpha\, f\\ -\sin \alpha\, f\end{pmatrix}\begin{pmatrix}\cos h\,\alpha\, f\\ -\cos \alpha\, f\end{pmatrix}+\begin{pmatrix}\sinh 2\,\alpha\, f\\ +\sin 2\alpha f\end{pmatrix}}{\cosh 2\,\alpha\, f - \cos 2\,\alpha\, f} \qquad \textbf{(26)}$$

and R is the equation of the ratio for the average loss of all conductors in the same slot.

Considering the case in Fig. 2 limited to infinite stranding and no phase displacement in currents in the conductors shown in the top and bottom coils in the same slot and we find

$$\frac{d}{dx}\Big[\Sigma\, V - \gamma\, n\, \rho\, \Delta\Big] = o \text{ or } \Sigma\,\frac{d\,v}{d\,x} \qquad n\,\rho\,\frac{d\,\Delta}{d\,x}$$

$\frac{d^2\, V}{dx^2} = 4\,\pi\,\gamma\,\frac{d\,\Delta}{dt}\,10^{-9}$ as before and our limiting equations are $\Delta_o \sin\,\omega\, t = \frac{1}{f}\int_o^f \Delta\, d\,x$ and $\Sigma\,\frac{d\,v}{d\,x}$ for lower half of slot when $X = O$ or $-$

$$10^{-9}\,\frac{4\,\pi}{\alpha}\,\omega\, J \cos\omega\, t \left[0 + 1 + 2 + \qquad + \left(\frac{n}{2}-1\right)\right]$$

and $\Sigma\frac{d\,v}{d\,x}$ for upper half of slot when $x = f$ or

$$-10^{-9}\,\frac{4\,\pi}{\alpha}\,\omega\, J \cos\omega\, t \left[n + (n-1) + (n-2) + \quad + \frac{n}{2} + 1\right].$$

Therefore $\gamma\,\rho\,\frac{d\,\Delta}{d\,x} = 10^{-9} - \left(\frac{n+2}{4}\right) J \cos\omega\, t\, \frac{4\,\pi\,\omega}{\alpha}$ **(27)**

These limits result in a solution for R the same as (22) except that $-\dfrac{n+2}{4}$ replaces $(m-1)$and R is the equation for the ratio of the loss of all conductors in the same slot—

$$R=\frac{\alpha f\left(\dfrac{n^2-4}{16}\right)4\begin{pmatrix}\sinh\alpha f\\ -\sin\alpha f\end{pmatrix}\begin{pmatrix}\cosh\alpha f\\ -\cos\alpha f\end{pmatrix}+\begin{pmatrix}\sinh 2\alpha f\\ +\sin 2\alpha f\end{pmatrix}\alpha f}{\cosh 2\alpha f-\cos 2\alpha f} \qquad (28)$$

In both of these cases $\dfrac{f}{\sqrt{\gamma}}$ is used in place of f to get the proper loss ratio.

The development of this case of infinite stranding has been gone through is in order to show that R is of the same general form for all cases considered and that an equivalent depth of conductor can be used.

5—APPROXIMATE FORMULAS FOR LOSSES IN SOLID BARS

From formula (21)

$$R = \left[\frac{I}{J}\cos(\theta-\phi)+\frac{I^2}{J^2}\right]$$
$$\left[4\alpha f\frac{(\sinh\alpha f-\sin\alpha f)(\cosh\alpha f-\cos\alpha f)}{\cosh 2\alpha f-\cos 2\alpha f}\right]$$
$$+\left[\frac{\alpha f\sinh 2\alpha f+\sin 2\alpha f}{\cosh 2\alpha f-\cos 2\alpha f}\right]$$

We can for convenience let

$$M_s = 4\,\alpha f\frac{(\sinh\alpha f-\sin\alpha f)\,(\cosh\alpha f-\cos\alpha f)}{\cosh 2\alpha f-\cos 2\alpha f} =$$
$$2\alpha f\frac{(\sinh\alpha f-\sin\alpha f)}{\cosh\alpha f+\cos\alpha f} \quad O_s=\alpha f\frac{\sinh 2\alpha f+\sin 2\alpha f}{\cosh 2\alpha f-\cos 2\alpha f}$$

Let us investigate these equations for M_s and O_s.
If we develop sinh x_1 sin x etc. in series we get

$$\text{Sinh } x = x+\frac{x^3}{\lfloor\underline{3}}+\frac{x_5}{\lfloor\underline{5}}+\frac{x^7}{\lfloor\underline{7}}\text{ etc.}$$

$$\text{Sin } x = x-\frac{x^3}{\lfloor\underline{3}}+\frac{x^5}{\lfloor\underline{5}}-\frac{x^7}{\lfloor\underline{7}}\text{ etc.}$$

$$\text{Cosh } x = 1+\frac{x^2}{\lfloor\underline{2}}+\frac{x^4}{\lfloor\underline{4}}+\frac{x^6}{\lfloor\underline{6}}\text{ etc.}$$

$$\text{Cos } x = 1-\frac{x^2}{\lfloor\underline{2}}+\frac{x^4}{\lfloor\underline{4}}-\frac{x^6}{\lfloor\underline{6}}\text{ etc}$$

If for convenience we use x for αf, M becomes

$$M_s = \frac{2\left[\frac{x^3}{\lfloor\underline{3}} + \frac{x^7}{\lfloor\underline{7}}\text{etc.}\right] x}{1 + \frac{x^4}{\lfloor\underline{4}} + \frac{x^8}{\lfloor\underline{8}}\text{ etc.}} = \frac{x^4}{3} - 0.01x^8 \text{ approx.} \tag{29}$$

For all practical purposes when x is less than 1

$$M_s = \frac{x^4}{3} = \frac{1}{3}(\alpha f)^4$$

Similarly for O_s

$$O_s = \frac{\left[2x + 8\frac{x^3}{\lfloor\underline{3}} + \frac{32x^5}{\lfloor\underline{5}}\text{etc.}\right]x + \left[2x - \frac{8x^3}{\lfloor\underline{3}} + \frac{32x^5}{\lfloor\underline{5}}\text{ etc.}\right]x}{\left[1 + \frac{(2x)^2}{\lfloor\underline{2}} + \frac{(2x)^4}{\lfloor\underline{4}}\text{etc.}\right] - \left[1 - \frac{(2x)^2}{\lfloor\underline{2}} + \frac{(2x)^4}{\lfloor\underline{4}}\text{etc.}\right]}$$

or since $x = \alpha f$

$$O_s = 1 + \frac{8}{90}(\alpha f)^4 \tag{30}$$

for all practical purposes when $\alpha f < 1$

In modern day machines we will find that αf is less than unity $\alpha^2 = \frac{2\pi\gamma_1}{\rho} 2\pi \sim 10^{-9}$ in *c.g.s.* system $\rho = 625\ 10^{-9}$ per cu. in. at zero deg. cent.

$$\alpha^2 = \frac{4\pi^2 \times 2.54}{625} \sim \gamma_1 \text{ at } 0^\circ \quad \alpha = \frac{.4}{\sqrt{1 + .00427\,\theta}} \sqrt{\sim \gamma_1}$$

for θ deg. cent.

For high voltage machines γ_1 is approximately 0.6 and for $\sim = 60$ α at 100 deg. cent. would be $0.334 \times 6 = 2$ abt.

αf is < 0.75 for a ⅜ in. deep strand even in case of 60 cycles.

Therefore for all cases which we care to consider values of O_s and M_s can be taken from formulas (29) and (30).

6—SOLUTION FOR CURRENTS IN STRANDS

In the paper up to the present stage we have considered only the two limiting cases of a stranded conductor, that is the solid bar and the infinitely laminated bar. We have found that an equivalent solid bar can be substituted for an infinitely laminated one, and that the equivalent depth of substituted bar is $\frac{1}{\sqrt{\gamma}}$ times the depth of the stranded conductor; γ is the ratio of total length of the conductor to the embedded length.

If we express in rectangular coordinates this relation using the values of the equivalent depth of bars as ordinates and the ratio of strand depth to conductor depth as abscissas we obtain two limiting points for each value of γ. It is reasonable to assume that these terminal points can be connected with a curve at least under certain limiting conditions and we shall now investigate this possibility.

So far we have a formula from which we can calculate the density of current at any point of a conductor carrying a current J of known value and of known phase θ provided we know the value of the current I below J and the phase of I or ϕ.

This formula is (10).

$$\Delta = 2\,\epsilon^{ax}\left[\alpha_1 \sin(\alpha x + \omega t) + a_2 \cos(\alpha x + \omega t)\right]$$

$$+\,2\,\epsilon^{-ax}\left[\alpha_3 \sin(\alpha x - \omega t) + a_4 \cos(\alpha x - \omega t)\right]$$

a_1, a_2, a_3, and a_4 are constants dependent on the value of the current J and its phase angle θ in the conductor or strand and of the current I and its phase angle ϕ for the current below the strand. When we know J, θ, I and ϕ we can write the loss ratio in the strand from formula (21)

$$R = \frac{1}{J^2}\left[\left\{J\,I\cos(\theta - \phi) + I^2\right\}M_s + J^2\,O_s\right]$$

If we take the case of a stranded conductor where the number of strands is finite we can calculate the currents and phase displacements for the strands and the relative loss in each strand. Let us take the phase angle for a conductor current as θ_0 and the current as J_0. Let the conductor have any number of strands (say n) and designate the sin and cos components of these strands as $J_n \sin\theta_n$, $J_{n-1}\sin\theta_{n-1}$, $J_n\cos\theta_n$, $J_{n-1}\cos\theta_{n-1}$, etc. For convenience we will write these respectively ${}_sJ_n$, ${}_sJ_{n-1}$, ${}_cJ_n$, ${}_cJ_{n-1}$, etc.

In any stranded conductor between the points of common short circuit of all strands which go to make up a conductor, if any two strands are adjacent at any point they will be adjacent throughout their entire length. It is possible therefore to sum all the voltages in adjacent strand edges and equate them since the insulation thickness between strands can be taken as negligible. In this way 2 (n-1) equations can be obtained in solving 2 n unknown quantities (${}_sJ_n$ ${}_sJ_{n-1}$ - - - - ${}_cJ_n$ ${}_cJ_{n-1}$ - - - etc.) and the other two equations needed to evaluate all unknown quantities are $\sum_1^n {}_cJ_x = J_0 \cos \theta_0$ and $\sum_1^n {}_sJ_x = J_0 \sin \theta_0$. We have therefore a method for obtaining R for any stranded conductor provided we are able to manipulate it.

In Part 4 we considered the case of infinitely stranded coils and in Figs. 1 and 2 illustrated "straight up" and "turned over" coils. It may be well to point out here that there are various modifications of these types; for example, all strands which go to make up a conductor can be short circuited at the end of a half turn, a full turn or only at the end of a coil, where the coil may consist of one or more turns; also in the case of coils shown in Fig. 2, Part 4, the currents in the conductors A'' B'', etc., may be of different phase from those in A B, or in the event of only one coil per slot turned over there will not be any conductors A'' B''.

Treatment of all these factors will be considered in the remainder of this paper. The final formulas will give the equivalent loss ratio for such a group of conductors as will be included in a common insulating cell from ground. That is in the case of one coil side per slot the ratio for the entire slot will be given. While in the case of two coil sides per slot the upper and lower coil sides of this will be given separately.

Let us investigate the case of a coil wound straight up. The strand arrangement is then as shown in Fig. 1, Part 4. The winding order is strand 1 coil A to strand 1 coil A' etc. We can consider a full turn as our unit or since each conductor has relatively the same position in the second slot we can consider a single wire as the unit. Let l_t be length of the conductor, l_s the length of the iron, and $\frac{l_t}{l_i} = \gamma$. Then the length of the end winding $= \frac{l_t - l_i}{l_i} = \gamma - 1$.

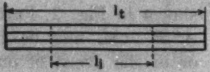
In the bottom edge of the n^{th} strand and the top edge of the (n-1) strand the summation of e.m.fs. is zero, provided we neglect the flux that leaks through the insulation between strands. We will also assume that no unbalanced e.m.fs. are induced in the strands beyond the iron length of the conductor. Let V be the induced e.m.f. per unit length in the core; use ${}_{n-1}V_t$ to represent this e.m.f. at the top of the (n-1) strand, and ${}_nV_b$ to represent this e.m.f. at the bottom of the n^{th} strand, etc.

Then $l_i\, {}_{n-1}V_t + \rho\, {}_{n-1}\Delta_t\, l_i + J_{n-1} \sin(\omega t + \theta_{n-1})\, \rho \frac{l_t - l_i}{a \gamma_1 h} =$

$$l_i\, {}_nV_b + \rho\, {}_n\Delta_b\, l_i + J_n \sin(\omega t + \theta_n)\, \rho \frac{l_t - l_i}{a \gamma_1 h}$$

or ${}_{n-1}\Delta_t + J_{n-1} \sin\ (\omega t + \theta_{n-1}) \frac{\gamma - 1}{a \gamma_1 h} = {}_n\Delta_b + J_n \sin$

$$(\omega t + \theta_n) \frac{\gamma - 1}{a \gamma_1 h} \qquad (31)$$



Where a is the width of the slot h is the depth of the strand, $a\, \gamma_1\, h$ is the section of the strand.

Equation (31) can be combined with equations (10) and (18) to give us relations involving J_{n1} and $J_n\ \theta_{n-1}$ and θ_n. From the general equation for Δ (10) we can conveniently express ${}_{n-1}\Delta_t$ and ${}_n\Delta_b$ at the two intervals of time represented by $\omega t = 0$ and $\omega t = \frac{\pi}{2}$. At the top of the strand in (10) αx is replaced by αh therefore $\Delta_t =$ $2 \sin \alpha h\, (\epsilon^{\alpha h} a_1 + \epsilon^{-\alpha h} a_3) + 2 \cos \alpha h\, (\epsilon^{\alpha h} a_2 - \epsilon^{-\alpha h} a_4)$ when $\omega t = 0$ **(32)**

$\Delta_t = 2 \cos \alpha h\, (\epsilon^{\alpha h} a_1 + \epsilon^{-\alpha h} a_3) - 2 \sin \alpha h\, (\epsilon^{\alpha h} a_2 + \epsilon^{-\alpha h} a_4)$ when $\omega t = \frac{\pi}{2}$

At bottom of the strand in (10) αx is zero.

$\Delta_b = 2\,(a_2 - a_4)$ when $\omega t = O$ $\Delta_b = 2\,(a_1 + a_3)$ when $\omega t = \frac{\pi}{2}$ **(33)**

We can now put the values of a_1, a_2, a_3, and a_4 in equations (32) and (33).

In equation (18) we had formulas for a_1, a_2, etc. The value of J in (18) is the strand current and I is the total current below the strand. It will be noticed that these equations can be rewritten as follows:

$$\begin{aligned}
a_1 = K \Big\{ & (p-q)\,{}_sI_t + (p+q)\,{}_cI_t - [(p+q)\,\epsilon^{-ah}\sin\alpha h \\
& + (p-q)\,\epsilon^{-ah}\cos\alpha h]\,{}_sI_b + [(p-q)\epsilon^{-ah}\sin\alpha h - \\
& (p+q)\,\epsilon^{-ah}\cos\alpha h]\,{}_cI_b \\
a_2 = K \Big\{ & (p+q)\,{}_sI_t - (p-q)\,{}_cI_t + [(p-q)\,\epsilon^{-ah}\sin\alpha h \\
& - (p+q)\,\epsilon^{-ah}\cos\alpha h]\,{}_sI_b + [(p+q)\,\epsilon^{-ah}\sin\alpha h \\
& + (p-q)\,\epsilon^{-ah}\cos\alpha h]\,{}_cI_b \qquad (34) \\
a_3 = K \Big\{ & (p-q)\,{}_sI_t + (p+q)\,{}_cI_t + [(p+q)\,\epsilon^{ah}\sin\alpha h \\
& - (p-q)\,\epsilon^{ah}\cos\alpha h]\,{}_sI_b - [(p-q)\,\epsilon^{ah}\sin\alpha h \\
& + (p+q)\,\epsilon^{ah}\cos\alpha h]\,{}_cI_b \\
a_4 = K \Big\{ & -(p+q)\,{}_sI_t + (p-q)\,{}_cI_t + [(p-q)\,\epsilon^{ah}\sin \\
& \alpha h + (p+q)\,\epsilon^{ah}\cos\alpha h]\,{}_sI_b + [(p+q)\,\epsilon^{ah}\sin \\
& \alpha h - (p-q)\,\epsilon^{ah}\cos\alpha h]\,{}_cI_b
\end{aligned}$$

Where ${}_sI_t$, ${}_cI_t$ are the sin and cos components respectively of all currents from the bottom of slot up to and including the current in the strand under consideration. Similarly I_b is the summation of currents below the strand under consideration.

By substitution (34) in equation (32) we get

$$\begin{aligned}
& 2\sin\alpha h\,[\epsilon^{ah} a_1 - \epsilon^{-ah} a_3] + 2\cos\alpha h\,[\epsilon^{ah} a_2 - \epsilon^{-ah} a_4] \\
& \quad = 4K\,[\tfrac{1}{2}(d+e)\,{}_sI_t - \tfrac{1}{2}(d-e)\,{}_cI_t - (p+q) \\
& \quad {}_sI_b + (p-q)\,{}_cI_b] = \Delta_t,\ \omega t = 0 \\
& 2\cos\alpha h\,[\epsilon^{ah} a_1 + \epsilon^{-ah} a_3] - 2\sin\alpha h\,[\epsilon^{ah} a_2 + \epsilon^{-ah} a_4] \\
& \quad = 4K\,[\tfrac{1}{2}(d-e)\,{}_sI_t + \tfrac{1}{2}(d+e)\,{}_cI_t - (p-q) \\
& \quad {}_sI_b - (p+q)\,{}_cI_b] = \Delta_t,\ \omega t = \frac{\pi}{2} \\
& 2(a_2 - a_4) = 4K\,[(p+q)\,{}_sI_t - (p-q)\,{}_cI_t - \tfrac{1}{2}(d+e) \\
& \quad {}_sI_b + \tfrac{1}{2}(d-e)\,{}_cI_b] = \Delta_b,\ \omega t = o \\
& 2(a_1 + a_3) = 4K\,[(p-q)\,{}_sI_t + (p+q)\,{}_cI_t - \tfrac{1}{2}(d-e) \\
& \quad {}_sI_b - \tfrac{1}{2}(d+e)\,{}_cI_b] = \Delta_b,\ \omega t = \frac{\pi}{2} \qquad (35)
\end{aligned}$$

In these equations d is $\sin 2\alpha h = 2\sin\alpha h\cos\alpha h$
e is $\sinh 2\alpha h = 2\sinh\alpha h\cosh\alpha h$

also

$$\begin{aligned}
(d+e) &= 2\,[\sinh\alpha h\sin\alpha h\,(p-q) + \cosh\alpha h \\
& \quad \cos\alpha h\,(p+q)] \\
-(d-e) &= 2\,[\sinh\alpha h\sin\alpha h\,(p+q) - \cosh\alpha h \\
& \quad \cos\alpha h\,(p-q)]
\end{aligned}$$

The above expressions (35) for Δ_t and Δ_b when ωt is zero and $\frac{\pi}{2}$ are general equations and are true for any strand provided the proper currents are used in obtaining the values of ${}_sI_t$, ${}_cI_t$ and ${}_sI_b$, ${}_cI_b$. Consider the case of an even number of conductors per slot say m when we have two coil sides per slot. Let us evaluate the expression for the x and $(x-1)$ strands of the y^{th} coil above the center line of the slot. Where the currents in the bottom half of the slot are displaced by the angle β

$$\left.\begin{aligned}
{}_sI_t &= \sum_o^x {}_sJ + \frac{m}{2} J_o \sin\beta + (y-1)\,{}_sJ_o \\
{}_cI_t &= \sum_o^x {}_cJ + \frac{m}{2} J_o \cos\beta + (y-1)\,{}_cJ_o \\
{}_sI_b &= \sum_o^{x-1} {}_sJ + \frac{m}{2} J_o \sin\beta + (y-1)\,{}_sJ_o \\
{}_cI_b &= \sum_o^{x-1} {}_cJ + \frac{m}{2} J_o \cos\beta + (y-1)\,{}_cJ_o
\end{aligned}\right\} \text{If } \theta_o \text{ is taken as zero, then } {}_sJ_o = 0,\ {}_cJ_o = J_o. \qquad (36)$$

At such time as ωt is zero, if we substitute values from (35) and (36) in (31) we obtain

$$-(p+q+2c)\,{}_sJ_x - (p+q-2c-d-e)\,{}_sJ_{x-1} - (2p+2q-d-e)\sum_o^{x-2} {}_sJ + (p+q)\,{}_cJ_x + (p-q-d+e)\,{}_cJ_{x-1} + (2p-2q-d+e)\sum_o^{x-2} {}_cJ = \Big[2(p+q)-(d+e)\Big]\Big[\frac{m}{2} J_o \sin\beta\Big] - \Big[2(p-q)-(d-e)\Big]\Big[(y-1)J_o + \frac{m}{2} J_o \cos\beta\Big]$$

At such time as $\omega t = \frac{\pi}{2}$ we get

$$-(p-q)\,{}_sJ_x - (p-q-d+e)\,{}_sJ_{x-1} - (2p-2q-d+e)\sum_o^{x-2} {}_sJ - (p+q+2c)\,{}_cJ_x - (p+q-d-e-2c)\,{}_cJ_{x-1} - (2p+2q-d-e)\sum_o^{x-2} {}_cJ = \Big[2(p-q)-(d-e)\Big]\frac{m}{2} J_o \sin\beta + \Big[2(p+q)-(d+e)\Big]\Big[(y-1)\,{}_cJ_o + \frac{m}{2} J_o \cos\beta\Big]$$

$$\text{Where } 2c = \frac{\gamma-1}{\alpha\gamma_1 h}\,\frac{1}{4K} = \frac{\gamma-1}{\alpha h}(p^2+q^2) \qquad (36A)$$

These two equations can be simplified and rewritten

$$\left.\begin{aligned}&(p+c)\,{}_sJ_x-(d-p+c)\,{}_sJ_{x-1}-(d-2p)\sum_{o}^{x-2}{}_sJ\\&+(q+c)\,{}_cJ_x-(e-q+c)\,{}_cJ_{x-1}-(e-2q)\sum_{o}^{x-2}{}_cJ\\&=K_1\quad(q+c)\,{}_sJ_x-(e-q+c)\,{}_sJ_{x-1}-(e-2q)\\&\sum_{o}^{x-2}{}_sJ-(p+c)\,{}_cJ_x+(d-p+c)\,{}_cJ_{x-1}+(d-2p)\\&\sum_{o}^{x-2}{}_cJ=K_2\end{aligned}\right\}\quad(37)$$

$$\left.\begin{aligned}\text{Where } K_1&=(d-2p)\left[\frac{m}{2}J_o\sin\beta\right]+(e-2q)\\&\left[{}_cJ_o(y-1)+\frac{m}{2}J_o\cos\beta\right]\\K_2&=(e-2q)\left[\frac{m}{2}J_o\sin\beta\right]-(d-2p)\\&\left[{}_cJ_o(y-1)+\frac{m}{2}J_o\cos\beta\right]\end{aligned}\right\}\quad(38)$$

Since we have taken any two strands the x and the $(x\text{-}1)$ in the y^{th} conductor above the center line of a slot where there are two coil sides per slot carrying current with phase displacement or $\beta°$, therefore, we have a perfectly general solution for a strand in any position above the center line of a slot, for a coil wound straight up. For a strand below the center line y is counted from the bottom of the slot and terms $J_o \sin\beta$ and $J_o \cos\beta$ are zero. So we can write for this case

$$\left.\begin{aligned}&K_1-(e-2q)\left[(y-1)\,{}_cJ_o\right]\quad(e-2q)\left[\left(\frac{m}{2}-\right.\right.\\&\qquad\left.y)\,{}_cJ_o\right]\\&\qquad\qquad\text{or}\\&K_2=-(d-2p)\left[(y-1)\,{}_cJ_o\right]-(d-\\&\qquad2p)\left[\left(\frac{m}{2}-y\right)\,{}_cJ_o\right]\end{aligned}\right\}\quad(38\text{A})$$

if y is counted from center line of slot.

Obviously these same limitations apply to a case of one coil per slot where y is counted from bottom. It is also quite apparent that if we strand continuously that the only term which will change in K_1 and K_2 is the coefficients of ${}_cJ_o$ and that this will have the average value of $\sum_1^m (y-1)$ or $\frac{m-1}{2}$ in the case of one coil per slot.

Equations (37) and (38) with modifications enable us to solve for the sin and cos components for any strand in any coil wound straight up.

The turned over coil can be solved in exactly the same manner. In Fig. 3 we show the relative position of the x and $(x-1)$ strands in the usual type of coil where two coil sides per slot are used.

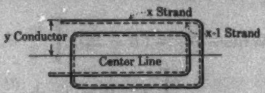


Fig. 3

Consider all strands which go to make up a conductor short-circuited each end of a full turn and assume that the currents in the bottom half of any slot are $\beta°$ out of phase with the currents in the top of the slot.

We get an expression for the e.m.fs. in the bottom edge of the x and top edge of the $(x$-1) strand summed around closed circuit similar to (31).

$$
{}_{n-1}\Delta_t + {}_{n-1}\overline{\Delta_b} + 2\,J_{n-1} \sin(\omega t + \theta)_{n-1} \frac{\gamma - 1}{a\gamma_1 h} = {}_n\Delta_b + {}_t\overline{\Delta_n} + 2\,J_n \sin(\omega t + \theta_n) \frac{\gamma - 1}{a\gamma_1 h} \tag{39}
$$

in which ${}_{n-1}\overline{\Delta_b}$ is the density in the bottom of the $(n$-1) strand of the y^{th} conductor below the center line of slot, etc.

This equation can be combined with (35), using proper values for ${}_sI_t$, ${}_sI_b$, etc., for the strands above and below the center line of the slot.

In the case of a turned over coil these values for the x^{th} strand of the y^{th} conductor above center line are

$$
{}_sI_t = \sum_o^x {}_sJ + (y-1)\,{}_sJ_o + \frac{m}{2} J_o \sin\beta
$$

$$
{}_cI_t = \sum_o^x {}_cJ + (y-1)\,{}_sJ_o + \frac{m}{2} J_o \cos\beta
$$

$$
{}_sJ_b = \sum_o^{x-1} {}_sJ + (y-1)\,{}_sJ_o + \frac{m}{2} J_o \sin\beta
$$

$$
{}_cI_b = \sum_o^{x-1} {}_cJ + (y-1)\,{}_sJ_o + \frac{m}{2} J_o \cos\beta
$$

and for the same strand below the center line

$$
\begin{gathered}
{}_sI_t = \left(\frac{m}{2} - y + 1\right) {}_sJ_o - \sum_0^{x-1} {}_sJ \\
{}_cI_t = \left(\frac{m}{2} - y + 1\right) {}_cJ_o - \sum_0^{x-1} {}_cJ \\
{}_sI_b = \left(\frac{m}{2} - y + 1\right) {}_sJ_o - \sum_0^{x} {}_sJ \\
{}_cI_b = \left(\frac{m}{2} - y + 1\right) {}_cJ_o - \sum_0^{x} {}_cJ, \text{ etc.}
\end{gathered}
$$

Substituting these values in 39 and combining with 35 we obtain when $\omega t =$ zero.

$$
\begin{gathered}
-(p + q + 2c)\, {}_sJ_x - (p + q - d - e - 2c)\, {}_sJ_{x-1} - \\
(2p + 2q - d - e) \sum_0^{x-2} {}_sJ + (p - q)\, {}_cJ_x + (p - q - \\
d + o)\, {}_cJ_{x-1} + (2p - 2q - d + e) \sum_0^{x-2} {}_cJ = \Big[2(p + q) - \\
(d + e) \Big] \left[\frac{m}{4} J_o \sin \beta \right] - \Big[2(p - q) - (d + e) \Big] \\
\left[(y - \frac{m}{4} - 1)\, {}_cJ_o + \frac{m}{4} + J_o \cos \beta \right]
\end{gathered}
$$

and when $\omega t = \frac{\pi}{2}$

$$
\begin{gathered}
-(p - q)\, {}_sJ_x - (p - q - d + e)\, {}_sJ_{x-1} - (2p - 2q - \\
d + e) \sum_0^{x-2} {}_sJ - (p + q + 2c)\, {}_cJ_x - (p + q - d - e - 2c)\, {}_cJ_{x-1} \\
- (2p + 2q - d - e) \sum_0^{x-2} {}_cJ = \Big[2(p - q) - (d - e) \Big] \\
\left[\frac{m}{4} J_o \sin \beta \right] + \Big[2(p + q) - (d + e) \Big] \Big[(y - \frac{m}{4} - 1)\, {}_cJ_o \\
+ \frac{m}{4} J_o \cos \beta \Big]
\end{gathered}
$$

These two equations rearranged and rewritten are the same as (37) except the values of K_1 and K_2 will be for a turned over coil which is short-circuited at the end of each turn.

$$
\left.
\begin{gathered}
K_1 = (d - 2p) \left[\frac{m}{4} J_o \sin \beta \right] + (e - 2q) \Big[(y - \\
\frac{m}{4} - 1)\, {}_cJ_o + \frac{m}{4} J_o \cos \beta \Big] \\
K_2 = (e - 2q) \left[\frac{m}{4} J_o \sin \beta \right] - (d - 2p) \Big[(y - \\
\frac{m}{4} - 1)\, {}_cJ_o + \frac{m}{4} J_o \cos \beta \Big]
\end{gathered}
\right\} \quad (40)
$$

For the case of one coil per slot turned over short-circuited at the end of each turn.

$$\left.\begin{aligned} K_1 &= (e-2q)\left[y-\frac{m}{4}-1\right]{}_cJ_o \\ K_2 &= -(d-2p)\left[y-\frac{m}{4}-1\right]{}_cJ_o \end{aligned}\right\} \quad \begin{array}{l}\text{Since } J_o \sin\beta \\ \text{and } J_o\cos\beta \text{ are} \\ \text{zero, } \frac{m}{2} \text{ in this} \\ \text{case is number of} \\ \text{conductors per} \\ \text{slot.}\end{array} \quad (40A)$$

For continuous stranding two coils per slot, the coefficient of ${}_cJ_o$ is the only term to change and this becomes the average value of $\sum_o^{\frac{m}{2}} (y - \frac{m}{4} - 1)$

$$\left.\begin{aligned} K_1 &= (d-2p)\frac{m}{4}J_o\sin\beta + (e-2q)\left[\frac{m}{4}J_o\cos\beta \right. \\ &\quad \left. - \tfrac{1}{2}\,{}_cJ_o\right] \\ K_2 &= (e-2q)\frac{m}{4}J_o\sin\beta - (d-2p)\left[\frac{m}{4}J_o\cos\beta \right. \\ &\quad \left. - \tfrac{1}{2}\,{}_cJ_o\right] \end{aligned}\right\} \quad (40B)$$

And for one coil per slot continuous stranding,

$$K_1 = (e-2q)\left[-\tfrac{1}{2}\,{}_cJ_o\right] \quad K_2 = -(d-2p)\left[-\tfrac{1}{2}\,{}_cJ_o\right] \qquad (40C)$$

We have now determined the equations necessary for the solution of the sin and cos components for any strand in any of the coil arrangements of the types shown in Figs. 1 and 2. The solution is made at once from equation (37) and the conditions that $\sum_o^n {}_sJ = \text{zero}$ and $\sum_o^n {}_cJ = J_o$ by the aid of determinants.

Equation (37) will be simplified if we note that when $c = 0$ we have a solid bar and that when c is not zero that $(p + c)$ and $(q + c)$ are substitutes for p and q and $(d + 2c)$ and $(e + 2c)$ for d and e. We can omit all reference to c, therefore provided we introduce it later on as indicated above.

Our determinant for two strands would be.

$$\left\| \begin{array}{ccccc} p, & -(d-p), & q, & -(e-q) & K_1 \\ I, & 1 & o, & o & o \\ q, & -(e-q), & p, & (d-p) & K_2 \\ o, & o, & 1, & 1 & J_o \end{array} \right\|$$

And the solution is

$$\left.\begin{aligned}
{}_sJ_2 &= \frac{K_1\,d + K_2\,e + J_o(p\,e - d\,q)}{d^2 + c^1} \\
{}_sJ_1 &= -\,{}_sJ_2 \\
{}_cJ_1 &= \frac{-\,K_{1\,e} + K_2\,d + J_o(d\,p + e\,q)}{d^2 + e^2} \\
{}_cJ_2 &= J_o - {}_cJ_1
\end{aligned}\right\}\text{(41)}$$

Where K_1 and K_2 come from terminal conditions for three strands our determinant is

$$\left\|\begin{array}{ccccccc}
p & (d-p) & -(d-2\,p) & q & -(e-q) & -(e-2\,q) & K_1 \\
o & p & -(d-p) & o & q & -(e-q) & K_1 \\
1 & 1 & 1 & o & o & o & o \\
q & -(e-q) & -(e-2\,q) & p & (d-p) & (d-2p) & K_2 \\
o & q & -(e-q) & o & -p & d-p & K_2 \\
o & o & o & 1 & 1 & 1 & J_o
\end{array}\right\|$$

We will use the following expressions in the solution

$E = d\,p + e\,q$
$F = p\,e - d\,q$
$Q = d^2 + e^2$
$R = p^2 + q^2$
$V - 2\,E\,F$
$W - F(Q + R)$
$X = Q^2 + R^2 + 2\,F^2 - Z\,E^2$
$Y = F^2 + R^2 - E^2$
$Z = E(Q - R)$

In the equations for E, F, Q, R, etc. we must remember that for laminated windings we have to add c to p and q and 2 c to d and e in evaluating these symbols.

With this understanding we give the equations for the sin and cos components of the three strands. The evaluation of these terms depends on the terminal conditions affecting K_1 and K_2

$$\left.\begin{aligned}
{}_sJ_3 &= \frac{W\,J_o + [K_1(d-p) + K_2(e-q)]\,[Q+R+2\,E]}{X} \\
{}_sJ_2 &= -\,\frac{W - V}{X}\,J_o \\
{}_sJ_1 &= -\,\frac{V}{X}\,J_o - \frac{[K_1(d-p)+K_2(e-q)]\,[Q+R+2E]}{X} \\
{}_cJ_3 &= \frac{X-Z}{X}\,J_o + \frac{[K_1(e-q)-K_2(d-p)]\,[Q+R+2E]}{X} \\
{}_cJ_2 &= \frac{Y+Z}{X}\,J_o \\
{}_cJ_1 &= -\,\frac{Y}{X}\,J_o - \frac{[K_1(e-q)-K_2(d-p)]\,[Q+R+2E]}{X}
\end{aligned}\right\}\text{(42)}$$

If we let Jx represent the current in any strand the *xth* and J_o the conductor current then we can write from (21)

$\frac{J_x^2}{n^2} R = \left[\left\{ J_x I_b \cos(\theta_x - \phi_b) + I_b^2 \right\} M_s + J_x^2 O_s \right]$ as loss ratio for *xth* strand and for the conductor

$$\left.\begin{array}{l} \frac{J_o^2}{n^2} R = \frac{1}{n} \sum_o^n \left\{ \left[{}_sJ_x\, {}_sI_b + {}_cJ_x\, {}_cI_b + {}_sI_b^2 + {}_cI_b^2 \right] M_s + \right. \\ \quad \left[{}_sJ_x^2 + {}_cJ_x^2 \right] O_s \end{array} \right\} (43)$$

By substituting the values of sin and cos components of the strand currents from equations (41) and (42) in equation (43) we can secure the average value of the losses for various stranding conditions shown by different values of K_1 and K_2

7—EDDY CURRENTS IN CONDUCTORS OF TWO STRANDS

The average loss in top and bottom coils in a slot having two coil sides is given below, covering the case where the conductors are solid, and where the currents in the bottom half of the slots are displaced from those in the top half by the angle β.

The expression for loss in any one bar is

$$R = \frac{1}{J_o^2} \left\{ \left[J_o I_b \cos(\theta_o - \phi_b) + I_b^2 \right] M_s + J_o^2 O_s \right\} \quad (44)$$

If the bar is located above the center line of the slot in the position y, then

$$J_o \cos\theta_o = J_o \qquad J_o \sin\theta_o = 0$$

$$I_b \cos\phi_b = \frac{m}{2} J_o \cos\beta + (y-1) J_o \quad I_b \sin\phi_b = \frac{m}{2} J_o \sin\beta$$

$$R = \left\{ \frac{m}{2} \cos\beta + (y-1) + \frac{m^2}{4} + m(y-1)\cos\beta \right.$$

$$\left. + (y-1)^2 \right\} M_s + O_s \quad (45)$$

This is the equation for the loss ratio in the yth bar above the center line of the slot.

The loss ratio for the top half of the slot is $\frac{2}{m} \sum_o^{\frac{m}{2}} R = R_t$

$$R_t = \left[\frac{m^2 - 1}{3} + \frac{m^2}{4} \cos\beta \right] M_s + O_s \quad (46)$$

The general equation for the loss ratio in the bottom half of the slot is the same as (**45**) except that the terms containing sin β and cos β are zero.

$$R_b = \left[\frac{m^2-4}{12}\right] M_s + O_s \qquad \textbf{(47)}$$

The $\sum\limits_{1}^{m} (x) = \dfrac{m(m+1)}{2}$

and the $\sum\limits_{1}^{m} x^2 = \dfrac{1}{6}\, m(m+1)(2m+1)$

We have developed the formulas (46) and (47) as we wish to use them for standard reference as regards the form for the equations developed for conductors of two strands.

Calculation of the loss ratio for any conductor, say the y conductor, above the center line of the slot, with m total conductors, each consisting of two strands in parallel, is given below. The general case is chosen where the strands are short-circuited at the end of each conductor, and when there are two coil sides per slot with the current in the bottom half of the slot, β degrees out of phase with the currents in the top half. The loss ratio for any strand is given in (44). We have, therefore, for the first and second strands of the y conductor above the center line of the slot

$$\begin{aligned}
&\mathrm{M}_s \Big[{}_sJ_2 \left({}_sJ_1 + \frac{m}{2}\, J_o \sin\beta\right) + {}_cJ_2 \left({}_cJ_1 + \frac{m}{2}\, J_o \cos\beta \right.\\
&+ (y-1)\, J_o + \left({}_sJ_1 + \frac{m}{2}\, J_o \sin\beta\, \mathrm{M}^2\right)\\
&+ \left({}_cJ_1 + \frac{m}{2}\, J_o \cos\beta + (y-1)\, J_o\right)^2 \Big]\, {}_sJ^2_2 + {}_cJ_2^2]\, O_s\\
&+ M_s \Big[{}_sJ_1 \left(\frac{m}{2}\, J_o \sin\beta \right) + {}_sJ_1 \left(\frac{m}{2}\, J_o \cos\beta \right.\\
&\left. + (y-1)\, J_o \right) + \left(\frac{m}{2}\, J_o \sin\beta \right)^2 + \left(\frac{m}{2}\, J_o \cos\beta \right.\\
&\left. + (y - 1J_o) \right)^2 \Big] + \Big[{}_sJ_1 + {}_cJ_1 \Big]\, O_s = R\,\frac{J_o^2}{n}
\end{aligned}$$

Since ${}_sJ_1 + {}_sJ_2 + = O$ and ${}_cJ_1 + {}_cJ_2 = J_o$ we can reassemble the above and simplify to an expression in which we keep separate the terms which involve cos β and sin β

$$\frac{R\,J_o^2}{n} = M_s \Big[J_o\, {}_cJ_1 (2y-1) + J_o^2 (2y^2 - 3y + 1)$$

$$+ J_o m \left({}_cJ_1 \cos\beta + {}_sJ_1 \sin\beta \right) + J_o^2 \left(\frac{m}{2} \cos\beta \right.$$

$$\left. + \frac{m^2}{2} + 2(y-1)\, m \cos\beta \right) \Big]$$

$$+ \left[J_o^2 - 2\, J_o\, {}_cJ_1 + 2\, {}_cJ_1^2 + 2\, {}_sJ_1^2 \right] O_s \tag{48}$$

In evaluating the above ${}_sJ_1$ and ${}_cJ_1$ are taken from (**41**) and k_1 and k_2 are taken from (**38**) for the top half of the slot.

Let $\dfrac{p\,e - d\,q}{d^2 + e^2} = H$ and $\dfrac{d\,p + e\,q =}{d^2 + e^2} = L$

Then (**48**) after substitution becomes

$$R\, \frac{J_o^2}{n} = {}_1J\, Ms \Big\{ (L\, m^2 + 2\, L\, m \cos\beta + L)$$

$$+ (y-1)(4\, L\, m \cos\beta + 4\, L) + (y-1)^2\, 4\, L \Big\}$$

$$+ J_o^2 \Big\} 1 + m \cos\beta - 4\, L\, m \cos\beta - 2\, L$$

$$+ 2\, H^2 m^2 + \frac{m^2}{2} + 2\, L^2 m^2 - 2\, L\, m^2 + 2\, L^2$$

$$+ 4\, L\, m^2 \cos\beta + 2\, H^2 + 4\, H^2 m \cos\beta$$

$$+ (y-1)(2\, m \cos\beta - 8\, L\, m \cos\beta - 8\, L$$

$$+ 8\, L^2 m \cos\beta + 8\, L^2 + 8\, H^2 m \cos\beta$$

$$+ 8\, H^2 + 2) + (y-1)^2 (2 + 8\, L^2 - 8\, L + 8\, H^2) \Big\} O_s$$

and $\quad R_t = \dfrac{m}{2} \overset{\Sigma}{\underset{1}{}} R \quad$ which reduces to

$$\frac{R_t}{n} = \left[\frac{4}{3} L\, m^2 + L\, m^2 \cos\beta - \frac{1}{3} L \right] M_s$$

$$+ \Bigg[(H^2 + L^2) \left(\frac{8}{3} m^2 + 2\, m^2 \cos\beta - \frac{2}{3} \right) - \left(\frac{8}{3} L\, m^2 \right.$$

$$\left. + 2\, L\, m^2 \cos\beta - \frac{2}{3} L \right) + \frac{2}{3} m^2 + \frac{m^2}{2} \cos\beta$$

$$\left. + \frac{1}{3} \right) \Bigg] Os$$

If we reduce this formula to the same form as (**46**)

$$\frac{R_t}{n} = 4 \left[\frac{m^2 - 1}{3} + \frac{m^2}{4} \cos\beta \right] \Big[L\, M_s \Big] + L\, M_s$$

$$+ 4 \left[\frac{m^2 - 1}{3} + \frac{m^2}{4} \cos\beta \right] \Big[2(H^2 + L^2) - 2\, L$$

$$+ \tfrac{1}{2} \Big] O_s + \Big[2(H^2 + L^2) - 2\, L + 1 \Big] O_s$$

$$R_t = \left[\frac{m^2 - 1}{3} + \frac{m^2}{4} \cos\beta\right] 4\,n \left[L\,M_s\right.$$

$$\left. + 2(H^2 + L^2)\,O_s - 2\,L\,O_s + \tfrac{1}{2}\,O_s\right]$$

$$+ \,n \left[L\,M_s + 2(H^2 + L^2)\,O_s - 2\,L\,O_s + O_s\right] \qquad (49)$$

Let $4\,n \left[L\,M_s + 2(H^2 + L^2)\,O_s - 2\,L\,O_s + \tfrac{1}{2}\,O_s\right] = M^1{}_c$

Let $n \left[L\,M_s + 2(H^2 + L^2)\,O_s - 2\,L\,O_s + O_s\right] = O^1$

$$R_t = \left[\frac{m^2 - 1}{3} + \frac{m^2}{4} \cos\beta\right] M^1{}_c + O^1{}_c$$

Similarly, for the bottom half of the slot, our general term is the same as (48), except that those terms which have a factor sin β or cos β are zero.

$$R\,\frac{J_o^2}{n} = M_s\left[J_o\,{}_cJ_1(2\,y - 1) + J_o^2(2\,y_2 - 3\,y + 1)\right]$$

$$+ \left[J_o^2 - 2\,J_o\,{}_cJ_1 + 2\,{}_cJ_1^2 + 2\,{}_sJ_1^2\right] O_s$$

s the general term for the bottom half of the slot. In evaluating this expression, ${}_sJ_1$ and ${}_sJ_2$ are taken from (41), and K_1 and K_2 from (38A).

$$R_b = \left[\frac{\mathrm{M}m^2 - 4}{12}\right] M^1{}_c + O^1{}_c$$

which is of the same form as equation (47). Equations (49) and (50) can be simplified by following out the same method of development which we have used in Part 5 and by limiting $\alpha\,h$ to values less than unity.

$$H = \frac{(p + c)(e + 2\,c) - (q + c)(d + 2\,c)}{(d + 2c)^2 + (e + 2\,c)^2}$$

$$L = \frac{(p + c)(d + 2\,c) + (q + c)(e + 2\,c)}{(d + 2\,c)^2 + (e + 2\,c)^2}$$

$$2\,c = \frac{\gamma - 1}{\alpha\,h}(p^2 + q^2) = 2\alpha\,h(\gamma - 1)$$

$$p^2 + q^2 = \frac{\cosh 2\alpha\,h - \cos 2\alpha\,h}{2}$$

$$= \tfrac{1}{2}\left[\left(1 + \frac{2^2(\alpha\,h)^2}{\underline{|2}} + \frac{(2\alpha\,h)^4}{\underline{|4}} + \right)\right.$$

$$-\left(1-\frac{(2\alpha h)^2}{\lfloor\underline{2}}+\frac{(2\alpha h)^4}{\lfloor\underline{4}}-1\right)\Bigg]$$

$$=2(\alpha h)^2+\frac{8}{90}(\alpha h)^6,\text{ etc.},=2(\alpha h)^2\text{ approx.}$$

$$H \qquad =\frac{4(\alpha h)^4\gamma}{8(\alpha h)^2\gamma^2}=\tfrac{1}{2}\frac{(\alpha h)^2}{\gamma}$$

$$L \qquad =\frac{4(\alpha h)^2\gamma^2}{8(\alpha h)^2\gamma^2}=\tfrac{1}{2}$$

$$p \qquad =\cosh\alpha h\sin\alpha h.$$

$$q \qquad =\sinh\alpha h\cos\alpha h.$$

$$d \qquad =\sin 2\alpha h.$$

$$e \qquad =\sinh 2\alpha h.$$

When the stranding of the straight up coil is continuous, the general term is the same as equation (48), omitting the β terms and counting y from the bottom of the slot.

$$\mathrm{R}\frac{J_o^2}{n}=M_s\Big[J_o\,{}_cJ_1(2y-1)+J_o^2(2y^2-3y+1)\Big]$$

$$+\Big[J_o^2-2J_o\,{}_cJ_1+2\,{}_cJ_1^2+2\,{}_sJ_1^2\Big]O_s$$

$$K_1=(e-2q)\left(\frac{m-1}{2}\right)J_o$$

$$K_2=-(d-2p)\left(\frac{m-1}{2}\right)J_o$$

$$R_{av}\frac{J_o^2}{n}=\frac{1}{m}\sum_1^m R$$

$$R_{av}=n^2\left[\frac{m^2-1}{12}\right]M_s+\left[\frac{m^2-1}{4}\right]M^1_c+O^1_c$$

For a coil of two strands "turned over" and short-circuited at the end of each full turn with currents in the bottom half of the slot displaced $\beta°$ K_1 and K_2 are taken from (40). The general equation for the loss ratio in the y conductor above the center line is (48), and after substituting and $\sum\limits_1^{\frac{m}{2}} R$, the average value in the top of the slot will be

$$R_t=n\,M_s\left[\frac{H\,m^2}{4}\sin\beta\right]+n^2M_s\left[\frac{m^2-1}{3}-\frac{m^2-4}{12}\right.$$

$$\left.+\frac{m^2}{4}\cos\beta\right]+\left[\frac{m^2-4}{12}\right]M^1_c+O^1_c$$

The general equation for R in bottom of the slot is the same as (48), except that J_1 and J_2 are interchanged, and that all terms containing $\sin\beta$ or $\cos\beta$ are zero.

$$R_b = n\,M_s\left[-\frac{H\,m^2}{4}\sin\beta\right] + M_e\left[\frac{m^2-4}{12}\right] + O_e$$

In the case of turned-over coils, where the stranding is continuous from the start to the finish of the coil, and there are two coils per slot, K_1 and K_2 are taken from (40B).

$$R_t = n\,M_s\left[\frac{H\,m^2}{4}\sin\beta\right] + n^2\,M_s\left[\frac{m^2-1}{3} - \frac{m^2-4}{16} + \frac{m^2}{4}\cos\beta\right] + \left[\frac{m^2-4}{16}\right]M^1_c + O^1_c$$

$$R_b = n\,M_s\left[-\frac{H\,m^2}{4}\sin\beta\right] + n^2\,M_s\left[\frac{n^2-4}{12} - \frac{n^2-4}{16}\right] + \left[\frac{m^2-4}{16}\right]M^1_c + O^1_c$$

In the case of one coil per slot turned over, short-circuited at the end of each turn, K_1 and K_2 are from (40A); $\frac{m}{2}$ is the number of conductors per slot.

If $2\,m_1 = m$ where m_1 is the number of conductors per slot, then

$$R_{av} = n^2\,M_s\left[\frac{m_1-1}{3} - \frac{m_1-4}{12}\right] + \left[\frac{m^2-4}{12}\right]M^1_c + O^1_c$$

For one coil per slot turned over, and stranded continuous, K_1 and K_2 are taken from (40C).

$$R_{av} = n\,M_s\left[\frac{m^2-1}{6}\right] + n\,\tfrac{1}{2}\,O_s.$$

Where m is twice the number of conductors per slot,

$$R_{av} = M_s\left(\frac{s^2-1}{3}\right) + O_s$$

Where s is the total number of strands per coil depthwise.

8—EDDY CURRENTS IN CONDUCTORS OF THREE STRANDS

Consider the case of two coils per slot where there are m total conductors and the currents in the bottom conductor are displaced by an angle from those in the top conductors, let each conductor consist of three strands depthwise and let all strands be short-circuited at each end of the conductor. Take the case of the yth conductor above the center-line of the slot

$$\begin{aligned}
&M_s \biggl\{ {}_sJ_3 \biggl[{}_sJ_2 + {}_sJ_1 + \frac{m}{2} J_o \sin \beta \biggr] + {}_cJ_3 \biggl[{}_cJ_2 + {}_cJ_1 \\
&+ (y-1) J_o + \frac{m}{2} J_o \cos \beta \biggr] + \biggl[{}_sJ_2 + {}_sJ_1 \\
&+ \frac{m}{2} J_o \sin \beta \biggr]^2 + \biggl[{}_cJ_2 + {}_cJ_1 + (y-1) J_o \\
&+ \frac{m}{2} J_o \cos \beta \biggr]^2 \biggr\} + \biggl\{ {}_sJ_3^2 + {}_cJ_3^2 \biggr\} O_s + M_s \biggl\{ {}_sJ_2 \biggl[{}_sJ_1 \\
&+ \frac{m}{2} J_o \sin \beta \biggr] + {}_cJ_2 \biggl[{}_cJ_1 + (y-1) J_o + \frac{m}{2} J_o \cos \beta \biggr] \\
&+ \biggl[{}_sJ_1 + \frac{m}{2} J_o \sin \beta \biggr]^2 + \biggl[{}_cJ_1 + (y-1) J_o \\
&+ \frac{m}{2} J_o \cos \mathrm{B} \biggr]^2 \biggr\} + \biggl\{ {}_cJ_2^2 + {}_sJ_2^2 \biggr\} O_s \\
&+ M_s \biggl\{ {}_sJ_1 \biggl[\frac{m}{2} J_o \sin \beta \biggr] + {}_cJ_1 \biggl[(y-1) J_o \\
&+ \frac{m}{2} J_o \cos \beta \biggr] + \biggl[\frac{m}{2} J_o \sin \beta \biggr]^2 + \biggl[(y-1) J_o \\
&+ \frac{m}{2} J_o \cos \beta \biggr]^2 \biggr\} + \biggl[{}_sJ_1^2 + {}_cJ_1^2 \biggr] O_s = \frac{R J_o^2}{n}
\end{aligned}$$

Since ${}_sJ_1 + {}_sJ_2 + {}_sJ_3 = 0$ and ${}_cJ_1 + {}_cJ_2 + {}_cJ_3 = J_o$ we can rewrite this

$$\begin{aligned}
&M_s \biggl\{ \biggl[- {}_sJ_1 \, {}_sJ_3 + J_o (2 \, {}_cJ_1 + {}_cJ_2) - {}_cJ_1 \, {}_cJ_3 \\
&+ 2 (y-1) J_o (2 \, {}_cJ_1 + {}_cJ_2) + (y-1) J_o^2 \\
&+ 3 (y-1)^2 J_o^2 \biggr] + \biggl[(2 \, {}_sJ_1 + {}_sJ_2) \, m J_o \sin \beta
\end{aligned}$$

$$+ \tfrac{3}{4} m^2 J_o^2 (\sin^2 \beta + \cos^2 \beta) + \frac{m}{2} J_o \cos \beta$$

$$\left. \left. + (2\, {}_cJ_1 + {}_cJ_2)\, m\, J_o \cos \beta + 3\, m\, (y - 1)\, J_o \cos \beta \right] \right\}$$

$$+ \left\{ {}_sJ_1^2 + {}_sJ_2^2 + {}_sJ_3^2 + {}_cJ_1^2 + {}_cJ_2^2 + {}_cJ_3^2 \right\} O_s = \frac{R\, J_o^2}{n} \quad (60)$$

This equation is for the loss in the yth conductor above the center-line of the slot; y is counted from the center-line upwards. If we want to consider a term below the center-line it is the same for straight up coil except that y is counted from the bottom, the β terms are zero. For a turned over coil below the center-line the expression changes in the M_s term due to the fact that the position of J_3 and J_1 are interchanged, that $\left(\frac{m}{2} - y\right)$ replaces $(y - 1)$ and also that β terms are zero.

Bottom term is

$$M_s \left\{ - {}_sJ_1\, {}_sJ_3 + J_o (2\, {}_cJ_3 + {}_cJ_2) - {}_cJ_1\, {}_cJ_3 \right.$$

$$+ 2 \left(\frac{m}{2} - y\right) J_o (2\, {}_cJ_3 + {}_cJ_2) + \left(\frac{m}{2} - y\right) J_o^2$$

$$\left. + 3 \left(\frac{m}{2} - y\right) {}^2J_o^2 \right\} + \left\{ {}_sJ_1^2 + {}_sJ_2^2 + {}_sJ_3^2 \right.$$

$$\left. + {}_cJ_1^2 + {}_cJ_2^2 + {}_cJ_3^2 \right\} O_s = \frac{R\, J_o}{n} \qquad (61)$$

For the case of a conductor short-circuited at each end we have the values of ${}_sJ_1$, ${}_sJ_2$, etc., from formula (42) where K_1 and K_2 are defined by (38) for the top half of the slot, and by 38-a for the bottom half.

The following transformations are convenient

$$\left[(d - p)(d - 2p) + (e - q)(e - 2q) \right]$$

$$\left[Q + R + 2E \right] = X + Y - Z$$

$$\left[(d - p)(e - 2q) - (e - q)(d - 2p) \right]$$

$$\left[Q + R + 2E \right]. = W + V$$

$$\frac{X+Y-Z}{X} = 1 - \frac{Z-Y}{X}$$

$$\frac{Z-Y}{X} = \frac{E\,(Q-R) - (F^2 + R^2 - E^2)}{Q^2 + R^2 + F^2 - 2\,E^2}$$

Where d and e are written for $(d + 2\,c)$ and $(e + 2\,c)$ and p and q for $p + c$ and $q + c$. By developing in series it will be found that the above reduce to

$$\frac{Z-Y}{X} = \frac{4\,(\alpha\,h)^2\gamma^2\Big[8\,(\alpha\,h)^2\gamma^2 - 2\,(\alpha\,h)^2\gamma^2\Big] + 16\,(\alpha\,h)^4\gamma^4 - 4\,(\alpha\,h)^4\gamma^4}{64\,(\alpha\,h)^4\gamma^4 + 4\,(\alpha\,h)^4\gamma^4 - 32\,(\alpha\,h)^4\gamma^4} = 1$$

$$\text{and } \frac{X+Y-Z}{X} = 0$$

The average value $\sum\limits_{1}^{\frac{m}{2}} R$ in formula (60) gives

$$n\,M_s\left\{\frac{WV+XY-YZ}{X^2} + \left(\frac{W+V}{X}\right)^2\left[\frac{m^{2-1}}{3} + \frac{m^2}{4}\cos\beta\right] + m^2 + \frac{3}{4}\,m^2\cos\beta\right\} + \left\{\left(\frac{X-Z}{X}\right)^2 + \left(\frac{Y+Z}{X}\right)^2 + \left(\frac{W-V}{X}\right)^2 + \frac{W^2+V^2+Y^2}{X^2} + 2\left(\frac{W+V}{X}\right)^2\left(\frac{m^2-1}{3} + \frac{m^2}{4}\cos\beta\right)\right\}O_s\,n = R_t$$

$$\left[\frac{m^2-1}{3} + \frac{m^2}{4}\cos\beta\right]\left\{M_s\,n\left[\left(\frac{W+V}{X}\right)^2 + 3\right] + 2\left(\frac{W+V}{X}\right)^2 O_s\,n\right\} + \left\{\frac{WV+XZ-YZ}{X^2}\,M_s + \left[\left(\frac{Y+Z}{X}\right)^2 + \left(\frac{X-Z}{X}\right)^2 + \left(\frac{W-V}{X}\right)^2 + \frac{W^2+V^2+Y^2}{X^2}\right]O_s\right\} n = R_t \qquad (61)$$

Let $n\left\{M_s\left[\left(\frac{W+V}{X}\right)^2 + 3\right] + 2\left(\frac{W+V}{X}\right)^2 O_s\right\} = M''_s$

$$\text{and}\left\{\frac{WV+XZ-YZ}{X^2}+\left[\left(\frac{Y+Z}{X}\right)^2+\left(\frac{Y-Z}{X}\right)^2\right.\right.$$

$$\left.\left.+\left(\frac{W-V}{X}\right)^2\frac{+W^2+V^2+Y^2}{X^2}\right]O_s\right\}n=O''_e$$

$$\text{Then } R_t=\left[\frac{m^2-1}{3}+\frac{M^2}{4}\cos\beta\right]M''_e+O''_e \tag{62}$$

Which is of the same form as formula (51) for two strands. Similarly

$$R_b=\left(\frac{m^2-4}{12}\right)M''_e+O''_e \tag{63}$$

The same form as (52)

For three strands wound continuous in a straight up coil with one coil per slot use equation (60) except that the β terms are zero and that Y is measured from the bottom of the slot.

To get the average term we have $\frac{1}{m}\sum_1^m R$ and the values of k_1 and k_2 are taken from

$$k_1=(e-2q)\left(\frac{m-1}{2}\right)J_o,\quad k_2=-(d-2p)\left(\frac{m-1}{2}\right)J_o$$

The equation reduces to

$$n^2M_s\left(\frac{m^2-1}{12}\right)+\left(\frac{m^2-1}{4}\right)M''_e+O''_e=R \tag{64}$$

For three strands per conductor with a turned over coil short-circuited at end of each turn. k_1 and k_2 are taken from (40).

R for top of the slot is taken from (60) and for bottom of slot is taken from (61).

$$\left.\begin{aligned}R_t&=n\,M_s\left\{\frac{W+V}{X}\,\frac{m^2}{4}\sin \mathrm{B}\right\}+n^2M_s\left\{\frac{m^2-1}{3}\right.\\&\quad\left.-\frac{m^2-4}{12}+\frac{m^2}{4}\cos\beta\right\}+\frac{m^2-4}{12}M''_e+O''_e\\R_b&=n\,M_s\left\{-\frac{W+V}{X}\,\frac{m^2}{4}\sin\beta\right\}+\frac{m^2-4}{12}M''_e+O''_e\end{aligned}\right\} \tag{65}$$

For three strands, turned over coil with continuous stranding when there are two coils per slot take k_1 and k from 40A. R for the top half of the slot is taken from (60) and for the bottom from (61).

$$\left.\begin{aligned} R_t &= n\,M_s\left\{\frac{W+V}{X}\,\frac{m^2}{4}\sin\beta\right\} + n^2 M_s\left\{\frac{m^2-1}{3}\right. \\ &\quad \left. -\frac{m^2-4}{16} + \frac{m^2}{4}\cos\beta\right\} + \frac{m^2-4}{16}M_c'' + O_c'' \\ R_b &= n\,M_s\left\{-\frac{W+V}{X}\,\frac{m^2}{4}\sin\beta\right\} + n^2 M_s\left\{\frac{m^2-4}{12}\right. \\ &\quad \left. -\frac{m^2-4}{16}\right\} + \frac{m^2-4}{16}M_c'' + O_c'' \end{aligned}\right\} \quad (66)$$

For three strands, turned over coil, short-circuited each end with one coil per slot take

$$k_1 = (e-2\,q)\,(Y-\frac{m}{4}-1)\,J_o$$

$$k_2 = -(d-2\,p)\,(Y-\frac{m}{4}-1)\,J_o$$

$$\frac{m}{2} = \text{number conductors}$$

$$R = n^2 M_s\left\{\frac{m^2-4}{12} - \frac{m^2-16}{48}\right\}$$

$$+\frac{m^2-16}{48}M_c'' + O_c'' \quad \text{if } 2\,m_1 = m$$

$$R = n^2 M_s\left\{\frac{m_1^2-1}{3} - \frac{m_1^2-4}{12}\right\}$$

$$+\frac{m_1^2-4}{12}M_c'' + O_c'' \qquad (67)$$

For continuous stranding, three strands per conductor, one coil per slot and turned over, take

$$k_1 = (e-2\,q)\,(-\tfrac{1}{2})\,J_o$$
$$k_2 = (d-2\,p)\,\tfrac{1}{2}\,J_o$$

The average value of R is

$$nM_s\left[\frac{WV+XZ-YZ}{X^2}+\frac{m}{4}-1-\tfrac{1}{4}\left(\frac{W+V}{X}\right)^2\right]$$

$$+\left[\left(\frac{X-Z}{X}\right)^2+\left(\frac{Y+Z}{X}\right)^2+\left(\frac{W-V}{X}\right)^2\right.$$

$$\left.+\frac{W^2+V^2+W^2}{X^2}-\tfrac{1}{2}\left(\frac{W+V}{X}\right)^2\right]nO_s \qquad (68)$$

To reduce the above expression to a convenient form it is desirable to use the development in series for $p - q - d - e$, etc., and to limit αh to values < 1. The following formula will be found convenient:

$$\begin{aligned}
E &= 4(\alpha h)^2\gamma^2\\
F &= 4(\alpha h)^4\gamma\\
Q &= 8(\alpha h)^2\gamma^2\\
R &= 2(\alpha h)^2\gamma^2\\
V &= 32(\alpha h)^6\gamma^3\\
W &= 40(\alpha h^6\gamma^3\\
X &= 36(\alpha h)^4\gamma^4\\
Y &= -12(\alpha h)^4\gamma^4\\
Z &= 24(\alpha h)^4\gamma^4
\end{aligned}$$

$$\begin{aligned}
&\frac{X+Y+Z}{x}=0\\
&\frac{W+V}{x}=2\frac{(\alpha h)^2}{r}\\
&\frac{Y+Z-Y}{x}=2\\
&\frac{X-Z}{X}=\tfrac{1}{3}\\
&\frac{W-V}{X}=\frac{2}{9}\frac{(\alpha h)^2}{\gamma}\\
&\frac{W}{X}=\frac{10}{9}\frac{(\alpha h)^2}{\gamma}
\end{aligned}$$

$$\begin{aligned}
&\frac{V}{x}=\frac{8}{9}\frac{(\alpha h)^2}{r}\\
&\frac{Y}{x}=-\tfrac{1}{3}\\
&\frac{WV}{X^2}=\frac{80}{81}\frac{(\alpha h)^4}{\gamma^2}\\
&\frac{XZ}{X^2}=\tfrac{2}{3}\\
&\frac{-YZ}{X^2}=\tfrac{2}{9}
\end{aligned}$$

$$\begin{aligned}
M_s &= \tfrac{1}{3}(\alpha h)^4\\
L &= \tfrac{1}{2}\\
H &= \frac{(\alpha h)^2}{2\gamma}\\
O_s &= 1+\frac{8}{90}(\alpha h)^4
\end{aligned}$$

Substituting in (68) we get.

$$3M_s\left[\frac{m^2}{4}-\frac{1}{9}\right]+\left[1+\frac{2}{9}\frac{(\alpha h)^4}{\gamma^2}\right]O_s = R$$

$\frac{m}{2}$ is the number of coils per slot. If S is the number of strands per slot depthwise then $m = \frac{2}{3}S$ $m^2 = \frac{4}{9}S^2$ and

$$M_s\left[\frac{s^2-1}{3}\right]+O_s = R \text{ average,}$$

where S is number of strands depthwise. (69)

9—SUMMARY OF FORMULAS

We have shown that the formula for the loss in windings due to eddy currents has the same general form for solid bars and bars of two and three strands and for infinite number of strands.

Consider whether we can develop the equivalent depth of a solid conductor to replace the stranded one

$$M'_c = 4\,n\,[L\,M_s + 2\,H^2\,O_s]$$

$$M'_c = n\left\{M_s\left[\left(\frac{W+V}{X}\right)^2 + 3\right] + 2\left[\frac{W+V}{X}\right]^2 O_s\right\}$$

$$M' = 4\left[M_s + \frac{3\,M_s}{\gamma_2} + \frac{24}{90}\,\frac{M^2_s}{\gamma_2}\right] \text{ approx.}$$

$$M''_c = 3\,M_s\left[\frac{12\,M_s}{\gamma^2} + 3\right] + 72\,\frac{M_s^2}{\gamma^2}$$

Since $M_s = \frac{(\alpha h)^4}{3}$ let $M = n^4\,M_s$ where M corresponds to $n\alpha\,h$. Then $M = M_c\left[\frac{1}{n^2} + \frac{n^2-1}{n^2}\,\frac{1}{\gamma_2}\right]$

$$M''_c = M\left[\frac{1}{n^2} + \frac{n^2-1}{n^2}\,\frac{1}{\gamma^2}\right]$$

We have therefore a value from our M curve which is to be multiplied by $\left[\frac{1}{n^2} + \frac{n^2-1}{n_2}\,\frac{1}{\gamma^2}\right]$ or we can use the value $\alpha\,f\left[\frac{1}{n^2} + \frac{n^2-1}{n_2}\,\frac{1}{\gamma^2}\right]^{\frac{1}{4}}$ to get the equivalent solid conductor to replace the stranded one. O'_c and O'' follows the same law, a proof is not readily demonstrated but in any event the error is small.

If we consider the limiting conditions where the number of strands are one or infinite we find that our equivalent depth fits both cases. For a solid bar the term containing γ is zero and for an infinite number of laminations $\frac{\alpha f}{\sqrt{\gamma}}$ is the equivalent solid bar a result found previously in Part 4. As a check on the approximations made, the writer calculated the values M'_c, M''_c, O^c_1 and O''_c for two values each of $\alpha\ h$ 0.3 and 0.4 and for $\gamma = 1, 2$ and 3. The tabulated results of these figures which were made on a calculating machine and are correct to five decimal places are tabulated for comparison with the results taken from curves and approximate methods outlined above.

		Calculated $M'c$			Curve Value $M'c$	
		$r = 1$	$r = 2$	$r = 3$	$r = 2$	$r = 3$
0.3	0.6	0.042838	0.018883	0.014390	0.0189	0.0144
0.4	0.8	0.134074	0.059240	0.045107	0.0590	0.0455
		Calculated $M''c$			Curve Valu $M''c$	
0.3	0.9	0.21345	0.07436	0.04570	0.0732	0.0456
0.4	1.2	0.63709	0.23439	0.14299	0.2320	0.1440
		Calculated $O'c$			Curve Value $O'c$	
0.3	0.6	1.011413	1.005425	1.00430	1.0053	1.0040
0.4	0.8	1.035742	1.017034	1.013501	1.0160	1.0120
		Calculated $O''c$			Curve Value $O''c$	
0.3	0.6	1.05671	1.02036	1.01347	1.0195	1.0120
0.4	1.2	1.17066	1.06326	1.04231	1.0620	1.0390

These results show a sufficiently close agreement to permit of their use in any calculations.

It may be of interest to develop the formula for the loss in the end windings under the assumptions which we have made.

It will be noted that the loss in the ends is proportional to

$$[{}_sJ_1^2 + {}_sJ_2^2 + {}_sJ_3^2 + \quad + {}_cJ_1^2 + {}_cJ_2^2 + {}_cJ \ + +]$$

and that this is most readily evaluated as follows:

For two strands we have

$$[K'] \, [L \, M_s + 2 \, H^2 \, O_s] \, 4n + n \, [L \, M_s + 2 \, H^2 \, O_s + \tfrac{1}{2} \, O_s]$$

as loss ratio

Where $[K']$ is coefficient of M'_c and the above can be written $[K'] \; M'_c + O'_c$. The loss on the ends is proportional to the coefficient of O_s and is for two strands,

$$\frac{O_c}{O_s} - \frac{n \, L \, M_s}{O_s} + 4 \, n \, 2 \, H^2 \, [K'] = \text{loss ratio on ends}$$

$$= \frac{O_c}{O_s} + \frac{n^2 - 1}{n^2} \frac{M}{\gamma^2} [K'] \;\; M \text{ is curve value for } \alpha f.$$

For three strands,

$$\frac{O_c}{O_s} - \frac{3 \, M_s}{O_s} \left[\frac{80}{81} \frac{(\alpha h)^4}{\gamma^2} + \frac{8}{9} \right] + \frac{n^2 - 1}{n^2} \frac{M}{\gamma^2} \; [K'] \text{ which is}$$

the same as the above, since $3 \, M_s \left[\frac{80}{81} \frac{(\alpha \, h)^4}{\gamma^2} + \frac{8}{9} \right]$

is negligible. Checking up on this value we find that for one strand per conductor our loss is unity and for infinite strands is of the same value as in the straight part of the coil.

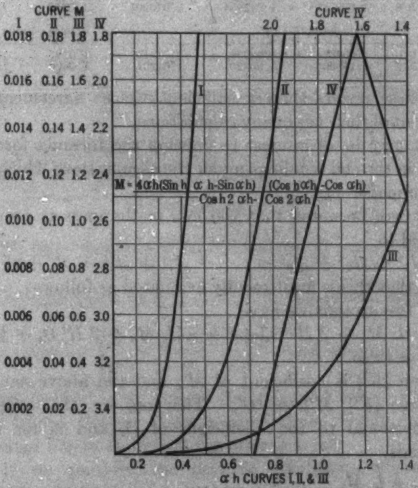
Approximate loss on ends is

$$\frac{O_e}{O_s} + \frac{n^2 - 1}{n^2}\frac{M}{\gamma^2}\left[K^1\right] = \text{ratio}$$

K^1 is coefficient of M_e in loss for embedded length

M is curve value for αf

O_e is curve value for αf equivalent



O_s is curve value for αh

A general summary for loss in the embedded part of coil is

m = the no. of conductors depthwise in a slot

n = the no. of strands depthwise per conductor (equal strands)

γ = the ratio of total conductor length to the embedded part

s = the total number of strands, depthwise

$\alpha = \frac{0.4}{\sqrt{(1 + .00427\,\theta)}}\sqrt{\sim r_1}$ = ratio of copper width to slot width

θ = total temperature of the copper above zero

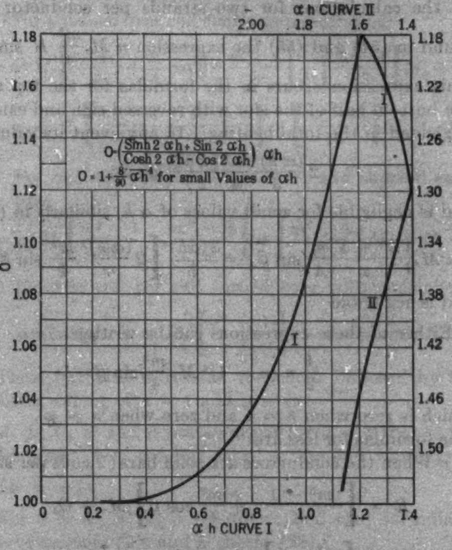
h = the depth of the strand in inches

f = the depth of the conductor in inches

M_s = the curve value for αh

M_e = the curve value for the equivalent value of αf

O_e = the curve value for the equivalent value of αf



β = the angular displacement between the conductor currents of the top and the bottom coils in a slot.

M = the curve value for αf

K^1 = the coefficient M_e

$$M_s = \frac{(\alpha h)^4}{3} \text{ for small values of } \alpha h$$

$$O_s = \text{curve value for } \alpha h = 1 + \frac{8}{90}(\alpha h)^4$$

The equivalent value of a conductor is

$$\alpha f_e = \alpha f \left[\frac{1}{n^2} + \frac{n^2 - 1}{n^2} \cdot \frac{1}{\gamma^2}\right]^{\frac{1}{2}}$$

and is given in the curve.

R_t is the average loss ratio in the top half of a slot when there are two coils per slot.

R_b is the average loss in the bottom.

R is the average loss for the entire slot.

In the calculations for two strands per conductor we found in (54) and (55) the expression $n\,M_s \frac{m^2}{4} H \sin\beta$.

This expression occurs in the formulas for the top and the bottom half of the slot with reversed sign and can be neglected in the total heating. In any event its value is

$$2\,\frac{(\alpha h)^4}{3}\,\frac{m^2}{4}\,\frac{(\alpha h)^2}{2r}\sin\beta$$

and is negligible for small values of αh; similarly in (65)

$$n\,M_s\left\{\frac{W+V}{x}\,\frac{m^2}{4}\sin\beta\right\} = \frac{3(\alpha h)^4}{3}\left[2\,\frac{(\alpha h)^2}{r}\,\frac{m^2}{4}\sin\beta\right]$$

and is negligible

Either of these expressions can be written

$$\frac{1}{2}\,n\,(n-1)^2\,M_s\,\frac{m^2}{4}\sin\beta$$

which is zero when $n = 1$ and zero when $n = \infty$.

The formulas for loss are

(A) When the conductors are solid bars (2 coils per slot)

$$R = \left[\frac{m^2 - 1}{3} + \frac{m^2}{4}\cos\beta\right] M_e + O_e$$

$$R_b = \left[\frac{m^2 - 4}{12}\right] M_e + O_e$$

(B) When the conductor is stranded and shorted at each half turn

$$R_t = \left[\frac{m^2 - 1}{3} + \frac{m^2}{4}\cos\beta\right] M_e + O_e$$

$$R_b = \left[\frac{m^2 - 4}{12}\right] M_e + O_e$$

(C) For a continuous stranded coil wound straight up

$$R = n^2 \left[\frac{m^2 - 1}{12}\right] M_s + \left[\frac{m^2 - 1}{4}\right] M_e + O_e$$

(D) For a turned over coil—shorted each full turn with two coils per slot

$$R_t = n^2 M_s \left[\frac{m^2-1}{3} + \frac{m^2}{4}\cos\beta - \frac{m^2-4}{12}\right] + \left[\frac{m^2-4}{12}\right] M_e + O_e$$

$$R_b = \left[\frac{m^2 - 4}{12}\right] M_e + O_e$$

(E) For a turned over coil continuous stranding

$$R_t = n^2 M_s \left[\frac{m^2-1}{3} + \frac{m^2}{4}\cos\beta - \frac{m^2-4}{16}\right] + \left[\frac{m^2-4}{16}\right] M_e + O_e$$

$$R_b = n^2 M_s \left[\frac{m^2 - 4}{12} - \frac{m^2 - 4}{16}\right] + \left[\frac{m^2 - 4}{16}\right] M_e + O_e$$

(F) For one coil per slot turned over and shorted each turn

$$R = n^2 M_s \left[\frac{m^2 - 1}{3} - \frac{m^2 - 4}{12}\right] + \left[\frac{m^2 - 4}{12}\right] M_e + O_e$$

(G) For one coil per slot turned over with continuous stranding $R = M_s \left[\frac{s^2 - 1}{3}\right] + O_e$ (approx.)

These formulas will stand the check imposed by the limiting values of $n = 1$ and $n = \infty$.

A, B, C, D and E are all equal for $n = 1$ that is for solid bars. Also the loss ratio on the end windings is 1 for $n = 1$.

For $n = \infty$, $n^2 M_s = O$ and we find that formula C checks value in (26) and E checks (28).

Also we see D and E are equal for $m = 2$ and the loss ratio on the ends for $n = \infty$ is the same as the loss ratio in the slots.

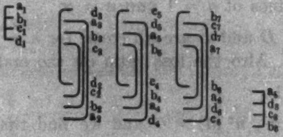
A study of these formulas shows that for a given total copper depth in a slot with two coils per slot that the minimum loss is secured by increasing m and n to the limit.

n is limited by mechanical considerations in manufacture and *m* for machines with a great many poles can be controlled to some extent by the use of multiple turn coils and by transposing the winding in parallel circuits. Such a procedure is in common use but introduces the danger of a line voltage between adjacent conductors with the increased chance of break down. For turbo generators, where only two or four poles are common, a transposition scheme has been developed to give the effect of increasing *m* without introducing multiple circuits. This arrangement consists in subdividing the strands of a conductor depthwise into a number of groups which is a multiple of the slots per pole per phase then interconnecting these groups in such a way that equal voltages will be produced in all the various circuits in parallel.

Consider the case of four slots per pole per phase shown below:

1 3 5 7
top coil *x x x x o o o o . . . . x x x x*
bottom coil *x x x x*
2 4 6 8

The successive positions of a coil are shown as 1, 2, 3, 4, 5, 6, 7, and 8 this being the sequence of conductors in series. Each single conductor is then arranged in four groups of strands, so that a section of the coil group would show as below.

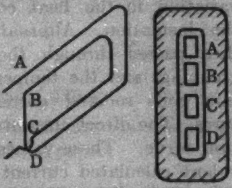


The voltage of the groups *a*, *b*, *c*, *d*, taken through eight conductors are all equal, and a little consideration will show that equal currents must flow. Hence we have the effect in two coils per slot of securing the same current distribution as if we had had eight conductors, each carrying one-quarter of the total current. The loss is given by formula (*D*), Part 9, and is only slightly greater than the loss for (*E*).

This method of winding has been found very useful in reducing the eddy losses in coils of large current capacity.

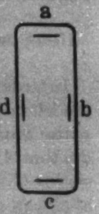
10—COMPARISON OF CALCULATIONS AND TESTS

In order to compare the formulas with test results a model was made up with an adjustable coil and tested with various combinations of windings. Four conductors *A B C D* each consisting of six strands in parallel were wound as shown in the sketch, that is, one end of each coil open and one closed. An iron core surrounded the conductors as shown with this arrangement the m.m.f. of *A B* opposed that of *C* and *D* and coils *A* and *B* have the same relation to each other as the top and bottom coils in a slot.



There is no iron loss except that due to the transverse flux across the slots. With the six strands in each conductor were provided connectors so that the following possible arrangements could be secured: *A* coil of six turns of one strand, three turns of two strands, two turns of three strands or one turn of six strands. In addition, the connectors on the open end were provided with taps so that it was possible to get the effect of all strands of a conductor in series throughout the multiple turn coil or else a short circuit could be made at the end of each full turn or one-half turn. By adjusting the length of the iron we could introduce the factor γ into our formula as a variable. By changing the frequency we could vary $\alpha\, h$ and $\alpha\, f$. So on a single model we were able to check all the formulas except the case of one coil per slot and those covering two coils per slot where the currents in the top and bottom coils were not in phase.

Tests were made as follows: the coils were provided with thermocouples inside on the copper around the periphery of the coil and direct current was passed through the coil until constant temperatures were reached. A calibration curve between temperature and current was then made up. The average temperature of the coil was taken by multiplying the temperature at (*a*) by the surface at (*a*) temperature at (*b*) by surface (*b*), etc., and using the average temperature so obtained. This procedure eliminates any variation in the heat conduction through the insulation. Alternating current was then passed through the coil with various groupings and the temperatures of the top and bottom coils secured corresponding to these temperatures. The direct currents were taken from the calibration curve. These values are compared with the equivalent calculated current obtained from the loss ratio R_t or R_b as the case may be. Since the iron temperature varies with the alternating currents a correction is necessary; the correction formulas are given below. The relation between current and temperature for a given iron temperature is



$$I_2 = I_1 \left[\frac{\theta_2\{1 + (\alpha + \theta_1)\, 0.00427\}}{\theta_1\{1 + (\alpha + O_2)\, 0.00427\}}\right]\frac{1}{2}$$

I_1 is known amperes corresponding to total temperature $(\alpha + \theta_1)$.

α is iron temperature. θ_1 is rise above iron with amperes I_1.

θ_2 is rise above iron for same unknown current and I_2 is unknown current.

Neglecting conduction of heat along the copper the current corresponding to some other iron temperature is

$$I_2 = I_1 \left[\frac{1 + 0.00427\,(\alpha_1 + \theta)}{1 + 0.00427\,(\alpha_2 + \theta)}\right]\frac{1}{2}$$

Where I_2 is desired current for θ_o rise above its iron temperature α_2 and I_1 is the known current for the iron temperature α_1 and rise θ above the iron. For a 10 deg. change in iron temperature, this correction is about 1 per cent. The agreement between the current value taken from the calibration curve and that calculated from the formulas is unusually close, and indicates that the formulas can be relied upon in practise.

The physical dimensions of the test model were as follows:

Each strand $\frac{3}{16}$ in. by $\frac{5}{8}$ in., copper six strands per coil.

Iron length was 40 in. when $r = 2$.

Iron length $26\frac{3}{4}$ in. when $r = 3$.

Opening in iron widthwise was 1.178 in.

The curves for M_s, M_c, O_s, O_c and for the evaluation of αf_s are attached as a part of the paper.

It is evident that these formulas can be used in a number of ways for the special investigation of particular problems in design. For example, curves can be made up showing that beyond a certain point it is useless to increase the copper depth in a slot in order to secure a lower temperature. The heating effect of third harmonics in delta-connected windings can be investigated. Also, such problems as are involved in the calculation of the losses in the damper windings of single-phase machines, and phase-converting apparatus. The applications are for the most part obvious.

Current Connection Short. Circ. Value of r.	D-C Series	60 Series	60 Two Strands			60 Three Strands		
	None 2	None 2	None 2	1T 2	½T 2	None 2	1T 2	½T 2
Amperes............	250	210	420	420	385	600	600	500
Temp. Rise..........	78.5°	71.9°	73.3°	74.6°	75.2°	71.7°	74°	74°
Iron Temp..........	53°	49°	44°	44.5°	49°	48°	54°	48°
D.C. Amps..........	....	242	495	496	494	725	731	736
Temp. Correct........	....	1.8°	1.8°	1.8°	2.3°	1.8°	1.8°	3.7°
Calc. R.............	....	1.402	1.428	1.43	1.695	1.497	1.495	2.25
D.C. Equiv..........	....	245	497	500	500	735	738	752
Amps. from R........	....	249	502	502	501	734	733	750
Temp. Rise..........	71°	52.3°	53.6°	54.8°	46.2°	52.7°	55°	34.2°
D.C. Amps..........	....	222	450	454	416	666	675	552
Temp. Correct........	....	1.8°	1.8°	1.8°	2.3°	1.8°	1.8°	3.7°
Calc. R.............	....	1.065	1.10	1.108	1.173	1.168	1.179	1.214
D.C. Equiv..........	....	217	444	445	410	655	660	.....
Amps. from R........	....	216.6	440.5	442.5	406	650	651	551
Diff. Temp..........	7.5°	19.6°	19.7°	19.8°	29°	19°	19°	40°

Current Connection Short Circ. Value of r.	D.C. 60 6 Strands		60 3 Strands		60 6 Strands		40 6 Strands	
	None 2	½T 2	1T 3	½T 3	1T 3	½T 3	1T 3	½T 3
Amperes............	1140	730	624	565	1210	900	1330	1100
Temp. Rise..........	78°	70°	76°	78.5°	81.9°	82.5°	83.8°	80.3°
Iron Temp..........	47.5°	49°	47.5°	54°	53°	54.5°	62°	62°
D.C. Amps..........	1500	1430	745	749	1524	1522	1526	1492
Temp. Correct........	1.8°	4.3°	1.9°	3.1°	2.2°	4.4°	1.9°	3.8°
Calc. R.............	1.718	4.225	1.444	1.741	1.521	2.79	1.232	1.771
D.C. Equiv..........	1524	1480	752	761	1524	1558	1535	1520
Amps. from R........	1492	1500	750	746	1494	1502	1477	1465
Temp. Rise..........	58.5°	23.5°	54.7°	43.5°	57.5°	33.4°	63.1°	37.8°
D.C. Amps..........	1392	933	678	611	1375	1082	1418	1133
Calc. R.............	1.47	1.491	1.122	1.124	1.23	1.241	1.101	1.115
D.C. Equiv..........	1380	846	669	593	1350	1013	1393	1082
Amps. from R........	1382	891	662	600	1342	1002	1396	1162.
Diff. Temp..........	19.5°	46.5°	21.5°	35°	24.4°	49°	20.7°	42.5°

Temperature correction in degrees is given by $\frac{1}{2}$ $\frac{D \times 0\ 0.625.}{3.5}$ D is the difference in temperature rise between the top and bottom coil; 0.625 is the copper surface in the top coil adjacent to the bottom coil; 3.5 is the periphery of the coil

The comparison of the calculated current from the formula to the corrected current value taken from the calibration curve is secured by taking the values for the d-c. equivalent and for amperes from R and R

DISCUSSION ON "EDDY CURRENT LOSSES IN ARMATURE CONDUCTORS" (GILMAN), WHITE SULPHUR, W. VA., JUNE 30, 1920.

S. L. Henderson: The eddy current theory as worked out in Mr. Gilman's paper has made possible a much more accurate study of the losses in armature coils. He has developed formulas covering all the most commonly used types of coils, and it is possible therefore, to calculate the loss in different coil arrangements and to find for a given application that coil giving the minimum temperature drop through the insulation. As an example, in the application of these formulas we will consider several coil designs for a 31,250-kv-a., 13,200-volt, three-phase machine on both sixty and twenty-five cycles, and from this study several interesting results can be shown.

A number of curves have been worked up showing the relation between loss and temperature drop with varying depth of copper. These curves have been calculated on the basis of a rise on the iron surrounding the coil of 55 deg. cent., and a temperature of inlet air of 40 deg. cent., giving a total temperature of iron of 95 deg. cent. The length of the machine being great enough, it was assumed that there is no heat transfer from the point under discussion to the end windings and all the heat conducted through the insulation. The assumption has also been made that the watts per square inch transmitted is uniform around the slot. For purposes of calculating the temperature drop, the surface through which the heat flows has been taken as the mean between the bare copper and the periphery of the slot. The temperature of the iron and the thermal conductivity of the insulation has been taken constant at all points. Since the temperature drop is the unknown and this must be known in order to calculate the eddy current factor and the temperature coefficient of the copper, it was first necessary at each point to calculate a preliminary curve of three points by assuming three temperature drops and working backward to find the loss and current for each temperature. A curve was then plotted between current and temperature drop and from this curve the temperature drop was taken for the rated current.

In connection with the assumption of the temperature of the iron, this has little effect on the loss in the coil, and the consequent drop through the insulation, because with decrease in temperature, for instance, the temperature coefficient of the copper decreases, but the eddy factor increases and results in the product of the two being very nearly constant. The figures of temperature drop are practically independent of ventilation.

In Fig. 1 is shown a number of curves of a two-turn per coil winding, two coils per slot. Each conductor consists of three in width of 0.182 in. wide by 0.091 in. thick. Starting in with eight strands in depth, and increasing in steps by one strand on each conductor up to thirteen strands in depth per conductor, the losses and temperature drop at each point have been plotted

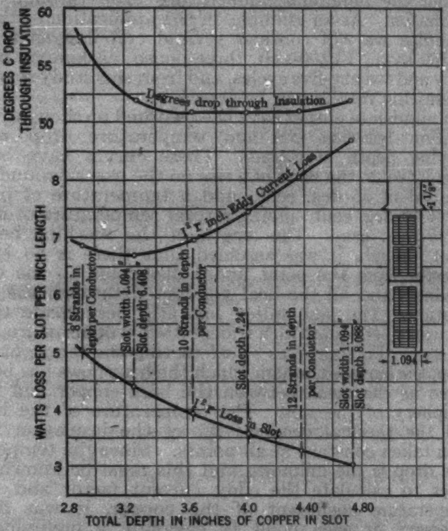


Fig. 1—Based on 31,250 Kv-a., 13,200 Volts, 3-Phase, 60 Cycles, 1370 Amperes. Two Parallels

against total copper depth in the slot. From these curves it is evident that beyond a certain point the temperatures are not decreased by increasing the copper depth, and that a point may be reached where an increase in copper really increases the copper temperature. In contrast, if this same coil were used on twenty-five cycles, we reach mechanical limitations in the coil with increasing coil depths before reaching a point of constant temperature. The curves for this coil on twenty-five cycles are shown in Fig. 2. The depth of the slot given on the curves includes the dimension of 1⅛ in. covering the wedge and open section of slot above the wedge.

If now we had used a four-turn coil, which would be possible by connecting the coils per phase in twice as many parallels, at considerable gain in temperature drop, as shown in Fig. 3, is obtained, being approximately 19 deg. improvement at a point of 4.36 in. total copper depth of slot.

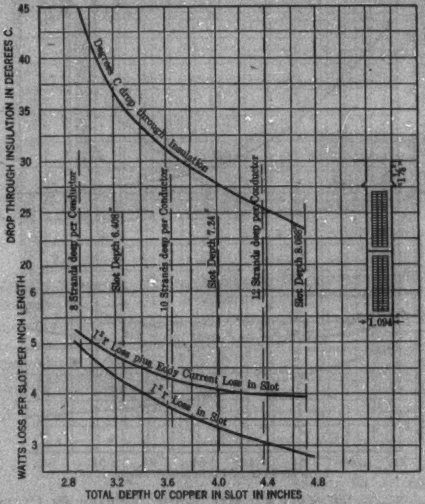


FIG. 2—31,250 KV-A., 13,200 VOLTS, 3-PHASE, 25 CYCLES. FOUR CONDUCTORS PER SLOT. STRANDING CONTINUOUS

One other possibility in the design of the coil lies in the use of a transposed coil, instead of a multi-turn coil. In this type of coil, the strands must be a multiple of the slots per pole per phase, in this case six. Fig. 4 is, therefore, calculated at 18, 24, and 36 strands in depth per coil. This coil is slightly better than the four-turn coil, and also reaches a point beyond which there is no gain in temperature. The point of 24 strands in depth per conductor in the transposed coil can be compared with the four-turn coil and six strands per conductor, or the two-turn coil with twelve strands in depth per conductor, as all three coils have the same copper depth. The four-turn coil and the transposed coil have practically the same temperature drop, differing only because of a slight difference in thickness of insulation, due to the absence of conductor insulation in the transposed coil. Either of these arrangements is considerably better than the two-turn coil. All these curves show an increase of total loss in the coil before a corresponding increase occurs in the drop through the insulation which means that for a period the surface of the coil increases at a faster rate than the loss.

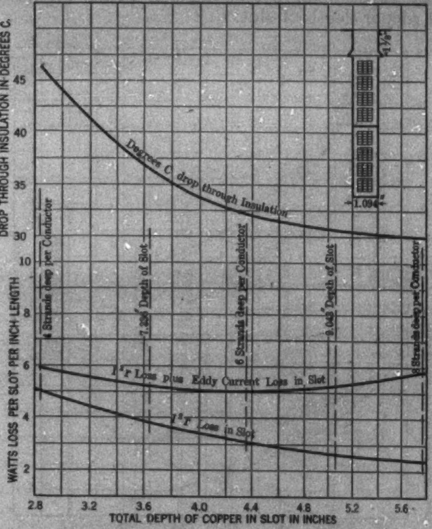


FIG. 3—31,250 KV-A., 13,200 VOLTS, 3-PHASE, 60 CYCLES. EIGHT CONDUCTORS PER SLOT. STRANDING CONTINUOUS. FOUR PARALLELS

All these curves have been carried farther than it is mechanically possible to build coils for large machines. Being continuous coils, not made in halves and connected together in front and back, they become too heavy to be handled without danger of injuring the insulation. Also, the great depth of the coil results in a considerable difference in length of chord in the bottom and top of the slot and makes it almost impossible to enter the coil at the top of the slot without seriously twisting it, and damaging the insulation.

It would be possible to use coils of this great depth if they were formed in halves and then connected together in front and back. It is practically impossible, however, to connect each strand individually, owing to the large number of strands, and if all the strands are connected solidly together on both the front and rear ends, the eddy current loss is increased tremendously. For instance, if the four-turn coil of seven strands in depth per conductor were connected front and back, the eddy current factor would be increased from 2.12 for a continuous coil to 5.7, and the drop through the insulation increased from 31 deg. cent. to 93 deg. cent. This is for a depth of slot of 9.048 in. which is deeper than has been used, and, were

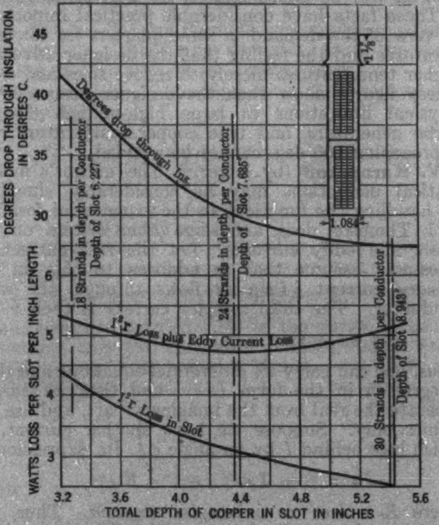


Fig. 4—31,250 Kv-a., 13,200 Volts, 3 Phase, 60 Cycles. Two Conductors per Slot. Each Conductor Transposed.

it used, would probably necessitate a two-piece coil.

From a consideration of the curves, it is theoretically possible on sixty cycles, and with 13,200-volt insulation, to obtain a drop through the insulation of approximately 30 deg. cent. as a minimum. Practically, however, this result cannot be obtained because of mechanical limitations. So far, a slot deeper than 6 to 6.75 in. has not been used, and for such a slot, and by using a transposed coil or a multi-conductor coil, the temperature drop from bare copper to iron will be approximately 40 deg. cent.

These facts have considerable practical importance in view of the demand in some quarters for low temperatures and the feeling that the designer advocates higher temperatures merely to reduce sizes and costs. These figures show that the designer faces definite physical limitations in large high-voltage 60-cycle turbo generators, and true copper temperature rises approaching 100 deg. cannot be avoided.

V. Karapetoff (by letter): The author's mathematical deductions may be considerably simplified by handling the functions in the exponential form, as J. J. Thomson does, and some other writers on skin effect and eddy currents. Let the *real* current be $I \cos \omega t$; assume that in addition to this an *imaginary* current $j I \sin \omega t$ flows through the same conductor. The total *complex* current is then

$$I (\cos \omega t + j \sin \omega t) = I \epsilon^{j\omega t} \tag{a}$$

Thus, one can carry all differential equations and their integration in the form $I \epsilon^{j\omega t}$, and then *in the end* separate the real from the imaginary part, and use the former only. Suppose we have another current, J, which lags behind I by an angle ϕ. Its expression is

$$J \epsilon^{j(\omega t-\phi)} = J \epsilon^{-j\phi} . \epsilon^{j\omega t} = K \epsilon^{j\omega t} \tag{b}$$

where $K = J \epsilon^{-j\phi}$ is a *complex constant.* Thus, by using *complex amplitudes,* quantities differing in phase from one another can be represented by means of the same exponential $\epsilon^{j\omega t}$.

Consider Mr. Gilman's eq. (3) and let the density be expressed as

$$\Delta = u \epsilon^{j\omega t} \tag{c}$$

where u is a *complex function* of x, but does not contain t. This means that the eddy currents are everywhere of the frequency ω, that their phase and amplitude depend upon x, and that in addition to the real currents we assume certain imaginary currents to flow, which are disregarded in the result. Substituting the value of Δ from eq. (**c**) in eq. (**3**) we get, after reduction,

$$\frac{d^2 u}{d x^2} = 2 j \alpha^2 u \tag{d}$$

One can readily see how much simpler this differential equation is than the simultaneous equations which the author gets on p. 1002. The well known solution of (*d*) is

$$u = P \epsilon^{\beta x} + Q \epsilon^{-\beta x} \tag{e}$$

where $\beta = \alpha \sqrt{2 j}$ and P and Q are complex constants of integration. This simple deduction takes the place of all the mathematics on pp. 1002 to 1004.

The expression $\sqrt{j}$ corresponds to the rotation of a vector by 45°, or

$$\sqrt{j} = \cos 45° + j \sin 45° = \frac{1}{\sqrt{2}} (1 + j);$$

hence

$$\beta = \alpha \sqrt{2 j} = (1 + j) \alpha \tag{f}$$

Thus expression (**e**) becomes, after being combined with (*c*);

$$\Delta = \epsilon^{j\omega t} u = P \epsilon^{\alpha x} \epsilon^{j(\alpha x + \omega t)} + Q \epsilon^{-\alpha x} \epsilon^{j(-\alpha x + \omega t)} \tag{g}$$

This is identical with eq. (10) in the paper, although it contains only two constants of integration. Each constant being of the complex form $a + j b$, there are really four *scalar* constants. Eq. (11) becomes

$$u_0 f = \int_0^f u \, d x \tag{h}$$

where

$$u_0 = u_0{}^1 \epsilon^{j \theta} \tag{i}$$

and $u_0{}^1$ is the scalar density (average).

Integrating eq. (**h**) we get

$$\beta f u_0 = P (\epsilon^{\beta f} - 1) - Q (\epsilon^{-\beta f} - 1) \tag{j}$$

which gives one relationship between the constants P and Q.

To deduce eq. (**12**) we have, in accordance with the author's expression near the middle of p. 1005:

$$\beta_1 = \frac{4 \pi}{10 a} I \epsilon^{j(\omega t + \phi)}$$

and

$$\rho \left(\frac{d\,\Delta}{d\,x} \right)_{x=o} = \frac{d\,\beta_1}{d\,t}\,10^{-8}$$

Hence, omitting the factor $\epsilon^{j\omega t}$ on both sides, we get

$$\rho\,\beta\,(P - Q) = j\,\frac{4\,\pi\,\omega}{a}\,I\,\epsilon^{j\phi}\,10^{-9} \qquad \textbf{(k)}$$

From eqs. (**k**) and (**j**) the constants P and Q may be readily computed, thus making pp. 1006 and 1007 unnecessary. The trick is not to use trigonometric functions explicitly, although they are understood throughout the foregoing deduction.

Coming now to the power computations, on p. 1008, no integration is needed with respect to t, since the current is sinusoidal at all points. Expression (**e**) may be readily changed to the form $u_r\;\epsilon^{j\psi}$, where u_r is a *real* quantity and a function of x, and where angle ψ is also a function of x. The value of u_r is then the amplitude of the current density, so that

$$(\text{effective current})^2 = 1/2 \int_0^f u_r^2\,d\,x \qquad \textbf{(l)}$$

This will do away with the long expressions on pp. 1008 and 1009.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 30, 1920.

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THE CORONA VOLTMETER AND THE ELECTRIC STRENGTH OF AIR

A Natural Secondary Standard of Voltage

BY J. B. WHITEHEAD
Professor of Electrical Engineering, Johns Hopkins University

AND T. ISSHIKI
Engineer, Shibaura Engineering Works

An improved form of the corona voltmeter is described. Precision measurements of crest values of high alternating voltage taken in the high-tension circuit are compared with the indications of the corona voltmeter.

The law of corona has been determined to a higher degree of accuracy, and a modification in the form of the law as heretofore accepted is revealed.

As based on the precision voltage measurements the corona voltmeter is proposed as a natural secondary standard of high voltages. Its advantages as a standard, and its practical operation are described.

I. INTRODUCTION

THE corona voltmeter is an instrument for measuring accurately the crest values of high alternating voltages. It makes use of the fact that corona forms on a clean round wire in air at a sharply marked definite value of voltage dependent in a simple relation on the density of the air. The range of the instrument using a single wire is extended to wide limits by enclosing the wire and varying the density of the air.

The essential elements of the instrument are a central rod or wire on which corona forms, an outer concentric cylinder forming the opposite terminal, an outer air-tight containing case in which the air pressure may be varied, and convenient means for determining accurately the first appearance of corona. The principle and method of operation, including the use of gaseous ionization and sound as corona indicators, and two earlier forms of the instrument have been described in an earlier paper.[1] An improved type of the instrument for voltages in the neighborhood of 150,000 volts is described below and shown in Figs. 9 and 10.

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[1] A bilbiography of all references will be found at the end of the paper.

The principal object of this paper is to describe a series of experiments in which the values of corona forming crest voltages have been determined by precision measurements made in the high-voltage circuit. Also to show that the law followed is so definite, and the indications of the instrument so constant, that it constitutes not only an accurate measuring instrument, but also through the results of the present investigation, a natural secondary standard of high voltage possessing many advantages over others at present in use.

An important result of the work is the discovery of an interesting modification of the law of corona formation.

The various precautionary and check measurements taken to ensure the accuracy of the final readings constitute in themselves prime evidence of the accuracy of the corona as a measure of voltage, and also of its constancy and reliability in operation in the corona voltmeter. In addition some further notes on the operation of the voltmeter are given towards the end of the paper.

II. THE CORONA AS A STANDARD OF VOLTAGE

Two striking properties of the high-voltage corona in air have led to the suggestion of its use for the measurement of voltage and to the development of the corona voltmeter. The first is the remarkable constancy of the value of voltage at which, under fixed conditions, the corona appears on a round wire or rod; and the second is the simplicity of the law connecting the critical or corona-forming voltage with the diameter of the rod and the condition of the surrounding gas.

The former of these properties has been noted by a number of observers and in particular by one of the present authors in the first of a series of papers on the electric strength of air,[2] and again especially in a paper on the corona voltmeter.[1] Using a clean round rod and the best type of portable voltmeter in the low-tension circuit, on repeated raising and lowering of the voltage corona appears sharply at exactly the same value throughout, that is, at a value constant to within say one-tenth or one-quarter per cent. Under more refined conditions the constancy is shown to be even closer.

The empirical law connecting the critical or corona-forming voltage gradient E in kilovolts per cm., at the surface of the wire, the radius of the wire r in centimeters, and the relative density of the gas δ, is usually stated in the form

$$E = A\,\delta\left(1 + \frac{B}{\sqrt{\delta\, r}}\right) \tag{1}$$

A more convenient form for our present purposes is

$$E/\delta = A + \frac{B'}{\sqrt{\delta\, r}} \tag{2}$$

which gives a linear relation between E/δ and $\frac{1}{\sqrt{\delta\, r}}$; obviously $B' = A\,B$.

The value of δ is given by

$$\frac{3.92\,p}{273 + t} \tag{3}$$

in which p is the pressure in centimeters of mercury and t is the temperature in degrees centigrade.

The above relatively simple relations have now been corroborated by a number of observers and with quite close agreement as to the values of A and B. The influence of the diameter of the wire on corona-forming voltage was first emphasized by H. J. Ryan,[3] who was also the first to point out the possibilities of the corona as a voltage indicator. The exact nature of this influence and the presence of the two constants A and B were first shown by one of the present authors.[4] The precise influence of the density of the air was first shown by F. W. Peek, Jr.,[5] in one of the most important contributions yet made to the knowledge of the subject. Moisture in the air has no effect on the critical intensity.[2]

The form of the above law is the same for both continuous voltages and crest values of alternating voltages. With continuous voltage, however, there are appreciable differences in the values of the constants A and B, as between positive and negative corona-forming wire, the form of the law in each case remaining the same.[6] One of the most important results of the present work is the fact that this difference between positive and negative corona is reflected in the alternating corona, and that the law as given by formulas (**1**) and (**2**) must be modified. Briefly stated, the modification consists in the use of different values of the constants A and B above and below a definite value of $\frac{1}{\sqrt{\delta r}}$, the form of the law, however, remaining the same in each case, as will be seen below.

It has generally been accepted that within the commercial range frequency has no influence on the corona-forming voltage. Observations with the accurate methods used in the experiments show a slight influence of frequency within the range mentioned.

Since corona formation through the constancy of its appearance and the simplicity of its law offers a ready means for the measurement of high voltage, it is important that the constants A and B be determined accurately. When this is once done, such an instrument as the corona voltmeter has a calibration dependent only on its dimensions, and so constitutes a natural secondary standard of voltage.

Nearly all determinations of alternating corona voltages have been based on observations of voltage and crest factor taken in the low-tension circuit, and computed from transformer ratios. As is well known, this method is subject to serious errors on both accounts. If therefore advantage is to be taken of the constancy of corona voltage as a standard and as a method of measurement, it is necessary that the constants A and B be determined by direct measurement in the high-voltage circuit of the crest values of corona voltage, and to a relatively high degree of accuracy in terms of accepted standards. These determinations once made over a sufficiently wide range of values of δ, corona formation, by reason of the simplicity of the relation of formula (**1**), and its freedom from outside influence, becomes a far more reliable standard than the sphere gap, the potential transformer, or any other standard at present proposed.

III. PRECISION MEASUREMENT OF CORONA CONSTANTS

For the determination of the values of A and B we must (1) measure accurately the crest value of alternating voltage at which corona appears, (2) be able to observe to as small a difference of voltage as possible the first appearance of corona, and (3) measure δ and provide a wide range of its values.

The crest value of voltage (1) may be determined from the average value of the charging current of an air condenser in the high-voltage circuit. This method first used by Chubb and Fortescue,[7] was modified by Whitehead and Gorton,[8] and is now further improved as described below.

For (2) the accurate observation of the first appearance of corona, two methods are used,—(a) the telephone for detecting the sound of the corona, and (b) the galvanometer for detecting the conductivity of the air caused by the corona. Both methods are used in the corona voltmeter and are described in detail in an earlier paper;[1] further observations are reported below. The visibility of corona is neither convenient nor accurate as a means of determining its first appearance.

The corona voltmeter with its air-tight outer casing provides the method (3) for the observation of δ, the relative air density, and its variation over a wide range. Pressure and temperature are read and the pressure may be adjusted to any chosen value, thus permitting setting for any value of δ.

III. 1. MEASUREMENT OF VOLTAGE

If an alternating voltage of maximum value E volts and frequency f be impressed on a condenser of capacity C, the average charging current is

$$i = 4 f C E; \tag{4}$$

if f and C are known and i is measured E the maximum for the maximum value of charging voltage is determined. When used for the high values of voltage pertaining to corona formation, one side of the condenser is grounded and the charging current measured in the ground connection. Since the condenser must withstand the full maximum voltage and have no dielectric or other loss, the most convenient form is that of concentric cylinders with wide radial separation, and with air as dielectric. This, however, means small capacity per unit axial length, and small total capacity if the outside dimensions are to be kept within reasonable limits. Consequently the use of this method involves the use of a large air condenser of small capacity and a determination of the value of the capacity.

Chubb and Fortescue[7] constructed a cylindrical condenser consisting of two wooden forms, each covered with sheet metal surfaces. The diameters of the two members were 60 cm. and 162.8 cm. respectively, and the outer member was provided with two flaring guard ring ends. The capacity between the inner member and the central section of the outer member was calculated as 2.65×10^{-11} farad, no attempt at measurement being made, doubtless owing to the difficulty of measuring so small a value. Chubb and Fortescue measured the charging current in the ground connection of the central section of the outer member of the condenser by means of a d'Arsonval galvanometer and a synchronous commutator connected as a shunt suppressor.

In the present experiments the same type of condenser is used, *i. e.*, the cylindrical guard ring type with voltage applied to the inside member and charging current measured in the ground connection of the central section of the outside member. The capacity, however, was measured, as described below. Further, the charging current was measured by the use of two rectifying kenotrons, thus obviating the irregularities and uncertainties of the synchronous commutator. The commutator was, however, frequently used for comparison and certain auxiliary tests.

A diagram of the principal connections is shown in Fig. 1. Voltage is applied from the transformer *A* to the corona voltmeter *B* and the air condenser *C*. The charging current of the central section of the latter passes to ground in alternate half waves through the resistances and kenotrons R_1, K_1, and R_2, K_2. The currents in R_1 and R_2 are therefore pulsating but unidirectional and so may be read by a continuous-current instrument in series or in shunt, as shown in Fig. 1, G_1 being a sensitive d'Arsonval galvanometer critically damped. A second galvanometer G_2 and a telephone T are used to detect the first appearance of corona on the central rod of the corona voltmeter as described below. A number of auxiliary circuits have been omitted from Fig. 1 and will be referred to in connection with the various measurements.

We will now describe in turn the methods of measuring the charging current, the frequency, and the capacity of the condenser, together with the precautions taken, the limits of accuracy, and all leading to the determination of the value of voltage present on the first appearance of corona in the corona voltmeter B.

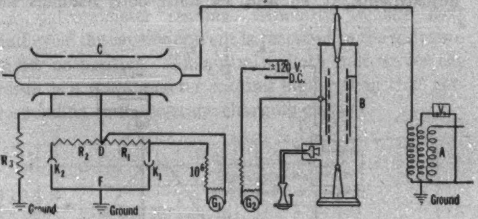

FIG. 1—PRINCIPAL CONNECTIONS

(*a*) *Charging Current.* Balance in kenotron circuit. In formula (4) i is the average value of the charging current. In Fig. 1 all positive half waves will pass through one kenotron and all negative half waves through the other. The d'Arsonval galvanometer in shunt to the resistance R_1 will therefore receive a pulsating unidirectional current and show a deflection proportional to its average value. Obviously the combination may be calibrated directly in terms of continuous current in R_1 and in terms of such a calibration the galvanometer will read one-half the average value of the charging current. In view of the foregoing it is of first importance that when no charging current is passing there be no continuous current flowing in the closed circuit $K_1 D K_2 F$. This condition was realized by the adjustment of the point of connection to the filament exciting circuits of the kenotrons as indicated at $P_1 P_2$ in Fig. 2. This in effect interposes a small adjustable e. m. f. counter to the normal direction of conductivity, at each kenotron. If this is not done the normal leak of electrons from the filament, particularly at its negative end, results in a small current in the circuit $K_1 D K_2 F$. In making these adjustments the galvanometer G_1 was connected first in series in the circuit $K_1 D K_2 F$ and then between the points D and F, repeating with adjustments at P_1 and P_2 until both readings are simultaneously zero. After balancing the kenotron circuit, as above, it was connected into the condenser ground circuit and with alternating voltage on was further balanced for equal resistance in the two branches by adjustment for zero current in the galvanometer connected across $D F$; on removal of the alternating voltage the circuit $K_1 D K_2 F$ is still balanced. Without these adjustments a small error is possible, the galvanometer G_1 in the connection of Fig. 1 showing at times a deflection of 0.5 mm.; after the adjustments mentioned no deflection can be detected.

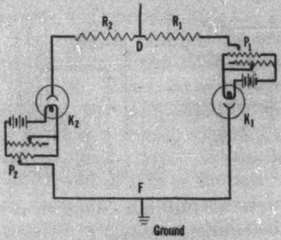

FIG. 2—KENOTRON CONTROL

(*b*) *Influence of Wave Form.* The use of the kenotrons for rectifying the charging current introduces an error if the wave form of voltage is not smooth, *i. e.*, if it has more than one maximum in each half wave. In this case there is a reversal of condenser current following every such maximum or elevation in the wave and since the kenotron passes current in only one direction, this reverse current passes through the opposite kenotron and so does not contribute to the galvanometer reading. Similarly in the next half cycle the reverse current is recorded in the galvanometer as positive. Thus due to both half waves the result is a galvanometer reading higher than that corresponding to the average charging current.

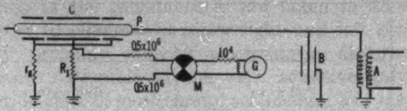

FIG. 3—MEASUREMENT OF CONDENSER CHARGING CURRENT WITH SYNCHRONOUS COMMUTATOR

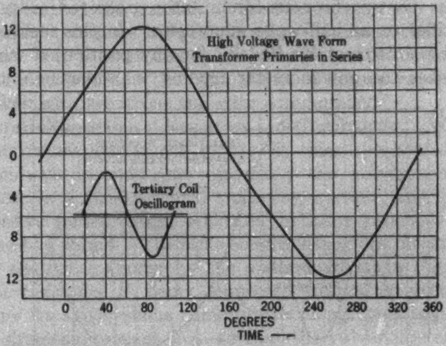

FIG. 4—HIGH-VOLTAGE WAVE FORM

The generator used in the experiments has a surface wound armature and shows a smooth wave on an oscillogram. The inserts of Figs. 4 and 5 show the voltage waves as taken from a low-tension tertiary coil on the transformer *T*, Fig. 1, for series and parallel connections respectively of the two primary coils. In order, however, to answer this question definitely, the wave form of the voltage at the high-tension terminals was taken by the method indicated in Fig. 3, in which

TABLE I.
WAVE FORM OF HIGH VOLTAGE

Brushes degrees	Galvanometer deflection cm.					
	Full wave			Half wave		
	Left	Right	Mean	Left	Right	2 *X* Mean
22.5	7.80	7.76	7.78	3.72	4.10	7.82
30	8.41	8.47	8.44	4.09	4.39	8.48
37.5	9.14	9.10	9.12	4.42	4.70	9.12
90	11.91	11.87	11.89	5.87	6.03	11.90
120	10.68	10.60	10.64	5.32	5.30	10.62

M is the synchronous commutator connected as full rectifier or as half-wave suppressor. In this method first pointed out by Bedell,[9] for any position of the brushes the galvanometer reading is proportional to the average value of the charging current for any particular half-wave interval between the brushes, and this in turn is proportional to the instantaneous value of the voltage on the condenser. Figs. 4 and 5 show

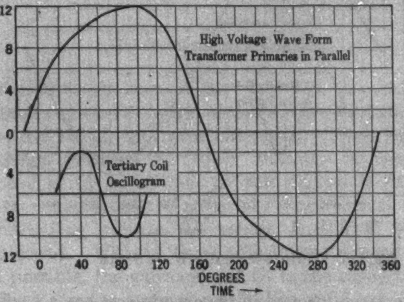

FIG. 5—HIGH-VOLTAGE WAVE FORM

the wave forms so taken for series and for parallel connections respectively of the transformer primary coils, which together with Table I, giving a section from the complete sheet of readings, indicate the conditions of accuracy. The mean values of right and left readings of the galvanometer are taken in all cases in order to eliminate a slight right and left dissymmetry probably due to electrostatic disturbance, generally noticeable in the very sensitive galvanometer, in spite of most careful screening. For obvious reasons this disturbance is more pronounced in the half-wave measurements in which the galvanometer is used as a half-wave suppressor.

The curves of Figs. 4 and 5 were each taken at the critical or corona-forming voltage using the same corona rod and equal values of air density. Although there are noticeable differences in wave form and in the values of effective voltage at the terminals of the tertiary coil (38 volts and 34.5 volts respectively) it is seen that the maxima of the two waves have very closely the same values. Further evidence that no error was present due to irregularities of wave form is found below in the comparison of corona readings taken with kenotrons and with commutator.

(c) *Comparison of Kenotrons with Commutator*. With the connections shown in Fig. 1, since the kenotron conducts in only one direction, the galvanometer receives a unidirectional pulsating current, the successive pulses being separated by time intervals of one-half period. The same conditions may be obtained in the galvanometer by the method shown in Fig. 3, the resistance R_1 being connected straight to ground and the synchronous commutator being connected as a shunt suppressor, *i. e.*, so that the galvanometer is short-circuited during alternate half-cycles. With fixed conditions in the high-tension circuit therefore both these methods should give the same galvanometer reading. In the experiments recorded in the following Table II, the voltage was set at the corona-forming value for a 0.955-cm. (0.376-in.) diameter rod in the corona voltmeter, for each of the two wave forms pertaining to the two methods of connection of the transformer primaries. The table gives for each method of connection first the readings leading to the commutator setting for maximum galvanometer deflection, and this is done using the galvanometer both as complete rectifier and as half-wave suppressor. It is seen that these two sets of readings are closely in the relation 2 to 1, and that the commutator setting is indicated to within one and one-half electrical degrees. The readings at maximum setting are then given and directly below them the corresponding kenotron

TABLE II.

COMPARISON OF READINGS WITH KENOTRONS AND WITH COMMUTATOR.

Brush setting degrees	Transformer primaries in parallel					
	Full wave			Half wave		
	Left	Right	Mean	Left	Right	Mean
90	11.99	12.01	12.00	6.07	5.98	6.02
93	12.00	12.07	12.03	6.09	5.99	6.04
96	12.00	12.07	12.03	6.09	5.98	6.03
99	11.93	12.02	11.97	6.01	5.99	6.00
94.5	11.99	12.07	6.01 × 2	6.08	5.98	6.03
With kenotrons				5.99	6.01	6.00
	Transformer primaries in series.					
72	11.82	11.90	11.86	6.01	5.89	5.95
75	11.92	12.02	11.97	6.04	5.93	5.98
78	11.98	12.07	12.03	6.08	5.94	6.01
81	11.97	12.03	12.20	6.07	5.97	6.02
84	11.89	12.00	11.94	6.02	5.93	5.97
79.5	11.96	12.07	6.01 × 2	6.07	5.97	6.02
With kenotrons						6.01
				5.99	6.01	6.00
				5.98	6.02	6.00

readings. It is seen that there is excellent agreement particularly when the full rectification readings are included. A further interesting observation was made in connecting the galvanometer and commutator as shunt suppressor across each of the resistances R_1 and R_2 of Fig. 1 in turn, using the maximum brush settings and other conditions of Table II. For one of the resistances the full galvanometer deflections of Table II were obtained, but for the other the mean of the right and left readings of the galvanometer was accurately zero, as is to be expected since for the half wave during which the kenotron conducts, the galvanometer is short-circuited by the commutator. This observation also indicates the absence of reverse currents due to inequalities in the voltage wave.

(*d*) *Calibration of Galvanometer*. The galvanometer used for measuring the charging current of the air condenser was a late American type d'Arsonval read by telescope and scale. Its constants were as follows: resistance 115 ohms; sensitivity 40 megohms; free period 9.5 seconds; critical damping resistance 560 ohms. Throughout all the observations the galvanometer was shunted with 560 ohms and the combination used in series with 10^6 ohms for measuring the potential drop across the resistances R_1 and R_2 in Fig. 1. Since the maximum voltage is measured through the current in R_1 or R_2, it is obviously of the first importance that the galvanometer be accurately calibrated. The calibration directly in amperes was effected by passing continuous current through R_1 or R_2 and measuring this current through the resulting potential drop over a resistance of 499 ohms always in this auxiliary circuit; the potential drop was measured on a precision potentiometer in terms of a Weston cell. The value of the resistance was determined to within 1/25 of 1 per cent by comparison with certified laboratory standards. Two certified Weston cells were used, one checking the constancy of that in use with the potentiometer; at the end of the observations their values were equal to the fourth decimal.

The galvanometer was calibrated for every series of observations and usually at approximately the scale reading pertaining to the particular charging current being measured; see Table VII.

In order to investigate a possible error due to the pulsating character of the galvanometer current when the instrument is calibrated for continuous current, an extensive series of observations was made using the commutator for breaking up continuous current and for cutting out alternate half waves of alternating current. These experiments have been described in another paper,[10] and show that when the galvanometer is connected as in Fig. 1 or Fig. 3 the calibration with continuous current is accurately the same as that with pulsating current whether rectangular or of approximately sine shape.

(*e*) *Resistance of Ground Connection.* Since the central section of the air condenser is connected to ground through the resistance and kenotron circuits of Fig. 1, two questions arise as to the effect of the resistance of these circuits on the charging current of the condenser. First, if the resistance of this circuit is sufficiently high the voltage across the terminals of the condenser may be appreciably lower than the total voltage between high-tension terminal and ground; and second, if the resistance in question is sufficiently high, suitable adjustment must be made to ensure equal potential between the central section of the condenser and the guard rings, or otherwise the current in the kenotron circuit would not be that due to the capacity between the high-voltage member of the condenser and the central section of the grounded member alone.

With reference to the first of these questions, the calculated capacity of the central member of the condenser is 8.262×10^{-11} farads. The measured value is 8.286×10^{-11} farads, (see paragraph (h) below). The corresponding reactance at 60 cycles is therefore 3.2×10^7 ohms. The resistances R_1 and R_2, Fig. 1, were 2000 ohms each throughout the experiments. The equivalent resistance of one kenotron varies with its filament current and also with the current that the kenotron is transmitting. The values of these resistances were determined by taking the volt-ampere characteristics of the kenotrons with continuous currents. It is not thought necessary to reproduce these readings here, as the characteristics of the kenotron are well-known. The maximum value of kenotron resistance obtaining in the experiments was approximatley 4000 ohms.

The filament currents of the two kenotrons were always adjusted to the same value as indicated by a direct-reading continuous-current instrument. Slight variations in the filament current have no effect on the transmitting power of the kenotron and only produce small variations in the equivalent resistance of the kenotron.

Consequently the maximum aggregate resistance in the ground circuit of the central section at any time was approximately 6000 ohms. This non-inductive resistance is in series with the capacity reactance of 3.21×10^7 ohms of the central section of the condenser and has therefore a quite negligible effect either in elevating the potential of the outer member of the condenser above ground, or in reducing the voltage at the condenser terminal below the full applied value.

The accuracy of the above deductions was tested by connecting the central section of the condenser to ground through resistance only and similarly the two guard rings in parallel to ground through resistance only. The voltage drop over these resistances was studied by means of the commutator and galvanometer. The results showed that the voltage drop over each resistance was accurately proportional to its value. They also showed that the values so measured on either resistance were independent of the value of the other resistance within the range 0 to 10,000 ohms, the study not being carried further.

With reference to the second question raised above, namely, as to the effect of a difference of potential between the guard rings and the central section on the values of the charging current to ground from the central section, it would appear from the foregoing that, since the difference of potential between the two members is negligibly small, and furthermore since the capacity between them is also small, no influence on the

TABLE III.

INFLUENCE OF RESISTANCE IN GROUND CONNECTION OF CONDENSER.

R_3	Volts over *D-F.*			Volts over R_2			Volts over R_1			Volts over R_3		
	Left	Right	Mean	Left	Right	Mean	Left	Right	Mean	Left	Right	Mean
9999	10.7	10.73	10.71	3.3	3.35	3.32	3.3	3.35	3.32	17.9	18.1	18.0
5970	10.73	10.82	10.77	3.29	3.36	3.32	3.29	3.36	3.32	10.68	10.8	10.79
0	10.73	10.82	10.77	3.29	3.36	3.32	3.29	3.36	3.32	0.1	0.1	0.

$R_1 = R_2 = 2000$ ohms. Filament current 2.75 amperes.

charging current of the central section would be found even if the guard rings were connected directly to ground. However, it was thought best to investigate the matter experimentally and also to determine the proper value of resistance to connect between the guard rings and ground in order to ensure that the guard rings are at the same potential as the central member. The results of this study are given in Table III.

Referring to Table III and Fig. 1, the first column gives the value of the resistance R_3 between guard rings and ground; the next three columns right, left and mean readings of the galvanometer connected between D and F of Fig. 1 using the commutator for rectification; the next three columns voltage over R_2; and the next three that over R_1 both without use of commutator; and the last three columns the voltage over R_3 with the aid of the commutator. These results show that the voltage between the point D and ground and the voltages over the resistances R_1 and R_2 are all independent of the value of the resistance R_3 up to 10,000 ohms. Furthermore they show that the guard rings are brought to the same potential as the central section when connected to ground through a resistance R_3 of 5,970 ohms. R_3 was kept at this value throughout the course of the work.

(*f*) *Measurement of Frequency*. The value of frequency enters directly from Formula (4) into the expression for the maximum value of voltage. Constancy of frequency therefore and as accurate a determination of its value as possible are highly essential to an accurate determination of the maximum voltage.

As regards constancy, the 5-kw. single-phase generator was driven by a continuous-current motor run as the only load on a large storage battery. This constant source of supply was further supplemented by an automatic speed control illustrated in Fig. 6. In this method the ultimate source of constant speed is an electrically operated tuning fork. A small rotary converter, R, of the same frequency as the tuning fork is driven from the direct-current end and is loaded with a resistance on the alternating-current end, the load circuit being taken through a pair of contacts on the tuning fork, contact being made once during each half wave. The time interval of contact by the tuning fork is a fraction of the whole alternating-current period. If the speed rises, contact is made at an instant when there is a greater electromotive force and current, thus resulting in a greater load on the converter. If the speed goes down, the conditions are reversed, consequently the tendency of the change of load is to maintain the speed of the converter constant. This tendency can be made greater or more positive by inserting resistance in the armature circuit of the continuous-current end of the converter.

The shaft of the small converter and that of the larger machine which is to be controlled are each supplied with a crown commutator, K_1 and K_2. The number of commutator segments for each is chosen so that by their speeds the frequencies of reversal of the two commutators are the same. The commutators are connected together electrically, as indicated in Fig. 6, in such a way as to short-circuit a small resistance in the armature circuit of the main driving motor for a greater or less period, according as the commutators depart more or less from the position of exact coincidence of phase. The machines automatically find such a relative commutator phase relation that an increase in speed decreases the duration of short circuit of the armature resistance, and vice versa, so that the average voltage on the motor armature is such as to more or less exactly maintain the speed in a constant relation to that in the small converter, which in its turn is maintained constant by the tuning fork. The

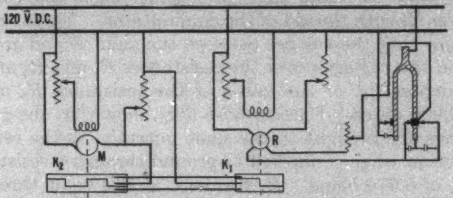

FIG. 6—CONTROL OF FREQUENCY BY TUNING FORK

introduction of the small converter is necessary, since the tuning fork contacts will not carry the large currents interrupted in the control of the considerably larger direct-current motor.

The use of the foregoing method prevented slow changes of speed due to temperature changes in the motor, or to variations of applied voltage, etc. Occasionally changes of this character were sufficiently great to overcome the regulating power of the tuning fork and the resulting upset in frequency was immediately indicated by the "beats" between the two frequencies detected by means of a pair of stroboscopic disks, one on each machine and through either of which the tuning fork might be viewed. In this way it was possible to tell at any instant by a glance whether the frequency was constant.

Observations taken with the stroboscopic disks indicate that by the above method the frequency was maintained constant at within 0.5 per cent. In this connection it is to be noted, however, that as read by the galvanometer, the charging current of the air condenser is read as an average value. Consequently, so long as the frequency is kept to an average constant value, momentary changes of frequency will not be registered in the galvanometer. This was borne out by the general character of the galvanometer reading, this reading being always absolutely stationary and not subject to any momentary variations which could be detected.

(*g*) *Value of Frequency.* Most of the measurements described in this paper were made at 60 cycles and this frequency is to be understood unless particular note is made of some other value. The frequency of the tuning fork was measured each day by bringing the small rotary converter into synchronism with the fork by the method described above and then taking the speed of the rotary by means of a "tachascope" or, combined revolution counter and stop watch. The speed of the rotary corresponding to 60 cycles for the alternating generator was 1800 rev. per min. The usual method of observation was to take the number of revolutions of the rotary within a period of three minutes, taking this observation three times and taking the mean of these for the determination of frequency. Table IV gives an example of the measurements which indicate that the average frequency was determined to within 0.04 per cent.

TABLE IV.
MEASUREMENT OF FREQUENCY.

Time	Revolutions of control rotary		
3 min.	5400 + 4.5	5400 − 4	5400 − 6
3 min.	5400 − 1	5400 − 3	5400 − 2
3 min.	5400 + 3.5	5400 + 5	5400 + 2
Average...............	5400 + 2.3	5400 − 2	5400 − 2
Average frequency.....	60.02	59.97	59.98

(*h*) *Capacity of Air Condenser.* As already stated, the air condenser was of concentric cylinder type. It had a continuous inside member and an outer member consisting of a central section protected at each end with guard rings. A photograph is shown in Fig. 7. Both members were made from standard cast iron pipe.

Fig. 7—Air Condenser

The surface of the inner member was turned off so as to have a uniform diameter and to be free from surface irregularities. The ends were filled with rounded wooden plugs covered with tin foil and the whole inner member was supported by a 1½ in. pipe through central holes in these wooden plugs. The guard ring ends of the outer member consisted of straight sections on the outer ends of which were mounted standard flanges screwed on, the inside surfaces being turned off so as to provide a smooth flaring end to each guard ring.

The central or inner member was supported on dry oak posts boiled in paraffin. Sliding and screw adjustments at the top and bottom of these posts permitted accurate centering in relation to the outer member. The two guard ring ends were tied together by means of four stout oak pieces maintaining them in line. The central section of the outer member was supported on eight small plate glass insulators mounted in the four oak pieces mentioned. The whole was then supported in a wooden cradle, as indicated in Fig. 7.

Following are the principal dimensions of the air condenser:

Diameter of inner member	29.50 cm.	(11.61 in.)
Average diameter of outer member	49.30 cm.	(19.42 in.)
Length of central section of outer member	76.20 cm.	(30.00 in.)
Length of guard ring ends	30.5 cm.	(12.00 in.)

The inside diameter of the outer member was not strictly uniform. It was measured in twelve places, the extreme variation being between 49.20 cm. and 49.51 cm. The calculated value of the capacity between the central section of the outer member and the inner member, based on the average diameter of the outer member given above, is 8.249×10^{-11} farads. In view of the uncertainty introduced by slight irregularities of this character, it was decided to measure the capacity.

The measurement of so small a value of capacity is a matter of considerable difficulty and requires much care. Maxwell's bridge method was used, following in general the experimental method of Rosa and

Dorsey[11]. We were fortunate in being able to borrow from the National Bureau of Standards the special commutator constructed by them.

The diagram of the connections of this method is shown in Fig. 8. The principle is well-known and consists in replacing one of the resistances in a Wheatstone bridge with a condenser, and a suitable device for rapidly charging and discharging the condenser. In the arrangement shown in Fig. 8 the charging current only is used, the condenser being short-circuited in the alternate intervals. If, as in our case, the condenser has guard rings, it is necessary that they follow the same cycle of potential as the central section to be measured. This is accomplished by the double bridge shown in Fig. 8. *R S* and *R′ S′* are contacts carried on the special form of rotating commutator constructed by Rosa and Dorsey for effecting the simultaneous charge and discharge of the central and guard ring members.

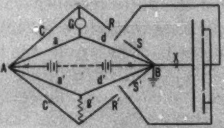

Fig. 8—Measurement of Capacity of Air Condenser (8.28 × 10^{-11} Farads)

The bridge is balanced by varying the arms *a* and *a′* and when in proper adjustment corresponding arms in the two bridges must have identical values. The value of the capacity between the inner member and the central section of the outer member is

$$C = \frac{a\,F}{n\,c\,d} \tag{5}$$

a, *c* and *d* are values of resistance; *n* is the number of charges of the condenser per second; and *F* is a correction factor depending on the relative values of the resistances, and differing but little from unity; in our work by about 3 parts in 10,000.

The values used by us were as follows:

$$c = c' = 521{,}000 \text{ ohms}$$
$$d = d' = 50{,}000 \text{ ohms}$$
$$g = g' = 1{,}124 \text{ ohms}$$

The high resistances were of manganin made up of specially wound non-inductive units lent by the Bureau of Standards. The remaining resistances were of standard, high-grade, laboratory type, and all values as given were measured in terms of laboratory standards to 0.04 per cent.

For the determination of n, the number of charges per second, the commutator which had sixteen segments was geared directly to the small rotary converter controlled by a tuning fork by the method of Fig. 6, as already described. Two speeds differing by the factor 2 were possible with this arrangement, control of the machine speed by the fork being effective at either speed. Moreover, observation by means of the stroboscopic disk as to the constancy of the speed was also possible. The fork thus served for maintaining the frequency of charge and discharge at the average uniform value pertaining to either of two speeds. The value of this frequency, and thereby the value of n, was determined by a contact-making device on the commutator and a stop watch. The normal speed of the rotary was 1800 rev. per min. and this corresponded to the value of $n = 480$. The accuracy of the speed determination and so of the value of n was therefore within 0.04 per cent.

In measuring the capacity it was necessary to make a correction due to the capacity of the lead wires between the condenser and the bridge. This was done by measuring the capacity first with the condenser in and then with the connection to the central member of the condenser opened at the point X, Fig. 8. Two precautions have to be taken following this method; first, as to the insulation resistance of the capacity to be measured and the insulation resistance of the central section of the outer member to ground; and second, the capacity between the bridge wiring and the central member when the connection at X is opened.

With reference to the insulation resistances mentioned, it was found that in moist weather the values were relatively low, say of the order a few hundred megohms, doubtless owing to the rather large surfaces of support of the heavy parts. However, in clear dry weather the resistances were greater than one-half million megohms, a figure which reduces the possible error on this account to quite negligible proportions. These figures include the commutator insulation and were checked at each series of observations for the measurement of the capacity.

As to the possible error due to a charging current from the open contact at X through the capacity to the central member and thence to the outer member, this error was avoided by completely enclosing the projecting ends of the central member of the condenser by means of large cones made of galvanized sheet iron, the connection to the central member being carried through a small opening in one of the cone ends. The opening X was made at this place and under these circumstances the interior central member was entirely screened during the measurement for the correction due to the capacity of all the auxiliary wiring.

The most difficult part of this measurement is the adjustment of the brushes on the commutator for simultaneous charge and discharge of the central section and the guard ring ends. This requires very careful study of the conditions of contact of the brushes on the commutator sections, so as to prevent mechanical vibration and to ensure a reasonable permanence of a proper adjustment when once it is effected. Probably the best test of these conditions is by means of an auxiliary circuit through the brush contacts, indicating by means of a galvanometer the ratio of the running to the standstill deflections. By taking the two sets of brushes in turn singly and then in series, a quite accurate idea of the conditions is obtainable. Throughout the measurements of capacity, observations of this character were taken both before and after the capacity readings, thus ensuring satisfactory conditions. As an additional precaution in this direction a double set of readings was always taken, the duplicate contacts of the commutator being exchanged for the two readings between the central section of the condenser and the guard rings. Table V gives the readings which were selected as having been taken under the most favorable conditions, that is to say, those in which there was least difference between the two values of capacity corresponding to the exchange of the two sets of brush contacts.

TABLE V.
MEASUREMENT OF CAPACITY OF AIR CONDENSER.

	Half speed $n = 240.2$ $ncd = 62.56 \times 10^{11}$ A positive		Full speed $n = 480.3$ $ncd = 125.12 \times 10^{11}$ A positive	
X	Left	Right	Left	Right
Closed	940			1851
Open	419	417	805	810
Closed	939	934	1839	1854
Open	419	417	805	814
Closed		934	1835	
Difference	520.5	517	1032	1040.5
Capacities	8.320	8.264	8.248	8.316
Mean	8.296×10^{-11}		8.282×10^{-11}	
X	A negative		A negative	
Closed		930		1867
Open	420	415	826	828
Closed	941	929	1864	1864
Open	420	415	826	828
Closed	942		1864	1858
Difference	521.5	514.5	1038	1035
Capacities	833.6	8.224	8.296	8.272
Mean	8.280×10^{-11}		8.289×10^{-11}	

Mean of means 8.286×10^{-11} farads.
Calculated value 8.262×10^{-11} farads.

The figures given in Table V are the values of the resistance "*a*" in ohms. The words "right and "left" refer to the particular set of commutator brushes used. It will be noted also that readings were taken for two speeds and for each speed a reading for a reversal of the polarity of the battery of the bridge. The value of the capacity in each case is calculated by means of Formula (**5**). The final average value of these readings is 8.286×10^{-11} farads. The calculated values based on independent measurements of the dimensions by each of the authors were 8.28×10^{-11} and 8.245×10^{-11} farads.

In computing the maximum value of voltage the measured value of the capacity, 8.286×10^{-11}, has been used.

(*i*) *Electrostatic Screening.* In view of the small value of the current to be measured, especial care had to be taken for the elimination of all electrostatic inductive effects between the high-tension circuit, the outer surface of the condenser, and the various measuring circuits in the ground connection of the condenser. With the sensitive galvanometer used it was found that the unscreened exposure of relatively small and distant portions of the wiring could introduce considerable error. It was necessary for the elimination of errors of this character to completely enclose in grounded casing all of the low-voltage measuring circuits of Fig. 1, including the various auxiliary circuits not shown; thus the kenotrons, the resistances, R_1, R_2 and R_3 and their various control circuits were enclosed in one sheet-iron box. The entire equipment of the observer's station, including the galvanometers for the measurement of the charging current and for the detection of corona and the various calibrating circuits were all enclosed in a small chamber completely surrounded with wire mesh. The condenser itself was surrounded with an outer screen of wire mesh. All connections between these various parts were likewise carried in metal conduit. All of the screening coverings described were connected together and to ground.

The test for completeness of electrostatic screening consisted in opening the high-voltage connection to the central member of the air condenser at *P*, Fig. 3, applying the voltage, and observing the galvanometer in the method of connection of Fig. 3. In this observation the value of R_1 was 9900 ohms and is about five times the value used in the charging current measurements. The sensitivity of the galvanometer used was 1250 megohms and no deflection could be detected. The galvanometer could be read to 0.2 millimeter which, in accordance with the magnitude of the deflections corresponding to the charging currents measured, means that an error on account of electrostatic induction, if present, is less than 0.1 or 0.2 per cent.

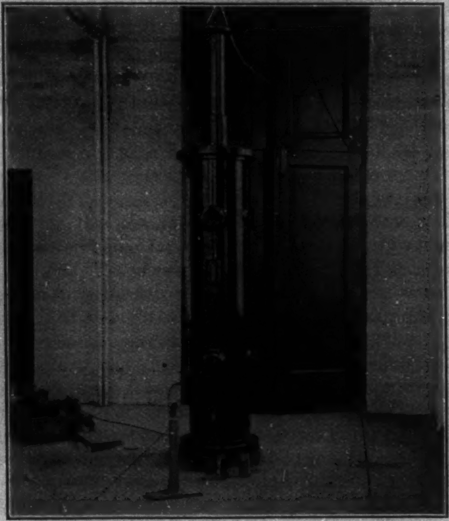

FIG. 9—CORONA VOLTMETER—200,000 VOLTS

III. 2. DETECTION OF CORONA

All of the corona observations on which the measurements are based were taken with the corona voltmeter shown in Fig. 9. A vertical section (not to scale) is shown in Fig. 10. The principal dimensions are,—height overall 9 ft. 10 in.; outside diameter 22 in.; diameter of grounded electrode cylinder 24.67 cm. (9.715 in.); length 60.95 cm. (24 in.). This cylinder was perforated over its whole surface with 0.952-cm. (0.375-in.) diameter holes on 1.27-cm. (0.5-in.) centers. The corona rods, 11 in number, of diameters ranging from 0.1038 to 1.2665 cm. were of tool steel polished and nickel-plated, and were all of the same length and equipped with similar threaded end fittings. A small opening *A* in the top cover permits easy insertion or removal of a rod, attachment of fittings to top and bottom insulators being made through two hand holes in the sides (see *A*, *B*, *B*, Fig. 10). These openings are closed with air-tight covers. The cover at *A* has a glass window permitting observation of visual corona. The locations of the thermometer *M*, the telephone, the connections *T* and *G* for telephone and galvanometer, and other auxiliaries are indicated in Fig. 10.

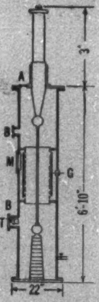

FIG. 10—CORONA VOLTMETER—200,000 VOLTS

(*a*) *Visual Corona.* Attention was first drawn to the phenomenon of the high-voltage corona by the power loss between transmission lines. This was promptly found to be accompanied by the visual corona around the conductors. All of the early studies of corona formation, of which the most notable were those of Ryan, gaged the first presence of corona by visual means. The visual method, however, is neither convenient nor accurate. It necessitates working in a dark room and is subject to error, in that accurate observation depends upon the state of the eye as regards its recent usage, fatigue, etc.

Nevertheless, under carefully controlled conditions using long time intervals for eye rests, it is possible to secure consistent observations with the visual corona. In the earlier papers of one of the authors comparisons of corona voltages as between the visual and other methods have been recorded showing that they have identical values. In the present work observations of this character have been repeated in some of the auxiliary experiments and they show the identity of corona voltages as observed by the visual method and by the far more convenient and accurate galvanometer. The visual method has, however, not been used for the measurements, the telephone and galvanometer being far more accurate and convenient.

(*b*) *The Telephone as Detector*. The presence of corona may be detected by its sound even in an open space. If the corona conductor is surrounded by an outer casing, such as a cylinder forming the opposite conductor, the sound within this enclosing space is confined and intensified. It may be conveniently used as a detector of the first presence of corona by means either of a direct tube connection between the ear and the enclosing chamber, or by means of a telephone transmitter within the enclosing chamber and a receiver at the ear. The latter method is necessary if the air pressure is to be varied and so is used in the corona voltmeter.

A number of experiments have been recorded in earlier papers, showing the identical values of corona voltages as observed visually, with the telephone, and with the galvanometer method described below. During all of the present work the telephone and galvanometer have been used simultaneously and the work throughout makes it certain that either of these methods may be used for detecting the first presence of corona.

The present work, however, has revealed that the character of the note in the telephone is different under different conditions as regards the size of the corona conductor and the density of the air surrounding it. This feature has considerable value after slight experience in the operation of the corona voltmeter, as it gives the observer information as to the conditions under which he is working. As regards its bearing on the accuracy of the telephone as a method of telling the first presence of corona, it is only necessary to note that the first uniform continuous note, whether it be faint and high or considerably louder and of lower tone, is to be taken as the signal of the presence of corona. The first type mentioned pertains to large wires at low pressure and the second to smaller wires and higher pressures.

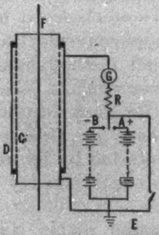

Fig. 11—The Galvanometer as Detector of Corona

Irregularities or other surface imperfections of the corona rod can usually be detected in the telephone by a characteristic crackling note quite distinct from that of corona.

(*c*) *The Galvanometer as Detector.* The essential elements of the galvanometer method of detecting corona are shown in Fig. 11, in which *C* is the perforated cylinder in the corona voltmeter which is connected to ground and to one side of the voltage to be measured; *D* is a surrounding cylinder only slightly larger in diameter than *C*, from which it is carefully insulated. The remaining elements of the circuit are obvious from Figs. 1 and 10. When corona appears on the central rod *F*, the surrounding air is copiously ionized and this ionization extends through the perforations to the space between the cylinders *C* and *D* which thus becomes highly conducting, resulting in a deflection of the galvanometer *G*.

An extensive series of observations has been made with corona rods of various size and under different conditions as to temperature and pressure on the relation between the voltage on the corona rod *F* and the resulting galvanometer deflections. A characteristic series of observations are given in Table VI, and the corresponding curves are plotted in Figs. 12, 13 and 14. In connection with these observations it may be noted that the sensitivity of the undamped galvanometer was 1280 megohms, and when critically damped with a shunt of 3400 ohms the sensitivity was 428 megohms. The resistance *R* was 50,000 ohms and the battery ± 115 volts. From the dimensions of the corona voltmeter already given, the length of the cylinder *C* was 60.95 cm., its diameter 24.67 cm., and its space separation from the cylinder *D* 0.317 cm.

TABLE VI.

GALVANOMETER AND TELEPHONE AS DETECTORS OF CORONA.

0.314 cm. diameter rod.

Positive electrode		Negative electrode		Electrode zero potential	
T. C. volts	Deflection cm.	T. C. volts	Deflection cm.	T. C. volts	Deflection cm.
0	0.0	0	0.0	0	0
31.4	0.0	31.4	0.0	31.4	0
31.5*	0.0	31.5*	11.7	31.5*	1.4
31.7	0.0	31.6	16.7	32	5.1
31.8	0.0			33	7.3
31.9	0.06			34	8.7
32	0.1			35	9.1
32.1	0.18			36	8.4
32.2	0.58			37	6.3
32.5	1.08			38	3.6
33	5.9			39	0.1
34	19.8			40	-3.9

*Telephone.

From these constants and from the observations the resistance of the space between *C* and *D* when in the initial conducting condition corresponding to the start of corona is about 1600 megohms.

Figs. 12, 13 and 14, pertaining to three different sizes of corona rod, are plotted with galvanometer deflections in centimeters as ordinates and transformer tertiary coil volts as abscissas. Each contains three sets of curves taken at different values of air density. The upper smaller curves are taken with the value of galvanometer sensitivity used throughout the voltage measurements. The larger curves are taken with galvanometer sensitivity reduced to 1/10 in order to extend the curves. Three curves were taken at each pressure, one each for the electrode *D* of Fig. 11 at 115 volts positive potential, one at 115 volts negative potential, and one at ground potential. The value of voltage at which the telephone is first heard is also indicated. It is to be noted that in all of these curves negative potential on the electrode *D* is best for the detection of the first presence of corona in that the curve rises most sharply. This is especially

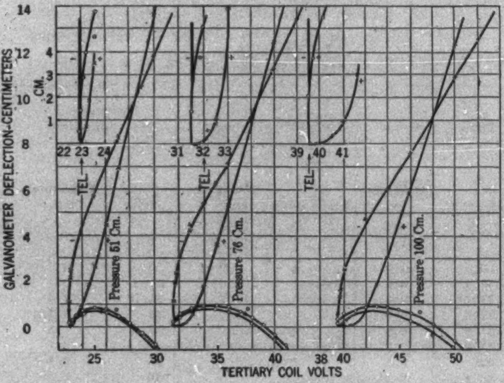

Fig. 12—Galvanometer as Detector of Corona—0.314-Cm. Diam. Rod

noticeable with small rods. With larger rods the advantage of negative over positive potential holds at low pressure, but tends to disappear at high pressures.

With larger rods at low pressures where the negative electrode should be used for first detection of corona, the telephone gives a pure high note of relatively faint volume within the interval of voltage between the curves of negative and positive electrodes, the full volume of sound appearing when the latter curve begins. At high pressures the curves come together; and with either positive or negative electrode the first corona is accompanied by a full sound in the telephone. With the smaller rods, although there is also a lag of the rise of the curve of positive electrode behind that of negative electrode, the faint initial high tone in the telephone is absent and except at very low pressures there is no marked variation in the telephone note, this note being clear and full with the first appearance of corona with negative electrode.

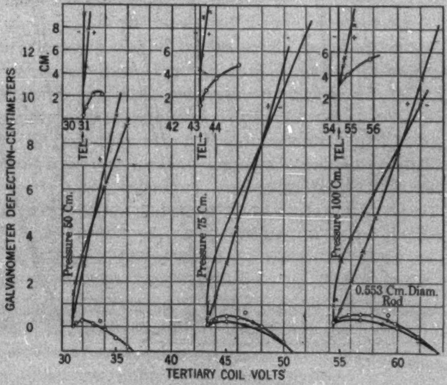

FIG. 13—GALVANOMETER AS DETECTOR OF CORONA—0.553-CM. DIAM. ROD

Considering the foregoing, therefore, from the standpoint of accuracy of determination of the first appearance of corona, negative potential should be used on the electrode *D* in all cases. Conditions of observation with both telephone and galvanometer are better at values of air density above that of normal atmosphere, rather than below, if large rods are used. Consequently, for reading low voltages better conditions are obtained by using a small rod rather than by using a large rod with reduced air density.

The difference in shape of the curves of positive and negative electrode has been noted in an earlier paper.

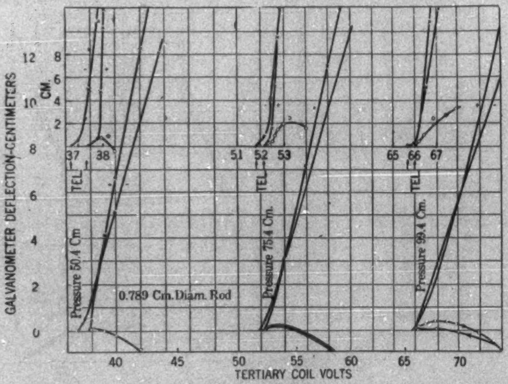

Fig. 14—Galvanometer as Detector of Corona—0.789-Cm. Diam. Rod

The greater sensitivity of the negative electrode is obviously due to the fact that corona formation or ionization of the air occurs first due to the motion of the negative electrons. The acceleration of the electron is greatest when it is moving toward the positively charged corona conductor. Under these circumstances the positive ions, as products of the process of ionization, would be repelled and would therefore give maximum current in the galvanometer circuit of Fig. 11 when the electrode *D* is negative. The exact shape of the curves probably depends on the wave form of voltage, moisture content of the air, and possibly on the frequency. It is only the initial slope of the most sensitive of these curves that is of importance in the detection of corona; their shape above the region of starting is of no present interest. It may be noted in passing, however, that it is possible to eliminate the voltage for the electrode *D* entirely and still observe the beginning of corona, although at a considerably lessened sensitivity. The curves showing the galvanometer deflection when the electrode *D* is at ground potential are seen to show reversal of galvanometer deflection at the voltage at which the curves of positive and negative electrodes intersect.

With reference to the actual degree of accuracy to which the beginning of corona could be observed, it will be noted from Fig. 12 that the galvanometer deflection begins and increases so sharply as to be practically instantaneous. Thus, for example, in Table VI and Fig. 12 the galvanometer deflection increases from 0 to 11.7 cm. within the voltage interval 31.4 to 31.5, the telephone coming in sharply at the same point. This indicates an accuracy of a very small fraction of 1 per cent. With the larger rod in Fig. 14 at high pressure we have a deflection of 1.4 cm. within the voltage interval 65.9 to 66. At low pressures the deflection is 2.4 cm. within the voltage interval 36.7 to 37.2. From this it will be seen that the sensitivity of corona detection is still quite high even under the unfavorable conditions of large rod at low pressure.

Throughout the observations the telephone and the galvanometer were read simultaneously, each checking the other. If an appreciable galvanometer deflection occurred without a corresponding clear telephone indication, or vice versa, an explanation could usually be found in a local spark or other surface impurity on the rod. We believe, in view of the foregoing that throughout the work the accuracy with which the beginning of corona has been read is better than 1/10 of 1 per cent. In this connection it may be pointed out that the initial flat portion or low rate of rise of the negative curves with large wires can only be detected by use of a very sensitive instrument. These initial portions are probably due to slight surface imperfections rather than to full corona. This is borne out by the fact that the full telephone note comes out at a point corresponding to the steep portion of the curve. With an instrument of lower sensitivity, such as would normally be used with the corona voltmeter, these initial portions of the curve cannot be detected and the instrument takes an initial sharp deflection accompanied by the simultaneous telephone note.

III. 3 MEASUREMENT OF AIR DENSITY

The relative air density δ as given in Formula (3) is

$$\delta = \frac{3.92 \times p}{273 + t} \tag{3}$$

p being the absolute pressure in centimeters of mercury. and t the temperature in degrees centigrade. Thus δ has the value 1 at 76 cm. mercury and 25 deg. centigrade.

(*a*) *Pressure.* In the experiments the pressure was read on an open mercury manometer, (see Fig. 10), the accuracy of observation therefore being to about 0.2 millimeter. The usual correction for temperature was applied. All of the observations leading to the precision formulas (7) and (8) for the electric strength of air were taken within the range of pressures 25 cm. and 139 cm. absolute, all of which was covered by the mercury manometer. A number of observations studying the performance of the corona voltmeter at higher pressures were made with a standard direct reading gage by Schaeffer & Budenberg, calibrated within their common range in terms of the mercury manometer.

At the highest and the lowest pressures slight leaks in the outer casing of the voltmeter were detected. These leaks were, however, quite slow and pressure readings were taken at the beginning and end of each series of voltage readings. The average value of pressure within the interval was usually taken in these cases and an inspection of Table VII indicates that the error on this account was negligible. In calculating δ, p is the absolute pressure in centimeters of mercury.

The open mercury manometer reads pressure with relation to the atmosphere. Atmospheric pressure was determined from the laboratory standard barometer with the usual correction for temperature.

(*b*) *Temperature*. The temperature within the voltmeter casing was read on an ordinary laboratory centigrade thermometer hung within the casing near its wall and so that it could be viewed through a small glass window. The thermometer could be read to within about 0.2 degree. From an inspection of the value of δ this indicates that the absolute temperature was read to within less than 1/10 per cent.

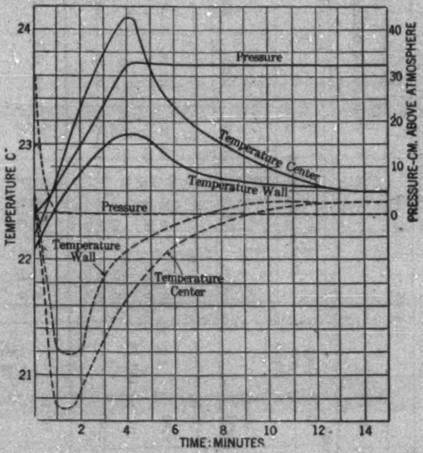

Fig. 15—Temperature at Surface of Corona Rod

The determination of the temperature in this way assumes that the temperature of the air at the surafce of the corona rod is the same as that near the outer wall of the voltmeter. There is an obvious possibility of error here, so the matter was investigated experimentally by comparing the thermometer mentioned above with another hung immediately adjacent to the corona rod and viewed by telescope through another glass window. The curves of Fig. 15 show the differ-

TABLE VII.
CORONA VOLTAGE READINGS.

			Press. Guage		Corr. Abs. Press.	Voltage Galvanometer							Ter. Coil Volts
0.4765 cm. diam. rod						At start of Corona			Calibration				
Freq.	Bar. Press.	Temp.	Left	Right		Left	Right	Mean	Left	Right	Volts 499 ohms	Milamp. per div.	
60.03	76.56	18.9	72.30	8.60	139.74	10.22	10.30	10.26	10.26	10.29	0.3890	0.07590	65.9
						10.23	10.30	10.26					65.95
						10.22	10.30	10.26					65.95
		19.0	72.00	8.90		10.22	10.30	10.26	10.23	10.30	0.3891		65.95
60.04	76.12	19.8			76.12	6.22	6.23	6.22	6.19	6.21	0.2348	0.07591	40.2
						6.19	6.23	6.21					40.1
						6.19	6.22	6.20					40.1
		20				6.19	6.22	6.20	6.18	6.22	0.2348		40.05

60.03	76.50	18.8	19.97	60.48	36.25	3.46	3.48	3.47	3.42	3.44	0.1308	0.07642	22.5
						3.47	3.49	3.48					22.2
						3.48	3.49	3.48					22.3
		18.7	20.07	60.33		3.48	3.50	3.49	3.42	3.44	0.1308		22.4
0.7109 cm. diam. rod													
60.03	76.02	20.8	60.30	19.85	116.13	11.01	11.09	11.05	11.01	11.10	0.4189	0.07584	70.9
						11.01	11.08	11.04					70.95
						11.01	11.10	11.05					70.95
		20.8	60.12	20.05		11.00	11.09	11.04	11.01	11.10	0.4189		70.95
60.11	75.97	21.7	30.02	50.17	55.98	6.08	6.12	6.10	6.01	6.03	0.22785	0.07585	39.05
						6.08	6.11	6.095					39.05
						6.08	6.11	6.095					39.1
		21.7	30.08	50.08		6.08	6.12	6.10	6.00	6.04	0.22785		39.1
60.03	75.07	21.7	20.35	60.10	35.60	4.17	4.29	4.23	4.18	4.30	0.1605	0.07588	27.1
						4.18	4.29	4.235					27.1
						4.19	4.3	4.245					27.3
		21.7	20.49	60.00		4.19	4.32	4.255	4.18	4.30	0.1605		27.4

ence in temperature between these two thermometers resulting from rapid expansion and compression of the air within the voltmeter. They indicate that on compression from atmosphere to 30 cm. above atmosphere there is a resulting difference of temperature of about 1 deg. and on expansion of approximately ½ deg. between the two thermometers. In the former case the two thermometers reached the same temperature within five or six minutes and within a shorter interval in the latter case. In the observations no such sharp changes of pressure occurred, the common maximum change being about 10 cm. In all cases, however, sufficient time was allowed to ensure that the observed temperature was sensibly the same as that near the corona rod.

IV. EXPERIMENTAL OBSERVATIONS

Many hundreds of observations were taken with eleven different sizes of corona rod of diameters, as follows: 1.266, 0.955, 0.790, 0.710, 0.654, 0.5536, 0.4765, 0.3142, 0.2060, 0.1197, 0.1038 cm. The figures for the diameters given in the tables and computations are the average values taken from 20 micrometer measurements on each rod. Except in the cases of the smallest rods, the maximum variations from the mean diameters as given were quite small, that is, in the neighborhood of 0.2 or 0.3 per cent. The rods were all of tool steel and nickel-plated after polishing, this material and treatment yielding the most accurate cylindrical shape and smoothest surface that we have found. The extremes of absolute pressure reached in the precision determinations were 25 cm. and 139 cm. of mercury, although not all the rods were carried through the entire range.

The usual sequence in taking observations was as follows: Adjust frequency control and read its value; set pressure in corona voltmeter to desired value; adjust filament current and kenotron circuit for zero current in the closed kenotron circuit; read temperature and pressure in voltmeter; raise voltage slowly until corona begins, as indicated by galvanometer and telephone; read value of charging current of condenser by galvanometer.

One observer took the temperature and pressure and read the charging current galvanometer. A second observer, who raised the voltage, was equipped with telephone head-piece and read the galvanometer indicating the beginning of corona. This observer also took a reading of an ordinary direct-reading voltmeter connected to the tertiary coil of the high-tension transformer. This last reading was useful as a check of the constancy of circuit conditions and for rough comparisons, but its readings were not used in any computations. At the instant that the presence of corona was detected by galvanometer and telephone the slow elevation of voltage would be stopped and the first observer would take the reading of the charging current galvanometer. Obviously the voltage elevation could be made as slow as desired and frequent check readings were taken which do not appear in the record. The degree of constancy of the corona voltage is discussed in the following paragraph. After each series of observations the charging current galvanometer was calibrated at a deflection approximately equal to that of the observation.

Table VII gives six typical sets of readings of the principal data, these sets being selected at random from the observation sheets. The first six columns give the frequency, temperature and pressure, leading to the corrected absolute pressure in column 6. Columns 7 and 8 give the right and left readings of the galvanometer measuring the condenser charging current. It will be noted that there are four pairs of such readings for each value of pressure. Between the readings of each pair and between each of the pairs the voltage was lowered below corona value and raised again. The readings of these columns therefore are a good indication as to the accuracy with which corona formation repeats itself.

Since the beginning of corona is indicated by the telephone and by a sudden sharp deflection of the galvanometer rather than by the magnitude of the deflection of the latter, this latter reading was not recorded. (See however Figs. 12, 13 and 14.) A direct relative indication of the alternating voltages at which successive coronas start is available, however, in the tertiary coil voltmeter and the readings of this instrument are given in the last column of Table VII.

Referring to the readings of the pressure, it will be noted that there is usually a difference in the net readings of the mercury manometer at the beginning and at the end of each series of four pairs of corona readings. These differences are due to slight leakage of the voltmeter casing and are therefore greatest for the highest and lowest absolute pressures. The differences are never very great and when they occur they are followed by a corresponding change in the corona voltage as indicated by the condenser charging current. Therefore in taking for each set of readings the average value of the pressure and the average value of the galvanometer reading no appreciable error is introduced.

Three sets of auxiliary observations should be recorded here bearing, respectively, on the perforation of the grounded cylinder, the influence of frequency and the permanence of the surface of the corona forming rod.

As regards the holes in the grounded cylinder, a number of observations from time to time have shown that these perforations have no effect on the value of the corona-forming voltage; for example, in the paper "The Electric Strength of Air.—III"[13] a number of experiments were conducted with an outer cylinder made of wire mesh of a quite wide opening. The values of corona voltage observed in this case were sensibly the same as those obtained with cylinders with continuous walls. Further, numerous readings taken with outer cylinders having continuous walls have shown that the voltages at which visual corona appears are in accord with those observed in the corona voltmeter. In order, however, to make a direct test of this question, two series of observations of visual corona voltage were taken, using in the two cases outer cylinders cut from the same length of brass tubing, one piece being perforated and the other not perforated.

The two tubes were each 9.5-cm. inside diameter and of length 24 cm. One was perforated with 0.27-cm. diameter holes drilled as closely as possible to each other and in a number of cases being actually tangent to each other. A clean rod 0.315 cm. in diameter was centered in each of these tubes in turn and visual corona observations taken with a rested eye in a darkened room. The corona voltages as measured on the transformer tertiary coil in the two cases were: with perforated cylinder 23.5, 23.4, 23.4; with unperforated cylinder 23.4, 23.5, 23.6, 23.5, 23.5. There appears therefore no reason to question that the perforations in the grounded cylinder have no effect on the value of corona-forming voltage.

As regards the influence of frequency on corona-forming voltage, in the paper "The Electric Strength of Air.—II",[14] experiments were described which indicated a slight lowering of corona voltage with increasing frequency between the range 10 and 100 cycles. Subsequently Whitehead and Gorton[8] have extended the range of frequency up to 3000 cycles and have shown that within the range 500 to 3000 cycles there is practically no influence of the frequency on the corona-forming voltage. They noted, however, that the value of corona voltage within this range was from 3 to 4 per cent lower than at 60 cycles. F. W. Peek records that within the range of commercial values the frequency has no influence on corona-forming voltage.

Having at hand the accurate method of measurement already described, a series of observations was taken on the influence of frequency between the values 20 and 90 cycles. The readings were taken on a rod 0.48 cm. in diameter and at atmospheric pressure. Several readings at each frequency were taken and all were reduced to the same value of $\frac{1}{\sqrt{\delta r}}$. The corresponding values of E/δ are given in Table VIII.

The above figures indicate that there is a small influence of frequency on corona-forming voltage within the range 20 to 90 cycles. For example, the corona-forming voltage at 25 cycles is shown to be about 0.8 per cent higher than at 60 cycles. All of the other observations of this paper were taken at 60 cycles and therefore the laws of corona formation, as discussed below, pertain to that frequency.

As regards the permanence of the surface of the corona-forming rod, experiments have shown that a practically indefinite number of corona observations may be taken without deterioration of the surface of the nickel-plated steel rods used. These experiments consisted in maintaining corona on a rod for a long period of time, interrupting it at regular intervals and taking the corona-forming voltage. One of several such tests consisted in maintaining a 0.476-cm. diam. rod at a voltage 2.5 per cent above the corona-forming value for one hour, and taking the corona-forming voltage six times during the interval. At start the corona-forming voltage as indicated at the tertiary coil terminals was 40.1; at successive intervals through the hour the values were 40.05, 40.05, 40.05, 40.15, 40.2, 40.1.

TABLE VIII.
INFLUENCE OF FREQUENCY.

Frequency cycles per second	Values of $\frac{E}{\delta}$ at $\frac{1}{\sqrt{\delta r}} = 2.03$	Aver.
20	50.41, 50.50	50.45
40	50.25, 50.25, 50.39, 50.5	50.35
60	49.95, 50.07, 50.13	50.05
90	49.24	49.24

If the voltage is carried higher, increasing the volume of corona discharge, the surface will ultimately develop local spark points, leading to local sparks at voltages lower than corona-forming values. Normally, however, with an initially clean rod and dust-free air inside the voltmeter, the surface of the rods in the corona voltmeter shows a most satisfactory degree of permanence.

V. SUMMARY OF OBSERVATIONS

From the complete data of which Table VII is a small portion, the values of the relative density δ are computed from formula (**3**). The charging current galvanometer is calibrated directly in terms of current in the resistances R_1 and R_2 of Fig. 1. As already described, the readings of this galvanometer then lead through formula (**4**) to the crest value of alternating voltage. From this value and the dimensions of the rod and the inner cylinder of the corona voltmeter, the critical or corona-forming electric intensity in kilovolts per centimeter at the surface of the conductor is readily computed. Some of these steps are given in Table VII and the more important ones are collected in Table IX.

Table IX shows about one-fourth of the total number of derived values for computing the law of corona. The summary of all these values is given in Table X. Each reading of voltage in Table IX corresponds to four pairs of right and left galvanometer readings at each pressure, as set forth in Table VII. It will be noted that on the average Table IX presents three sets of readings for each value of pressure. This means that for each pressure there are 12 observations of corona forming voltage. Table IX also includes both observed and computed values of $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta r}}$, the latter based on the law of corona deduced from all the observations as set forth in the following section.

VI. THE LAW OF CORONA

As the work proceeded the values of $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta r}}$ were plotted. It was found that the resulting curve was a straight line, in accordance with formula (**1**), for the larger values of $\frac{1}{\sqrt{\delta r}}$. However, on extending the study to larger corona rods, and especially at the higher pressures, it was found that the points departed from the straight line indicated for smaller rods and lower pressures. This fact at first was quite disturbing as it suggested either a departure from the simple law

TABLE IX.
COMPUTED VALUES FOR LAW OF CORONA.

Rad. r cm.	Temp. t deg. cent.	Press. p cm.	δ	Kv. max.	Surf. int. E kv/cm max.	$\frac{E}{\delta}$	$\frac{1}{\sqrt{\delta r}}$	$\frac{E}{\delta}$ calc.	Diff. per cent.
0.2383	18.0	46.92	0.6322	32.51	34.57	54.69	2.577	55.02	−0.60
"	18.0	47.20	0.6360	32.71	34.79	54.71	2.569	54.95	−0.44
"	18.1	47.37	0.6381	33.00	35.10	55.01	2.565	54.92	+0.16
"	18.7	35.97	0.4835	26.42	28.09	58.11	2.946	58.17	−0.10
"	18.75	36.25	0.4872	26.73	28.42	58.35	2.935	58.08	+0.47
"	20.3	26.65	0.3563	20.61	22.14	62.16	3.432	62.33	−0.27
"	20.5	27.06	0.3615	21.06	22.39	61.96	3.408	62.13	−0.27
0.1571	22.4	115.29	1.5301	52.24	76.21	49.80	2.040	50.11	−0.62
"	22.3	114.76	1.5238	52.20	76.15	49.97	2.044	50.15	−0.36
"	22.2	114.32	1.5185	52.09	75.99	50.04	2.048	50.19	−0.30
"	22.8	96.10	1.2739	45.00	65.65	51.53	2.235	52.05	−1.00
"	22.8	95.88	1.2710	45.04	65.69	51.69	2.238	52.08	−0.75
"	22.7	95.70	1.2690	45.05	65.72	51.78	2.240	52.09	−0.60
"	21.85	76.00	1.0107	37.37	54.51	53.94	2.510	54.45	−0.90
"	22.0	75.98	1.0099	37.54	54.76	54.23	2.511	54.46	−0.42
"	22.1	75.95	1.6092	37.54	54.76	54.27	2.512	54.46	−0.35
"	22.9	55.20	0.7315	28.98	42.28	57.80	2.950	58.21	−0.71
"	23.05	55.39	0.7336	29.17	42.35	58.00	2.946	58.27	−0.29
"	23.2	55.50	0.7347	29.24	42.66	58.06	2.944	58.25	−0.15
"	23.4	45.19	0.5978	24.89	36.31	60.74	3.263	60.90	−0.26
"	23.45	45.38	0.6002	25.03	36.51	60.83	3.257	60.84	−0.02
"	23.4	45.48	0.6016	25.04	36.52	60.70	3.253	60.81	−0.02
"	18.2	35.66	0.4802	21.15	30.86	64.26	3.641	64.13	+0.22
"	18.3	36.19	0.4871	21.42	31.25	64.14	3.615	63.91	+0.38
"	18.3	36.33	0.4890	21.46	31.30	64.01	3.608	63.85	+0.27
0.1030	26.0	115.48	1.5144	40.88	82.94	54.78	2.532	54.64	+0.26
"	26.0	115.01	1.5082	40.91	82.98	55.03	2.537	54.68	+0.64
"	26.0	114.68	1.5039	40.75	82.75	55.03	2.541	54.71	+0.59
"	20.6	114.80	1.5331	40.85	82.86	54.05	2.517	54.51	−0.85
"	20.75	113.97	1.5213	40.86	82.89	54.49	2.526	54.59	−0.18
"	20.8	113.70	1.5174	40.86	82.89	54.63	2.530	54.61	+0.04
"	26.7	96.05	1.2566	35.25	71.52	56.90	2.780	56.75	+0.26
"	26.6	95.77	1.2534	35.26	71.55	57.07	2.783	56.78	+0.51
"	26.6	95.62	1.2514	35.25	71.52	57.14	2.785	56.80	+0.60
"	19.9	76.05	1.0181	29.88	60.62	59.54	3.088	59.40	+0.24
"	20.0	76.03	1.0174	29.97	60.80	59.76	3.089	59.41	+0.59
"	20.0	76.00	1.0170	30.00	60.86	59.85	3.090	59.41	+0.74

TABLE X.

No.	Rad. cm.	$\frac{1}{\sqrt{\delta r}}$	$\frac{E}{\delta}$		Per cent discrep.
			obs.	calc.	
1	0.633	1.253	42.38	42.29	+0.21
5	"	1.386	43.71	43.61	+0.23
2	0.477	1.282	42.56	42.58	−0.05
4	"	1.353	43.02	43.28	−0.6
6	"	1.43	43.92	44.05	−0.3
7	"	1.431	44.20	44.06	+0.32
10	"	1.530	44.99	45.04	−0.11
12	"	1.661	46.28	46.34	−0.13
15	"	1.852	48.22	48.24	−0.04
16	"	1.870	48.57	48.42	+0.31
	"	2.117	50.79	50.88	−0.18
20	"	2.493	54.1	54.29	−0.35
3	0.355	1.349	43.24	43.24	0.0
8	"	1.480	44.60	44.55	+0.11
13	"	1.667	46.58	46.40	+0.39
	"	1.944	49.41	49.15	+0.53
19	"	2.418	54.01	53.66	+0.65
18	"	2.379	53.49	53.32	+0.32
9	0.238	1.503	44.55	44.78	−0.51
11	"	1.653	46.28	46.26	+0.04
14	"	1.798	47.70	47.70	0.0
	"	2.035	49.94	50.05	−0.22
	"	2.045	49.91	50.15	−0.48
	"	2.194	51.31	50.64	−0.65
17	"	2.375	53.07	53.29	−0.41
24	"	2.570	54.79	54.95	−0.29
26	"	2.941	58.22	58.11	+0.19
30	"	3.413	62.06	62.16	−0.16
	0.157	2.044	49.94	50.15	−0.42
	"	2.238	51.67	52.08	−0.79
21	"	2.511	54.15	54.45	−0.55
27	"	2.946	57.95	58.17	−0.38
29	"	3.257	60.76	60.84	−0.13
33	"	3.621	64.14	63.94	+0.31
23	0.103	2.537	54.95	54.68	+0.49
22	"	2.524	54.39	54.56	−0.31
25	"	2.783	57.04	56.78	+0.46
28	"	3.089	59.72	59.40	+0.54
32	"	3.610	64.25	63.87	+0.6
35	"	4.024	67.38	67.40	−0.03
38	"	4.562	72.21	72.00	+0.29
36	0.0598	4.041	67.99	67.54	+0.67
39	"	4.716	73.33	73.31	+0.03
41	"	5.22	77.48	77.63	−0.19
43	"	5.983	83.42	84.16	−0.88
31	0.0519	3.537	63.77	63.24	+0.84
34	"	3.865	66.57	66.04	+0.81
37	"	4.356	70.23	70.24	−0.01
40	"	5.097	76.46	76.58	−0.16
42	"	5.529	79.93	80.28	−0.44
44	"	6.254	85.60	86.47	−1.01

of formula (1) or the presence of some error in method or observation. Many readings were repeated but resulted only in confirming the earlier ones.

Further study and investigation of the foregoing interesting results led to the work of Whitehead and Brown[6] on "The Corona at Continuous Voltages," which shows that while both the positive and negative corona obey a law of the form of formula (1), yet the constants A and B are different in the two cases. This means that if the law for each case is put into the form of the linear relation between $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta r}}$ the two lines will intersect and that below the point of intersection negative corona appears first, while above the point of intersection positive corona appears first.

Extending the foregoing to corona formation at alternating voltage, we should find, if we plot between $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta r}}$ that below the value $\frac{1}{\sqrt{\delta r}} = 1.895$, representing the intersection of the positive and negative corona curves, the alternating corona should obey the same law as the negative continuous corona and that above that value it should obey the law found for positive corona. This is exactly the result that we have found and it therefore constitutes a necessary and in fact important, modification of the law of corona.

The foregoing conclusions are immediately obvious if all of the observations are plotted, and as the results are very consistent throughout, a quite close approximation to the exact values of the constants of formula (1) is possible by this graphical method. However, it is obviously better to derive the values of these constants from the figures themselves, and for this purpose the "Sigma-Delta" method[12] for evaluating the constants, has been used.

If we attempt to apply the Sigma-Delta method to the entire set of observations, a part of which are given in Table IX, it becomes very laborious indeed. It appeared to us, therefore, that this could be avoided by a still further averaging of the results for one pressure corresponding to each of the groups of readings in Table IX, this averaging being done on the values of $\frac{E}{\delta}$ and $\frac{1}{\sqrt{\delta r}}$. There being a linear relation between these two quantities, no error is thereby introduced. In this way the values of Table X are reached. The first column of Table X gives the sequence numbers as used in the Sigma Delta method; the third and fourth columns the mean values of $\frac{1}{\sqrt{\delta r}}$ and $\frac{E}{\delta}$; the fifth column the calculated value of $\frac{E}{\delta}$ as derived from the Sigma-Delta method; and the sixth column the error as between observed and calculated values of $\frac{E}{\delta}$ expressed in per cent.

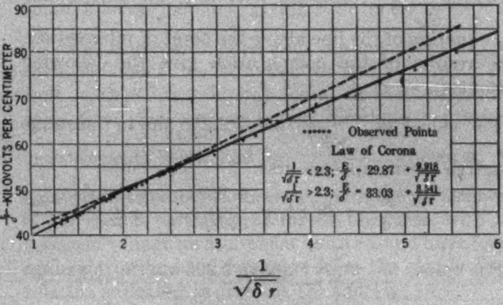

FIG. 16—THE LAW OF CORONA

In applying the Sigma-Delta method to the figures of Table X it was thought best, in view of the uncertainty as to the exact point of intersection of the two straight lines referred to above, to omit the points in the immediate neighborhood of this point of intersection. Consequently the points were plotted, as shown in Fig. 16, and the approximate position of the point of intersection of the two lines thus determined roughly. This being done, the readings corresponding to values of $\frac{1}{\sqrt{\delta r}}$ between 1.9 and 2.3 were omitted from the computations for the reason mentioned. In this way the equation of the line below the point of intersection was determined from the first sixteen readings of Table X, comprising the interval 1.253 to 1.870, for $\frac{1}{\sqrt{\delta r}}$. There are twenty-eight readings above the point of intersection comprising values from 2.375 to 6.254 for $\frac{1}{\sqrt{\delta r}}$. These were used for determining the equation of the line above the point of intersection. The results of the computation give the following formulas:

For values of $\frac{1}{\sqrt{\delta r}}$ below 2.295 and in this range negative corona appears first:

$$\frac{E}{\delta} = 29.84 + \frac{9.938}{\sqrt{\delta r}} \tag{7}$$

For values of $\frac{1}{\sqrt{\delta r}}$ above 2.295 and in this range positive corona appears first:

$$\frac{E}{\delta} = 32.96 + \frac{8.559}{\sqrt{\delta r}} \tag{8}$$

The point of intersection of the two lines is $\frac{1}{\sqrt{\delta r}} = 2.26$ corresponding to a value of $\frac{E}{\delta} = 52.39$.

It is interesting to note in connection with Fig. 16 that observations on two of the rods used (0.238-cm. and 0.157-cm. radii) give several points each on both sides of the point of intersection of the two lines.

The extension of the observations in the direction of larger values of $\frac{1}{\sqrt{\delta r}}$ was readily accomplished by using smaller rods and lower pressures. The extreme values in this direction were reached in using a 0.0519-cm. radius rod and an absolute pressure of 36 cm. of mercury giving the value 6.28 for $\frac{1}{\sqrt{\delta r}}$.

In the opposite direction, *i. e.*, smaller values of $\frac{1}{\sqrt{\delta r}}$ there is also a wide range possible in increasing the pressure and using larger rods. Our largest rod was 0.633-cm. radius and our greatest pressure about 135-cm. mercury, giving 1.25 for $\frac{1}{\sqrt{\delta r}}$. As indicated elsewhere, larger rods are not desirable, but obviously much higher pressures can be used. Our limit in this direction was found in the break-down voltage of the air condenser, precluding precision measurements below 1.25 for $\frac{1}{\sqrt{\delta r}}$. However, we have made a number of observations of the performance of the corona voltmeter at higher pressures, with the precision voltage measurement omitted, and have found nothing to suggest a departure from the simple linear relation indicated in formula (7) and Fig. 16.

VII. ADVANTAGES OF THE CORONA VOLTMETER AS A STANDARD

The law expressed in formulas (7) and (8) constitutes a definite standard over a wide range of voltage. The range has been tested by precision measurements up to the neighborhood of 150,000 volts and with every evidence that the law continues beyond that value. The only quantities which enter are the radius of the corona rod, the radius of the outer cylinder, and the density of atmospheric air. This is equivalent to saying that the calibration of the corona voltmeter depends only on its physical dimensions.

As regards its availability in practical measurements, a workable corona voltmeter may be set up in practically any laboratory with very little trouble. A straight clean wire stretched on the axis of a surrounding metal cylinder will give very reliable indications in a darkened room with a visual observation of corona formation. With little additional trouble a galvanometer may be used as corona indicator. A considerable although not a continuous, range may be had by using corona wires of different diameters.

The construction of the complete corona voltmeter itself, moreover, is a relatively simple and inexpensive matter. Up to 100,000 volts it may be readily constructed in any well-equipped laboratory and for higher ranges offers no serious difficulties. With the complete instrument a wide and continuous voltage setting is available using a single rod and observations sharply marked may be taken with an ordinary laboratory galvanometer or with a telephone receiver.

The usual method of setting for a definite voltage is to read the temperature inside the instrument, set the pressure at a particular value based on the value of δ, corresponding to the desired voltage setting, and which may be read from a table based on formulas (**7**) and (**8**) and then slowly raise the voltage until corona appears, as indicated by telephone or galvanometer. While this would probably be the more common usage in connection with insulation testing and other similar service, the instrument may also be used for measuring an unknown voltage. In this case the pressure would be set for a voltage known to be higher than that to be measured and the pressure then allowed to fall slowly, its value being read at the instant corona appears. Formulas (**7**) and (**8**), or tables computed from them, would then give the value of the voltage.

The corona voltmeter would appear to have several advantages over the needle and sphere gaps; among them may be mentioned the following:

(a) Freedom from disturbance by proximity of neighboring conductors or extraneous electrostatic fields.

(b) A 2 per cent inaccuracy is the minimum claimed for the sphere gap. With careful manipulation and good circuit conditions it is certain that a corresponding figure of better than 0.5 per cent is possible with the corona voltmeter.

(c) No manipulation of high-voltage circuit. Each change of setting of the sphere gap requires altering the distance between the discharge spheres. The setting of the corona voltmeter for different voltages requires the change of air pressure only.

(d) No discharge of high-voltage circuit and no series resistance necessary. The reading of the corona voltmeter is continuous and stationary and draws no current from the high-voltage circuit.

(e) Measurement of an unknown voltage. This cannot be done with the sphere gap except through repeated opening of circuit and successive approximation.

(f) All parts of the corona voltmeter are grounded except the leading-in wire of the high-tension terminal. All dimensions remained fixed. All auxiliary circuits are at low values of continuous voltage.

(g) Permanence. The surface of a corona-forming rod remains unaffected under the continuous application of initial corona over long periods.

As regards outside dimensions, the earlier paper[1] on the corona voltmeter described a corona voltmeter for voltages under 50,000 having dimensions of 76 cm. in length and 24 cm. in diameter. The instrument shown in Fig. 9 is 9 ft. 10 in. high (of which 3 ft. is in the insulating bushing) and 1 ft. 10 in. in diameter. The inside cylinder forming the grounded terminal is 24.6 cm. in diameter and 61 cm. long. The most convenient diameters of corona rod are between 0.3 cm. and 0.9 cm. This instrument has been used for voltages up to 150,000 volts without sign of distress, this being the maximum voltage obtainable under the conditions of test. It was used for this voltage at an internal air pressure of about 135 cm. of mercury absolute, *i. e.*, not quite 15 lb. per sq. in. above atmosphere. Pressures three or four times this value could readily be used. The limiting voltage would probably be found in the flash-over voltage of the insulating terminal bushing. This bushing has a normal rating of

150,000 volts effective. It is probable therefore that the instrument shown in Fig. 9 may safely be operated without trouble to 200,000 volts maximum value.

An instrument rated normally at 300,000 volts, and which it is expected may reach 400,000 volts, is now in process of construction.

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DISCUSSION ON "THE CORONA VOLTMETER AND THE ELECTRIC STRENGTH OF AIR" (WHITEHEAD AND ISSHIKI), WHITE SULPHUR, W. VA., JUNE 30, 1920.

C. L. Fortescue: In regard to the use of corona voltmeter as a secondary standard, there are a number of things to be said. One is that for practical work it is very hard to use sensitive galvanometers, on account of vibration and other disturbances. It is difficult to get men with sufficient judgment to note such a thing as a corona point, and even to use a device like the telephone. It is much easier to have something that they can see, like a voltmeter.

Now, in using the sphere gap, the fact that you cannot measure varying voltages is not important. In nearly all practical work we have to test apparatus for a certain maximum voltage, and it is only necessary in making a test, to have the apparatus connected in, set the sphere gap to the value you want to use, spark over the gap, and note the reading of the voltmeter on the secondary winding of transformer or tertiary winding, and after resetting the sphere gap for a slightly higher spark-over voltage test the apparatus by holding the voltage by means of the voltmeter; in other words, note the correspondence, and keep the apparatus at that voltage. It is evident therefore that the sphere gap does not offer any difficulties in that respect.

As regards the accuracy of the sphere gap, which is in the neighborhood of two per cent in practical work, that is a sufficient degree of accuracy. While I think Dr. Whitehead's corona voltmeter would have other uses; than for practical work I still think that the sphere gap is a better secondary standard for that purpose.

Now possibly the method that Dr. Whitehead used for calibrating his voltmeter might be used for a secondary standard as well as for a primary standard—in fact, we have been using that method for a crest voltmeter measuring device, in conjunction with the kenotron, and found it quite reliable within certain limits. Dr. Whitehead defined those limits; they depend on the following characteristics of the vacuum valve, so long as the voltage wave does not have more than one maximum, that is to say, so long as the slope of the voltage wave does not become zero at more than one point in a half wave—the kenotron device will give accurate results. There is a prospect of using the commutator device instead of the kenotron. We encountered the same difficulties in regard to that, that I have also mentioned in regard to observations—the ordinary tester is not a very good observer, and you cannot depend on his judgment a great deal. The main difficulty with the commutator device is the angle setting, the setting of the angle of the commutator so that you get the maximum value. It requires good judgment in order to do that properly. I still have hopes that we may be able to apply the commutator absolute measurement scheme also for secondary standards.

A. E. Kennelly: I would like to ask Dr. Whitehead whether he does not look on his galvanometric method as the primary method, and merely checks it up with his telephone to make sure that it is functioning properly?

Joseph Slepian: I would like to ask Dr. Whitehead, now that he has found that the corona makes a good peak voltmeter, whether he concludes that, like the sphere gap, it has a unity impulse ratio, or whether under periodic voltage, which exceeds a definite peak, the corona begins to develop through several cycles?

This would be of importance in considering the application of impulse discharges, such as are made in tests on insulators for lightning impulses.

L. W. Chubb: The point that Dr. Slepian just brought up is a very pertinent one; in some cases we do need a big voltmeter that has an impulse ratio of 1, in other words, it has a great discharge, and that is one of the disadvantages of the sphere gap in ordinary testing. If there is a slight surge it sparks over and ruins your test in a great many cases. The calibration curves on an ordinary circuit are regular gunshot diagrams. and it seems as though Dr. Whitehead's method, which checks so very close should be given consideration as a standard for the Institute. I think the other method of integrating the current in an air condenser is probably more practical as a working standard, but this is a very accurate method of detecting corona, and its constancy makes it a rather good, you might say, sub-primary standard.

F. W. Peek, Jr.: It is interesting to point out that high voltages anywhere may be readily determined by means of a piece of wire and a formula which involves a simple arithmetical calculation.

Dr. Whitehead has developed the corona voltmeter to a high state. There will undoubtedly be certain fields in which it will prove useful. It rarely happens that any method replaces all others. The accuracy of the corona voltmeter should be of the same order as that of the sphere gap. The phenomena is the same.

Corona is spark-over to space. In the sphere the spark occurs from metal to metal.

In my work I have found that in making investigations at operating frequencies a voltmeter coil in the transformer is by far the most convenient method and gives the best results. If a sine wave is not available a crest voltmeter should be used on the voltmeter coil.

When transient or lightning voltages are studied the sphere gap is practically the only available method. It also has the additional advantage that it measures to a high degree of accuracy voltages varying from direct current to transients of a fraction of a microsecond duration. Our knowledge of transients has been greatly extended during the last few years because of these characteristics of the sphere gap.

Transient voltages can also be measured by means of the corona voltmeter but to a less degree of accuracy than with the sphere. There is a difference in the appearance between the corona on a wire when it is (+) or (—). In my investigations I found that corona produced by transient voltages lasting less than a millionth of a second could not only be readily seen but that the eye could distinguish between a (+) and (—) wire.[1] For the smaller wires there is lag and the starting voltage is higher than at the lower frequencies

J. B. Whitehead: As regards the objection to the use of the galvanometer, the question raised by Mr. Fortescue, I may say that since the experiments described in this paper were completed, we have used an ordinary indicating instrument, a milammeter or voltmeter, as corona detector in place of the galvanometer. We do this with vacuum tube amplifiers. The additional equipment is not serious for a standard instrument. As to the sphere gap and the standard air condenser used directly as working methods, I think that the advantages that can be offered for the corona voltmeter are its greater accuracy and the facts, that it is compact in shape, that there are no exposed high-tension terminals or working parts except the top end of the insulating bushing, and the fact that it is entirely free from disturbances due to external fields.

I do not desire to make the suggestion that the corona voltmeter is any more suitable for shop testing than existing methods, but it does appear to me, that in view of the advantages I have mentioned, and especially the fact that as regards accuracy, it certainly far exceeds any figures which have been offered for the sphere gap, there is a great deal to be said for it as an ultimate secondary standard.

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[1] Peek, Effect of Transient Voltages on Dielectrics, TRANS. A. I. E. E., 1915, Vol. XXXIV, p. 1857.

As regards Prof. Kennelly's question as to the relative value of the telephone and galvanometer, as indicators, certainly in the hands of the man in the machine shop, the telephone appeals to me as being the better. It is very positive indeed in its indications, and does not involve the use of any instrument. On the other hand, the galvanometer is certainly the better instrument for laboratory work.

Mr. Peek has kindly answered for me the question raised by Mr. Slepian, as to the value of the corona voltmeter for measuring transients, and their impulse ratio. As the observations by telephone and galvanometer are based on the continuous presence of corona, the voltmeter would probably not respond to transients of short duration. Possibly the telephone might.

It is reassuring to know, as stated by Mr. Peek, that the corona due to the transient voltage may be seen. If that is the case, there is no reason why the peekhole in the top of the case of the corona voltmeter should not be used for detecting the corona due to the transient voltage. I am interested in the statement of Mr. Peek, in terms merely of the crest values. I do not recall the work which led to that conclusion, but I would be interested, if he would give me, the reference.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs W. Va., June 30, 1920.

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REACTIVE POWER AND MAGNETIC ENERGY

BY JOSEPH SLEPIAN

Research Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

The relation between reactive power and magnetic and electrostatic energies is stated and proved and its utility is illustrated by deriving the connection between reactive power and the size of machines, between the magnetic energy of an a-c. system and the field excitations of the synchronous machines therein, and by giving the physical significance of power factor under unbalanced conditions.

IT is a matter of common knowledge that where a magnetic field is maintained by alternating currents, reactive or wattless power must be supplied. The exact quantitative relation between the reactive power and the magnetic field is not so generally understood, probably because the proportionality is between reactive power and total magnetic energy, and this last quantity is not often used by electrical engineers. In this paper a quantitative statement of the relation is given, and its utility is illustrated by several examples.

The relation which is given here is subject to certain limitations which are brought out in the appendix where the proof of the relation is given. For practical purposes the relation may be said to hold for all combinations of condensers, resistors, and inductive apparatus, stationary, oscillating, rotating, single-phase or polyphase, balanced or unbalanced, excepting commutator and homopolar machines.

To state the relation, suppose we have any combination of apparatus of the kind just described, with an arbitrary number of terminals having certain periodically varying potentials, and carrying certain periodically varying currents. These periodically varying potentials and currents may be resolved in the usual way into pure sine-wave components of different frequencies. For each frequency, the reactive power at each terminal may be calculated in the usual way. If now the reactive powers of each frequency are divided by the frequency, and the whole added together, the result equals 4 π times the mean magnetic energy minus the mean electrostatic energy.

$$\sum \frac{\text{Reactive power}}{\omega} = 2\,(T_m - T_e)$$

If there is only one frequency, the relation reduces to

$$\text{Total reactive power} = 2\,\omega\,(T_m - T_e)$$

which has been derived for static apparatus before.[1] The mean is taken with respect to time. In balanced polyphase machines the total magnetic and electrostatic energies are nearly constant at all parts of a cycle, but in unbalanced polyphase and single-phase machines the electrostatic and magnetic energies have large pulsations with frequency twice that of the supply line.

Where there is a d-c. component of voltage and current, both the reactive power and the frequency are zero, so that indeterminate forms appear in the relation as given. In this case, as is shown in the appendix, these terms should be replaced by $\Sigma\, \Phi\, I \,.\, \frac{1}{10^8}$ where I is the d-c. component of current flowing in any coil or branch, and Φ is the d-c. component of flux linking that coil or branch.

REACTIVE POWER AND THE SIZE OF MACHINES

Magnetic energy is believed to reside wherever there is a magnetic flux, the energy being distributed throughout the magnetic field. The magnetic energy density is given by $\frac{1}{8\,\pi} \,.\, \frac{B^2}{\mu}\ 10^{-7}$ joules per cm.^{3}, where B is the flux density and μ is the permeability. This gives a direct relation between the reactive power taken by an induction machine without d-c. excitation, and its general dimensions. For such a machine, if currents of one frequency are supplied, we have

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[1] Inherent Limitations in Transformations Possible by Static Apparatus, J. Slepian, A. I. E. E. TRANSACTIONS, 1919.

Total reactive power = 2 ω (Mean magnetic energy)
On the other hand,
Mean magnetic energy

$$= \frac{10^{-7}}{8\pi}\left[\frac{(\text{Volume of iron}) \times (\text{Mean } B^2 \text{ in iron})}{\mu} + (\text{Volume of air gap}) \times (\text{Mean } B^2 \text{ in air gap})\right]$$

Hence,
Total reactive power

$$= \frac{10^{-7} f}{2}\left[\frac{(\text{Volume of iron}) \times (\text{Mean } B^2 \text{ in iron})}{\mu} + (\text{Volume of air gap}) \times (\text{Mean } B^2 \text{ in air gap})\right]$$

This relation was discovered independently by Mr. H. G. Jungk, from calculations on a line of railway motors.[2]

A similar relation connects the reactive power supplied by a condenser, and the volume of its dielectric. Since electrostatic energy density is given by

$$\frac{10^{-7} D^2}{8\pi\epsilon} \text{ joules/cm.}^3$$

where D is the electric flux density, and ϵ the dielectric constant, we have
Reactive power taken by condenser

$$= f/2\left(\frac{\text{Volume of dielectric} \times \text{Mean } D^2}{\epsilon}\right) . 10^{-7}$$

ENERGY OF AN A-C. SYSTEM AND THE CURRENT FLUX LINKAGES OF ITS FIELD CIRCUITS

An interesting connection between the total magnetic energy of an alternating-current system and the exciting

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[2] The relation as found by Mr. Jungk is
Total magnetising kv-a. = $B^2 S g f \times$ sat. factor $\times 10^{-11}$
where B = max. density lines /in^2
$S = \pi D L$, air gap surface in square inches.
g = effective gap, one side, in inches.
f = frequency

$$\text{sat. factor} = \frac{\text{ampere turns, Iron + gap}}{\text{ampere turns, gap}}$$

direct currents may be found by applying the principle of this paper to a whole a-c. system. In making this application, we must exclude the exciters, because they are commutator machines to which the principle does not apply. Hence the field terminals of the generators, and synchronous motors are the terminals to which the principle refers, the circuits of all other current-carrying members being closed in the system itself, thus giving no external terminals. We get then

$$\frac{1}{10^8} \Sigma \Phi I \text{ for all d-c. fields in system}$$

$$= 2 \text{ (Total Mean Magnetic Energy of System Minus Total Electrostatic Energy of System)}$$

Let us take a specific example. Suppose we have a generator whose field is controlled so as to give constant voltage. This means of course, that the main flux is nearly constant. On open circuit, the only magnetic energy is that corresponding to the main flux, and the field current is such as to make $\Sigma \Phi I$ take the appropriate value,

$$\frac{1}{10^8} \Sigma \Phi I = 2 \times \text{(Energy of main flux)}$$

Suppose now a unity power factor load is drawn from the generator. The main flux must be increased somewhat to keep the voltage at the armature terminals constant, and at the same time, additional magnetic energy appears in the form of leakage flux of the armature. The corresponding field current satisfies the relation

$$\frac{1}{10^8} \Sigma \Phi I = 2 \text{ (Energy of main flux + Energy of leakage flux)}$$

If a zero per cent power factor lagging load is substituted for the unity power factor load just considered, the exciter current must be further increased due to the magnetic energy of the load.

$$\frac{1}{10^8} \Sigma \Phi I = 2 \times \text{(Energy of main flux + Energy of leakage flux + magnetic energy of load.)}$$

If a condenser load is drawn, the exciter current must be decreased corresponding to the equation

$$\frac{1}{10^8} \Sigma \Phi I = 2 \times (\text{Energy of main flux} + \text{Energy of leakage flux} - \text{electrostatic energy of load.})$$

If the generator is self-exciting on the capacity load, I is zero, and from the last relation we get as a necessary condition for self-excitation,

Electrostatic Energy of Capacity Load
= Magnetic Energy of System.

Consider a generator A, supplying a synchronous motor B. If the voltage at the generator is kept constant, the magnetic energy in each machine is nearly constant, varying somewhat by the amount of leakage flux. If B is without excitation, we must have

$$\frac{1}{10^8} \Sigma \Phi_A I_A = 2 \text{ (Magnetic energy of } A + \text{Magnetic energy of } B)$$

When B is excited, the field current of A must receive a corresponding diminution as we must have

$$\frac{1}{10^8} \Sigma \Phi_A I_A + \frac{1}{10^8} \Sigma \Phi_B I_B = 2 \text{ (Magnetic energy of } A + \text{Magnetic energy of } B.)$$

The mutual dependence of the field currents of synchronous machines operating in parallel on a system is thus clearly shown.

POWER-FACTOR UNDER UNBALANCED CONDITIONS. VECTORIAL KV-A. SUM FORMULA

Under balanced conditions power factor is defined as the cosine of the acute angle of a right triangle having for adjacent and opposite sides respectively, the total true power and total reactive power of the circuits being measured (Fig. 1). For unbalanced circuits no definition has been generally agreed upon.

It has been proposed as one definition, to construct kv-a. triangles for each phase, using line current and voltage to neutral, then to add the hypotenuses of these triangles vectorially, and to take the cosine of the angle of this resultant as the power factor. It is clear from Fig. 2 that the triangle corresponding to this resultant has for its base the sum of the true powers supplied to the individual phases, and for its altitude the sum of the reactive powers supplied to the individual phases.

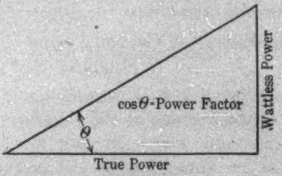

FIG. 1

This definition has the advantage that the quantities it uses have actual physical significance. The base of the triangle gives the total real power supplied to the load being measured, and the altitude gives the excess of the mean magnetic energy of the load over the mean electrostatic energy. A pure resistance load according to this definition will always have one

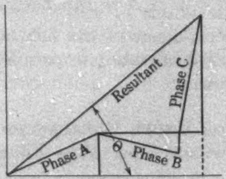

FIG. 2

hundred per cent power factor irrespective of the unbalance of the load resistances or impressed voltages.

Under this definition, however, all apparatus will not have this property of giving the same power factor irrespective of voltage unbalance. Rotating induction machines are particularly sensitive in this respect, and when measured according to this definition would show very low power factor when operated under unbalanced voltages. The conditions in a rotating induction machine can best be understood and analyzed by resolving the unbalanced quantities into polyphase or symmetrical components.

For clearness let us consider a three-phase induction motor, with low-resistance secondary, running at small slip, with ungrounded neutral, so that voltages tending to produce neutral currents may be neglected. It has been shown[3] that the voltages impressed upon the motor terminals, however unbalanced, may be regarded as two component systems superimposed or simultaneously impressed; the component systems are each individually balanced three-phase voltages, but one is of normal or positive phase sequence, while the other is of opposite, or negative, or counter-rotational phase sequence. A given unbalanced three-phase voltage can be resolved into these symmetrical components in only one way. In ordinary slightly unbalanced systems, the negative or counter-rotational component of voltage is small. It has been shown for the induction motor[4] that each symmetrical component of voltage may be considered as acting as if the other component were absent. Thus to calculate the currents in the different phases of the motor under unbalanced voltage, we may calculate the currents which each symmetrical component of voltage would produce were it acting alone, and then add these two current systems together as they appear in the different phases. The problem of operation under unbalanced voltage is thus reduced to two problems under balanced voltages. The direct-rotational or positive phase sequence voltage produces direct-rotational currents; the counter-rotational voltage produces counter-rotational currents. These are the two symmetrical components of the resultant unbalanced currents. To get the unbalanced resultant currents we add together in each phase the currents corresponding to the two symmetrical components.

As with currents, so with fluxes. Each component of voltage produces flux as if the other component of voltage were absent. The resultant flux distribution under unbalanced voltage may be obtained by combining the fluxes corresponding to each component of voltage taken separately. It may be shown[5] that the mean magnetic energy of the resultant flux is equal to the sum of the magnetic energies of the fluxes corresponding to the polyphase components taken separately.

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[3] Fortescue, A. I. E. E. TRANS. Vol. XXXVII p 1027, 1918

[4] Fortescue loc. cit.

When running at a small slip, the direct-rotational flux consists mostly of the synchronous direct-rotating main flux and a small leakage flux. Because most of the magnetic energy here resides in the main flux, a relatively high direct-rotational voltage is necessary to maintain it. The counter-rotational components, however, are all at nearly 200 per cent slip relative to the rotor, and therefore, owing to the magnetic damping of the rotor, the counter rotational flux consists mostly of leakage flux between primary and secondary, and to a small extent of counter-rotating main flux. A relatively small counter-rotational voltage therefore may produce considerable counter-rotational magnetic energy.

The conditions for reducing the magnetic energy, and thus giving good power factor, under direct-rotational voltage, *e. g.* small air gap, good magnetic coupling between primary and secondary windings, etc., are just the conditions for giving excessive magnetic energy and low power factor under counter-rotational voltage. An induction motor designed for a high power factor under normal balanced voltage conditions, would, if measured according to the vectorial kv-a. definition, show a low power factor under voltages which are only slightly unbalanced.

A case in point was brought to the attention of the author, where a synchronous converter was sold with the guarantee of giving 100 per cent power factor at rated load. The leads from the transformers to the slip rings were installed by the customer in an unsymmetrical way, causing a considerable unbalance in the voltages at the slip rings. The power factors taken from the three diametrical pairs of rings were all different, and the question arose as to what would represent 100 per cent power factor. It was finally agreed that the guarantees would be met if 100 per cent were maintained on one diameter, with equal power factors on the other two, leading on one, and lagging on the other. Since the kv-a. in all the leads were nearly equal, this was evidently equivalent to adopting the vectorial kv-a. definition of power factor.

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[5] Slepian, A. I. E. E. TRANS. Vol. XXXVII, 1918 Discussion, p. 661.

It is clear from the preceding discussion that this was unfair to the manufacturer. Under the test conditions agreed upon the field excitation was raised high enough to have given leading power factor if the voltages had been balanced. With the voltages unbalanced the rotary was taking a large counter-rotational magnetic energy, and therefore correspondingly large counter-rotational reactive current. It was then called upon to give an equal amount of direct-rotational reactive current, but leading, which meant leading direct-rotational power factor and higher field excitation than the manufacturer had anticipated. Stated in another way, at unity power factor under this definition we have if i is the field current

$$\frac{1}{10^8}\Sigma\,\phi\,i = 2\ (\text{Total magnetic energy})$$

If the voltages are balanced, all this energy is direct-rotational,

$$\frac{1}{10^8}\Sigma\,\phi\,i = 2\ (\text{Direct-rotational magnetic energy})$$

A slight voltage unbalance will give considerable counter-rotational magnetic energy, and this calls for a considerable increase in field current since

$$\frac{1}{10^8}\Sigma\,\phi\,i = 2\ (\text{Direct-rotational magnetic energy} + \text{counter-rotational magnetic energy})$$

POWER FACTOR UNDER UNBALANCED CONDITIONS. DIRECT-ROTATIONAL COMPONENT DEFINITION

If rotating machines are to be given a power factor descriptive of their operation under normal conditions, it is evident that some definition other than that of the preceding section must be used when measurements are made under unbalanced conditions. In the light of the preceding discussion, the following would appear to be a satisfactory definition of power factor, under unbalanced voltage conditions, of a balanced rotating machine. Power factor is the cosine of the angle of lag between the direct-rotational component of voltage and the direct-rotational component of current. Methods for determining from the observed unbalanced voltages and currents, the direct-rotational components and the angle between them are beyond the scope of this paper. With this definition, a balanced rotating machine will show the same power factor irrespective of the voltage unbalance.

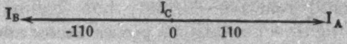

FIG. 3

What is the state of affairs, however, when this definition is applied to apparatus which is inherently unbalanced? This is best brought out by a simple example. A three-phase generator, $\frac{110}{\sqrt{3}}$ volts to neutral, is supplying a pure resistance load of one ohm, across one phase. What are the line voltages, currents, and the power factor under the last definition in this case? The power factor under the first definition, of course must be 100 per cent. The current through the resistance will be 110 amperes. The currents in the three lines will be as shown in Fig. 3, line A, $I_A = 110$ amperes; line B, $I_B = -110$ amperes; line C, $I_C = 0$ amperes. Resolving these currents into symmetrical components, we get for the direct-rotational component (heavy lines, Fig. 4) line A,

$I_{A1} = 55 + 55/\sqrt{3}\,j$ amperes; line B, $I_{B1} = -55 + 55/\sqrt{3}\,j$ amperes; line C, $I_{C1} = -110/\sqrt{3}\,j$ amperes. For the counter-rotational component we have (dotted lines, Fig. 3) line A, $I_{A2} = 55 - 55/\sqrt{3}\,j$ amperes; line B, $I_{B2} = -55 - 55/\sqrt{3}\,j$ amperes; line C, $I_{C2} = +110/\sqrt{3}\,j$ amperes.

The counter-rotational currents, (I_{A2}, I_{B2}, I_{C2}) when drawn from the generator, produce counter-rotational voltages at the terminals (E_{A2}, E_{B2}, E_{C2}), which lag nearly 90 deg. behind the currents, see Fig. 5. The magnitude of the counter-rotational voltage will depend upon the closeness of coupling between the armature of the generator and the damper winding on its field. The magnitude of the direct-rotational voltage depends upon the field excitation. Suppose this is adjusted to give 110 volts total between lines *A* and *B*. From this condition, since further this voltage must be in phase with the current through the resistance, the direct-rotational voltage (E_{A1}, E_{B1}, E_{C1}) is readily determined (Fig. 6.). In Fig. 6, we see that the direct-rotational current lags behind the direct-rotational voltage so that the pure resistance load actually draws direct-rotational reactive power. Thus its power factor according to the direct-rotational component definition would be less than 100 per cent. From Fig. 5 we see that all the counter-rotational power is leading reactive power. This might have been foreseen, for since the total reactive power taken by the resistance load is zero, whatever is taken in direct-rotational form must be returned to the line in counter-rotational form.

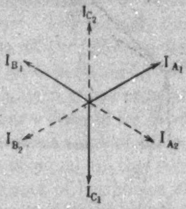

FIG. 4

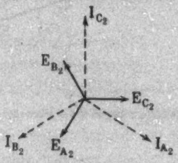

FIG. 5

The field excitation will be that corresponding to the direct-rotational voltage, current, and power factor. It is thus larger than that which would be required for an equal load which is balanced. This is because with the unbalanced load there is actually more magnetic energy in the system. The unbalanced resistance load, while containing no magnetic energy itself, causes counter-rotational magnetic energy to exist in the system. This affects the field excitation according to the equation,

$$\Sigma \phi i = 2 \ (\text{Direct-rotational magnetic energy} + \text{counter-rotational magnetic energy.})$$

where $\Sigma \phi i$ represents the total flux-current linkages of the field of the generator. By the direct-rotational component definition, in this case, the power factor of the unbalanced resistance load is dependent upon the constants of the generator.

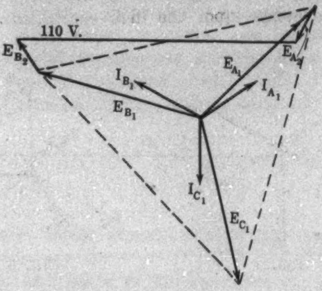

FIG. 6

It is not intended that this paper shall advocate any particular definition of power factor for unbalanced conditions, but merely to point out the meaning of two definitions which have been proposed. However, there seems little doubt that for rotating balanced machines, the direct-rotational component definition is the best. For unbalanced stationary apparatus, for example polyphase furnaces, it seems likely that the vectorial kv-a. formula is preferable. For this class of apparatus, a further term, an unbalance factor, would appear desirable for giving adequate description.

Power factor has been used in this paper as characterizing a piece of apparatus. It is also widely used to describe a load from the point of view of the cost of its generation and transmission. Here the considerations are somewhat different, but here also the principle connecting wattless power with magnetic energy will prove useful in discussions.

APPENDIX

PROOF OF RELATION CONNECTING REACTIVE POWER AND MAGNETIC AND ELECTROSTATIC ENERGIES

A few well-known propositions concerning the mean value of products of periodic quantities, and some expressions for magnetic and electrostatic energy which are used in the proof will first be given.

Expansion of a Periodic Quantity into a Fourier Series. Any periodic quantity can be expanded into a series of the following form,

$$y = a_0 + a_1 \sin(\omega t + \alpha_1) + a_2 \sin(2\,\omega t + \alpha_2) + \ldots .$$

Each term of the expansion is called a harmonic.

Mean Value of the Product of Two Periodic Quantities. The mean value of the product of two periodic quantities is equal to the sum of the mean values of the products of harmonics of the same frequencies in the Fourier expansions of the two periodic quantities. Thus if

$$\begin{aligned} y &= a_0 + a_1 \sin(\omega t + \alpha_1) + a_2 \sin(2\,\omega t + \alpha_2) \\ &\qquad + a_3 \sin(3\,\omega t + \alpha_3) + \ldots \\ z &= b_0 + b_1 \sin(\omega t + \beta_1) + b_2 \sin(2\,\omega t + \beta_2) \\ &\qquad + b_3 \sin(3\,\omega t + \beta_3) + \ldots \end{aligned}$$

then

$$\begin{aligned} \text{Mean of } y\,z &= a_0 b_0 + \text{mean of } a_1 b_1 \sin(\omega t + \alpha_1) \\ &\qquad\qquad \sin(\omega t + \beta_1) \\ &\quad + \text{mean of } a_2 b_2 \sin(2\,\omega t + \alpha_2) \\ &\qquad\qquad \sin(2\,\omega t + \beta_2) \\ &\quad + \text{mean of } a_3 b_3 \sin(3\,\omega t + \alpha_3) \\ &\qquad\qquad \sin(3\,\omega t + \beta_3) \\ &\quad \text{etc.} \\ &= a_0 b_0 + \tfrac{1}{2} a_1 b_1 \cos(\alpha_1 - \beta_1) \\ &\quad + \tfrac{1}{2} a_2 b_2 \cos(\alpha_2 - \beta_2) \\ &\quad + \tfrac{1}{2} a_3 b_3 \cos(\alpha_3 - \beta_3) \\ &\quad \text{etc.} \end{aligned}$$

Magnetic Energy. The magnetic energy of the field produced by a system of currents is given by

$$t_m = \frac{1}{2.10^8} \Sigma \phi i$$

$$= \frac{1}{2.10^8} (\phi_1 i_1 + \phi_2 i_2 + \phi_3 i_3 + \ldots) \text{ joules}$$

where each term is the product of i_K the current in a branch or circuit of the system, and ϕ_K the number of flux linkages of that branch or circuit; the summation is taken for all the branches and circuits of the system.

Electrostatic Energy. The electrostatic energy of a condenser is given by $t_e = \frac{1}{2} \frac{q^2}{C}$ joules where q is the charge in coulombs and C the capacity of the condenser in farads.

Proof of the Relation. For any branch or circuit of the system we have

$$e = r\,i + \frac{1}{10^8} \frac{d\phi}{dt} + \frac{q}{C} \qquad (\mathbf{1})$$

where i is the current in the branch or circuit, ϕ the flux linking the branch or circuit, q the charge on the condenser in that branch or circuit, and e the difference of potential between the ends of the branch or circuit. If the circuit is a closed loop, $e = 0$. The quantities in equation (**1**) are periodic, and their Fourier expansions in general will contain constant terms. Suppose these constant terms subtracted out from equation (**1**) leaving

$$e_A = r\,i_A + \frac{1}{10^8} \frac{d\phi_A}{dt} + \frac{q_A}{C} \qquad (\mathbf{2})$$

where e_A, i_A, ϕ_A, and q_A are the purely alternating parts of e, i, ϕ, and q respectively remaining after the constant terms of the Fourier expansions of these quantities have been subtracted.

Now integrate equation (**2**) taking that integral which is purely alternating. (This merely fixes the constant of integration.)

$$\int e_A\,dt = r\int i_A\,d\,t_A + \frac{1}{10^8}\phi_A + \frac{1}{C}\int q_A\,dt \qquad (\mathbf{3})$$

Multiplying through by i_A and remembering that

$$i_A = \frac{d\,q_A}{d\,t}$$

$$i_A \int e_A\,d\,t$$

$$= r\,i_A \int i_A\,d\,t + \frac{1}{10^8}\,\phi_A\,i_A + \frac{1}{C}\,\frac{d\,q_A}{d\,t}\int q_A\,d\,t \quad (4)$$

Suppose equations similar to (**4**) found for all the branches or circuits of the system, and that they are all added together, giving

$$\Sigma\,i_A \int e_A\,dt = \Sigma\,r\,i_A \int i_A\,dt + \frac{1}{10^8}\,\Sigma\,i_A\,\phi_A$$

$$+ \Sigma\,\frac{1}{C}\,\frac{d\,q_A}{dt}\int q_A\,dt \qquad (5)$$

We may now take the mean value of each term of (**5**) getting

Mean value of $\Sigma\,i_A \int e_A\,dt$

$$= \text{mean value of } \Sigma\,r\,i_A \int i_A\,dt$$

$$+ \text{ " " " } \frac{1}{10^8}\,\Sigma\,\phi_A\,i_A \qquad (6)$$

$$+ \text{ " " " } \Sigma\,\frac{1}{C}\,\frac{d\,q_A}{dt} \cdot \int q^A\,dt.$$

We now proceed to evaluate each term of (**6**).

The first term

$$\Sigma i_A \int e_A\,dt.$$

Let

$$i_A = i_1 \sin(\omega\,t + \alpha_1) + i_2 \sin(2\,\omega_2\,t + \alpha_2) + \ldots;$$

let

$$e_A = e_1 \sin(\omega\,t + \beta_1) + e_2 \sin(2\,\omega\,t + \beta_2) + \ldots$$

Then

$$\int e_A\,dt = -\,\frac{e_1}{\omega}\,\cos(\omega\,t + \beta_1)$$

$$-\,\frac{e_2}{2\,\omega}\,\cos(2\,\omega\,t + \beta_2)\ldots$$

Hence it follows that the mean of

$$i_A \int e_A\,dt = \Sigma\,\frac{\text{Reactive Power}}{\omega}$$

supplied to the branch under consideration. It is readily seen that when we sum up for all the branches we get $\Sigma \dfrac{\text{Reactive Power}}{\omega}$ for the terminals of the system. Hence

$$\text{Mean value of } \Sigma\, i_A \int e_A\, dt = \Sigma\, \frac{\text{Reactive Power}}{\omega}.$$

The next term

$$\Sigma\, r\, i_A \int i_A\, dt.$$

Let

$$i_A = i_1 \sin(\omega t + \alpha_1) + i_2 \sin(2\,\omega t + \alpha_2) + \ldots$$

Then

$$\int i_A\, dt = -\frac{i_1}{\omega} \cos(\omega t + \alpha_1)$$

$$- \frac{i_2}{2\,\omega} \cos(2\,\omega t + \alpha_2) \ldots$$

Evidently then the mean value of $i_A \int i_A\, dt$ is zero. Hence

$$\text{Mean value of } \Sigma\, r\, i_A \int i_A\, dt = 0.$$

The next term

$$\frac{1}{10^8}\, \Sigma\, i_A\, \phi_A.$$

The instantaneous magnetic energy t_m is given by

$$t_m = \frac{1}{2.10^8}\, \Sigma\, \phi\, i = \frac{1}{2.10^8}\, \Sigma\, (\phi_D + \phi_A)\,(i_D + i_A)$$

$$= \frac{1}{2.10^8} \left[\Sigma\, i_D\, \phi_D + \Sigma\, \phi_A\, i_A + \Sigma\, \phi_A\, i_D + \Sigma\, \phi_D\, i_A\right]$$

where ϕ_D and i_D are the constant terms of the Fourier expansions of ϕ and i. Clearly the mean values of $\Sigma\, \phi_A\, i_D$ and $\Sigma\, \phi_D\, i_A$ are zero, and hence the mean magnetic energy

$$T_m = \text{mean value of } t_m$$

$$= \frac{1}{2.10^8} \left[\Sigma\, \phi_D\, i_D + \text{mean value of } \Sigma\, \phi_A\, i_A\right]$$

Hence

$$\text{Mean value of } \frac{1}{10^8}\, \Sigma\, \phi_A\, i_A = 2\, T_m - \frac{1}{10^8}\, \Sigma\, i_D\, \phi_D$$

The next term

$$\Sigma\, \frac{1}{C}\, \frac{d\, q_A}{dt} \int q_A\, dt.$$

Let

$$q = q_1 \sin\,(\omega\, t + \gamma_1) + q_2 \sin\,(2\, \omega\, t + \gamma_2) + \ldots$$

Then

$$\frac{d\, q_A}{dt} = q_1\, \omega \cos\,(\omega\, t + \gamma_1) + q_2\, 2\, \omega \cos\,(2\, \omega t + \gamma_2) + \ldots;$$

$$\int q_A\, dt = -\, q_1/\omega \cos\,(\omega\, t + \gamma_1) - \frac{q_2}{2\, \omega} \cos\,(2\, \omega\, t + \gamma_2)$$

Hence mean value of

$$\frac{d\, q_A}{dt} \int q_A\, d\,t = -\frac{q_1^2}{2} - \frac{q_2^2}{2} - \frac{q_3^2}{2} \;\ldots.$$

On the other hand mean value of

$$q_A^2 = + \frac{q_1^2}{2} + \frac{q_2^2}{2} + \frac{q_3^2}{2} + \ldots.$$

Hence mean value of

$$q_A^2 = -\text{ mean value of } \frac{d\, q_A}{dt} \int q_A\, dt.$$

The instantaneous electrostatic energy t_e is given by

$$t_e = \frac{1}{2}\, \Sigma\, \frac{q^2}{C} = \frac{1}{2}\, \Sigma\, \frac{(q_D + q_A)^2}{C}$$

$$= \frac{1}{2} \left[\Sigma\, \frac{q_D^2}{C} + \Sigma\, \frac{q_A^2}{C} + 2\, \Sigma\, \frac{q_D\, q_A}{C} \right]$$

where q_D is the constant term in the Fourier expansion of q. Since mean value of

$$\Sigma\, \frac{q_D\, q_A}{C} = 0,$$

mean value of $t_e = T_e$ = mean value of

$$\frac{1}{2}\left[\Sigma\frac{q_D^2}{C} + \Sigma\frac{q_A^2}{C}\right]$$

Hence

Mean value of $\Sigma \frac{1}{C}\frac{d\,q_A}{dt}\int q_A\,dt = 2\,T_e - \Sigma\frac{q_D^2}{C}$

Substituting into equation (**6**) we have finally

$$\Sigma\frac{\text{Reactive Power}}{\omega} + \frac{1}{10^8}\,\Sigma\,\phi_D i_D - \Sigma\,\frac{q_D^2}{C}$$
$$= 2\,(T_m - T_e)$$

The detailed steps in passing from $\Sigma\,\frac{\text{Reactive Power}}{\omega}$ taken for each branch to $\Sigma\,\frac{\text{Reactive Power}}{\omega}$ taken only for the terminals are fairly obvious and so were omitted in the above proof. There is one point however, which should be brought out in this connection, as it shows a limitation ruling out the homopolar machine from the relation. It will be readily seen that where slip rings are involved, all parts must be at the same potential, and the flux linkages between any point fixed on a ring and one of the brushes must be negligible. This last rules out the homopolar machine.

DISCUSSION ON "REACTIVE POWER AND MAGNETIC ENERGY" (SLEPIAN), WHITE SULPHUR SPRINGS, W. VA., JUNE 30, 1920.

C. L. Fortescue: I want to bring out one point which I do not think is brought out in the paper by Dr. Slepian, that is, a mathematical discussion of the relation between magnetic energy and reactive power. I want to point out that there are practical uses for this method of obtaining reactive power from the magnetic energy. It offers a very simple way of calculating the reactive power of induction motors and rotating machines, and it points out quite clearly the relation between reactive power delivered by a machine and the amount of excitation required, and I think it is a step in the right direction to recognize these relations.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 30, 1920

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THEORY OF SPEED AND POWER FACTOR CONTROL OF LARGE INDUCTION MOTORS BY NEUTRALIZED POLYPHASE ALTERNATING-CURRENT COMMUTATOR MACHINES

BY JOHN I. HULL
Designing Engineer, General Electric Co., Schenectady, N. Y.

Theory of induction motor control, discussing single-range (below synchronism only) speed and power factor control by means of a constant-speed series commutator motor, by means of a constant-speed shunt commutator motor, by means of a constant-speed compound excited commutator motor; double range (all speeds above or below synchronism) speed and power factor control by means of a constant-speed shunt commutator motor; and double-range (either above or below synchronism) operation remote from synchronism. The discussion is illustrated in detail, special attention being paid to the circle diagram.

A STATIONARY polyphase wound-rotor induction motor is merely a static transformer arranged so that the primary coils are all on one part of the magnetic circuit, and the secondary coils on another part of the magnetic circuit, the two parts thus being arranged so as to permit relative motion. The reluctance of the magnetic circuit is kept as low as possible by imbedding both primary and secondary winding in slots, thereby permitting the "teeth" between the slots of the primary iron to come as close to the "teeth" of the secondary iron as safe mechanical clearance permits. The necessity of some clearance or "air gap" makes the reluctance, hence, the magnetizing current and kv-a. larger than for the static transformer of similar capacity, voltage and frequency, while the separation of the primary winding from the secondary winding and the imbedding of both in slots make the leakage reactances larger than for the corresponding static transformer. It is thus evident that the induction motor may logically be considered from the point of view of a transformer so arranged as to permit the forces set up in the secondary conductors to cause rotation, at the cost of an increase in the magnetizing current and the leakage reactance.

The flux common to both and set up by the resultant of the primary and secondary magnetomotive forces is the link of mutual influence between the primary and secondary. This influence manifests itself as electromotive forces set up in proportion to the effective number of turns, the flux, and the frequency in the circuit in question. (Of course, this is rigorously true only with the usual assumptions of sine wave distribution, etc.)

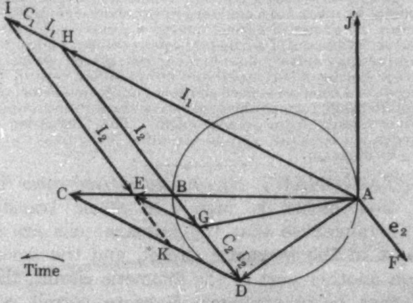

Fig. 1—The Circle Diagram of an Induction Motor

The torque is proportional to the summation of the products of the secondary turns, and the components of current in quadrature with the common flux of each, and to the common flux.

The usual theory of the induction motor takes into account only the phenomena within the motor itself, performance with adjustable external secondary resistance being analyzed by considering it to be merely an addition to the normal resistance of the secondary.

In Fig. 1, we reproduce a circle diagram, a vector diagram for analyzing induction motor performance, which lends itself to the introduction of electromotive forces etc., of concatenated machines.

$A\,H = I_1$ (proportional to primary current) represents the flux linking primary and secondary which would be produced by the primary current alone. (Saturation neglected)

$H\,I = C_1\,I_1$ represents primary leakage flux.

$H\,G = I\,E = I_2$ (proportional to the secondary current) represents the flux linking secondary and primary which would be produced by the secondary current alone. (Saturation neglected)

$G\,D = C_2\,I_2$ represents secondary leakage flux.

$A\,E$ is thus the resultant of all the primary flux and that secondary flux, linking the primary, so that neglecting primary resistance drop, E is a fixed point for constant line voltage and frequency, as $A\,E$ is the flux which generates the counter e. m. f. to balance the applied voltage e_1.

$H\,G$ intersects $A\,E$ at B, and as

$$A\,B/A\,E = \frac{I_1}{I_1\,(1 + c_1)},$$

we see that $A\,B$ is constant, making B also a fixed point. $A\,D$ is resultant of I_1 and $I_2 + C_2\,I_2$ and therefore generates all secondary electromotive forces except resistance drop, which is, therefore in phase opposition to the voltage e_2, set up in the secondary in quadrature to flux $A\,D$. This makes $A\,F$ parallel to $H\,D$ and to $I\,E$ and further makes $A\,D\,H$ a right angle, which taken in connection with the fact that, as shown above, A and B are fixed points, demonstrates that the curve traced by point D is the arc of a circle.

A line parallel to $A\,I$ from point D intersects prolongation of $A\,E$ at C

$$B\,D = H\,D - H\,B = I_2\left((1 + C_2) - \frac{1}{1 + C_1} \right)$$

$$= \frac{I_2}{1 + C_1}\left(C_1 + C_2\,(1 + C_1) \right)$$

Since

$$\frac{H\,B}{I_2} = \frac{I_1}{I_1\,(1 + C_1)}$$

$$\frac{C\,D}{A\,I} = \frac{B\,D}{I\,E}$$

$$C\ D = I_1\ (1 + C_1) \times \frac{I_2\,[C_1 + C_2\,(1 + C_1)\,]}{I_2\,(1 + C_1)}$$

$$= I_1\,[C_1 + C_2\,(1 + C_1)\,]$$

$E\,A$ is the flux whose counter e. m. f. balances all the applied line voltage as noted above. Let it be designated I_m. $B\,A$ is the *mutual flux* at running light and, if denoted by I_0, we have $I_0 = \dfrac{I_m}{1 + C_1}$.

$$C\,B = C\,D\ \frac{I_m}{I_1\,(1 + C_1)}$$

$$= \frac{I_m}{1 + C_1}\ [C_1 + C_2\,(1 + C_1)\,]$$

$$= I_0\,[C_1 + C_2\,(1 + C_1)\,]$$

$$C\,A = E\,A + C\,E = I_m\,(1 + C_2)$$

$$= \frac{I_m\,(1 + C_2)}{C_1 + C_2\,(1 + C_1)} \times [C_1 + C_2\,(1 + C_1)\,]$$

Since

$$\frac{C\,E}{E\,K} = \frac{I_m}{I_2}$$

$$C\ E\ = I_m\ \frac{I_2\,C_2}{I_2}$$

C is thus a fixed point,

As $C\,A$ = constant for I_m constant.

With proper scale, I_m could be made to represent the magnetizing current for the whole of flux I_m, (which is the quantity commonly calculated, as the primary reactance and resistance drop are usually omitted) I_0 could be made to represent the true running light current, primary reactance drop considered, I_1 the primary current and I_2 the secondary current. If we now change the scale of the diagram by the factor $C_1 + C_2\,(1 + C_1)$, we may say that magnetizing i_m divided by $1 + C_1$ equals $C\,B$, equals true running light current i_0; primary current i_1 equals $C\,D$; secondary current divided by $1 + C_1$ equals $D\,B$.

At standstill, with zero secondary resistance $A\,D$, the resultant secondary flux must, of course, be zero, as its generated voltage is zero, which means that D coincides with A and $C D = C A$, so that we have the ideal short-circuit or standstill current with zero secondary resistance equal to $C A$.

As $C_1 I_1$ is defined as primary leakage flux, the primary reactance drop with current i_m is $C_1 e_1$, since i_m produces the whole of flux whose e. m. f. is equal to e_1. The primary reactance drop is further equal to $i_m X_1$ if X_1 be the primary reactance, thus:

$$C_1 e_1 = i_m X_1$$

$$C_1 = \frac{i_m X_1}{e_1}, \text{ and similarly,}$$

$$C_2 = \frac{i_m X_2}{e_1}$$

Thus, to draw the diagram of the motor, we need to know the primary and secondary reactances X_1 and X_2 and the nominal magnetizing current i_m. We need then only so much of Fig. 1 as is shown in Fig. 2.

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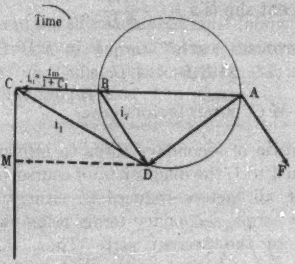

Fig. 2—Simplified Diagram

Having chosen a scale, lay off:

$C B$ equal to i_o

$$C A \text{ equal to } \frac{i_m (1 + C_2)}{C_1 + C_2 (1 + C_1)} \text{ and}$$

draw a circle with $B A$ as diameter. $C M$ is the in phase or watt component of input current for any considered load. $M D$ parallel to $C A$ then locates D and the remainder of the diagram. The primary current is then $i_1 = C D$ and the secondary current

i_2 is then $(1 + C_1)\,D\,B$, or if we use as the unit for i_2 the unit denoting the other currents divided by $1 + C_1$, we can let $D\,B = i_2$.

To find the secondary voltage $e_2 = A\,F$, we can first determine its value reduced to full frequency. The voltage generated by flux $A\,E$ of Fig. 1 is e_1, and that generated by $A\,B$ is, therefore, $\frac{e_1}{1 + C_1}$. So in Fig. 2, knowing e_1, we can say that $A\,B$ units of length correspond to $\frac{e_1}{1 + C_1}$ volts, and can regard $A\,B$ etc. as measures of voltage. So $A\,D$ is $\frac{A\,D}{A\,B}$ times $\frac{e_1}{1 + C_1}$ volts at standstill frequency. If the secondary resistance is known, the actual value of secondary induced voltage e_2 is, of course, $i_2\,r_2$, so that per cent slip is $s = \frac{i_2\,r_2}{A\,D}$.

"Synchronous watts" torque is $B\,D$ times $A\,D$, output is $(1 - S)\,B\,D \times A\,D$, efficiency $(1 - S)\,B\,D \times \frac{A\,D}{e_1}\,M\,C$, power factor $\frac{M\,C}{C\,D}$

If the ratio of secondary turns to primary turns is other than 1 to 1, the diagram is, of course, of necessity drawn for all factors reduced to either primary or secondary terms, secondary terms being usually used for work of the present sort. Thus, the primary voltage to be expressed in terms of secondary must, of course, be multiplied by ratio of secondary to primary effective terms, primary reactance by the square of this ratio, etc.

In the demonstration of Figs. 1 and 2, it was pointed out that point D traces the arc of a circle whose diameter is $B\,A$, because $A\,D\,B$ is a right angle, due to the phase opposition of e_2 and $i_2\,r_2$ when the only e. m. f. in the secondary circuit, other than that induced by the total secondary flux, is resistance drop. If, as in Fig. 3, another e. m. f. than the resistance drop as e_r be introduced, then D will still trace the circle with the diameter $A\,B$ when and only when the introduced e. m. f. is in phase with or in phase opposition to e_2. In this case, for given values of i_1, i_2 etc., e_2 must be equal and opposite to the algebraic sum of $i_2\,r_2$ and the introduced voltage; hence, since the inducing flux of e_2 is determined by the currents,—its inducing frequency and the slip and speed must follow variations in the algebraic sum of $i_2\,r_2$ and the introduced voltage. It is, therefore, evident that varying the introduced voltage, while maintaining it in phase with e_2 gives a means of varying the speed of the motor without effecting its power factor torque etc.

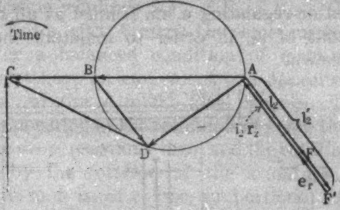

FIG. 3—INDUCTION MOTOR DIAGRAM
With regulating voltage e_r introduced into secondary in phase opposition with total induced e. m. f.

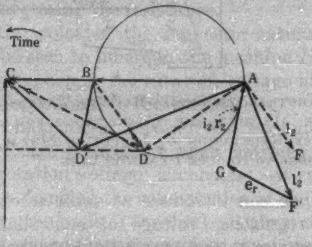

FIG. 4—INDUCTION MOTOR DIAGRAM
With regulating voltage e_r introduced into secondary out of phase with total induced e. m. f.

If now, as in Fig. 4, the introduced e. m. f., e_r be of different phase from that of e_2, point D departs from the circumference of the circle whose diameter is

B A, as shown at D' because i_2 is no longer in phase opposition to e_2, hence, $A\,D'\,B$ is no longer a right angle. It is seen that in addition to regulating the speed, the power factor of the motor may also be regulated by proper selection of phase as well as magnitude of the introduced e. m. f.

It is clear, of course, that the frequency of the introduced or regulating e. m. f. must at all times be exactly that of e_2, in order to maintain the phase relation shown.

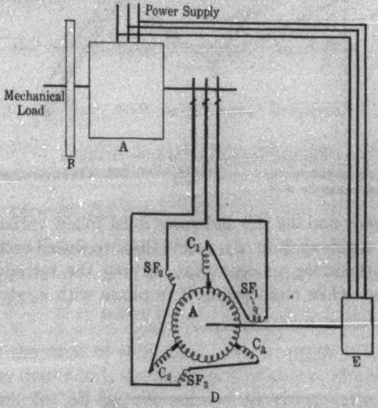

Fig. 5—Neutralized Series-Excited Three-Phase A-C. Commutator Machine and Connections for Automatic Single-Range Regulation of Induction Motor Equipped with Flywheel to Reduce Peaks on Line

Thus, if we can introduce at exact secondary frequency a regulating voltage of controllable phase with respect to e_2 and controllable magnitude, we shall be able to regulate either speed, power factor or both.

SINGLE-RANGE, (BELOW SYNCHRONISM ONLY) SPEED AND POWER FACTOR CONTROL BY MEANS OF A CONSTANT-SPEED, SERIES COMMUTATOR MOTOR

In Fig. 5, we show schematically at *D*, a three-phase series neutralized commutator machine whose terminals are connected to the secondary slip rings of main motor *A*. The speed of *D* is held practically constant by generator *E*.

Neutralizing winding C_1, C_2, C_3 balances the armature reaction (magnetomotive forces) of armature *A*, and so of necessity neutralizes the e. m. fs. set up in *A* by the transformer action of the fluxes induced by series exciting windings $S\,F_1$, $S\,F_2$, $S\,F_3$. (C_1, C_2, C_3 being in series with *A* carry the same currents as *A*, hence, for a balanced condition of magnetomotive forces must have an equivalent and opposite number of turns, so the e. m. fs. also cancel). Thus the e. m. fs. appearing at the terminals of *D* are the leakage reactance drop, resistance drop and the rotation e. m. f. induced by the rotation of the armature *A*. The rotation e. m. f. is, of course, proportional to the flux and the speed of rotation, the flux, neglecting saturation being proportional to the main currents which flow through series exciting windings $S\,F_1$, $S\,F_2$, $S\,F_3$. This arrangement can then be seen to be such that the speed of *A* will be reduced with the increase of load, provided the rotation voltage, as e_r in Figs. 3 and 4, be given a suitable component in phase with the resistance drop, thereby having the same effect on the main motor speed, as increasing the resistance.

Up to the point of the magnetic saturation, two laws may be seen to inhere in the machine *D*.

1. The flux, hence, the rotation voltage at constant speed, is proportional to the current.

2. The phase angle between the current and the rotation voltage (hence, the angle between resistance drop and rotation voltage) is constant, (it can only be changed by changing the construction of the machine).

These are the basis of the circle diagram of Fig. 6:

Points *A*, *B* and *C* are determined exactly as in Fig. 1, except that for X_2 we now substitute X_{2+c+c_s} where $X_{2+c+c_s} = X_2 + X_c + X_{c_s}$ and X_c = leakage reactance of regulating motor at primary frequency

$$X_{c_s} = \frac{\text{kv-a. required to excite regulating motor}}{i_2^2 \times \sqrt{3}}$$

The kv-a. is at primary frequency and unsaturated iron of regulating motor is assumed. Obviously the performance of an induction motor is not changed for our purposes, having a part or all of the rotor leakage reactance external to the machine. Angle $\alpha = F\,G\,A$, between resistance drop $F\,G$ and rotation e. m. f. $G\,A$ is constant by law No. 2, and $G\,A$ is by law No. 1 proportional to $B\,D$ and hence to $F\,G$. For these reasons angle $G\,F\,A$ is constant and since angle $B\,D\,A = 90$ deg. − angle $G\,F\,A$, we see that β = angle $B\,D\,A$ is also constant.

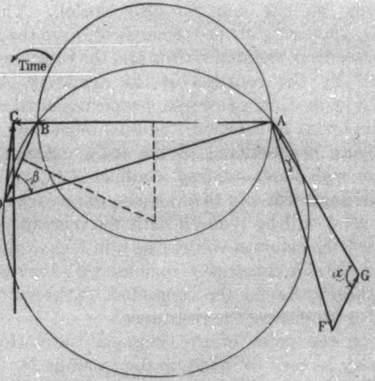

Fig. 6—Circle Diagram of Induction Motor with Constant-Speed Series Commutator Regulating Machine

Thus, as A, B, and C are fixed points, point D traces a circle whose center must be at the intersection of the perpendicular bisectors of $A\,B$ and $B\,D$.

We have remarked that Figs. 1, 2, 3, 4 and 6 are rigorous when and only when the iron of the machine is unsaturated, that is, when the flux may be regarded as proportional to the ampere turns. This condition is closely enough approximated in the main induction motor so that saturation may be neglected without much loss of accuracy. For the series machine of Fig. 6, however, to be of economical proportions, considerable saturation will be attained within the working range; hence, it becomes desirable to investigate its effects. In Fig. 7, A, B, C has been determined as was done for Fig. 6, and $B\,D\,A$ is the corresponding circle determined by angle $B\,D\,A$, which is determined by ratio $\frac{A\,G}{F\,G}$ and design angle α, saturation neglected.

Now with current $B\,D$, we can calculate a new value

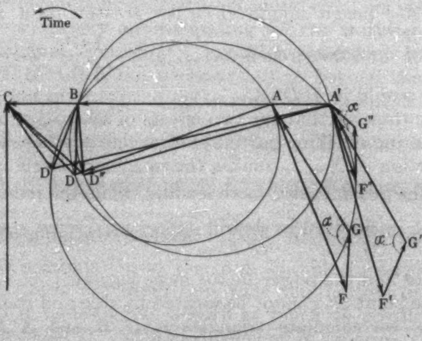

FIG. 7—DIAGRAM OF INDUCTION MOTOR AND CONSTANT-SPEED SERIES COMMUTATOR REGULATING MACHINE
Taking account of saturation of iron of regulating machine.

for X_{2+c+c_s} which will hold only for this one current, since in expression

$$X_{c_s} = \frac{\text{kv-a. to excite regulating motor}}{i_2^2 \times \sqrt{3}}$$

we can determine kv-a. (at full frequency, of course) from the known or assumed magnetizing curve of the regulating machine. With this new (and decreased) value of X_{2+c+c_s} we calculate $A'\,C$ instead of $A\,C$. If the new value of X_{2+c+c_s} were constant, our new circle would be $B\,D'\,A'$, but as it varies, D' may be the only load point upon it. Triangle A', F', G^1 is, of course, equal to triangle $A\,F\,G$, and angle $B\,D'\,A'$ is equal to angle $B\,D\,A$.

A second effect of the saturation is that the ratio of rotation e. m. f. to field current (field current being the same as the main current for a series machine) is reduced, so considering this, $A'\,G''\,F''$ is the e. m. f. triangle with angle $A'\,G''\,F''$ still equal to angle $A\,G'\,F'$ and $A\,G\,F$.

Since $A'\,G''$ is less than $A'\,G'$ and $G''\,F'' = G'\,F'$ with constant α we see that angle $G''\,F''\,A^1$ is less than angle $G'F'A'$, hence, angle $B\,D''\,A'$ greater than angle $B\,D'\,A'$ and the circle if X_{2+c+c_s} and $\frac{A\,G''}{G''\,F''}$ were constant would be $B\,D''\,A'$.

We thus see that the two effects of saturation of the regulating machine partially offset one another, as the reduction of X_{2+c+c_s} makes the imaginary circle larger and the power factor more leading, while the reduction of ratio $\frac{A'\,G'}{G'\,F'}$ to $\frac{A'\,G''}{G''\,F''}$ makes the imaginary circle smaller and the power factor more lagging.

The point D'' cannot be located by rule and compass unless we calculate triangles $A'\,D''\,B$ and $A'\,F''\,G''$ which can be done as follows:

$A'\,G''$, $G''\,F''$ and angle $A'\,G''\,F''$ are known.

$$A'\,F'' = \sqrt{\overline{A'\,G''}^2 + \overline{G''\,F''}^2 - 2 \times A'\,G'' \times G''\,F'' \times \cos A'\,G''\,F''}$$

$$\operatorname{Sin} A'\,F''\,G'' = \frac{A'G'' \times \sin A'\,G''\,F''}{A^1\,D''}$$

Angle $F''\,A'\,G''$

$$= 180 \text{ deg.} - (\text{angle } A'\,F''\,G'' + A'\,G''\,F'')$$

determining triangle $A'\,F''\,G''$.

In triangle $B\,D''\,A'$, $B\,A'$ and $B\,D''$ are known and angle $B\,D''\,A^1 = 90$ deg. $-$ angle $A'\,F''\,G''$.

$$\operatorname{Sin} B\,A'\,D'' = \frac{B\,D'' \times \sin B\,D''\,A'}{B\,A'}$$

Angle $D'' B A'$

$$= 180 \text{ deg.} - (\text{angle } B A' D'' + \text{angle } B D'' A')$$

$$A' D'' = \frac{B D'' \times \sin D'' B A'}{\sin B A' D''} \quad \text{or} \quad \frac{A' B \sin D'' B A'}{\sin B D'' A'}$$

Knowing, thus, $A D''$ and $B D''$, we can find point D'' with compass.

We can now construct the curve traced by D'' by assuming values of current $B D''$, calculating for each value $A' B$, $A' G''$ and $A' D''$ as described.

For the designs ordinarily encountered, this yields a curve so closely approximating for the working load the original circle $B D A$ in which saturation is neglected, that it is not necessary to go beyond the construction of $B D A$ to get a good idea of the characteristics except slip which is $\frac{A' F''}{A' D''}$. If the scale used for $A' F''$ is not that of $A' D''$, then, of course, slip is $\frac{A' F''}{A' D''}$ multiplied by the proper ratio of scales.

The combination in Fig. 5 is suitable to service in which there are rapid and wide fluctuations in load which it is desired to absorb as much as possible by the flywheel B. This arrangement is superior to the use of a resistance across the slip rings because instead of being wasted as in the resistance, the slip energy can all be returned to the power system except for the machine losses of D and E. When applied to a motor with secondary resistance the flywheel reduces the peak loads by delivering torque as it is retarded. The return of most of the slip energy to the line by the regulating set decreases the peak loads still more. A further advantage for the regulating set is the means which it affords of materially improving the power factor of the main motor.

SINGLE-RANGE (BELOW SYNCHRONISM ONLY), SPEED AND POWER FACTOR CONTROL BY MEANS OF A CONSTANT-SPEED, SHUNT COMMUTATOR MOTOR

The series regulating set is, of course, the simplest form, but it is not adjustable without tapping the field winding or external apparatus, and as it imparts to the main motor the characteristic of a material reduction of speed with the assumption of load, it is not suited to the majority of industrial uses in which variable speed from large induction motors is required. In the greater number of cases, it is desired to adjust the speed to a value suited to the momentary requirement of the process, and have the speed remain at approximately the adjusted value irrespective of load variation.

The total induced secondary e. m. f. of an induction motor including the secondary reactance drop is proportional to the "rotor field" (See *A D* of Fig. 1) and the slip. So, as is well appreciated, within the working range the slip is about proportional to the torque as the torque is about proportional to the rotor current, the current being proportional to the total induced rotor voltage. If at a given load we obtain speed reduction by an increase in resistance or by the use of a "series" regulating set, in which cases an increase in secondary induced e. m. f., hence, slip is required to overcome the additional resistance drop, or the rotational e. m. f. of regulating set, plus resistance drop, then, as soon as the load disappears, the main motor speeds up to synchronism, since the secondary resistance drop and the rotational e. m. f. of the regulating motor vanish with the current. If the rotational e. m. f. of the regulating motor could be made independent of the load and of the slip, then with the departure of load it would remain constant, so that the speed would only rise enough to make the total induced secondary e. m. f. equal to the rotational voltage of the regulating motor, leaving no resultant to circulate load current. With load fluctuations, the speed would then fluctuate by only such small amounts as to cause at all times the small difference between total induced secondary e. m. f. and the rotational e. m. f. of the regulating motor to overcome the small resistance drop of the windings; usually only a few per cent of synchronous speed.

In Fig. 8 is shown an arrangement to approximate these conditions. *"A"* is the main motor, *"D"* a neutralized three-phase shunt commutator motor, whose speed is held practically constant by generator *E*, returning the energy derived from *D* to the line. "*B*" is an auto transformer excited from the slip ring circuit and provided with suitable taps to apply predetermined percentages of the slip ring e. m. f. to the shunt exciting windings F_1, F_2, F_3. Assume that the resistance drop in the F_1, F_2, F_3 circuit is negligible

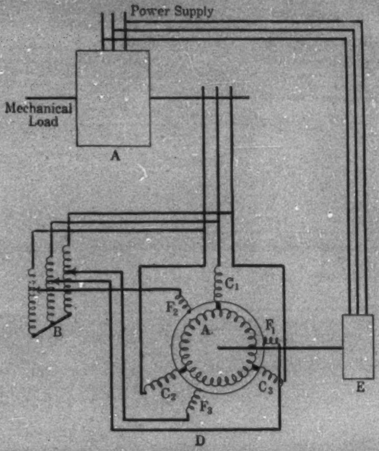

Fig. 8—Neutralized Three-Phase Shunt Constant-Speed A-C. Commutator Machine and Connections for Adjustable-Speed Control of Induction Motor below Synchronism

and that "*B*" applies to F_1, F_2, F_3 the selected percentage of the total secondary induced e. m. f. This with the further assumption that the reactance drop of the regulating motor is included in the slip ring e. m. f. (which is supplied to "*B*") and that the resistance drop of the main motor rotor is not included in the slip ring e. m. f., will give what may be termed for our purposes, "pure shunt excitation." The effects of these assumptions will be pointed out later. The counter e. m. f. of F_1, F_2, F_3 thus consists of the e. m. f.

set up by the flux excited by it, and is, therefore, proportional to the flux and the frequency (frequency itself being proportional to slip s).

$$i.\ e.,\quad e_f = k \times \phi \times s$$

The e. m. f. applied to the field is proportional to the total induced e. m. f., as remarked above, and the total induced e. m. f. is equal to its own standstill value for the main motor current (hence torque) in question times the slip, so we see that,

total induced e. m. f. at standstill $\times s = k \times \phi \times s$.

ϕ is thus independent of frequency and proportional

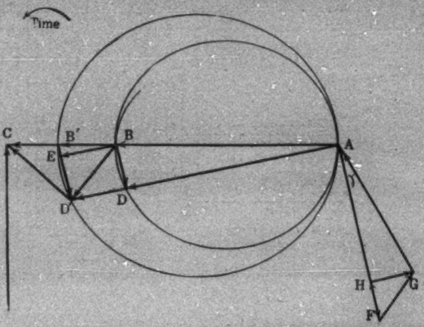

Fig. 9—Circle Diagram of Induction Motor and Pure Shunt Excited Constant-Speed Commutator Machine

only to total standstill induced e. m. f. Referring to Fig. 9, A, B, C is constructed exactly as Fig. 1, including the leakage reactance of the regulating motor with that of the secondary of the main motor, so that instead of X_2, we shall have $X_{2+e} = X_2 + X_e$ and instead of C_2, we shall have C_{2+e}. The circle B, D, A would thus apply with regulating set D and E of Fig. 8, stationary. With the regulating set running, we get secondary current $B\,D'$, total induced e. m. f., $A\,F$ of main motor secondary proportional to slip and to $A\,D'$ and rotation e. m. f. of regulating motor $A\,G$ proportional to $A\,D'$ and at a constant angle γ from $A\,F$, angle γ being determined by the connections of the transformer B and exciting winding F_1, F_2, F_3 of Fig. 8.

Resolve resistance drop $F\,G$ into the component $F\,H$ in phase opposition to $A\,F$ and $H\,G$ in quadrature to $A\,F$, the corresponding components of secondary current $B\,D'$ being $B\,D$ and $D\,D'$.

$D\,D'$ is proportional to $H\,G$, $H\,G = A\,G \sin \gamma$, so $H\,G$ is proportional to $A\,G$ (γ constant) which is proportional to $A\,D'$, hence, $D\,D'$ is proportional to $A\,D'$.

$A\,D = A\,D' - D\,D'$, hence, $A\,D$ is proportional to $A\,D'$.

$B\,D$ and $B'\,D'$ are both perpendicular to $A\,D'$, hence, $\frac{A\,B'}{A\,B} = \frac{A\,D'}{A\,D}$ = constant, and B' is fixed point. So curve traced by D' is a circle.

Slip "s" is equal to $\frac{A\,F}{A\,D'}$.

At running light (zero torque) $B\,D$ and $H\,F$ become zero. (For proof of this see Fig. 1. The torque of the motor is proportional to the mutual flux $A\,G$ and the component of secondary current in quadrature with it. This is the same thing as the total secondary current and the component of the mutual flux in quadrature to the current, which component is equal to $A\,D$, the "rotor field." The torque is, therefore, zero when $B\,D$ is zero, and in Fig. 9, $B\,D$ is the torqe producing component of $B\,D'$.)

Thus, running light slip $s_0 = \frac{A\,H}{A\,D'}$, and the additional slip s_1, due to the load is thus, $\frac{H\,F}{A\,D'}$.

It is thus seen that at running light the main motor runs at slip s_0, determined by the angle γ, and the ratio $\frac{A\,G}{A\,D'}$, which conditions are adjusted by the connections at B, Fig. 8. The load slip s_1 is the same for all values of s_0, provided angle γ be so chosen that $\frac{H\,G}{A\,D'}$ remains constant, and is the same as would obtain for a normal motor whose circle is $B'\,D'\,A$ and whose short-circuited secondary has the resistance corresponding to the current $B' D'$ and the drop $H F$. Thus it is evident that the main motor, regulated as in Figs. 8 and 9 would retain practically the same load slip-torque, power factor-torque and input-torque characteristics as with short-circuited slip rings, but would have no-load speeds equal to the synchronous speed $\times \frac{1 - s_0}{1}$.

It will be noted from Fig. 9 that the primary power factor can readily be improved and that at the same time the pull-out torque of the main motor can be increased.

SINGLE-RANGE (BELOW SYNCHRONISM ONLY) SPEED AND POWER FACTOR CONTROL BY MEANS OF A CONSTANT SPEED, COMPOUND EXCITED COMMUTATOR MOTOR

Occasionally, in processes where the peak loads are high and of brief duration and of sufficient magnitude in proportion to the capacity of the supply system to be objectionable, it becomes desirable to have a larger drop in speed due to load than would be obtained with a shunt commutator motor, so that a fly-wheel can be effectively added to smooth out the peak loads, and at the same time retain the adjustability of the speed. Fig. 10 illustrates a method of compounding the regulating motor.

In the shunt excitation, neglect the same factors as in the case of the shunt regulating motor, (the secondary resistance drop of the main motor and the absence of the reactance drop of the regulating motor). In order to get the compounding action it would not do merely to put in some series turns as in a d-c. machine, since there is applied to the shunt field a fixed percentage of the total induced secondary e. m. f., which means a flux proportional to the rotor field of the main motor as already pointed out. Thus, the ampere turns of the series winding would merely be balanced by a change in the shunt current, F_1, F_2, F_3 serving as the primary of a transformer. It, therefore, becomes necessary to change the field voltage in response to load, in order to change the flux and hence the rotation e. m. f. This is done by the series transformer *H*, which has an air gap in the magnetic circuit, so that its flux is proportional to the resultant of the primary and secondary ampere turns. Of course, a proportionate effect upon the flux of the regulating motor, will accompany any changes in the flux in the series transformer (slip transformer) since the alteration in flux produces a proportionate alteration in voltage applied to regulating motor field, and

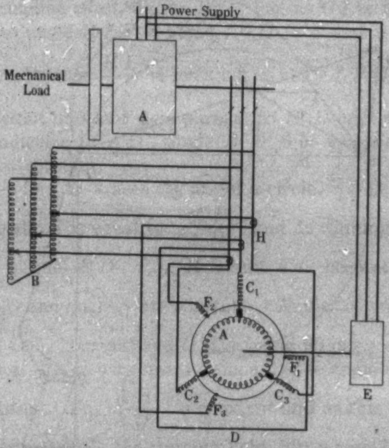

Fig. 10—Neutralized Compound-Excited Constant-Speed Three-Phase A-C. Commutator and Connection for Adjustable-Speed Single-Range Speed Control of Induction Motor Giving Automatic Drop in Speed With Increase of Load.

this in turn a proportionate change in flux thereof. This then means that the rotation e. m. f. will contain a component proportional to the resultant ampere turns of the series transformer, which component, of course, may itself be resolved into components corresponding to the components of the resultant ampere turns of the series transformer.

The performance of the motor, controlled as shown in Fig. 10, is illustrated in Fig. 11, in which *A*, *B* and *C* are again found as for Fig. 1, including the reactance of machine *D* with that of the secondary of *A*. *B* is the no-load point, such that *B B′* is the no-load secondary current, and *D′* is the load point under consideration. Resolve secondary current *B D′* into component *B E* parallel to *A D′* and *E D′* part of *B′ D′*. Component *B E* produces no torque. *I A* is the rotational e. m. f. of regulating motor at constant angle from total induced secondary e. m. f. *A F* of main motor and bearing constant ratio to *A D′* being due to the application of total induced e. m. f. *A F* to the shunt field circuit of *D*, Fig. 10. *H G** is resistance drop of *B E* and *I G* is

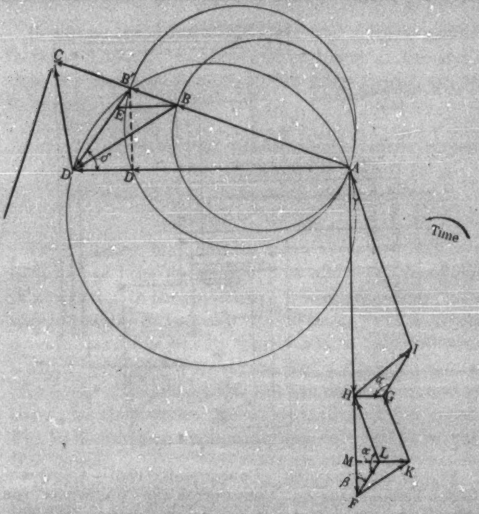

FIG. 11—CIRCLE DIAGRAM OF INDUCTION MOTOR WITH COMPOUND-EXCITED CONSTANT-SPEED A-C. COMMUTATOR REGULATING MACHINE

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*Point *H* is on *A F* because at zero load *B E* becomes *B B′*, the total secondary current and *H G* and *G I* the only e. m. fs. to close the gap between *I A* and *A F*. As *B E* is proportional to *A D′* and hence *A I* and at constant angle thereto, *H* must remain on *A F*.

the rotation e. m. f. produced by D of Fig. 10 by the voltage introduced into the field circuit by the existance of current $B\,E$ in primary of H. The angle α $(I\,G\,H)$ is determined by design and is, of course, constant, as is the ratio $\frac{G\,I}{H\,G}$.

$F\,K$ is total resistance drop, and $F\,L$ and $L\,K$ are components, due to $E\,D'$ and $B\,E$. $K\,G = L\,H$ is compounding effect, due to $E\,D'$, just as $G\,I$ is due to $B\,E$, so angle $H\,L\,F$ = angle $I\,G\,H = \alpha$ and is constant. Further $\frac{H\,L}{L\,F}$ is constant, so angle $L\,F\,H = \beta$ is constant and as $F\,L$ is parallel to $B'\,D$ and $A\,F$ is perpendicular to $A\,D'$, angle $B'\,D'\,A = \delta = 90$ deg. $- \beta =$ constant.

Therefore, D' traces the arc of a circle.

The slip S is equal to $\frac{A\,F}{A\,D'}$ and at running light (zero torque) $E\,D'$, $F\,L$, $H\,L$ and $K\,G$ become zero $\frac{A\,I}{A\,D'}$ and angle γ are constant. Angles of triangle $I\,H\,G$ being constant and $G\,H\,A$ being 90 deg., we see that $I\,H\,A$ is also a constant angle; hence angle $A\,I\,H$ is constant. So $\frac{A\,H}{A\,D'}$ is constant and as this is the expression for S_0, the running light slip where $H\,F$ is zero, we see that the slip, due to load $S_1 = \frac{HF}{A\,D'}$, consisting of $L\,F$, due to the resistance and $H\,L$, due to the compounding action of the slip transformer. We thus see that the running light slip S_0 is adjustable by means of B in Fig. 10, while the load slip S_1 has been increased from $\frac{M\,F}{A\,D'}$ to $\frac{H\,F}{A\,D'}$ by the slip transformer. Further, it is apparent that by controlling the angle α we can make the power factor get more leading or more lagging as load comes on, and thus also, increase or decrease pull-out torque of the motor.

In defining the conditions assumed for Fig. 10, we mentioned that the leakage reactance drop of the regulating motor was supposed to be included in the voltage applied to the exciting winding. The actual effect of excluding it from this circuit can now be shown in Fig. 11-A, a modification of part of Fig. 11. The reactance in the regulating motor is, of course, not applied to its field, and hence the actual rotation

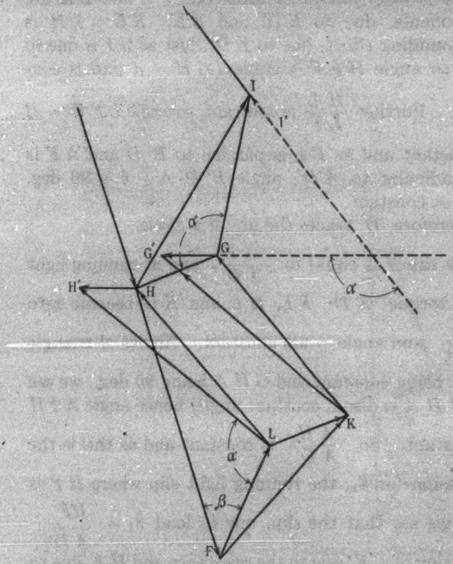

FIG. 11-A—EFFECT ON FIG. 11 OF CORRECTLY LOCATING RESISTANCE OF MAIN MOTOR SECONDARY AND REACTANCE OF REGULATING MACHINE

e. m. f. for shunt excitation should not include $I' I$, the rotation e. m. f., due to the application of reactance drop of $B E$ to the shunt field. Note that $I' A$ is the rotational e. m. f. of pure shunt excitation, and that as triangle $B E B'$ is similar to triangle $A D' B'$, $\frac{B E}{A D'} = \frac{B B'}{A B'}$, hence $I' I$ is proportional to $I A$ and to

A D′ so that *A H* is still proportional to *A D′*. *G G′* and *H H′* are the rotational e. m. f., due to application of reactance drop of *E D′* to shunt field, and hence proportional to *E D′*, so they may be excluded from *K G′* and *L H′*, and for them may be used instead, *K G* and *L H*. As angle *G′ G K* and *H′ H L* are fixed and as *H H′* and *G G′* are proportional to *H L* and *G K*, we see at once that *H L* is proportional to *L F*, angles *H L F* ($=$ angle $H L H' + \alpha$) and *H F L* are constant, hence, $\delta = B' D' A = 90$ deg. $- \beta$ is constant and D^1 still traces a circle.

Thus when we consider the actual effect of the leakage reactance of the regulating motor we see that it is merely to alter the amount of compounding. Hence, to consider this in the case of Fig. 9, would mean to change it to a diagram like Fig. 11, with a small amount of compounding, the "pure shunt excitation" being only a hypothetical condition.

The magnetizing current of the regulating motor has so far been neglected. Neglecting regulating motor saturation, this is proportional to and in phase with *A D′* of Fig. 11. As it flows through the armature and compensating windings of the regulating motor only, its reactance drop can be added to the compounding just as was done in Fig. 11-A at *I I′* and its resistance drop, proportional to *B E* can be added to the resistance drop of *B E*. Thus, we still would get our circle diagram. However, it does not usually pay to consider so small an element except as an interesting theoretical consideration.

The effect of the inclusion of the main motor resistance drop in the voltage applied to the regulating motor field of Figs. 8 and 9 may be treated similarly, providing we confine ourselves to operation so far from synchronism and at such a range of loads, that s_1 is a fairly small part of s_0, in which case the component of rotation e. m. f. caused by the resistance drop is approximately proportional to the current components.

Further the accuracy of the diagram developed so far hinges on the assumption that the values of S (distance from synchronous speed) are so great as to cause the variations in the relative values of the resistance and reactance drops of the shunt field circuit to be relatively small, which will mean a small variation in *H G* of Fig. 9, since variations in the phase relation of field current and "total induced e. m. f." *A D′* mean variations in angle γ. As the resistance drop is larger and larger compared to the reactance drop the smaller the slip and frequency, it therefore appears that Figs. 9 and 11 are accurate only at fairly large values of slip, and small ratios of resistance drop to reactance drop in the field circuit, becoming inaccurate as synchronism is approached. Consideration of these effects has lead the writer to the use of a constant voltage frequency changer and adjustable resistance for overcoming and regulating the resistance drop of the field circuit, and of an auto-transformer with taps (and alternative devices) for overcoming the reactance drop, leading in turn to a feasible way of regulating the main motor through and above its synchronous speed as well as below.

DOUBLE-RANGE (ALL SPEEDS ABOVE OR BELOW SYNCHRONISM) SPEED AND POWER FACTOR CONTROL BY MEANS OF A CONSTANT-SPEED SHUNT COMMUTATOR MOTOR

Several advantages of regulating the main motor speed above as well as below its synchronous value appear at once. The capacity of the regulating set for a given maximum speed variation and maximum speed is reduced 50 per cent, provided the synchronous speed of the main motor is half way between the extremes. For if, S_{max}, S_{min} and S_s represent the maximum, minimum and synchronous speeds of the main motor and HP_{max} be the horsepower capacity at speed S_{max}, we have for single range,—

$S_s = S_{max}$ and capacity of set is:—

$$HP_{set} = HP_{max} \times \frac{S_{max} - S_{min}}{S_{max}}$$

Now for double range, as above, we have:—

$$2\,(S_{max} - S_s) = S_{max} - S_{min}$$

$$HP_{set} = HP_s \times \frac{S_{max} - S_s}{S_s} = \frac{HP_s}{S_s} \times \frac{S_{max} - S_{min}}{2}$$

But as $\frac{H\,P_s}{S_s} = \frac{H\,P_{max}}{S_{max}}$ we have,—

$$H\,P_{set} = 1/2\,H\,P_{max} \times \frac{(S_{max} - S_{min})}{S_{max}}$$

showing that the capacity of the double-range set is one-half that of the single range. Thus, not only will the first cost be materially less, but the machine losses will also be greatly decreased.

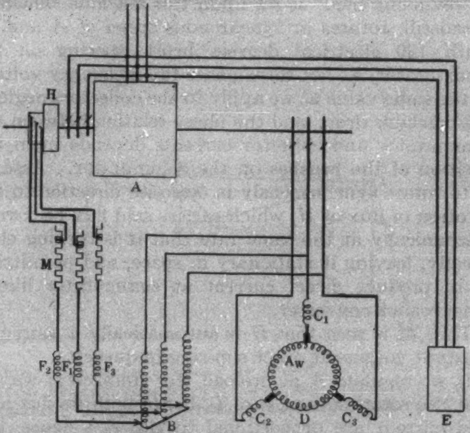

Fig. 12—Neutralized Three-Phase Shunt A-C. Commutator Machine and Connections for Adjustable-Speed Control of Induction Motor for Operation Both Above and Below Synchronism.

A second important advantage is that the synchronous speed of the main motor is in the middle of the speed range, so that often times many processes may be carried out running as plain induction motor with the set shut down, with consequent saving of wear and tear on it.

The apparatus, shown in Fig. 12, is the same as Fig. 8, except that instead of going to a star point the ends of shunt field coils F_1, F_2, F_3 are carried through the adjustable resistance *M*, to the frequency changer *H* mounted upon the shaft and wound for the same number of poles as main motor *A*. This machine has a single primary winding connected to a commutator exactly as in the armature of a d-c. machine, and has collector rings tapped in at points 120 electrical degrees apart (for three-phase power). The secondary is a smooth laminated ring without windings which may or may not rotate with the primary. Obviously, a "revolving field" is set up in this machine, which at standstill, rotates at synchronous speed of *A* and *H*. With 120 electrical degrees brush spacing on the commutator, we get three-phase full frequency voltage of the same value as we apply to the collectors neglecting machine drop, and the phase relation between the commutator and collector currents depends upon the position of the brushes on the commutator. Assume *A* to rotate synchronously in opposite direction to the rotation of flux of *H*, which carries said flux backward mechanically at the same rate that it is turning electrically, leaving it stationary in space, and permitting *H* to produce direct current at commutator like a synchronous converter.

Thus, it is seen that *H* is automatically a source of constant voltage at exact slip-ring frequency.

If we regulate *A* at no-load (for simplicity) we see that the rotation e. m. f. of *D*, hence both its flux and field current are proportional to slip *S*. Hence the reactance drop component of the impedance drop of the field circuit, being proportional to frequency as well as flux is proportional to S^2 while the resistance drop is merely proportional to the field current and to *S*. By connecting to taps of *B* whose distance from the star point is proportional to *S*, we get a voltage proportional to S^2, since the total e. m. f. of *B* is itself proportional to *S*. By changing taps on resistance *M* so that the entire resistance of the circuit is proportional to $1/S$, we just permit constant voltage frequency changer *H* to supply the resistance drop balancing e. m. f., while auto-transformer *B* furnishes reactance drop balancing e. m. f. In practise, one set of switches can be arranged to vary both *M* and *B* simultaneously.

With *M* operating at a considerable distance from synchronous speed, the field resistance drop can be exactly balanced for a given load by *H*, so that as *B* supplies the reactance drop, the conditions previously assumed are attained. From Fig. 9 it will be noted that the phase of *A F* alters with load, while that of the voltage from *H* in Fig. 12 remains fixed. This only introduces a comparatively small discrepancy for working loads, the main effect being a slight alteration of the load slip.

Let us now, on the other hand, consider the case of running near synchronism, where the reactance drop of the field, varying as S^2, and very nearly balanced by *B* has become practically ineffective. Fig. 13 is the simplest circle diagram for these conditions, the constant excitation of *D*, Fig. 12, from the frequency changer *H* being so chosen that, in Fig. 13, rotational e. m. f. *H A* is perpendicular to line *A B C* found as usual. *A F* is the total induced e. m. f. of main motor secondary, *F H* the resistance drop in main secondary circuit for secondary current *B D′*.

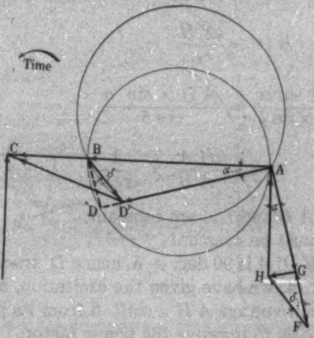

Fig. 13—Circle Diagram of Induction Motor Running below Synchronism with Regulating Machine Receiving Constant Shunt Excitation without No-Load Power Factor Improvement

Resolve *F H* into components *F G* in phase opposition to *A F* and *G H* perpendicular to *A F*, and *B D′*

into corresponding components $B\,D$ and $D\,D^1$, (Point D, thus traces the circle of the main motor with regulating motor reactance included, but with $A\,H$ left out of the circuit)

$$G\,H = H\,A \sin \alpha$$

$$F\,H = \frac{G\,H}{\sin \delta} = \frac{H\,A \sin \alpha}{\sin \delta}$$

$$B\,D = A\,B \sin \alpha$$

$$B\,D' = \frac{B\,D}{\cos \delta} = \frac{A\,B \sin \alpha}{\cos \delta}$$

Further, $$B\,D' = \frac{F\,H}{r_{2+c}}$$

So $$\frac{H\,A \sin \alpha}{r_{2+c} \times \sin \delta} = \frac{A\,B \times \sin \alpha}{\cos \delta}$$

And $$\tan = \frac{H\,A}{A\,B} \times \frac{1}{r_{2+c}}$$

As $H\,A$, $A\,B$ and r_{2+c} are constant,
Angle δ must be constant.

Angle $B\,D'\,A$ is 90 deg. $+\ \delta$, hence D' traces a circle.

In Fig. 14, we have given the excitation, and hence, the rotation voltage $A\,H$ a shift β from its position in Fig. 13, so as to improve the power factor.

Resolve the resistance drop $F\,H'$, of the secondary current $B'\,D'$ (A, B' and C being the usual fixed points) into $H'\,H$ perpendicular to $H\,A$ perpendicular to $A\,B$, and $F\,H$, corresponding current components being $B\,B'$ and $B\,D'$. Since $H'\,A$ and angle β are constant, H and B are fixed points. Now resolve $F\,H$ into $F\,G$ along $A\,F$ and $G\,H$ perpendicular to $A\,F$, also $B\,D'$ into corresponding components $B\,D$ and $D\,D'$. D' may now be shown to trace arc of circle $B\,D'\,A$ as in Fig. 13. We note the power factor and pull-out torque are better than for Fig. 13.

As we are considering operation at rather small values of slip where the shunt excitation is all from H and M of Fig. 12 and is not effected by B, we can compound by the use of plain series windings as with d-c. machines without interference by transformer action.

In Fig. 15, we have shown the effect of such compounding, brought about by making changes in the neutralizing winding C_1, C_2, C_3. It can be demonstrated that a polyphase armature turning above its own synchronous speed in the field, set up by its own reaction generates a rotation voltage leading the current by 90 deg. Hence, by weakening $C_1\, C_2\, C_3$, we can get this sort of compounding in any desired degree.

$A\ B'\ C$ denote the usual fixed points of the main

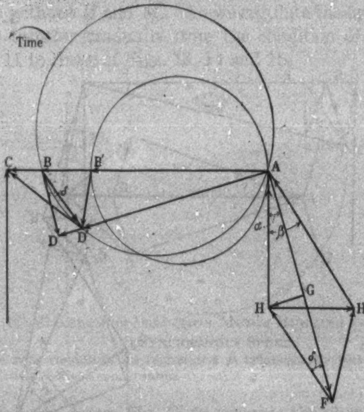

Fig. 14—Circle Diagram of Induction Motor Running Below Synchronism with Regulating Machine Receiving Constant Shunt Excitation, such as to Give No-Load Power Factor Improvement

motor with reactance of regulating motor included. $H'\ A$ is the fixed excitation from frequency changer and rheostat and $B'\ B$ is running light secondary current, due to $H'\ I$, $B'\ D$ being load current considered. $A\ F$ is the total secondary induced e. m. f.

The secondary current $B'\ D$ consists of a constant component $B'\ B$ and variable component $B\ D$. $I\ H'$ is the constant resistance drop and $H\ I$ the constant leading rotation e. m. f., due to component $B'\ B$, making H a fixed point. $G\ H$ is resistance drop and $F\ G$

leading rotation e. m. f. of variable component $B D$. Now the resistance drop and the variable rotation e. m. f. are each proportional to the current (the iron of the regulating motor is always at low densities for these near synchronism conditions) hence, angle $H F G = e$ is constant. Since H is a fixed point, angle

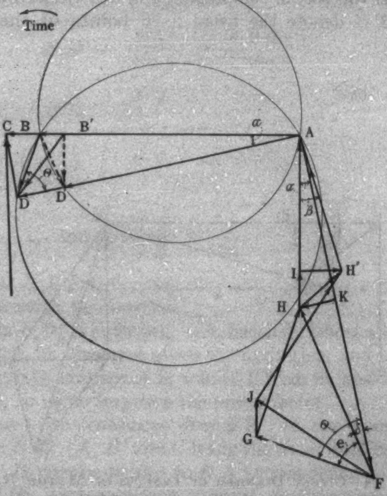

Fig. 15—Circle Diagram of Induction Motor Running Below Synchronism with Regulating Machine Compound-Excited, the Shunt Excitation Being Constant and Having No-Load Power Factor Improvement, and the Series Excitation Yielding E. M. F. 90 Deg. Ahead of the Current

$H F K = \delta$ can be shown to be constant as in Fig. 13. Angle $K F G = \theta = c + \delta =$ constant. Angle $B D A$ is also equal to θ, since $A D$ is perpendicular to $A F$ and $B D$ is perpendicular to $F G$. So D traces arc of a circle, passing through A and B.

It will be seen then that the power factor and pull-out torque can be improved as well as the speed regulated by this method, when regulating near synchronism, as well as remote therefrom. When we regulate the speed, we so adjust the taps of *B*, Fig. 12, as to get the desired percentage of slip voltage from $F_1 F_2 F_3$ to overcome the reactance drop and then so adjust resistance *M* that the field current corresponding to the desired conditions will have a resistance drop equal to the voltage supplied by *H*. As the field current is about constant over the working range of loads we can thus get an even better approximation to Figs. 9 and 11 than without *H* and *M*. As we regulate the speed, we thus transfer gradually from the condition of Figs. 9 and 11 to those of Figs. 13, 14 and 15.

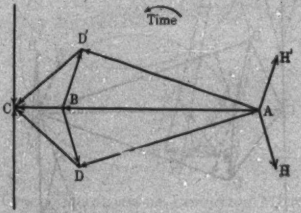

FIG. 16—DIAGRAM OF INDUCTION MOTOR RUNNING LOADED AT SYNCHRONOUS SPEED

The same conditions are represented as infinitesimally below and as infinitesimally above synchronism.

We have drawn Fig. 16 to examine the phenomenon of regulating the speed of the main motor while loaded from an infinitesimal amount below synchronism to an infinitesimal amount above synchronism. As the slip is negligible, the total induced e. m. f. is also negligible and the rotational e. m. f. of the regulating set *A H* just supplies the resistance drop *H A*, the main motor being assumed to be a trifle below synchronism. Let us now assume it to be an infinitesimal amount above synchronism.

All vectors are referred to the secondary whose phase rotation has been reversed, although the physical conditions in the motor remain unchanged. If we select the phase of *A C* as the phase of reference for both phase rotations, then the components of all vectors in phase with it will not be altered by reversal of the phase rotation, but the quadrature components of all vectors will be reversed, as a vector which would not reach its maximum until 90 deg. after *A C* will, in reversed phase rotation, reach its 90 deg. ahead of *A C*. This law yields us *H′ A D′ B C* to represent the same phenomena in terms of reversed phase rotation as are shown by *H, A, D, B, C* with original phase rotation in the secondary.

In Fig. 17, *D* is a load point with motor nearer

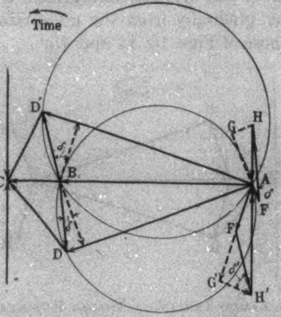

Fig. 17—Circle Diagram of Induction Motor and Constant-Excitation Regulating Machine with One Value of Excitation such that Speed for the Load Point Shown is Nearer Synchronism than the Natural Slip Value and Another Value of Excitation such that Speed for the Load Point Shown is Above Synchronism

synchronism than its natural slip, as the rotation e. m. f. of the regulating motor *H A* has been reversed so as to have a large component in phase with the total induced e. m. f., *A F*. The bulk of the resistance drop *F H* is, therefore, supplied by *H A*, so that *A F* and consequently the slip are reduced. In these conditions the motor would pass through and above synchronism as the load dropped off.

Let us now increase *H A* until the main motor runs above synchronism (with reversed secondary phase rotation). As *H A* was in quadrature to *A C*, the line of the phase of reference, its new value will be shown

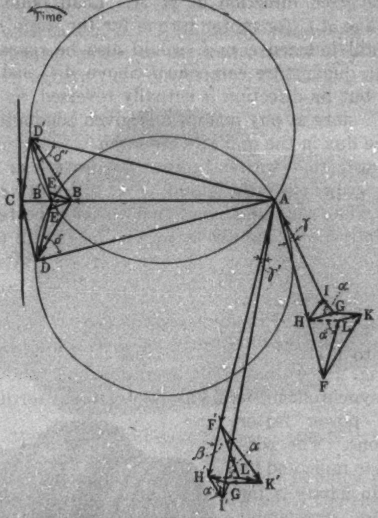
Time
D′
δ″
E′
C
B′
B
A
E
δ
D
γ
γ′
I
α
G
H
K
L
α
F
F′
β
α
L
H′
K′
α
G
I′

FIG. 18—CIRCLE DIAGRAMS OF INDUCTION MOTOR RUNNING BELOW SYNCHRONISM AND ABOVE SYNCHRONISM WITH THE SAME CHARACTERISTICS, CONTROLLED IN EACH CASE BY A COMPOUND CONSTANT-SPEED REGULATING MACHINE

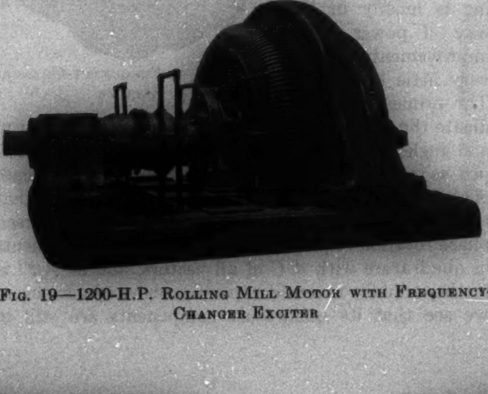

FIG. 19—1200-H.P. ROLLING MILL MOTOR WITH FREQUENCY-CHANGER EXCITER

with reversed direction at $H' A$. Load point D' is above line $A C$ for motor torque for the same reason. The total induced e. m. f. would also be represented with its quadrature component above $A C$ instead of below, but its direction is actually reversed, as shown at $A F'$, since at any instant any given conductor now cuts the flux in the opposite direction.

We note that with no initial quadrature or power factor component in $H A$, and $H' A$, the motor characteristic when running above synchronism is better than below in respect to power factor and maximum torque, while for the generator characteristic the converse is true.

DOUBLE-RANGE (EITHER ABOVE OR BELOW SYNCHRONISM) OPERATION REMOTE FROM SYNCHRONISM

In Fig. 18, we represent operation both above and below synchronism, with the same speed—torque and speed — power factor conditions. The configuration indicated by the plain letters is for using a compound commutator motor similar to that of Fig. 11, except that angle α has been decreased so that the compounding is mostly in the way of power factor improvement and adds very little to the slip. The primed letters indicate the relations for operating above synchronism and angle $A D' B'$ equals δ can be shown to be constant just as in the case of angle $A D B'$. Keeping the phase of $A C$ as the phase of reference we note as before that the representation in secondary terms with reversed phase rotation requires the reversal of the components in quadrature with $A C$ of all vectors, and as $A F'$ is further actually reversed in passing above synchronism, we see that its quadrature components are still in

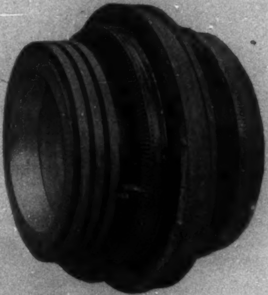

FIG. 20—A FREQUENCY-CHANGER EXCITER

phase with that of $A\ F$. But as the regulating machine must furnish power to the main motor secondary in order to satisfy the conservation of energy, the total current $B\ D'$ must have a component in phase with the rotation e. m. f. $I'A$, which we see is the case, thus requiring that $I'A$ be larger than $A\ F'$ fulfilling the condition that the regulating machine function as a generator.

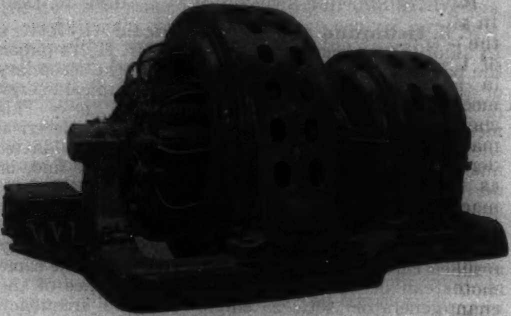

FIG. 21—SPEED-REGULATING SET FOR A 1600-H.P. MOTOR

NOTE:—For the development of the circle diagram, particular mention should be made of the works of Behrend, Blondel and Arnold-LaCour. Meyer-Delius has also written concerning what the writer has termed Single-Range Regulation.

DISCUSSION ON "THEORY OF SPEED AND POWER FACTOR CONTROL OF LARGE INDUCTION MOTORS BY NEUTRALIZED POLYPHASE ALTERNATING-CURRENT COMMUTATOR MACHINES" (HULL), WHITE SULPHUR SPRINGS, W. VA., JUNE 30, 1920.

R. E. Hellmund and C. W. Kincaid (by letter): In all of the arrangements shown in Mr. Hull's paper, the power derived from, or furnished to the secondary of the main motor is changed to mechanical energy in an auxiliary commutator machine *"D"*. This mechanical energy is retransformed into electrical energy by an induction machine *"E"*. The latter machine returns this electrical energy to the line, or derives the necessary excess energy from the line, as the case may be, depending whether the main machine is working below or above synchronism. In other words, the transformation of the rotor energy is taken care of by a motor-generator set. The same results can be obtained by applying, in place of the motor-generator set, a single machine frequency changer in which the change of frequency is accomplished the same as in the well known synchronous converter in which an a-c. frequency is changed to zero frequency, that is, to direct current. The advantages of using such single machine frequency changers in the secondary of the main motor, for the purpose of changing the rotor frequency to the line frequency, are the same as those obtained by the use of the well known rotary converter in place of a motor-generator set. This simply means that we have a lower first cost, as well as improved efficiency in the frequency changer proper. With such a machine, there is, however, a definite relation between the voltage of the output and input frequency, the same as in the alternating-current—direct-current rotary converters. In other words, the line frequency voltage of the machine is governed by the main motor secondary voltage and it is therefore necessary to apply a transformer for changing this voltage to the line voltage, in order to make a power exchange possible. We find, therefore, that part of the gain in first cost and efficiency is lost by the necessity of using a transformer, the capacity of which is the same as that of the auxiliary machine. This is again similar to the conditions often encountered in connection with a-c.—d-c. converters, which are usually operated in connection with a transformer, while in the case of motor-generator sets, such transformers can often be omitted. In other words, the problem of converting the power from the rotor frequency to that of the line is very similar to the familiar problem of converting certain a-c. frequency to zero frequency or direct current. In either case, it is possible to use a single machine, usually in connection with a transformer of the capacity of such machine, or a motor-generator set, in which case no transformer is required unless the line voltage is exceptionally high. Numerous control problems arise in either case in order to give the desirable speed and power factor control of the main motor. In the case of the single machine frequency changer, it is usually necessary to vary the voltage of the line frequency impressed upon such machine by shifting the taps on the transformer. In addition thereto, means for changing the speed of the frequency changer are required. This means that a double adjustment is required, the same as in the arrangement in Fig. 12 of the paper. The small transformer "*B*" of Fig. 12 should not be confused with the transformer mentioned in connection with the single machine arrangement. As mentioned before, the latter is of the full capacity of the auxiliary machine, while the transformer "*B*" of Fig. 12 furnishes only the excitation of the commutator machine "*D*". Similarly, the switches in connection with the small transformer "*B*" carry only a relatively small exciting current, while the switches for the transformer of the single machine arrangement carry the full-load current. The latter feature further tends to decrease the difference in favor of the single machine arrangement; nevertheless, it should be expected that comparison of the entire equipment, including all the apparatus, is somewhat in favor of the single machine arrangement. Further experience along this line, as well as the particular conditions applying to various applications, will largely govern the choice between the two alternatives. Those interested in the details of the single machine arrangements will find a number of possibilities along this line described in a paper read by Mr. Sykes and Dr. F. Meyer some years ago.

The arrangements described in the paper, as well as those with a single machine frequency changer, give inherently a constant torque for the main machine, that is, the rated torque for the main machine is approximately the same for all speeds. Many applications require, however, increased torques with the lower speeds and frequently it is desirable to have constant horse power output for all speeds. Such arrangements are possible if the rotor energy furnished to the commutator machine of the various arrangements shown in the paper is transferred into mechanical energy and directly applied to the shaft of the main motor. In other words, by omitting from the arrangements shown the induction machine "*E*" and by coupling the commutator machine "*D*" to the shaft of the main motor, we obtain constant horse power arrangements. The working principles of such combinations are, in general, rather similar to those described in the paper, although the speed characteristic is somewhat affected by the fact that the speed of the auxiliary commutator machine has to vary with that of the main machine. The only handicap of such an arrangement is that the commutator machine usually becomes rather large and expensive. With the arrangements shown in the paper, the speed of the commutator machine can be chosen at will to best suit its design, while with the commutator machine coupled to the main machine, the speed of the latter has to be applied and since it is rather low in many cases, it increases the size of the commutator machine. In order to avoid the use of a large commutator machine and the difficulties encountered in connection with the design thereof, an arrangement as shown in Fig. 1, has been devised for the purpose of giving a practical constant horse power arrangement. The main induction motor "*A*" is coupled to a synchronous machine "*S*". The secondary leads of the induction motor are connected to the commutator of a single machine frequency changer "*D*", the commutator end of which is connected to the synchronous machine. The frequency changer is driven by a small synchronous

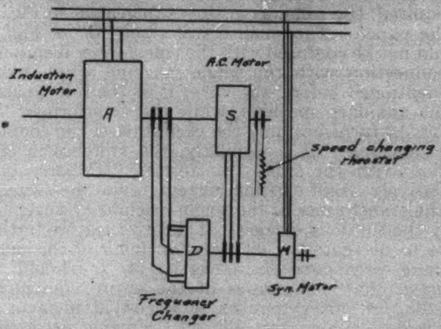

FIG. 1

machine "*M*" which is just large enough to furnish the no load friction energy of the frequency changer. If "*S*" is the slip of the main motor, the frequency of the synchronous machine "*S*" is $1 - S$. Since the auxiliary set runs at synchronous line frequency, while the slip-ring frequency being that of the alternator is $1 - S$, the difference of the two, as appearing on the commutator, is $1 - (1 - S) = S$. In other words, the commutator frequency of the frequency changer is always the same as the rotor frequency of the main rotor. The speed regulation can be very readily accomplished by simply varying the excitation of the synchronous machine "*S*", which in turn varies the voltage impressed upon the slip rings of the frequency changer, and therefore the voltage of the commutator and the slip rings of the main motor. Phase adjustment can be accomplished by either shifting the brushes on the commutator or by shifting the excitation of the small synchronous machine "*M*". The latter is in practise the simplest and can be accomplished by very small field rheostats. An arrangement of this kind can be operated equally well below and above synchronism and offers no difficulty in going through synchronism. The speed characteristic of the main set depends largely upon the inherent voltage regulation of the synchronous machine "*S*". It has been brought out in the paper that as long as the voltage impressed upon the induction motor secondary is constant, the speed regulation is similar to that of the straight induction motor. If the synchronous machine is designed for good voltage regulation, its voltage will remain nearly constant and we will obtain a flat speed characteristic. By designing the alternator for substantial voltage drops with increased load, it is easy to arrange a dropping voltage at the slip rings of the main motor. Consequently, it is possible in this manner to obtain a rather steep speed characteristic for the main set. Since both the phase and voltage adjustment is accomplished by means of d-c. rheostats, the control is as simple as it can possibly be conceived, at least simpler than any control requiring a change of a-c. voltages, which usually calls for auxiliary transformers and preventive coils, etc. The arrangement just described has been built and tested and met all requirements satisfactorily.

For the sake of completeness, it may be advisable to mention the possibility of directly connecting a single machine frequency changer to the shaft of the main motor for the purpose of transferring the rotor energy to the line or vice versa. This arrangement is essentially the same as in the case of separate single machine frequency changers, but usually it is not nearly as advantageous, on account of the fact that the frequency changer has to have the speed of the main machine, which leads to rather uneconomical designs. This latter arrangement has of course constant torque, because the frequency changer does not furnish any mechanical power.

John I. Hull: The paper was intended to cover only theoretical and explanatory aspects of the controlling of large induction motors by one particular method—the use of the constant speed, concatenated polyphase neutralized a-c. commutator machine. Its scope was not intended to include a comparison of the several schemes for accomplishing the results.

So far, although it has been known for many years no manufacturer has to our knowledge commercially exploited the single machine frequency changer coupled to the main motor, or otherwise driven at speeds proportional to it. This is due, no doubt to design difficulties as well as the obstacles to controlling the power between the frequency changer and line which Mr. Hellmund and Mr. Kincaid allude to. The writer's opinion is that at the present state of the art, the "single machine frequency changer" will not compare favorably to the two systems now in general use, unless perhaps in smaller sizes where the frequency changer can be built in its simplest form and still possess good commutation.

If we compare the two established schemes of speed control of induction motors, the "Kraemer" or rotary converter system, the "Scherbius" (as modified for double range) and the frequency changer analogy to the "Kraemer System" described by Mr. Hellmund and Mr. Kincaid, it will be observed that from the point of view of number of rotating power carrying machines, and total kv-a. machine capacity the double range Scherbius involves the least apparatus of the three. In the appended tables we show these comparisons.

Of course, to get apparatus which when working up to capacity at all speeds gives constant torque, the auxiliary machines must exchange the slip energy with the line, while for constant horse power at all speeds they must exchange the slip energy with the main motor shaft. Thus constant torque requires the slip energy to be converted to line voltage and frequency, but constant horse power requires it to be converted to mechanical power at shaft. In such cases, the low speed is really the determining speed rather than the high speed, since the ventilation is usually less. Thus, starting with a given torque capacity at low speed, each of the three systems must have this total as the total capacity for the machines on the main motor shaft, either in a constant horse power or torque arrangement. The constant torque arrangements permit the horse power to rise directly as the speed rises, while the constant horse power variations on the other hand limit the horse power to its low speed values, as the torque of the auxiliary machine on main shaft diminishes when the speed rises, and for double range operation reverses above synchronism.

It is often desirable to keep all the regulating machines off the main shaft, even in constant horse power drives, from other considerations than cost, so that drives have often been installed to deliver constant horse power with separate regulating sets which might have been a little cheaper with direct connected regulating motor.

Comparing from the point of view of control, it is seen that an exciter not shown in Mr. Hellmund and Mr. Kincaid's diagram is necessary with its little rheostat and control unless the drive is to be subjected to the obvious danger of shut down not only when the main a-c. supply fails, but also when the d-c. mill supply fails. Further, there is the danger that the loss of excitation on machines *S* & *M* when the d-c. fails might easily be the cause of a failure of the main a-c. supply, as well. Mr. Hellmund and Mr. Kincaid refer to a double adjustment for the field control of the Scherbius set as illustrated in Fig. 12 of the paper. This double adjustment has no physical existence, but was only shown in Fig. 12 for clearness of explanation. Actually in practise, the resistance *M* serves the additional purpose of limiting switching current, making the service very easy on the contactors. There is only one set of contactors each lying between a tap of the auto transformer and of the resistance. Thus it will be seen that there is actually only a single adjustment. In the scheme given by Mr. Hellmund and Mr. Kincaid, however, a double adjustment will be needed; the speed rheostat for *S*, and the power factor rheostat for *M*, or else the brushes of *D*.

TABLE I

Comparison as to Number Power Machines, Total Number Rotating Machines and Total Power Machines Capacity at Low Speed (Determining Condition) of Double Range Scherbius, Kraemer and Frequency Changer (Analogous to Kraemer) System of Induction Motor Control. Speeds 135/75. 25-cycle power.

System	Low speed capacity on main motor shaft		Separate regulating machines			Total capacity of power machines	Total number power machines	Number exciters & small synchro-nizing machines	Total number power machines and exciters	Base speed main motor alone
	Main motor	Aux. power machines	Unit No. 1	Unit No. 2	Unit No. 3					
CONSTANT TORQUE SERVICE AND DESIGN.							$\frac{1250 \text{ HP}}{750}$		$\frac{125}{75}$	R P M
Scherbius	750	....	250	250	....	1250	3	1	4	100
Kraemer	750	....	500	500	500 1250 at 25 cycles	2250 3000	4	1*	5	125
F. Changer	No Inherently Const. Torque Arrangement.									
CONSTANT HORSE POWER SERVICE AND DESIGN.							$\frac{750 \text{ HP}}{750}$		$\frac{125}{75}$	RPM
Scherbius	600	150	....	....	....	750	2	1	3	93.7
Kraemer	450	300	....	....	300 750 at 25 cycles	1050 1500	3	1*	4	125
F. Changer	600	150	....	....	250 at 33.3 cycles	1000	3	2*	5	93.7

TABLE I—*Continued*

System	Low speed capacity on main motor shaft		Separate regulating machines			Total capacity of power machines	Total number power machines	Number exciters & small synchro-nizing machines	Total number power machines and exciters	Base speed main motor alone
	Main motor	Aux. power machines	Unit No. 1	Unit No. 2	Unit No. 3					
CONSTANT TORQUE DESIGN USED ON CONST. HP SERVICE								$\frac{750}{750}$ HP	$\frac{125}{75}$	RPM
Scherbius	750	....	250	250	....	1250	3	1	4	100
Kraemer	750	....	500	500	500 1250 at 25 cycles	2250 3000	4	1*	5	125
F. CHANGER	No Inherently Const. Torque Arrangement.									
CONSTANT HORSE POWER DESIGN. USED ON CONST. TORQUE SERVICE.							$\frac{1250}{750}$ HP		$\frac{125}{75}$	P P M
Scherbius	1000	250	....	....	....	1250	2	1	3	93.7
Kraemer	750	500	....	....	500 1250 at 25 cycle	1750 2500	3	1*	4	125
F. Changer	1000	250	....	....	416 at 33.3 cycles	1665	3	2*	5	93.7

*By making apparatus dependent on mill d-c. as well as mill a-c. supply, one exciter may be omitted in Kraemer and frequency changer systems.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., June 30, 1920.

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HIGH-TENSION INSULATOR PORCELAIN

BY W. D. A. PEASLEE

Electrical Engineer, Jeffery Dewitt Insulator Co., Huntington, W. Va.

Porcelain used in the manufacture of high-tension insulators must meet certain requirements as to mechanical strength, ability to resist, sudden changes of temperature, porosity, homogeneity and temperature, coefficient of resistivity. A further suggestion as to the influences of the Piezo electric effect and the deterioration of seemingly perfect porcelain is presented with a brief discussion of the degree of progress made in the art to date.

INTRODUCTION

WHILE in the case of transmission line insulators it is true that the design of the insulators insofar as the utilization of the material is concerned is a matter of most efficient design of air insulators, that is, the boundary surface between the air and the supporting dielectric, the material of the insulator itself becomes of greater importance as the voltage of the line increases.

The shape design of the insulator will not be discussed here as that subject is reserved for a later communication. It is hoped that the following discussion of the factors affecting the material employed in the manufacture of high-tension insulators as a dielectric, may lead engineers to a more serious consideration of this vitally important question. The discussion is confined to porcelain as for many reasons this material has come to be accepted as the best for high-tension insulators.

FACTORS TO BE CONSIDERED IN THE SELECTION OF A PORCELAIN BODY AS A MATERIAL FOR HIGH-TENSION INSULATORS

Mechanical Strength. It is perfectly obvious that mechanical strength is one of the prime factors to be considered in an insulator material. One of the first duties of an insulator is to support the conductor, and it must do so under any conditions not severe enough to destroy some other element of the transmission line construction.

Insulators made today either in pin or suspension types, use porcelain either in compression or tension or both, and so it is of great importance to know the strength of porcelain under these conditions.

Porcelain being a brittle or rigid substance like cast iron, has no yield point as commonly known, and its first yield is complete rupture. There seems to be no indication that the stressing of porcelain to a point close to its ultimate strength injures it either mechanically or electrically. Indeed, the accumulated evidence of a large number of tests covering combined electrical and mechanical tests, fatigue tests and high-frequency tests indicate that such stressing has no effect whatever upon these properties. It is probably quite safe to say that properly vitrified porcelain must be stressed beyond its ultimate strength to rupture it even under repeated strains. Tension and compression tests both confirm this statement.

It is, therefore, immaterial whether the porcelain be used for compression or tension, provided that maximum momentary stress does not reach the ultimate strength of porcelain. For engineering reasons an ample safety factor must always be provided for. Ordinary porcelain, made up of the usual three ingredients,—clay, flint and feldspar— has been found by several investigators to have strengths reaching as a maximum 40,000 lb. per sq. in. for compression and 1500 lb. per sq. in. for tension. Naturally different proportions of these ingredients produce a porcelain, or to use a ceramic term, "Body," (which covers all bodies made up of the above or similar ingredients and vitrified) of somewhat different mechanical characteristics, but these figures cover rather the upper limit of strength for a body having the required characteristic as to dielectric qualities and ability to withstand sudden temperature changes.

The wide difference between these values has led many engineers to distrust porcelain used under tension, but provided the stresses are kept within the proper limits, from an engineering safety factor point of view, there is no more valid ground for this attitude than there is to condemn cast iron whenever used in tension.

Under the stimulus of the demand by transmission engineers for better insulation a great deal of work has been done on porcelain mixtures or bodies, and certain types are now available, whose strength runs up to 65,000 lb. per sq. in. in compression and 12,500 lb. per sq. in. in tension. This, indeed, is not the ultimate limit, as indications point to the commercial production of bodies with even greater strength which will at the same time retain the other necessary requirements.

In making tension and compression tests on porcelain it is very necessary that the application of the stress be made in such a manner as to place the porcelain either in pure compression or tension as desired. The compression tests are made on small blocks and the pressure applied through lead or blotting paper disks. The tension tests are made on test pieces consisting of a short, straight section of accurately determined area between two conical end pieces. These conical end pieces are gripped in a specially designed multiple part clutch faced with soft lead or blotting paper sheets; and remarkably consistent results are obtained in this manner.

Ability to Resist Sudden Changes in Temperature. A great many insulator failures are traceable directly to the inability of certain porcelain bodies to resist sudden changes in temperature. The first rays of sunlight on a frosty morning have often been the signal for insulator failures directly attributable to this weakness.

Also a body sensitive to this change is more difficult to manufacture reliably as it will develop internal strains which, added to the applied service strains, will produce rupture of the porcelain at very low applied loads. The existence of these strains has been shown by polarized light under microscopic examination.

Any mixture or body that in the shape and size employed will not, when completely equipped with hardware, withstand an indefinite number of alternate immersions in boiling and freezing water, should never be employed in the manufacture of high-tension insulators. This test should be insisted upon by purchasing engineers. Such bodies are made today, and some are made that will stand even greater ranges. At least two are known which can be heated red hot and thrown into a bucket of water without cracking.

Porosity. Porous porcelain is responsible to a large degree for the unsatisfactory condition of the insulator situation of today. One of the greatest insulator manufacturers has recently stated in a published article that non-porous porcelain cannot be produced.

This statement is challenged as the writer's investigations, both in this country and in the laboratories of France, indicate that it can be produced and under manufacturing conditions. Porcelain can be, and is, produced today that is in an engineering sense, absolutely non-porous. This statement is made after a great deal of careful research and on the strength of many porosity tests both by the impregnation method and the method used in ceramic analysis with a psychometer and a sample crushed to a 100-mesh fineness. The impregnation test has been carried in our laboratories to very high pressures, and is so penetrating that, under microscopic examination of the penetration of the colorant material, it has been observed in the cleavage cracks of microscopic quartz crystals.

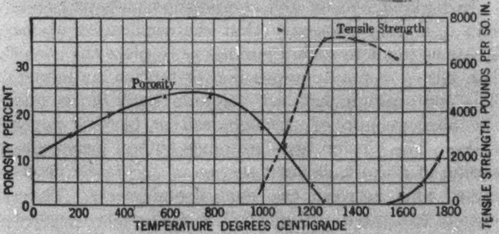

FIG. 1—POROSITY AND TENSILE STRENGTH CHARACTERISTICS—SPECIAL PORCELAIN BODY

Furthermore, a body having the slightest porosity, as indicated by the psychometer test, shows a decided penetration under the impregnation test.

Porosity is of two kinds, discussed in ceramic language as open pore and closed pore types. In the open pore type the pores or voids are connected by capillary passages, while in the second, or closed pore type, the voids or pores are isolated. Though the percentage of porosity may be the same in the two cases, it is obvious that the general character of porosity and its effect on insulators is different in the two cases. In general, the first form is a product of under-firing and the second of over-firing, though many instances have been found wherein over-firing produces the first type of porosity. There are probably many ceramists even today, who will dispute the statement that over-firing will produce porosity, but it can be very readily demonstrated by proper tests. In this connection the curve of Fig. 1 shows the behavior of one mixture or body and indicates very well the effect of the temperature of firing on the porosity of the body. The porosity developed on over-firing on this body was of the closed pore type and on under-firing of the open pore type. In securing the data for making this curve the samples were fired to given temperatures and cooled, and the porosity and tensile strength values were taken at room temperature. In other words the points on the curve represent maximum temperature of firing of each sample and not the temperature of the test. The porosity determinations were made by the psychometer method, the impregnation method being in any case merely qualitative.

In practise the effect of open pore type of porosity is too well-known to discuss here. The development of megger and buzz stick tests are ample evidence of the degree to which this factor has entered into the troubles encountered in the insulator field. Tests to determine porosity in units at the factory are needed, and the man who develops a method whereby we can detect porous insulators at the factory without destroying them, will be a true benefactor of the transmission engineers and porcelain manufacturers.

In the production of insulators a method has been developed that is very valuable in production control testing. A solution of fuchsine dye in wood alcohol is used and unglazed pieces placed in it under pressure. The slightest degree of porosity of the open pore type is indicated by a deep penetration of the dye into the body of the test piece. Indeed, as mentioned before, it is so penetrating that the microscope has shown it forced into the cleavage cracks of minute quartz crystals. If test pieces of the same shape and volume as the insulators being fired are properly distributed in the kiln the fuchsine test on these pieces will furnish a very reliable indication of the condition of neighboring pieces as to porosity.

Some very interesting developments have recently been brought out by the use of very high pressures on the solubility of porcelain in water under certain conditions, and it is probable that considerable light will soon be thrown upon certain types of insulator depreciation as a result of these developments.

It has been found possible to produce insulators of non-porous porcelain within the ordinary limits of quantity manufacture and by means of this method of control to prevent the porous insulators, a few of which are unavoidable in commercial manufacture, from going to the customers. Closed pore porosity is commonly indicated by a swelling of the insulators, and can be watched closely by gauges applied to the finished product.

Temperature Resistivity Coefficient. The temperature resistivity coefficient of porcelain is large and negative. The curves of Fig. 2 give an idea of this characteristic and of the improvement that has been made in it. The curve marked "Conventional Porcelain" in Fig. 2 is the standard mixture of clay, flint and feldspar, and the curve marked "New Body" is one of the recent developments, the formula for which naturally cannot be disclosed. In this curve the points on the curve represent measured resistance attained at the temperatures noted by the abscissas of the curve. For instance, a certain sample of conventional porcelain will have a resistance of two megohms at slightly over 700 degrees fahr. (371 deg. cent.)

whereas the "New Body" in an exactly similar piece has a resistance of two megohms at 1160 degrees fahr. (627 deg. cent). These curves will give some idea of the progress that has been made in this respect. The importance of this feature has been discussed by various writers, and it must be stated here that this characteristic may be improved but will always be negative even in pure quartz. It has an important bearing on the mechanism of insulator failures under transient voltages. This has been discussed by the writer in previous papers before the institute.[1]

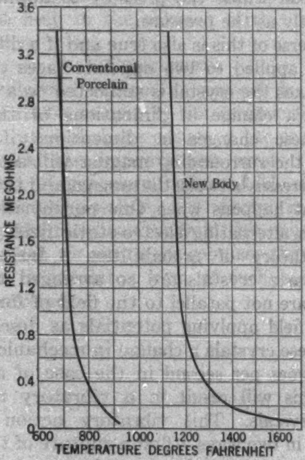

FIG. 2—TEMPERATURE RESISTANCE CHARACTERISTICS OF CONVENTIONAL AND NEW BODIES

Piezo Electric Effect. One of the most baffling difficulties in the insulator situation is the deterioration of seemingly perfect units after a time, and the acquiring by non-porous porcelain of a certain porosity.

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[1] "Insulator Failures Under Transient Voltages," W. D. A. Peaslee, A. I. E. E. TRANSACTIONS, page 1237, Vol. 35; "Insulator Situation....," W. D. A. Peaslee, A. I. E. E. TRANSACTIONS, page 401, Vol. 36.

Some recent investigations indicate that this may be intimately connected with Piezo electric qualities of quartz crystals. Under the microscope it will be seen that porcelain as ordinarily made, consists of rather large particles of quartz in a vitreous magma. It is well-known that if certain crystals such as quartz are subjected to a pressure on two diametrically opposite faces which are parallel with the major axes, a potential difference is set up on the faces perpendicular to those upon which the pressure is applied, which varies directly as the pressure.

The converse of this is also true and if a difference of potential is applied to two opposite faces parallel to the major axes the crystal is subjected to a squeezing action and a change in dimensions of the crystal results. These changes in dimensions if strongly resisted by the surrounding magma will set up enormous local stresses between the magma and the crystals.

Now what happens when this porphyritic mass is placed in an alternating electro-static field? According to the theory of probabilities a large number of these quartz crystals are so arranged that their major axes are not parallel to the field of force. This alternating field applying potentials as described will set up in these crystals a change in mechanical dimensions 120 times per second in the case of a 60-cycle system. This will result in a vibratory movement of these crystals. This vibratory action may be detrimental in two ways,—first a rupture of the crystal itself, along cleavage planes, and second a rupture between the crystals and the surrounding magma.

A concentrated leakage current results through the spaces between the magma and the crystals when the crystal is at its greatest deformation and the potential differences are at maximum. It will also tend towards making the porcelain porous in an entirely different manner (that is, by cracks and fracture voids) from the types before mentioned and the vibratory action of the crystals may aid in the introduction of moisture into the body. This decreases its value as a dielectric and as mentioned before the solvent action of such moisture may have an accelerating effect upon the deterioration of the porcelain.

Regardless of the degree to which this effect contributes to the deterioration of the porcelain when in service on high-tension lines the body to be sought is one wherein the quartz is dissolved as completely as possible in the feldspathic magma.

Great progress has been made in this direction and in a later communication some data, which are now being prepared, may be given that it is hoped will be of some help in the solution of this ever present problem.

CONCLUSION

The insulator problem at present is one whose solution is to be sought in the ceramic field. Aside from more rational shape design on the part of electrical engineers, the improvement must come in the development of porcelain bodies which meet, to the greatest degree possible, the above requirements. Great progress has been made in this respect and the statement that we may very soon see the commercial production in insulator form of such bodies is now amply warranted.

DISCUSSION ON "HIGH-TENION INSULATOR PORCELAIN" (PEASLEE), WHITE SULPHUR SPRINGS, W. VA., JUNE 30, 1920.

H. B. Vincent: It seems to me there is too much insulation between the manufacturer and the electrical engineer, and not enough cooperation by the operating engineer in respect to keeping the records of practise and experience with insulators under operating conditions.

I had occasion within the last year to go into the insulator problem pretty thoroughly, due to the fact that quite a large order had to be placed for high-tension insulators. Trouble had been experienced on some insulators which had been in service quite a while, but fortunately I had kept individual records of every broken high-tension insulator for six years, and on visiting the manufacturers, I found that record very valuable in coming to a conclusion as to what kind of insulator, I should buy. For example one insulator manufacturer, tried to impress very strongly on me that I should buy a certain type of insulator which he manufactured, mentioning the fact that some large company, which had been using it for years, had just duplicated the order. That would have made a very strong impression on me, had not my records failed to entirely substantiate his contention, probably due to the fact that the conditions under which I was operating were different from the conditions under which the other man was operating. I think the operating man should not lose sight of the fact that he is doing probably fifty per cent of the work of improving the product by keeping records, so that he can cooperate with the electrical engineers of the manufacturers, by giving them data of the actual performance of their insulator.

As another illustration I talked with an electrical engineer of a large operating company within the last month and was surprised to find that he was keeping no records, the apparent reason being he was not having much trouble, but I am inclined to feel, from my experience that in later years he may have trouble with his insulators, and then he will regret that he did not keep these records.

C. L. Fortescue: The insulator problem is quite a complicated one. It needs the cooperation of all the parties interested in obtaining good insulators. In dealing with the insulator problem we have to consider not only the problem of a clean insulator, but also the problem of the insulator which has been in service for a certain length of time consideration must be given also to the conditions of the line, etc.

There is one feature of the theoretical design that fits in admirably with this question of insulators that have been in service for sometime, that is, if the insulator is designed according to these principles, the intensity is tangential to the actual working surface, and consequently dirt does not accumulate thereon because there is no force tending to hold the particles of dirt onto the surface,—the actual forces being such as to cause them to simply slide off the surface, and accumulate on the petticoats, what might be called the protective surfaces. Consequently, an insulator designed according to these principles will keep clean, and is to that extent self protective. For that reason besides many others the theoretical design produces a very good insulator.

I wish to emphasize one point brought out by the last speaker namely that we ought to take great pains to keep a record of insulators. There are many things that go towards making a good insulator, the quality of the porcelain is as important as the design and we should take pains not to allow prejudices to upset our judgment in respect to this important commodity. Let us see to it that we all cooperate.

R. M. Spurck: Mr. Peaslee has discussed a number of characteristics of porcelain as they relate to insulators. These characteristics, mechanical strength, ability to resist sudden temperature changes, porosity and temperature resistivity coefficient are all so closely related that all of them must be considered in any discussion of the characteristics of any specific sample of porcelain. For instance, a sample of porcelain with a high mechanical strength might be produced by sacrificing desirable porosity characteristics or ability to resist sudden changes of temperature. A discussion of the above mentioned characteristics especially as they apply to insulators should therefore include the characteristics as they apply to samples of identical materials, mixtures, manipulation, shape, drying and firing. To be a practical manufacturing proposition the mixture must produce, when all other conditions remain the same, a uniform product under a fairly broad range of firing temperatures. The importance of such characteristics is evident when it is known that kilns for commercial work cannot be depended upon to produce the same temperature in all parts, nor can they be fired to give exactly the same temperature at the same place for all runs.

The mechanical strength of porcelain both in tension and compression is difficult to determine. The values obtained depend not only on the fundamental characteristics of the material as determined by the *materials*, mixture, manipulation and firing but also upon the size and shape of the test piece. Tensile strength tests on a very short thin sample will give unit values that cannot be applied to longer samples of larger diameter. Unit tensile strength values obtained from samples where extra precautions were taken to eliminate bending stresses must not be applied to practical designs where it is not possible to take such precautions. Tests and samples of porcelain where 3000 lb. per sq. in. were obtained have been reported.[1] The samples tested were 16 in. between grips and one inch in diameter. It is very probable that shorter samples would have given much higher values by the elimination of bending stresses. However, it is not believed that the unit stresses there obtained should be extended for use on samples of a much larger diameter.

The ability to resist such temperature changes is not only dependent upon the material itself, but also upon the shape and mass of the piece. This characteristic is also directly related to the temperature coefficient of expansion. The Bureau of Standards[2] have recently published results of tests of a wide variety of porcelain mixtures on which the temperature coefficient of expansion from room temperature to a temperature of 200 deg. cent. varied from a minimum of 1.6×10^6 to a maximum of 19.6×10^6. It is interesting to note that the clay content of both samples was the same but the sample having the lowest coefficient contained 35 per cent of beryl.

The need for a method of determining the porosity of an insulator without destroying it is almost too evident to be discussed. There is also a need for a standard of porosity. Various investigators speak of a percent porosity but make no statement as to how the value is to be obtained.

The temperature resistivity coefficient of porcelain varies with the temperature of the material and also with the impressed voltage and time of application of voltage. Its independent relation to porcelain for line insulators appears to be somewhat doubtful. The fact that a piece of porcelain has a high specific resistance under certain conditions does not necessarily imply that the porcelain is suitable for high-voltage insulators. The converse also appears to be true. Although it may be desirable for porcelain for line insulators to maintain an exceedingly high resistance at temperatures above 100 deg. cent., this seems to be improbable because there are no conditions of service where the insulators are operated at such a temperature.

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[1] "Boyd Elasticity and Strength of Porcelain and Stoneware." *Journal of the American Society of Mechanical Engineers*, March 1916.

[2] Bulletin 352. "Thermal Expansion of Insulating Material."

Mr. Peaslee's curves in Fig. 2 show a wide variation in the temperature coefficient of resistance for two samples of porcelain. It is impossible, however, to compare his values with values obtained by other investigators because unit values are not given and the conditions of measurement are not listed. I would be interested to know how the values were obtained, especially the data at 1700 deg. fahr.

The Piezo Electric effect, I believe was first discussed by C. Treischel[3] who says in part:

"Now then what happens when we subject this porphyritic structure to the influence of alternating high potential? The theory of probabilities will uphold the statement that a number of these quartz crystals are arranged so that their major axes are at right angles to the direction of the flow of the electric current. The current itself is changing its polarity in accordance with the cyclic change, and during each cyclic change there are points of maximum and zero potential difference. If the current is of 60 cycles there will be 120 maximum potential differences and 120 zero potential differences per second. The effect on each individual crystal, which is arranged with its axis in the right direction, will be a vibratory movement parallel to the direction of the flow of the displacement current. This effect could cause break down of the dielectric in three different manners: first, a rupturing of the crystals, second, a rupturing of the glassy magma surrounding the crystals; and third, a leakage through the voids between the crystals and the magma when the crystal is at its greatest deformation and the potential difference is at its maximum."

G. I. Gilchrest: I believe that Mr. Peaslee's paper indicates the problem to be more narrow than it is. I am commenting briefly on several portions of his paper.

The tension tests by Mr. Peaslee were made on test pieces with conical ends. Although it is possible to make tension tests on short specimens with conical ends, it is not very satisfactory, as it is difficult to obtain samples that will give uniform results. Various cushioning materials, such as lead, blotting paper, etc., will help to distribute the stress. However, from a consideration of test values obtained from bending tests, tension tests will only indicate about 30 to 50 per cent of the actual tensile strength of the material. Small bars of rectangular cross section tested under bending give the most uniform results.

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[3] "A Possible Cause for the Dielectric Failure of Porcelains which are apparently free from Mechanical Defects." *Journal of the American Ceramic Society* Vol. 2, No. 2, February 1919.

We recently obtained excellent comparative results from impact tests. The test pieces consisted of small cups having a diameter of 3 in., height of 2½ in., and thickness of wall of ½ in. The cups were placed on side in a V-shaped groove, a round nose plunger free to move in a vertical slide rested on the upper side of the cup. A small weight was dropped on the plunger the height of drop being increased by equal increments until the cups shattered. The tests pieces were easy to manufacture, grinding was not necessary, and the cups were convenient to test. The results are, of course, entirely comparative but where an extensive body investigation is being made, comparative and not quantitative results are desired.

Mr. Peaslee suggests that two bodies are known which can be heated red hot and plunged into water without cracking. Considerable experimental work has been done during the past few years to determine the best body compositions for porcelain used in electrical appliances. There are many bodies being manufactured at the present time that will stand being heated red hot and plunged into water. These bodies, however, are not vitrified to the same degree as the body of our high-tension porcelain insulators. The material is entirely satisfactory for its application, since its function is more that of a mechanical separater than dielectric separater.

Mr. Peaslee suggests detecting over-fired insulators by means of a gage. This method is not practical in an electrical porcelain factory. During firing, the body continues to shrink until it reaches the point of over-vitrification. When the piece is on the edge of over-vitrification, its dimensions are slightly less than when the piece is properly vitrified. When over-vitrification commences, the body starts to boil and the material becomes flaky. This condition can be readily detected by the eye.

The degree of porosity can be determined by means of a psycometer or by means of penetration of a colorant material. There is, however, a degree of porosity which, apparently, cannot be determined by any known laboratory method. Professor H. J. Ryan of Leland Stanford University obtained some very interesting results which are published in the TRANSACTIONS of the Institute for 1917. Porcelain insulators were found which the megger indicated as porous. These insulators could be dried so that the megger registered infinity. Thereafter, moisture could not be forced back into some of these insulators by any laboratory method tried by Professor Ryan.

Ceramic engineers who have had considerable experience in the production of electrical porcelain, feel that in order to avoid placing material improperly vitrified on the market, a careful check should be kept of the ingredients of the body. It is necessary to keep a careful check on each shipment of raw materials, a check on the prepared body and extremely close control of the firing temperatures. The proper temperature to vitrify the body sufficiently to not allow measurement of porosity should be determined. The temperature at which over-vitrification commences should be determined. The firing control must then be sufficiently close to lie between these two temperatures.

As suggested in Mr. Peaslee's conclusions, the ceramic engineer has his problems before him. He should, however, have the closest cooperation with electrical engineers, as a thorough appreciation of the application of the product is necessary.

W. D. A. Peaslee: I wish to indorse very strongly Mr. Vincent's remarks regarding co-operation between the manufacturer and the electrical engineer. It is very often found that the selection of insulators for a particular condition can be vastly aided by a careful study of operating conditions of lines working under similar conditions, but unfortunately, such operating records are seldom available. It is to be hoped that engineers will keep more and more complete records as time goes on.

With regard to Mr. Spurcks remarks as to the influence of the size and shape of test pieces on the units strength indicated, while it is true that there is a considerable influence, nevertheless, experience has shown that there is a very definite relation between such indications, and that this relation can be determined and relied upon to considerable accuracy.

As to the definition of porosity percentage and means of determining this percentage, it may be remarked that the American Society for Testing Materials, Committee on Electrical Insulating Material, is at present actively working on this very problem and I believe it will be of considerable benefit to the industry when these definitions and methods are properly defined.

The figures as to temperature coefficients of resistance are given here merely to show the degree of difference obtainable under identical conditions. The absolute values and precise methods are available but not for publication at present.

With regard to Mr. Gilchrist's statements as to tension tests and the difficulty of obtaining satisfactory results, I cannot agree with him at all, as we have obtained remarkably consistent and satisfactory results with equipment in the hands of untrained laboratory assistants and have not found the bar test in bending any more reliable than the properly conducted tension test.

With regard to the bodies that will withstand extreme temperature changes, I wish to correct Mr. Gilchrist's impression that these are not vitrified with the same degree as the body of his high-tension porcelain insulator. The bodies referred to are vitrified to the same degree, fired at just as high, if not higher, temperatures and make a superior insulating body to any that I have ever seen. The one difficulty in the employment of these bodies for insulators that would be superior to any on the market is the cost. Mr. Gilchrist has certainly received the impression that the body referred to here is a type such as is used for resistance elements, but this is not the case.

With regard to Mr. Gilchrist's remarks on gaging for over vitrification, I think this is one of the subjects to take up in the future to be standardized more fully, for when a ceramic engineer speaks of a vitrified body, unless his standard of vitrification is known, it is a little difficult to tell exactly what he means.

In closing I wish to urge upon all operating engineers the importance of keeping complete and accurate operating records of their insulators and when such records are available on a given type of insulator, these records should be made available to the manufacturer of that insulator. It is only by this sort of co-operation that we can, as insulator manufacturers, gain the most value from experience in the use of our product.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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THE USE OF REACTORS ON LARGE CENTRAL STATION SYSTEMS

(Introductory to paper on "The Stability of Operation of High-Power Generating Systems" by Dr. Steinmetz)

BY R. F. SCHUCHARDT
Commonwealth Edison Co., Chicago

THE use of current-limiting reactors on a large central station system is not a new subject as it is more than a decade since the need for such coils was first emphasized. The same is true with reference to reactors placed in tie lines between separate generating stations to improve the parallel operation of these stations. However, the literature on this subject bearing on experience with reactors is still somewhat limited and it is therefore thought to be of interest to give the experience of a large company having a capacity of upwards of half a million kilowatts, on which system reactors are used in part. This will be the more interesting because the occurrences which are here described have demonstrated the desirability of further installation of reactors.

The system in question is that of the Commonwealth Edison Company of Chicago. The principal supply of this company is obtained from three large power houses—namely, Fisk Street, Quarry Street, and Northwest Stations. In each of these there are both 25-cycle and 60-cycle turbo-generator units, but as the experience to be related in the following was had on the 25-cycle system, that system only will be considered in this paper.

The Fisk Street Station, with twelve 25-cycle units, is operated in two sections of approximately one-half of the total 25-cycle capacity of 180,000 kw. on each section. Quarry Street, with a 25-cycle capacity of 56,000 kw. in four units, is operated as one section, as is also Northwest Station with three 25-cycle units aggregating 85,000 kw. The generator and the bus-bar voltage is 9000 and all of the units are star-wound. Each of the units has an oil switch in the neutral, and this switch is closed on one of the operating units in each section. The neutral bus is connected to earth through a 2.5-ohm, non-inductive iron-grid resistance. The generators, which range from 12,000 to 35,000 kw. in capacity, are provided with reactors in the phase leads (with the exception of one 4500-volt, 20,000-kw. unit at Northwest Station which has an auto transformer to step up to 9000 volts). The total reactance of the various units, including the external coils, is from 11 to 14 percent, the higher figure applying to the larger units.

Quarry Street is connected to each of the Fisk Street sections through a short tie line of 750,000 cir. mil cross-section and each of these tie lines contains a reactor having 1.75 ohms reactance. The two Fisk Street sections were planned to have a reactor tie at the time the coils were placed in the Quarry tie lines in 1911 but for financial reasons at the time they were omitted and it was decided to operate in open ring for a while, the two sections being tied together solidly at light load periods when the capacity of the operating units was below 50,000 kw. A few years ago the coils for this tie connection were purchased but they were not actually installed until last year.

Northwest Station has a number of transmission lines which also connect to Section B of Fisk Street. Thus far no tie reactors have been installed in these connections. While it was appreciated that such reactors would ultimately be required for satisfactory parallel operation at times of abnormal condition, the decision to install them had been postponed until the ultimate capacity of Northwest Station, and with it the arrangement of transmission lines between Northwest and Fisk Street, was more nearly fixed. That time has now arrived. The postponement of this decision was made easy by the very satisfactory operation during normal conditions and even at times of mild disturbances.

Feeder reactors offer very desirable further protection and will also make the action of disconnective devices, *i. e.*, relays and switches, more satisfactory. However, after generator and bus sectionalizing reactors had been installed it was hoped that the very heavy expenditures necessary for the coils and their housing for the 150-odd lines in the existing stations could be avoided, but plans for a new station included provision for such feeder reactors.

Previous to last year, cable failures were generally at some distance from the station and the relays promptly disconnected lines when such failures occurred. The disturbance on the system was thus very slight. During the past year, however, there were several failures very close to the busbars and these had a fairly widespread effect. Judging from the nature of the disturbances it appears that these would probably have been very much minimized, if not entirely avoided, had there been reactors in the feeders and in those station ties which are not yet provided with them. It is because of this fact that these disturbances are here described.

The most serious disturbance occurred on September 18th, 1919. A few days later it was our good fortune to find Dr. Steinmetz in Chicago to address a convention in session at that time. Advantage was taken of the opportunity to ask Dr. Steinmetz to a conference with our engineers and as a result the Doctor was invited to make extended calculations to determine the best electrical dimensions of reactors for the system. As Dr. Steinmetz's paper which will follow this immediately, gives the detailed data with the discussion and the conclusions based on his study of the problems involved in this Chicago system, this present paper will be limited to a brief description of the operating conditions at the time of the disturbances—merely as an introductory statement to the valuable contribution of Dr. Steinmetz.

On May 19th, 1919, at 7:25 a. m., a burn-out occurred in a 12,000-kw. generator on Section A at

Fisk Street. (Fig. 1 shows a simplified diagram of the connections at the time). This unit is one of the earlier, vertical type, and is not provided with the usual balanced relay protection. It was, however, very promptly cleared from the system by the switchboard operator. Immediately an unstable condition resulted and lasted for eighteen minutes. The voltage charts, see Fig. 2, showed a constant swinging of the pressure during this time, the maximum being about 9100 volts and the minimum, except for the initial drop, about 7000 volts. The wattmeters and ammeters on tie lines *A* and *B* between Fisk and Quarry and on the lines from Fisk to Northwest swung continuously from one extreme to the other. The frequency varied from 24½ to 26½ cycles. Units No. 2 and No. 7 at Fisk Street tripped their throttles from overspeed. A number of converters in the various substations tripped out. At 7:43 a. m. the disturbance suddenly ceased and the system pressure rose to about 9400 volts but was immediately brought to normal.

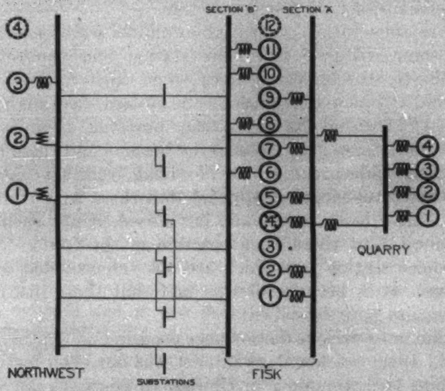

Fig. 1

This experience pointed to the need of more reactance in the tie connections between Fisk and Northwest Stations and also to the desirability of installing the Section A-B tie reactors which had been purchased some time previously but the actual installation of which had been postponed. Arrangements were started to install the *A-B* tie coils and, pending the determination of the size of coils for the Fisk-Northwest tie

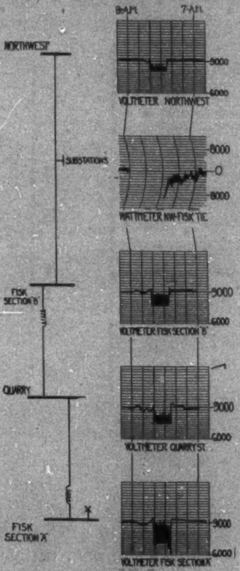

Fig. 2

lines, operating instructions were issued to promptly open these lines in the event of trouble producing a condition of instability.

On September 18th, 1919, at 3:47 p. m., when the 25-cycle system load was approximately 200,000 kw., a transmission line fed from Section B at Fisk Street broke down within 200 feet of the station busbars. Fig. 3 shows the diagrammatic location of the fault and a simplified diagram of the connections at the time. Because of the closeness to the busbars of this cable failure the voltage dropped to so low a point that all synchronous substation apparatus on Section B and on the Northwest Station (which connects directly with Section B as per description given above) dropped off and also some of the substation units on the Quarry Street section and on Section A of Fisk

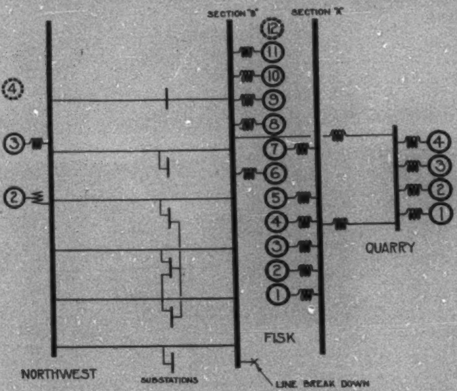

Fig. 3

Street, due largely to communication through the direct-current connections. These latter units, however, were quickly restored to service. The load thus being lost on Section B and on Northwest, all of the generating units involved speeded up and the emergency steam valve was tripped. Due to an incidental happening, the 25,000-kw. unit No. 11 operating on Section B was disconnected, leaving four of the 12,000-kw. vertical units on the *B* bus. These units as well as the two which had been in operation at Northwest Station were then out of synchronism with the units on those sections remaining normal; that is, Quarry Street and Section A of Fisk Street. Governor control was soon restored on these units but the out-of-synchronism condition continued for approximately thirteen minutes. After the first seven minutes had elapsed, the reactors on tie line *B* at Fisk Street showed evidence of excessive current flowing and the tie line meters swung from one end of the scale to the other in irregular beats. Thereupon tie line *B* switch was opened. Northwest Station and Section B at Fisk Street still remained out of synchronism, however,

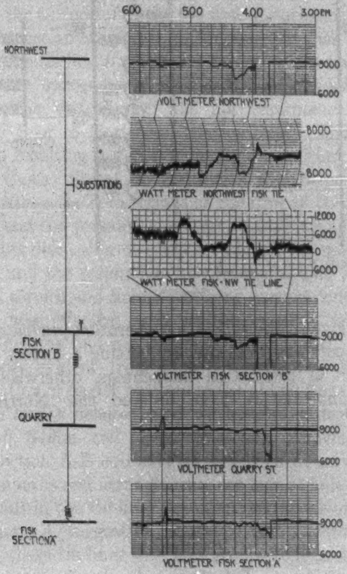

Fig. 4

and the pressure of Section B and Northwest buses remained near zero during the entire thirteen minutes. Fig. 4 shows the pressure charts for the period covering this and the succeeding disturbance.

A number of the lines at Fisk Street were still equipped with some of the earlier forms of overload relays and when this very severe shock due to the line breakdown came on, a number of these relays operated.

The lines opening as a result of this faulty relay operation together with the considerable direction neces-

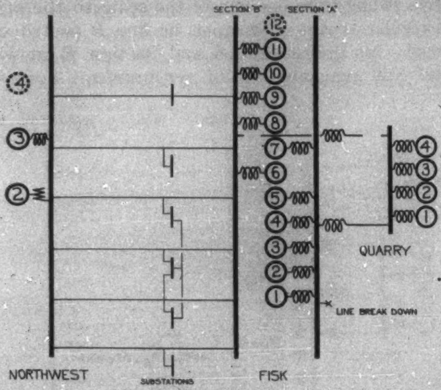
SECTION 'B'
SECTION 'A'
12
11
10
9
8
7
6
5
4
3
2
1
4
3
2
4
3
2
1
QUARRY
LINE BREAK DOWN
NORTHWEST
SUBSTATIONS
FISK

Fig. 5

sary to give to the substation operators prevented the load dispatchers from quickly getting a correct interpretation of the actual happenings, otherwise they would immediately have ordered the Northwest-

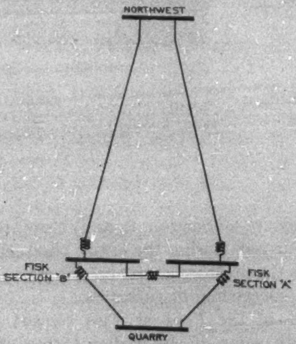
NORTHWEST
FISK SECTION 'B'
FISK SECTION 'A'
QUARRY

Fig. 6

Fisk Street tie lines opened up. The adjustment of the relays on these tie lines has now been so made that these lines will open automatically in the event of a similar disturbance.

While the restoration of service following the disturbance just described was in progress and getting on fairly well, another line broke down very close to the busbars. This line had carried an unusually heavy load throughout the day and an excessive load during the period following the first disturbance. The line in question was fed from Section A at Fisk Street, and as the tie line *B* to Quarry Street, which had been previously opened as above noted, had not yet been closed, the disturbance was not communicated to Section B or to Northwest Station lines. There was also no resulting out-of-synchronism condition of any of the Quarry Street or Section A Fisk Street units. Fig. 5 shows the connections at the time of this occurrence, and the pressure conditions are shown on Fig. 4.

In this discussion of station stability the secondary effects and the means adopted for continuing at least partial service and finally restoring normal conditions, which was effected early in the evening, have no direct bearing and are therefore not included.

It was evident from these disturbances that the installation of feeder reactors in the old stations should not be longer delayed, and we have now decided to use a 0. 5-ohm coil for all 9000-volt, 25-cycle lines. The Section A-B coil at Fisk has been in service for some months and arrangements are being made to provide coils in the tie lines between Fisk and Northwest Stations. A suggestion made by Dr. Steinmetz to divide these tie lines into two groups, one group connecting with Section A and the other with Section B at Fisk, has been adopted, so that the bus connections of the three stations will be a figure eight, that is, a double loop, as shown in Fig. 6.

We were glad to have the assistance of Dr. Steinmetz in calculating the proper amount of reactance for these lines and know that his paper containing the discussion of the problem will be of considerable value and interest to electrical engineers.

DISCUSSION ON "THE USE OF REACTORS ON LARGE CENTRAL STATION SYSTEMS (SCHUCHARDT), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

C. P. Steinmetz (read by R. E. Doherty): From the equations given in my paper on "Power Limitation and Stability of Electric Generating Stations," and the numerical constants of the machines and circuits, it is possible now to calculate approximate numerical values of the voltages and currents, which may be expected in the different parts of the system, during the troubles described by Mr. Schuchardt, under the various assumptions as to what has happened or may have happened, and by comparing these calculated values with the observed values, arrive at a fair conclusion of the events.

1. Trouble of September 17th 1919. A. Relation between Fisk Street B and Northwest Station. If these stations had remained in synchronism with each other, considerable voltage would have remained on the station busbars after the opening of the short circuit, no matter what happened. Since the voltage remained at zero for about a quarter of an hour, it must be assumed that these stations had broken synchronism with each other, and remained out of synchronism during this time. This is the more to be expected, as the connection between the two stations, through six cables, contains considerable resistance and almost no reactance, and the synchronizing power between these stations thus is very small.

Assuming thus:

(1) The two machines in Northwest Station are in synchronism with each other, and the four machines in Fisk Street B, also are in synchronism with each other, but Fisk Street B is out of synchronism with Northwest Station. This gives:

Terminal voltage:...........................	2550 volts
Current between stations, maximum effective value:..	6400 amperes
Max. pulsating power transfer, per phase:....	6150 kw.
Steady power transfer, per phase:.............	770 kw.
Critical slip, that is, limits of synchronizing power:..	1.47 per cent

These values do not well agree with the observations recorded, and while so many assumptions have to be made, that the exact numerical values cannot be relied upon too closely, nevertheless, the general magnitude of the numerical values can generally be relied upon.

It is probable that a terminal voltage of 2550, though below the scale of the recording meter, would not have escaped notice in the indicating meters. A slowly pulsating power transfer of 3 × 6150 = 18,450 kw. would have shown in the wattmeter record as a bad fluctuation, while the wattmeter record during this period is unusually steady, showing an apparent steady flow of about 1000 kw. only. A fluctuating interchange current between the stations, rising to 6400 amperes, would probably have shown marked distress in the cables.

It must therefore be assumed that the generators in the same stations did not stay in synchronism with each other. The behavior of the 20,000 and the 30,000-kw. machines in the Northwest Station is an indication in the same direction.

(2) With the 20,000 and the 30,000-kw. units of Northwest Station out of synchronism with each other on the busbars of this station, the following relations pertain:

Induced voltage:	1400 volts
Max. effective interchange current:	3200 amperes
Max. pulsating power transfer, per phase:	2230 kw.

Thus with the breaking of the synchronism between these machines, the voltage and the circulating current materially dropped, and with it also the current circulating between these machines and Fisk Street B station.

The 30,000-kw. machine was then shut down, leaving only the 20,000-kw. machine operating in the Northwest Station, drifting out of synchronism with Fisk Street B station.

(3) Assuming one 20,000-kw. machine in the Northwest Station, out of synchronism with the four 12,000-kw. machines in Fisk Street B Station, but the latter in synchronism with each other, gives:

Terminal voltage	1820 volts
Interchange current	3600 amperes
Fluctuating power per phase:	2750 kw.

While these values are much lower than those in (1), they are much larger than the values recorded. Furthermore, it is not probable that at this terminal voltage the machines in Fisk Street B Station would remain in synchronism with each other, under existing conditions of steam supply.

Assuming then that not only the two stations, but also the individual machines in either station had broken synchronism with each other, and were drifting past each other, the magnitude of voltages, currents and power is:

(4) Two 12,000-kw. alternators in Fisk Street B Station out of synchronism with each other:

Induced voltage, maximum effective:...........	2700 volts
Current:..	1820 amperes
Fluctuating power per phase:...................	1410 kw.

(5) The one 20,000-kw. alternator in the Northwest Station, and one 12,000-kw. alternator in the Fisk B Station:

Terminal voltage:................................	1640 volts
Current:..	2150 amperes
Fluctuating power per phase:...................	1690 kw.

This means, with all the alternators in Fisk B and in Northwest Station out of synchronism with each other, the terminal voltage will be between 1640 and nothing. The interchange current circulating between them will reach maximum amplitudes not exceeding 1820 to 2150 amperes effective. The I^2R loss in the tie cables then would fluctuate between maximum values of 1500 to 2200 kw. and nothing, with a probable average of about 900 kw. This agrees with the observation that the wattmeter in the tie lines was very steady showing about 1000 kw. and the voltage was inappreciable.

The conclusion therefore is inevitable, that in this trouble, not only the Northwest Station and the Fisk Street B station had broken synchronism with each other, but that the individual generators in the Fisk Street B Station and in the Northwest Station had broken synchronisms with each other also, and were drifting past each other out of synchronism.

The important question then is, what caused these alternators and stations to break synchronism.

As the tie cables which connect the busbars of the Northwest Station and the Fisk Street B Station with each other, contain appreciable resistance but practically no reactance, and the synchronizing power depends on the reactance, there can be only very little synchronizing power between these stations, and the voltage drop due to the short circuit in Fisk Street B Station caused the loss of synchronism with the Northwest station.

The alternators in Fisk Street B, have considerable reactance, and no resistance between each other. The reason for their breaking synchronism with each other must be found in the great drop of voltage resulting first from the short circuit and then from the break of synchronism between Fisk Street B and Northwest Station. The synchronizing power is proportional to the square of the voltage. With Fisk Street B and Northwest Station out of synchronism with each other but the individual alternators, in either station still in synchronism with each other, (Case 1), the actual induced voltage in these two stations drops to an average of $E_o = 1910$ volts per phase. The synchronizing power between two machines, for instance, two of the 12,000-kw. alternators of Fisk Street B, reaches a maximum at a phase displacement between two machines of 90 degrees, and then is:

$$p = \frac{E_o}{z} = 2100 \text{ kw. per phase}$$

or a total of 6300 kw. or about half load, and still less after the 30,000-kw. unit had been shut down.

Thus, if the load is suddenly released on these machines, unless the steam supply can be reduced almost instantly to less than half load, these machines will be torn out of synchronism with each other. But by breaking synchronism between machines in the same stations—as the 20,000 and the 30,000 in the Northwest Station—the voltage is still further lowered and thereby the synchronizing power reduced, so that, if one machine breaks out of synchronism under these circumstances, all will break. This must have happened on September 18th, 1919. In other words, the break of synchronism between Fisk Street B and Northwest Station lowered the voltage so that there was not sufficient synchronizing power left to keep the individual machines in each station in synchronism with each other.

B. Relations between Fisk Street B Station and Fisk Street A and Quarry Street Stations. Four 12,000-kw. alternators in Fisk B, out of synchronism over a power limiting reactor with three 14,000-kw. alternators in Quarry Street, the latter in synchronism with each other, and over a second power limiting reactor, with six 12,000-kw. alternators in Fisk A, gives:

Terminal voltage:...........................	6600 volts
Interchange current:........................	3900 amperes
Power fluctuation per phase:.................	8300 kw.

As seen, the calculated value of the terminal voltage of Quarry Street, 6600, agrees as closely with the observed value of 6800 volts as can be expected from such approximated calculations, especially when considering that some synchronous machines had been lost by Quarry Street in the substations, and the load thereby reduced, which would result in an increase of voltage.

As the total impedance of Fisk Street A is about 1.1, and it is connected to Quarry Street by a reactor of 1.75 ohms, the voltage of Fisk Street A should be higher than Quarry Street in the proportion of 1.1 to 1.1

+ 1.75. This gives for Fisk Street A a terminal voltage of 8100 volts. This well agrees with the average of the voltage record of Fisk Street A during the first seven minutes of the disturbance.

Allowing for the continuously low voltage maintained at Fisk Street B by the out of synchronism condition with Northwest Station, the fluctuating current over the reactor B between Fisk Street B and Quarry Street would vary approximately between 1200 and 2700 amperes. This would give an estimated final temperature rise of 600 to 800 deg. cent., so that it should be expected that after seven minutes, when this reactor was disconnected, it would be very hot. The reactor *A*, between Quarry Street and Fisk Street A, should carry a current of 500 to 1000 amperes, thus would not become heated.

The conclusion then is: As the calculated numerical values agree with the records of observation as closely as can be expected from such necessarily approximated calculations, it appears safe to accept the correctness of the explanation that—

Fisk Street A and Quarry Street remained in synchronism with each other, but Fisk Street B and Northwest Station broke out of synchronism with Quarry Street and with each other, and the individual machines of these latter two stations broke out of synchronism with each other, and were unable to pull into step, due to frequency differences greater than permissible by the smaller synchronizing power existing between these machines at the low voltage existing due to the break of synchronism between the stations.

II. Troubles of May 19th, 1919. Assuming first that all the four stations had dropped out of synchronism with each other, due to the trouble in Fisk Street A Station, and were drifting past each other. This would give the terminal voltages recorded in the following table. Comparing these with the observed terminal voltages, which also are recorded in the table, it is seen that the calculated values are much lower than the observed values. More particularly, at the low terminal voltage of 2650, in Fisk Street B and Northwest Stations, under the conditions existing, the individual machines would not stay in synchronism with each other, but would break out of synchronism. The relatively high terminal voltage recorded in these two stations shows however, that the individual machines had remained in synchronism with each other, and if this was the case, then the two stations, Fisk B Street and Northwest Station, also must have remained in synchronism with each other.

Considering thus the second assumption: Fisk Street B Station in synchronism with the Northwest Station, but out of synchronism with the Quarry Street Station, and the Quarry Street Station out of synchronism with the Fisk Street A Station. This gives the calculated terminal voltages recorded in the 3rd line of the following table.

TABLE OF TERMINAL VOLTAGES

	Fisk St. A.	Quarry St.	Fisk St. B.	Northwest
Observed values:	7800 to 8200	8000 to 8300	8300 to 8400	8400 to 8500
All 4 stations out of synchronism:	6100	5400	2650	2650
Only Fisk B and Northwest in synchronism:	6100	5400	7350	7350

As seen, the calculated values are uniformly very much lower than the observed values, and while no very great accuracy can be attributed to such approximated calculations, the difference is too great to be accounted for, and it must therefore be concluded that in this trouble none of the stations had broken synchronism, but all the four staions had kept in synchronism with each other.

Assuming then as reasonable, that in the trouble of May 19th, 1919, the four station sections had kept in synchronism, the question remains to account for the great drop and fluctuations of voltage, which were greatest at the source of the trouble, Fisk Street A, and decreased towards the other end of the station chain whether due to the hunting of the stations against each other, or due to excessive load of lagging current, caused by starting of synchronous machines or due to some other cause.

a. Assuming that the stations hunted against each other. From the observed voltage drop of the stations then can be calculated the approximate value of the surging current between the stations, the phase angle or amplitude of the hunting, etc.

b. Assuming that the fluctuating voltage drop was due to the starting of numerous synchronous machines. From the total amount of synchronous machine load lost in the trouble, can be estimated their starting current and its reaction on the generating system. As the substation operators would start their converters as soon as the voltage comes up, a number of successive voltage drops and recoveries would be recorded, giving the appearance of hunting.

Either of these two assumptions gives values which are reasonable and in accordance with the observed data. A decision, whether it was real hunting, or the reaction of the generating system on excessive lagging load could only be made by knowing the phase angles between the voltages of different stations, and their variation during the disturbance. That is, phase indicators, showing and preferably recording at any moment the phase angle of the station voltage against the voltage of any other station connected into the system, would therefor be desirable in the operation of such high power systems.

Alex. E. Bauhan: I notice in Mr. Schuchardt's paper, that when some of the machines fell out of synchronism, they dropped out on the steam end. It is unfortunate that the emergency trips must be set so low that machines will trip out on the steam end when they fall out of step. In Baltimore some steam turbines which had their emergency trips set at 12 per cent, on various occasions fell out of step due to the same cause as in Chicago, but remained in on the steam end. Some other turbines on which the emergency trips had been set down to 6 per cent, would repeatedly trip out on the steam end whenever they fell out of step, necessitating an additional delay of about two minutes in restoring service. This is a point which I think should be kept in mind in setting emergency trips and in the design of turbines.

Robert A. Hentz: It may be of interest to outline what is being done in Philadelphia to minimize the injurious effects of severe short circuits by the use of current-limiting reactors.

We operate two systems, 25 and 60 cycles, with a total capacity of about 300,000 kv-a., most of which is located in three large generating stations, "Schuylkill", "Chester", and "Delaware", the latter two of which are exclusively 60-cycle plants. The 25-cycle machines whose capacity is about 25 per cent of the above total, are all located in the Schuylkill Station and are tied in with the 60-cycle system through 22,500 kv-a. of frequency-changer capacity. The generation and transmission of both systems is 13,200 volts, three phase.

In the case of the 60-cycle system where the disturbances are more serious than in the other system, the practise is to provide reactance in the generators themselves, between bus sections, on the lines between generating stations and on outgoing transmission feeders. The generator reactance is provided entirely within the machines themselves, no external reactors being required. The capacity of the bus sections which are separated by bus reactors, is in every case limited to 66,666 kv-a., although in the new Delaware Station we are arranging to initially operate as high as 100,000 kv-a. on one bus section. The feeder reactors are rated at 3 per cent at 6800 kv-a. in all circuits, except in the case of certain older transmission lines of smaller than 350,000 cir. mils capacity, and in the case of tie lines between generating stations where the proper power transfer and other considerations indicate a lower value to be advisable.

The 350,000 cir. mils circuits are rated at 6000 kv-a. and the additional 800 kv-a. in the reactor capacity is a factor of safety, which is felt to be advisable.

The arrangement of the 25-cycle system is very similar, excepting that its smaller capacity has not made it necessary for us to install any bus reactors as yet. The generators are provided with external reactors as sufficient reactance could not be obtained within the machines themselves. The feeder reactors are, as on the 60-cycle system, 3 per cent.

Reactors installed on feeders are of greater benefit in minimizing the damaging effects of short circuits than those installed elsewhere, as they isolate the severe voltage disturbances resulting from trouble beyond them to such an extent that other feeders are but slightly effected. Furthermore, a given percentage reactance voltage drop installed on a 6000-kv-a. feeder will be five times as effective as the same percentage on a 30,000-kv-a. generator. Consequently, this use of reactors is considered to be most advantageous.

D. C. Jackson: Referring to Dr. Steinmetz's discussion we all know that if the voltage falls to zero at the busbars, the generators will have no synchronizing power on each other, and if the prime movers are not set so that they run at exactly the same speeds under different loads, they will probably fall out of step far enough so they can not bring themselves back into synchronism if the reduction of voltage lasts more than a short time. That is fundamental.

It is rather startling, however, to find that Dr. Steinmetz is suggesting in his paper that the large machines under consideration in this case will not hold themselves in synchronism even provided the voltage does not fall to less than 50 per cent of normal. Now, of course, such a drop on a great power system is unusual but it cannot be unexpected with present practise. We also must recognize that the synchronizing power of the machines depends upon the structure of the machines and likewise upon the impedance and the ratio of reactance to resistance of the other parts of the armature loop from busbar to busbar.

If 50 per cent, or less than 50 per cent of drop in the voltage is going to disturb the synchronizing power of the big generators sufficiently so that they may not be expected to hold in step, then it is desirable in the design of stations to give a great deal of consideration to that question of the reactance in the layout, and the effect of the introduction of reactors thereon.

Of course, with the big machines reactors are necessary, at least as far as our present light goes, but are they at times going to introduce more difficulties than they save? The question then is: Are the machines adequately designed which will perhaps drop out of control of each other with 50 per cent or less drop of voltage at the busbars? I think that is a pretty important matter for every central station engineer and every generator designer to have in his mind.

Philip Torchio: I want to say, first, that in regard to the efficiency of bus-tie reactors, or rather inter-station reactors, we can give, in our system in New York, the proof of their efficacy for at least eight years in the operation of two stations of about 150,000 kv-a. each, inter-connected, with five per cent 1,000-ampere reactors. We have had several cases of bus short circuits. In one case, two men went, under the instruction of the switchboard man, to take off the ground on a switch on a feeder that had been under repairs and came back with the report that the ground was removed, but when the operator closed the switch the ground was still there and made a dead short circuit on the bus.

The switches on the ties between the stations opened and the rotary converters on the unaffected station continued in operation regularly, so that shortly the system was put in regular function. We have had several similar cases, which absolutely prove that the reactors between the stations, with the proper relay protection, will be effective.

Prof. Jackson and one other speaker questioned the necessity of reactors on modern generators.

We have to recognize that we must have reactors to reduce the kv-a. in a short circuit when the switches are limited in their rupturing capacity. Therefore, we introduce the reactors to protect the switch, to bring the current within the breaking capacity of the switch. Now, if we want to protect the system bus against short circuits on the generators themselves, we must consider the rupturing capacity of the switch and the use of reactors. The same reasons which compel the use of feeder reactors require reactors for the generators on a large system, where the bus may give several hundred thousand kv-a.; otherwise, you will shut down the system, so that the purpose of the generator reactors is not to protect the generator but to protect the bus and the system, the same as the purpose of the feeder reactors is to protect the system.

James Lyman: The Commonwealth Edison Company was, I believe, the first large company to install current limiting reactors in their power stations. These were connected in the leads of their 25-cycle generators and in the tie lines between power stations.

Professor Jackson has referred to a paper entitled—"Protective Reactances for Feeder Circuits of Large City Power Systems," by Messrs. Lyman, Perry and Rossman, read before the Institute October, 1914. The same authors presented a paper February 1914 on "Protective Reactances in Large Power Stations." These papers were the result of a study of conditions incident to the design of the West End Power Station and distribution system at Cincinnati.

To keep within the rupturing capacity of the largest oil circuit breakers manufactured by the G. E. Company and the W. E. & M. Co. we were obliged to provide current limiting reactors in the main ring bus between adjacent generators and also on all feeder circuits. Each bus reactor is provided with a short-circuiting oil circuit breaker which is interlocked with the circuit breaker of its adjacent generator, so that when the generator circuit breaker is closed the short-circuiting reactor breaker is automatically opened and vice versa. Thus there is never more than one bus reactor in circuit between any two running generators.

Three per cent feeder reactors materially reduce the flow of the circuit, in case of a dead short circuit. Five per cent bus reactors, based on the current rating of one generator, limit the current flow under any condition of short circuit in generator or bus to the guaranteed capacity of the oil circuit breakers manufactured by either of the above companies when the ultimate eight generators are installed. Reactors similar to those at Cincinnati have been installed in the Windsor Power Station, Windsor, W. Va., and in the Northeast Power Station at Kansas City, Mo. All of these stations are designed for an ultimate capacity of from 200,000 to 250,000 kv.

The generators have an inherent reactance of from 12 to 16 per cent and require no reactors in their leads. It is interesting now, after these stations have been in operation from one to four years, to report that with the many incidental short circuits that necessarily occur in any large distribution system, the reactors have shown their value in limiting the current, and that in most instances of short circuits the bus voltage has not been seriously affected. The feeder circuit breakers act promptly and with only a momentary voltage fluctuation. We shall, no doubt, continue the installation of feeder reactors and bus reactors in power stations having generators of from 20,000 to 30,000 kw. capacity.

C. J. Holslag: The practical results of introducing reactors in bus lines and feeders may be existence of voltages at points and times not ordinarily encountered. For instance, it is common practise for the workman to ground a line he is repairing, the line being tagged by the station operator. In the case of reactors in this line he should ground it between himself and the station. In the case of bus lines he should ground on both sides of himself, to prevent a high voltage existing where he is working, if the line should be accidentally made alive. Perhaps a better method than the indefinite grounding by the workman, would be deliberate grounding through the use of double pole switches, at the point the feeder is disconnected at the station. The point to be here brought out is that grounding both sides of the location where work is to be done, is necessary.

B. G. Jamieson: I would like to call attention to the application of reactors beyond their use in generating stations. I refer to the case of a customer having a load of from 10,000 to 20,000 kw. In such cases, there are so many parallel ties issuing from the bus or from the busses of several stations and substations, that it becomes necessary to protect that customer from losing his bus pressure in the event of cable failures, just as it becomes necessary to prevent the bus system proper at the generating station from being lowered. Unless we get relays and switches for removing the faults before they become sufficiently great to cause a drop in voltage it suggests reactors in substations or switching centers on the premises of large customers.

Our own transmission system has become largely a distribution system, and it is very easy to see that perhaps reactors may have to be applied in considerable numbers at points outside the generating station, wherever we wish to preserve the integrity of the generating (or distribution) pressure.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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POWER CONTROL AND STABILITY OF ELECTRIC GENERATING STATIONS

By CHARLES P. STEINMETZ
Chief Consulting Engineer, General Electric Co.

WITH the increasing use of electric power, the size of electric generating systems has steadily increased, from the small electric lighting stations of the early days, with two or three, 30- or 60- kw. high-frequency alternators, to the huge metropolitan systems, with several hundred thousand kilowatts of steam turbine alternators.

The problem of close inherent regulation of the generators, that is, of constancy of voltage under sudden changes of load, has ceased, since no possible sudden change of load—short of short circuit—is sufficient to appreciably affect the voltage of these big systems. The reverse problem however has become serious, that of limiting the power which can accidentally be concentrated at any point of the system, and its destructiveness.

With the increasing size and extent of systems, they were divided into a number of generating stations, more economically to cover the territory, as under present conditions there seemed to be no material gain in going much over 100,000 kw. in one station. Thus usually two to four or more main generating stations are used, and a number of smaller secondary generating stations to stabilize the power at the end of long feeders, in outlying centers of distribution, etc.

Economy and reliability of operation demands parallel operation of the entire system, and synchronous operation of all the generating stations is thus the universal custom.

In the former 250-volt direct-current generating systems, from which most of the large metropolitan systems have developed, sub-division in a number of generating stations limited the power which could be developed at any point, and thereby its destructiveness, by the resistance of feeders and mains. In the present three-phase systems, interconnected by and distributing through underground cables, at 6600 to 22,000 volts, the impedance of these cables is entirely insufficient to limit the power concentration possible at any point of the system, and special means of limiting the possible power concentration in these systems thus became necessary. This problem became aggravated by the inherent characteristics of high-speed steam turbine alternators, which have completely superseded the former low-speed engine driven machines.

In the belt driven 60 cycle alternators of former days, the output was from 15 to 30 kw. per machine pole; in the 25-cycle slow-speed engine driven multipolar alternators such as were installed in the first Metropolitan Railway station of New York City, etc., the output was about 100 to 125 kw. per machine pole, while in the modern high-speed steam turbine alternator values of 15,000 to 20,000 kw. per machine pole have become necessary. This meant enormously larger magnetic fluxes and correspondingly larger armature reactions per pole. But with increasing output per pole, the effective or equivalent reactance of armature reaction (which is not instantaneous, but requires several seconds to develop) increases at a faster rate than the true or self-inductive reactance of the armature (which latter is instantaneous, and thus the only reactance which limits the momentary short-circuit current of the machine). Thus, while in the early high-frequency alternators the ratio of effective reactance of armature reaction, to true self-inductive armature reactance, was less than 0.5 to 1, it has risen in the large low-frequency turbo alternators to values of 20 to 1, and more. That is, while in the early high-frequency turbo alternators the momentary short-circuit current was very little larger than the permanent short-circuit current, in large low-frequency turbo alternators the momentary short-circuit current may be 20 or more times the permanent short-circuit current. Thus in a high-power system of several hundred thousand kilowatt connected steam turbine generator capacity, without power limiting devices, the momentary short-circuit current may represent several million kilovolt-amperes, with corresponding electrical, thermal and magnetic stresses. It is not the question whether a circuit breaker can be designed at all to open safely such power, but it is the fact that such a circuit breaker would in size and cost be economically impracticable, when considering that with the hundreds of feeder cables and interconnecting cables of such systems, several hundreds of such circuit breakers would be required. The practise of giving the circuit breakers a considerable time limit so that they open only after the momentary short-circuit current has greatly decreased, greatly relieves the circuit breakers, but at the expense of the system which is exposed to the full momentary short-circuit stresses, and usually shut down. The use of group circuit breakers in series to the circuit breakers in the individual feeders (and usually of larger interrupting capacity than the latter) reduces the number of high-power circuit breakers required and increases the reliability, by having two circuit-interrupting devices in series, and thus is extensively used, but by itself does not solve the problem, as the large number of group circuit breakers places an economic limit on their interrupting capacity, and the required time limit of their operation leaves the system exposed to the full destructive effect of the momentary short circuit.

Thus power limiting devices have become necessary and are universally used in all modern high-power systems, in some form or another. Such power limiting devices comprise:

1. *Power-Limiting Generator Reactors.* Besides designing the generator for the highest possible internal self-inductive reactance which can be given to it without serious sacrifice of its other characteristics, reactors are inserted into the leads between generator and busbars, so as to limit the power, which the generator can feed into the busbars in case of short circuit at or near the busbars, and to limit the power which the busbars can feed back into the generator in case of accident to the generator.

Such power-limiting generator reactors in some form or another are used wherever the internal self-inductive reactance cannot be made sufficiently high (10 to 15 per cent). The latter frequently is the case with 60-cycle machines.

Internal reactance of the generator, wherever it can be secured without material sacrifice of its other characteristics, has the advantage of saving the space and cost of the external reactance, but is not quite as good in protective value, since in case of an accident in the generator, its internal reactance is more or less eliminated, and thus does not protect against the busbars feeding back into the generator.

The amount of generator power-limiting reactance necessarily is limited to that value which does not materially increase the total (or synchronous) reactance, of the generator. Thus, with many generators running in parallel in the system, even with the power limitation of the individual generators, the total power which may be developed in case of a short circuit on or near the busbars becomes excessive. The economic limit of generator power, which may be concentrated on one busbar, probably is between 50,000 and 100,000 kw. Beyond this, it becomes necessary to cut, or divide the busbars, and since parallel operation is necessary, this may be done by:

2. *Power-Limiting Busbar Reactors.* These are reactors inserted into the busbars so as to limit the power which can flow along the busbars from one side to the other side of the reactor, without interfering with the flow of such current along the busbars, as may be required under the variations of load, for synchronous operation, etc.

Economically, the busbars are naturally arranged so as to require the minimum average flow of power along the busbars. That is, the feeders which carry power from the busbars to the load intermingle with the leads which bring power from the generators to the busbars. The power flowing along the busbars thus is the difference between the incoming and the outflowing power. Theoretically, with a ring bus, cut into sections by power limiting reactors, the maximum power which may have to flow over any busbar reactor, is one-quarter that of the smallest alternator connected to the section adjoining the reactor, and may rise to twice as much, if the busbar sections are not connected into a closed ring, but into an open chain.

The transfer of power from one busbar section to another, over the dividing reactance, does not mean a drop of voltage; but with the same voltage on two busbar sections the power transfer occurs by a phase displacement between the voltages of the two sections. That is, if the load on one busbar section increases beyond the output of the generators connected to it, or decreases below it, power begins to flow over the busbar reactors connecting it with the adjoining sections. The voltages of the adjoining sections however are kept constant by the control of the alternator field excitation, at the same value e, and the reactance voltage $i x$ of the current i, passing over the busbar reactance x, thus forms an equilateral triangle with the two voltages e of the adjoining busbar sections. (See Fig. 1.) That is, $i x$ is approximately in quadrature with the section voltages e, and as $i x$, as reactance voltage, is in quadrature with the current i, the current i is (approximately) in phase with the generator voltages e, that is, it is an energy current. The phase angle $2\ \omega$ between the two voltages e of the two adjoining busbar sections then is given by—

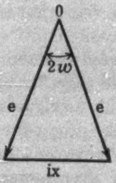

FIG. 1

$$\sin \omega = \frac{ix}{2e}$$

As the synchronizing power between the adjoining generator sections is a maximum for $2\,\omega = 90$ deg., and decreases beyond this, no danger of breaking out of synchronism exists, as long as $2\,\omega$ is materially less than 90 deg. Thus with a phase angle between the generator section voltages e, of $2\,\omega = 30$ deg., that is, fairly small phase displacement, it is:

$$\frac{i\,x}{2\,e} = \sin 15° = 0.26,$$

or

$$\frac{i\,x}{e} = 0.52$$

As theoretically i may be limited to one-quarter of the full-load current i_0 of the smallest generator on the section, it would be:

$$\frac{i_0 x}{e} = 2.08$$

that is, the maximum theoretically permissible busbar reactance, at a maximum of 30 deg. phase displacement between the busbar sections, would be 200 per cent, referred to the smallest generator on the section, as far as energy transfer from section to section, with negligible phase displacement — 15 degrees—is concerned.

As the power-limiting generator reactances were 10 to 15 per cent, or an average of 12.5 per cent, it is seen that much larger reactances may safely be used in power limiting busbar reactors than are permissible in power-limiting generator reactors.

It is advisable to use as large busbar reactances as possible, to limit the shock of a short circuit at or near a busbar section as much as possible to this section, that is, to affect the rest of the system as little as possible.

Where a number of stations are connected together, operating into the same system, that is, tied together by interconnecting cables into one bus, preferably a ring bus, it is advisable as far as possible to locate the power-limiting busbar reactors in the connections between the stations, that is, tie the stations together over power-limiting reactors. In this case it is advisable to install one-half of each of the busbar reactors at each end of the interconnecting cable, since the probability of short circuits in the interconnecting cables is far greater than the probability of short circuits at the busbars, and the division of the reactor into one half at each end of the cable, limits the effect of a short circuit in this cable on the generating stations connected together by it.

3. *Feeder Reactors*. Even with generator power-limiting reactors, and busbar-dividing reactors, the effect of a short circuit at or near the busbars is very severe, at least on that section of the system operated from this busbar, and will probably shut down this section. However, short circuits on the busbars are very much less frequent than short circuits in cables. The installation of proper feeder power-limiting reactors, by eliminating the short circuits on feeders, even when occurring very

near the busbars, from directly affecting the busbars, thus eliminates most of the severe short-circuit shocks from the generator sections, and is therefore economically very desirable. While the reactance of the feeder reactor may be only a small percentage of the feeder rating, it usually is very much larger than the combined reactance of the generators feeding into the sections, and the short circuit beyond even a small (in percentage) feeder reactor thus involves a very much smaller short-circuit current than would occur without the feeder reactor, and thus very greatly reduces the shock. Furthermore, without the feeder reactor, a short circuit in a cable near the busbars means zero voltage (or practically so) at the busbars, that means dropping out of synchronous apparatus (generators, synchronous motors, converters, etc.). With a short circuit beyond a feeder reactor, however, considerable voltage is retained at the busbars on the affected generating station, so that synchronous apparatus is not affected, that is, such a short circuit passes without material effect on the system, especially if the circuit breakers are set with short time-limit, which is permissible due to the greatly reduced current which they have to open.

By the proper use of power-limiting reactors in generator leads, busbars, and feeders it has become possible to operate the modern huge power systems with a high degree of safety, by limiting the maximum power concentration which can, in cases of accident, occur at any point of the system, and to give the possibility of unlimited extension of the system; that is, a power system of several million kilowatts of connected generator capacity will be just as safe in the limitation of the possible destructiveness of short circuits and other accidents, as a system of less than 100,000 kw. generator capacity.

When thus sectionalizing the system in installing reactors between the generators, stations, or station sections, these reactors are very low in reactance, absolutely (of the magnitude of an ohm), and thus permit ample current to flow over them for all requirements of the shifting load, without giving appreciable voltage drop or phase displacement between the sections. But relative to the station capacity they are, and must be very high to fulfill their function, in power limitation. Thus a reactor of 1.75 ohms reactance, connecting a 9000-volt station section of 72,000-kw. generator capacity, passes a maximum, at the limits of synchronizing power, of 45,000 kw. of energy, that is, materially less than the rated generator capacity.

The question then arises, what effect this necessary sectionalizing of the system by reactors has on the synchronizing power of the system and thus on the stability of operation, the more so as in case of accidents or disturbances a local and temporary drop of voltage may occur, and a corresponding decrease of synchronizing power.

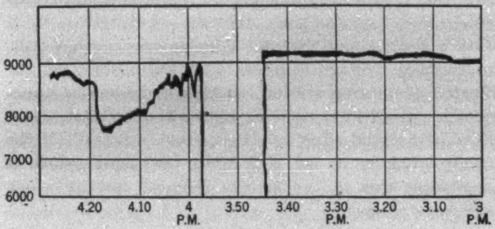

FIG. 2

In illustration of this, Figs. 2 to 5 show the voltage record during a trouble on September 18, 1919, in the Commonwealth Edison Company in Chicago, taken from

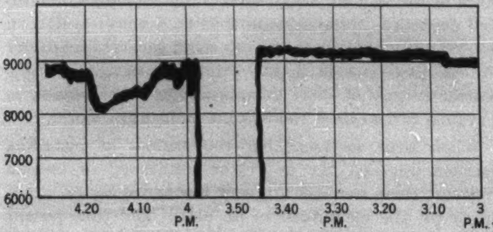

FIG. 3

Mr. Schuchardt's paper. Fig. 6, gives the diagram of station connections. The system consisted of four sections, *A*, *B*, *C*, and *D*, interconnected in chain connection, from *A* to *C* and from *C* to *B* by power-limiting reactors of 1.75 ohms per phase; from *B* to *D* by six underground cables of 0.31 ohm joint resistance and 0.074 ohm reactance per phase. Busbar voltage, 9000; load almost entirely 25-cycle synchronous converters. Connected generator capacity during the trouble: 237,000 kw., nearly full load. A dead short circuit close to the busbars of section *B* dropped out the converters on sections *B* and *D*, and some converters on sections *A* and *C*: the circuit breakers in the substations opened promptly and cut off the substations, and the short circuit was opened in a very few seconds, so that the system was clear again in three to four seconds, and the voltage should have come back. But it did not come back, but stayed at zero in both stations *B* and *D* (Figs. 2 and 3)*, and showed a permanent great drop in *C* station (Fig. 4), and a lesser drop in *A* station (Fig. 5). Interesting also

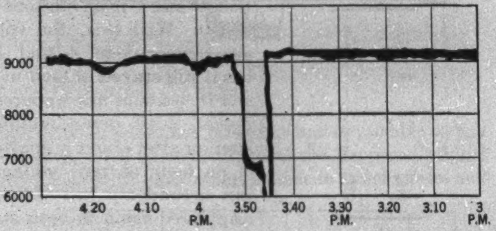

FIG. 4

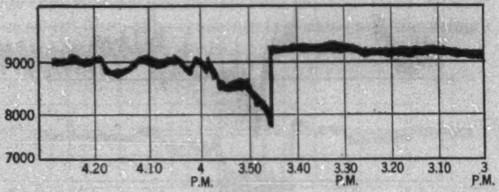

FIG. 5

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*While the charts do not read below 6000 volts, the station voltmeter showed that there was no appreciable voltage during the entire period.

is the wattmeter record of the power exchange between stations over the tie cables between *B* and *D* (Fig. 7): while usually considerable, practically no power or current exchange occurred during the trouble. An excessive current however flowed over the power-limiting reactor between *B* and *C*. This reactor was opened after 7 minutes, thus cutting off stations *A* and *C* from stations *B* and *D*. With this, the voltage recovered in *A* and *B*, but it still stayed at zero in *B* and *D*, without any apparent reason, until seven minutes later, or after about a quarter of an hour of zero voltage, just as suddenly full voltage reappeared again in both stations *B* and *D*, without any apparent reason.

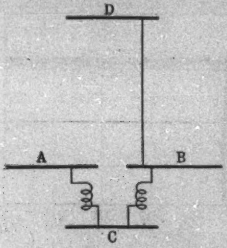

Fig. 6

What happened in this case was, as the investigation showed, that under short circuit the stations *B* and *D* momentarily dropped to zero voltage and lost their

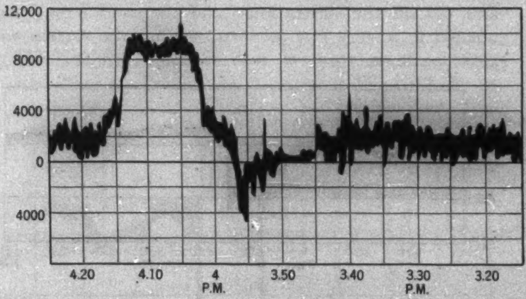

Fig. 7

synchronizing power. The steam turbines speeded up, cut off steam by closing their emergency valves, but were put back on the steam governors; their speeds however were already too far apart to pull each other into step promptly, and while the unaffected stations *A* and *C* stayed in step with each other, the stations *B* and *D* not only broke out of synchronism with each other and with *A* and *C*, but the individual machines in *B* and *D* broke out of synchronism with each other. The stations *B* and *D*, and the individual machines in these stations then kept drifting past each other indefinitely, unable to pull into step, until some of the machines happened to drift into phase with each other, caught in synchronism and thereby established some voltage, and then quickly pulled all the other machines into step, and the voltage then came back suddenly.

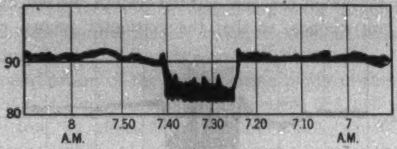

Fig. 8

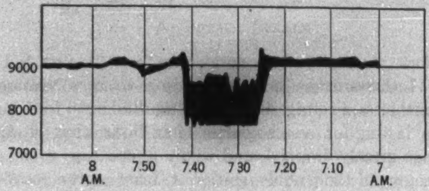

Fig. 9

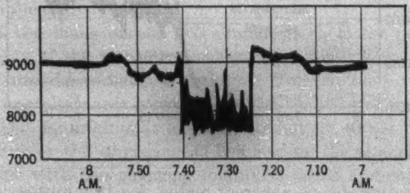

Fig. 10

Figs. 8 to 11 show the voltage records of the same four stations during a trouble on May 19th, 1919, and Fig. 12 the wattmeter record of the tie cables between *B* and *D*. The station arrangement was the same, the connected generator capacity 250,000 kw.—about two-thirds load.

In this case, a generator short-circuited in section *A*, pulling the voltage down to practically zero, but was cut off by the circuit breakers and the system cleared in less than two seconds, so that the voltmeter record of *A*

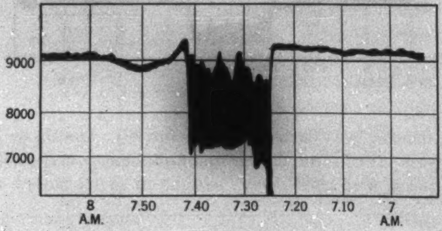

Fig. 11

Fig. 11 shows only a momentary drop to zero voltage. Nevertheless, a voltage disturbance resulted in all four stations, lasting for over a quarter of an hour; that is, the voltage greatly dropped, and wildly fluctuated; most at the source of the trouble, station *A*; least at the remote end, in Station *D*, and the voltage remained low and

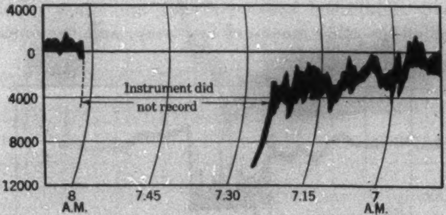

Fig. 12

fluctuating, for no apparent reason, for 18 minutes, and then suddenly recovered and steadied down, without any apparent reason also. An excessive current passed during the disturbance between stations *D* and *B* as shown by the wattmeter record going off the scale, and an excessive current between *B* and *C* station shown by the heating of the reactor. In this case, the stations did not break out of step with each other, but stayed in synchronism. In appearance these records look very much like hunting, or surging of the stations against each other, and thus are rather disquieting to the station operation. It is questionable however, whether it is real hunting.

The question of the synchronizing power of these big stations and in general of all phenomena of synchronous operation, as affected by the impedance between the machines, thus is of fundamental importance for the safe operation of our modern large power systems.

II. PARALLEL OPERATION OF SYNCHRONOUS MACHINES

A. Steady Strain

Let two alternators or groups of alternators, such as stations or stations sections, of the same terminal voltage, be connected with each other through a reactance, or more general, through an impedance, and in synchronism with each other.

We may assume the alternators of equal voltage, since a voltage difference merely superimposes on the synchronizing or energy current flowing between the alternators, a reactive magnetizing current, without materially changing the energy relations, and the equations thus are of the same general characteristics, merely a little more complicated.

If the loads on the two alternators equal the power output of the respective machines, no power flows over the impedance between them. If however, the load on the one alternator is greater, that on the other alternator by the same amount less than its output, power must flow over the impedance. The load on the alternators varies with the changing conditions in the system; the relative output of the alternators or group of alternators however is fixed by the speed governors of their prime movers and can be varied only in steps, by shutting down a machine or starting an additional machine. Thus the output of each generating section cannot always equal the load on it, and an exchange of power must occur between the generating sections, that is, power flow over the impedance between the generating sections.

Let P = the power flowing from the underloaded to the overloaded alternator, over a circuit of the impedance z, and let

$2\,\omega$ = the phase displacement between the two alternators, caused by the flow of power.

The e.m.fs. of the two alternators then may be represented by

$$\left.\begin{aligned} e_1 &= e_0 \cos(\phi - \omega) \\ e_2 &= e_0 \cos(\phi + \omega) \end{aligned}\right\} \qquad (1)$$

where e_0 = maximum value of e.m.f., and $\phi = 2\pi f t$. The resultant e.m.f., acting in the circuit between the two alternators then is

$$\begin{aligned} e &= e_1 - e_2 \\ &= e_0 \left\{ \cos(\phi - \omega) - \cos(\phi + \omega) \right\} \qquad (2) \\ &= 2\, e_0 \sin\omega \sin\phi \end{aligned}$$

that is, in quadrature with the average voltage of the two alternators. The interchange current between the two alternators then is

$$\begin{aligned} i &= /\frac{e}{z}/ \\ &= \frac{2\, e_0}{z} \sin\omega \sin(\phi - \alpha) \end{aligned} \qquad (3)$$

where

$$\begin{aligned} z &= \sqrt{\mathrm{r}^2 + \mathrm{x}^2} \\ r &= \text{resistance} \\ x &= \text{reactance} \end{aligned}$$

of the circuit between the two alternators, including their internal resistances and reactances, and the phase angle α is given by

$$\tan\alpha = \frac{\mathrm{x}}{\mathrm{r}}$$

The effective value of the current i is then given by

$$I = \frac{\sqrt{2}\, e_0}{x} \sin\omega$$

or, if E = effective value of generator e.m.f.,

$$e_0 = E\sqrt{2}, \text{ and}$$

$$I = \frac{2\,E}{z} \sin\omega$$

The power consumed in the resistance r of the circuit is

$$\begin{aligned} P &= I^2 r \\ &= \frac{4\,E^2\,r}{z^2} \sin^2 \omega \\ &= \frac{4\,E^2}{z} \sin^2 \omega \cos \alpha \qquad (5) \end{aligned}$$

The power of the first alternator then is

$$\begin{aligned} p_1 &= e_1 i \\ &= \frac{2e_o^2}{z} \sin \omega \sin (\phi - \alpha) \cos (\phi - \omega) \qquad (6) \end{aligned}$$

and the power of the second alternator,

$$p_2 = - \frac{2e_o^2}{z} \sin \omega \sin (\phi - \alpha) \cos (\phi + \omega) \qquad (7)$$

The sum of the powers of the two alternators then is

$$\begin{aligned} p' &= p_1 + p_2 \\ &= \frac{2e_0^2}{z} \sin \omega \sin (\phi - \alpha) \,[\cos (\phi - \omega) - \cos(\phi + \omega)] \\ &= \frac{4e_o^2}{z} \sin^2 \omega \sin \phi \sin (\phi - \alpha) \\ &= \frac{2e_o^2}{z} \sin^2 \omega \quad [\cos \alpha - \cos (2\,\phi - \alpha)] \end{aligned}$$

and its average value thus is

$$\begin{aligned} \text{avg. } p' &= \frac{2\,e_o^2}{z} \sin^2 \omega \cos \alpha \\ &= \frac{4\,E^2}{z} \sin^2 \omega \cos \alpha \\ &= P' \end{aligned}$$

that is, the sum of powers of the two alternators is the power consumed in the resistance of the circuit between them as obvious.

The difference of the powers of the two alternators is

$$\begin{aligned} 2\,p &= p_1 - p_2 \\ &= \frac{2e_o^2}{z} \sin \omega \sin (\phi - \alpha) \,[\cos(\phi - \omega) + \cos(\phi + \omega)] \\ &= \frac{4\,e_o^2}{z} \sin \omega \cos \omega \cos \phi \sin (\phi - \alpha) \end{aligned}$$

$$2\,p = - \frac{e_o^2}{z} \sin 2\,\omega \,[\sin \alpha - \sin (2\,\phi - \alpha)] \qquad (8)$$

and its average value is

$$\text{avg. } 2\,p = -\frac{e_0^2}{z} \sin 2\,\omega \sin \alpha$$

$$= -\frac{2\,E^2}{z} \sin 2\,\omega \sin \alpha$$

$$= 2\,P \tag{9}$$

The power transfer therefore between the two alternators (or generating stations or sections of generating stations) is

$$P = \frac{E^2}{z} \sin 2\;\omega \sin \alpha \tag{10}$$

and the leading alternator, e_2 delivers power to the lagging alternator, e_1.

The power P is thus zero for $\omega = 0$, and it increases, reaching a maximum

$$P'_m = \frac{E^2}{z} \sin \alpha \tag{11}$$

for $\omega = 45$ deg., or 90 deg. phase displacement between the alternators, and then decreases again to zero at $\omega = 90$ deg., or phase opposition of the alternators.

Beyond $\omega = 90$ deg. the synchronizing power P_m becomes negative, with the same values, that is, the alternators synchronize at the next pole.

The synchronizing power P is zero for $\alpha = 0$, that is, if the circuit between the alternators contain no reactance, but only resistance, and is a maximum when the resistance is negligible compared with the reactance, that is $\alpha = 90$ deg.

$$P''_m = \frac{E^2}{x} \sin 2\,\omega \tag{12}$$

Substituting in (10),

$$\sin \alpha = \frac{x}{z}$$

gives

$$P = \frac{E^2}{z^2}\; x \sin\; 2\,\omega \tag{13}$$

that is, with a given impedance z, and thus given synchronizing current between the alternators, the synchronizing power P is directly proportional to the reactance x of the circuit between the alternators.

The maximum synchronizing power between the alternators thus occurs at phase angle $\omega = 45$ deg., that is, 90 deg. phase displacement, and neglibible resistance, and is

$$P_m = \frac{E^2}{x} \tag{14}$$

at current (effective):

$$I_m = \frac{E \sqrt{2}}{x} \tag{15}$$

and resultant e.m.f.

$$E_m = E \sqrt{2} \tag{16}$$

In this case, the phase angle $2\,\omega$ between the alternators or station sections is constant during operation, but varies with change of load between the station sections, and can be kept very small by properly apportioning the number of generators in operation in each section, to the respective load on this section.

B. OSCILLATION

Consider again two alternators or groups of alternators, such as stations or station sections, which are running in synchronizm with each other, having the same frequency f, but connected together while out of phase with each other by an angle $2\,\omega$, or thrown out of phase by some sudden change of load, momentary short circuit, etc. As is well known, the alternators then oscillate against each other, with (practically) constant frequency of oscillation $p\,f$, and gradually decreasing amplitude of oscillation, and finally steady down in phase with each other, at the constant phase angle ω deg., determined by the condition of steady power transfer between the alternators.

Since, under normal conditions of operation, the steady phase angle ω deg. must be small, we may assume that the oscillation occurs symmetrically around the position of the alternators in phase with each other, that is, the one alternator has the phase $\phi - \omega$, when the other has the phase $\phi + \omega$.

The same equations then pertain as in section A, that is, The e.m.fs. of the two alternators are

$$\begin{aligned} e_1 &= e_o \cos \phi - \omega) \\ e_2 &= e_o \cos(\phi + \omega) \end{aligned} \qquad (1)$$

The e.m.f. acting in the circuit between the two alternators

$$e = 2\, e_o \sin\omega \sin\phi$$

with effective value

$$E^o = 2\, E \sin\omega \qquad (17)$$

The current flowing between the two alternators is,

$$i = \frac{2\, e_o}{z} \sin\omega \quad \sin(\phi - \alpha) \qquad (3)$$

with effective value of

$$I = \frac{2\, E}{z} \sin\omega \qquad (4)$$

Where z is the impedance of the circuit between the two alternators or groups of alternators, including their internal impedance; and the power transferred between the alternators is

$$p = \frac{E^2}{z} \sin 2\,\omega\, [\sin(2\,\phi - \alpha) - \sin\alpha] \qquad (8)$$

The first term of (8) is of double frequency, $2\,f$. It thus does not represent energy transfer between the alternators, but merely represents the energy storage and return, twice per cycle, occurring in any inductive circuit. It thus is of no further interest, and is:
Power transfer between the alternators

$$\begin{aligned} P &= \frac{E^2}{z} \sin 2\,\omega \sin\alpha \qquad (10) \\ &= \frac{E^2\, x}{z^2} \sin 2\,\omega \qquad (12) \end{aligned}$$

In this case, however, the phase angle ω of the e.m.f. is not constant, but pulsates with approximately constant frequency of the beat, and decreasing amplitude.
Let $\omega_o = \omega_{oo}\, \epsilon^{-at}$ (18)
be the maximum value of the phase angle during each oscillation (decreasing from its initial maximum value ω_{oo} by the exponential of time ϵ^{-at}).

We may then represent the gradually decreasing amplitude of the phase angle ω by

$$\left.\begin{aligned} \omega &= \omega_o \sin p\,\phi \\ &= \omega_{oo}\, \epsilon^{-at} \sin p\,\phi \end{aligned}\right\} \qquad (19)$$

where $p f$ = frequency of the beat, or the (complete) periodic variation of the phase angle ω.

In reality, the equations (3), (4), (8), (10) of section A are not strictly correct for the conditions under investigations in section B, since in the derivation of these equations, in A, ω has been assumed as constant. In the case of B, oscillation of the alternators, ω varies periodically, is a function of ϕ and thus additional terms appear in these equations. Since however the frequency of variation of ω is very low compared with the frequency of f, p = a small quantity; these additional terms are small, and the above equations thus are correct with sufficient approximation, especially in the present case, where we are essentially interested in the magnitude of the power relations.

Substituting (19) into (10), gives as the periodically varying power transfer or synchronizing power,

$$P = \frac{E^2}{z} \sin \alpha \sin 2(\omega_o \sin p\, \phi) \qquad (20)$$

where ω_o is the maximum amplitude of this oscillation.

The average value of P during the half cycle of oscillation may be represented by

$$\begin{aligned} P_o &= \text{avg. } P \\ &= \frac{E^2}{z} \sin \alpha \; \frac{1\text{-}\cos 2\, \omega_o}{2\, \omega_o} \qquad (21) \end{aligned}$$

and as the duration t_o of one-half cycle of oscillation—during which the power transfer remains in the same direction—is given by half a cycle of $p\phi$, that is,

$$\begin{aligned} p\, \phi &= 2 \pi p f t_o = \pi \\ t_o &= \frac{1}{2 p f} \qquad (22) \end{aligned}$$

and the *energy transfer* between the two machines or groups of machines, during each half cycle of oscillation, thus is given by

$$\begin{aligned} W_o &= t_o P_o \\ &= \frac{E^2}{4\, p f \omega_o z} \sin \alpha (1 - \cos 2\, \omega_o) \qquad (23) \end{aligned}$$

This is a maximum for ω_o = 90 deg. $= \frac{\pi}{2}$, and then,

$$W_m = \frac{E^2}{\pi p f z} \sin \alpha \qquad (24)$$

W_m thus is the maximum energy which can be absorbed by the machine or group of machines, without being thrown out of synchronism. In other words, if a sudden demand greater than W_m is made on the machine, or if more energy than W_m is given by the steam supply to the machine or group of machines, after the load has been thrown off and before the steam has been cut off, the machine is thrown out of synchronism; otherwise it remains in synchronism and after an oscillation settles down again in phase.

As seen from the equations, during each complete cycle of oscillation, of frequency, $p f$, the current twice rises and falls, thus reaching two maxima, and the power P twice reverses, so that the energy W flows one way during half the cycle, and in opposite direction during the other half cycle of oscillation. The frequency of the rise and fall of the current thus is $2\ p f$.

Curves *I* and *II* in Fig. 13, show the current i and the voltage e_1 of the oscillation, for the (exaggerated) value $p = 0.1$, and for $\omega_o = 45$ deg. and $\omega_o = 90$ deg.]

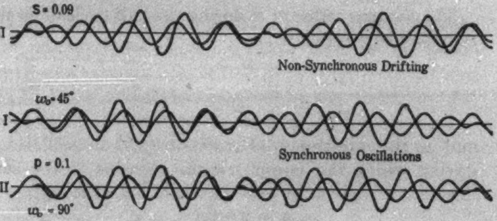

FIG. 13

It is interesting to note from equation (20) that the power transfer P reverses twice per cycle of oscillation (for $p\phi = 0$ and 180 deg.). If $\omega_o = 45$ deg. or less, that is, 90 deg. or less maximum phase displacement during the oscillation, then the power P has two maxima at the maximum phase displacement midway between the reversal of power, as seen in Curve *I* of Fig. 14. If however $\omega_o > 45$ deg., that is, more than 90 deg. phase displacement, then the power transfer decreases again at maximum phase displacement midway between the reversals of power, and the power transfer has four maxima, separated by two reversals and two minima, as seen by Curve II of Fig. 14; and finally at $\omega_o = 90$, (Curve III in Fig. 14), the power reaches four maxima and four zero values during each cycle of oscillation, but reverses only twice. That is, at the moment when the two alternators are in phase, the power transfer is zero, the power reverses, and the current is zero, and in phase with the voltage. With increasing phase displacement, power and current increase, the power reaches a maximum at 90 deg. phase displacement between the machines, where the current is 45 deg. out of phase with the voltage. With further increase of phase displacement during the swing of oscillation, the power decreases again, to zero at 180 deg. phase displacement or phase opposition; but the current continues to increase and reaches a maximum at phase opposition, with the phase angle between voltage and current steadily increasing, to 90 deg., or zero power, in phase opposition. Then, without reversal of the flow of power, the phase angle between voltage and current again decreases, the current

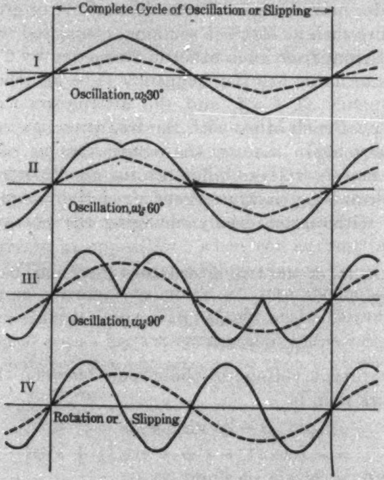

FIG. 14

decreases, but the power increases again in the same direction as before, to the second maximum in the same half cycle, at 90 deg. phase displacement, and then the power decreases again, to the reversal. Figs. 13 and 14 well illustrate this.

C. SLIPPING

Consider now the case of two alternators, or groups of alternators such as stations sections, connected together while different from each other in frequency by 2 s; that is, one alternator has the frequency $(1 - s)f$, the other the frequency $(1 + s)f$, and the alternators thus are slipping past each other with the frequency $2\,s\,f$.

We may again assume the alternators as of equal voltage, since a voltage difference merely superposes on the synchronizing energy current a reactive magnetizing current, without materially changing the energy relations.

The e.m.fs. of the two alternators then may be represented by

$$\left.\begin{aligned} e_1 &= e_o \cos(1 - s)\phi \\ e_2 &= e_o \cos(1 + s)\phi \end{aligned}\right\} \qquad (25)$$

The resultant voltage in the circuit between the two alternators then is

$$\begin{aligned} e &= e_1 - e_2 \\ &= e_o\left[\cos(1 - s)\phi - \cos(1 + s)\phi\right] \\ &= 2e_o \sin s\phi \sin\phi \\ &= 2\,E\,\sqrt{2} \sin s\phi \sin\phi \end{aligned} \qquad (26)$$

and its effective value

$$E_o = 2\,E \sin s\phi \qquad (27)$$

where E = effective value of generator e.m.f.

Assuming now that s is a small quantity (just as we assumed in section A, that p is a small quantity), that is, that the two alternators have nearly the same frequency. The change of $\sin s\phi$ then is slow compared with that of $\sin\phi$, and for all phenomena of the frequency f, $\sin s\phi$ may be assumed as constant, and the reactance of the circuit may be assumed as the same, $x = 2\pi f L$, for both component e.m.fs., e_1 and e_2; that is, for both frequencies $(1 - s)f$ and $(1 + s)f$.

The interchange current between the alternators then is

$$i = \frac{2\,e_0}{z} \sin s\phi \sin (\phi - \alpha)$$

$$= \frac{2\,E\,\sqrt{2}}{z} \sin s\phi \sin (\phi - \alpha) \qquad (28)$$

hence, the effective value is

$$I = \frac{2\,E}{z} \sin s\phi \qquad (29)$$

$$\text{where } z = \sqrt{r^2 + x^2}$$

$$\tan \alpha = \frac{x}{r}$$

With regards to the e.m.f. of one of the alternators, for instance, e_1, this current always lags. Its lag is 90 deg. when the current is a maximum. With the decrease of current, the lag decreases from 90 deg. in the one, and increases in the next beat, and approaches in-phase respectively in opposition, when the current is a minimum. The power factor thus varies from zero at maximum current, to unity at zero current, and its average thus is low. Fig. 13 shows in Curve *III* the relation of e_1 to i for the exaggerated value $s = 0.09$.

The power of the one alternator then is given by

$$p_1 = e_1\, i$$

$$= \frac{2\,e_0^{\,2}}{z} \sin s\phi \sin (\phi - \alpha) \cos (1-s)\phi$$

$$= \frac{4\,E^2}{z} \sin s\phi \sin (\phi - \alpha) \cos (1-s)\phi \qquad (30)$$

and that of the other alternator by

$$p_2 = e_2\, i$$

$$= -\frac{4\,E^2}{z} \sin s\phi \sin (\phi - \alpha) \cos (1+s)\phi \quad (31)$$

and the power transfer between the two alternators is given by

$$2\,p = p_1 - p_2$$

$$= \frac{8\,E^2}{z} \sin s\phi \sin (\phi - \alpha) \cos s\phi \cos\phi$$

$$= \frac{2\,E^2}{z} \sin 2\,s\phi\ [\sin (2\phi - \alpha) - \sin\alpha] \quad (32)$$

The first term, with $\sin (2\phi - \alpha)$, is again a double frequency term representing the periodic storage and return of the energy during the half cycle of voltage and thus does not represent any power transfer, and the power transfer between the alternators is given by

$$P = \frac{E^2}{z} \sin 2\, s\phi \sin\alpha \tag{33}$$

Usually it is approximately, $\alpha = 90$ deg., that is, the reactance is large compared with the resistance, and equation (33) then becomes

$$P = \frac{E^2}{z} \sin 2\, s\phi \tag{34}$$

During each cycle of the frequency $s\,f$, of the slip from synchronism or average frequency, the amplitude of the current i twice becomes zero and in phase, and twice reaches a maximum, when the alternators are in opposition, and the power p reaches a maximum four times and becomes zero and reverses four times, twice when the current comes into phase with the e.m.f., when the current becomes zero, and twice when the current is a maximum, but in quadrature with the e.m.f. and the power becomes zero. The power transfer between the alternators thus reverses four times per complete cycle of slip, $s\,f$; that is, it is of the frequency $2\,s\,f$, with two positive and two negative maxima.

The average value of the power is

$$\frac{2}{\pi}\, p = \frac{2\,E^2}{\pi\, z} \sin\alpha \tag{35}$$

and as the duration of one-quarter cycle of slip is $t^\circ = \frac{1}{4\,s\,f}$, the *energy transfer* between the two machine, during a quarter cycle of slip is

$$\begin{aligned} W &= \frac{1}{4\,s\,f}\,\frac{2}{\pi}\, p \\ &= \frac{E^2}{2\,\pi\, s\, f\, z} \sin\alpha \end{aligned} \tag{36}$$

There is a difference between the slipping of alternators past each other out of synchronism, and the oscillation of the alternators against each other at synchronism (A). In slipping, the power fluctuation and the reversal of the energy is of twice the frequency of the current fluctuation, while in the oscillation of the alternators against each other at synchronism, the power fluctuation or reversal of energy flow is of the same frequency as the current fluctuation.

If two alternators are connected together while out of synchronism, and slowly slip past each other, during each half cycle of slip, or beat, while the two machine e.m.fs. pass from in-phase, to in-opposition, to in-phase again, a periodic energy transfer takes place. During one quarter cycle of slip (that is, while one alternator e.m.f. slips behind, the other pulls ahead of the mean frequency by one-quarter cycle, and the two alternators e.m.fs. thus slip against each other by one-half cycle), the alternators are partly in phase with each other, and the slower machine receives energy from the faster machine. The two machines are thereby brought nearer to each other in speed; pulled towards synchronism. During the next quarter-cycle of slip, however, the two alternators are partly in opposition, and the faster machine receives energy from the slower one. The faster machine then speeds up, the slower machine slows down, and the two machines pull apart again by the same amount by which they pulled together in the preceding quarter cycle of slip (if their e.m.fs. are constant). Thus the two machines can pull into step only if the energy transferred during one-quarter cycle of slip, W, is larger than the energy required to speed up the momentum, that is, the kinetic energy M of the machine, to full synchronism.

Due to the energy transfer W between the machines, resulting in an alternate speeding up and slowing down, the slip s is not constant, but pulsates periodically, between the minimum value $s - s_1$, at the end of the quarter cycle during which the machines pull together, and beginning of the quarter cycle during which the machines pull apart, and a maximum value $s + s_1$, at the end of the quarter cycle during which the machines pull apart, and beginning at the quarter cycle during which the machines pull together—where s_1 is the amplitude of the pulsation of slip. As the energy required to accelerate the momentum M of the machine by the speed $2\,s_1$ is $4\,s_1\,M$, it follows that

$$W = 4\,s_1\,M$$

or

$$s_1 = \frac{W}{4\,M}$$

$$s_1 = \frac{E^2 \sin \alpha}{8\pi\, s f z M} \qquad (37)$$

is the amplitude of the speed fluctuation of the two alternators during the slipping past each other, out of synchronism with the slip s.

$s_1 = s$ gives as minimum slip, $s - s_1 = 0$, that is, the machines pull into synchronism.

The maximum slip s_1 from which the two machines pull into synchronism with each other, is given by substituting $s_1 = s$ in (37)

$$s_0 = \frac{E}{2} \sqrt{\frac{\sin \alpha}{2\,\pi f z M}} \qquad (38)$$

s_0 thus is the *limit of synchronizing power*.

As illustrations, Fig. 14, shows four curves of power and of current (effective value), the former in solid and the latter in dotted lines; for oscillation, $\omega_0 = 30$ deg., 60 deg., 90 deg. and for slipping, *I*, *II*, *III* and *IV*.

As seen, the single maximum power Curve *I*, with increasing swing of the oscillation becomes a double maximum with a minimum between the maxima, *II*, the minimum then decreases to zero, *III*, at the limits of synchronizing power, and the power curve then overturns, *IV*; that is, the alternator, instead of swinging back into phase again, continues to slip and drops into phase again by skipping one cycle, etc., and thereby the power transfer curve doubles its frequency by one of the two lobes of *III* overturning, while the current curve remains the same, at the frequency of the beat or slip.

D. PULLING IN STEP

With the two machines out of synchronism with each other by a greater speed difference $2\,s$, than that from which the machines can pull each other into synchronism within one-quarter cycle of slip, from the equations of *C* it would follow, that the machines can never pull each other into synchronism, if the voltage E is constant, but must indefinitely continue to slip past each other, coming nearer together during one-quarter cycle of slip, and dropping apart again by the same amount during the next quarter cycle of slip.

This, however, is under the assumption that the machine e.m.f., E is constant. In reality, however, E is not constant, but varies periodically with the same frequency that the current fluctuates. The current in the circuit between the machines, and thus the armature reaction in the machine, varies in amplitude and in phase difference against the machine voltage, and the machine voltage varies with the amplitude and the phase of the armature reaction.

Consider, as an approximation, the armature reaction as proportional to the quadrature component of the current. The e.m.f. of the machine would then be expressed by an approximate equation of the form:

$$E' = E\,[1 - c \sin s\phi \sin \delta] \tag{39}$$

where c is a constant and δ is the phase angle between the current and the e.m.f. and $s\phi$ represents the amplitude of the current pulsation, by (29) thus $\sin s\,\phi$ $\sin \delta$ represents the quadrature component of the armature current.

We have, however, from (25) and (29)

$$\begin{aligned} \delta &= (\phi - \alpha) - (1 - s)\phi - 90^\circ \\ &= s\phi - \alpha + 90^\circ \end{aligned}$$

$$\text{thus } E' = E\,\{1 + c \sin s\phi \cos (s\phi - \alpha)\} \tag{40}$$

Substituting (40) into the expression of the power of the alternator (33), the equations still remain alternating, that is, there is no resultant synchronizing power, but equal positive and negative values of power alternate.

However, (40) assumes that the magnetizing effect of the armature reaction is instantaneous, that is, that the e.m.f., E, at any moment is the value corresponding to the armature reaction existing at this moment. This, however, is not the case, and the armature reaction is not instantaneous, but requires an appreciable time—several seconds—to develop, and the magnetizing or demagnetizing effect of the armature reaction on the voltage therefore materially lags behind the armature reaction.

Let σ = angle of lag of the voltage change behind the armature reaction which causes it. Then

$$E' = E\,\{1 + c \sin s\phi \cos (s\phi - \alpha - \sigma)\} \tag{41}$$

and substituting (41) into (33), gives the *power transfer* between the machines:

$$P = \frac{E^2}{z} \sin 2\,s\phi \sin\alpha \left\{1 + c \sin s\phi \cos (s\phi - \alpha - \sigma)\right\}^2$$

or approximately, considering c as a small quantity,

$$P = \frac{E^2}{z} \sin 2\,s\phi \sin\alpha + \frac{2\,c\,E^2}{z} \sin 2\,s\phi \sin\alpha \sin s\phi \cos$$
$$(s\phi - \alpha - \sigma) \qquad (42)$$

The first term, $\frac{E^2}{z} \sin 2\,s\phi \sin\alpha$, is the slowly alternating energy transfer between the machines, discussed in section C, which causes their speed to fluctuate, but does not permanently bring them nearer to each other; that is, exerts no synchronizing power unless, during these speed fluctuations they reach complete synchronism and then fall into step.

The second term,

$$P^1 = \frac{2\,c\,E^2}{z} \sin 2\,s\phi \sin\alpha \sin s\phi \cos (s\phi - \alpha - \sigma)$$
$$= \frac{c\,E^2}{z} \sin 2\,s\phi \sin\alpha\, [\sin (\alpha + \sigma) + \sin (2\,s\phi - \alpha - \sigma)]$$
$$= \frac{c\,E^2}{z} \sin 2\,s\phi \sin\alpha \sin (\alpha + \sigma) + \frac{c\,E^2}{2\,z} \sin \quad [\cos$$
$$(4\,s\phi - \alpha - \sigma) + \cos (\alpha + \sigma)]$$
$$= \frac{c\,E^2}{z} \sin 2\,s\,\phi \sin\alpha \sin (\alpha + \sigma) + \frac{c\,E^2}{2\,z} \sin\alpha \cos$$
$$(4\,s\phi - \alpha - \sigma) + \frac{c\,E^2}{2\,z} \sin\alpha \cos (\alpha + \sigma) \qquad (43)$$

The first two terms also are slowly alternating, at double and quadruple frequency of slip, as they contain terms with $2\,s\phi$ and $4\,s\phi$ and thus represent no continuous power transfer; the third term, however,

$$P_o = \frac{c\,E^2}{2\,z} \sin\alpha \cos (\alpha + \sigma) \qquad (44)$$

is constant, that is, represents a continuous synchronizing power.

If $\alpha = 90$ deg., that is, the resistance is negligible compared with the reactance, it is,

$$P_o = \frac{c\,E^2}{2\,z} \sin \sigma \qquad (45)$$

If thus two alternators or station sections are considerably out of synchronizm with each other, they continue slipping past each other, with large fluctuating currents flowing between them, and the speeds of the machines fluctuating with the fluctuations of the current. These currents do not decrease in amplitude, but remain of practically constant value, but their period of fluctuation gradually gets slower, that is, the fluctuation gradually becomes slower, while currents slowly pull the machines nearer into synchronism with each other, or decrease their frequency difference, until the critical frequency $2\,s_0$ is reached (where the acceleration during a quarter-cycle of slip, $2\,s_1$, reaches full synchronism). Then the machines suddenly drop into synchronism, but oscillate in phase against each other, with an approximately constant frequency of oscillation, but with a current fluctuation, which steadily (and usually rapidly) decreases, until steady conditions of speed, current and voltage are reached.

The armature reaction of the alternator is represented by the difference of the synchronous reactance x_0 and the true reactance x_1, that is, by an effective *reactance of armature reaction.*

$$x_2 = x_0 - x_1$$

The coefficient c in the synchronizing power, P_0 (44) is that fraction of the reactance of the armature reaction x_2, which appears during the short time of the current fluctuation. Thus c is the larger, the slower the fluctuation, that is, the less s. In other words, c increases with decreasing slip, or, increasing approach to synchronism.

Inversely, since σ is a maximum and practically 90 deg. for large values of s, where the voltage fluctuation lags practically 90 deg. behind the fluctuation of the armature reaction, and decreases with decreasing s, that is, increasing approach to synchronism, $c \sin \sigma$ and thus the synchronizing power P_0 (44), should be a maximum at some moderate slip s, and decrease for larger as well as smaller slips.

Assuming that it takes t_0 seconds for the field to build up to correspond to the armature reaction. With the current fluctuating with the frequency $2\,s\,f$, and assuming that the magnetizing effect of the armature reaction is sinoidal, it would be,

$$c = \frac{1}{4\,s\,f\,t_o}$$

and $$\sin\sigma = \sqrt{1 - \left(\frac{1}{4\,s\,f\,t_o}\right)^2}$$

thus $$P_o = \frac{E_o^2}{8\,z\,s\,f\,t_o}\sqrt{1 - \left(\frac{1}{4\,s\,f\,t_o}\right)^2} \qquad (46)$$

However, secondary effects occur and more or less modify the value P_o, such as the effect of secondary currents, induced in the field structure by that component of the armature current which is due to the e.m.f. of the other machine, and which gives an induction motor torque, tending to pull the machines together into synchronism.

E. EQUATIONS

$z = \sqrt{r^2 + x^2}$ = total impedance of circuit between alternators. $\tan\alpha = \frac{x}{r}$

ω = phase angle between alternator e.m.fs.

$p\,f$ = frequency of oscillation.

$s\,f$ = frequency of slipping.

	A. Continuous Power Transfer	B. Ocsillation in Synchronism	C. & D. Slipping Out of Synchronism
Alternator e. m. fs.			
$e_1 =$	$e_0 \cos(\phi - \omega)$	$e_0 \cos(\phi - \omega)$	$e_0 \cos(1 - s)\,\phi$
$e_2 =$	$e_0 \cos(\phi + \omega)$	$e_0 \cos(\phi + \omega)$	$e_0 \cos(1 + s)\,\phi$
Eff. $E =$	$\frac{e_0}{\sqrt{2}}$	$\frac{e_0}{\sqrt{2}}$	$\frac{e_0}{\sqrt{2}}$
Resultant e. m. fs.			
$e =$	$2\,e \sin\omega \sin\phi$	$2\,e \sin\omega \sin\phi$	$2\,e \sin s\,\phi \sin\phi$
Eff. $E_0 =$	$2\,E \sin\omega$	$2\,E \sin\omega$	$2\,E \sin s\,\phi$
Resultant Current			
$i =$	$\frac{2\,e_0}{z} \sin\omega \sin(\phi - \alpha)_0$	$\frac{2\,e_0}{z} \sin\omega \sin(\phi - \alpha)$	$\frac{2\,e_0}{z} \sin s\,\phi \sin(\phi - \alpha)$
Eff. $I =$	$\frac{2\,E}{z} \sin\omega$	$\frac{2\,E}{z} \sin\omega$	$\frac{2\,E}{z} \sin s\,\phi$

Continuous Power Transfer

$$P_o = \frac{E^2}{z} \sin 2\,\omega \sin \alpha \qquad \frac{c\,E^2}{2\,z} \sin \alpha \cos(\alpha + \sigma)$$

Low Frequency Power Fluctuation

$$p = \qquad \frac{E^2}{z} \sin 2\,\omega_o \sin \alpha \quad \frac{E^2}{z} \sin 2\,s\,\phi \sin \alpha$$

Low Frequency Energy Transfer

$$W = \qquad \frac{E^2}{z} \sin \alpha \, \frac{1 - \cos 2\,\omega_o}{4\,p\,f\,\omega_o} \quad \frac{E^2}{2\,\pi\,s\,f\,z} \sin \alpha$$

Attenuation

$$\omega = \qquad \omega_o \sin p\,\phi$$

$$= \omega_{oo}\,\epsilon^{-at} \sin p\,\phi$$

Pulsation of Slip

$$s_1 = \qquad \frac{E^2 \sin \alpha}{8\,\pi\,s\,f\,z\,M}$$

Critical Slip

$$s_o = \qquad \frac{E}{2} \sqrt{\frac{\sin \alpha}{2\,\pi\,f\,z\,M}}$$

Pulsation of Armature Reaction

$$c = \qquad \frac{1}{4\,s\,f\,t_o}$$

Lag of Armature Reaction

$$\sin \sigma = \qquad \sqrt{1 - c^2}$$

It is interesting to note that the limit case of W, in B for $\omega_o = \frac{\pi}{2},$ and in C, for $s = s_o$, must coincide. $W_B = 2\,W_C$. This gives,

$$\Big/ \frac{E^2}{z} \sin \alpha \, \frac{1 - \cos 2\omega_o}{4\,p\,f\,\omega_2} \Big/_{\omega_o = \frac{\pi}{2}} \quad \Big/ \frac{E^2}{2\,\pi\,s\,f\,z} \sin \alpha \Big/_{s = s_0}$$

hence $p = s_o$

and, substituting for s_o

$$p = \frac{E}{2} \sqrt{\frac{\sin \alpha}{2\,\pi\,f\,z\,M}}$$

is the frequency of oscillation.

III. DISCUSSION OF E.m.fs.

The foremost difficulty and uncertainty in the application of the preceding equations is found in the selection of the proper values of the machine e.m.f., E. E is not the terminal voltage; by slipping past each other without external impedance, the terminal voltage of the alternators goes down to zero. Neither is E the "nominal induced voltage," as this has no actual existence, but is the voltage which would be induced by the field excitation if the saturation curve of the machine continued as a straight line. It appears to me that E must be considered as the "true induced voltage," or actual induced voltage, that is, the voltage induced by the actual field flux, or the field flux due to the resultant field excitation and armature reaction. The armature reaction, however, fluctuates with the current between zero and a maximum, while the actual field flux often may be assumed as practically constant, since the magnetic field cannot follow the relatively rapid fluctuations of armature reaction.

The magnetic effect of the armature reaction is represented electrically in the synchronous reactance x_o. The synchronous reactance thus consists of a true self-inductive reactance x_1, which is instantaneous, and an effective reactance of armature reaction x_2, which requires appreciable time to develop, and does not correspond to any real magnetic flux.

$$x_o = x_1 + x_2$$

In turbo-alternators, x_2 usually is very much larger than x_1.

Electrically, the actual induced e.m.f. thus should be the nominal induced voltage e_o, which corresponds to the field excitation, less the reactance drop of the average current in the effective reactance of armature reaction, x_2.

If then I = maximum effective value of the fluctuating current, then average current is $\frac{I}{2}$, and the actual induced voltage is

$$E = e_o - \frac{I x_2}{2}$$

However, in two alternators connected together out of synchronism, through an additional reactance

$$2E = I(2x_1 + x)$$

where x is the additional reactance through which the alternators of actual induced voltage E and true self-inductive reactance x_1 are connected together, while running out of synchronism with each other.

From these two equations it follows that:

Maximum (effective) value of the fluctuating interchange current,

$$I = \frac{2e_o}{2x_1 + x_2 + x} \quad (47)$$

and, actual induced voltage,

$$E = e_o \frac{2x_1 + x}{2x_1 + x_2 + x} \quad (48)$$

where e_o = nominal induced voltage, effective value.

If the alternators are connected through an inpedance z, z takes the place of x, combining vectorily with x_1 and x_2.

In this calculation, the armature reaction has been assumed as demagnetizing, and the impedance voltage therefore subtracted from the nominal induced voltage. This appears correct, as the interchange current between the alternators out of synchronism with each other, is essentially a lagging current throughout, as illustrated in Fig. 13.

If the two alternators are in synchronism, but out of phase with each other by a maximum phase displacement angle $2\omega_o$, it is

$$2E \sin \omega_o = I(2x_1 + x)$$

and again assuming the armature reaction as demagnetizing,

$$E = e_o - \frac{I x_2}{2}$$

thus, the maximum (effective) value of the fluctuating exchange current is,

$$I = \frac{2 e_o \sin \omega_o}{2x_1 + x + x_2 \sin \omega_o} \quad (49)$$

and, actual induced voltage is,

$$E = e_o \frac{2x_1 + x}{2x_1 + x + x_2 \sin \omega_o} \quad (50)$$

where e_o is the nominal induced voltage, effective value.

However, in this case of alternators in synchronism but oscillating against each other, at least for small and moderate values of ω_o the interchange current I is essentially an energy current with regards to the machine voltage, and the reactive component of this current alternately changes between lag and lead, that is, between demagnetizing and magnetizing. Therefore, the correctness is doubtful of subtracting the impedance voltage from the nominal induced voltage to get the induced voltage, but it would be

$$E = \sqrt{e_o^2 - i^2 x_2^2}$$

and as i varies between O and I, the average E would be the mean between e_o and $\sqrt{e_o^2 - I^2 x_2^2}$. Thus, combining with the equation,

$$2 E \sin \omega_o = I (2 x_1 + x)$$

gives

$$I = \frac{2 e_o (2 x_1 + x) \sin \omega_o}{(2 x_1 + x)^2 + x_2^2 \sin^2 \omega_o} \tag{51}$$

$$E = e_o \frac{(2 x_1 + x)^2}{(2 x_1 + x)^2 + x_2^2 \sin^2 \omega_o} \tag{52}$$

It is probable that the true value of E lies between (**50**) and (**52**), but nearer to (**52**).

Substituting these values (**48**), (**50**), (**52**) into the equations of A, B, and C, and substituting $z = 2 x_1 + x$ in these equations, as the impedance of the circuit between the two alternators, gives the equations referred to the nominal induced voltage, e_o, that is, the field excitation.

The nominal induced e.m.f., e_o, is derived by combining the terminal voltage e with $i z$, where z is the total impedance inside of the terminals, true reactance as well as effective reactance of armature reaction. For non-inductive load—and synchronous machine load may be assumed as approximately non-inductive—this gives,

$$\begin{aligned} e_o &= \sqrt{e^2 + (i x)^2} \\ &= e \sqrt{1 + \xi^2} \end{aligned} \tag{53}$$

where ξ is the percentage reactance, and the resistance is neglected as small compared with the reactance.

However, this expression neglects the change of reactance with increase of magnetic saturation, increase of magnetic leakage between field poles, etc., and therefore, especially in turbo-alternators with their enormous magnetic fields, high saturation and high field leakage, this expression is not very accurate, and is reasonably reliable only in the mean range of current and voltage.

In *C* and *D*, the case of two alternators or groups of alternators out of synchronism with each other, the equations of synchronizing power, energy and critical slip: p, p_o W, s_o contain the term

$$\frac{2\,x_1 + x}{(2\,x_1 + x + x_2)^2}$$

thus are a maximum, if this term is a maximum. This is the case if $x_2 = 2\,x_1 + x$

or $x = x_2 - 2\,x_1$ **(54)**

The synchronizing power between alternators out of synchronism with each other is a maximum, and the frequency difference from which they pull each other into synchronism, is greatest, if the alternators or groups of alternators are connected together through a reactance which is equal to the effective reactance of armature reaction, less twice the self-inductive reactance of the circuit between the alternators or groups of alternators. With two alternators or groups of alternators connected together without any external reactance this means if the self-inductive reactance of the alternators or groups of alternators is one-third the synchronous reactance. With turbo-alternators, the self-inductive reactance usually is much less, and with such machines the synchronizing power is increased by the insertion of external reactance.

Substituting the above relation into the equations of *C* and *D*, gives as the expression for the case of maximum synchronizing power:

Actual machine e.m.f.: $E = \dfrac{e_o}{2}$

Resultant e.m.f.: $E_o = e_o \sin s\,\phi$

Resultant current: $I = \dfrac{e_o \sin s\,\phi}{x_2}$

Power fluctuation of low frequency: $P = \dfrac{e_o^2 \sin 2\,\phi \sin \alpha}{4\,x_2}$

Energy transfer of low frequency: $W = \dfrac{e_o^2}{8\,\pi\,s\,f\,x_2} \sin \alpha$

Continuous power transfer: $$P_o = \frac{c\, e_o^2 \sin\alpha \cos(\alpha + \sigma)}{8\, x_2}$$

Critical slip: $$s_o = \frac{e_o}{4 \cdot \sqrt{2\pi f x_2 M}}$$

IV. FEEDER REACTORS

A. General

Economy in cost and space makes it desirable to use the smallest feeder reactor which is reasonably safe, the more so as the number of feeder reactors required to protect every feeder going out from the generating station is usually much larger than that of the generator and busbar reactors.

Any reactance inserted into the system increases the reactive lagging volt-amperes and therefore, if the load on the system is lagging, lowers the power factor, the more, the greater the reactance of the feeder reactor. In 25-cycle systems, this is of no moment, as the load usually is almost exclusively synchronous machines, equally operative with leading as with lagging current, so that even with large values of feeder reactance, the system operates at unity power. In 60-cycle systems, however, a considerable part of the system usually comprises induction machines and other apparatus which produce lagging current; the power factor thus is below unity, lagging, and much additional reactance is therefore undesirable. An at least approximate investigation of the relations between the size of the feeder reactance and the disturbance in the generating station caused by a short circuit at the generating end of the feeder is thus desirable.

The function of a feeder reactor is three-fold:

(1) It reduces the short-circuit current on the generating station in case of a breakdown of the feeder near the generating station, and thereby reduces the shock on the system.

(2) It permits setting the feeder circuit breakers for a much shorter time of opening, due to the lesser short-circuit current which they have to open, and thereby reduces the time during which the system is exposed to the short-circuit stresses.

(3) It keeps at least partial voltage on the busbars of the generating station during the feeder short circuit, and thereby reduces the liability of the generators, stations and substations falling out of synchronism with each other.

Without a power-limiting reactor in the feeder, a short circuit in the feeder near the generating station —which is much more liable to occur than a short circuit on the busbars—is practically a short circuit at the busbars. The short-circuit current thus is the maximum which the generators can give, and its momentary or initial value (about 8 times the final value, with the usual amount of generator reactors) is so great as to make it necessary to set the circuit breakers for a considerable time limit so as to allow the momentary excess current to die out. During the short circuit, the busbar voltage is zero or practically so, thus there is no synchronizing power between the generators of the affected station section, between this station section and the other station sections, and between the synchronous converters of the substations fed by the affected generating station sections, and as due to the time limit of the circuit breakers the short circuit lasts an appreciable time, it is probable that during the short circuit the synchronous machines in the substations, and the generators have drifted out of step with each other so much, that at the opening of the short circuit they do not catch into synchronism any more, and a shut-down of a considerable part of the system results.

At the moment where a short circuit begins, the alternator field and thus the machine voltage still has full value, and the inductive short-circuit current thus is limited by the true self-inductive reactance only—the external and internal reactance of the generators, and the reactance of the feeder reactor, where such is used. At the moment when the short circuit begins, the busbar voltage drops from its normal previous value to zero, if no feeder reactor is used, or to the reactance voltage of the feeder reactor under the momentary short-circuit current, which may be a considerable part of the normal busbar voltage. If then the short circuit could be opened instantly before the alternator field can build down under the demagnetizing action of the inductive short-circuit current, the busbar voltage would recover instantly, to its previous value. If however the short circuit lasts any appreciable time, the alternator fields gradually—and rather rapidly—build down; the machine e.m.f. and the short-circuit current decrease (and the busbar voltage, with feeder reactor; without feeder reactor the busbar voltage is zero, as stated). If now the short circuit is opened, the busbar voltage does not instantly recover, but jumps up only to the value corresponding to the then prevailing field flux, and then only gradually—and rather slowly—recovers by the field flux again building up under the effect of the exciter voltage.

In turbo-alternators, the rate is very high; at which the machine fields build down under dead short circuit and thus the busbar voltage decreases which appears at the moment of opening the short circuit that is, the field is demagnetized in about half a second, so that with the delayed opening of the circuit breakers, the field has practically been demagnetized before the short circuit is opened; but the rate at which the voltage of the machine recovers after the opening of the short circuit, is rather slow; from three to five seconds or more (depending on the existing field exciting current and thus on the load previously on the machine.

With a power-limiting feeder reactor however, of a reactance which though small with regard to the rating of the feeder, is considerable compared with the reactance of the generating station (internal and external generator reactances), the rate of demagnetization of the field flux is greatly slowed down, due to the lesser demagnetizing action of the smaller short-circuit current; that is, the time required for the demagnetization of the machine field is of the magnitude of one and a half seconds. It is the larger, the higher the feeder reactance and the greater the number of generators connected to the busbars; it is smaller with lower feeder reactance and fewer generators on the busbars. If then the circuit breakers can be adjusted to open quicker, which appears feasible at the lesser short-circuit current, most of the field flux will still be there at the opening of the short circuit, and the voltage thus would, at the opening of the short circuit, jump back to nearer full value. Considering that even during the short circuit of the feeder cable, considerable voltage remains on the busbars, and that the duration of the short-circuit period is greatly reduced by the permissible quicker opening of the circuit breakers, it appears feasible, with a moderate value of feeder reactance, to limit the voltage drop and its duration in the affected station so that all or at least most of the synchronous apparatus on this station section will remain in step.

B. ARMATURE AND FIELD TRANSIENTS OF SYNCHRONOUS MACHINES

1. If p = number of poles
n_o = number of field turns per pole
i_{oo} = exciting current at no load, and
Φ_o = magnetic flux per pole

then $p\, n_o\, \Phi_o$ is the total number of interlinkages, and

$$\frac{p\, n_o\, \Phi_o}{i_{oo}}$$

the flux interlinkages per unit current; that is, the inductance of the field circuit in standard units,

$$L_o = \frac{p_o\, n_o\, \Phi_o}{i_{oo}}\, 10^{-8}\, h. \qquad (55)$$

is the inductance of the field.

If e_o is the voltage of the exciting current and i_o the (permanent) field current, the resistance of the field circuit is,

$$r_o = \frac{e_o}{i_o} \qquad (56)$$

this is the total resistance, field winding as well as external rheostat, etc., as both have the same action in the field transient. The duration of the field transient then is,

$$t_{oo} = \frac{L_o}{r_o} \qquad (57)$$

$$\text{or; } a_o = \frac{i}{t_{oo}} = \text{attenuation constant, and} \qquad (58)$$

$$\begin{aligned} i_1 &= I_o\, \epsilon^{-a_o t} \\ E &= E_o\, \epsilon^{-a_o t} \qquad (56) \end{aligned}$$

is the field discharge, or the transient by which the field current and thus the field magnetism and the induced voltage decrease on withdrawal of the exciting e.m.f., and

$$i_2 = i_o\,(1 - \epsilon^{-a_0 t})$$

$$E = E_o\,(1 - \epsilon^{-a_0 t}) \qquad (60)$$

is the charging transient of the field or the starting current of the field; that is, the transient by which field current and field magnetism and thus the induced voltage rises on the application of the exciting voltage, or recovers after the demagnetizing action of an excessive lagging current, such as a short circuit.

2. On inductive load, the armature current of an alternator demagnetizes, and to give the same field flux, the field exciting current thus has to be increased to counteract the demagnetizing armature reaction.

In a three-phase alternator, if n = number of armature series turns per pole per phase and I = armature current per phase (effective), the armature reaction per pole is,

$$F = 1.5\ \sqrt{2}\,n\,I \text{ ampere turns.}$$

and

$$n_o\,i = 1.5\sqrt{2}\,n\,I$$

thus gives the field current

$$i = 1.5\sqrt{2}\,\frac{n}{n_o}\,I \qquad (61)$$

$$= c\,I$$

$$\text{where } c = 1.5\ \sqrt{2}\,\frac{n}{n_o} \qquad (62)$$

is the reduction factor from armature to field.

Thus if i_o = field exciting current, and an additional inductive load of I amperes is put on the alternator, to keep the same magnetic flux and thus the same voltage, the field exciting current has to be increased from i_o to $i_o + c\,I$.

[This does not consider the change of the magnetic flux distribution resulting from the inductive armature current I, such as the increase of leakage flux, corresponding increase of saturation, etc., which requires a somewhat greater increase of field excitation. That is, c is somewhat greater than given by equation (**62**).]

3. Let E_o be the voltage produced by the no load field excitation i_{oo}. An inductive load of current I requires an

increase of the field excitation $c\,I$. This additional field current $c\,I$ would produce (assuming straight line saturation curve, that is below saturation) a voltage

$$E_2 = \frac{c\,I}{i_{oo}} E_o$$

thus gives an apparent internal reactance of the machine,

$$x_2 = \frac{E_2}{I}$$

or

$$x_2 = \frac{c\,E_o}{i_{oo}} \tag{63}$$

This is the effective or equivalent reactance of armature reaction. It is not a true reactance, and differs from the true self inductive reactance, that the latter is instantaneous, while the effective reactance of armature reaction, x_2, requires some time to develop.

Or, if I_o = full load or rated armature current, the effective reactance of armature reaction, given as a fraction (or in per cent), is

$$\xi_2 = \frac{x_2\,I_o}{E_o} = \frac{c\,I_o}{i_{oo}} \tag{64}$$

that is, the ratio of the field equivalent of the armature current, $c\,I_o$, to the no load field current i_{oo}, is the effective reactance of armature reaction, as a fraction.

Substituting (62) into (63) gives:

$$x_2 = 1.5\,\sqrt{2}\,n\,\frac{E_o}{n_o\,i_{oo}}$$

$$= 1.5\,\sqrt{2}\,n\,\frac{E_o}{F_o} \tag{65}$$

where F_o are the no load field ampere turns per pole, which give the voltage E_o.

4. Let E_o = voltage and i_o = field exciting current of an alternator. Let then an inductive load of current I_o be suddenly thrown on the alternator, for instance by a short circuit beyond a feeder reactance, or on the busbars. If then the reactance (true self-inductive reactance) of the circuit of this inductive load is x_1, the current is

$$I_o = \frac{E_o}{x_1} \tag{66}$$

This current I_o however, demagnetizes with the field equivalent $c\,I_o$ and the magnetic field flux of the machine, and thus the voltage must therefore decrease. The field flux, however, cannot change instantly, as in changing it induces a voltage and therefore produces a current in the field circuit, which opposes the change. That is, the field flux begins to decrease at such a rate as to induce in the first moment a voltage in the field winding, increasing the field current by $c\,I_o$, the field equivalent of the armature current.

That is, in the moment when the inductive load current I_o is thrown on the alternator armature, the alternator field current jumps from i_o to $i_o + c\,I_o$.

As, however, the exciting voltage e_o can maintain only the current i_o in the field circuit, the momentary excess field current $i_o + c\,I_o$ gradually decreases, down to the permanent value i_o, and with it decreases the field flux and the voltage of the machine, from the initial values Φ_o respectively E_o, to the final values,

$$\Phi_1 = \frac{i_o}{i_o + c\,I_o}\,\Phi_o = b\,\Phi_o \qquad (67)$$

and $$E_1 = \frac{i_o}{i_o + c\,I_o}\,E_o = b\,E_o \qquad (68)$$

and with it decreases the current, from the initial value I_o, to:

$$I_1 = \frac{i_o}{i_o + c\,I_o}\,I_o = b\,I_o \qquad (69)$$

where $$b = \frac{i_o}{i_o + c\,I_o} \qquad (70)$$

At the first moment the field flux is still Φ_o, the field exciting current however, is $i_o + c\,I_o$.

Field flux Φ_o and no load field exciting current i_{oo} give the field inductance L_o. Field Φ_o and field exciting current $i_o + c\,I_o$ thus give an apparent or equivalent or effective field inductance:

$$L = \frac{i_{oo}}{i_o + c\,I_o}\,L_o = b_o\,L_o \qquad (71)$$

where $$b_o = \frac{i_{oo}}{i_o + c\,I_o} \qquad (72)$$

That is, when throwing an inductive load on an alternator, field flux and voltage decrease by the demagnetizing armature reaction, and during the field transient, the mutual inductance of the armature current on the field reduces the field self-inductance from the true self-inductance L_o to an apparent or effective inductance $L = b_o L_o$.

As the resistance of the field circuit remains the same, the duration of the transient resulting from a sudden inductive load, such as a short circuit, is given by,

$$\begin{aligned} t_o &= \frac{L}{r_o} \\ &= b_o \frac{L_o}{r_o} \\ &= b_o t_{oo} \end{aligned} \tag{73}$$

and the attenuation constant is:

$$a = \frac{1}{t_o} = \frac{a_o}{b_o} \tag{74}$$

and the equations of the transient thus are as follows:

The armature current, changing from

$$I_o = \frac{E_o}{x_1}$$

to $I_1 = b\, I_o$

is $I = I_1 + (I_o - I_1)\, \epsilon^{-at}$ (75)

or $I = I_o \;\left\{ b + (1 - b)\, \epsilon^{-at} \right\}$ (76)

and the voltage then is

$$E = x_1 I \tag{77}$$

and, if of the reactance x_1, the part x is external, $x_1 - x$ internal in the machine or station, the terminal voltage is

$$E = x\, I \tag{78}$$

5. Equations and Notations.

r_o = resistance of exciting circuit

$$= \frac{e_o}{i_o} \text{ ohms} \tag{56}$$

e_o = exciter voltage

i_{oo} = field current at no load

i_o = field current at full load

L_o = true inductance of field exciting circuit.

$$= \frac{p\, n_o\, \Phi_o}{i_{oo}} 10^{-8} h \tag{55}$$

p = number of poles

n_o = number of field turns per pole

Φ_o = magnetic flux per field pole, produced by exciting current i_{oo}

t_{oo} = duration of field transient

$$= \frac{L_o}{r_o} \tag{57}$$

a_o = attenuation constant of field transient

$$= \frac{1}{t_{oo}} \tag{58}$$

$E = E_o \epsilon^{-a_o t}$

= field discharge transient (59)

$E = E_o (1 - \epsilon^{-a_o t})$

= starting transient of field (60)

x_1 = total self inductive reactance of alternator circuit

x = external part of this reactance

E_o = machine voltage before closing the circuit on reactance x_1

I_o = initial (or momentary maximum) value of the current in reactance x_1 (effective value)

$$= \frac{E_o}{x_1} \tag{66}$$

I_1 = final (or permanent) value of current

$$= b\, I_o \tag{69}$$

$$b = \frac{i_o}{i_o + c\, I_o} \tag{70}$$

$$c = 1.5 \sqrt{2} \frac{n}{n_o}$$

= reduction factor from armature to field circuit.

n = number of series armature turns per pole per phase.

$$I = I_o \left[b + (1 - b)\, \epsilon^{-at} \right] = I_1 + (I_o - I_1)\, \epsilon^{-at} \quad (75)\ (76)$$

= armature transient.

a = equivalent attenuation constant of transient

$$= \frac{1}{t_o} \tag{74}$$

$$t_o = b_o\, t_{oo} \tag{73}$$

$$b_o = \frac{i_{oo}}{i_o + c\, I} \tag{72}$$

E = total voltage

$$= x_1 I \tag{77}$$

E' = terminal voltage (78)

$= x\, I$

6. From these equations, and the numerical constants of the alternators, it is possible to calculate the action of short circuit or similar disturbance on the system, and the effect thereon of the reactance of the feeder reactor, by calculating, and plotting with the time as abscissas, the transients of induced voltage, current and terminal voltage resulting from the application of a short circuit. This gives for the moment of the opening of the circuit breaker the values of current, terminal voltage and induced voltage, and from the latter the value of the terminal voltage immediately after the moment of the opening of the circuit breaker. Calculating then, and plotting, with the latter as initial values, the field transient, gives the voltage recovery curve of the system. From the drop of voltage, and its duration, can be estimated whether any synchronous apparatus such as converters, operated from the affected generating station, are liable to be thrown out of synchronism, and whether by the voltage drop the synchronizing power of the station against other stations tied to it by busbar reactors is sufficiently lowered to fail to keep in step, and whether in this case the duration of the voltage drop is sufficient for the machines or stations to drift far enough apart so that at the voltage recovery they are not able to pull each other into step.

C. NUMERICAL CALCULATIONS

The constants of some typical steam turbine alternators of large size, three-phase machines of 25 and 60 cycles, are given in the following Table I.

Considering now as a numerical instance the effect of a feeder short circuit close to the busbars, on a 25-cycle, 9000-volt generating station of 60,000 kw. steam turbine alternator capacity, without and with feeder reactors, assuming that the short circuit is opened after one second. Assuming as fair average an equivalent effective reactance of armature reaction of 85 per cent; a true self-inductive internal reactance of the alternators of 6.8 per cent, and an external reactance, as power-limiting generator reactor, of 6 per cent.*

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*A reactance of *n* per cent means *n* per cent of the value of $\frac{\text{rated voltage}}{\text{rated current}}$.

TABLE I. CONSTANTS OF THREE-PHASE STEAM TURBINE ALTERNATORS

	25 Cycles					60 Cycles				
Rating, kw.	12,000	14,000	20,000	30,000	35,000	12,500	14,000	20,000	20,000	30,000
Speed, rev. per min.	750	750	750	1500	1500	1800	720	720	1200	1800
Mech. momentum, mega joules 3 M =	150	200	233	262	204	97	220	258	310	190
No. of poles, p =	4	4	4	2	2	4	10	10	6	4
Volts, terminal	9000	9000	9000	9000	9000	12,000	12,000	12,000	12,000	12,000
Amperes, full load	770	900	1280	1925	2250	750[1]	900[2]	1200[1]	1200[1]	1600[3]
Synchronous reactance, x_0, %	89	85	129	108	147	120	74	125	73	130
Internal true reactance, x_{11}, %	5.4	4.6	6.8	9.0	11.4	12	9	9	13	15
Eff. react. of armat. reaction, x_2, %	83.6	80.4	122.2	99	135.6	108	65	116	60	115
External reactance, x_{12}, %	6	6	4.13	6.25	....	....	....	....	....	....
Regulation, nonind. load, %	21	22	29	24	28	31	12	16	15	25
Field.										
No. of turns per pole, n_0 =	95	96	78	198	243	150	39	36	84	95
Amperes, no load: i_{00} =	250	307	328	226	185	122	385	348	286	235
Full nonind. load i_0 =	333	388	522	335	320	190	463	517	367	368
Exciter voltage, e_0 =	125	125	125	250	250	125	125	125	230	230
Flux per pole, ml. Φ_0 =	111	135	137.5	276	252	71.5	68	68	95.5	117.5
Armature.										
No. series turns per pole per phase, n =	12	10	10	12	12	10	4	4	5	6
$c = 1.5\sqrt{2}\frac{n}{n_0}$ =	.268	.221	.271	.129	.105	.141	.217	.236	.126	.134
$L_0 = \frac{p\,n_0\,\Phi_0}{i_{00}}10^{-8}$ (henrys)	1.69	1.69	1.30	4.84	6.62	3.53	.662	.703	1.80	1.90
No-load.										
$r_0 = \frac{e_0}{i_{00}}$ = (ohms)	.5	.408	.385	1.11	1.35	1.02	.325	.36	.806	.98

[1] at 80 % power factor. [2] at 75 % power factor. [3] at 90 % power factor.

TABLE I. CONSTANTS OF THREE-PHASE STEAM TURBINE ALTERNATORS—*(Continued)*

	25 Cycles					60 Cycles				
$t_{00} = \frac{L_0}{r_0}$ = (seconds)	3.38	4.14	3.38	4.36	4.91	3.45	2.04	1.96	2.23	1.94
$a_0 = \frac{1}{t_0}$ =	.296	.241	.296	.229	.204	.29	.49	.51	.448	.515
Full load.										
$r_0 = \frac{e_0}{i_0}$ = (ohms)	.375	.322	.24	.747	.781	.66	.27	.242	.628	.625
$t_{00} = \frac{L_0}{r_0}$ = (seconds)	4.51	5.25	5.42	6.48	8.47	5.35	2.45	2.91	2.92	3.05
$a_0 = \frac{1}{t_{00}}$ =	.222	.191	.185	.154	.118	.187	.408	.344	.342	.328
Short circuit after full load.										
Initial current, $I_0 = \frac{E_0}{x_1}$ =	6700	8450	11,700	12,700	15,200	6200	10,0 0	13,300	9200	10,600
$b = \frac{i_0}{i_0' + c\,L_0}$ =	.156	.172	.141	.170	.167	.180	.176	.142	.240	.206
Final current: $I_1 = b\,I_0$ =	1050	1450	1650	2160	2530	1120	1760	1880	2200	2180
$b_0 = \frac{i_{00}}{i_0 + I c_0}$ =	.117	.137	.089	.114	.0965	.115	.146	.095	.187	.137
$L = b_0 L_0$ = (henrys)	.198	.232	.116	.551	.638	.405	.097	.067	.337	.260
$t_0 = \frac{L}{r_0}$ = (seconds)	.527	.72	.482	.737	.816	.615	.358	.276	.547	.418
$a = \frac{1}{t_0}$ =	1.90	1.40	2.08	1.35	1.23	1.63	2.77	3.63	1.83	.239

Let the duration of the field transient (full-load condition) be $t_{oo} = 4.51$ sec., and the field attenuation constant thus $a_o = 0.222$

The field transient then is given by

$$e = e_1 + (e_o - e_1)\,\epsilon^{-a_o t} \tag{79}$$

where e_o = voltage of the machine at the moment $t = o$, for instance, initial voltage in the moment when a short circuit has been opened.

e_1 = machine voltage corresponding to the exciter voltage impressed upon the machine, that is, final voltage of the machine.

Consider the three cases:

a. No feeder reactor, thus dead short circuit on the busbars.

b. A feeder reactor of 0.5 ohms per phase, or 2.9 times the true reactance of the generating station, or 37 per cent.

c. A feeder reactor of 0.7 ohms per phase, or 4.05 times the true reactance of the generating station, or 52 per cent.

1. With 12.8 per cent self-inductive reactance, the momentary or initial short-circuit current, as a fraction of the rated current of the station, is given by

$$i_o = \frac{1}{.128} = 7.8$$

From the machine constants, it follows that

$$b = 0.172$$
$$b_o = 0.129$$

Thus, final short-circuit current, as a fraction of rated current is

$$i_1 = b\, i_o = 1.34$$

and, duration of the short-circuit transient is

$$t_o = b_o\, t_{oo} = .582 \text{ sec.}$$

Thus, attenuation constant is

$$a = \frac{1}{t_o} = 1.72$$

and, equation of the short-circuit current transient,

$$\begin{aligned} i &= i_1 + (i_o - i_1)\,\epsilon^{-at} \\ &= 1.34 + 6.46\,\epsilon^{-1.72t} \end{aligned} \tag{80}$$

This current is plotted in Fig. 15, in dotted lines, as i_1.

Proportional hereto is the induced generator voltage, which is given, as a fraction of the induced voltage immediately before the short circuit, by the transient,

$$\begin{aligned} e &= b + (1 - b)\,\epsilon^{-at} \\ &= 0.172 + 0.828\,\epsilon^{-1.72t} \qquad (81) \end{aligned}$$

plotted as curve e_1 in Fig. 15.

The terminal voltage is zero during the short circuit; at one second, with the opening of the short circuit, the terminal voltage jumps back to the same fraction of the terminal voltage before the short circuit, to which the induced voltage has dropped, that is, to the point D_1 of curve e_1, 32.2 per cent of the previous terminal voltage.*

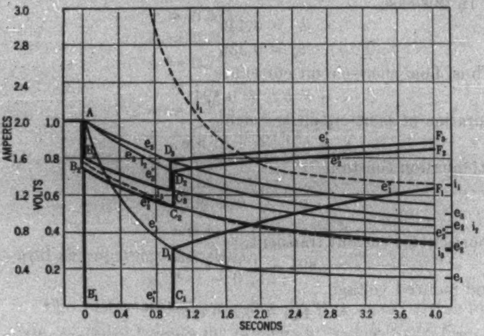

Fig. 15

From this point, of 32.2 per cent voltage at one second, the voltage now gradually recovers on the field transient, equation (73)

$$\text{for } e_1 = 1:\quad e_o = 0.322:\quad a_o = 0.222, \text{ thus,}$$
$$e^o = 1 - 0.678\,\epsilon^{-0.222t}$$

During the short circuit, the terminal voltage thus traverses the values of A, B_1, C_1, D_1, F_1 in Fig. 15. As seen, the voltage recovery is very slow, and it is not probable that any synchronous apparatus will remain in step.

The short-circuit current after one second—which the circuit breaker has to open—is 2.5 times the rated current, or 150,000 kv-a.

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*Assuming that the conditions of the external load have not materially changed or have no material effect, which latter may be assumed approximately, since the short-circuit currents are very large compared with the normal load currents.

2. With 12.8 per cent self-inductive generator reactance, and 37 per cent feeder reactance, the total reactance is 49 per cent, the initial short-circuit current is thus:

$$i_o = \frac{1}{0.498} = 2.01$$

In this case,

$$b = 0.446$$
$$b_o = 0.335$$

Thus, final short-circuit current is

$$i_1 = b\, i_o = 0.894$$

duration of short-circuit transient,

$$t_o = b_o\, t_{oo} = 1.51 \text{ sec.}$$

attenuation constant,

$$a = \frac{1}{t_o} = 0.662$$

short-circuit current transient,

$$i = .894 + 1.107\, \epsilon^{-662t}$$

and induced voltage,

$$e = 0.446 + 0.554\, \epsilon^{-662t}$$

The terminal voltage during the short circuit now is not zero, but is the same fraction of the induced voltage, as the reactance of the feeder reactor is to the total self-inductive reactance.

$$\frac{x}{x + x_1} = \frac{37}{37 + 12.8} = 0.744$$

Thus, the terminal voltage during the short circuit is,

$$e^o = 0.744e = 0.332 + 412\, \epsilon^{-662t}$$

The transients of short-circuit current, induced voltage and terminal voltage are shown in Fig. 15 by the curves i_2, e_2 and e_2^o.

As seen, at the moment of short circuit, the terminal voltage makes a sudden drop to curve e_2^o, to 0.744 of the previous value, then follows the curve e_2^o for one second; then on opening the short circuit suddenly jumps, from 0.543 on e_2^o to 0.732 on e_2, and then gradually recovers on the field transient given by the equation,

$$e_o = 1 - 0.268\, \epsilon^{-0.222t}$$

The terminal voltage thus traverses the broken curve $A\, B_2\, C_2\, D_2\, F_2$.

As seen, while the voltage recovery after the short circuit is slow, the terminal voltage even during the short circuit remains above half value.

The current after one second, when opening the short circuit, is only 1.46 times full-load current.

c. In the same manner the curves are calculated for the 52 per cent feeder reactance, giving,

$$i_o = 1.55$$
$$b = 0.511$$
$$b_o = 0.383$$
$$i_1 = 0.79$$
$$t_o = 1.73 \text{ sec.}$$
$$a = 0.578$$
$$i = 0.79 + 0.76\,\epsilon^{-.578t}$$
$$e = 0.511 + 0.489\,\epsilon^{-.578t}$$
$$\frac{x}{x + x_1} = 0.802$$
$$e_o = 0.41 + 0.392\,\epsilon^{-0.578t}$$

and the recovery transient,

$$e_o = 1 - 0.215\,\epsilon^{-0.222t}$$

The values are shown in Fig. 15 as i_3, e_3, e_3°, giving for the terminal voltage the broken curve $A\ B_3\ C_3\ D_3\ F_3$.

The short-circuit current when the circuit breaker opens, after one second, is only 9 per cent above full-load current.

Table II gives the numerical values of voltage and current, at the beginning of short circuit, after one second and final, as fractions of rated voltage and current.

TABLE II. SHORT CIRCUIT ON 60,000-KW. 25-CYCLE 9000-VOLT STATION

			Resistance of Feeder Reactor		
			(1) None	(2) 0.5 ohms	(3) 0.7 ohms
Duration of Field Transient, seconds		$t_{00} =$	4.51	4.51	4.51
		$a_0 =$	0.222	0.222	0.222
Duration of Armature Transient, seconds		$t_0 =$	0.582	1.51	1.73
		$a =$	1.72	0.662	0.578
		$b =$	0.172	0.446	0.511
		$b_0 =$	0.129	0.335	0.383
Short-Circuit Current	Initial	$i_0 =$	7.8	2.01	1.55
	After 1 sec.	$i =$	2.5	1.46	1.09
	Final	$i_1 =$	1.34	0.804	0.79
Induced Voltage	Initial	$e_0 =$	1.00	1.00	1.00
	After 1 sec.	$e =$	0.322	0.732	0.785
	Final	$e_1 =$	0.172	0.446	0.511
Terminal Voltage	Initial Before	$e_0 =$	1.00	1.00	1.00
	Initial After	$e_0^0 =$	0	0.744	0.802
	After 1 sec. Before	$e^0 =$	0	0.543	0.63
	After 1 sec. After	$e =$	0.322	0.732	0.785
	Final	$e_1^0 =$	0	0.322	0.41

The question then arises of the bearing of these voltage curves, Fig. 15, on synchronous operation.

During the period of dead short circuit or zero terminal voltage, $B_1 C_1$, there is no synchronizing power. There is no load on the generators beyond the $i^2 r$ and the load losses which are moderate even at the initial high momentary short-circuit current, and rapidly decrease with the decreasing short-circuit current. Thus the alternators speed up, until the governor shuts off steam or the emergency governor trips. The former necessarily must take an appreciable time to avoid trouble from steam governor hunting. Usually, the speeding up will occur until the emergency trips and cuts off steam, about 10 per cent above normal speed. Then slowing down occurs until the machines are again put on their governors. The speeding up however occurs at different rates, due to the differences in the momentum of different machines; the speed of tripping cannot be exactly the same, as absolute reliability rather than exactness of speed is the main requirement of the emergency cut off, and furthermore, some speeding up will continue after the closing of the governor, due to the steam contained between the cut off and the turbine, and in the turbine.* Thus if during this period the machines do not have sufficient power to keep each other rigidly in step, at the time when the short circuit is cleared and the voltage returns, the machines probably have drifted so far apart that they cannot pull each other in step again but continue slipping out of synchronism, short-circuiting each other and keeping zero terminal voltage indefinitely.

Let P = load on the machine before the short circuit. With the load taken off, the power P then accelerates the momentum M of the machine, until the steam is cut off. This means

$$2\, s\, M = P\, t \qquad (82)$$

where s is the increase of speed in fraction, and t the time, or more accurately:

$$M\left[(1 + s)^2 - 1\right] = P\, t \qquad (83)$$

however, (82) is sufficiently accurate for our purposes.

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*One cubic meter of steam at 14 atmospheres (200 lb.) retained between the turbine and the steam cut off, would speed up a 35,000-k.w. steam turbine alternator by more than one per cent, after the cutting off of the steam.

Thus, $s = \frac{P}{2M} t$ (84)

Substituting the values of P and M from Table I, gives the acceleration curves. In Fig. 16 are given such curves for four 25-cycle machines, the 12,000, 14,000, 30,000 and 35,000 kw., as (1), (2), (3) and (4). As seen, the acceleration is very rapid, from 3.5 to 8.6 per cent per second.

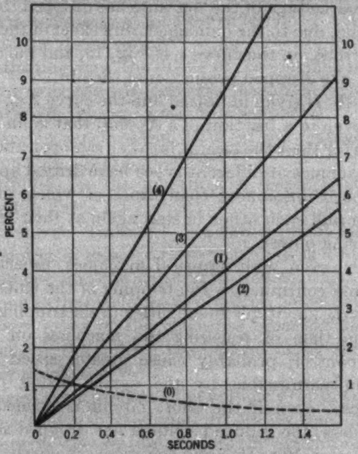

Fig. 16.

The limits of synchronizing power, that is, the maximum speed difference from which the machines can pull each other into step promptly, is given by equation (38) as,

$$2 s_0 = E \sqrt{\frac{\sin \alpha}{2 \pi f z M}}$$

Chosing the same values as in Fig. 15, that is, per 10,000 kw. rated machine capacity,

$z = 2 x_1 = 2.08$ ohms.

$3 M = 125 \times 10^6$ joule

$f = 25$ cycles

$\alpha = 90$ deg.

$$E = \frac{9000}{\sqrt{3}} = 5200 \text{ volts.}$$

gives $$2\,s_0 = 1.4 \text{ per cent.}$$

In the moment however, when the short circuit opens, the induced voltage of the machine has dropped from the initial value, due to the demagnetizing effect of the short-circuit current, on the curve e_1 of Fig. 15, and the critical speed $2\,s_0$ has dropped proportional thereto.

In Fig. 16, is given in dotted line the curve $2\,s_0$, as (0). As seen, even in a fraction of a second, that is, in a time much shorter than the circuit breaker can open the short circuit, machines of different types have drifted apart by greater speed differences than those at which the machines can pull each other in step again at their reduced synchronizing power.

However, even with identical machines, especially if the speeding continues to the tripping of the emergency steam valves, inevitable inequalities in the tripping speed and in the time of restoring the machines on steam governor control, probably cause greater speed differences than permissible by the synchronizing power. Furthermore, even if the short circuit is opened in a second or less, the induced voltage has dropped so considerably—e_1 in Fig. 15—and the recovery curve—e_1^0 in Fig. 15—is so slow, that the machines cannot immediately take load, and speeding up continues for some time.

Thus it may be expected that with a dead short circuit at or near the busbars of a high-power steam turbine station, the generators drop out of synchronism and are not able to pull back promptly into synchronism, but begin to drift indefinitely, slipping past each other at zero voltage.

For a machine to remain in synchronism with other machines, with full load steam supply and the load thrown off by a short circuit, etc., the machine must be able to transfer full load to other machines, within its limits of synchronizing power, that is, with a phase displacement not exceeding 90 deg.

The maximum power transfer between two machines is given by equation (11) as

$$P = \frac{E^2}{z} \sin \alpha$$

where z is the total impedance between two machines, and α may be assumed as 90 deg. This gives

$$E = \sqrt{z P} \tag{85}$$

as the minimum voltage E, at which the machine will keep in synchronism at a power difference P between the load and the steam supply.

Substituting for P the rating of the machine per phase, and for z twice the self-inductive reactance (external and internal), per phase, gives the minimum voltages of remaining in synchronism, that is, the voltage limit of synchronizing power.

This gives, for the machines in Table I, the values recorded in Table III.

As seen from Table III, the voltage limit of synchronizing power in most of these machines is a little below half voltage, and the conclusion follows that:

As long as the machines do not drop below half voltage, no danger exists of the machines breaking out of synchronism by the sudden loss of load under short circuit or other accidents.

If a feeder reactor limits the voltage drop of the station, due to a feeder short circuit, to 50 per cent or less, the machines in the station remain in synchronism, even when speeding up due to the release of load, when tripping their emergency steam cut offs, etc., and the voltage thus recovers immediately on the opening of the short circuit.

As seen from Fig. 15, this is the case even with the smaller feeder reactor, and under the given conditions, the smaller feeder reactor thus should offer complete protection against loss of synchronism of the station as result of feeder short circuit.

Similar relations then exist between generating station and synchronous machine loads, such as converters and synchronous motors.

The synchronous converter probably represents by far the most frequent synchronous machine load. Its internal characteristics are somewhat similar to those of the steam turbine alternator, that is, high effective reactance of armature reaction and low true self-inductive reactance, and it therefore is probable that about the same numerical relations pertain.

TABLE III. VOLTAGE LIMIT OF SYNCHRONIZING POWER

	25 Cycles					60 Cycles				
Rating, kw	12,000	14,000	20,000	30,000	35,000	12,500	14,000	20,000	20,000	30,000
Speed, rev. per min	750	750	750	1500	1500	1800	720	720	1200	1800
Volts per phase e =	5200	5200	5200	5200	5200	6900	6900	6900	6900	6900
Rated current i =	770	900	1280	1925	2250	750	900	1200	1200	1600
$\frac{e}{i}$ = (ohms)	6.76	5.78	4.07	2.71	2.32	9.2	7.67	5.76	5.76	4.32
x_1 = (%)	11.4	10.6	10.93	15.25	11.4	12	9	9	13	15
$s = 2x_1$ = (ohms)	1.54	1.225	.89	.823	.528	2.21	1.38	1.04	1.495	1.30
P per phase (watts) 10^6	4.0	4.67	6.67	10.0	11.67	4.17	4.67	6.67	6.67	10.0
$E = \sqrt{Ps}$ = (volts)	2480	2390	2430	2870	2480	3030	2530	2630	3150	3610
E, in %	47.8	45.0	46.9	55.2	47.8	43.8	36.6	38.0	45.6	52.1

Such values of feeder reactors, which are sufficient to guard the generating stations against loss of synchronism by maintaining the station under feeder short circuits, above the voltage limit of synchronizing power, may in general be expected also to guard the synchronous converters in the substations against being thrown out of step, that is, shut down, by the shock on the system due to feeder short circuit.

DISCUSSION ON "STABILITY OF HIGH POWER GENERATING STATIONS" (STEINMETZ), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

R. E. Doherty: It has been my experience, and perhaps that of many others, in listening to discussions of synchronizing power, and reading some papers about the values of reactance and resistance in the circuit to get the maximum synchronizing power, that there has been a good deal of confusion as to what is meant. In addition to that, in connection with Dr. Steinmetz' paper, I wish to call attention to some equations he has given regarding the relations between the feeder connection of the armature and the field circuit.

In studying the several conditions of parallel operation of two groups of alternators as discussed in the paper, the question arises, what shall be taken as the synchronizing power. Is it the power delivered by the generator, or a group of generators, which is ahead in phase, or is it the power received by the generator, or group, which is behind? Obviously if there is negligible resistance in the circuit connecting the two groups, the power delivered from the first group must be the power received by the second. The question arises only when there is a significant $I^2 R$ loss in the connecting circuit.

An alternator is connected through an impedance $r + j x$ to a bus of infinite capacity. The governor ceases to function, and the prime mover tends to drive the alternator ahead. The power which tends to hold the machine in step must obviously include the $I^2 R$ loss in the circuit, as well as the power delivered to the system. That is, the maximum synchronizing power, must be the maximum total electrical power generated by the alternator. Suppose, on the other hand, that a negative torque is applied to the shaft tending to pull the alternator back out of step. Then, obviously, the power lost in $I^2 R$ although included in the above case, cannot in this instance produce torque in the alternator, and therefore cannot be included in the power which tends to hold it in step. What remains, is the power of the lagging alternator. Thus, again, the maximum synchronizing power must be the maximum power of the alternator; in this case, the maximum power which is received by the alternator and converted to mechanical power. In either case, therefore, it is the electrical power of the machine under its particular conditions; that is

$$P = E\,I \operatorname{Cos} \theta$$

Where

E = actual generated voltage corresponding to the magnetic flux

I = current

θ = phase displacement between E and I.

Going a step further, suppose in the above case, that another alternator, duplicate of the first, is substituted for the infinite system, and that the two machines connected through the impedance are pulled apart in phase. What is now the synchronizing power? If we say it is the electrical power of the alternators, as above, we shall have two values. If, on the other hand, we say it is the power holding the two alternators together, then what is that power? Thus it is obviously a matter of definition. The present paper defines it in equation (**9**) as neither the power delivered nor the power received but the average of the two; that is, the power delivered by the first, less half the $I^2 R$ loss; or the power received by the second, plus half the $I^2 R$ loss; in other words, the power which a wattmeter would indicate if placed at that point in the circuit where the resistance on either side is half the total resistance.

However, from the point of view that the synchronizing power is the power that holds the alternator, or stations, or station sections together, it might logically be considered as that power which leaves the first and is received by the second; that is, as the power received by the lagging generator or station bus. Because the problem is often stated thus: What is the maximum power which can be transmitted across a given circuit? The interest, therefore, in such a case, is less in how much power can be delivered to that circuit than in how much can be put across it. However, when that maximum is reached, there is a definite value, not a maximum, of the power delivered to the circuit, and therefore a definite average. Thus, so far as getting the facts is concerned, it makes little difference which one is calculated. It all depends on what the problem is. The present problem is largely the calculation of power exchanges under unstable conditions in which a given group of alternators is alternately ahead and behind another group, or in which the two groups are slipping by each other; and, therefore, in which the machines are alternately delivering and receiving power. It is therefore more satisfactory to deal with the average.

Consider briefly a few criterional relations between resistance and reactance for maximum synchronizing power, P_m under different assumed conditions:

1. Two buses each held at constant potential E, connected by a circuit of impedance $r + j\,x$.

(a) From equation (**13**), P_m occurs, for any given value of r, when

$$x = r$$

If P_m were considered as power received by the lagging bus (equation **6**) instead of the average power, the criterion would be

$$x = \sqrt{3}\,r$$

(b) From equation (**13**), P_m occurs, for any given value of x, when

$$r = 0$$

The question of what voltage or what reactance should be used in these relations is entirely a matter of what voltage is going to be considered. If you take the terminal voltage of the alternator and hold that constant, then obviously you must consider not the internal voltage of the machine, but only the impedance between the buses which are held at the voltage of the machine. On the other hand, if you assume that the magnetic flux in the machine is constant, then you must take, not the external impedance only but also the internal impedance, that is, the true leakage reactance.

If you go a step further and consider, not that the flux is constant, but that the nominal electromotive force (a fictitious voltage in proportion to the field excitation) is constant, then you must consider not only the external impedance and the internal true leakage but also the armature reaction. Depending upon which one of these voltages you take, you come to the proper value of reactance to be considered in the criterional relation.

2. Two alternators or two groups of alternators with equal "nominal e. m. f.," equal leakage reactance x_1 and equal effective reactance of armature reaction x_2, connected by a circuit of reactance x.

(a) Slipping by each other out of synchronism. The total effective reactance (including internal) between the alternators is, from equation (**47**)

$$2\,x_1 + x_2 + x$$

and P_m is obtained when (by equation **54**) the external reactance is

$$x = x_2 - 2\,x_1$$

(b) Steady strain. The total effective re-reactance in this case is

$$2\,x_1 + 2\,x_2 + x$$

For any given value of external reactance x, the maximum power transfer, neglecting resistance, is

$$P_m = \frac{e_0^2}{2\,x_2 + 2\,x_1 + x}$$

The value of external reactance x for maximum P_m in this case is obviously zero, although for the above case of slipping it was

$$x = x_2 - 2\,x_1$$

The reason is, if two machines are slipping by each other, then the total magnetic flux in the machines is determined by the leakage reactance. If you have no reactance, you have no magnetic flux, and therefore no torque, whereas if you have too much reactance you limit the flow of current, so that a compromise between the two gives the most favorable value.

Thus, summing up, the synchronizing power is always increased by a further decrease in the resistance of the circuit; in almost all cases it is increased by a further decrease in the reactance. Two exceptions to the latter are:

1. In slipping by each other out of synchronism, the machines are practically short-circuited on each other. Hence with negligible reactance there could be no magnetic flux, and therefore no torque; on the other hand, with too much reactance, the current is limited. Therefore there is a most favorable value of reactance as given in equation (**54**) of the paper.

2. There must, of course, always be some resistance in the circuit. The maximum synchronizing power will be increased by decreasing the reactance until the latter is of practically the same magnitude as the resistance. Beyond this, the gain in lower impedance is more than offset by loss in phase displacement.

There are some cases in which the criterional relations between x and r for maximum synchronizing power involve the true reactance only; and others in which not only the true reactance but also the much larger effective reactance of armature reaction must be taken in relation to r. The former includes those cases where, by operating conditions, the buses of two stations, or of two station sections, paralleled over a circuit of impedance Z, are actually held at the same constant potential to the limit of synchronizing power. Here the circuit reactance and resistance only should be considered. It also includes those cases where the magnetic flux of the alternators can be assumed to be constant, under which conditions, the true leakage reactance of the alternator, as well as the external circuit reactance should be taken. In these cases there is some probability of the particular value of resistance, settled upon by considerations of economy or regulation, becoming a significant factor in synchronizing force, on account of the relatively low value of the reactance involved.

However, in those cases where, instead of voltage, the field m. m. f., that is the "nominal e. m. f.," is held constant, and the voltage is allowed to drop, the large "synchronous reactance," including armature reaction, must be considered. In such a case, the effect of r is usually negligible—not because the effect of r has been changed, but because the armature reaction, assumed to be neutralized in the above cases by increased field current, is in this instance, free to act; and it simply overshadows the other factors, and seriously reduces the synchronizing power. However, the criterion still holds. If such a case existed in which the resistance were greater than the synchronous reactance, then it would help matters to add reactance.

Illustrating three cases:

1. Equal Bus Potentials Held Constant.

Assume that a 25 per cent reactor is inserted in the connecting circuit, which itself contains 2 per cent resistance, 1 per cent reactance, based on the normal rating of the line equal to 1000 kw. Thus $x = 26$ per cent, $r = 2$ per cent. The maximum power which can be transmitted from one bus to the other is, by reduction of equation (**6**),

$$P_m = \frac{E^2}{Z} \sin^2 \alpha$$

If the average power, given by equation (**9**) is considered, the maximum would be, by equation (**11**),

$$P_m^1 = \frac{E^2}{Z} \sin \alpha$$

For $\alpha = 90$ deg., that is, negligible resistance, these are of course identical. However, taking the former case,

$$\alpha = \tan^{-1} 13$$

Practically $\alpha = 90$ deg. and $Z = X$

$$P_m = \frac{1.0^2}{0.26} \times 1000 = 3850 \text{ kw.}$$

Taking out the reactor, the circuit is: 1 per cent reactance, 2 per cent resistance,

$$\alpha = \tan^{-1} 0.5 = 26.5°$$

$$Z = 2.23 \text{ per cent}$$

$$P_m = \frac{1.02}{0.0223} \times 0.39^2 \times 1000 = 6870 \text{ kw.}$$

Thus, higher than before; but by increasing x from 1 per cent to

$\sqrt{3}\, r = 3.46$ per cent

P_m is further increased.

$\alpha = \tan^{-1} 1.73 = 60$ deg. and $Z = 4$ per cent

$$P_m = \frac{1.0^2}{0.04} \times 0.866^2 \times 1000 = 18{,}750 \text{ kw.}$$

An increase or decrease of reactance from the above value will decrease P_m. This illustrates the particular case where r becomes a significant factor—the case, where the circuit reactance is low, and the alternator reactance is not considered.

2. Same case as above, except that the true leakage reactance of the alternator, or the resultant reactance of a group, based on the line rating of 1000 kw. is 5 per cent, and that the magnetic flux of the alternator is kept constant. In this case the alternator reactance must be considered.

The total impedance is now: $x = 36$ per cent, $r = 2$ per cent; or without reactor, $x = 11$ per cent, $r = 2$ per cent, $z = 11.15$ per cent.

For first case:

$$P_m = \left(\frac{1.0^2 + 0.05^2}{0.36}\right) \times 1000 = 2780 \text{ kw.}$$

Second case:

$$P_m = \left(\frac{1.0^2 + 0.05^2}{0.1115}\right) \times 1000 = 8950 \text{ kw.}$$

3. Holding Constant Field Excitation: Here, both terminal voltage and the magnetic flux decrease. Assuming the synchronous reactance, including true leakage reactance as well as the effect of armature reaction, is 50 per cent based on line rating, then the nominal e. m. f. is

$e_0 = \sqrt{1.0^2 + 0.5^2} = 1.12\, E$

The impedance is: $x = 126$ per cent; $r = 2$ per cent; or without reactor, $x = 101$ per cent, $r = 2$ per cent.

For the first case,

$P_m = 1.12^2 \times 1000 = 1000$ kw.

For the second case,

$$P_m = \frac{1.12^2}{1.01} \times 1000 = 1240 \text{ kw.}$$

Regarding the latter part of the paper covering "Armature and Field Transients of Synchronous Machines" attention should be called to an approximation which, although justified, of course, by the character of the present problem cannot be thus applied in general. Equation (**61**) assumes equality between the effective m. m. f. of armature reaction, due to inductive armature current, and the field m. m. f. required to balance it. This, of course, assumes perfect magnetic coupling. Actually, the leakage flux modifies the relation.

Considering that the machine is operating on sustained short circuit—as in the ordinary synchronous impedance test—then the armature flux is zero (neglecting resistance drop), and is

$$(n_0\, i)\, M + (1.5\, \sqrt{2}\, n\, I)\, L_1 = O$$

where M and L_1 are respectively the permeance of the path of mutual flux, and of the path of the total armature flux.

Hence equation (**61**) would become

$$i = 1.5\, \sqrt{2}\, \frac{n}{n_0}\, \frac{L_1}{M}\, I$$

and (**62**)

$$c = 1.5\, \sqrt{2}\, \frac{L_1}{M}\, \frac{n}{n_0}$$

That is, the equations are modified by the ratio

$$\frac{L_1}{M}$$

This ratio is of the order of 1.1 to 1.2 in large generators. In the generators under consideration, it is near 1.1.

This relation holds also for sudden short circuit, since, neglecting the armature transient, as assumed in the paper, the distribution of flux in the several leakage paths is the same provided there is no damping winding in the pole face and that the rotor is laminated. Otherwise, part of the secondary induced ampere turns will be in the massive steel rotor or in the damping winding, and therefore will not appear in the main field winding. This, of course, would change the flux distribution, sending more of the flux through the armature leakage paths than would exist there (assuming the same total flux) under the condition of sustained short circuit.

V. Karapetoff: The problem of computations relating to the stability and oscillations of a large electrical system is not new. It used to be an important problem in the days of reciprocating engines, both steam and gas, especially in Europe. Quite a number of articles appeared in those days in various European magazines, and the gist of it all has been incorporated by Dr. E. Arnold in the Fourth Volume of his "Wechselstromtechnik." That theory, I think, might well be revived now, in so far as the method of attack is concerned, compared with Dr. Steinmetz' present method of attack. In a large system, such as the one here under discussion, the energy relations are much more general and safe to go by than are the voltage and current relations. Electrical oscillations, from a formal mathematical point of view, are analogous to mechanical oscillations, and electrical engineers interested in the problems of protective apparatus and disturbances on the systems, would do well to get posted on the old classical methods of dealing with mechanical oscillations.

Take a comparatively simple case of an alternator working parallel with what may be called a system of limitless capacity, so that this alternator can hunt without affecting the voltage or the frequency of the rest of the system. You have to consider variations in several kinds of energy, *viz.*, stored kinetic energy, mechanical energy, stored magnetic energy, stored electrostatic energy and also energy dissipation into heat. At any instant there is a balance of power corresponding to these amounts of energy, so that if you start with an equation containing a balance of energy you are on a more secure ground than by beginning with components of energy such as voltage or current.

The equation that expresses oscillations of an alternator against an infinite system contains a term proportional to the instantaneous angular velocity; a term that is the first derivative of that angular velocity with respect to time, also an integral of the same velocity with respect to time. On the other side of the equation are the forces which disturb the original equilibrium.

Now, if you compare this equation with that of a simple electric system, containing capacity, reactance and resistance, with an applied non-sinusoidal electromotive force, the analogy is so complete that it is not necessary to solve the differential equation of oscillation of the mechanical system. We can write down the solution directly and we can also draw the locus of the vector of one of the variables in the problem, such as capacity and inductance. Dr. Arnold whom I mentioned before, has worked out the method in a very elaborate way in his book. I have presented the essentials of this method for English speaking readers in an article on "Hunting and Parallel

Operation of Synchronous Machines," Sibley; *Journal of Engineering*, March 1920.

Harry R. Woodrow: I do not agree with Mr. Doherty on the question of synchronizing power as synchronizing power should be defined as the amount of power that one generator can receive from a system. The two definitions give entirely different conditions of stability of the system. Taking the ratio of x divided by r as abscissa and synchronizing power as ordinates the curves given by the two definitions are shown on Fig. 1. From these curves it is evident that the power received is a maximum when the ratio x/r is equal to $\sqrt{3}$ with a given value of resistance, whereas, the maximum power delivered by the leading unit occurs when the ratio x/r is equal to zero, that is, zero reactance.

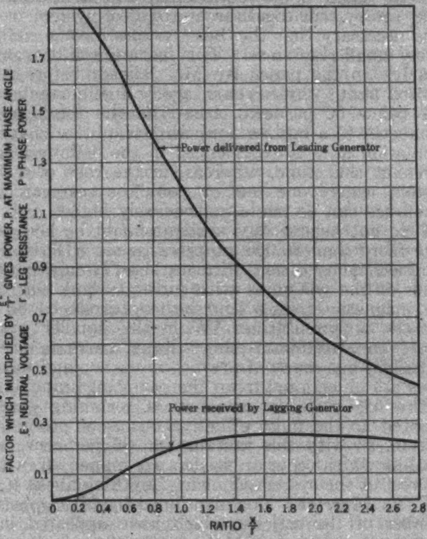

Fig. 1

For instance, if we have two systems connected together without reactance we know it is impossible to hold the two systems in step, whereas, the synchronizing power is high by definition of power sent out from the leading unit. By the definition of the amount of power received, it is clear that there is no synchronizing power under this condition and the amount of reactance should be increased to a point where it approaches the $\sqrt{3}$ times the resistance for stable operation. This plan has been followed on tying systems together in New York with good results.

There is a considerable difference in the adaptability of bus reactors to systems which are dependent on the character of the system. First, if we take a system which is interconnected, and it is desired to hold the system together at times of system trouble, it is necessary that the bus reactors have a comparatively small reactance. This feature was brought out clearly in the paper by Mr. Johnson before this Institute about three years ago. If the reactance is not below 50 per cent based on the total capacity connected to a bus section, there is quite a chance for oscillation as was experienced in the definite example given at that time, whereas, in the case of a radial system, where one section could be segregated, the reactance can be made considerably higher.

I do not believe any system should be laid out to give more than 80,000 amperes (mean effective value symmetrical condition) into any short circuit that could occur on the system as with currents above this value, the magnetic stresses and heating produced on conductors is prohibitive. With the smaller systems where the maximum short-circuit currents are not excessive, bus reactors are not necessary and in many cases are objectional from the operating standpoint.

The use of feeder reactors is becoming generally adopted as the most effective means of protecting a system, as with this installation, 95 per cent of the troubles which occur on feeders are nipped off, without the rest of the system knowing anything about it.

E. G. Merrick: As mentioned by Mr. Doherty, a number of the articles which have appeared in the past have so confused the terms "synchronous reactance" and "instantaneous reactance," that the formulas derived have not been of much value.

In regard to the point of machines getting out of step, and staying out of step for considerable periods, that is a condition which can be made worse if one or more generators in a plant are being operated under hand control. In one of the large hydroelectric plants a field killing device is used in connection with a short-circuit suppressor, and tests have shown that when two machines are operating in parallel and one is hand controlled, the time required to bring the machines back into synchronism is increased very considerably. Hand control is usually a temporary condition, and not of very great interest; however, in tying together plants in the large systems that are contemplated, it has been considered advisable, especially where automatic stations are concerned, to operate certain units without governors in order to simplify the equipment. This may lead to some difficulty under short-circuit conditions.

In regard to the use of reactors, no one system of application can be considered the best—each installation must be studied on its own merits, and proper distribution of reactors made.

The results obtained from the actual experience of operating companies have demonstrated the value of reactors—plants which have outgrown their original contemplated capacity have been able to continue the use of original equipment without overtaxing the buses, switches, etc.

There is no question but that stresses are reduced in buses by the use of reactors, and there is no question but that the duty on switches can be reduced by reactors if properly applied. Within certain limits, however, it is not entirely certain how much effect the reactor has. The Institute has standardized the rupturing duty of switches as being the current interrupted at normal voltage. If a short circuit occurs close to the generator, without reactance interposed, and is maintained for a certain period, the re-established voltage is certainly not normal, whereas, with infinite capacity and large enough reactance, the voltage rises instantly to normal after the interruption of the short-circuit. Between these limits of low voltage—high current and high voltage—low current there must be some point where the reactor does not give an appreciable decrease in the true rupturing duty of the switch.

Morgan Brooks: In view of the enormous size to which these reactors have grown, it might be interesting to say a few words in regard to the relation of the resistance to the reactance in these coreless coils. Putting in a small coil, such as may be used for a gasoline ignition, weighing a pound or two, it is impossible to get the reactance in ohms for moderate frequency, like 25, equal to the resistance of that coil in ohms, that is, the power factor of the coil would be rather good, that is, it has too much resistance in it.

You want a very low power factor in a reactance coil to use for synchronizing. If you take a coil and wind twice as many turns on it, a rough approximation would be that you multiply the inductance by four and the resistance by two. That is not quite right, because of your not having all the turns in the same spacing, and if you could maintain the turns in the same spacing, it would be true—the reactance increases faster than the resistance, assuming constant arc over in a large coil.

In the very large coils we get extremely low power factor, which is what we want, from five per cent or ten per cent. While early experiments on small machines seem to show it is absolutely necessary to have a coreless coil—because if you have a core reactance coil you get resistance for the power required equivalent to making a coreless coil of some conductor, such as German silver. It is out of the question. In the large coils we may be willing to use a reactance or coil of 20 per cent power factor, instead of getting one of five per cent.

It seems to me there is a possibility of introducing a small amount of core—much smaller than you would consider in transformer design, of course—and therefore get the reactance at a somewhat less cost, I should say, for the coil itself. However, please bear in mind that the reactance, due to the winding itself, with the coreless coil, is instantaneously available.

J. A. Johnson: I wish to say a word in support of Prof. Karapetoff's suggested method of attacking the problem which appeals to me as having much merit in a problem of this kind.

Most of the discussion and the papers today are based on an occurrence, or several occurences, which took place in a steam turbine driven installation. Upon the occurrence of trouble, in a plant of this kind, quite frequently the machine trips off on the steam end, so that there is no further energy supplied from the steam turbine to the system. Now, in the case of an hydraulic installation that is not true. Upon the occurrence of trouble, or a disturbance of the system, it is not possible to cut off the energy supply instantaneously. The governor reacts to the oscillations, and if there is considerable stored energy in the hydraulic system, there will be, accompanying the oscillations of the generators, an induced oscillation in the supply of power to the system from the hydraulic portion of the plant, tending to maintain the oscillations of the generators.

On page 1220 of Dr. Steinmetz' paper he states that "the maximum theoretically permissible busbar reactance, at a maximum of 30 deg. phase displacement between the busbar sections, would be 200 per cent, referred to the smallest generator on the section, as far as energy transfer from section to section, with negligible phase displacement—15 degrees."

Now, in two very large hydraulic plants, with which I have had experience, namely, the Ontario Power Co., and the former Hydraulic Power Co., bus bar reactors were installed, of only 12 per cent reactance based on the capacity of the smallest generator on the section, and in both of these cases unstable conditions resulted between sections of the busbars; not complete instability, but continued hunting; and in both cases it was corrected by a reduction in the amount of reactance.

On page 1231 Dr. Steinmetz says: "As is well known the alternators then oscillate against each other, with (practically) constant frequency of oscillation, and gradually decreasing amplitude of oscillation, and finally steady down in phase with each other, at the constant phase angle ω deg., determined by the condition of steady power transfer between the alternators." Now, in the case where there is no continued energy supply to the system, that, of course, is true, but in a hydraulic plant such as I am speaking of, where there is oscillating energy supplied to the system, by the forced harmonic oscillation of the turbine governors in synchronism with the swing of the generators, they do not settle down; they continue to oscillate, and in the instance of which I speak, that oscillation continued with no diminution in amplitude until the switches between the sections were opened.

I bring this point up at this time because this paper by Steinmetz will undoubtedly become a classic paper, and will be used not only by the engineers of the steam turbine plants, but also by those of the hydroelectric plants. For this reason I cannot refrain from again calling attention to this limitation which may occur in the hydraulic plant, and entering a protest against the idea that you can put a 200 per cent reactance, based on the smallest unit, in the bus bars of an hydroelectric plant of moderately high head and get away with it.

D. C. Jackson: Mr. Doherty has classified synchronizing power by the logic of his equation.

There are really two situations in respect to synchronizing power, one is the case where machines are running at exactly the same speed, but are slightly out of phase. Under these conditions, whether one considers a single machine with respect to a bus bar, or a pair of machines with respect to each other, an unbalanced voltage is set up out of phase with the main voltage. In this case, the $I^2 r$ of the current set up by this unbalanced voltage has no influence in bringing forward the lagging machine, but tends to retard the prime mover of the leading machine. Its effect is on the prime mover of the leading machine, to load the prime mover a little more, and as far as the synchronizing power of the generator *per se* is concerned, the $I^2 R$ loss does not come into play if we define that synchronizing power as the electromagnetic gripping of one machine on the other. On the other hand, when the machines are slipping past each other, running at slightly different frequencies, this induces a transfer of energy, and that leads Mr. Doherty to make an exception in his principal rules in respect to the relation of resistance and reactance in affecting synchronizing effort.

Another instance is referred to by Mr. Johnson, *i. e.*, the addition of forced vibration from outside influences. It is quite true, as Prof. Karapetoff has suggested, that energy equations may be set up which include the effects of force vibrations along with those of natural vibrations, but the solution of the energy equations which have thus far been developed for mechanical conditions do not approach conditions of complexity equal to those that are found in the electromagnetic circuit associated with a number of generators attached to one system.

I have the highest confidence in solutions of the energy equations where they can be applied. On the other hand, I am in accord with the manner in which Dr. Steinmetz and Mr. Doherty have dealt with the particularly complex example described in the paper, although their approximate solutions perhaps may ultimately be followed by solutions of energy equations which will give us more light. In other words, we must make the advance in these complex matters by first using approximate analyses, which are followed up by the full analyses.

D. W. Roper: On page 1249, of Dr. Steinmetz paper about the middle of the page, he states the conditions for maximum synchronizing power. I should like to inquire what is the order of this maximum synchronizing power under some of the conditions that have been discussed in these papers? Is it twice the power of the turbine, or ten times the capacity of the turbine, or how big is the maximum synchronizing power?

Secondly, on page 1263, in the figure representing the current and voltages, that occur when a short circuit occurs on the line beyond the reactance, it shows the voltage as dropping to zero at the time the line fails. In such cases there is always an arc between the two conductors, or between a conductor and ground. The New York Edison Company has, I believe, made some experiments and tests to determine under similar conditions the voltage across the arc, and I should like to ask Mr. Torchio if he thinks from these experiments, that the assumption that, after the arc occurs the voltage is zero, as made by Dr. Steinmetz, is entirely warranted?

Philip Torchio: The experiments Mr. Roper refers to were made on low-tension cables and they were carried out to determine the maximum amount of current that would flow into a short-circuited low-tension feeder, consisting of 1,000,000 circular mils concentric cable. We found that by making a short circuit close to the busbar, by driving a spike into the cable, the first rush of current was 8000 amperes. When the spike burned out, the current promptly dropped to about 3000 amperes, and held around that value for about one minute. Subsequently for quite a considerable time, I think about eight minutes, the value of the current was always under 2000 amperes, so we figured that the reason for that reduction in current, without any corresponding drop in the voltage in the bus, was due principally to the arc.

This is an important fact, of great value to engineers studying systems of distribution.

With regard to the matter concerning high-tension short circuits, if I am right, I think that the arc voltage drop in a short-circuited high-tension cable would be limited to a relatively small percentage of the total voltage, while on a 240-volt d-c. system, if we have an 80 or 90-volt arc, that will limit the current quite a great deal. On a high-tension system I imagine that the arc voltage is in the order of a small percentage of the total voltage on the circuit, so I believe Dr. Steinmetz is right in considering the voltage as practically zero.

H. R. Summerhayes: On page 1263, the curves shown give the voltage of the machine during a short circuit and before the time the switch is opened. I think that is rather important, as establishing the relation of the feeder reactor to the circuit breaker, which was adjacent, and with regard to the setting time of the relay. That is, these things are interrelated in that if we have a certain size of reactor, we can use a circuit breaker which will interrupt a certain current, and we can set it so that the voltage at any time will fall below the point which will cause the machines to drop out of synchronism.

R. E. Dougherty: In my discussion of Dr. Steinmetz's paper, I mentioned the confusion which exists in a great many discussions of synchronizing power, and I think that that point has been illustrated very forcibly by the present discussion about it. I will take first, the question of synchronizing power.

Mr. Woodrow says he does not agree with my definition. He means that he does not agree with Dr. Steinmetz's definition. I did not define it; I said that we must define it if we are going to talk about it. Otherwise, we would be talking about different things. I said that it depends upon what the problem is, if one is calculating the power received by the circuit, if that is what one wants to find out, and call that the synchronizing power, then the relation $\frac{x}{r} = \sqrt{3}$ is correct, provided however, that the voltages of the two buses are kept equal to each other. If the buses under consideration are not kept at the same potential, then that relation is no longer correct.

With reference to Mr. Johnson's paper, which was presented some two years ago: if I remember rightly that was one of the particular instances where the reactance of the alternators was considered in the calculation of synchronizing power, and, at the same time, the assumption was made that the bus voltages were equal and constant. Obviously if the constant bus voltages are used in the calculation, the reactance of the alternator does not enter.

Prof. Jackson said exactly what I tried to say in my discussion, about the effect of the I^2R loss, when it should be included and when it should not be. In considering the question of synchronizing power, decide what power you are going to consider, define it, and then it is perfectly easy to decide what reactance and what voltage to use. It all depends upon assumptions, and for the reason that these assumptions are not always made clear, there is confusion in the result.

Mr. Torchio brought up a point regarding external generator reactance—that its function is not to protect the generator but the bus. I think everybody will agree that it is to protect the bus, but I think you must also grant that in the case of older machines, particularly, its function is also to protect the generator.

Prof. Karapetoff and Dr. Jackson, I believe, mentioned the desirability of using energy equations instead of those of volts and amperes in the consideration of these problems. I do not know how Dr. Steinmetz would answer that point, but I think it would probably be along this line: the character of this problem is not one of extreme accuracy, that is, to define within a few per cent what is going on; and even if it were, I think it would be impossible to do so. It is rather hard to find an answer that will give some idea of the magnitude of the quantities involved. Ten per cent or fifteen per cent is not of serious consequence. It may be that if we were considering oscillation only, it would be more simple to use the form of equation which Prof. Karapetoff mentions, but here the principal problem is not oscillation. This has not been touched upon to any great extent. The principal problem is that of alternators slipping by each other out of synchronism, and how much synchronizing power is available under that condition, and I question whether it would be advisable in a problem of this sort to go to the extent of attempting a rigid solution. The method given in the paper gives symmetrical equations for all of the conditions and makes the calculation fairly simple and sufficiently accurate. It is true that many rather rough assumptions are made, but they are justified by the character of the problem.

Mr. Johnson brought out the difference between the steam and hydraulic station. There is a real difference. I think it is true that governors on modern steam turbines are probably less liable to sustain any oscillation that may be started than the hydraulic governors, although I have been told that hydraulic governors of modern design will not sustain oscillations. The use of a larger reactance might, after all, have helped Mr. Johnson in the difficulty which he experienced. He used 12 per cent reactance and had serious trouble from oscillation; and as I gather from his paper there was a fairly close relation between the natural oscillating frequency of these generators and the frequency of the hydrualic governor, that period being of the order of one second. It is not improbable that if that reactance had been 5 or 6 times as much instead of the negligible value of 12 per cent compared with the synchronous reactance of these machines (thereby changing the relation of the periods), the oscillation would have died out of its own weight instead of increasing to a serious magnitude.

Chairman Roper asked about the order of magnitude of the synchronizing power during slipping. The numerical data are given in Dr. Steinmetz's discussion of Mr. Schuchardt's paper, in which different assumptions and calculations are made giving the magnitude of this power.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July, 1920

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VOLTAGE STRESSES IN REACTORS IN SERVICE

BY F. H. KIERSTEAD AND ROYAL MEEKER
Both of the General Electric Co., Pittsfield, Mass.

IN this paper the writers wish to present some experimental data which indicate the voltage stresses that exist in current-limiting reactors under different conditions. We have endeavored to give a physical explanation for the phenomena revealed by our data. We have purposely abstained as far as possible from mathematics and abstruse theoretical considerations. We hope to interest those to whom experimental facts explained in simple physical language appeal. We expect that engineers who most readily get their conceptions of electrical phenomena through mathematical expressions will not care for this article. However, we find continually active engineers, busily engaged putting electrical apparatus into service, who have misconceptions of the proper use and care of such apparatus because, with the time available to them, they have not been able to interpret correctly the highly technical articles written on the subject. We have striven to write our paper so that "he who runs may read."

The function of a current-limiting reactor is to limit the current which will flow into a short circuit. It bears the same relation to the electric system that an emergency governor does to the turbine. On the other hand, a great difference exists as to the manner of predetermination of the ability of these two protective devices to perform their functions.

In case of the emergency governor the means of determining its protective value consists in assembling the governor, bringing the speed of the turbine up by means of the hand valve and watching the speed indicator. If the governor is properly set it trips out the turbine at the speed limit. Otherwise the turbine is tripped out by hand, the governor is readjusted or replaced by another one and the trial is repeated, this process being kept up until a governor operates properly.

The case of the reactor is quite different for two reasons, as follows:

1. It is almost never possible to obtain a generator for testing reactors large enough to maintain, across the reactor, the circuit voltage for which the reactor was designed, when delivering the short-circuit current.

2. The voltage stresses that obtain across a reactor, in service, in a system containing large power, long underground cables and highly concentrated inductances may be far more severe than would obtain with a large generator alone discharging into a short circuit through a reactor. Manufacturers would, of course, desire to short-circuit some feeder of a large system through their reactors in order to observe the stresses across the reactors. Generating companies, however, without exception feel that they have sufficient troubles on their feeders that cannot be avoided, without introducing additional ones. Therefore the reactor is seldom tested under service conditions. This is the equivalent in the case of the emergency governor of not being able to test the governor at the speed it must operate.

The pre-determination of a reactor's current-limiting ability is a very simple calculation but the pre-determination of the voltage stresses that it may be called upon to stand is extremely difficult. On the other hand if the reactor fails to withstand the voltage stresses placed upon it in service it will fail to limit the short-circuit current and will be therefore useless. The presenting of experimental data indicating the magnitude of these voltage stresses is the purposes of this paper.

SHORT-CIRCUIT TESTS

In order to produce if possible high-voltage stresses in reactors, comparable with the stresses to be expected under service conditions and to be able to observe the effects of these voltages upon reactors, a large number of short-circuit tests were made.

The generator used was a 10,000-kv-a., 10,000-volt, 40-cycle turbo generator. The reactors tested had an inductance of 0.02 henry and a current rating of 115 amperes.

The voltage across the reactors and the current passing through them were measured by means of oscillograph records. A diagram of the connections used in the tests is given in Fig. 1.

A large number of three-phase short-circuit tests were made with the generator over-excited for a voltage of 12,000 volts at the instant of short circuit and, in order to give the reactors a more severe test, single-phase short circuits at 12,000 volts were made through one reactor. In these single-phase tests the reactor in the *B* phase of Fig. 1 was removed and the short-circuiting was done by closing *A* and *B* switches only.

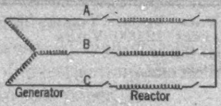

Fig. 1

Although over a hundred oscillograph records of these tests were taken not any of the records showed the presence of voltage stresses greater than the normal reactive drop due to the short-circuit current at the instant the short circuit occurred. In some cases there was found a rise in the voltage at the instant the current was interrupted but this rise occurred when the normal reactive voltage had been reduced due to the decay of the current and the records did not show that it reached a value as high as the reactive voltage at the instant of short circuit.

Since the general characteristics of all the records taken are the same, we have shown in Fig. 2 and Fig. 3 one record, each, of the three-phase and single-phase tests at 12,000 volts. The records show simply the well known discharge of a generator into a short circuit through an inductance.

The conclusions we have drawn from the records are, that if excessive voltages did exist across the reactor, they existed for such a brief instant that the oscillograph could not record their full magnitude. Whether excessive voltages existed between the turns or layers due to a non-uniform distribution of the voltage stresses we had no means of detecting, except that there was no arcing between turns or layers.

FIG. 2—THREE-PHASE SHORT-CIRCUIT TESTS AT APPROXIMATELY 12,000 VOLTS

Upper Curves—Line Current
Lower Curves—Voltage

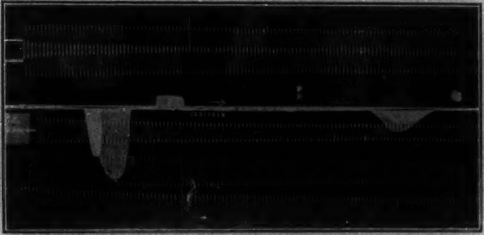

FIG. 3—SINGLE-PHASE SHORT-CIRCUIT TESTS AT APPROXIMATELY 12,000 VOLTS

Upper Curves—Line Current
Lower Curves—Voltage

In the short-cirucit tests above described, any high-voltage stresses, which may occur, appear at the time of making the short circuit or at the time of interrupting the short-circuit current. Between these two periods, there exists simply the reactive voltage due to the short-circuit current.

We have investigated the voltage stresses, at the instant of making short circuits, and also at the instant of the interruption of short-circuit currents, and have recorded the results of these investigations in this paper.

Reactors, in service, may have voltage stresses in them, greater than those that could be caused by the short-circuit conditions indicated above. The reason for this is two-fold: first, they may be subjected to induced high-voltage impulses due to lightning discharges near the overhead lines; and, second, reactors may be subjected to high voltages due to resonance, as a result of being placed in systems of high power and having the large capacitance of a network of underground cables.

Both of these causes of high-voltage stresses have been the subjects of investigation, the results of which are also included in this paper.

A—Voltage Stresses at the Instant of Short Circuit

At the instant a short-circuit occurs on an electric circuit that has been raised to some definite voltage, the voltage at the point of short circuit drops to a zero value, while the voltages at all other points on the circuit, except those immediately adjacent to the short circuit, are still at full value. This results in a steep front traveling wave.

To illustrate our conception of how a steep front wave is produced and propagated we have recourse to a hydraulic analogy. We do not hold that this hydraulic analogy truly represents the electrical phenomenon in all its phases. However, the wave produced in the manner about to be described, is a true analogue of an electric wave of a steep front.

Fig. 4 shows a reciprocating pump connected to a long pipe line. At a distance from the pump, a diaphragm in the pipe prevents free flow of the liquid. The pipe line and the diaphragm are stressed as indicated (13 and 26 lbs. respectively) and the diaphragm is nearly at the rupture point.

Fig. 5 shows the conditions at a later instant. During the interval of time which has elapsed the pump piston has traveled upward and increased the pressure (28 lb.) on the diaphragm sufficiently to rupture it. Through this ruptured diaphragm the liquid starts to flow due to both the pressure exerted on it by the expanded pipe on one side of the diaphragm and by the suction exerted on it by the contracted pipe on the other side of the diaphragm. The pipe resumes its normal size as fast as the liquid leaves the expanded portion and enters the contracted portion. The sudden rupture of the diaphragm therefore, causes two steep waves which travel back toward the pump each with an amplitude of one-half the rupture pressure. (One-half of 28, or 14 lb.)

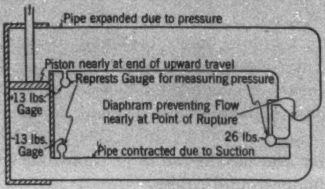
Pipe expanded due to pressure
Piston nearly at end of upward travel
Represts Gauge for measuring pressure
+13 lbs.
Gage
Diaphram preventing Flow
nearly at Point of Rupture
-13 lbs.
Gage
26 lbs.
Pipe contracted due to Suction

FIG. 4—SYSTEM UNDER STRESS BEFORE RUPTURE

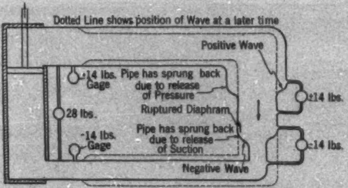
Dotted Line shows position of Wave at a later time
Positive Wave
+14 lbs.
Gage
Pipe has sprung back
due to release
of Pressure
+14 lbs.
28 lbs.
Ruptured Diaphram
Pipe has sprung back
due to release
of Suction
-14 lbs.
Gage
-14 lbs.
Negative Wave

FIG. 5—SYSTEM UNDER STRESS AFTER RUPTURE

We believed that the same sort of a wave would be produced on an electric circuit if the insulation at some point were ruptured. In order to prove whether or not this would be so, we arranged a circuit as shown in Fig. 6 with the sphere gaps, at *B*, the point at which the insulation was ruptured. We caused the spheres to arc over at 41,000 volts and measured across the loop of wire 30 feet long a difference of potential of 15,000 volts. Now since the circuit had a resistance of 350,000 ohms while the inductance and the resistance of the loop of wire were very low, we believe the fact that 15,000 volts could be measured between ends to be clear proof that waves similar to those described in the hydraulic analogy and shown in Fig. 5 did exist in this electric circuit.

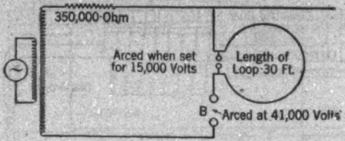

FIG. 6—CIRCUIT USED TO PRODUCE STEEP WAVE FRONTS

As in the hydraulic circuit so in this electric circuit waves traveling back from the point of rupture, have a maximum amplitude of one-half the rupture voltage.

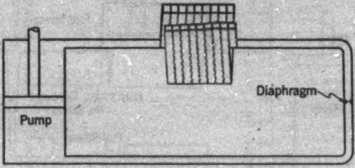

FIG. 7—NO STRESS ON SYSTEM

Therefore, the 15,000 volts measured indicate that the wave length was but a little longer than the length of the loop which was 30 feet.

Having shown the manner of production and propagation of steep wave fronts on a circuit we wish next to show their effect in producing voltage stresses in a reactor.

To obtain a mental picture of the manner in which these voltage stresses arrange themselves in the reactor, we again present a hydraulic analogy. In Fig. 7 we show a pump with a pipe line and a long cylindrical ring, having a hollow, rectangular cross-section. The ring contains a continuous helical diaphragm so arranged in the ring to give the effect of a number of turns of pipe having the diaphragm as a common wall to adjacent turns. In Fig. 8 we show this same system subjected to stress. When the diaphragm in the pipe preventing flow of the liquid, is ruptured, steep front waves are started, as before described. This is shown in Fig. 9.

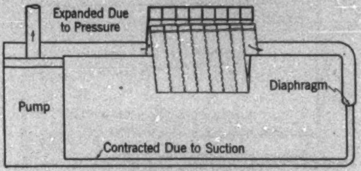

FIG. 8—SYSTEM UNDER MAXIMUM STRESS

In Fig. 10 we show the effect of the steep wave front on the ring. It will be noted that those portions of the helical diaphragm between the turns nearest to point of rupture are most severely strained, while each of the succeeding ones are progressively less strained.

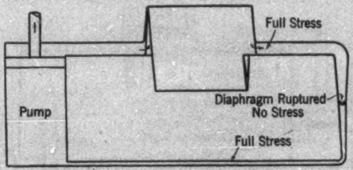

FIG. 9—SYSTEM AT THE INSTANT OF DIAPHRAGM RUPTURE

Now when a steep wave front strikes a reactor in an electric circuit the effect of the wave on the reactor is very similar to the effect of the wave on the ring above described. We have confirmed these conceptions and conclusions by measuring the voltage stresses between the layers and between the turns of the end layers nearest to the point of rupture, of a reactor, at the instant a short circuit is made near the reactor.

In our investigation, the tests were made with one side of the circuit grounded. With this condition, it appeared to us that there would be greater voltage stresses in the event that the rupture occurred at the end nearer to ground, as shown in Fig. 11A, than there would be if the rupture occurred at the end farther from ground, as shown in Fig. 11B.

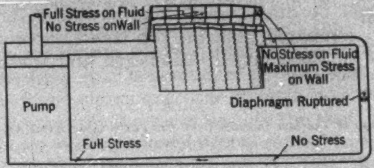
Full Stress on Fluid
No Stress on Wall
No Stress on Fluid
Maximum Stress on Wall
Diaphragm Ruptured
Pump
Full Stress
No Stress

Fig. 10—System After Diaphragm Has Ruptured and Wave Has Entered Coil

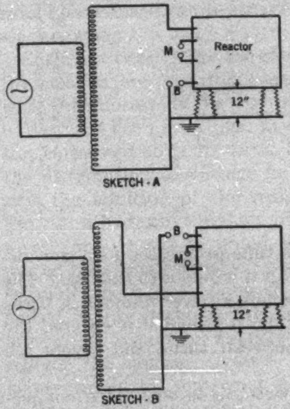
M
Reactor
B
12"
SKETCH - A
B
M
12"
SKETCH - B

Fig. 11

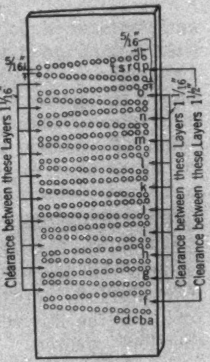
5/16"
5/16"
t s r q p
o
n
m
l
k
j
i
h
g
f
e d c b a
Clearance between these Layers 1 1/16"
Clearance between these Layers 1 1/16"
Clearance between these Layers 1 1/2"

Fig. 12

Our reason for believing this to be so was that the capacity of the bottom layers to ground is greater than that of the top layers to ground. Therefore the bottom layers store a greater quantity of electricity. Now when a wave strikes a reactor, the rate at which it progresses inward depends upon the amount of electricity that must be discharged. The bottom layers of the reactor, compared with the top layers, have a greater amount of electricity to discharge and therefore the wave in a given length of time progresses a shorter distance into the bottom than it would into the top. The result is that the wave spans a fewer number of turns if it enters the bottom than it would if it entered the top, and the stresses between turns and layers are therefore greater.

TABLE I

Distribution of Voltage Stresses between Turns and Layers at the Instant of Short Circuit

Position on reactor See Fig. 12	With rupture at bottom terminal of reactor See Fig. 11-A	Position on Reactor See Fig. 12	With rupture at top terminal of of reactor See Fig. 11-B
a − *b*	6.4	*p* − *q*	4.9
b − *c*	7.2	*q* − *x*	5.3
c − *d*	6.8	*r* − *s*	5.3
d − *c*	6.4	*s* − *t*	5.1
a − *f*	23.1	*o* − *p*	20.0
f − *g*	12.1	*n* − *o*	12.0
g − *h*	9.7	*m* − *n*	9.1
h − *i*	7.9	*l* − *m*	8.4
i − *j*	7.5	*k* − *l*	7.8
j − *k*	7.9	*j* − *k*	6.7
k − *l*	6.4	*i* − *j*	6.2
l − *m*	5.7	*h* − *i*	5.5
m − *n*	5.7	*g* − *h*	4.6
n − *o*	5.2	*f* − *g*	4.4
o − *p*	5.2	*a* − *f*	3.7
a − *p*	50.5	*a* − *p*	50.5

For this reason tests were made with both connections *"A"* and *"B"* of Fig. 11.

The sphere gap at *B* Fig. 11 was spaced to arc over at 70,000 volts, and the voltage stresses between the various parts of the winding, were measured, by the sphere gap *M*, at the instant of spark-over at gap *B*.

In Table I is given the results of the test and in

Fig 12 are shown the positions where the voltages were measured.

It will be noted that Table I shows all the layers stressed to some degree, while in the hydraulic analogy we have shown in Fig. 10, only part of the layers are stressed. This is explained by the fact that the table gives the maximum stresses that existed during the time the wave was progressing through the reactor, while the figure shows only, the stresses we conceive to exist at the instant the wave reaches the ring. At some later instant the wave would have moved further into the ring and the stresses between the first two layers would be reduced, while stresses would appear between all the layers.

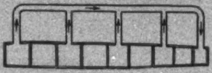

Fig. 13—Effect of a By-Pass on Distribution of the Stress

Since our object was the study of the reactor under the most severe conditions, we made our further tests with the rupture at the end of reactor nearer ground.

We have been interested in determining how a resistor, connected so as to shunt the reactor, would effect the distribution of voltage in the reactor. To represent the addition of the resistor we have shown in Fig. 13, a hydraulic analogy of the reactor shunted by a resistor. It will be noted that the resistor is represented by a small bypass. The effect of this bypass is to allow the upper layers to discharge some of their stored energy without discharging through the lower layers. It would be expected therefore, that in the reactor the voltage stresses between end turns and layers would be somewhat reduced by the shunting resistor. We have made measurements of the distribution of voltage stresses, with a resistor shunting the reactor, and have shown the results in Table II in comparison with the results of the measurements on the reactor not shunted by a resistor. In Table II is shown, also, voltage measurements on the reactor with

TABLE II (Refer to Fig. 12)

Position	1 No shunt resistor (from Table I)	2 Shunt resis. connected at ends only	3 Shunt resis. connected at intermediate points	Per cent change	Per cent change	
				1 to 2	1 to 3	2 to 3
$a-b$	6.4	5.7	4.9	− 11	− 23	− 14
$b-c$	7.2	6.6	4.9	− 8	− 32	− 26
$c-d$	6.8	6.1	4.9	− 10	− 28	− 20
$d-e$	6.4	5.7	4.1	− 11	− 36	− 28
$a-f$	23.1	19.2	11.9	− 17	− 48	− 38
$f-g$	12.1	7.2	5.9	− 40	− 51	− 18
$g-h$	9.7	5.9	6.1	− 39	− 37	+ 3
$h-i$	7.9	3.9	4.4	− 51	− 44	+ 13
$i-j$	7.5	3.9	4.2	− 48	− 44	+ 8
$j-k$	7.9	1.8	2.4	− 77	− 70	+ 33
$k-l$	6.4	4.4	2.9	− 31	− 55	− 30
$l-m$	5.7	3.5	1.8	− 39	− 68	− 49
$m-n$	5.7	4.7	3.2	− 18	− 44	− 32
$n-o$	5.2	4.7	1.6	− 10	− 69	− 66
$o-p$	5.2	9.7	2.7	+ 86	− 48	− 72
$a-p$	50.3	15.1	15.1	− 70	− 70	0

intermediate points of the resistor connected to corresponding points on the reactor. The resistor used was a carborundum resistance of 1200 ohms at 500 volts, having the characteristic of presenting higher resistance to low voltages than to high voltages.

It will be noted that with the resistor connected across the terminals of the reactor there is a reduction of concentration of voltage on the end turns and layers and that with the intermediate connections between resistor and reactor a further reduction is obtained. The per cent reduction for each measurement is also shown in the the table.

Summarizing we have shown the following:

First. That steep waves may be produced by the sudden drop in voltage caused by a short circuit.

Second. That the initial distribution of a steep wave across an inductance is determined by the amount of energy stored up in its capacity to ground which must be discharged in order for the wave to progress through the inductance and by the capacity between turns and layers of the inductance through which the stored energy must discharge.

Third. That a resistance shunting an inductance reduces the voltage stresses caused by steep waves.

B—VOLTAGE STRESSES AT THE INSTANT OF THE INTERRUPTION OF A SHORT-CIRCUIT CURRENT

There appears to be no definite method of predetermining the voltage stresses that may occur in a reactor due to an interruption of a short circuit. The voltage that may occur across the reactor depends on the switch which interrupts the current, the value of the current interrupted, the voltage of the circuit interrupted, and the amount of inductance and electrostatic capacity in the circuit interrupted.

Probably in the majority of cases the switch itself is the safety valve limiting the voltage in the following manner.

When the blades of the switch open an arc bridges the gap until a favorable moment and then the arc goes out. If the magnetic energy stored in the system is still high the voltage across the switch blades will rise and, because arcing occurred an instant before between the blades, the arc is reestablished without excessive voltage rise and this cycle of events continues until the stored magnetic energy has become so low and the distance between the blades so great that the arc is not reestablished and the circuit is interrupted without excessive rise in voltage across the switch or reactor.

In order to obtain a conception of the manner in which the different elements affect the voltage stresses across reactors, tests were made as follows:

1. To determine the variation in voltage rise across a reactor with the amount of current interrupted.

2. To determine the effect of different speeds of interruption of the current on voltage rise.

3. To determine the variation of the voltage rise with the point on the current wave at which the switch starts to open.

The first and second tests were made using direct current for the reason that it was of interest to note the effect of interrupting current without the presence of a large applied voltage.

The method of test consisted in establishing a current in a reactor, then suddenly interrupting the current by opening an oil switch and measuring the maximum voltage that existed across the reactor during the time the switch was opening.

The voltage was measured by means of sphere gaps and also by oscillograph records. In order to multiply the voltage to be read with sphere gaps we placed a coil of many turns inside of the reactor. This coil was linked with part of the reactor flux and the voltage across the coil bore a constant relation to the voltage across the reactor. This relation was determined by arcing spheres across the reactor and the coil simultaneously. Fig. 14 is a diagram of connections of the circuit used.

The reactor used for the tests had an inductance of 0.01 henry. It would introduce ten per cent reactance in a three-phase, 60-cycle, 13,200-volt, 200-ampere circuit.

(1) *Variation in voltage rise across a reactor with the amount of current interrupted.*

The method of making the tests consisted in holding various values of current in the circuit shown in Fig. 14 and interrupting the current by the opening of an oil switch. Voltages were measured by means of sphere gaps. Oscillographs recorded the current in the circuit, the voltage across the reactor and a 420-cycle timing wave to determine the length of time to interrupt the circuit. Two oscillograph records were taken for each value of current.

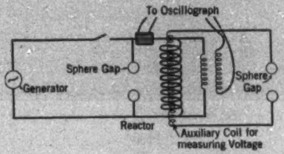

FIG. 14—CIRCUIT USED FOR CURRENT INTERRUPTION TESTS

The sphere gap voltages were obtained by determining the maximum gap at which the spheres would arc.

Attention is called to the fact that great variation existed in the voltages measured across the reactor under apparently constant conditions. It was found that after continued operation of the switch the voltage measured would be considerably decreased as is shown in Fig. 15. Our investigation further showed that if the switch stood idle for an hour the voltage reading would be high again and we came to the conclusion that heating of the oil lowered the voltage. We encountered other variations in the voltage measured which we

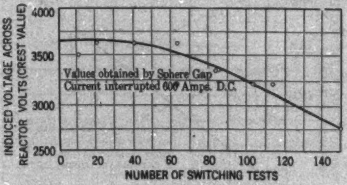

FIG. 15—VARIATION OF VOLTAGE WITH REPEATED SWITCHING

could not explain. This necessitated making many more tests than would otherwise have been required.

In Table III we have given the results of all our sphere gap tests. In Fig. 16 we show plotted the values recorded in Table III.

TABLE III

Sphere Gap Results of Rises Obtained by Interrupting Different Values of Direct Current

Current	No. of trials	Voltage crest value
66	50	1700
134	50	2000
200	50	2300
300	50	2300
400	50	2780
500	50	2420
600	150	3560
700	50	2780

It will be noted that the results do not follow any regular curve but they indicate that the voltage rises do not increase proportionately with the current. We believe therefore that the current is interrupted only after a succession of fluctuations of the current during which time the current is being reduced and that it requires a longer time to interrupt large currents than it does small currents. The oscillograms taken bear out the above conclusions.

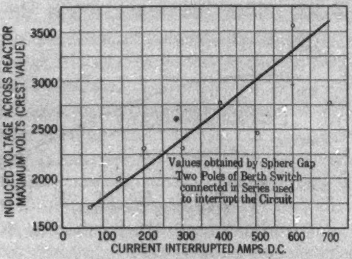

FIG. 16—VARIATION OF INDUCED VOLTAGE WITH AMOUNT OF CURRENT INTERRUPTED

In Fig. 17 we have shown one of the oscillograph records taken and in Fig. 18 we have plotted the results shown by all the records. Two points in contrast to the sphere gap results will be noted.

First that the oscillograph data follow much closer a regular curve than do the sphere gap data and second that the voltages given by the sphere gaps are higher. The only explanation of the former point is that the oscillograph curves are the result of two readings for each current while the sphere gap points are the maximum of many readings. We believe that the reason for the lower voltages recorded by the oscillograph is that the inertia of the moving element of the machine did not allow it to follow the steep waves completely.

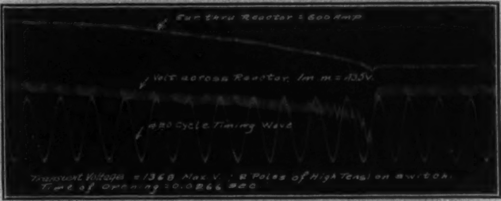

FIG. 17—VOLTAGE ACROSS REACTOR WHEN LINE SWITCH OPENS

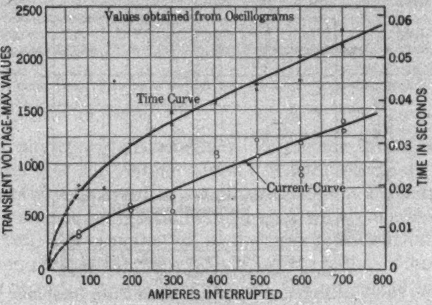

FIG. 18—VOLTAGE ACROSS REACTOR WHEN LINE SWITCH OPENS

The conclusions to be drawn from these results are that there is not a corresponding increase in voltage for a given increase in current interrupted. In other words for similar conditions the voltage obtained with 600 amperes interrupted is not twice as great as for 300 amperes.

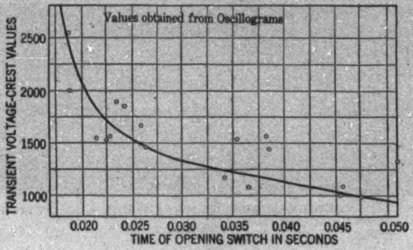

FIG. 19

(2) *Determination of the Effect of Different Speeds of Interruption of the Current on the Voltage Rise.* This investigation consisted in measuring the voltage rises

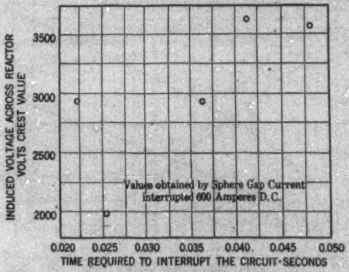

FIG. 20—VARIATION OF VOLTAGE WITH TIME REQUIRED TO INTERRUPT THE CIRCUIT

due to interruptions of the current with slow, medium and quick acting switches. To obtain the different speeds of operation we used several types of switches.

We varied the speed of operation of a standard three-phase switch by connecting all three phases in series thereby inserting three gaps in the circuit when the switch operated and also by connecting only two phases in series.

We measured the voltages by sphere gaps and also by oscillograms. Fig. 19 gives the results shown by the oscillograms. Table IV gives the sphere gap results and in Fig. 20 these values of sphere readings are plotted

The oscillograms indicate that the voltage increases with the speed of the switch which is what should be expected. On the other hand the sphere gap measurements which are comprised of many more tests do not show any consistent tendency. We come to the conclusion that the voltage across a reactor due to an interruption of current by an oil switch is not a constant and varies in an erratic manner that we are unable to explain. An examination of the records shows that at earlier periods the current was on the point of being interrupted. A small change in the oil might have completed the interruption at the earlier period, accompanied by consequent higher voltages than actually was the case.

TABLE IV

Sphere Gap Results of Voltages Obtained by Interrupting 600 Amperes Direct Current with Different Switches

Switch used	No. of trials	*Seconds to open	Voltage crest value
Standard switch with three phases in series	50	0.0425	3620
Standard switch with two phases in series	50	0.0480	3620
Experimental switch with loose spring	50	0.036	2920
Experimental switch with tight spring	50	......	2920
High-voltage switch	50	0.025	1960

*The time required to open switch was obtained from the oscillograph records.

(3) *Determination of the Variation of the Voltage rise with the Point on the Current Wave at which the Switch Starts to Open.* These tests were made with alternating current at 25 cycles. The circuit of the trip coil of

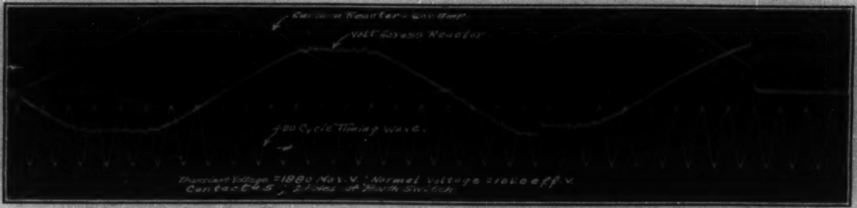

FIG. 21—VOLTAGE ACROSS REACTOR WHEN LINE SWITCH OPENS

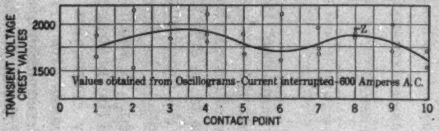

FIG. 22

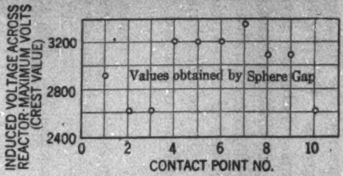

FIG. 23—VARIATION OF VOLTAGE ACROSS REACTOR WITH OPENING OF SWITCH AT DIFFERENT POINTS ON CURRENT WAVE

the oil switch was so arranged that a finger attached to the shaft of the generator in revolving made contact and completed the circuit of the trip coil causing the circuit to open. The finger was arranged to take 10 different angular positions on the shaft, each corresponding to 18 electrical degrees apart. By this means it was possible to open the circuit at points on the current wave 18 degrees apart.

The voltage across the reactor was measured with the current interrupted at each of these 10 points by oscillograms and by sphere gaps. The voltage held across the reactor was 1000 volts and the current flowing was 600 amperes. The frequency was 24 cycles. Fig. 21 shows one of the oscillograms taken and Fig. 22 shows the results of the oscillograms plotted. Fig. 23 shows the sphere gap results plotted.

The results of these tests were much more consistent than those of the direct-current tests. We did not find any definite relation between the various timings of the opening of the switch and the voltage rise. Apparently the voltage measured was the reflection of the applied voltage—1000 volts $\sqrt{2} \times 2$ or 2840 volts crest value.

It is to be noted that the oscillograms all show that the current was finally interrupted just after passing through the zero and that one-half a cycle earlier it had tried to open. The conclusions from these tests are that the crest voltage across the reactor rises to two times the crest value of the applied voltage at the time of interruption of the current.

The reason that double voltage occurs across the reactor at the instant of the interruption of the circuit we believe to be as follows:

The circuit is interrupted when the current is nearly zero and, therefore, changing at its most rapid rate. For this reason the impressed voltage is at its maximum value and is all being consumed in overcoming the counter voltage of the inductance of the circuit. At the instant the current is interrupted the impressed voltage is available for charging up the capacity of the switch and in doing so draws charging current through the inductance of the circuit. When the switch is charged up to the full circuit voltage the charging current is maximum and, due to the inertia of the inductance through which it passes, continues to flow until the switch is charged up to double normal voltage. In the resultant oscillation due to the switch discharging from double voltage to normal voltage, double voltage normal voltage is placed across the reactor.

Our conclusions drawn from this investigation of voltage stresses due to interruption of short-circuit currents are as follows:

First. The voltage obtained across a reactor due to the interruption of direct current is independent of the applied voltage and depends on the current to be interrupted and the rate at which it is interrupted.

Second. The voltage across a reactor due to the interruption of alternating currents with commercial switches is in general independent of the current interrupted and depends on the impressed voltage. If, however, the circuit is opened by a switch so quick acting that the current is interrupted before it can reach its zero value in accordance with its normal curve then the voltage will depend on the current interrupted and the rate at which it is interrupted.

C. VOLTAGE STRESSES ON REACTORS DUE TO IMPULSE WAVES INDUCED ON A TRANSMISSION LINE BY LIGHTNING DISCHARGES

Before considering the results obtained during these investigations we wish to show the analogy between the circuit used in the tests and a hydraulic system.

In Fig. 24 is shown a pipe system, under pressure from a pump. Arranged as shown are three diaphragms. Two of these diaphragms D' D' are equally flexible and the third D'' is relatively stiff. In the pipe line between, the flexible diaphragms is shown a long cylindrical ring analogous to a reactor. Pressure on the system is increased to the rupture point of the stiff diaphragm D''. When D'' ruptures, the immediate effect is to reduce the difference of pressure at this point to zero. The pressure further away has not yet changed due to the inertia of the water in the pipes. As the water progressively gets in motion, to bring about a balanced condition again, waves are formed, propagate along the pipe to the flexible diaphragms and relieve the stress on the pump side of each. Then due to the tendency of the diaphragms to take an unstressed position the pipes on the far side of the diaphragms are stressed and the waves continue to travel on toward the cylindrical ring.

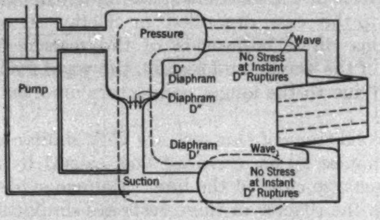
Pressure
Wave
No Stress
at Instant
D" Ruptures
Pump
D'
Diaphram
Diaphram
D"
Diaphram
D'
Wave
Suction
No Stress
at Instant
D" Ruptures

Fig. 24—Hydraulic Analogy of Circuit Used to Produce Impulse Voltages

Heavy line shows condition of stress at instant of rupture. Dotted line shows condition of stress when wave is about to enter reactor.

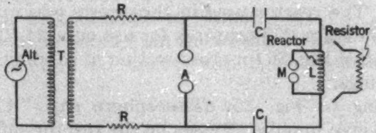
R
C
Reactor
Resistor
Alt.
T
A
M
L
R
C

Fig. 25—Diagram of Circuit Used to Produce Impulse Voltages

T — 60-cycle transformer.
R R — Protective resistance for transformer.
A — Sphere gap—75 cm. diameter.
C C — Condensers—0.0012 microfarad each.
M — Sphere gap—25 cm. diameter.
Gap *A* adjusted for voltage stress on circuit.
Voltage raised on *T* to spark-over of *A*.
Gap *M*—adjusted to measure voltage at reactor.

Fig. 25 shows the electric circuit used in the tests, of which the pipe system described above is analogous. When the gap "*A*" arcs, waves are set in motion which propagate back, relieving the stress on all wires on the transformer side of the condensers and stressing the wires and reactor, hitherto unstressed to the full voltage to which the condensers had been charged.

The effect of the waves in stressing up the reactor, is similar to the case described in Section A, with two exceptions—first, both ends of the reactor feel the impact of the waves, and second, the wave front is not so steep due to the longer conductors involved in this case.

The conditions of this test are very similar to those of an impulse on an overhead line, caused by a lightning discharge close to the line. Voltage stresses produced in the reactor in these tests are similar to those caused by lightning impulses.

This investigation of the effects of impulse waves applied to a reactor is divided as follows:

1. Effect of shunt resistance on the voltage across a reactor.
2. Effect of shunt resistance on the distribution of voltage stresses in the reactor.
3. Effect of extra end turn and layer spacings on the stresses in reactors.

1. *Effect of Shunt Resistance on the Voltage across a Reactor.* The reactor used in these tests was rated 60 cycles, 705 volts, 177 amperes for use on a 24,400-volt system. See Fig. 26 for cross-section of winding showing spacings.

Referring to Fig. 25. The sphere gap "*A*" was spaced to give impulse stresses on the reactor sufficient to cause sparking between layers. With the same spacing held on gap "*A*" various resistors were connected across the terminals of the reactor and the voltage across the reactor was measured by means of the sphere gap "*M*".

We have, in Section A, described the function of the resistor and in Fig. 13 have illustrated its use.

The following resistors were used.

1. A water resistor of 3000 ohms.
2. A non-inductive metallic resistor of 1190 ohms.
3. A non-inductive metallic resistor of 780 ohms.

4. A resistor made up from carborundum rod units of 2400 ohms at low voltages.

In Table V are given the values of voltage measured across the reactor, with each of the various resistors shunting the reactor, and also the voltage across the reactor without shunt resistance.

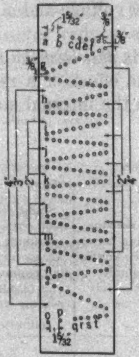

FIG. 26

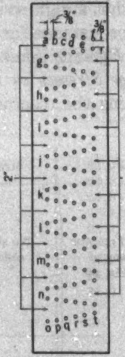

FIG. 27

It will be noted from the results in Table V that the carborundum rod resistor, although it had a high resistance, effected the greatest reduction of the impulse voltage across the reactor.

TABLE V

Shunt Resistor	Ohms	Voltage across reactor
No. resistor	....	360 kv.
1	3000	330 "
2	1190	312 "
3	780	300 "
4	2400	83 "

The reason for this is that this resistance is manufactured from finely divided conducting and non-conducting materials. The conducting particles are to a great extent held apart by the non-conducting material. For low voltage, the current takes a devious path through the rod along which the carbon particles make contact, while at high voltage, the current takes a direct path, arcing between conducting particles. This resistor, then, has a high resistance for low voltage and a much lower resistance for high voltages.

2. *Effect of Shunt Resistance on the Distribution of Voltage Stresses in Reactors.* The effect of a resistor shunting the reactor is to reduce the internal stresses considerably and the stress across the whole winding to a greater degree. This may be readily explained by a reference to the hydraulic analogy shown in Section A, Fig. 13. The bypass pipe as shown has a small cross-section and a short length. The volume of water in the pipe is relatively small and is easily set in motion. This will reduce the difference of pressure between the ends of the bypass. On the other hand, due to the comparatively small cross-section of the bypass pipe it does not drain the water from the connected chambers as quickly as the water drains from the chamber which the wave first reaches. Nevertheless, the reduction which is effected is considerable.

TABLE VI

Stresses Between Layers

Part of reactor measured see Fig. 26	No shunt resistor	Shunt resistor 3000-ohm water resistance across terminals of reactor only	Shunt resistor 2400 ohms carborundum rods. Across terminals and connected in at middle of reactor winding
Top			
a – g	49.5	42.0	14.7
g – h	53.2	32.7	15.8
h – i	40.7	29.0	15.5
i – j	36.2	23.5	12.4
j – k	42.7	24.5	14.7
k – l	36.5	24.5	10.8
l – m	38.5	25.4	13.5
m – n	52.4	30.0	14.5
n – o	50.1	41.0	14.8
a – o	320	200	83

We show in Table VI the data obtained from measurements of internal voltage stresses on the reactor shown in Fig. 26 with and without resistance shunting the reactor.

The conclusions that we draw from the values in the table are twofold. First, that the resistance shunting the reactor reduces the voltage stresses and second, that a resistor with the characteristics of smaller resistance for high voltages than for low voltages gives greater reduction than one with a constant resistance.

TABLE VII

Part of reactor	Reactor 1	Reactor 2
Top two layers	33.6 kv.	32.0 kv.
3d and 4th layers	23.4	26.0
5th and 6th layers	21.6	26.0
7th and 8th layers	22.4	21.0
9th and 10th layers	24.4	28.2
11th and 12th layers	22.4	23.0
13th and 14th layers	24.4	25.0
15th and 16th layers	27.3	25.0
Bottom two layers	40.4	31.0
Whole winding	191	191
Top layer		
1st turn	6.8	4.7
2nd turn	6.8	5.8
3rd turn	6.4	4.2
4th turn	3.4	3.6
5th turn	3.4	3.6
Bottom layer		
1st turn	7.2	6.3
2nd turn	8.6	6.8
3rd turn	6.8	4.7
4th turn	6.4	3.9
5th turn	3.4	3.9

3. *Effect of Extra End Turn and Layer Spacing on the Stresses in Reactors.* We have measured the distribution of voltage stresses produced by impulse voltages in two reactors, one having extra spaced end turns and layers and the other having uniformly spaced layers and turns. Fig. 26 is a cross-section showing the spacing between turns and layers, of the reactor with extra spaced turns and layers which we will hereafter call No. 1 and Fig. 27 is a similar cross-section for the other reactor which we will call No. 2.

In Table VII are given the voltage stresses between layers and turns in reactors No. 1 and No. 2. The connections of the circuit used in these tests are shown in Fig. 25.

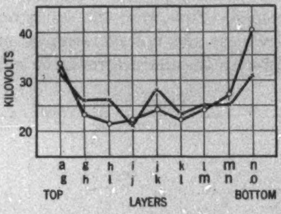

Fig. 28—Voltage Distribution Between Layers Due to Impulse Wave

Points *o* — **Reactor with extra spaced end turns and layers.**
Points *x* — **Reactor with uniform turn and layer spacings.**

The data of table VII are plotted in curves Fig. 28 and 29.

The explanation of the difference of distribution of stresses in the two reactors, is, that the extra spacing by reducing the capacitance between the layers and turns caused a greater proportion of the voltage to be consumed across the extra spaced portions than would have been the case if they had not been extra spaced.

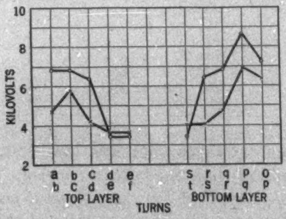

Fig. 29—Voltage Distribution Between End Turns Due to Impulse Wave

Points *o* — **Reactor with extra spaced end turns and layers.**
Points *x* — **Reactor with uniform turn and layer spacings.**

The extra spacing of turns and layers is equivalent, in the hydraulic analogy Fig. 10 to increasing the stiffness of the diaphragm at the ends, thereby increasing the stress at these points.

We also investigated the effect of extra end turn and layer spacing on the strength to withstand impulse voltages. The tests were made using the circuit shown in Fig. 25. The two reactors, No. 1 and No. 2 were connected in parallel and impulse waves applied to them. Gap *A* spacing was adjusted to cause sparking between layers. The performance of each reactor was noted. The results are shown in table VIII.

TABLE VIII

Impulse Voltages across reactors	Arcing between layers	
	Reactor No. 1	Reactor No. 2
405 kv.	arcs	arcs
371 kv.	arcs	few arcs
360 kv.	arcs	no arcs
336 kv.	no arcs	no arcs

Although the difference indicated in Table VIII is not large it is in favor of the uniformly spaced reactor and when it is realized that the reactor with extra spaced layers and turns actually had greater insulation clearances than did the other reactor, the value of extra spaced end layers and turns appears questionable. For reactor No. 1, the sum of the layer spacings is 24 in. For reactor No. 2 the sum of the layer spaces is 18 in.

The conclusions we have drawn from these impulse wave tests are as follows:

First, impulse waves may cause concentrated stresses in a reactor. The distribution of these stresses between turns and layers of the reactor depends upon the distribution of capacitance between layers and turns of the reactor and upon the wave shape.

Second, resistance connected to the reactor winding effects a reduction of these voltage stresses. The amount of reduction depending upon the magnitude of the resistance.

Third, extra spacing of layers and turns at the ends of reactor windings causes greater concentration of voltage stress on those extra spaced parts. Apparently no increased insulation strength is gained.

D. VOLTAGE STRESSES IN REACTORS DUE TO RESONANCE

Before looking into the theory of electrical resonance let us review a mechanical resonance familiar in everyday life. Let us take the case of a slender flag pole with a ball on top of it, and with a rope attached to the upper part. Now suppose a boy exerts a feeble pull on the rope the result will be a slight bending of the pole. If he slackens his pull on the rope the pole will tend to spring back to its original position, but due to its momentum, will bend in the reverse direction almost to the same extent that the boy bent it in the first direction, and then will again tend to spring to its original position. Suppose at this time the boy exerts another pull on the rope, then his pull be timed so as to add to the amount the pole bent due to his original pull. It is clear then that by a series of well-timed but very small pulls the boy will be able to cause the pole to vibrate to and fro very violently, and that the ultimate bending would be far in excess of the bending the boy could produce by exerting a steady pull on the pole. It is apparent, therefore, that strains could be caused by the boy's feeble pulls which would cause a pole to break although it would safely withstand the very much greater steady pressure put upon it by the wind.

Let us now turn our attention to the subject of electrical resonance, and consider just what such resonance is. Let us first take the simplest case which would consist of a generator with an inductance and capacity in series across its terminals, and draw an analogy between this electrical circuit and the mechanical resonance case described in the above paragraph. The boy pulling the string in the previous case is in this case the generator. The ball on top of the flag pole is in this case the inductance, and the elasticity causing the flag pole to spring back when the tension on the cord is relaxed is the capacity. Now as in the case of the flag pole which had a definite rate of vibration depending upon the elasticity of the pole and the weight of the ball so likewise this electric circuit has a definite rate of oscillation depending upon the value of its inductance and capacity. The rate at which the boy pulls the string in the first case is in this case the frequency of the generator. In the first case when the boy timed his pulls in accordance with the natural vibration of the pole then he caused excessive stresses to be placed upon the pole. In this latter case if the frequency of the generator is timed in accordance with the natural frequency of oscillation of the inductance and capacity excessive voltages will appear across each.

Up to now we have not considered at all the nature of the generator. It is clear, however, that this is not important so long as it generates alternating or pulsating current at the natural frequency of the inductance and capacity. If a large condenser be charged up and then allowed to discharge through an inductance it will be a generator of alternating or pulsating current, the frequency of which may be made any value depending on the inductance through which it is discharged. Therefore, if a circuit containing a capacity in series with an inductance is placed across the condenser just described we have all the conditions necessary to produce resonance. In other words we have a generator of pulsating or alternating current placed across an inductance and capacity in series, and all that is necessary to produce voltages higher than the applied is to vary the frequency of the generator until it is approximately equal to the natural frequency of the inductance and capacity. (See Fig. 30).

Now in case of the flag pole we have three limiting conditions: First, the boy may continue to pull and increase the amplitude of the vibration of the pole until the pole breaks, second, the pole may be strong enough so that it continues to increase its amplitude of vibration up to the point that the friction of the bending of the pole is equal to the energy delivered by each pull of the rope, and third, the boy may become exhausted before either of the first two conditions are reached. In our case of electrical resonance there are also these three limiting conditions, first, the insulation of the inductance or capacity may break down, and therefore relieve the strain, second, the resistance loss may become equal to each individual impulse of energy delivered by the generator, and third, the generator may become exhausted by the discharging of the condenser before the first two conditions are reached.

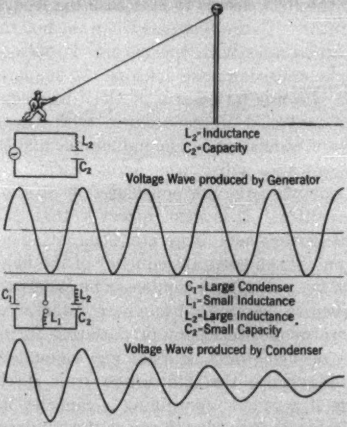
L₂
C₂
L₂-Inductance
C₂-Capacity
Voltage Wave produced by Generator
C₁
L₂
L₁
C₂
C₁-Large Condenser
L₁-Small Inductance
L₂-Large Inductance
C₂-Small Capacity
Voltage Wave produced by Condenser

FIG. 30—THREE MEANS OF PRODUCING RESONANCE

Let us now consider how these resonating circuits can be obtained in an ordinary generating system. For the sake of simplicity let us take a single-phase system. The phenomenon holds equally well for a three-phase system but involves a more complicated diagram.

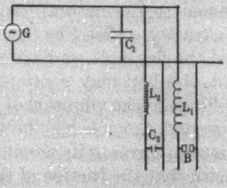
G
C₁
L₂
L₁
C₂
B

FIG. 31—DIAGRAM OF RESONATING CIRCUIT

Referring now to the diagram in Fig. 31 we have indicated by G a generator, by C_1 the capacity of a long underground cable line leading for example from a generating station to a substation, by L_1 the inductance of a feeder, not having a reactor in it, by B a short circuit on the feeder causing arcing, by L_1 a feeder reactor in another feeder and by C_2 the capacity of this feeder. Now C_1 is a very large condenser, and when arcing occurs at B this condenser discharges through L_1 and B in an oscillatory manner and therefore becomes a generator of alternating or pulsating current, the frequency of which depends upon the value of L_1. The circuit containing the inductance L_2 and the capacity C_2 is across this generator and, if the frequency of this generator is approximately equal to the natural frequency of the circuit containing L_2 and C_2, then resonance occurs and voltages will be built up across L_2 and C_2 in accordance with the three conditions described above, namely: First, until C_2 or L_2 breaks down relieving the strain, or second until the resistance loss in the circuit L_2 C_2 is equal to the energy of each impulse delivered to it, or third, until the condenser C_1 is entirely exhausted by being discharged. Therefore, either C_2 or L_2 must break down if the capacity of C_1 is high enough and the resistance of the circuit is low enough.

It is important to note that C_1 in Fig. 31 may be made up of the capacity of several underground feeders and tie lines not having reactors in them. Another diagram of a circuit in which resonance can be obtained is shown in Fig. 32 in which C_2 is the capacity of the busbars and generator, L_2 the inductance of the feeder reactor, C_1 the capacity of the feeder cables and L_1 the inductance of a current transformer. In this case C_1 is the condenser acting as the generator and delivers impulses of energy to the circuit containing C_2 and L_2.

The remedy for preventing high resonant voltages is apparent from consideration of the method of preventing the flag pole referred to above from being thrown in such violent vibrations. This obviously is obtained by making the energy loss during each vibration equal to the energy of each pull. For instance, if it were possible to immerse the flag pole in water, the energy loss in the pole whipping backward and forward would be very large and with very small amplitude would equal the pull of the boy. The same result is accomplished in the electrical resonance by shunting the reactor with resistance. For as the voltage commences to rise across the reactor, and, therefore, across the resistor, the energy loss in the resistance becomes high and soon the energy loss per oscillation reaches the energy input per oscillation from the condenser.

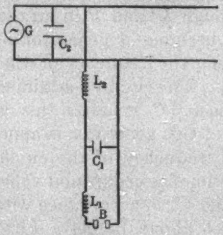

Fig. 32—Diagram of Resonating Circuit

It will be remembered that we have stated that one of the conditions limiting the extent to which the resonant voltage might rise was the exhaustion of the condenser that acts as the generator; that is, C_1 in Fig. 31. We wish now to explain briefly how this occurs and how the limiting voltage may be arrived at.

We have seen how the voltage across C_2 and L_2 in Fig. 31 has increased, due to resonance, above the applied voltage. Now if C_2 has increased in voltage it also has had the energy stored in it increased because the energy stored in a condenser is given by the equation,

$$W = 1 / 2\, E^2 C$$

where W is the energy stored
E is the voltage across the condenser.
C is the capacity of the condenser.

Therefore, the energy stored has increased as the square of the increase in voltage.

Now this increased energy must come from C_1 just as in the increased energy of the vibrating flag pole comes from each of the boy's pulls on the rope. Furthermore, just as the boy cannot increase the amplitude of vibration of the flag pole, after he is exhausted, so neither can C_1 increase the voltage across C_2 after it has exhausted all the energy stored in it. Therefore the limit of voltage increased would be when all the energy stored in C_1 has been delivered to C_2.

The energy stored in C_1 is given by the equation

$$W_1 = \frac{1}{2} E_s^2 C_1$$

where W_1 is the energy stored in C_1 and E_s is the instantaneous voltage at the instant the discharge starts.

The energy stored in C_2 at the start of the discharge is

$$W_{2s} = \frac{1}{2} E_s^2 C_2$$

where W_{2s} is the energy stored in C_2 at the start of the discharge.

When all the energy stored in C_1 has been delivered to C_2 the energy stored in C_2 is as follows.

$$W_{2m} = W_1 + W_{2s} = \frac{1}{2} E_s^2 (C_1 + C_2) = \frac{1}{2} E_{2m}^2 C_2$$

where W_{2m} is the maximum energy stored in C_2

E_{2m} is the maximum instantaneous voltage across C_2.

The maximum instantaneous voltage across C_2 can be expressed as follows:

$$E_{2m} = \sqrt{\frac{2W_{2m}}{C_2}} = E_s \sqrt{\frac{C_1 + C_2}{C_2}}$$

and the ratio of the maximum instantaneous voltage across C_2 to the instantaneous voltage across C_2 at the start of discharge is given in the equation below

$$\frac{E_{2m}}{E_s} = \sqrt{\frac{C_1 + C_2}{C_2}}$$

Now it may be possible that the generator might be able to furnish energy for the increase in resonant voltages (in which case the above ratio would be much too low) but we believe that this is not so, and experimental data which we will show later seem to confirm this.

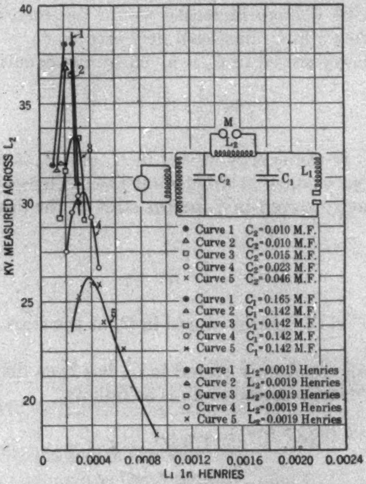

FIG. 33—RESONANCE CURVES

On the other hand, it would never be possible in practise for C_1 to deliver all its energy to C_2 because in order for it to deliver any energy it is necessary for C_1 to discharge in an oscillation and during this oscillation a large part of the energy stored is dissipated as $I^2 R$.

Let us now turn our attention to the experimental data which we have obtained and note how it accords with the discussion above.

Tests have been made both with circuits as shown in Fig. 31 and also with circuits as shown in Fig. 32. The generator was rated three-phase, eight poles, 60 cycles

TABLE IX
Resonant Voltages Obtained with Circuits Shown in Figs. 31 and 32

C_1 M. F.	L_1 henrys	C_2 M. F.	L_2 henrys	Applied Voltage	Voltage across L_2		Voltage across C_2	
					Without resistor	With resistor	Without resistor	With resistor
0.165	0.00022	0.010	0.0019	13,000	38,000	....	38,000	....
0.142	0.00022	0.010	0.0019	13,000	37,000	....	37,000	....
0.142	0.00027	0.015	0.0019	13,000	33,000	....	33,000	....
0.142	0.00034	0.023	0.0019	13,000	30,000	....	30,000	....
0.142	0.00038	0.046	0.0019	13,000	26,000	....	26,000	
0.180	0.00008	0.003	0.0058	10,000	50,000	....	50,000	....
0.262	0.00008	0.003	0.0083	10,000	59,000	6,500	59,000	*10,000
0.427	0.000028	0.0018	0.0102	10,000	104,000	6,000	104,000	*10,000

*With resistor shunting the reactor the applied voltage is placed across C_2.

500 kv-a., 900 rev. per min., 2300 volts. The transformer was rated single-phase, 60 cycles, 500 kv-a., 11,000–2300 volts. C_1, C_2, L_1 and L_2 all were varied and their values are given with voltages measured. The method of test consisted in setting the gap *B* for a definite spark-over voltage and then raising the voltage until spark-over occurred and reading the voltages by sphere gaps across C_2 and L_2.

In Fig. 33 we have shown five resonant voltage curves taken under different conditions as noted on the figure.

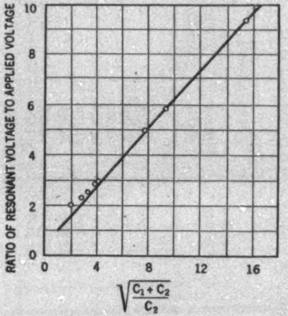

FIG. 34—CURVE SHOWING INFLUENCE OF CAPACITY ON RESONANT VOLTAGES

It will be noted that the circuit shown is identically the same as the one shown in Fig. 32 before referred to as a resonating circuit. The applied voltage in these tests was 13,000 volts and the highest voltage measured across L_2 was 38,000 volts. These curves are of interest as showing how the voltages vary with the different constants. It is to be further noted that although the curve does not show it the voltage across C_2 under conditions that made the voltage across L_2 maximum was always equal to the L_2 voltage.

In Table IX we have given the resonant voltages which we have obtained with circuits shown in Fig. 31 and Fig. 32 having various values of L_1, L_2, C_1 and C_2, both with and without a carborundum resistor of 2400 ohms shunting the reactor.

It will be noted that we have measured voltages across L_2 and also C_2 as high as 9.4 times the applied voltage and that with a carborundum resistor shunting the L_2 the voltage measured across L_2 was less than the applied voltage and that across C_2 was equal to the applied voltage.

In Fig. 34 we have plotted the ratio of the resonant voltage to the applied voltage against $\sqrt{\frac{C_1 + C_2}{C_2}}$. It will be noted that the ratio of voltage is lower than $\sqrt{\frac{C_1 + C_2}{C_2}}$ and bears a relation to it. This is in accordance with the discussion above.

The outstanding features of the investigation covered in this section of our paper are threefold.

1. Resonant voltages were obtained with circuits similar to those used in electric generating and distributing systems.

2. Resonant voltages in circuits having high capacity were enormous and the higher the value of

$$\sqrt{\frac{C_1 + C_2}{C_2}}$$

the higher was the voltage.

3. High resonant voltages were entirely eliminated by a carborundum resistor having 2400 ohms resistance. In closing we wish to call attention to a series of events that are linked together and which will cause destructive stresses in reactors.

Let us consider a system having a circuit similar to that shown in Fig. 31. Now suppose a short circuit occurs at B and the oil switch starts to open. Voltages equal to two times the normal voltage may pile up on the switch and also charge up all condensers connected to the busbars to double voltage; that is, C_1 is charged up to two times normal. The arcing may be reestablished between the blades of the switch due to double normal voltage. The short circuit is completed again by the arcing and the capacities all discharge again from double normal voltage; that is, C_1 discharges through B from double voltage. High resonant voltages may be built up across the reactor. Suppose the constants of circuit L_2 and C_2 are right for five times the applied voltage and since the applied voltage is now two times normal because C_1 is charged up to double voltage, ten times normal voltage is placed across L_2 and also across C_2. If due to this abnormal voltage C_2 should puncture steep waves would enter L_2 as described in section A (that is, waves due to a sudden drop in voltage) resulting in tremendous stresses on L_2.

The conditions on such a system may be fatal but can be rendered safer by the installation of current-limiting reactors in places indicated by L_1 which represents the inductance of the line. Furthermore, on the assumption that L_1 is replaced by current-limiting reactors, the factor of safety of the entire system can be increased to a considerable degree, by shunting the reactors with resistors.

DISCUSSION ON "VOLTAGE STRESSES IN REACTORS IN SERVICE" (KIERSTEAD AND MEEKER), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

J. F. Peters: If I understand the authors, it is their principal intention to show the voltage stresses that are developed in reactors, under abnormal conditions. It is my personal feeling that too much stress is placed on conditions that may never be met in practise. In fact, in several instances I feel that they are conditions that can not be met on an underground distributing system, even though a direct effort be made to get the worst possible combination.

Before explaining my views on that, I would like to refer to several other points in the paper. Referring to Fig. 6, it is not clear to me why that combination of circuits will give a surge having a wave of 30 ft. in length. As far as I can see that is a case of constantly applied voltage to an inductance and a resistance in series, in which case the voltage across the inductance decreases according to a logarithmic law.

The reason, in my opinion, as to why the total of 41,000 volts was not measured across the small gap is due to the small inductance of the loop and the relatively large capacitance between the terminals of the loop. I believe that practically any voltage, less than the 41,000 could be obtained across the small gap by changing the electrostatic conditions around the terminals of the gap. I can see no relation between the length of the loop and the length of the wave, for undoubtedly the length of the wave would be dependent on the inductance of the circuit. Let us assume, instead of being a loop it is 15 feet out and 15 feet back on itself, the loop still remaining 30 feet in length. Do you still have a voltage wave of 30 feet?

Referring to Section C, and particularly to Fig. 25, in all probability if that circuit was applied to a commercial system, there would be some kind of apparatus just back of the inductance, perhaps a generator, and if there is going to be any 360,000-volt surges coming in over the line, it probably would be a sad story to tell if the surge was permitted to bump into the inductance of the generator. It would undoubtedly be vastly better to have a reactor in the circuit that would reflect the surge back on the line, and leave it to dissipate its energy in the resistance of the line. If a reactance was placed in the circuit and shunted by a resistance, such as the authors indicate in No. 4 of Table V, when a 360,000-volt surge comes in over the line, and 83,000 volts across the reactance, what became of the other 277,000 volts? The answer is obvious

—that voltage is permitted to pass by the reactance and permitted to bump into the apparatus back of the reactance. It is easier to insulate a reactance for surges that come in over the line than to insulate generators or transformers for corresponding surges. Referring to the top of page 1313, concerning the 2400-ohm resistance that was used, Mr. Kierstead stated that it is 2400 ohms at 500 volts. I would like to ask Mr. Kierstead what the ohmic value of the resistance would be with 7000 volts across it, that is, the voltage that would be across it if it were installed in a 13,000-volt 3-phase system, under short circuit? Of course, under those conditions, the function of the reactor is to limit the current, and it is desirable to make the combined impedance-ohms under those conditions as high as possible, and I was wondering if the resistance would appreciably decrease at 7000 volts and by-pass much current?

Referring to the extra spacing on the end turns of reactors, it is interesting to note that as the spacing between end turns is increased the voltage across these end turns also increases. The increase in voltage is not in proportion with the increase in spacing. For instance, referring to Table VII, Coil No. 1 has 6.8 kv. across the first turn, with a spacing of one and five thirty-seconds inch, coil No. 2, 4.7 kv., with a spacing of 3/8 in., the ratio of the two voltages is 1.45 and the ratio of the spacing is 3.1, so that the improvement in the insulation of the end turn is a little better than double.

Referring to Section D, the subject of resonance is a matter that we should look into very carefully. On a distributing system we can get many combinations of circuits, and sometimes very peculiar things happen. However, I believe that in many cases we accuse and convict resonance of things for which it is not at all responsible. In investigating the conditions on a complicated distributing system, it is practically out of the question to get a mathematical analysis, since many of the factors are indeterminates. So we are left to work it out experimentally, and in making up our circuits and in working it out experimentally, it is very important that we get conditions that actually represent conditions on the system.

Now referring to Fig. 31, I claim that this is not a condition that will be met in practise. For instance, there are two feeders from one bus one having a current limiting reactor L_2 and the other no current limiting reactor. If it is necessary to install a reactor in one, it is necessary to install a reactor in both, and if reactors are installed in both the condition of resonance has disappeared, because then L_1 will approximately equal L_2 and C_1 will approximately equal C_2 and there is no building up of voltage in transferring from one condenser to the other.

However, if we assume the condition shown in Figs. 31 and 32 can be produced, still I claim that the conditions given in the paper will not be obtained, especially on an underground system, because in building up that voltage, the current involved in transferring the voltage from one condenser to the other, passes through quite a number of cycles, and in doing so, considerable energy is absorbed in the dielectric losses of the cable before the energy is transferred and for every cycle a large amount of energy is absorbed, and before the energy is transferred to the other condenser it is practically all absorbed.

It is very difficult in the laboratory to study circuits that represent commercial circuits, because it is difficult to imitate the capacitance and dielectric losses of lead covered cables. To do that, and represent a commercial circuit would probably require 20 miles of cable or perhaps 100 miles of cable, and it is not possible to represent these cables, by any other kinds of condensers, since the loss of the cable goes up, not only with voltage but also with frequency, and to my mind it is rather clear, from Fig. 33, that the circuit set up did not have anything of the order of dielectric loss in the condensers that would be met with in commercial circuits.

Philip Torchio: On this question of surge voltages on systems, especially on underground systems, from our experience it is practically proved that breakdowns are ultimately, if properly analyzed, due to some defect in the piece of apparatus and not to surges and steep wave voltages. During the experience we have had with the operation of reactors, we have never had any instance of trouble due to steep wave voltages on reactors. However, this subject of protection, as Messrs. Kierstead and Meeker suggest is very interesting.

My attention was first called to this system of protection at the time of the International Electrical Congress in Turin in 1911. Mr. Campos had developed a system based on the possibility of using shunting resistances and shunting condensers to protect the apparatus against high-frequency surges. At that time the suggestion was to protect against lightning. Personally, I have given a lot of thought and study, for many years, to this subject, and I doubt whether that system would be practicable as protection against lightning. However, if there are going to be steep wave surges in our systems, a protection of this kind may help a great deal, and I think we ought to give it wide application, and try to get the best knowledge from practical experience with it.

I think there is a misunderstanding about the action of these resistors by and some of the speakers, when they state that the voltage is shunted across the resistance and put on the generators and the apparatus back of it. With a high-frequency surge it will not go through the reactor but through the resistors, and the energy is therein expended, so that the apparatus behind would get only a fraction; at least, that is the theory of the people who have developed this system of protection. Mr. Campos' idea was to protect the high-voltage lines by using, for 500 feet near the receiving station or generating station, copper wire coated with nickel, so that the high-frequency current that flows on the surface of the wire and not in the wire itself, would expend its energy in the resistance of this 500 feet of nickel.

Harry R. Woodrow: The results that have been given on voltage stresses in reactors are theoretically correct, but there is a question if experience has shown the necessity of installing this additional equipment which may give a greater amount of trouble than the possible increase in protection.

The resistance shown here, which is 2400 ohms, and probably at 8000 volts, would go down to 1000 ohms would have a loss of 64 kw. on short circuits. That is an important feature for consideration in the design of the resistor.

There is one point, however, which is not brought out in this paper, in regard to the voltage stress on the end turns. The end-turn voltage stress is dependent upon the leakage reactance, between the end turn and second turn, and the capacitance between these turns. This plays a very important part in the ability of the end turns to withstand surges.

Some reactances have been made with insulation, on the end turns which are laid close together and therefore have a high capacity and a low leakage reactance, which have not shown any high potential between end turns. The results in this paper show that an increase in spacing between end turns does not increase the factor of safety. Therefore would it not be more desirable to insulate the end turns than install additional resistor?

C. L. Fortescue: The function of a reactor is narrowed by the authors to that of limiting the value of the short-circuit current, whereas it would seem that its proper function should be much broader namely, that of protecting the supply system from the result of accident of any kind occurring under normal operating conditions.

The sudden redistribution of charges that take place due to these accidents naturally give rise to abnormally high electromotive forces across any inductance in the path of the adjustment currents, and, therefore, if any portion of the protective inductance is prevented from absorbing its share of the electromotive forces set up, other portions of the system will have to absorb more than their share. It is true, of course, that due to the absorption of the energy by the protective devices the decrement of the surge is increased, but nevertheless, there will be a considerable increase in stress on the apparatus which the reactor is intended to protect. If the reactor, itself, cannot stand the gaff, how can the supply apparatus be expected to do so?

In dealing with this matter we should consider the economical aspects, that is, the relative amounts of the investments, generating apparatus and reactors, and find out whether it would be more logical to obtain better protection by building better reactors, rather than by protecting the reactors, risking breakdown of the generators.

H. Bristol Dwight: May I ask the authors to give, if they have not done so in the paper, some description of the injuries which the reactors have suffered from the high voltages? The paper seems to describe certain voltages, as though they had a suspicion of them. Have they had really practical trouble from these voltages.

A. Nyman: The function of current limiting reactors of absorbing the surges seems to be the one that is limited by putting a resistor across it. When it has to meet steep wave front current rushes, the current limiting reactors will smooth it out, and the apparatus behind it will receive only a very slow surge. Now, when you put a resistance across it the steepness is not reduced by the same amount, it will jump around the reactor and come right on the apparatus behind, so that, as Mr. Peters, Mr. Fortescue and some other gentlemen have pointed out, the voltage that exists in the surge is thrown over the reactor to the machinery behind it. Really, the resistor, instead of protecting the protector, shifts the potential of the protector onto the generator or other machinery.

Another factor I would like to point to is the hydraulic analogy. The analogy is very pretty, and they give a conception of what is happening during the surges, but it is rather risky to draw conclusions of the operation of these surges from such an analogy simply because the quantities you are dealing with are not the same. If the hydraulic surge through an instrument like that shown in some of the pictures creates a pressure disturbance throughout the apparatus, it does not necessarily mean you have the same pressure disturbance throughout the reactance coil.

Joseph Slepian: In regard to the last point of Mr. Torchio, the resistor cannot absorb energy, unless current is flowing through it. The only current that can flow through it is due to the charge carried by the steep high-frequency wave passing through the resistor to the generator, and it follows if the resistor is functioning at all, that there is a current passing through it, and there is a rapidly moving charge passing to the generator, which is going to be held back by the inductive windings and puncture the insulation.

F. H. Kierstead: I see there is quite a difference of opinion here. I just want to call attention to one of the different phases of the action of reactors that Mr. Woodrow referred to, that in high frequency it acts as a capacitant and not as a reactor, and therefore a large amount of energy passes through it. The amount of energy that would leak through in the resistor is very small in comparison to the amount of energy that goes through it as a capacity. The energy that goes through the resistor becomes absorbed as I^2r, and the energy that goes through as capacity is not absorbed.

I want to dwell on that just to bring out the point that we cannot always expect that the reactor in the circuit is going to round out the peaks, because if the peak is high enough, the capacity between the layers is sufficient to pass the high-frequency voltage through, and the peak will pass into the other inductive apparatus beyond it. If you do have a voltage built up across the reactors, you have it also built up across the resistor, and some of the energy will be absorbed into the system that is likely to cause trouble.

A question has been asked about the value of the resistance at 7000 volts. Mr. Woodrow answered that question for me—in this 2400-ohm resistor, it is between 1000 and 1500 ohms, so it is a very high barrier at normal frequency.

In regard to the capacities that we used, I think one of the speakers referred to that as being equal to 20 miles of cable. The capacity we used in our resonant test was comparable to a mile of cable. We do not need to use a very high capacity to get a voltage nine times the applied voltage in the resonant test. The resistor reduced that voltage to a very low voltage.

What Mr. Woodrow says in regard to the increase of insulation between the end turns without increasing the space—really increasing the capacity—does reduce the voltage across the end turns, and I agree with Mr. Woodrow in the statement that the voltage divides across the reactor in accordance with the capacity of the reactor.

Mr. Dwight asked a question as to why we thought there are high voltages across reactors. We have had reactors that flashed over, and we placed across them in a test, much higher voltages than we could ever conceive of actually happening in normal operating conditions, therefore, it leads us to believe there are very high-voltage stresses across reactors.

Philip Torchio: We have had the first reactance coil I built some years ago tested in an experimental test, and it flashed over at the ends. There was no extra voltage there—it was the moisture that caused it. As I said before, we have not had any experience with failures due to flash over, that we know of.

The first trouble we had was flash over and then we reinforced the insulation. I do not know whether, the insulation increases the capacitance of the circuit or removes the moisture, and it is difficult to say, if you have flash over in service, whether it is due to overvoltage or due to a poor coil or dust or moisture, or some other cause.

F. H. Kierstead: We have reactors around in our shop that we have given no particular attention to. They are thrown around wherever the craneman wants to put them, and we put them back into service and we do not find that the insulation strength has been deteriorated to an amount that can be discovered by any voltage test we put across them.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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CALCULATION OF MAGNETIC FORCE ON DISCONNECTING SWITCHES

BY HERBERT BRISTOL DWIGHT

Canadian Westinghouse Co., Ltd., Hamilton, Canada

ABSTRACT OF PAPER

The practical problem of finding the magnetic force tending to open a disconnecting switch has seldom been the subject of discussion in the past, but it is a very useful problem to solve. The result is expressed in concise form in formulas (**20**) and (**21**), and their derivation and the assumptions on which they are based are given.

Curves are also presented in Figs. No. 3 and No. 4 from which the force may be found without using the formulas.

IT is a well-known feature connected with short circuits in large electric power systems, that disconnecting switches are sometimes forcibly opened by the magnetic repulsion caused by the large currents flowing. This is recognized by the designers of disconnecting switches, who provide the switches subject to heavy duty with latches to hold them closed.

In this paper are given formulas for the magnetic force acting on a disconnecting switch, which can be applied to the various types of circuits usually used with such switches. Curves are given in Figs. 3 and 4 from which values of magnetic force can be taken without using the formulas. It is believed that the formulas or the curves give the value of the magnetic force within a very small percentage. The formulas should be useful not only to the designers of switches who must design the parts so as to withstand the maximum force to be expected, but also for purposes of comparison when circuits are being designed containing disconnecting switches, for it is often desirable to choose a form of circuit which will produce the least possible magnetic force on the switch. The formulas and methods of calculation may also be used for calcu-

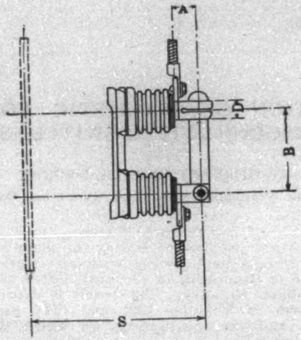

Fig. 1—Front-Connected Disconnecting Switch

lating the forces on different parts of circuit breakers and other types of apparatus.

In Figs. 1 and 2 are shown typical disconnecting switches. It is evident that with the front-connected switch especially, the parts of the circuit parallel to the blade can have a large proportionate effect on the magnetic force acting on the blade, and should not be neglected.

In Fig. 5 there is shown a typical electric circuit

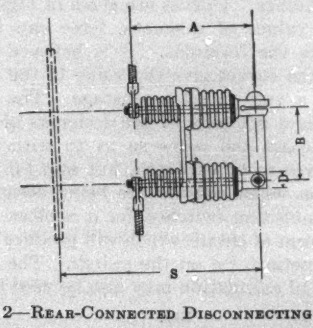

Fig. 2—Rear-Connected Disconnecting Switch

which approximates very closely to a disconnecting switch. The jaws are represented by round rods of diameter $D = 2r$, and the blade is a flat strap of width $2c$.

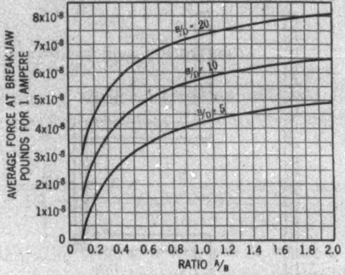

Fig. 3—Repulsion in Disconnecting Switches

The force varies as the square of the current.
Force due to return circuit not included.
B = Length of blade.
D = Width of break-jaw.

The mechanical force on the blade is proportional to the current in the blade and to the magnetic lines of force cutting it. The mechanical force acts in a direction perpendicular to the blade.

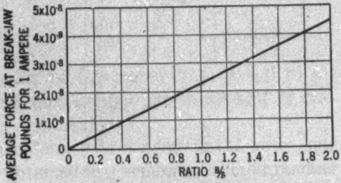

Fig. 4—Repulsion Due to Return Circuit

The force varies as the square of the current.
B = Length of blade.
S = Distance from blade to return conductor.

The first part calculated will be the force on the part of the blade not inside the break-jaw, produced by magnetic lines due to the current in the upper arm of length *A*. The current in the arm, which tapers off gradually for the distance $2c$ at the jaw, may be assumed to be full strength as far as the middle of the blade. The magnetic field produced by current in a round rod is in circles around it, and the strength of the field is the same, outside the metal, as if the current were concentrated at the axis of the rod. Therefore, if the rear part of the arm is of a smaller diameter than D, the magnetic field at the blade will not be changed on that account.

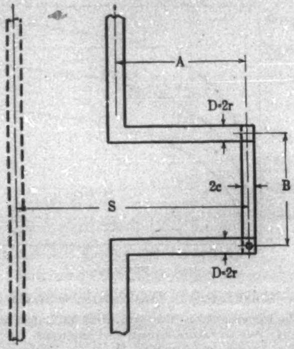

FIG. 5—TYPICAL CIRCUIT

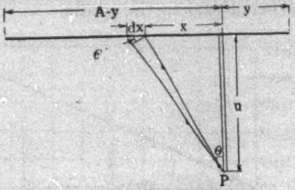

FIG. 6

The flux density at a point P, Fig. 6, caused by a current of I absamperes in the element of conductor

dx is $\dfrac{I\,dx\cos\theta}{u^2 + x^2}$ [1]lines per square centimeter (1)

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[1] "Elements of the Mathematical Theory of Electricity and Magnetism," by J. J. Thomson, Fourth Edition, Art. 211, page 356.

$$= \frac{I\,u\,dx}{(u^2 + x^2)^3/_2} \quad \text{lines per square centimeter} \qquad \textbf{(2)}$$

The flux density at P due to the length $A - y$ is obtained by integrating expression (**2**) from

$x = 0$ to $x = A - y$ and is

$$\frac{I\,(A - y)}{u\,\sqrt{u^2 + (A - y)^2}} \text{ lines per sq. cm.} \qquad \textbf{(3)}$$

By adding a similar expression obtained for the length y, the flux density at P due to the conductor of length A is

$$\frac{I\,(A - y)}{u\,\sqrt{u^2 + (A - y)^2}} + \frac{I\,y}{u\,\sqrt{u^2 + y^2}} \text{ lines per sq. cm.} \qquad \textbf{(4)}$$

By integrating from $u = r$ to $u = B$, where y is constant, the flux in an elementary strip of the blade is found to be

$$I\left[\operatorname{logh}\left\{\frac{(A - y) + \sqrt{(A - y)^2 + r^2}}{r}\right\} - \operatorname{logh}\left\{\frac{(A - y) + \sqrt{(A - y)^2 + B^2}}{B}\right\} + \operatorname{logh}\left\{\frac{y + \sqrt{y^2 + r^2}}{r}\right\} - \operatorname{logh}\left\{\frac{y + \sqrt{y^2 + B^2}}{B}\right\}\right] dy \text{ lines} \qquad \textbf{(5)}$$

This must be integrated from $y = 0$ to $y = C$ to obtain the flux cutting the inner half of the blade, outside of the break-jaw.

The flux density at a point in the outer half of the blade, is obtained in a somewhat similar manner and is equal to

$$\frac{I\,(A + y)}{u\,\sqrt{u^2 + (A + y)^2}} - \frac{I\,y}{u\,\sqrt{u^2 + y^2}} \text{ lines per sq. cm.} \qquad \textbf{(6)}$$

By integrating this from $u = r$ to $u = B$, and then from $y = 0$ to $y = c$ the flux cutting the outer half of the blade is obtained.

Thus, the flux cutting the entire blade outside the break-jaw, produced by the conductor of length A is found to be

$$
\begin{aligned}
I \Bigg[& (A + c) \operatorname{logh} \left\{ \frac{(A + c) + \sqrt{(A + c)^2 + r^2}}{r} \right\} \\
& - (A + c) \operatorname{logh} \left\{ \frac{(A + c) + \sqrt{(A + c)^2 + B^2}}{B} \right\} \\
& - (A - c) \operatorname{logh} \left\{ \frac{(A - c) + \sqrt{(A - c)^2 + r^2}}{r} \right\} \\
& + (A - c) \operatorname{logh} \left\{ \frac{(A - c) + \sqrt{(A - c)^2 + B^2}}{B} \right\} \\
& + \sqrt{(A + c)^2 + B^2} - \sqrt{(A + c)^2 + r^2} \\
& - \sqrt{(A - c)^2 + B^2} + \sqrt{(A - c)^2 + r^2} \Bigg] \text{ lines} \qquad (7)
\end{aligned}
$$

The mechanical force on the blade in dynes, due to this flux is obtained by multiplying expression (7) by $\frac{I}{2\,c}$ where I is the current in absamperes.

Formula (7) is expressed more simply as a convergent series, but different series are required for the two cases in which B is less than A, and greater than A.

For this purpose, the following series are used:—

$$
\operatorname{logh} (x + \sqrt{1 + x^2}) = x - \frac{1}{2.3} x^3 + \frac{1.3}{2.4.5} x^5 - \frac{1.3.5}{2.4.6.7} x^7 + \ldots \qquad (8)
$$

where x^2 is less than 1,

and

$$
\operatorname{logh} (x + \sqrt{1 + x^2}) = \operatorname{logh} (2\,x) + \frac{1}{2.2} u^2 - \frac{1.3}{2.4.4} u^4 + \frac{1.3.5}{2.4.6.6} u^6 - \ldots \qquad (9)
$$

where x^2 is greater than 1 and where $u = \frac{1}{x}$.

Where B is less than A, the force due to the flux outside of the jaw, caused by the conductor of length A is

$$\frac{I^2}{100}\left[\operatorname{logh}\left(\frac{B}{r}\right)-\frac{1}{4}\frac{B^2}{A^2}+\frac{1}{4}\frac{r^2}{A^2}+\frac{3}{32}\frac{B^4}{A^4}\right.$$
$$\left.-\frac{1}{4}\frac{B^2\,c^2}{A^4}+\ldots\right]\text{dynes} \qquad \textbf{(10)}$$

where I is the current in amperes.

When B is greater than A the above force is

$$\frac{I^2}{100}\left[\operatorname{logh}\left(\frac{2\,A}{r}\right)-\frac{A}{B}-\frac{1}{6}\frac{c^2}{A^2}+\frac{1}{4}\frac{r^2}{A^2}\right.$$
$$+\frac{1}{6}\frac{A^3}{B^3}+\frac{1}{6}\frac{A\,c^2}{B^3}-\frac{3}{40}\frac{A^5}{B^5}$$
$$\left.-\frac{1}{4}\frac{A^3\,c^2}{B^5}+\ldots\right]\text{dynes} \qquad \textbf{(11)}$$

Some of these terms are small and may be omitted.

The second part of the force to be calculated is that due to the flux cutting the part of the blade inside the jaw of the switch.

The magnetic flux inside the metal of the round rod is in concentric circles around the axis. The flux density at any of the circular paths is that which would be produced by the current inside the circular path, for current outside cannot produce magnetic flux in a path which does not surround the current. Therefore, the flux density at radius u inside the metal is obtained by multiplying expression (**4**) by $\frac{u^2}{r^2}$ for the parts P and Q of the jaw and by multiplying expression (**6**) by $\frac{u^2}{r^2}$ for the parts R and S. Now the flux in the jaw does not produce the full amount of mechanical force on the blade, because the current in the blade tapers off gradually from the full value to zero in the distance $2\,r$ in the jaw. Consequently, the flux in the parts P and R must be multiplied by $\frac{(r+u)}{2\,r}$ to obtain the effective flux, and the flux in the parts Q and S must be multiplied by $-\dfrac{(r-u)}{2\,r}$, since the flux in the parts Q and S is in the opposite direction to that in P and R and is such as to tend to hold the blade closed.

By integrating the effective flux for each of the four parts of the jaw, the total force on the part of the blade in the jaw is found to be

$$\begin{aligned}\frac{I^2}{4\,r^3 c}\Bigg[\ &\frac{1}{4}\,(A-c)^4 \operatorname{logh}\left\{\frac{r+\sqrt{r^2+(A-c)^2}}{(A-c)}\right\}\\ &-\frac{1}{4}\,(A+c)^4 \operatorname{logh}\left\{\frac{r+\sqrt{r^2+(A+c)^2}}{(A+c)}\right\}\\ &+\frac{r}{4}\,(A-c)^2\,\sqrt{r^2+(A-c)^2}\\ &-\frac{r}{4}\,(A+c)^2\,\sqrt{r^2+(A+c)^2}\\ &-\frac{r}{2}\,\{r^2+(A-c)^2\}^{3/2}\\ &+\frac{r}{2}\,\{r^2+(A+c)^2\}^{3/2}\ \Bigg]\text{ dynes}\end{aligned}\qquad(12)$$

When this is expressed as a convergent series, it becomes

$$I^2\left(\frac{1}{3}-\frac{1}{10}\,\frac{r^2}{A^2}\right)\text{ dynes}\qquad(13)$$

This is, in an ordinary case, about 8 per cent of the total mechanical force on the switch blade. Since the result of this part of the calculation is such a small percentage of the total, the effect of the approximations used in making up the typical circuit, will be seen to be small. The fact that the current in the horizontal conductor tapers gradually to zero in the distance $2\,c$ and the fact that the section of the jaw may be approximately square instead of round, will not make an important difference in this small part of the total force. If the blade is made up of two parallel straps, which is often done to make a more rigid structure, as indicated by the dotted lines at M and N, Fig. 7, only the outermost lines of force will cut the blade. However, when the integration is carried out for this case, expression (**13**) is found to be reduced by only one-eighth of itself. This is only 1 per cent of the total force. Accordingly, only one formula is given for the

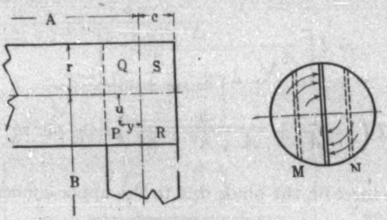

FIG. 7—SWITCH JAW

different types of disconnecting switches, and the formula has been calculated on the basis of a single-blade switch.

It is evident that the effective flux inside the metal is calculated quite differently from the method used

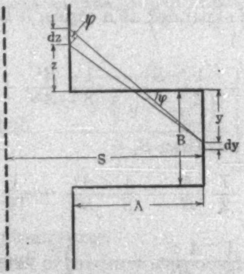

FIG. 8—TYPICAL CIRCUIT FOR CONNECTIONS PARALLEL TO BLADE

for calculating self-inductances, and therefore the geometrical mean distance of the section of the conductor should not be used in this problem.

In calculating the force due to the connections parallel to the blade, the flux density at $d\,y$, Fig. 8, due to the upper connection is in the opposite direction to the flux previously considered, and is

$$I \int_{z=0}^{z=\alpha} \frac{-\,d\,z\cos\phi}{A^2+(y+z)^2} = I \int_{z=0}^{z=\alpha} \frac{-\,A\,d\,z}{\{A^2+(y+z)^2\}^{3/2}}$$

$$= I\left[\frac{-1}{A\sqrt{1+\dfrac{A^2}{(y+z)^2}}}\right]_{z=0}^{z=\alpha}$$

$$= I\left(-\frac{1}{A}+\frac{y}{A\sqrt{A^2+y^2}}\right) \text{ lines per sq. cm.} \tag{14}$$

The force on the blade due to the upper connection is

$$I^2 \int_{y=0}^{y=B}\left(-\frac{1}{A}+\frac{y}{A\sqrt{A^2+y^2}}\right) d\,y$$

$$= I^2\left[-\frac{y}{A}+\frac{\sqrt{A^2+y^2}}{A}\right]_{y=0}^{y=B}$$

$$= I^2\left(-\frac{B}{A}+\frac{\sqrt{A^2+B^2}}{A}-1\right) \text{ dynes} \tag{15}$$

I being the current in absamperes.

When this is expanded as a series, it becomes, when *B* is less than *A*,

$$\frac{I^2}{100}\left(-\frac{B}{A}+\frac{1}{2}\,\frac{B^2}{A^2}-\frac{1}{8}\,\frac{B^4}{A^4}+\ldots\right) \text{ dynes} \tag{16}$$

and when *A* is less than *B*,

$$\frac{I^2}{100}\left(-1+\frac{1}{2}\,\frac{A}{B}-\frac{1}{8}\,\frac{A^3}{B^3}+\frac{1}{16}\,\frac{A^5}{B^5}-\frac{1}{8}\,\frac{A\,c^2}{B^3}+\ldots\right) \text{ dynes} \tag{17}$$

where *I* is the current in amperes. The term

$$-\frac{1}{8}\,\frac{A\,c^2}{B^3}$$

is obtained by integrating the formulas over the blade of width 2 *c*.

In some cases the return conductor is brought back directly behind the switch at a distance S, as indicated by the dotted lines in Figs. 1, 2, 5 and 8, and this adds to the force tending to open the switch. The flux density at $d\,y$, Fig. 8, due to the part of the return conductor above the center of the switch is

$$I\int_{z=-\frac{B}{2}}^{z=\alpha}\frac{S\,d\,z}{\{S^2+(y+z)^2\}^3/_2}$$

$$=I\left[\frac{1}{S\sqrt{1+\dfrac{S^2}{(y+z)^2}}}\right]_{z=-\frac{B}{2}}^{z=\alpha}$$

$$=I\left[\frac{1}{S}-\frac{\left(y-\dfrac{B}{2}\right)}{S\sqrt{S^2+\left(y-\dfrac{B}{2}\right)^2}}\right]$$

lines per sq. cm. **(18)**

The force on the blade due to this part of the conductor is

$$I^2\int_{y=0}^{y=B}\left[\frac{1}{S}-\frac{\left(y-\dfrac{B}{2}\right)}{S\sqrt{S^2+\left(y-\dfrac{B}{2}\right)^2}}\right]d\,y$$

$$=I^2\left[\frac{y}{S}-\frac{\sqrt{S^2+\left(y-\dfrac{B}{2}\right)^2}}{S}\right]_{y=0}^{y=B}$$

$$=I^2\,\frac{B}{S}\text{ dynes}\qquad\textbf{(19)}$$

where I is in absamperes.

This is the part of the force supported by the friction of the break-jaw, or the latch, and it agrees with the usual formula for the repulsion of two long parallel conductors.

The forces acting on the switch blade are in general symmetrical as regards the two ends of the blade. One-half of the complete total force is supported at the break-jaw and one-half at the hinge-jaw. Therefore, in calculating the force at the break-jaw, an addition is made of the forces acting on the blade due to one horizontal conductor, one vertical connection to the switch, and one-half of the return conductor, if it returns directly behind the switch. In cases where the horizontal connections are of unequal length, a good approximation to the value of the force may be obtained by using their average length for the dimension A in the formulas.

The force at the break-jaw, expressed as a series, is when B is less than A,

$$\frac{I^2}{4.45 \times 10^7}\left[2.30 \log_{10}\left(\frac{B}{r}\right) + \frac{1}{3} - \frac{B}{A} + \frac{1}{4}\frac{B^2}{A^2} + \frac{3}{20}\frac{r^2}{A^2} - \frac{1}{32}\frac{B^4}{A^4} + \frac{B}{S}\right] \text{ pounds} \quad (\mathbf{20})$$

When A is less than B, the force is,—

$$\frac{I^2}{4.45 \times 10^7}\left[2.30 \log_{10}\left(\frac{2A}{r}\right) - \frac{2}{3} - \frac{1}{2}\frac{A}{B} - \frac{1}{6}\frac{c^2}{A^2} + \frac{3}{20}\frac{r^2}{A^2} + \frac{1}{24}\frac{A^3}{B^3} + \frac{1}{24}\frac{A\ c^2}{B^3} + \frac{B}{S}\right] \text{ pounds} \quad (\mathbf{21})$$

where I is in amperes. If the circuit does not return behind the switch, the term $\frac{B}{S}$ is zero.

The average force in pounds acting along the center-line of the break-jaw, assuming that the circuit does not return behind the switch, is plotted in Fig. 3. If the circuit returns directly behind the switch, the force to be added can be taken from Fig. 4. These curves are more convenient for the solution of engineering problems than the formulas. In the curves, c is taken to equal r.

The dimensions used in the formulas and curves may be all in centimeters or all in inches, since only ratios of dimensions appear.

The force given by the formulas and curves is the average force or the steady push, exerted by the current, which is measured in effective amperes. It is the force which would be measured by a spring balance if the blade were free to move. However, it should not be forgotten that with alternating current the force rises to double the average value every cycle. This large momentary value of force must be used in some problems, for it is capable of overcoming initial friction or of cracking a latch which holds a switch closed. If I represents the peak value of the current, the formulas give the maximum momentary value of the force.

It would be desirable if the calculated solution of this problem were to be checked by laboratory measurements. No published measurements of the force on a disconnecting switch are known to the writer, but it should be possible to make such a measurement with an accuracy within a few per cent.

This problem was discussed by J. Loebenstein in the *Electrical World* of March 17, 1917, but the basis of his calculation was wrong and there were also errors in the ensuing mathematical operations, so that the final results were much smaller than they should have been.

An approximate solution of this problem was published by the writer in October, 1917,[2] the assumptions being made that the effective flux in the jaw was negligible and that the average flux density in the blade was equal to the density at the center line of the blade.

An approximate formula for the force of repulsion was given by L. B. W. Jolley in a letter to the *Electrical World*, page 381, Feb. 14, 1920. In this it was assumed among other things that the complete circuit was a rectangle, and the connections parallel to the blade, which make a large percentage effect on the result, were neglected.

Example. Find the force of repulsion at 30,000 amperes, effective, on a disconnecting switch in which $A = 15$ inches, $B = 20$ inches, width of blade $= 2c = 1.5$ inches and width of jaw $= D = 2r = 1$ inch.

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[2] "The Force of Repulsion in Disconnecting Switches," by H. B. Dwight, *The Electrical Review*, Oct. 20, 1917, page 684.

There is no return circuit behind the switch.

By formula (**21**), since A is less than B, the average force at the break-jaw is

$$\frac{9 \times 10^8}{4.45 \times 10^7}\left[4.094 - 0.667 - 0.375 + 0.018\right]$$

$$= \frac{90}{4.45} \times 3.07 = 62 \text{ pounds.}$$

Using the curves of Fig. 3, $\frac{A}{B} = 0.75$ and $\frac{B}{D} = 20$, and therefore the average force is $9 \times 10^8 \times 6.9 \times 10^{-8} = 62$ pounds.

The maximum momentary force at the peak of the current wave is 124 pounds.

Discussion on "Calculation of Magnetic Force on Disconnecting Switches" (Dwight), White Sulphur Springs, W. Va., July 1, 1920.

V. Karapetoff: There is an increasing tendency to handle problems involving force in terms of energy. It is a method that was brought up yesterday and also today, and I believe that Mr. Dwight's problem should be so handled rather than from the standpoint of the La Placian equation of an elementary force acting upon an infinitesimal length of current. The treatment is much more elegant and moreover the result leads to a method of experimental electrical determination of the mechanical forces tending to open a disconnecting switch. The particular method of expressing energy to which I have reference is based on one of Kelvin's laws, namely, that if a deformation takes place in an electric circuit without saturation, at a constant current, the mechanical work done constitutes only one half of the energy brought from the source, the other half increasing the stored energy of the circuit. I always remember this law and also tell my students to remember it in this way: A constant electric current always collects 50 per cent profit for doing mechanical work (Maxwell, *Electricity and Magnetism*, v. 2. page 225).

In figuring out the energy relations I shall use the principle of virtual displacements, that is, displacements which need not necessarily take place, but which might take place. As such a virtual displacement I shall consider an infinitesimal displacement $d\,s$ of the blade of the switch, parallel to itself, by an infinitesimal amount, and call F the mechanical force necessary to resist its motion. I shall assume that this deformation of the circuit takes place at a constant current I, that is, by some controlling arrangement, the current is prevented from changing its value. Then, in accordance with the above law of fifty per cent profit, I can figure out by what amount the stored energy is increased, and that is fifty per cent of the energy brought in, therefore the other fifty per cent is the mechanical work done.

Let the coefficient of self-induction of the whole disconnecting switch be L. The initial stored electromagnetic energy is $1/2\, I^2\, L$ and its increase due to the displacement is $1/2\, I^2\, d\,L$. The work done upon the resisting force is $F\,d\,S$, so that

$$F\,d\,s = 1/2\, I^2\, d\,L,$$

hence

$$F = 1/2\, I^2\, (d\,L \,/\, d\,s).$$

see V. Karapetoff's "Magnetic Circuit" page 251;

C. P. Steinmetz, A. I. E. E. TRANS., Vol. 30 (1911). This is all the general mathematics needed for this problem, or for almost any problem involving mechanical forces in an electro-magnetic field.

We find that the force is proportional to the rate of change of the inductance, with the independent variable, which ever variable it may be. Of course, in this case F is the total force acting on the blade, one half in the jaw, and the other half in the hinge. If you are interested in the force acting on the hinge, we have to divide it by two.

This method and this formula permit computation of mechanical forces in a circuit of any irregular shape. Of course it involves a detailed computation or at least an estimate of self induction but this is a subject on which we have a very extensive literature. Those of us who have to determine coefficients of self and mutual induction regularly in our business are reasonably sure of the results in dealing with a quantity of this kind.

The formula given above enables one to predetermine the mechanical force on a given switch by a simple experiment which involves electrical measurements only. Place the blade in a wrong position, not far from the correct position and measure the coefficient of inductance of the circuit by one of the well known methods. Shift the blade a little, and again measure the coefficient of inductance; repeat this for four or five positions, differing slightly from each other. Then plot a curve giving values of L for different positions of the blade adjacent to its actual position in the switch, and then determine the slope of the curve, which will give you the value of $d\,L/d\,s$. Substitute this value in the formula given above and you will have the mechanical force for any desired value of the current, thus doing away with the necessity of actually measuring this force with a spring balance. You, of course, realize that such a mechanical measurement of F would involve tremendous values of current in order to obtain any reliable results.

A. Nyman: I was much interested in Prof. Karapetoff's explanation of measurement of mechanical stresses. I had an opportunity about a year ago to make similar calculations on a circuit breaker, subject to short-circuiting stresses due to current of about 100,000 amperes, and this is exactly the method I employed. I found by using it I did not have to cover more than a couple of pages of mathematics to get all the stresses on the lower portion of the circuit breaker, on the arc tips, and the bending moment on the bars, coming down from the top of the case.

Fig. 1 is an approximate sketch of the circuit breaker, and the forces I considered were, first, the force through point *A*, the force tending to open out the arc tips, and the force tending to bend out the buses going down through the case.

In figuring these various forces, I not only plotted the formula as it stands here, but separated it into self-inductance and mutual-inductance, in other words, for the force on the bus *B*, I took the increase in self-inductance in the bar *A*, and the bar *B*, and also the increase in mutual inductance between these bars; in this way the result would give me the force on this individual bus, and similarly the force on the arc tips.

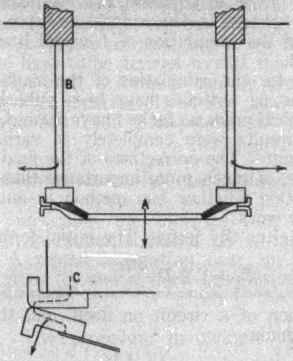

FIG. 1

I would assume the current path as an approximation of the actual path (as at *C*) and figure the increase in self inductance in this path and the change in mutual inductance between the various parts. This way of making the calculation is helpful when you have to figure out the bending moment, because you will find the force will be different for each point along the bus, and it is a matter of integration, if you know the forces at each point, to get the total bending moment.

E. G. Merrick: In applying these formulas, we should naturally consider the worst possible condition in a short circuit, which will be with a totally displaced wave. I believe, to be on the safe side, we should use the peak values of this wave. The inertia and elasticity are so small that they can be practically neglected, and the force considered as a hammer blow corresponding to the maximum peak value of current.

H. B. Dwight: In reply to the statement of Prof. Karapetoff that it would be better to use the well known formula in $\frac{d\,L}{d\,s}$ than equation (1) of my paper as the starting point of the calculation, I may say, that I have used both formulas in working on this problem at different times and I have found in equation (1) to be the proper one to use.

I am sorry that Prof. Karapetoff has not worked out his suggested method to a conclusion, for serious difficulties would be encountered in doing so. One difficulty would be in calculating the force in the switch jaw (given in equation **13**). As shown in the paper, the currents in the conductors taper gradually to zero, and the calculation of L would become very complicated.

Solutions for the calculation of the magnetic force in disconnecting switches have been published twice in the technical press, so far as I have noticed, and both times the results were completely at variance with my own result. The correctness of the final formulas is, therefore, of much more importance than the academic question whether one method or another will involve less work.

H. B Dwight: (by letter) The initial formulas

$$F = 1/2\, I^2 \frac{d\,L}{d\,s}$$

for the action of a circuit on itself, and the corresponding formula

$$F = I^2 \frac{d\,M}{d\,s}$$

for the action of one circuit on another, are well known. They are convenient to use where a formula for L or M may be taken with accuracy and safety from some publication, which is not the case with the present problem. The writer has used this method for calculating the force between reactance coils with parallel axes.*

An attempt to use the above method for calculating the force on a disconnecting switch was made by L. B. W. Jolley, as referred to at the close of my paper. In following the procedure of taking a published formula for L in order to find $\frac{d\,L}{d\,s}$, he was forced to use the formula for L of a rectangle since that is probably the only formula published, even in approximate form, which involves two conductors at right angles. He used practically the same formula as given in eq. (**107**) page 155, Scientific Paper No. 169 of the Bureau of Standards. However, this formula, while approximately correct for a rectangular circuit using a small conductor, involves serious error if used for a disconnecting switch. The connections at the rear of the switch are not considered and the formula applies only imperfectly to a rear-connected switch and not at all to a front-connected switch. Further, the formula practically neglects the flux in the corner represented by the switch jaw, and this produces an error of about 8 per cent in the final result.

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*Some New Formulas for "Reactance Coils" by H. B. Dwight, TRANSACTIONS A. I. E. E. 1919, page 1681.

It is therefore plainly evident that a published formula for L cannot be used, but a special formula for L would have to be derived to suit this particular problem. But such a formula can be derived only by starting with equation (**1**) of my paper, which is the very equation Prof. Karapetoff said should not be used.

If the suggestion to use $\frac{d\,L}{d\,s}$ be followed out in the only way practicable, the following comparison can be made: In my paper, equation (**1**) is integrated to give (4). A second integration over the length of the blade gives (5) and a third integration over the width of the blade gives (7), which is the expression for force. The procedure involving $\frac{d\,L}{d\,s}$ requires integrating (1) to give (4) A second integration is made to the length B. A third integration is made to the length A and a fourth integration to the length C. Then this expression for L must be differentiated to give $\frac{d\,L}{d\,s}$. Thus the suggested procedure involves four integrations and one differentiation, or two operations more than the procedure in my paper.

It is, therefore, even more absurd to state that the equation

$$F = 1/2\, I^2 \frac{d\,L}{d\,s}$$

is all the general mathematics needed for this problem, than it would be to say that equation (**1**) of my paper is all the general mathematics required.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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DESIGN OF CONSTANT-CURRENT GENERATORS FOR ARC WELDING

BY K. L. HANSEN

Designing Engineer, Mechanical Appliance Co., Milwaukee, Wis.

IN design of medium size direct-current machines the problem is usually to meet certain stable or permanent conditions, such as efficiency and temperature rises at a given load, voltage regulation of generators, speed regulation of motors, etc. However, as the requirements become more exact, transient conditions may increase in importance to the extent of becoming a factor in the design of a machine. For example, in a motor application where the starts and stops are very frequent, or the motor reverses direction of rotation at short intervals, the performance during the acceleration period may be of greater importance than when operating at a definite speed.

The conditions met with in the application of the generator to be discussed in this paper are such that the proportions of the various parts of the machine are determined almost entirely by consideration of the transient phenomena.

Due to the well-known characteristic of the electric arc, that its resistance decreases with increase in current and vice versa, a stabilizing agent such as a ballast resistance is required when the arc is operated from a constant-potential circuit. This results in a considerable loss of energy and consequently necessitates a generating equipment of larger capacity than the energy expended in the arc itself requires. Furthermore, with a constant-potential generator, auxiliary equipment in the form of grid resistances and a rather complicated system of switches for controlling the current are required. It is natural, therefore, that considerable effort should have been directed towards eliminating these obvious disadvantages, and it is to accomplish this that the variable-voltage or constant-current generators have been developed.

VOLT-AMPERE CHARACTERISTIC

It is evident that to overcome the inherent unstable characteristic of the arc, variations in the current should cause the terminal voltage of the generator to vary in opposite sense, but there seems to be quite a divergence of opinion in regard to the extent to which it should so vary; that is whether it should vary to maintain a practically constant current or substantially less than would be required to maintain constant current.

One theory advanced is that in order to maintain constant heat the wattage expended in the arc should be constant, or the product $E\,I$ equal a constant, where E is the arc voltage and I the current. The curve represented by this equation, when plotted with current as abscissas and the volts as ordinates, is a hyperbola approaching the X-axis asymptotically for zero resistance in the arc circuit, that is the current has a very large value on short circuit. (Curve *a* Fig. 1).

In the results of a large number of experiments, which the writer has recently had the opportunity to observe, he has found nothing to support the contention that this characteristic is particularly suited for arc welding. Indeed it can readily be seen that in the extreme cases of very large current at low voltage or small current at high voltage the theory would not hold, and the assertion is therefore made that it is necessary or desirable to maintain a constant product of volts and amperes for small variations from normal arc voltage only.

For this reason the characteristic obtained on actual machines is approximately as shown by curve *b*, which over a small range approximates curve *a*. Curve *b* is the curve that would be obtained in a generator with a separately excited field and a differential series field, and it also approximates the curve of a constant-potential generator with a ballast resistance.

The normal arc voltage is 18 to 22, and it will be noticed that even with a characteristic as shown by curve *b*, the short-circuit current has a considerably larger value than the welding current. In the opinion of many it is desirable that the starting current should be larger than the working current in order to heat the point of contact rapidly.

The writer's experience, on the contrary, has been that a large current at the instant of striking the arc merely increases the tendency of the electrode to freeze or stick. For example, less ballast resistance is required at 60 volts constant potential than at 75 volts, and the short-circuit current is therefore greater. As a result of the larger starting current the arc is appreciably more difficult to start at 60 volts than at 75, while the arc is almost as easy to maintain in one case as in the other.

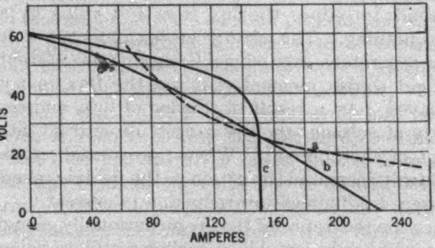

Fig. 1—Characteristic Curves of Variable Voltage Arc Welding Generators

It should moreover be borne in mind that in the case of the constant-potential system the value of the momentary short-circuit current is not greater than that of the permanent short-circuit current. As a matter of fact, due to the momentary drop in voltage that takes place in all d-c. generators with the sudden application of load, the short-circuit current may not even reach its permanent value if the electrode is quickly withdrawn. The greater the reactance of the main circuit the greater will be the drop in voltage and the longer will be the time required to reach the permanent value of current. Even a small reactance is therefore an effective aid in starting the arc on a low-voltage constant-potential circuit.

In a variable-voltage generator it can readily be seen, however, that the momentary short-circuit current is always greater than the permanent short-circuit current. When, for example, the generator is running on open circuit and the terminals are suddenly short-circuited, the rise of current in the series field tends to reduce the flux from full value to practically nothing. This change of flux induces a voltage in the separately excited field, causing the current in this field to rise momentarily, and the flux reduction is delayed. As a result of this lag of flux, and consequently of voltage, there is a rush of current at the instant of short circuit. When the external resistance is suddenly increased the action of the fields is reversed, and there is a momentary reduction in current.

Since the resistance of the arc is continually changing, due to its inherent instability, molten metal dropping off the electrode, unsteadiness of the operator's hand, uneveness of the material, etc., the generator must adjust its terminal voltage to meet these rapidly changing conditions.

From the preceding it will readily be seen that to take care of these transient conditions becomes a very important factor in the design. The writer's experience has been that, if these momentary fluctuations are kept within certain limits, a stable arc can be maintained with almost any fairly drooping volt-ampere characteristic. However, a stable arc does not necessarily mean satisfactory welding. The usual method of smoothing out the momentary fluctuations is by inserting a large reactance in the welding circuit. While a large reactance makes it easy to strike and maintain the arc, it seems that the penetration is seriously affected. Moreover, the long arc which can be drawn makes it possible for a careless or less skillful welder to hold the arc and deposit metal without insuring a good weld. Furthermore, a large reactance adds considerably to the cost and weight of the equipment and is also the source of considerable loss of energy and thus reduces the efficiency. The reactance should, therefore, be reduced to a minimum or, if possible, eliminated altogether.

As already stated, a large permanent short-circuit current is neither necessary nor desirable. On the contrary, even when the permanent short-circuit current has approximately the same value as the welding current (Curve *c* Fig. 1), the momentary short-circuit current has to be limited either by special design or an external reactance to keep the electrode from sticking.

All factors considered, such as penetration, ease of starting and maintaining the arc, ease of controlling the current, etc., the characteristic shown by curve *c* is, in the writer's opinion, the most desirable. It will be noticed that in this case the regulation of the machine is such as to keep the current approximately constant over the operating range, that is from short circuit up to about 30 volts, for a slowly varying external resistance. While it would be desirable to keep the current constant, even for the rapidly varying resistance of the arc circuit, this cannot be done, as already stated, but the machine should be designed to keep the fluctuations as small as possible.

Oscillograms are frequently taken to show the performance of a machine during the welding operation, and the smoothness of the current curve, which is an indication of a steady current, is always taken as a measure of the excellence of the machine. This would certainly indicate that an absolutely constant current, if attainable, would be the ideal condition in the welding circuit.

MOMENTARY CURRENT FLUCTUATIONS

The volt-ampere characteristic represented by curve *c* can be produced by an additional field in which the excitation varies with the terminal voltage of the machine. This can be accomplished by adding a self-excited field or by a number of interconnections of the fields, which will be discussed more fully later.

However, since the effect of the proportions of the machine on the transient currents is in general the same, no matter what scheme of field connections is used, the simplest case will be considered first for the purpose of a mathematical analysis of the current fluctuations when the resistance of the external circuit is suddenly increased or decreased.

Consider a generator supplied with a separately excited field and a differential series field shown diagrammatically in Fig. 2 and let

i_1 = current in main circuit
i_2 = current in separately excited field

The derivation of formulas for the instantaneous values of the currents in the main circuit and the separately excited field circuit is carried out in detail in the appendix. It must, of course, be understood that it is not attempted to calculate the current fluctuations in the arc circuit when welding is being performed, but merely the momentary increase or decrease in current that would take place if the resistance in the main circuit is suddenly changed from one value to another, as for example, from open circuit to short circuit. It is, however, fair to assume that any special features in the design of the machine which will reduce the fluctuations when calculated in this manner will also reduce the fluctuations during the welding operation and thus help to stabilize the arc.

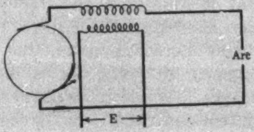

Fig. 2

Inspection of formulas 21 to 25 inclusive show that the expressions for the currents contain a term corresponding to the permanent value and one or more terms corresponding to the transient values. The latter terms have the factor ϵ to a negative exponent, which in magnitude varies directly with the time, and since the exponents are usually relatively large, these terms reduce to zero in a very short time.

As an illustration, formulas for the currents in the main circuit and separately excited field of an actual machine also have been calculated in the appendix and the curves of these equations are shown in Fig. 3. It will be noted that maximum overreach of the currents occurs 0.04 second after the short circuit and the main current exceeds the short-circuit current of 228 amperes by about 380 amperes.

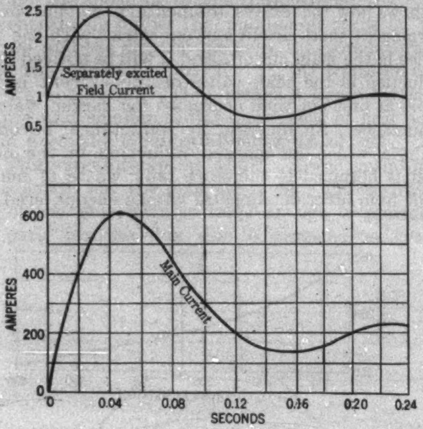

Fig. 3

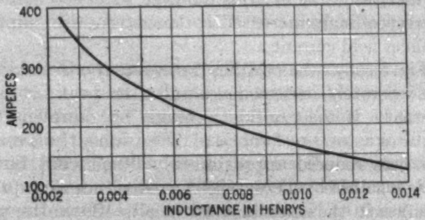

Fig. 4

For design purposes it is desirable to know how a change in one or more of the constants of the machine is going to affect the peak value of the current. For ready comparison formulas also have been derived by means of which the time at which the current reaches its peak value can be calculated. The curve in Fig. 4 shows how the excess momentary current over the permanent short-circuit current varies with the inductance in the main circuit, and it will be noticed that the reduction in the peak value of the current is rather rapid at first but then decreases for larger values of inductance. An increase in inductance beyond the point at which the marked reduction in the current peak is obtained is therefore only waste of material, aside from other detrimental effects encountered.

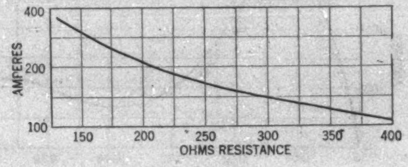

Fig. 5

Instead of varying the inductance let us assume that the resistance of the separately excited field is increased, the exciting voltage in every case being correspondingly increased to maintain the same permanent field current.

Fig. 5 shows the relation between the resistance of the separately excited field and the peak values of current. If the exciting voltage be doubled, which requires a total resistance of 260 ohms, the overreach has been reduced approximately 30 per cent, but very little is gained by increasing the resistance any further. The loss in the external resistance is 120 watts, which, however, is a small matter compared with the improvement in operation that actual tests have shown to be accomplished by this means.

FEATURES IN DESIGN

Curves similar to Figs. 4 and 5 can be made up, showing the relation between the various constants and the peak currents, and the machine proportioned to obtain the most favorable combination.

In general, the higher the number of turns on the armature and the weaker the field, the more favorable will be the conditions so far as the transient currents are concerned. That means a larger diameter armature and smaller radial dimensions of field poles than in a constant-potential generator of same rating. The circumferential width of the main pole body is small, but the pole arc is wide and the radial depth of the pole horn is large enough to provide a good path for the flux due to cross-magnetization.

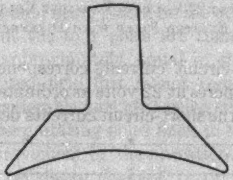

FIG. 6

(Fig. 6.) That is, distortion of the main field due to cross-magnetization is facilitated as much as possible to increase the self-induction of the armature to a maximum.

So far as the transient conditions are concerned the smaller the mutual induction between the series field and shunt field the better, and the number of turns of the series field should therefore be as low as possible. However, the series field supplies the magnetizing force opposing the separately excited field, and the stronger the series field the smaller will be the excess of permanent short-circuit current over the welding current, and from this standpoint it is desirable to have a large number of turns on the series field. A balance must be struck between these two opposites and may be arrived at as follows.

Fig. 7 shows a curve plotted with the number of turns per pole on series field as abscissas and the permanent short-circuit currents corresponding to a welding of 150 amperes at 22 volts as ordinates. Up to three turns per coil the short-circuit currents decrease rapidly, but beyond this point the gain is slight. Since a higher number of turns merely increases the mutual induction

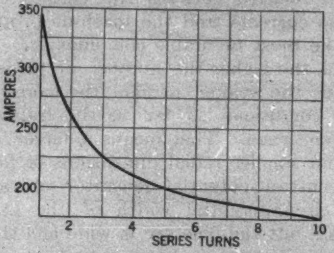

Fig. 7—Curve Showing Relation Between Short-Circuit Currents and Series Turns on Basis of Welding Current of 150 Amperes at 22 Volts in Variable Voltage Generator (Separately Excited Field with Differential Series)

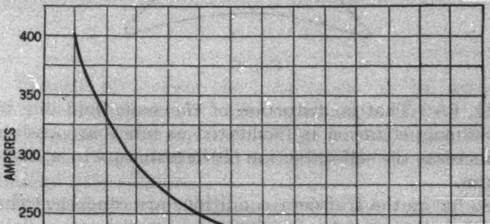

Fig. 8—Curve Showing Relation Between Short-Circuit Currents and Impressed Volts on Basis of Welding Currents of 150 Amperes at 22 Volts in Constant Potential System

between the two fields without obtaining a corresponding advantage, so far as the magnetizing force is concerned, the series field should not have more than three turns per coil for this particular combination of armature turns and speed.

It may be of interest to compare this with a similar method of obtaining the most desirable voltage in a constant-potential system. The curve in Fig. 8 shows there is a very marked increase in stabilizing effect in going from 40 to 60 volts, but only a comparatively slight gain in going from 60 to 80 volts.

The change in number of field interlinkages in going from open circuit to short circuit can be somewhat reduced by having the separately excited field coils of same polarity and placed on two diametrically opposite poles, the series field being placed on the other two poles. On short circuit the four poles are then of the same polarity and approximately equal in strength. The separately excited field flux is not reduced to zero, however, a considerable part of it being simply diverted through the commutating poles back to the frame. A noticeable improvement in the operation was observed when the field coils were changed to this arrangement from the usual way of placing coils of both fields on all four poles.

Due to the large current fluctuations and the rapidly varying field strength, commutating poles are required to insure absolutely perfect commutation at all times. Since most single-operator units are portable, it is desirable to keep the outside diameter of the frame as small as possible, and this, combined with the relatively large armature diameter, limits the radial distance from armature to frame; it is, therefore, desirable to keep the number of turns on the commutating poles down to a minimum.

As already stated, the number of turns on the armature is relatively large, and to keep down the armature ampere turns per pole requiring to be neutralized, an armature wound for four poles works out better than when wound for two poles. Also, since only half the number of compensating turns are required when four commutating poles are used instead of two, four main and four commutating poles would seem to be the most favorable combination.

GENERATORS WITH SEPARATELY EXCITED AND SELF-EXCITED SHUNT FIELDS AND DIFFERENTIAL SERIES FIELD

It has been previously stated that to obtain the characteristic curve *C* in Fig. 1 a self-excited field may be added to the separately excited field. If the number of turns per pole on this self-excited field equals n_3 and the field current equals i_3, we have for induced voltage

$$e = \frac{K\,(n_2\,i_2 + n_3\,i_3 - n_1\,i_1)}{r_1 + R}$$

The terminal voltage $= R\,i_1$ and the self-excited field current

$$i_3 = \frac{R i_1}{r_3}$$

Substituting this value of i_3 in above formula for i_1 and solving for i_1, we have

$$i_1 = \frac{K\,n_2\,i_2}{r_1 + K\,n + R\left(\frac{1 - K\,n_3}{r_3}\right)}$$

From this formula it will be seen that i_1 is independent of the external resistance when

$$1 - \frac{K\,n_3}{r_3} = 0 \text{ or } r_3 = K\,n_3$$

in which case

$$i_1 = \frac{K\,n_2\,i_2}{r_2 + K\,n_1}$$

These equations hold, of course, only over the straight part of the saturation curve, the voltage being limited by saturation above 35 volts.

To obtain formulas for the transient currents in this case requires the solution of three differential equations, and consequently involves the solution of a cubic. To derive a general algebraic formula becomes, therefore, complicated, but a solution can readily be effected by

Horner's method of approximation if numerical coefficients are substituted for literal ones. However, all

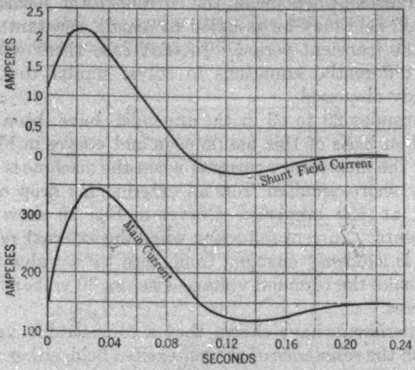

FIG. 9

the conclusions in regard to the proportions of the machine, that can be drawn from the formulas already

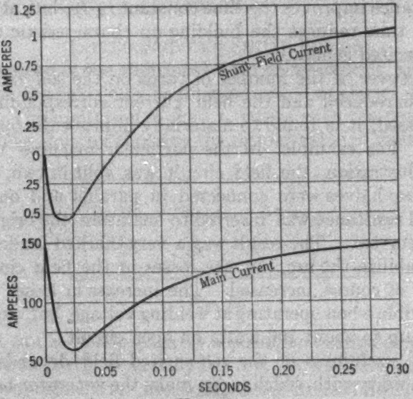

FIG. 10

derived, apply equally well if a self-excited field has been added.

If it is desired to obtain data on the proportions of the self-excited field that will make it suitable to meet the transient phenomena, the current in the separately excited field may be assumed to remain constant during the transient period. In that case there will be two differential equations to solve, similar to those already discussed.

Formulas 29 to 63 in the appendix have been derived on basis of this assumption and curves in Fig. 9 show the momentary currents when the machine is suddenly short-circuited from an external $R\,i$ drop of 30 volts at 150 amperes. Curves in Fig. 10 show the momentary current reduction when the external resistance is suddenly changed from zero to a value that will make the terminal voltage rise to 30 volts at 150 amperes.

Experiments have shown that a considerable variation in the resistance of the self-excited field, either way from the particular value which makes the generator regulate for constant current, other things remaining unchanged, has little effect on the stability of the arc. This was to be expected, because an increase in the resistance improves the time constant r_3/L_3, but at the same time reduces the building up characteristic as a self-excited generator.

However, if the number of turns on the self-excited field is reduced and the field current correspondingly increased, it is found to materially improve the operation. For example, in the particular machine used for illustration, the field circuit was split in two, and the two halves were connected in parallel and on external resistance was inserted to maintain the original field current. The result was a very marked tendency to stabilize the arc, but the losses in the field circuit were, of course, increased. The increase in losses was negligible when operating at welding voltage, but would come up to about 100 watts on open circuit.

The usefulness of the self-excited field depends on the rapidity with which it can make the generator build up its voltage so as to prevent too great a reduction in current when the external resistance is suddenly increased. In Fig. 11 is plotted a curve with number of turns of self-excited field as abscissas and current reduction in amperes as ordinates; the remaining constants of this particular machine are the ones used for illustration in the appendix.

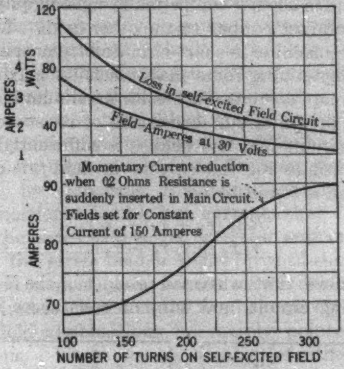

Fig. 11

Interconnected Shunt Fields

Instead of the additional field being entirely self-excited, the fields may be interconnected as shown in Fig. 12. The excitation of field f_3 varies with the terminal voltage, but by choosing the proper value of resistance

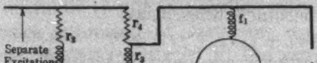

Fig. 12

r_4 it can be made entirely separately excited or partly self and partly separately excited. It showed a marked improvement over the straight self-excited field, probably because of the better time constant of the circuit $r_4 - f_3$.

Another connection that has been suggested is shown in Fig. 13. In this case the magnetizing force of field

f_3 can be made cumulative on open circuit and differential on short circuit by proper combination of resistances r_4 and r_5. The characteristic c of Fig. 1 can therefore be produced without a series field, or, if a series field is used, it can be made considerably smaller without adding copper to the other fields. Moreover, when the machine is short-circuited from open circuit, the demagnetizing force may reach its maximum before the main current reaches its maximum if the time constant of the circuit consisting of resistance r_4, the welding circuit and the field f_3 is sufficiently high.

The formula for the current in field f_3 is easily obtained and is found to be

$$i_3 = \frac{R\, i_1\,(r_4 + r_5) - E\, r_4}{r_3 r_4 + r_3 r_5 + r_4 r_5}$$

which shows that when the product of the $R\ i$ drop in the welding circuit and sum of resistances r_4 and r_5 is less than the product of the separate excitation voltage and the resistance r_4, the current in f_3 reverses and this field becomes differential.

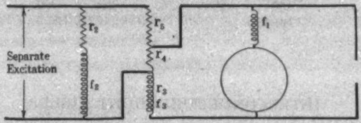

FIG. 13

Let $r_3 r_4 + r_3 r_5 + r_4 r_5 = C$
and solving for i_1 assuming no series field,

$$i_1 = \frac{C\, K\, n_2\, i_2 - K\, n_3\, E\, r_4}{C\, r_1 + R\,[C - K\, n_3\,(r_4 + r_5)\,]}$$

and if $C - K\ n_3\,(r_4 + r_5) = 0$, the main current is independent of the external resistance.

This combination is more sensitive to adjustment, but when correctly proportioned showed better results than any of the others.

Since the current in field f_3 is independent of the line current on short circuit, the line current becomes equal to zero when the current in field f_2 is reduced to such a value as to make the ampere turns in it equal to the ampere turns in field f_3. If the current in field f_2 is still further reduced, the machine reverses its polarity and builds up in opposite direction; it then becomes difficult to adjust the generator for low current values, but the difficulty is easily overcome. One way of preventing the reversal of polarity is by increasing the resistance r_5, which reduces the differential field f_3, but this is incorrect since it to a certain extent defeats the object of the connection, which is to produce a fairly strong differential field independent of the line current. Another more correct way is to increase the current in field f_2 by increasing the demagnetizing ampere turns proportional to the line current, that is by increasing the series field turns, or, if it is not convenient to change the series field, a slight forward shift of the brushes will in most cases furnish all the demagnetization required to overcome the trouble entirely.

These field combinations are merely pointed out as illustrations. There are a number of others, with and without interconnections of the fields, which will give the desired volt-ampere characteristic.

Generators without Separate Excitation

Due to the inherent tendency of a shunt generator to lose its excitation entirely when the terminals are short-circuited, a straight shunt machine with differential series field is not suitable for arc welding. If it is to work at all the pole pieces must be shaped so that a small part of the magnetic circuit in parallel with the main circuit becomes saturated at very low voltage and some external resistance inserted in the arc circuit to keep the field from collapsing altogether on short circuit.

However, the equivalent of separate excitation can be obtained by having voltages generated in the armature circuits which on short circuit neutralize each other to give practically zero potential at the terminals, but a difference of potential is maintained between one set of main brushes and a third brush from which the excitation is taken.

One method used in accomplishing this is to have the two sets of poles of a four pole generator of consequent polarity and the armature wound for two poles.

On short circuit the excitation is automatically changed so that half the conductors in series between the main brushes will pass in front of poles of opposite polarity to that of the other half. The strength of the different poles is adjusted so that just sufficient voltage is available at the terminals to force the desired current through the welding circuit, the proper excitation being obtained by a combination of armature reaction and a series field.

The shunt field may be placed on two diametrically opposite poles and the series field on the other two poles as shown in Fig. 14.

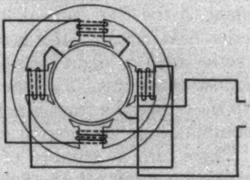

Fig. 14

The armature, being wound for two poles, has necessarily a relatively high number of ampere turns per pole and on account of the strong armature reaction a cumulative series field is usually required. Moreover, the strong magnetizing force of the armature requires a correspondingly large number of neutralizing turns on the commutating poles, as already pointed out, which necessitates a larger diameter frame than the four pole machine for same rating. Since there are only two sets of main brushes, the commutator must be longer than in the four-pole machine to keep the same current density in the brushes. On the whole the material is not so economically employed as in the four-pole generator with separate excitation.

Furthermore, since the series field turns generally have to be changed for different current values, either by shunting or by taps, the current adjustment is not comparable to the fine adjustment that can easily be obtained over a wide range in the separately exciter machine.

On the whole, the writer feels that the advantage obtained by eliminating the separate excitation is not sufficient to compensate for the disadvantages mentioned. Undoubtedly efforts will be continued in this direction, however, and if a generator can be developed possessing all the desirable features of the present separately excited machine without the need of separate excitation, the advantage of such a machine is obvious.

APPENDIX

Fig. 2 in the paper shows diagrammatically a generator with separately excited field and differential series field.

Let i_1 = current in main circuit.

i_2 = current in separately excited field.

n_1 = number of turns per pole on series field.

n_2 = number of turns per pole on separately excited field.

L_1 = coefficient of self-induction of main circuit.

L_2 = coefficient of self-induction of field circuit.

M = coefficient of mutual induction between main and separately excited field circuits.

E = volts impressed on separately excited field.

e = induced volts (by rotation).

Due to saturation the coefficients of self and mutual induction are not constants, but since the operating range is almost entirely over the straight part of the saturation curve, sufficiently accurate results will be obtained if they are considered so. The coefficients of self induction are the total number of interlinkages per unit current and include mutual as well as leakage fluxes.

At constant speed the induced voltage is directly proportional to the flux, and since the operating range is over the straight part of the saturation curve, the flux is proportional to the net ampere turns per pole. The induced voltage is therefore proportional to the ampere turns per pole and may be expressed by the formula

$$e = K\,(n_2\,i_2 - n_1\,i_1)$$

Volts consumed by resistance in main circuit $= (r_1 + R)$
where R is the resistance of external circuit.

Volts induced by self induction $= L_1 \dfrac{d\,i_1}{dt}$

Volts induced by mutual induction $= M \dfrac{d\,i_2}{dt}$

From Kirchhoff's law that the algebraic sum of the voltages in a closed circuit is equal to zero we have for differential equation of main circuit

$$L_1 \frac{d\,i_1}{dt} + (r_1 + R)\,i_1 - M \frac{d\,i_2}{dt} - K\,(n_2\,i_2 - n_1\,i_1) = 0 \tag{1}$$

In a similar manner we obtain for differential equation of separately excited field circuit

$$L_2 \frac{d\,i_2}{dt} + r_2\,i_2 - M \frac{d\,i_1}{dt} = E \tag{2}$$

In symbolic form with the operator D signifying the operation of differentiation the equations may be written

$$(L_1 D + r_1 + R + K\,n_1)\,i_1 - (M\,D + K\,n_2)\,i_2 = 0 \tag{3}$$

$$-\,M\,D\,i_1 + (L_2 D + r_2)\,i_2 = E \tag{4}$$

Let $r_1 + R + K\,n_1 = P$ and eliminating i_2 we have

$$\begin{vmatrix} (L_1 D + P) & -(M\,D + K\,n_2) \\ -M\,D & (L_2 D + r_2) \end{vmatrix} i_1 = \begin{vmatrix} 0 & -(M\,D + K\,n_2) \\ E & (L_2 D + r_2) \end{vmatrix} \tag{5}$$

Expanding the determinants

$$[\,(L_1 L_2 - M^2)\,D^2 + (P\,L_2 + r_2 L_1 - M\,K\,n_2)\,D + P r_2]\,i_1 = E\,K\,n_1 \tag{6}$$

The solution of this equation is

$$i_1 = A_1 + A_2\,\epsilon^{a_1 t} + A_3\,\epsilon^{a_2 t}$$

where A_1, A_2 and A_3 are constants of integration and a_1 and a_2 are the roots of the quadratic

$$(L_1 L_2 - M^2) D^2 + (P L_2 + r_2 L_1 - M K n_2) D + P r_2 = 0, \text{ hence}$$

$$a_1 = \frac{-(P L_2 + r_2 L_1 - M K n_2) - \sqrt{(P L_2 + r_2 L_1 - M K n_2)^2 - 4 (L_1 L_2 - M^2) P r_2}}{2 (L_1 L_2 - M^2)}$$

$$a_2 = \frac{-(P L_2 + r_2 L_1 - M K n_2) + \sqrt{(P L_2 + r_2 L_1 - M K n_2)^2 - 4 (L_1 L_2 - M^2) P r_2}}{2 (L_1 L_2 - M^2)}$$

The coefficients of the quadratic are in general positive and the roots, when real, are negative; there are consequently three cases to be considered when

$$(P\,L_2 + r_2\,L_1 - M\,K\,n_2)^2 > 4\,(L_1\,L_2 - M^2)\,P\,r_2,$$

the roots are negative, real and unequal;
when

$$(P\,L_2 + r_2\,L_1 - M\,K\,n_2)^2 = 4\,(L_1\,L_2 - M^2)\,P\,r_2,$$

the roots are negative, real and equal;
when

$$(P\,L_2 + r_2\,L_1 - M\,K\,n_2)^2 < 4\,(L_1\,L_2 - M^2)\,P\,r_2,$$

the roots are complex, imaginary numbers.

Case I, roots real and unequal.

$$i_1 = A_1 + A_2\,\epsilon^{a_1 t} + A_3\,\epsilon^{a_2 t} \tag{7}$$

From equation (2) we have

$$L_2 \frac{d\,i_2}{d\,t} + r_2\,i_2 = E + M \frac{d\,i_1}{d\,t}$$

Substituting the value of i_1 from (7) in this equation

$$L_2 \frac{d\,i_2}{dt} + r_2\,i_2 = E + M\,a_1\,A_2\,\epsilon^{a_1 t} + M\,a_2\,A_3\,\epsilon^{a_2 t} \tag{8}$$

a linear differential equation of first order, the solution of which is

$$i_2 = 1/L_2\epsilon^{-\frac{r_2}{L_2}t} \int \epsilon^{\frac{r_2}{L_2}t} (E + M\,a_1\,A_2\epsilon^{a_1 t} + M\,a_2\,A_3\,\epsilon^{a_2 t})dt$$

Integrating and simplifying

$$i_2 = \frac{E}{r_2} + \frac{M\,a_1\,A_2}{r_2 + L_2\,a_1}\,\epsilon^{a_1 t} + \frac{M\,a_2\,A_3}{r_2 + L_2\,a_2}\,\epsilon^{a_2 t} \tag{9}$$

Let $i_1^{(s)}$ be the main current just before the change in resistance takes place and $i_1^{(p)}$ the permanent value the current would reach if the new resistance remained unchanged. We then have from equation (7)

when $t = \infty\ A_1 = i_1^{(p)}$

In the separately excited field the current is E/r_2 before and after the change and from equations (7) and (9) follows that when $t = 0$.

$$A_2 + A_3 = i_1^{(s)} - i_1^{(p)}$$

$$\frac{M\,a_1}{r_2 + L_2\,a_1}\,A_2 + \frac{M\,a_2}{r_2 + L_2\,a_2}\,A_3 = 0$$

Solving these equations simultaneously for A_2 and A_3 we have

$$A_2 = \frac{a_2\,(i_1^{(s)} - i_1^{(p)})\,(r_2 + L_2\,a_1)}{r_2\,(a_2 - a_1)}$$

$$A_3 = \frac{a_1\,(i_1^{(s)} - i_1^{(p)})\,(r_2 + L_2\,a_2)}{r_2\,(a_2 - a_1)}$$

Substituting these values in equations (**7**) and (**9**) and reducing

$$i_1 = i_1^{(p)} + \frac{i_1^{(s)} - i_1^{(p)}}{r_2\,(a_2 - a_1)} \left[a_2\,(r_2 + L_2\,a_1)\,\epsilon^{a_1 t} - a_1(r_2 + L_2\,a_2)\,\epsilon^{a_2 t}\right] \tag{10}$$

$$i_2 = \frac{E}{r_2} + \frac{M\,a_1\,a_2\,(i_1^{(s)} - i_1^{(p)})}{r_2\,(a_2 - a_1)}\left[\epsilon^{a_1 t} - \epsilon^{a_2 t}\right] \tag{11}$$

Case II, Roots real and equal.

Let the two roots of the quadratic be expressed by the equations

$$a_1 = a - h \qquad\qquad a_2 = a + h$$

where $h =$

$$\frac{\sqrt{(P\,L_2 + r_2\,L_1 - M\,K\,n_2)^2 - 4\,(L_1\,L_2 - M^2)\,P\,r_2}}{2\,(L_1\,L_2 - M^2)}$$

when h becomes equal to zero the roots are equal

$$a = a_1 = a_2$$

Substituting $a - h$ for a_1 and $a + h$ for a_2 in equation (**10**) we have

$$i_1 = i_1^{(p)} + \frac{i_1^{(s)} - i_1^{(p)}}{2\,r_2\,h} \Big[\,(a + h)\,(r_2 + L_2\,a - L_2\,h)\,\epsilon^{(a-h)t} - (a - h)\,(r_2 + L_2\,a + L_2\,h)\,\epsilon^{(a+h)t}\Big]$$

$$= i_1^{(p)} + \frac{i_1^{(s)} - i_1^{(p)}}{2\,r_2} \left[\left(\frac{r_2\,a + L_2\,a^2}{h} + r_2 - L_2\,h\right)\epsilon^{-ht} - \left(\frac{r_2\,a + L_2\,a^2}{h} - r_2 - L_2\,h\right)\epsilon^{h\,t}\right]\epsilon^{at}$$

$$= i_1^{(P)} + \frac{i_1^{(S)} - i_1^{(P)}}{2\,r_2}$$

$$\Big[r_2\,\epsilon^{-ht} + r_2\,\epsilon^{ht} - L_2\,h\epsilon^{-ht} + L_2\,h\,\epsilon^{ht}$$

$$+ (r_2\,a + L_2\,a^2)\,\frac{\epsilon^{-ht} - \epsilon^{ht}}{h} \Big]\,\epsilon^{at}$$

when h approaches zero as a limit the expressions $L_2\,h\,\epsilon^{-ht}$ and $L_2\,h\epsilon^{ht}$ likewise become zero, the expressions $r_2\,\epsilon^{-ht}$ and $r_2\,\epsilon^{ht}$ approach r_2 as a limit, and the formula for i_1 reduces to

$$i_1 = i_1^{(P)} + (i_1^{(S)} - i_1^{(P)})$$

$$\left[1 + \frac{a\,(r_2 + L_2\,a)}{2\,r_2}\;\frac{(\epsilon^{-ht} - \epsilon^{ht})}{h} \right] \epsilon^{at} \quad (\mathbf{12})$$

When $h = 0$ the fraction $\dfrac{\epsilon^{-ht} - \epsilon^{ht}}{h} = \dfrac{0}{0}$, which is indeterminate. To evaluate this fraction the numerator and denominator are differentiated with respect to h thus

$$\frac{\dfrac{d}{d\,h}\,[\epsilon^{-ht} - \epsilon^{ht}]_{\,h\,=\,o}}{\dfrac{d}{d\,h}\,[h]}$$

$$= \frac{[-\,t\,\epsilon^{-ht} - t\,\epsilon^{ht}]}{1}\,h = 0 \;\; = -\,2\,t$$

Substituting in equation (**12**)

$$i_1 = i_1^{(P)} + (i_1^{(S)} - i_1^{(P)}) \left[1 - \frac{a\,t\,(r_2 + L_2\,a)}{r_2} \right] \epsilon^{at} \quad (\mathbf{13})$$

Substituting $a - h$ for a_1 and $a + h$ for a_2 in equation (**11**) we have

$$i_2 = \frac{E}{r_2} + \frac{M\,(i_1^{(S)} - i_1^{(P)})\,(a + h)\,(a - h)}{2\,r_2\,h}$$

$$[\epsilon^{(a-h)t} - \epsilon^{(a+h)t}]$$

$$= \frac{E}{r_2} + \frac{M\,(i_1^{(s)} - i_1^{(P)})\,a^2}{2\,r_2}\left(\frac{\epsilon^{-ht} - \epsilon^{ht}}{h}\right)\epsilon^{at}$$

$$- \frac{M\,(i_1^{(s)} - i_1^{(P)})\,h}{2\,r_2}\,(\epsilon^{-ht} - \epsilon^{ht})\,\epsilon^{at}$$

The last term is equal to zero when h is zero and the indeterminate fraction $\left[\frac{\epsilon^{-ht} - \epsilon^{ht}}{h}\right]_{h=o}$ may be evaluated as before and is equal to $-2\,t$ when $h = 0$, hence

$$i_2 = \frac{E}{r_2} - \frac{M\,a^2 t\,(i_1^{(s)} - i_1^{(P)})}{r_2}\,\epsilon^{at} \qquad \textbf{(14)}$$

Case III, Roots Imaginary.

When

$$4\,(L_1\,L_2 - M^2)\,P\,r_2 > (P\,L_2 + r_2\,L_1 - M\,K\,n_2)^2$$

the roots of the quadratic are complex imaginary numbers and may be represented by the expressions $p + j\,q$ and $p - j\,q$ where $j = \sqrt{-1}$.

The solution of equation (6) then is

$$i_1 = A_1 + A_2\,\epsilon^{pt}\;\;\epsilon^{jqt} + A_3\,\epsilon^{pt}\;\;\epsilon^{-jqt}$$

From the following relation existing between the exponential and trigonometric functions

$$\epsilon^{jqt} = \cos q\,t + j\sin q\,t \quad \epsilon^{-jqt} = \cos q\,t - j\sin q\,t$$

the above equation may be written

$$i_1 = A_1 + \epsilon^{pt}\,[(A_2 + A_3)\cos q\,t + j\,(A_2 - A_3)\sin q\,t]$$

Let $A_2 + A_3 = B_1$ and $A_2 - A_3 = j\,B_2$

$$i_1 = A_1 + \epsilon^{pt}\,(B_1\cos q\,t + B_2\sin q\,t) \qquad \textbf{(15)}$$

From equation (4)

$$L_2\,\frac{d\,i_2}{d\,t} + r_2\,i_2 = E + M\,\frac{d\,i_1}{dt}$$

Differentiating (15) and multiplying by M

$$M\,\frac{d\,i_1}{dt} = M\,\epsilon^{pt}\,[\,(p\,B_1 + q\,B_2)\cos q\,t$$

$$+ (p\,B_2 - q\,B_1)\sin q\,t]$$

Substituting we have for differential equation of separately excited field circuit

$$L_2\,\frac{d\,i_2}{dt} + r_2\,i_2 = E + M\,\epsilon^{pt}\,[\,(p\,B_1 + q\,B_2)\cos q\,t$$

$$+ (p\,B_2 - q\,B_1)\sin q\,t]$$

The solution of this equation is

$$i_2 = 1/L_2\, \epsilon^{-r_2/L_2\, t} \int \epsilon^{r_2/L_2\, t} \;\{E + M\, \epsilon^{pt}\; [\,(p\, B_1 + q\, B_2)$$

$$\cos q\, t + p\, B_2 - q\, B_1) \sin q\, t]\;\}\; dt$$

Integrating the first term and multiplying out

$$i_2 = E/r_2 + M/L_2\, \epsilon^{-r_2/L_2 \times t} \int \epsilon^{(r_2/L_2 + p)t}\, (p\, B_1 + q\, B_2)$$

$$\cos q\, t\, dt + M/L_2\, \epsilon^{-r_2/L_2 \times t} \int {}^{(r_2/L_2 + p)t}$$

$$(p\, B_2 - q\, B_1) \sin q\, t\, dt \tag{16}$$

These integrals are of the form $\int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x$ and $\int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$, integrating by parts

$$\int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x$$

$$= \frac{\epsilon^{\alpha x} \sin\, \beta\, x}{\alpha} - \frac{\beta}{\alpha} \int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$$

$$\int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$$

$$= \frac{\epsilon^{\alpha x} \cos\, \beta\, x}{\alpha} + \frac{\beta}{\alpha} \int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x$$

Transposing

$$\int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x + \beta/\alpha \int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$$

$$= \frac{\epsilon^{\alpha x} \sin\, \beta\, x}{\alpha}$$

$$-\, \beta/\alpha \int \epsilon^{\alpha x} \sin\, \beta x\, d\, x + \int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$$

$$= \frac{\epsilon^{\alpha x} \cos\, \beta\, x}{\alpha}$$

Eliminating $\int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x$ and simplifying we have

$$\int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x = \frac{\epsilon^{\alpha x}}{\alpha^2 + \beta^2}\, (\alpha \sin\, \beta\, x - \beta \cos\, \beta\, x)$$

Eliminating $\int \epsilon^{\alpha x} \sin\, \beta\, x\, d\, x$ we obtain

$$\int \epsilon^{\alpha x} \cos\, \beta\, x\, d\, x = \frac{\epsilon^{\alpha x}}{\alpha^2 + \beta^2}\, (\beta \sin\, \beta\, x + \alpha \cos\, \beta\, x)$$

The integrals of equation (**16**) can now be evaluated by means of these formulas as follows.

$$\int \epsilon^{(r_2/L_2+p)t} \sin q\, t\, d\, t$$

$$= \frac{L_2^2\, \epsilon^{(r_2/L_2+p)t}}{(r_2 + L_2\, p)^2 + L_2^2\, q^2} \left(\frac{r_2 + L_2\, p}{L_2} \sin q\, t - q \cos q\, t \right)$$

$$\int \epsilon^{(r_2/L_2+p)t} \cos q\, t\, dt$$

$$= \frac{L_2^2\, \epsilon^{(r_2/L_2+p)t}}{(r_2 + L_2\, p)^2 + L_2^2\, q^2} \left(q \sin q\, t + \frac{r_2 + L_2\, p}{L_2} \cos q\, t \right)$$

Substituting in (**16**)

$$i_2 = \frac{E}{r_2} + \frac{M}{L_2}\, \epsilon^{-r_2/L_2 \times t} \; \frac{L_2\, \epsilon^{(r_2/L_2+p)t}}{(r_2 + L_2\, p)^2 + L_2^2\, q^2}$$

$$[\,(p\, B_1 + q\, B_2)\, L_2 q \sin qt + (p\, B_1 + q\, B_2)\, (r_2 + L_2\, p) \cos qt + (pB_2 - q\, B_1)\, (r_2 + L_2\, p) \sin q\, t - (p\, B_2 - q\, B_1)\, L_2\, q \cos qt]$$

$$= \frac{E}{r_2} + \frac{M}{(r_2 + L_2\, p)^2 + L_2^2\, q^2}$$

$$\{\,[\,(p\, B_1 + q\, B_2)\, L_2\, q + (p\, B_2 - q\, B_1)\, (r_2 + L_2\, p)\,] \sin q\, t + [\,(p\, B_1 + q\, B_2)\, (r_2 + L_2\, p) - (p\, B_2 - q\, B_1)\, L_2\, q] \cos q\, t\}\; \epsilon^{pt}$$

$$i_2 = \frac{E}{r_2} + \frac{M}{(r_2 + L_2\, p)^2 + L_2^2\, q^2}$$

$$\{\,[\,(L_2\, (q^2 + p^2) + p\, r_2)\, B_2 - q\, r_2\, B_1] \sin q\, t + [\,(L_2\, (q^2 + p^2) + p\, r_2)\, B_1 + q\, r_2\, B_2] \cos q\, t\}\; \epsilon^{pt} \qquad \textbf{(17)}$$

From equation (**15**) we have when $t = \infty$ $A_1 = i_1^{(p)}$ and when $t = 0$, $B_1 = i_1^{(s)} - i_1^{(p)}$.

Since $i_2 = E/r_2$ when $t = 0$ we have from equation (**17**)

$$\frac{M\,\{[L_2\,(q^2+p^2)+p\,r_2]\,B_1+q\,r_2\,B_2\}}{(r_2+L_2\,p)^2+L_2^2\,q^2}=0$$

or
$$[L_2\,(q^2+p^2)+p\,r_2]\,B_1+q\,r_2\,B_2=0$$

Substituting $B_1 = i_1^{(s)} - i_1^{(p)}$

$$[\,L_2\,(q^2+p^2)+pr_2]\,(i_1^{(s)}-i_1^{(p)}) = -\,qr_2\,B_2$$

$$B_2 = -\,\frac{(i_1^{(s)}-i_1^{(p)})\,[L_2\,(q^2+p^2)+p\,r_2]}{q\,r_2}$$

Substituting in equations (**15**) and (**17**)

$$i_1 = i_1^{(p)} + \left\{ (i_1^{(s)}-i_1^{(p)})\cos q\,t - \frac{(i_1^{(s)}-i_1^{(p)})\,[L_2\,(q^2+p^2)+p\,r_2]}{q\,r_2}\sin q\,t \right\}\epsilon^{pt} \qquad \textbf{(18)}$$

$$i_2 = \frac{E}{r_2} - \frac{M\,(i_1^{(s)}-i_1^{(p)})\,\{[L_2\,(q^2+p^2)+p\,r_2]^2-q^2\,r_2^2\}}{q\,r_2\,[\,(r_2+L_2\,p)^2+L_2^2\,q^2]}\,\epsilon^{pt}\sin q\,t \qquad \textbf{(19)}$$

If the initial condition is open circuit $i_1^{(s)} = 0$ and the formulas in the three cases then reduce to

Case I.

$$i_1 = i_1^{(p)} + \frac{a_1\,i_1^{(p)}}{r_2\,(a_2-a_1)}\,(r_2+L_2\,a_2)\,\epsilon^{a_2t} - \frac{a_2\,i_1^{(p)}}{r_2\,(a_2-a_1)}\,(r_2+L_2\,a_1)\,\epsilon^{a_1t} \qquad \textbf{(20)}$$

$$i_2 = \frac{E}{r_2} + \frac{M\,a_1\,a_2\,i_1^{(p)}}{r_2\,(a_2-a_1)}\,(\epsilon^{a_2t}-\epsilon^{a_1t}) \qquad \textbf{(21)}$$

Case II.

$$i_1 = i_1^{(p)} + i_1^{(p)}\left[\frac{a\,t\,(r_2+L_2\,a)}{r_2} - 1\right]\epsilon^{at} \qquad \textbf{(22)}$$

$$i_2 = \frac{E}{r_2} + \frac{M\,a^2\,t\,i_1^{(p)}}{r_2}\,\epsilon^{at} \qquad \textbf{(23)}$$

Case III.

$$i_1 = i_1^{(P)} + \left\{ \frac{i_1^{(P)}\,[L_2\,(q^2 + p^2) + p\,r_2]}{q\,r_2} \sin q\,t - i_1^{(P)} \cos q\,t \right\} \epsilon^{pt} \tag{24}$$

$$i_2 = \frac{E}{r_2} + \frac{M\,i_1^{(P)}\{[L_2\,(q^2 + p^2) + p\,r_2]^2 - q^2\,r_2^2\}}{q\,r_2\,[\,(r_2 + L_2\,p)^2 + L_2^2\,q^2]}\,\epsilon^{pt} \sin q\,t \tag{25}$$

It will be noticed that in every case the current formula contains the permanent value of current and one or more transient terms. These terms contain the factor ϵ to a negative exponent, which in magnitude varies directly with the time, and since the exponents are usually relatively large the transient terms reduce to zero in a very short time. Since we desire to know the value of the peaks of these current fluctuations in the main circuit it will be of interest to know the time at which they attain their maximum value.

This is easily obtained by differentiating the current formula with respect to t and putting the derivative equal to zero and solving for t.

Differentiating equation (**20**) with respect to t and equating to zero.

$$\frac{a_1\,i_1^{(P)}}{r_2\,(a_2 - a_1)}\,(r_2 + L_2\,a_2)\,a_2\,\epsilon^{a_2 t} - \frac{a_2\,i_1^{(P)}}{r_2\,(a_2 - a_1)}\,(r_2 + L_2\,a_1)a_1\,\epsilon^{a_1 t} = 0$$

$$\frac{a_1\,a_2\,i_1^{(P)}}{r_2\,(a_2 - a_1)}\,[\,(r_2 + L_2\,a_2)\,\epsilon^{a_2 t} - (r_2 + L_2\,a_1)\,\epsilon^{a_1 t}] = 0$$

$$(r_2 + L_2\,a_2)\,\epsilon^{a_2 t} = (r_2 + L_2\,a_1)\,\epsilon^{a_1 t}$$

$$\frac{\epsilon^{a_2 t}}{\epsilon^{a_1 t}} = \frac{r_2 + L_2\,a_1}{r_2 + L_2\,a_2}$$

Taking logarithms of both sides

$$a_2\,t - a_1\,t = \log_e\,(r_2 + L_2\,a_1) - \log_e\,(r_2 + L_2\,a_2)$$

$$t = \frac{\log_\epsilon (r_2 + L_2 a_1) - \log_\epsilon (r_2 + L_2 a_2)}{a_2 - a_1}$$

If logarithms are taken to base 10 instead of base ϵ the numerator must be multiplied by $\frac{1}{\log_{10}\epsilon} = 2.31$ thus

$$t = \frac{2.31\,[\log_{10} (r_2 + L_2 a_1) - \log_{10} (r_2 + L_2 a_2)\,]}{a_2 - a_1} \tag{26}$$

Differentiating equation (**22**) with respect to t and equating to zero

$$\frac{r_2 + L_2 a}{r_2} + \frac{r_2 + L_2 a}{r_2}\, a\, t - 1 = 0$$

$$t = -\frac{L_2}{r_2 + L_2 a} \tag{27}$$

Since the time must be positive, the denominator of this fraction must be negative. The product of the negative root a and L_2 must therefore be greater than r_2 in order to have the condition of equal roots of the quadratic.

Differentiating equation (**24**) and equating to zero

$$\frac{[L_2\,(q^2 + p^2) + p\, r_2]}{q\, r_2}\, p \sin q\, t$$

$$+ \frac{[L_2\,(q^2 + p^2) + p\, r_2]}{q\, r_2}\, q \cos q\, t - p \cos q\, t$$

$$+ q \sin q\, t = 0$$

$$\left\{ \frac{[L_2\,(q^2 + p^2) + p\, r_2]\, p}{q\, r_2} + q \right\} \sin q\, t$$

$$= \left\{ p - \frac{[L_2\,(q^2 + p^2) + p\, r_2]}{r_2} \right\} \cos q\, t$$

$$\frac{[L_2\,(q^2 + p^2) + p\, r_2]\, p + q^2\, r_2}{q\, r_2} \sin q\, t$$

$$= \frac{p\, r_2 - [L_2\,(q^2 + p^2) + p\, r_2]}{r_2} \cos q\, t$$

$$\frac{\sin q\,t}{\cos q\,t} = -\frac{L_2\,q\,(q^2 + p^2)}{L_2\,p\,(q^2 + p^2) + r_2\,(q^2 + p^2)}$$

$$\tan q\,t = -\frac{L_2\,q\,(q^2 + p^2)}{(L_2\,p + r_2)\,(q^2 + p^2)} = -\frac{L_2\,q}{L_2\,p + r_2}$$

$$q\,t = \tan^{-1}\left(-\frac{L_2\,q}{L_2\,p + r_2}\right) \qquad \textbf{(28)}$$

The tangent is a periodic function and an unlimited number of values of t would satisfy this equation. However, the coefficient of $\sin q\,t$ in equation (**24**) is usually positive and greater than the coefficient of $\cos q\,t$, and since the factor $\epsilon^{p\,t}$ decreases with time the current usually reaches its maximum value while the $\sin q\,t$ passes through its first series of positive values, that is the first or second quadrants. Hence, if $\left(-\frac{L_2\,p}{L_2\,p + r_2}\right)$ is positive the angle $q\,t$ is in the first quadrant and if negative the angle is in the second quadrant for maximum values of line current.

These formulas have been derived on the supposition that the mutual induction due to eddy currents in the frame is negligible. This assumption is permissible when the frame section is relatively thin, and since the generators under discussion are single-operator units, they are built in small sizes only. In larger sizes this effect would have to be taken into account since it would be appreciable, unless the frame is laminated.

As illustration consider a 150-ampere 1750-rev. per min. generator with the following constants: $L_1 = 0.00245$. $L_2 = 14$, $r_1 = 0.03$, $K = 0.0825$, $M = 0.05$ r_2 (including rheostat) $= 130$ ohms, $n_1 = 3$, $n_2 = 800$, assuming a normal arc voltage of 22 volts, the field current required at 150 amperes normal voltage is 0.96 amperes, the permanent short-circuit current is 228 amperes. The open-circuit voltage being somewhat limited by saturation is 50 volts. External excitation voltage is 125. We wish to find the current peak when the machine is suddenly short-circuited from open circuit. Hence $R = 0$.

We then have

$$L_1 L_2 - M^2 = 0.0317 \qquad P = 0.278 \qquad P r_2 = 36$$

$$P L_2 + r_2 L_1 - M K n_2 = 0.898 \quad p = -14.2 \quad q = 30.5$$

$$\frac{228 \{14 (30.5^2 + 14.2^2) - 14.2 \times 130\}}{30.5 \times 130} = 798$$

$$\frac{0.05 \times 228 \{[14(30.5^2 + 14.2^2) - 14.2 \times 130]^2 - (130 \times 30.5)^2\}}{130 \times 30.5[(130 - 14.2 \times 14)^2 + (14 \times 30.5)^2]} = 2.73$$

$$i_1 = 228 + 798 \epsilon^{-14.2t} \sin 30.5 t + 228 \epsilon^{-14.2t} \cos 30.5t$$

$$i_2 = 0.96 + 2.73 \epsilon^{-14.2t} \sin 30.5 t$$

The curves of these equations are shown in Fig. 3, the maximum overreach of current occurs 0.04 seconds after the short circuit and is approximately 380 amperes.

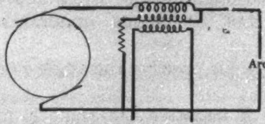

FIG. 15

GENERATORS WITH SELF-EXCITED FIELD AND SEPARATELY EXCITED FIELD

It was pointed out in the discussion of generators supplied with both separately excited and self-excited fields that the complete solution for the transient currents would in that case require the solution of three differential equations and therefore involves the solution of a cubic. However, all the conclusions in regard to the proportions of armature, separately excited and series field that can be drawn from the formula derived, apply equally well if a self-excited field is added.

It was also pointed out that to obtain data on the self-excited field, the current in the separately excited field may be assumed to remain constant during the transient period. There will then be two differential equations to solve similar to those already discussed except that the self-excitation brings in the building-up characteristic.

To obtain these data let us consider a generator with separately excited and self-excited shunt fields and a differential series field, and let

i_1 = current in main circuit.
i_2 = current in separately excited field.
i_3 = current in self-excited field.
n_1 = number of turns per pole on series field.
n_2 = number of turns per pole on separately excited field.
n_3 = number of turns per pole on self-excited field.
L_1 = coefficient of self-induction of main circuit.
L_3 = coefficient of self-induction of self-excited field.
M_1 = mutual induction between main and self-excited field circuits.

The e. m. f. induced by rotation then is

$$e = K\,(n_2 i_2 + n_3 i_3 - n_1 i_1)$$

and for differential equation of main circuit we have

$$L_1 \frac{d\,i_1}{dt} + (r_1 + R)\,i_1 - M_1 \frac{d\,i_3}{dt} - K\,(n_2 i_2 + n_3 i_3 - n_1 i_1) = 0 \tag{29}$$

and for differential equation of self-excited field circuit

$$L_3 \frac{d\,i_3}{dt} + r_3 i_3 - M_1 \frac{d\,i_1}{dt} - R\,i_1 = 0 \tag{30}$$

In symbolic form they are

$$(L_1 D + r_1 + R + K n_1)\,i_1 - (M_1 D + K n_3)\,i_3 = K n_2 i_2 \tag{31}$$

$$-\,(M_1 D + R)\,i_1 + (L_3 D + r_3)\,i_3 = 0 \tag{32}$$

Let $r_1 + R + K n_1 = P$ and eliminating i_3

$$\begin{vmatrix} (L_1 D + P) & -\,(M_1 D + K n_3) \\ -\,(M_1 D + R) & (L_3 D + r_3 \end{vmatrix} i_1 = \begin{vmatrix} K n_2 i_2 & -\,(M_1 D + K M_3) \\ 0 & (L_3 D + r_3) \end{vmatrix} \tag{33}$$

Expanding the determinants we have

$$[\,(L_1 L_3 - M_1^2)\,D^2 + (P L_3 + r_3 L_1 - M_1 K n_3 - M_1 R)\,D + (P r_3 - R K n_3)\,]\,i_1 = K n_2 i_2 r_3.$$

An equation of which the general solution is as before

$$i_1 = A_1 + A_2\,\epsilon^{a_1 t} + A_3\,\epsilon^{a_2 t}$$

Where A_1, A_2 and A_3 are integration constants and a_1 and a_2 the roots of the quadratic

$$(L_1 L_3 - M_1^2)\, D^2 + (P\, L_3 + r_3 L_1 - M_1 K\, n_3 - M_1 R)\, D + (P\, r_3 - R\, K\, n_3) = 0, \text{ hence}$$

$$a_1 = \frac{-\,(P\, L_3 + r_3 L_1 - M_1 K\, n_3 - M_1 R) - \sqrt{(P\, L_3 + r_3 L_1 - M_1 K\, n_3 - M_1 R)^2 - 4(L_1 L_3 - M_1^2)(P\, r_3 - R\, K\, n_3)}}{2\,(L_1 L_3 - M_1^2)}$$

$$a_2 = \frac{-(P\, L_3 + r_3 L_1 - M_1 K\, n_3 - M_1 R) + \sqrt{(P\, L_3 + r_3 L_1 - M_1 K\, n_3 - M_1 R)^2 - 4(L_1 L_3 - M_1^2)(P\, r_3 - R\, K\, n_3)}}{2\,(L_1 L_3 - M_1^2)}$$

It will be noted that if r_3 has a sufficiently small value relative to n_3 the expression $p\, r_3 - R\, K\, n_3$ becomes negative and the quadratic has a positive root. One of the transient terms in the general equation for i_1 then contains the factor ϵ to a positive exponent which means that the current would continue to increase. Such a low value of self-excited field resistance would not be used in practise, because the regulation of the machine would be very poor, and even if it were the current would be limited by saturation which has not been taken into account in above formula. There are consequently again three cases to be considered according to whether the expression under the radical sign is greater than, equal to, or less than zero.

Case I, Roots Negative, Real and Unequal.

$$i_1 = A_1 + A_2\, \epsilon^{a_1 t} + A_3\, \epsilon^{a_2 t} \tag{34}$$

From equation (**30**) we have

$$L_3 \frac{d\, i_3}{dt} + r_3\, i_3 = M_1 \frac{d\, i_1}{dt} + R\, i_1$$

Solving for i_3

$$i_3 = \frac{1}{L_3}\, \epsilon^{-r_3/L_3 \times t} \int \epsilon^{r_3/L_3 \times t} \left(M_1 \frac{d\, i_1}{dt} + R\, i_1 \right) dt \tag{35}$$

Substituting the value of i_1 from (**34**) in (**35**)

$$i_3 = \frac{1}{L_3}\, \epsilon^{-r_3/L_3 \times t} \int \epsilon^{r_3/L_3 \times t} [R\, A_1 + A_2\, (R + M_1\, a_1)\, \epsilon^{a_1 t}$$
$$+ A_3\, (R + M_1\, a_2)\, \epsilon^{a_2 t}]\, dt$$

Integrating and simplifying

$$i_3 = \frac{R\, A_1}{r_3} + \frac{A_2\, (R + M_1\, a_1)}{a_1\, L_3 + r_3}\, \epsilon^{a_1 t}$$
$$+ \frac{A_3\, (R + M_1\, a_2)}{a_2\, L_3 + r_3}\, \epsilon^{a_2 t} \tag{36}$$

As before let $i_1^{(s)}$ and $i_1^{(P)}$ be the currents in the main circuit at the initial and final conditions respectively and $i_3^{(s)}$ and $i^{3(P)}$ the corresponding currents in the field circuit, we then have from equations (**34**) and (**36**) when $t = \infty$

$$A_1 = i_1^{(P)} \qquad i_3^{(P)} = \frac{R\, i_1^{(P)}}{r_3}$$

and when $t = 0$

$$A_2 + A_3 = i_1^{(s)} - i_1^{(P)}$$

$$\frac{R + M_1\, a_1}{a_1\, L_3 + r_3}\, A_2 + \frac{R + M_1\, a_2}{a_2\, L_3 + r_3}\, A_3 = i_3^{(s)} - \frac{R\, i_1^{(P)}}{r_3}$$

Solving these equations for A_2 and A_3

Solving these equations for A_2 and A_3

$$A_2 = \frac{[\,(i_1^{(s)} - i_1^{(p)})\,(R + M_1\,a_2)\,r_3 - (i_3^{(s)}\,r_3 - R\,i_1^{(p)})\,(r_3 + a_2\,L_3)\,]\,(r_3 + a_1\,L_3)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}$$

$$A_3 = \frac{[\,(i_3^{(s)}\,r_3 - R\,i_1^{(p)})\,(r_3 + a_1\,L_3) - (i_1^{(s)} - i_1^{(p)})\,(R + M_1\,a_1)\,r_3]\,(r_3 + a_2\,L_3)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}$$

Substituting in formulas (**34**) and (**36**) we have

$$i_1 = i_1^{(p)} + \frac{[\,(i_1^{(s)} - i_1^{(p)})\,(R + M_1\,a_2)\,r_3 - (i_3^{(s)}\,r_3 - R\,i_1^{(p)})\,(r_3 + a_2\,L_3)\,]\,(r_3 + a_1\,L_3)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}\,\epsilon^{a_1 t}$$
$$+ \frac{[\,(i_3^{(s)}\,r_3 - R\,i_1^{(p)})\,(r_3 + a_1\,L_3) - (i_1^{(s)} - i_1^{(p)})\,(R + M_1\,a_1)\,r_3]\,(r_3 + a_2\,L_3)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}\,\epsilon^{a_2 t} \qquad \textbf{(37)}$$

$$i_3 = \frac{R\,i_1^{(p)}}{r_3} + \frac{[\,(i_1^{(s)} - i_1^{(p)})\,(R + M_1\,a_2)\,r_3 - (i_3^{(s)}\,r_3 - R\,i_1^{(p)})\,(r_3 + a_2\,L_3)\,]\,(R + M_1\,a_1)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}\,\epsilon^{a_1 t}$$
$$+ \frac{[\,(i_3^{(s)}\,r_3 - R\,i_1^{(p)})\,(r_3 + a_1\,L_3) - (i_1^{(s)} - i_1^{(p)})\,(R + M_1\,a_1)\,r_3]\,(R + M\,a_2)}{r_3\,(R\,L_3 - M_1\,r_3)\,(a_1 - a_2)}\,\epsilon^{a_2 t} \qquad \textbf{(38)}$$

It has been shown in the paper that when $r_3 = K\, n_3$ the machine regulates for constant current over the operating range in which case $i_1^{(S)} = i_1^{(P)}$ and $i_1^{(S)} - i_1^{(P)} = 0$, under these conditions formulas (**37**) and (**38**) reduce to

$$i_1 = i_1^{(P)} + \frac{(i_3^{(S)}\, r_3 - R\, i_1^{(P)})\,(r_3 + a_2\, L_3)\,(r_3 + a_1\, L_3)}{r_3\,(R\, L_3 - M_1\, r_3)\,(a_1 - a_2)}$$

$$[\epsilon^{a_2 t} - \epsilon^{a_1 t}] \qquad \textbf{(39)}$$

$$i_3 = \frac{R\, i_1^{(P)}}{r_3} + \frac{(i_3^{(S)}\, r_3 - R\, i_1^{(P)}}{r_3\,(R\, L_3 - M_1\, r_3)\,(a_1 - a_2)}$$

$$[\,(r_3 + a_1\, L_3)\,(R + M_1\, a_2)\,\epsilon^{a_2 t} - (r_3 + a_2\, L_3)$$

$$(R + M_1\, a_1)\,\epsilon^{a_1 t}] \quad \textbf{(40)}$$

Since the current remains constant over the operating range for one value of r_3 only, formulas (**39**) and (**40**) cannot be used to determine the effect which a change in the resistance of the self-excited field will have on the momentary value of the main current. Experiments have shown, however, that a variation in this resistance over quite a wide range, other things remaining unchanged, has practically little or no effect on the stability of the arc. This is indeed to be expected, because while an increase in the resistance improves the time constant r_3/L_3 it also reduces the building-up characteristic as a self-excited machine.

Moreover open circuit or infinite external resistance cannot be taken as one of the initial conditions as was done in the discussion of the separately excited machine, because the voltage formula is based on the straight part of the saturation curve only. Manifestly, if the saturation curve continued in a straight line, the voltage would be infinite for infinite external resistance.

The highest value we can assume the external resistance to have must be limited to the point where the volt-ampere curve begins to bend rapidly from the straight line, the lower limit is, of course, zero resistance or short circuit.

What we are particularly interested in is the current fluctuation in the main circuit when the external resistance is suddenly changed from one to the other of these two extremes.

If the machine is suddenly short-circuited $R = 0$ and formulas (**39**) and (**40**) become

$$i_1 = i_1^{(P)} + \frac{i_3^{(S)} (r_3 + a_2 L_3)(r_3 + a_1 L_3)}{-M_1 r_3 (a_1 - a_2)} (\epsilon^{a_2 t} - \epsilon^{a_1 t}) \tag{41}$$

$$i_3 = \frac{i_3^{(S)}}{-M_1 r_3 (a_1 - a_2)} [(r_3 + a_1 L_3) M_1 a_2 \epsilon^{a_2 t} - (r_3 + a_2 L_3) M_1 a_1 \epsilon^{a_1 t}] \tag{42}$$

If an external resistance R is suddenly inserted in the main circuit when the generator is running on short circuit $i_3^{(S)} = 0$ and the formulas become

$$i_1 = i_1^{(P)} - \frac{R i_1^{(P)} (r_3 + a_2 L_3)(r_3 + a_1 L_3)}{r_3 (R L_3 - M_1 r_3)(a_1 - a_2)} [\epsilon^{a_2 t} - \epsilon^{a_1 t}] \tag{43}$$

$$i_3 = \frac{R i_1^{(P)}}{r_3} - \frac{R i_1^{(P)}}{r_3 (R L_3 - M_1 r_3)(a_1 - a_2)} [(r_3 + a_1 L_3)(R + M_1 a_2) \epsilon^{a_2 t} - (r_3 + a_2 L_3)(R + M_1 a_1) \epsilon^{a_1 t}] \tag{44}$$

Case II, Equal Roots.

If the roots of the quadratic be expressed as follows

$$a_1 = a + h \qquad a_2 = a - h$$

where $h = \dfrac{\sqrt{(P\,L_3 + r_3\,L_3 - M_1\,K\,n_3 - M_1\,R)^2 - 4\,(L_1\,L_3 - M_1^2)\,(P\,r_3 - R\,K\,n_3)}}{2\,(L_1\,L_3 - M_1^2)}$

the roots are equal when h becomes zero and may then be expressed

$$a = a_1 = a_2$$

Substituting $a + h$ for a_1 and $a - h$ for a_2 in equation (**39**)

$$i_1 = i_1^{(p)} + \frac{(i_3^{(s)}\,r_3 - R\,i_1^{(p)})\,[r_3 + (a-h)\,L_3]\,[r_3 + (a+h)\,L_3]}{r_3\,(R\,L_3 - M_1\,r_3)\,2\,h}\,[\epsilon^{(a-h)t} - \epsilon^{(a+h)t}]$$

$$= i_1^{(p)} + \frac{(i_3^{(s)}\,r_3 - R\,i_1^{(p)})\,[r_3 + a\,L_3 + h\,L_3]\,[r_3 + a\,L_3 + h\,L_3]}{2\,r_3\,(R\,L_3 - M_1\,r_3)}\,\frac{(\epsilon^{-ht} - \epsilon^{ht})}{h}\,\epsilon^{at}$$

When $h = 0$ the indeterminate fraction may be evaluated as before and is equal to $-2t$ and we then have

$$i_1 = i_1^{(P)} - \frac{(i_3^{(S)}\, r_3 - R\, i_1^{(P)})\, [r_3 + a\, L_3]^2\, t}{r_3\, (R\, L_3 - M_1\, r_3)}\, \epsilon^{at} \tag{45}$$

When going from welding voltage to short circuit we have again $R = 0$ and

$$i_1 = i_1^{(P)} + \frac{i_3^{(S)}\, (r_3 + a\, L_3)^2\, t}{M_1\, r_3}\, \epsilon^{at} \tag{46}$$

and when going from short circuit to welding voltage $i_3^{(S)}\, r_3 = 0$ and

$$i_1 = i_1^{(P)} + \frac{R\, i^{(P)}\, (r + a\, L_3)^2\, t}{r_3\, (R\, L_3 - M_1\, r_3)}\, \epsilon^{at} \tag{47}$$

Substituting $a + h$ for a_1 and $a - h$ for a_2 in equation **(40)**

$$\begin{aligned} i_1 = {} & \frac{R\, i_1^{(P)}}{r_3} + \frac{(i_3^{(S)}\, r_3 - R\, i_1^{(P)})}{r_3\, (R\, L_3 - M_1\, r_3)\, 2\, h} \\ & \{ [r_3 + (a + h)\, L_3]\, [R + M_1\, (a - h)\, L_3]\, \epsilon^{(a-h)t} \\ & - [\, (r_3 + (a - h)\, L_3]\, [R + M_1\, (a + h)\, L_3] \\ & \qquad \epsilon^{(a+h)t} \} \end{aligned}$$

$$\begin{aligned} = {} & \frac{R\, i_1^{(P)}}{r_3} + \frac{(i_3^{(S)}\, r_3 - R\, i_1^{(P)})}{r_3\, (R\, L_3 - M_1\, r_3)\, 2\, h} \\ & \{ [\, (r_3 + a\, L_3 + L_3\, h)\, (R + M_1\, a - M_1\, h)\,]\, \epsilon^{-ht} \\ & - [\, (r_3 + a\, L_3 - L_3\, h)\, (R + M_1\, a + M_1\, h)\,] \\ & \qquad \epsilon^{ht} \}\, \epsilon^{at} \end{aligned}$$

$$\begin{aligned} = {} & \frac{R\, i_1^{(P)}}{r_3} + \frac{(i_3^{(S)}\, r_3 - R\, i_1^{(P)})}{r_3\, (R\, L_3 - M_1\, r_3)\, 2\, h} \\ & \{ [\, (r_3 + a\, L_3) + L_3\, h)\, (R + M_1\, a) - M_1\, h)\,]\, \epsilon^{-ht} \\ & - [\, (r_3 + a\, L_3) - L_3\, h)\, (R + M_1\, a) + M_1\, h] \\ & \qquad \epsilon^{ht} \}\, \epsilon^{at} \end{aligned}$$

$$\begin{aligned} = {} & \frac{R\, i_1^{(P)}}{r_3} + \frac{(i_3^{(S)}\, r_3 - R\, i_1^{(P)})}{r_3\, (R\, L_3 - M_1\, r_3)\, 2\, h} \\ & \{ [\, (r_3 + a\, L_3)\, (R + M_1\, a) + (R + M_1\, a)\, L_3\, h \\ & - M_1\, L_3\, h^2]\, \epsilon^{-ht} - [\, (r_3 + a\, L_3)\, (R + M_1\, a) \\ & \qquad - (R + M_1\, a)\, L_3\, h - M_1\, L_3\, h^2]\, \epsilon^{ht} \}\, \epsilon^{at} \end{aligned}$$

$$= \frac{R\, i_1^{(p)}}{r_3} + \frac{(i_3^{(s)}\, r_3 - R\, i_1^{(p)})}{r_3\,(R\, L_3 - M_1\, r_3)\, 2}$$

$$\Big\{ (R + M_1\, a)\, L_3\, \epsilon^{-ht} + (R + M_1\, a)\, L_3\, \epsilon^{ht}$$

$$+ (r_3 + a\, L_3)\,(R + M_1\, a)\left(\frac{\epsilon^{-ht} - \epsilon^{ht}}{h}\right)\Big\}\, \epsilon^{at}$$

and for limiting value as h approaches zero as a limit.

$$i_3 = \frac{R\, i_1^{(p)}}{r_3} + \frac{(i_3^{(s)}\, r_3 - R\, i_1^{(p)})\,(R + M_1\, a)}{r_3\,(R\, L_3 - M_1\, r_3)}$$

$$[L_3 - (r + a\, L_3)t]\, \epsilon^{at} \tag{48}$$

and when going from welding voltage to short circuit

$$i_3 = \frac{i_3^{(s)}\, a}{r_3}\, [\,(r_3 + a\, L_3)\, t - L_3]\, \epsilon^{at} \tag{49}$$

and from short circuit to welding voltage

$$i_3 = \frac{R\, i_1^{(p)}}{r_3} + \frac{R\, i_1^{(p)}\,(R + M_1\, a)}{r_3\,(R\, L_3 - M_1\, r_3)}$$

$$[\,(r + a\, L_3)\, t - L_3]\, \epsilon^{at} \tag{50}$$

Case III, Roots Complex Imaginary Numbers.

The roots may in this case be represented by the expressions.

$$p + j\, q \quad \text{and} \quad p - j\, q$$

and the general solution of (**29**) is

$$i_1 = A_1\, h + \epsilon^{pt}\,(B_1 \cos q\, t + B_2 \sin q\, t) \tag{51}$$

From equation (**30**)

$$L\, \frac{d\, i_3}{dt} + r_3\, i_3 = M_1\, \frac{d\, i_1}{dt} + R\, i_1$$

Substituting in this equation from (**51**) we have

$$L_3 \frac{d\, i_3}{dt} + r_3\, i_3 = R\, A_1 + \epsilon^{pt}$$

$$(M_1\, p\, B_1 + M_1\, q\, B_2 + R\, B_1) \cos q\, t + \epsilon^{pt}$$

$$(M_1\, p\, B_2 - M_1\, q\, B_1 + R\, B_2) \sin q\, t$$

Solving this equation

$$i_3 = \frac{1}{L_3}\, \epsilon^{-r_3/L_3 \times t} \int R\, A_1\, \epsilon^{-r_3/L_3 \times t}\, dt + \frac{1}{L_3}\, \epsilon^{-r_3/L_3 \times t}$$

$$\int \epsilon^{r_3/L_3 \times t} (M_1 \, p \, B_1 + M_1 \, q \, B_2 + R \, B_1)$$

$$\epsilon^{pt} \cos q \, t \, dt + \frac{1}{L_3} \epsilon^{-r_3/L_3 \times t} \int \epsilon^{r_3/L_3 \times t}$$

$$(M_1 \, p \, B_2 - M_1 \, q \, B_1 + R \, B_2) \, \epsilon^{pt} \sin q \, t \, dt$$

$$= \frac{1}{L_3} \epsilon^{-r_3/L_3 \times t} R \, A_1 \int \epsilon^{r_3/L_3 \times t} \, dt$$

$$+ \frac{(M_1 \, p \, B_1 + M_1 \, q \, B_2 + R \, B_1)}{L_3} \epsilon^{-r_3/L_3 \times t}$$

$$\int \epsilon^{(r_3/L_3 + p)t} \cos q \, t \, dt$$

$$+ \frac{(M_1 \, p \, B_2 - M_1 \, q \, B_1 + R \, B_2)}{L_3} \epsilon^{-r_3/L_3 \times t}$$

$$\int \epsilon^{(r_3/L_3 + p)t} \sin q \, t \, dt \qquad (52)$$

Evaluating the integrals by means of formulas

$$\int \epsilon^{\alpha x} \sin \beta \, x \, dx = \frac{\epsilon^{\alpha x}}{\alpha^2 + \beta^2} (\alpha \sin \beta \, x - \beta \cos \beta \, x)$$

$$\int \epsilon^{\alpha x} \cos \beta \, x \, dx = \frac{\epsilon^{\alpha x}}{\alpha^2 + \beta^2} (\beta \sin \beta \, x + \alpha \cos \beta \, x)$$

$$i_3 = \frac{R \, A_1}{r_3}$$

$$+ \frac{(M_1 \, p \, B_1 + M_1 \, q \, B_2 + R \, B_1) \, \epsilon^{-r_3/L_3 + t} \, \epsilon^{(r_3/L_3 + p)t}}{L_3 \left[\left(\frac{r_3}{L_3} + p \right)^2 + q^2 \right]}$$

$$\left[q \sin q \, t + \left(\frac{r_3}{L_3} + p \right) \cos q \, t \right]$$

$$+ \frac{(M_1 \, p \, B_2 - M_1 \, q \, B_1 + R \, B_2) \, \epsilon^{-r_3/L_3 \times t} \, \epsilon^{(r_3/L_3 + p)}}{L_3 \left[\left(\frac{r_3}{L_3} + p \right)^2 + q^2 \right]}$$

$$\left[\left(\frac{r_3}{L_3} + p\right) \sin q\,t - q \cos q\,t\right]$$

$$= \frac{R\,A_1}{r_3} + \frac{\epsilon^{pt}}{(r_3 + L_3\,p)^2 + L_3^2\,q^2}$$

$$\{\,[\,(M_1\,p\,B_1 + M_1\,q\,B_2 + R\,B_1)\,L_3\,q + (M_1\,p\,B_2 - M_1\,q\,B_1 + R\,B_2)\,(r_3 + L_3\,p)\,]\sin q\,t + [\,(M_1\,pB_1 + M_1\,q\,B_2 + R\,B_1)\,(r_3 + L_3\,p) - (M_1\,p\,B_2 - M_1\,q\,B_1 + R\,B_2)\,L_3\,q]\cos q\,t\}$$

Simplifying

$$i_3 = \frac{R\,A_1}{r_3} + \frac{\epsilon^{pt}}{(r_3 + L_3\,p)^2 + L_3^2\,q^2}$$

$$\{\,[M_1\,L_3\,B_2\,(q^2 + p^2) + M_1\,r_3\,(p\,B_2 - q\,B_1) + R\,L_3\,(q\,B_1 + p\,B_2) + R\,B_2\,r_3]\sin q\,t + [M_1\,L_3\,B_1\,(q^2 + p^2) + M_1\,r_3\,(p\,B_1 + q\,B_2) + R\,L_3\,(p\,B_1 - q\,B_2) + R\,B_1\,r_3]\cos q\,t\} \quad (\mathbf{53})$$

From equation (**51**) we have when $t = \infty$ $A_1 = i_1^{(P)}$ and when $t = 0$ $B_1 = i_1^{(S)} - i_1^{(P)}$ and from equation (**53**) when $t = 0$

$$\frac{M_1\,L_3\,B_1\,(q^2 + p^2) + M_1\,r_3\,(p\,B_1 + q\,B_2) + R\,L_3\,(p\,B_1 - q\,B_2) + R\,B_1\,r_3}{(r_3 + L_3\,p)^2 + L_3^2\,q^2} = \frac{i_3^{(S)}\,r_3 - R\,i_1^{(P)}}{r_3}$$

When the resistance of the self-excited field is adjusted so that generator regulates for approximately constant current we have

$$i_1^{(S)} - i_1^{(P)} = B_1 = 0 \text{ and}$$

$$\frac{M_1\,r_3\,q\,B_2 - R\,L_3\,q\,B_2}{(r_3 + L_3\,p)^2 + L_3^2\,q^2} = \frac{i_3^{(S)}\,r_3 - R\,i_1^{(P)}}{r_3}$$

Solving for B_2

$$B_2 = \frac{(i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,[\,(r_3 + L_3\,p)^2 + L_3^2\,q^2]}{r_3\,q\,(M_1\,r_3 - R\,L_3)} \quad (\mathbf{54})$$

Substituting in (**51**)

$$i_1 = i_1^{(P)} + \frac{(i_3^{(S)}\,r_3 - R\,i_1^{(P)})\,[\,(r_3 + L_3\,p)^2 + L_3^2\,q^2]}{r_3\,q\,(M_1\,r_3 - R\,L_3)}\,\epsilon^{pt} \sin q\,t \quad (\mathbf{55})$$

Substituting $B_1 = 0$ and

$$\frac{M_1 L_3 B_1 (q^2 + p^2) + M_1 r_3 (p B_1 + q B_2) + R L_3 (p B_1 - q B_2) + R B_1 r_3}{(r_3 + L_3 p)^2 + L_3^2 q^2} = \frac{i_3^{(s)} r_3 - R i_1^{(P)}}{r_3}$$

in equation (**53**) we have

$$i_3 = \frac{R i_1^{(P)}}{r_3} + \epsilon^{pt} \left\{ \frac{B_2 [M_1 L_3 (q^2 + p^2) + M_1 r_3 p + R L_3 p + R r_3]}{(r_3 + L_3 p)^2 + L_3^2 q^2} \sin q t + \frac{i_3^{(s)} r_3 - R i_1^{(P)}}{r_3} \cos q t \right\}$$

Substituting B_2 from equation (**54**)

$$i_3 = \frac{R i_1^{(P)}}{r_3} + \frac{(i_3^{(s)} r_3 - R i_1^{(P)})\, \epsilon^{pt}}{r_3} \left\{ \frac{M_1 L_3 (q^2 + p^2) + M_1 r_3 p + R L_3 p + R r_3}{q (M_1 r_3 - R L_3)} \sin q t + \cos q t \right\} \qquad (\mathbf{56})$$

When going from welding voltage to short circuit $R = 0$ and formulas (**55**) and (**56**) become

$$i_1 = i_1^{(P)} + \frac{i_3^{(s)} [(r_3 + L_3 p)^2 + L_3^2 q^2]\, \epsilon^{pt}}{q M_1 r_3} \sin q t \qquad (\mathbf{57})$$

$$i_3 = i_3^{(s)} \epsilon^{pt} \left\{ \frac{L_3 (q^2 + p^2) + r_3 p}{q r_3} \sin q t + \cos q t \right\} \qquad (\mathbf{58})$$

And when going from short circuit to welding voltage $i_3^{(s)} = 0$ and the formulas reduce to

$$i_1 = i_1^{(P)} - \frac{R i_1^{(P)} [(r_3 + L_3 p)^2 + L_3^2 q^2]\, \epsilon^{pt}}{r_3 q (M_1 r_3 - R L_3)} \sin q t \qquad (\mathbf{59})$$

$$i_3 = \frac{R i_1^{(P)}}{r_3} - \frac{R i_1^{(P)}}{r_3} \epsilon^{pt}$$

$$\left\{ \frac{M_1 L_3 (q^2 + p^2) + M_1 r_3 p + R L_3 p + R r_3}{q\,(M_1 r_3 - R L_3)} \sin q t + \cos q t \right\} \tag{60}$$

To find the time at which the current fluctuation becomes a maximum or a minimum we differentiate equation (**39**) with respect to t and put the derivative equal to zero as follows

$$\frac{d}{dt} \left\{ i_1^{(P)} + \frac{(i_3^{(S)} r_3 - R\, i_1^{(P)})(r_3 + a_2 L_3)(r_3 + a_1 L_3)}{r_3\,(R L_3 - M_1 r_3)\,(a_1 - a_2)} \left[\epsilon^{a_2 t} - \epsilon^{a_1 t}\right] \right\} = 0$$

$$\left[a_2\, \epsilon^{a_2 t} - a_1\, \epsilon^{a_1 t}\right] = 0$$

$$a_2\, \epsilon^{a_2 t} = a_1\, \epsilon^{a_1 t}$$

To solve this equation for t we take logarithms of both sides, and since a_1 and a_2 are in general negative it is preferable to make the following substitutions to avoid taking logarithms of negative numbers

$$a_1 = -\,a_1' \qquad a_2 = -\,a_2'$$ and we have

$$-\,a_2'\, \epsilon^{-a_2' t} = -\,a_1'\, \epsilon^{-a_1' t}$$

Changing signs and taking logarithms of both sides

$$\log a_2' - a_2' t = \log a_1' - a_1' t$$

$$(a_1' - a_2')\, t = \log a_1' - \log a_2'$$

$$t = \frac{\log a_1' - \log a_2'}{a_1' - a_2'}$$

And when logarithms are taken to base 10 instead of base ϵ

$$t = \frac{2.31\,(\log a_1' - \log a_2')}{a_1' - a_2'} \tag{61}$$

To find the time of maximum or minimum current fluctuation in the second case we differentiate equation (**45**) with respect to t and equate to zero.

$$\left\{ \frac{(i_3^{(S)} r_3 - R\, i_1^{(P)})\,[r_3 + L_3 a]^2}{r_3\,(R L_3 - M_1 r_3)} \left[\epsilon^{a t} + t\, a\, \epsilon^{a t}\right] \right\} = 0$$

$$\epsilon^{a t}\,(1 + t\, a) = 0, \text{ or } t = -\,\frac{1}{a} \tag{62}$$

In case III we differentiate (**55**) and equate to zero

$$\frac{(i_3^{(s)}\, r_3 - R\, i_1^{(p)})\,[\,(r_3 + L_3\, p)^2 + L_3^2\, q^2]}{r_3\, q\,(M_1\, r_3 - R\, L_3)}$$

$$\frac{d}{dt}\,(\epsilon^{pt} \sin q\,t) = 0$$

hence

$$\frac{d}{dt}\,(\epsilon^{pt} \sin q\,t) = 0$$

$$p\,\epsilon^{pt} \sin q\,t + q\,\epsilon^{pt} \cos q\,t = 0$$

$$p \sin q\,t = -\,q \cos q\,t$$

$$\frac{\sin q\,t}{\cos q\,t} = -\,\frac{q}{p}$$

$$q\,t = \tan^{-1}\left(-\,\frac{q}{p}\right)$$

The angle qt usually to be taken in the first quadrant.

Using the same illustration as before except assuming that the generator has been supplied with a self-excited field of the following constants $n_3 = 325$ turns, $r_3 = 27$ ohms, $M_1 = 0.0203$ henrys $L_3 = 2.5$ henrys.

Assume the separately excited field set for 150 amperes in the main circuit and an external resistance varying from 0.2 ohm to 0 and back to 0.2 ohm. The other constants of the machine are $L_1 = 0.00245$ $r_1 = 0.03$ $k = 0.0825$.

The curves in Fig. 9 show the current fluctuations in the first case when the machine is short-circuited and the curves in Fig. 10 show the momentary currents in the second case when the external resistance is suddenly inserted.

Discussion on "Design of Constant Current Generators for Arc Welding" (Hansen), White Sulphur Springs, W. Va., July 1, 1920.

R. W. Owens: In connection with Fig. 13 of Mr. Hansen's paper, in the statement that follows, he has said something with regard to the possibility of changing the characteristics of an arc welding machine of that type by changing the resistances. I would like to emphasize that point a little. By changing the values of the resistances r_5 and r_4 or by changing the value of the separate excitation, I believe that it is possible with that type of machine to duplicate the characteristics of any one of the various machines that are described in these papers.

Now, that leaves it open to the designer of a machine to pick out those characteristics which give the greatest ease of manipulation of the arc. That point cannot be too strongly emphasized for arc welding machines to be used for manual operation of the arc, even though the ease of manipulation is obtained at some sacrifice of efficiency, for the reason that in manual operation we are concerned with both labor costs and power costs. That is, if we obtain extreme ease of manipulation it is conceivable that it is possible to save in labor what might be sacrificed in the efficiency of the machine, or, in other words, in one hour's time, it will be possible for a man to deposit a greater amount of metal than if the machine were not so easy to handle.

S. R. Bergman: It was stated that by aid of the machine, characteristics similar to any of the machines described in these papers can be obtained. This may be possible if a steady load is used, for example, such load as is obtained on steady resistance load. But there is a distinct difference between the values obtained on a steady resistance load and the values obtained when the load changes abruptly as is the case in an arc-welding machine. The instantaneous values depend upon the inter-relationship between the armature and the different windings in the field. Therefore, since all of these machines employ different combination of windings such as shunt and series windings on the poles I predict that all of these machines will give different results when applied to a load that changes rapidly. I can, therefore, not agree with the statement that all of these machines can be made to give similar characteristics when applied to arc welding.

[illegible] able it agree [illegible]

[illegible] adequacy of any [illegible]

[illegible] one's time, it will [illegible]

[illegible] greater amount of [illegible]

[illegible]

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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AUTOMATIC ARC WELDING APPARATUS

BY S. R. BERGMAN AND R. L. UNLAND

Both of the General Electric Co.

AMONG the requirements which must be met by arc welding equipments are simplicity and reliability. Their importance is increased by the fact that in a great many cases these equipments are installed and operated in locations where no trained electrical help is available. A new type of arc welding generator is described which inherently possesses the electrical characteristics desirable for single operator arc welding generators. This results in the elimination of external resistors or other regulating devices since the generator delivers at its terminals the voltage required by the arc and the current for which the equipment is adjusted.

It has been found that a drooping volt-ampere characteristic or a circuit in which the current decreases as the voltage increases, or vice versa, is advantageous for successful electric arc welding. This has been obtained in the past by the use of constant potential generators with a resistance in series with the arc to provide the drooping characteristic in the arc circuit. It has also been accomplished by using differentially wound generators with a separate excitation source to provide a stable magnetic circuit to act as a base for the regulation of the generator. In the generator described this result is accomplished by what may be called a dual magnetic circuit.

In the generator described the design consists of a four-pole field structure and an armature wound for two poles. The field poles are paired to give two adjacent poles of each polarity. The opposite field poles have the same character and are similarly connected in the electrical circuit so that variations in the excitation occur simultaneously in the opposite poles and consequently the flux may be considered as two individual circuits, each of which passes through a pair of opposite poles, the armature and the field ring. These two fluxes are independent of each other as long as the magnetic structure common to both is not saturated. One of the magnetic circuits, referred to as the main field, is designed to provide constant flux and thereby maintain constant voltage in that portion of the armature under the influence of this field. The field poles in this circuit are designed to be saturated under normal conditions and therefore the flux will be very slightly affected by considerable variations in the electrical circuit. The other component of the field is at right angles to the first and is referred to as the cross field, and in this circuit the magnetic structure is not saturated. These two individual fields generate electromotive forces in the armature which under no-load conditions add arithmetically to provide the no-load voltage of the generator. As current is taken from the load brushes of the generator, however, an armature reaction is built up which may be resolved into two components at right angles, in line with the field fluxes described above. One component tends to increase the main field flux, the other component opposes the cross field flux. On account of saturation in the main field magnetic circuit further increase of flux is impossible, but in the cross field circuit the initial flux is reduced as the current and armature reaction increase and when the generator is finally short-circuited this flux is reversed to a value equal to the initial value. As the cross field is reduced and finally reversed it is obvious that the electromotive force generated in the armature by this flux also decreases and is finally reversed. The line voltage of the generator consists of the sum of these two voltage components. Under the no-load conditions they are equal and on short circuit the component due to the cross field has the same value as at no-load but is reversed so that the terminal voltage of the generator is zero.

A differentially connected series winding is placed on the cross field poles to assist the armature reaction when it is desired to reduce the current output of this generator.

It is a simple matter to produce a generator which under steady load conditions has the drooping characteristic considered desirable for arc welding but when used for arc welding such machines fail due to slow regulation and the lag between the sudden variation in the arc and the corrective electrical or magnetic adjustment in the machine. It should be borne in mind that the regulation of this generator is mainly produced by the armature itself. Since the armature is the seat of the induced voltage it is obvious that if the armature itself is the seat of the regulating power this action is as intimate as can be obtained.

A new development is also described which takes the form of a device for automatically feeding a bare wire electrode into the welding arc at the exact rate required to maintain constant electrical conditions in the arc. This device consists of a small direct-current motor geared to feed rolls and electrically connected across the welding arc through control with the result that the speed of the motor and consequently the rate at which the wire is fed into the arc varies with the voltage across the arc. The result is that practically constant voltage is maintained across the arc and therefore the current will be constant. The arc voltage may be maintained below any value which is possible with hand manipulation of the electrode, and due to the steadiness of operation the speed of welding may be greatly increased over that obtained by hand. The length of the arc can be maintained at a minimum with the result that the metal has little opportunity of being oxidized in passing through the arc and the metal deposited is therefore of a higher and more uniform quality than that found where hand operation is used. The field for this device is limited to manufacturing production where the number of duplicate welds is sufficient to warrant the making of special fixtures for holding the work and for facilitating handling. Illustrations of work performed and operating data are given.

DISCUSSION ON "AUTOMATIC ARC WELDING APPARATUS" (BERGMAN AND UNLAND), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

K. L. Hansen: It is stated in the paper that the armature is the seat of the induced e. m. f. and also the seat of the regulating power. Does that mean, for example, that when the flux is reduced by armature reaction the transient current can be different from what it would be when the reduction in flux is produced by a series field? If such is the case I would take exception to the statement, because there is a certain amount of energy stored in the field and any change in this energy is reflected in the armature circuit no matter what causes it.

The case is similar to that of an alternator on short circuit. The fact that in the alternator, the field flux is reduced by armature reaction to produce a steady short-circuit current, does not alter the fact that there will be a large transient current at the instant of short circuit, due to voltage induced in the field caused by the rapid change of flux.

S. R. Bergman: I will answer that question by stating that the further removed the field is from the armature, the more leakage and other conditions enter, which makes for conditions that obviously are further removed from the armature. The armature itself naturally is the seat of the electromotive force—the ampere turns of the armature are as nearly connected with that electromotive force as possible. Take a commutating field, for example, that commutating pole has larger leakage and saturation, of course, and different conditions enter, whereas the armature reaction itself is more instantaneous in its action than would be any field located outside of the armature. That can be illustrated by oscillographs and has been verified.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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ARC WELDING MACHINES OF THE WILSON WELDER AND METALS COMPANY

BY ALEXANDER CHURCHWARD

Engineer, Wilson Welder and Metals Co.

WHILE experimenting with alloy wire to get greater strength than that obtained with the ordinary low-carbon wire, it was discovered that with a long arc a high-carbon content wire lost most of its carbon while going through the arc.

Having the above in view, a system was developed, whereby a long arc cannot be drawn; 16 to 19 volts is accepted as a normal arc. There must be enough stabilizing resistance used to allow the arc momentarily to be drawn out to 22 volts, to take care of inequalities, of the piece to be welded, the burning off of the electrode and unsteadiness of the operator's hand. The lowest voltage to take care of this condition with ordinary resistance grids, non-automatic, was found to be 60, but this would give too long an arc with the ordinary welder, it gave a flexible arc to be sure, but with the ordinary welder, a flexible arc is dangerous if good welds are required.

Therefore, to get a normal arc without dangerous flexibility, it was found that 35 to 37.5 volts was the maximum that could be used. Now, 35 volts with a fixed resistance will not give a steady or constant current. It had also been determined that unless the current was constant good welds of the maximum strength could not be made. It was decided to devise an automatic resistance, namely the carbon pile.

The function of the carbon pile is briefly as follows:

First: The carbon pile is in series with the arc.

Second: A heavy spring compresses the pile to minimum resistance and adjustments may be made for current values required.

Third: A solenoid in series with the welding circuit, counteracts the spring pressure so that any predetermined current value can be maintained.

The generator is of the constant-potential low-voltage type (35 to 37.5-volt) flat compounded. When the electrode is short-circuited on the work the voltage of the generator remains constant, and does not drop to zero, to be built up when the arc is started, but the solenoid instantaneously functions, and releases the pressure on the carbon pile, thus, inserting the proper amount of resistance automatically, preventing the short-circuiting current on the surge from rising more than 10 per cent above the welding current. As the arc lengthens and requires more voltage, the carbon pile, controlled by the solenoid, instantaneously responds and the current when welding is kept constant within 5 per cent.

This type of control was selected because its time element was much less than some other types of control. It has especially proved itself in multiple arc machines, two or more welders operating from the same machine, sometimes on the same piece of work.

What is claimed for this system is that there is produced constant heat per unit area in the weld, *not in the arc.*

Standard machines are made in three capacities, one-,two- and four-operator machines for all available circuits, both a-c. and d-c. Also gas-driven one- and two-operator sets, and belted machines to be driven off line shafts.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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CHARACTERISTICS AND PERFORMANCE OF ARC WELDING MACHINERY

BY A. M. CANDY

Engineering Department, Westinghouse Electric & Mfg. Co.

THIS paper deals rather briefly at first with the earlier history of arc welding apparatus discussing the increased efficiency of the constant-potential welding circuit and decreased size and cost of the apparatus by changing the generated circuit potential from 75 to 60 volts. Curves are given showing the efficiency increase and also the disadvantages attending any material decrease of the circuit potential below 60 volts.

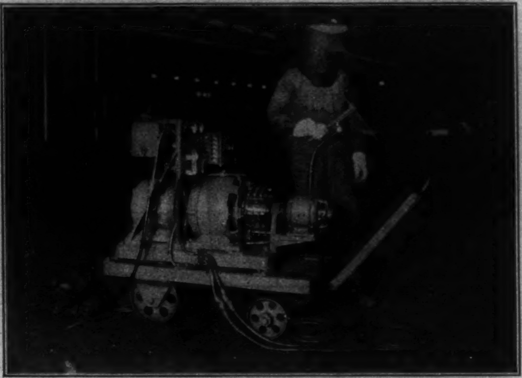

FIG. 7

The discussion then turns to the more recent developments of variable-voltage or constant-current equipment of both the alternating and direct current types. Although the a-c. arc welding transformer is about one-half as heavy and expensive as the d-c. equipment its use at present and in the immediate future at least will be relatively limited for several reasons.

1. To make the equipment commercially successful in the hands of the average welder it is necessary to use an especially prepared electrode.

2. The alternating-current arc is not so effective in fusing metal as the direct-current arc assuming the same current in amperes in each case. In other words, to obtain the same rate of fusion the ratio of alternating current to direct current required is in the order of 170 amperes alternating-current to 140 amperes direct-current.

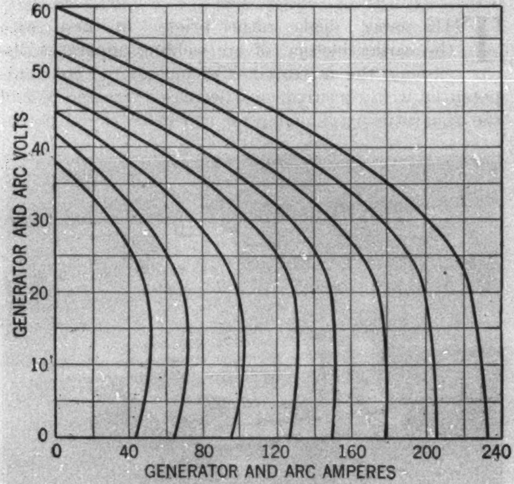

Fig. 5—Single-Operator Arc Welder—Volt-Ampere Characteristic

3. The transformer is inherently a single-phase low power factor load (20 per cent to 30 per cent maximum). This characteristic is necessary so that the arc will be reasonably stable.

The first two features make the operating cost so much higher than for the interconnected constant-current d-c. machine that the saving in operating cost per annum will be about 50 per cent interest on the difference of the first costs of the two types.

4. No thoroughly satisfactory means of limiting the open-circuit voltage to reasonably safe values (60 volts or less) has as yet been developed. To be commercially satisfactory the transformer must either actually develop or have the characteristics of an open-circuit potential of 135 volts minimum to 175 volts maximum.

This potential is higher than the operator should be subjected to as it is a real life hazard.

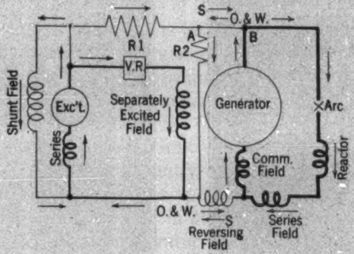

FIG. 8

The main portion of the paper discusses an interconnected constant-current variable-voltage d-c. generator and exciter (see Fig. 7) which has been developed and placed on the market quite recently. The generator is a commutating-pole machine provided with a series field winding, a shunt field winding separately excited continually by means of a small exciter coupled to the generator shaft, and a second shunt field winding connected to both the generator terminals, and the exciter terminals so that under open-circuit and normal welding conditions it is self-excited by the generator voltage, whereas under short-circuited arc condition it is excited in the opposite direction by current from the exciter terminals. This field winding is therefore logically called a reversing field. The series field is connected so that it always opposes the separately excited field. Under open circuit and nor-

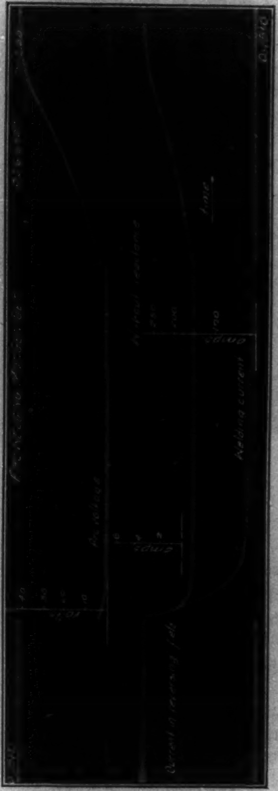
volts
40
30
20
10
Ac voltage
amps
6
4
2
Current in receiving field
amps
100
200
Welding current
time

FIG. 10

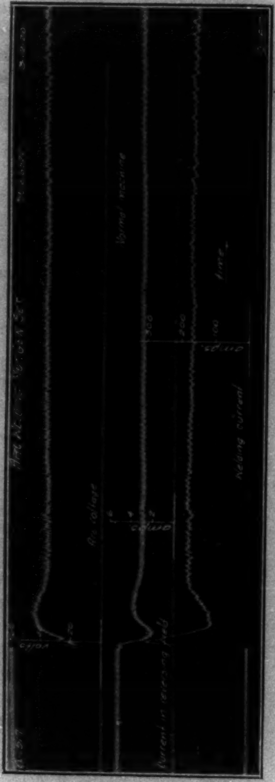
volts
20
Ac voltage
amps
6
4
2
Current in receiving field
amps
100
200
300
Welding current
time

FIG. 18

mal welding conditions the reversing field (self-excited by generator) assists the separate field in maintaining the generator voltage. Under short-circuited arc conditions the reversing field is excited by current from the exciter so that it functions as a separately excited field and assists the series in opposing the separate field thereby limiting the generator voltage to a value such that the short-circuit current is substantially the same as the normal welding current. This characteristic is illustrated by Fig. 5. The scheme of connections producing these characteristics is illustrated by Fig. 8. A very decided advantage of this interconnected scheme is the fact that the arc is given the advantage of the constant potential continually developed by the exciter although the generator terminal voltage is never above 60 and varies from that value down to practically zero when short-circuited.

A portion of the exciter voltage exists across the electrode terminals instantly when they are separated by the operator incident to striking the arc, thereby materially assisting him in starting the arc regardless of any lag in the generator building up its voltage. Furthermore, this impressed constant potential from the exciter circuit helps to stabilize the arc making it exceptionally tenacious. Both of these features are of very material assistance to the welding operator. The latter characteristic is illustrated by Fig. 10 where the kick in the generator terminal voltage is indicated when the arc current was broken suddenly by deliberately separating the electrodes quickly.

Another feature of this machine is that it can be successfully used for graphite electrode welding, delivering 150 amperes at 40 volts, as illustrated by Fig. 18. Where more than 150 amperes is required two or more equipments can be operated in parallel by simply connecting together the external leads of like polarity of the generators. Due to the characteristics of the generator no equalizing connections are necessary.

Due to the elimination of losses in resistances in series with the arc the operating cost for the interconnected constant-current machine is considerably less than for the 60-volt constant-potential equipments and of course proportionally smaller than for 75-volt constant potential equipments. A curve and typical example comparing the operating cost of six interconnected machines with that of a 1000-ampere, 60-volt machine feeding six welding circuits are given. The figures show that although there is a difference in first cost of approximately $2050 in favor of the 1000-ampere equipment, under normal average operating conditions where central station a-c. service is 3 cents a kw-hr. the annual saving resulting from the use of the interconnected equipments is equivalent to practically 60 per cent interest on the additional investment of $2050.

Summary of the advantages of the interconnected constant-current scheme,

1. Ease of striking arc.
2. Ease of holding arc due to increased stability.
3. Operating expense less than for other types of equipment.
4. Simplicity of obtaining current adjustment.
5. Generator polarity cannot be reversed even though exciter circuit should be accidentally opened.
6. Two or more generators may be operated in parallel for graphite welding without requiring equalizer connections.
7. A maximum of 225 amperes for metal electrode and 150 amperes for graphite can be obtained from one generator.

DISCUSSION ON "CHARACTERISTICS AND PERFORMANCE OF ARC WELDING MACHINERY" (CANDY), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

C. J. Holslag: This talk of constant current being the heat in the arc has appeared to me for the last six or seven years to be foolish, and yet nearly everyone seems to accept that belief.

I believe that watts constitute the rate of heat in the arc and I believe after it is pointed, out that you will all think that way. Certainly, $I^2 r$ at the tip of the electrode and $I^2 r$ at the point of the plate does not explain the real state of things. Those two $I^2 r$ are each made up of an $E. I.$ and an I and the conception of an E in the center of which the I will not combine with to make watts, is too much for anyone to grasp. Granted that watts represent the rate of heat supplied to the arc, the ideal condition is that this rate besupplied so as to approximate constant heat. Our idea of this is to supply constant rate of heat for any given setting for any given length of arc, with an increased rate allowing for radiation for longer arc lengths.

Another point, although the watts should be held constant for any given operation to properly meet the characteristics of welding, they should be made up for various electrodes and various operations of the two factors—volts and amperes in different quantities. For instance, the bare wire electrode might have best conditions with 18 or 20 volts across the arc and 200 amperes through it, while a heavily coated electrode would be most efficient with the same watts made up of 100 amperes and 36 to 40 volts across the arc. It would be impossible to get the best results with these conditions reversed for either electrode.

Now, as Mr. Candy has pointed out, the ideal welding machine must not only meet the two extremes, but all the range in between. There is no d-c. machine that I know of which will properly use all types of electrodes.

Another essential which is outlined in the abstract of my paper is that the arc voltage, while of certain values as given in the table herein, may be the same for different types of electrodes, yet this voltage may be made up of different factors for different electrodes or for different operations.

The idea I have tried to convey is that the arc voltage consists of three separate voltages, two of which are alike, and well known, and the third is the guardian or holding voltage which is different to anything ordinarily taught. For instance, no one before ever built an a-c. machine that would hold an arc at the impress voltage of the arc. The answer as I see it is that there is a voltage necessary to not only start the arc but to keep it from going out, which is ordinarily of such short duration that it does not add materially to the average voltage, and that some conditions require this voltage to be present a greater part of the time, but to be of such short time value that it does add materially to the average voltage.

The ideal machine must also control the length of arc and not have definite length either long or short, as different operations require different maximum lengths and the only possible way that I can see to accomplish, with my present knowledge, all of these requirements, *i. e.*, arc length control, constant rate of heat, correct sub-division of heat factors and voltage, is by means of an a-c. transformer, and for that reason and because the a-c. arc possesses many meritorious features impossible to direct current, I believe, alternating current to be the ultimate power supply for arc welding.

William O. Noble: The curves shown in Fig. 5 of Mr. Candy's paper were evidently taken by slowly changing the load by means of a water rheostat or some such arrangement. Mr. Candy brought out the point that if you do not have any increase in the current at the moment of short circuit, you do not have the tendency for the electrode to stick. In my opinion the curves shown in Fig. 5 do not represent the condition you get on short-circuiting the electrode at all, that is, when you short-circuit the machine, you must necessarily destroy the magnetic flux in the machine which, of course, takes a certain time. The momentary inrush on any of these self-regulating machines is probably four or five times the final steady short-circuiting value, and that momentary inrush is what tends to make the electrode stick.

I would like to point out the advantage of a more sloping characteristic in welding, in the increase of current with the decrease in the length of the arc automatically increases the rate of electrode consumption and tends to bring the arc back to normal. Increase in the length of the arc, with a consequent decrease in current, decreases the rate of electrode consumption and tends to compensate in this direction also.

S. R. Bergman: Mr. Candy first stated that any of these machines would work satisfactorily with a reactance. Immediately thereupon he states that a reactance is detrimental. I merely wish to point out that these statements are contradictory. The idea of employing a reactance is based upon the fundamental property of self-induction of opposing any change in the current. Therefore, it appears to me that a reactance is always helpful since it steadies the current and prevents the arc from breaking particularly when the operator of the arc-welding machine is not an expert.

I believe one of the difficulties in the past has been that the reactances employed have not been properly designed for arc-welding. Most arc-welding machines are designed for a large range of load currents, for example, from a minimum arc-current of 50 amperes to a maximum arc-current of 200 amperes. If the reactance is designed properly for a direct current of 50 amperes it becomes saturated for 200 amperes. On the other hand, if the reactance is properly designed for 200 amperes the reactance drop becomes too low when used for 50 amperes. This drawback may be overcome by aid of a simple arrangement which is shown in Fig. 1.

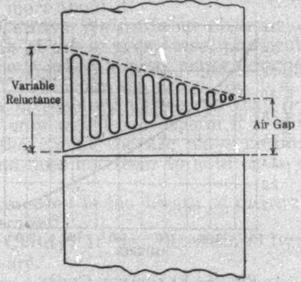

FIG. 1—VARIABLE CAP RESISTANCE

This reactance has a tapered gap and in addition there are slots in the iron as indicated. Such an arrangement is equivalent to an adjustable gap. For the lowest current value of 50 amperes the teeth are not saturated and the total reluctance is merely the reluctance of the air-gap. For higher current values saturation sets in and finally for the maximum current value all of the teeth are saturated. It may be easily understood that for higher current values we have the equivalent of a larger gap. Tests have clearly demonstrated that this reactance can be used over a wide range of load currents and experience has shown that this reactance is very useful in arc-welding.

A. M. Candy: Mr. Holslag dwelt a little on the question of constant current. The melting of the electrode, I think, practically all of us agree is the question of I^2r drop at the end of the electrode where the current is melting the electrode, in other words, it is the surface voltage which exists at the end of the arc. This is thoroughly covered in a great many papers on the subject of the arc. It is undoubtedly true that the I^2r melts the electrode at one end of the arc, and the plate at the other end of the arc. Incidentally, I will have to agree with Mr. Holslag, that if you get the alternating arc started when using bare wire, it is fairly easy to hold the arc, but the big trick comes in in getting the arc started.

Mr. Bergman in his discussion of reactor design has me somewhat out of my field since I do not pretend to be either a transformer or a reactor designer. However, if I understood Mr. Bergman correctly, I inferred that the inductance of a reactor could only be made variable by designing a special shaped air gap.

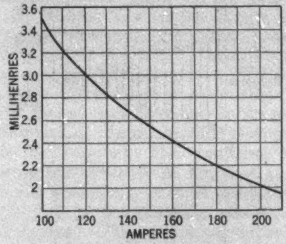

FIG. 2—REACTANCE CURVE

In the case of the reactor which we use, the air gap is uniform over the entire core section, but the inductance of our reactor varies with the current as indicated by the curve submitted by our design engineers which is approximately as indicated by Fig. 2. In other words, at 100 amperes, the inductance is approximately 3½ millihenries, whereas at 200 amperes it is approximately 2 millihenries. When I was referring to large and small reactors I was considering a small reactor, one capable of developing an inductance of approximately 3 to 4 millihenries versus one developing from 80 to 100 millihenries. Mr. Bergman did not specify the inductance of his reactor, and therefore, I do not know where it falls in this classification It would be interesting if he would give the inductance at the two current values which I mentioned above.

We found that in the case of our own particular machine, if we used an inductance of 80 to 100 millihenries, that we did not get as deep penetration or fusion of the deposited metal as we could get with the smaller reactor developing approximately 3½ millihenries. Therefore, we used the smaller reactor as I have indicated.

J. C. Lincoln: Mr. Candy states that an excessively large reactance gives poor penetration, and implied that the reason for the poor penetration is due to the presence of reactors. If that is true it is very interesting, but it is not probably due to the fact that with a large reactance it is possible to hold an arc, a longer arc, and poor penetration is not due to the increased reactance but due to the increased length of arc, which the larger reactor makes possible? If that is true, there is no mystery about it But if reactance really makes poorer penetration, I would like to know something more about it.

A. M. Candy: We made our observations with the same length of arc in every case, practically 18 to 20 volts. I have asked a great many designers the very question Mr. Lincoln has asked me, but I have not received a satisfactory answer to that question myself. The only way I can explain it is with the reactance in the circuit the arc plays around a greater surface, and does not seem to hold on to a particular point.

J. C. Lincoln: Is the length of the arc determined by the operator?

A. M. Candy: It was determined by a voltmeter across the arc.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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ARC WELDING MACHINERY OF THE U. S. LIGHT & HEAT CORPORATION

BY W. A. TURBAYNE

Electrical Engineer, U. S. Light & Heat Corporation

THE U. S. Light & Heat Corporation produces only the single-circuit or single-operator type of arc welding machine, wherein control of the current accompanying inverse changes in arc voltage and length is accomplished by inherent action of the machine windings, unaided by interposed circuit resistors.

In developing this equipment it was sought to reduce the machine and circuit losses to the lowest possible value compatible with practical operation, to obtain the quickest possible voltage response by the generator, or current converter, at the time of striking and manipulating the arc, and to insure the maintenance of a steady and stable arc, with a minimum amount of circuit reactance. It was also considered desirable to produce such a machine characteristic that the product of voltage and current would remain reasonably constant at the value best suited to the operation, despite the unavoidable variations in arc length incident to manipulation of the electrode in the hands of the operator. Provisions were incorporated, however, whereby by a simple adjustment, the slope of the volt-ampere characteristic could be altered in such a manner that practically a constant current would be maintained in the welding circuit, regardless of appreciable variations in arc length and voltage, this characteristic being favored by many operators.

Two distinct classes of arc welding machine were developed—one a *direct-current generator* adapted to be separately driven by any form of motor, determined by the available source of power supply, and the other a *direct-current converter* which is self driving when supplied from a direct-current source having from 100 to 125 volts pressure.

The *generator* is a self-exciting, cumulative compound-wound machine, and current regulation is effected solely by the reaction of the armature current upon the field flux.

The relations between pole span and pole pitch and between the shunt and series field ampere turns are so chosen, with respect to the armature structure that, while a pronounced drooping voltage characteristic results, perfect circuit stability obtains at any value of current within the working range of the machine. A definite open-circuit voltage setting is inconsequential as the machine has a pronounced series characteristic, and a moderate variation in the shunt field current does not materially affect the current setting although such adjustment of the shunt field current affords a ready means of modifying the current values determined more or less broadly by means of a current-adjusting switch provided for the purpose.

To insure quick response to circuit conditions, the magnetic structure throughout is laminated, and the sensibility of the machine is decidedly enhanced by the inductive or transformer action resulting from the close association of the shunt and series field windings.

For operation on electric supply circuits, a compact motor-generator is furnished, induction motors of suitable voltage, phase and frequency being provided for use on a-c. circuits, and direct-current motors of appropriate voltage for d-c. supply.

The motors are direct connected to the generator, the rotor being pressed on the elongated armature shaft thereby eliminating the necessity for bolted couplings.

The complete running gear is supported on annular ball bearings and before assembly is thoroughly checked as to static and running balance.

Generators are also provided, direct connected to gasoline engines, or supplied merely with pulleys, enabling them to be driven from any available or suitable source of power.

For locations where 100 to 125-volt d-c. supply is available the *direct-current converter* is recommended.

This machine has but a single magnet frame, armature winding and commutator, and combines within itself the functions of a shunt-wound motor and a variable-voltage generator. In size, weight and appearance it corresponds closely to the generator previously described. The complete field structure also is laminated.

The field system comprises two main poles supporting the shunt field winding and two smaller auxiliary poles, spaced at right angles thereto, which carry the regulating windings. These latter comprise a shunt winding connected across the supply circuit and an opposing series winding included in the welding circuit. A field rheostat is provided in the shunt circuit for purposes of adjustment.

The armature conductors are placed in slots located substantially 120 degrees apart around the periphery and are connected to the commutator to form an ordinary two-path winding with symmetrical end connections.

Four sets of brushes engage the commutator. Two sets, in line with the main poles and diametrically, opposed, admit current from the source to drive the machine. Two other sets, one each displaced 60 degrees from a main brush, together constitute one terminal of the welding circuit, the other terminal being the main brush midway between them.

Current regulation is effected by varying the flux distribution under the poles and, therefore, the voltage around the commutator by inherent action of the windings on the auxiliary poles. These auxiliary poles are of like polarity at any instant and, depending on the degree of their excitation, act to increase the flux density in one of the main poles while decreasing it in the other.

With the converter running idly as a motor, the open-circuit voltage effective on the secondary or working brushes is brought to the desired value by means of a field rheostat, provided for controlling the auxiliary shunt-field current. Upon closing the welding circuit, however, current traverses the auxiliary series opposing winding, resulting in a decrease or even reversal of the flux in the auxiliary poles, a simultaneous lowering of the flux in one of the main poles and a corresponding increase in the other. This transfer of flux causes an immediate drop in voltage on the working brushes and corresponding increase across the remainder.

Two-thirds of the armature conductors at any instant carry only the input current, the other third carrying the difference between the output, or working current, and the input current. The resultant distribution of current, therefore, is such that conductors of comparatively small section may be employed as compared with those necessary in a generator of equivalent capacity.

During operation the current in the welding circuit comes, partly from the supply mains, this being the input current which drives the machine, the balance being contributed by the machine through generator action.

As a welder the direct-current converter shows an efficiency of 65 to 70 per cent. With 200 amperes and 20 volts at the welding arc the current demand from a 120-volt source is substantially 25 per cent of the value of the welding current or 50 amperes.

Differing from a generator, in which the flux in the complete structure is varied to produce regulation, the converter flux is simply transferred from one portion to another. Consequently, a notable freedom from lag exists.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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RECENT DEVELOPMENTS IN ELECTRO-PERCUSSIVE WELDING

BY DOUGLAS F. MINER
Westinghouse Electric & Mfg. Co.

CONDENSER TYPE OF WELDER

FOLLOWING the original experiments of Mr. L. W. Chubb in 1905, machines were built for welding wires by the electro-percussive process and these have been used in a limited way for lamp leads, copper terminals on aluminum coils and similar applications.

These machines utilize the discharge of an electrolytic condenser to fuse the wires substantially simultaneous with a percussive engagement. With this equipment, perfect welds are made between like and unlike metals, even those of widely different physical characteristics, but the field of application has been narrow because of the state of development of condensers. A condenser of sufficient capacity to provide energy for welding large sections would be prohibitive in size.

ELECTROMAGNETIC TYPE OF WELDER

Within the last year, equipment has been developed which successfully welds stock up to ½ in. (1.27 cm.) diameter, and large sizes will apparently offer no difficulty. The same principles are used in this device, but stored electromagnetic energy replaces electrostatic energy. Establishment of a strong direct-current field in a reactance coil with a primary and secondary winding is followed by rupture of the primary current with the secondary circuit closed. Transfer of energy of the collapsing field to the secondary results and a subsequent separation of electrodes in this circuit establishes an intense arc. When the surfaces of the electrodes (pieces to be welded) are sufficiently melted, a hammer forges the parts together.

The total time of the above operations is of the order of 1/10 second. Thus the weld can be said to be practically instantaneous. In an experimental welder, these events were secured in proper sequence by apparatus represented diagrammatically in Fig. 2. A cycle of operations is somewhat as follows: The primary is energized by closing switch *D* which also raises the lower electrode C_2 into contact with C_1, by means of magnet *M*. Then operating switch *O S* is opened and magnet *W R* de-energized. The hammer falls and in its travel knocks out switch *T S*. This allows primary switch *P S* to open, kicking out trip *S T*, which then opens the secondary circuit by allowing the lower electrode to drop to the anvil. An arc is thereby established until the hammer strikes W_2, forging the parts together.

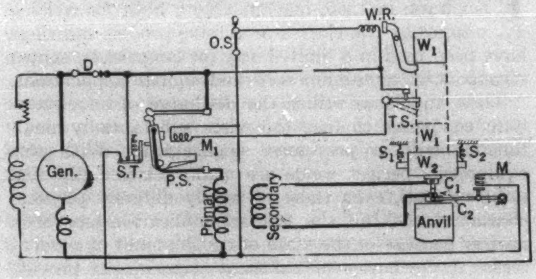

FIG. 2

A record of events is given by the oscillogram Fig. 5. This indicates secondary quantities for a weld of 3/8 in., diameter stock and shows the following: Maximum secondary current 2600 amperes, arc voltage 30 volts, maximum watts 60,000, average watts 29,600, total time 0.094 seconds, energy 2780 watt-seconds or 0.00077 kw-hr.

A few sample welds are shown in Fig. 9, illustrating a variety of work.

20. ¼ in. copper rod welded to steel disk.
21. ¼ in. steel rod welded to steel disk and tested in bending without failure.

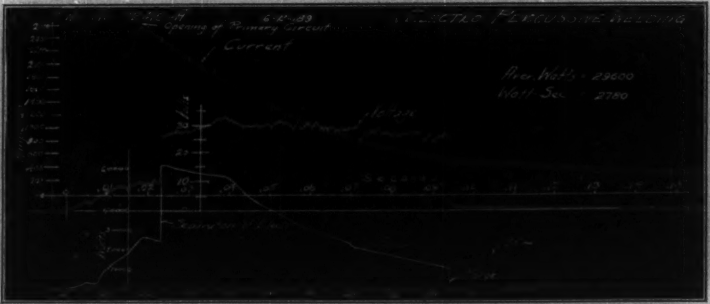

Fig. 5—Oscillogram of Electro-Percussive Weld Between $\frac{3}{8}$-in. Cold Rolled Steel and $\frac{3}{8}$-in. Copper

22. 5/16 in. copper-copper weld bent sharply without failure.
23. Nickel-steel valve head welded to cold rolled steel stem. Failed in bending outside weld.
24. T-weld of cold rolled steel.

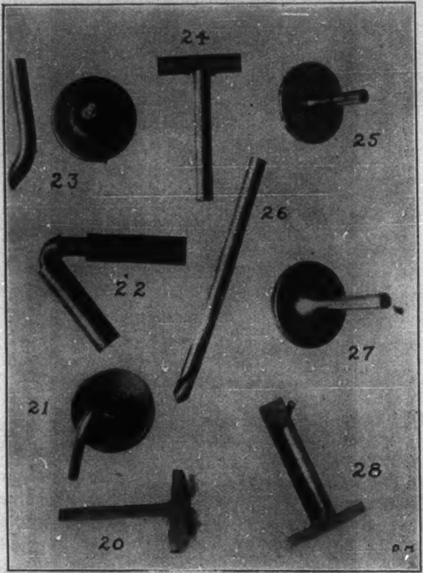

FIG. 9—EXAMPLES OF ELECTRO-PERCUSSIVE WELDING

25. Cold rolled steel rod and disk polished.
26. 5/16 in. drill lengthened with low carbon stock.
27. Nickel steel head to C. R. S. valve stem.
28. ½ in. hollow steel rod welded to steel plate.

Tests have shown a high strength of weld—96,000 lb. per sq. in. for steel-steel and 40,000 lb. per sq. in. for copper steel. Microphotographs confirm the quality of the weld and indicate an inter-penetration of metals, without visible alloying, and a thorough fusion without oxidation.

ADVANTAGES OF PROCESS

Some of the points of superiority of electro-percussive welding are:

(a) *Power Saved.* Power used in the weld is about 1/16 that required in butt-welding.*

(b) *Time Saved.* The operation is so rapid that the time of weld is practically negligible. Speed of production will, therefore, depend chiefly on time of handling pieces, and large output can be obtained by design of semi-automatic apparatus.

(c) *Welds of Unequal Sections.* Necessary energy is concentrated in a very small amount of material and not dissipated in heating the whole stock. Consequently welds of unequal section are possible without preparation of surfaces or preheating of large piece.

(d) *Welds of Unlike Metals,* with widely different physical characteristics are made possible.

(e) *Welds without Change of Condition.* Tempering or other treatments are not destroyed because heat is localized and rapid.

(f) *Welds Uniform.* After proper settings, unskilled labor can produce uniformly perfect welds.

(g) *Finishing* is unnecessary in some products and inexpensive in all because of small fin or flash.

Extension of the original process has met with gratifying success and when details of design are perfected, a wide field of application for electro-percussive welding is expected.

Credit is due C. F. Wagner and E. L. Hillstrom for aid in development embodying Mr. Chubbs adaptation of electromagnetic energy to his original process.

**Electric Welding;* D. Hamilton and E. Oberg, p. 49.

DISCUSSION ON "RECENT DEVELOPMENTS IN ELECTRO-PERCUSSIVE WELDING" (MINER), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

J. C. Lincoln: There is one question I would like to ask, and that is—For the oscillogram, in one case I remember, the voltage across the arc is given at about 30 volts, at the instant, I presume, just before the blow is struck and the forge weld made. I understand that the weld was steel to steel and the distance between the electrodes not over $^1/_{16}$ in. or possibly not over $^1/_{32}$ in. The voltage for the steel to steel, with the ordinary metallic arc, is in the order of 12 to 13 volts. How does it happen such a high voltage as 30 volts obtains with this excessively short arc?

D. F. Miner: I will answer Mr. Lincoln's question in this way—if I said it was steel to steel I made a mistake. It was a copper to steel weld which the oscillogram showed.

J. C. Lincoln: In the case of a steel to steel weld, what voltage does occur?

D. F. Miner: I have no data on the steel to steel weld, and so I could not say. I imagine it would be around the figures you mention, because the gap is approximately 1/10 in.

J. C. Lincoln: According to that the weld with steel to steel would be less than copper to steel?

D. F. Miner: That would be true, but the result of experiments seem to show that it requires a faster rate of dissipation of energy for a copper-steel weld than a steel-steel weld.

J. C. Lincoln: In the case of copper to steel weld did you notice any difference in the quality of the weld, as to whether the copper is made positive or negative with reference to the steel?

D. F. Miner: We have tried reversing the polarity of the electrodes and cannot detect any difference. Perhaps there may be a difference if we analyze it closely enough, but we have not been able to discern any difference.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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ELECTRIC ARC WELDING APPARATUS

BY ROBERT E. KINKEAD
Lincoln Electric Co.

THE characteristic of the arc obtained upon an electric arc welding machine determines to a large extent the utility of that particular machine for electric arc welding purposes. There has been considerable discussion concerning the relative merits of the "long arc" and the "short arc" machines.

Any arc which is sufficiently flexible to permit welding on a commercial scale can be held at such a length by the operator that poor welding will be the result. The 35-volt type of equipment with carbon pile rheostat, the self-excited individual units having a differential series field, and the alternating-current equipment have what is called a "short arc." It is a fact, however, that a sufficiently long arc can be held on any of these machines to give very poor welding. The term "short arc" is erroneously applied to this equipment for the reason that if the arc is suddenly lengthened it will be broken. On the other hand, if the arc is slowly lengthened these machines can furnish the increased voltage required to maintain the arc.

The variable-voltage type of equipment, having separately excited main field, differential series and stablizer is sometimes referred to as a long arc machine. In the case of this type of equipment, the arc will not be broken by suddenly increasing its length for the reason that the induced voltage from the stabilizer will keep the arc in operation until the increased voltage from the welding generator is obtained.

From the above analysis it is evident that the controversy over the "long arc" or the "short arc" as applied to welding machines is in fact a controversy over the relative ease of operation and speed of operation of the several types of welding equipment.

No entirely satisfactory method of rating electric arc welding machines has yet been evolved. In general there are two methods of rating welding machines at the present time. The older method of the two is to rate the machine in amperes the machine will deliver as continuously as is required for welding service without destructive heating of the machine. The later method is to rate the machine in amperes it will deliver for welding service for from thirty to sixty minutes.

It has been suggested that the welding machine be rated in kilowatt output at the arc. This plan does not overcome the difficulty, for the reason that it does not indicate how much work can be done with the equipment per unit time, nor does it indicate whether the machine is rated for metal electrode service or carbon electrode service or both.

It has been proposed that the welding machine be rated in heat units which the machine will make available for welding purposes. Since the welding machine is merely a device for converting electrical energy into heat, this plan is certainly logical.

Another proposal is that the welding machine be rated in number of pounds of metal deposited per hour. This would seem a logical basis upon which to rate a machine for the reason that practically all welding done with the electric arc process consists of merely depositing metal. A rating based on pounds of metal deposited per hour would be affected by the characteristic of the arc which would be desirable since it would give the user an idea of how much work he could expect from the machine.

From the above it is evident there are some serious difficulties in the way of attempting to rate a welding machine on the basis of how much work it will do, although it is certainly not beyond the range of possibility that this desirable end may ultimately be reached. It cannot be questioned, however, but that a distinct advantage would be gained by the users of welding equipment if the manufacturers of it would adopt some uniform method of rating the equipment.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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A-C. TRANSFORMERS FOR ARC WELDING

BY C. J. HOLSLAG
Chief Engineer, Electric Arc Cutting and Welding Co.

SPEAKING of the metallic arc; after careful compilation of a large number of voltage readings across the arc, d-c. and a-c., together with the amount of current through it, and the physical characteristics of the arc, the writer was forced to the conclusion that the current does not affect the arc voltage enough to influence an ordinary meter. The only factors which affect the voltage of the arc are distance, *i. e.*, length of arc, type of electrode and coating, if any.

The arc voltage consists of a foundation voltage together with an $I R$ drop and a guardian or instantaneous voltage. While not stopping here to discuss whether this foundation voltage is a counter e.m.f. or an $I R$ drop, it is a fact that there is a minimum voltage below which a metallic arc cannot be held. This minimum voltage lies between 10 and 11 volts. This is what is referred to as the foundation voltage above. There is added to this minimum voltage a constant drop which in the opinion of the writer is the resistance of the molten metal and occluded gases. This is not a pure $I R$ drop as it does not seem to vary at different current values. While the unit resistance probably varies, heavier currents mean larger cross-section of arc path and hence lower resistance. It appears to be a constant drop changed only by the characteristics of the electrode or its covering. This drop varies from one or two volts in bare wire at ordinary welding currents, to from fifteen to twenty volts with heavily coated electrodes. As this drop is constant for any given electrode, and the foundation voltage is constant in any apparatus designed to supply the characteristics of the metallic arc, the two can be treated as one.

The difference between most old-time apparatus and the latest developments is that a needlessly high voltage is supplied all the time to be used less than one per cent of the time, while modern a-c. apparatus supplies this voltage only when needed and in amounts as needed.

It is now practically admitted that the short arc is not only desirable, but necessary for good work. With a short arc there is less chance of the metal being oxidized, or what may be worse, nitrogenized. The metallurgists, as near as I can find out, are still at sea as to whether bad physical characteristics when present in welds are due to oxides, nitrides, or cyanides. The short arc, however, presents the minimum chance of any of the undesirable compounds being formed because of the less time and less area of contact with the air.

The condition to be satisfactory for good arc welding is that the metal of the electrode passes into the crater made fluid on the work by the arc. The short arc causes a maximum crater for any given number of watts, or rate of heat transfer, and hence there is much less chance that the metal of the electrode will pass on to the work at any place but into the crater, not only because the crater is the maximum, but because of the electrode being nearest to it so that there is no other place for the vapor globules to enter. There are other advantages of the short arc, which are known to all welders, one of them being control of the arc. If the arc is kept short within reasonable limits, it can be controlled.

One of the first conditions for good welding is control of the arc length. This can be accomplished, first, by lowering the impressed voltage until it is hard to hold the arc; second, by apparatus, such as relays across the arc, which will either shunt or open the arc at any predetermined voltage value. The first of these methods sacrifices penetration. It is the writer's opinion that this higher voltage, even where it is not of sufficient duration to register determines the penetration. The second method of limiting the voltage across the d-c. arc is the addition of rapidly moving parts such as relays, etc., which are entirely workable in a laboratory, and even there, if timed for one sequence, they may not be correct for some other. Also, the arc voltage must be made up differently for various electrodes and conditions of welding.

While an arc can be limited by either of the foregoing methods, there is a third and better method, and that is to so arrange this guardian voltage that it not only varies in value but swings in and out of phase with the main voltage even to the extent of lowering it when necessary.

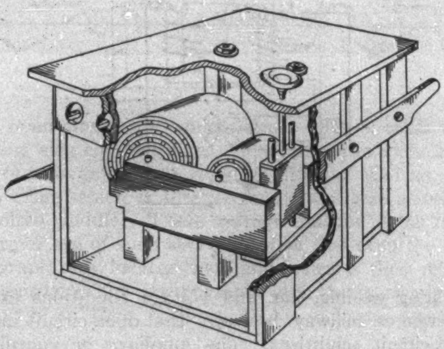

Fig. 1

Different definite arc lengths are necessary for different operations and in some cases variations are needed in different parts of the same job. The maximum length necessary must be available, leaving to the judgment of the operator the control of arc length within this limitation.

Transformer type of welding apparatus, as shown in Figs. 1 and 2 offers a solution of all of these length-of arc-problems. The foundation voltage can be varied by taps on the primary, by taps on the secondary, or by diverted or controlled flux. The guardian voltage can be added to the other separately and so arranged to swing in and out of phase as described, besides changing in value, and even the *I R* or resistance voltage which must be different for various types of electrodes, can be arranged by taps to have the proper value.

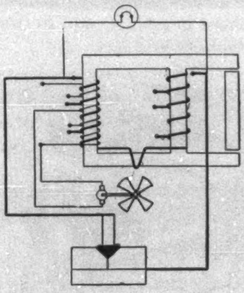

FIG. 2

During welding, flux and voltage conditions exist somewheres midway between the open-circuit and short-circuit conditions. The auxiliary or guardian voltage always standing ready to rise to its full value on open circuit or tendency for the arc to go out. As the current must go through zero twice per cycle the guardian voltage is kept in reserve to be ready to arrest any attempt of the arc to go out due to imperfections in running of electrode or work, operator or other feeding mechanism.

Figs. 6, 7, and 8 show what occurs, during welding, to the main and auxiliary turns with both coated, covered and bare wire electrodes. *X*–10 Fig. 6 shows the effect of the auxiliary coil to hold up successfully the secondary or arc voltage as this voltage went through zero. Fig. 6, *X*–5 shows a partial short circuit with voltage zero as does *X*–1, *X*–2, and *X*–3, Fig. 7. *X*–6 and *X*–7, also *X*–10, Fig 6, show by their perpendicular lines that the time that voltage is zero is of very short duration as is necessary to hold an arc in the vapor, gas and liquid path of a bare wire electrode. X–11, and X–12 are similar actions to that of X–10, Fig. 6. Fig. 6 X–15 shows the secondary voltage rising to meet a tendency to open-circuit. Each move shows these corrective kicks both upon tendency to open circuit and short circuit. Fig. 8, taken when using slag covered wires, shows the extra kicks necessary to overcome the resistive effects of the heavy slag covering which is forever trying to freeze across the molten puddle. These voltage changes in the transformer are of such short duration that even the oscillograph only locates them and indicates their relative intensity. The oscillograph also shows clearly the change in time position of the different secondary voltages under various loads showing the auxiliary voltage swinging away from the main voltage approximately proportional to load and the steadying turns following but at a more sober rate. Fig. 9, shows this action vectorially and as the current must necessarily pass through zero at a rate of twice per cycle the auxiliary voltage must make these rapid changes in

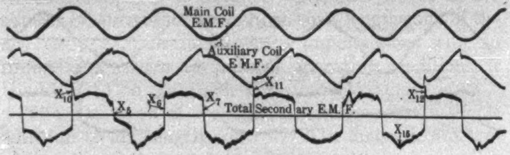

FIG. 6

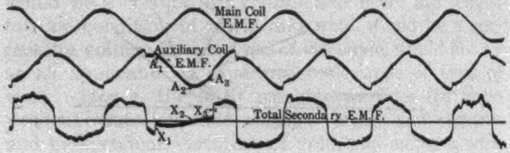

FIG. 7

time position at the same rate beside a few other corrective changes in between, if the arc tends to go out or to short-circuit meanwhile.

The same action which gives the arc holding effect in this type of welding apparatus also holds constant the rate of heat supplied to the metal arc. If a constant current is maintained together with a very short arc, *i. e.*, practically no variation in length of arc allowed, then of course, practically a constant rate of heat results. The failing case of the constant-current-limit-voltage machine is lack of voltage when needed, first, to get by any imperfections in the work or electrode, variable gaps in the parts to be joined, and second, lack of penetration, *i. e.*, with the minimum volts impressed that will hold the arc, the minimum penetration is obtained. In other words, penetration is sacrificed. This results in many disadvantages. With a-c. a definite length of arc can be maintained without loss of penetration due to the holding volts swinging in and out of phase with the main or foundation arc voltage, and the guardian voltage being always ready practically instantly to rise to its full value when needed. This provides a means of overcoming

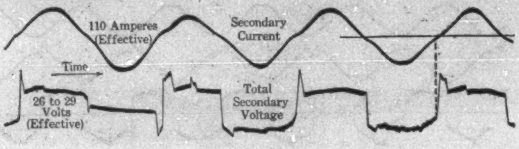

FIG. 8

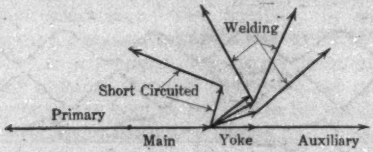

FIG. 9

imperfections in work or electrode, dirt, oil slag, etc., and allows enough flexibility and persistency to bridge over variable gaps and to allow for variations in the operator's handling of the electrode. It is important, moreover, while allowing enough flexibility in length of arc to take care of the variations that are ordinarily encountered, yet a definite maximum length of arc should be provided, and short time lengthening of the arc to overcome momentary imperfections with an immediate return to normal operating conditions, is the ideal to be obtained.

The a-c. type of welding apparatus offers a simple solution to arc length control. The magnetic flux providing the voltage return of the auxiliary or guardian coil volts can be by-passed by any simple means, electrically or mechanically, so that the amount of this guardian voltage can be in the hands of the foreman welder. Furthermore, as provided on a certain type of apparatus, the means for this adjustment is removable so as not to be in the hands of the operator. For the unavoidable variations in arc length, all below a length above which welding is poor, my idea is that the heat should be kept constant within these limits, with variations compensating for radiation losses of increased arc length. Furthermore, what would be a correct arc voltage and hence arc length for one condition of work, and what would be a correct arc voltage for one type of electrode, would not be at all applicable to some other electrode or type of work. Hence, the ideal machine provides for independent variation, not only of voltage and current, and independent variation of volts and amperes with respect to rate of heat, but also provides independent variation of the factors making up the arc voltage. Such variation of all of the factors named, holding any one constant, is entirely possible by the various taps on the main and auxiliary windings with fine adjustments by use of flux controller together with a few taps on the primary. It is true that an arc can be held on a-c. with a resistance or a reactance or both, but these makeshifts provide the proper welding characteristic about as well on a-c. as a water barrel resistance does on d-c.

DISCUSSION ON "ALTERNATING-CURRENT TRANSFORMERS FOR ARC WELDING" (HOLSLAG), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

William O. Noble: In Mr. Holslag's paper, just under Fig. 9, he makes a statement which implies that the open circuit or striking voltage on a welding system is affected by penetration. That is just as mysterious to me as reactance is to Mr. Unland. Penetration is simple depth of fusion in the work and depends on two things—the amount of current and the rate of speed under which the work is moving under the electrode or the electrode moving over the work. With a given current you can increase penetration by an increase in the rate of travel, and with a given rate of travel you can increase the penetration by increasing the current.

It is a well known fact in any direct-current arc the energy loss at the positive terminal is approximately twice the energy loss at the negative terminal, in d-c. welding the electrode is negative and the work is positive. I will represent the heat on the work by drawing the following diagram:

	Electrode	Work	Total
D. C.	1	2	3
A. C.	1.5	1.5	3
For same heating in work A. C	2	2	4

Describing this diagram, if we represent the heat at the negative terminal of the d-c. arc as 1, and the heat on the positive terminal as 2, the total is 3. In the a-c. arc, with the same total of 3, the average heat on both of the terminals must necessarily be the same on account of the rapid alternation of the current; that is, we have one and one-half on the electrode and one and one-half on the work. This means that with 75 per cent of the heating on the work we are depositing 50 per cent more electrode material. In order to get the same heating on work with alternating current that you get with a given direct current, it is necessary to increase these quantities 33⅓ per cent, and that brings the upper two to 2 and this to 4. This agrees with Mr. Candy's statement that 170 amperes alternating current is about equal to heating effect on the work to 130 amperes direct current.

This difference is responsible for the characteristic appearance of all a-c. welds we have seen. With the same heating on the work, and the same penetration, the a-c. arc deposits twice as much electrode material, giving the weld a different appearance. A large percentage of this excess metal is simply flowed on and not fused to the work.

J. C. Lincoln: Mr. Holslag states in the paper that the voltage required for the operation of the coated electrodes is higher than the voltage required in the operation of the bare electrodes. I have not had much experience in the use of coated electrodes, but I know that is approximately true in the operation of the quasi-arc. The question I want to ask is—Why will not a given current deposit more metal in a given time with a covered electrode than with a bare electrode? Increasing the voltage across the arc certainly increases the heat delivered to the work with a given amount of current, and on that basis it ought to increase the speed of the weld, and therefore it looks as if it should be a proper conclusion that coated electrodes ought to work faster with a given current, melt more metal in a given time with a given current with coated electrodes than bare electrodes.

C. J. Holslag: I would say that the trouble in fitting together the remarks of Mr. Noble, Mr. Candy, Mr. Lincoln and those of myself, is that the assumption is pretty generally made that current is the whole factor in melting the electrode in arc welding. I do not see how any one can get out of the broad straight way that watts determine the rate of heat in the arc and hence the amount of metal that is deposited. Of course, there are certain combinations of voltages and amperes that give best results for any set of conditions, but the watts determine the rate of heat, and watts generally consist of volts and amperes, being direct product except on alternating current where one must allow for a 90 per cent power factor of the metallic arc.

The theoretical answer to Mr. Noble's table and argument, and the practical answer as well is that although I wish that his table was true that alternating current could work twice as fast as direct current with the electrode negative, yet arc welding is generally a small electrode being welded to a large one, in which case the heat evidently cannot be evenly divided. For instance, a 5/32 in. electrode which is welded to a half inch plate, although on alternating current the heat is supposed to be evenly divided, yet because of the different sizes of the electrode, and because the heat which was in the depositing electrode at any instant is transferred to the plate, the heat transfer is direct from the electrode to the plate. Although the current and voltage may be alternating, it has been well known for many years that with two carbon electrodes of unequal size the heat does not divide equally.

The answer as to whether a-c. arc welding is equally as useful as d-c. was given in a test made by the Shipping Board in 1918, where nearly every electrical welding company and system in the United States was represented, and there were 20 using and promoting direct current to one using alternating current, and the alternating current made welds just as good, if not slightly better, than the best of the d-c. welds, and if there was anything to Mr. Noble's argument that part of the deposited metal was not fused, these results could not have been obtained.

The entire answer is that watts determine the heat and the heat transfer is direct from the electrode to the plate

J. C. Lincoln: I will ask my question over again. With a given current, say of 200 amperes, if bare wire takes 11, 12 or 13 volts, and the covered electrode takes an average of 20 volts, as stated in the paper, then there are more watts used in the arc for the covered electrode than the bare electrode. Here we have a case of covered electrodes using 75 per cent more watts than in the case of the bare electrode. Is it possible to deposit the metal faster with the covered electrode than with a bare electrode?

C. J. Holslag In answer specifically to Mr. Lincoln, the coating of the electrodes certainly increases the speed of deposition providing that the same current is held and providing that the current density of the electrode will allow of this greater rate of heat. The voltage across a coated electrode arc varies with the thickness of the coating. The well known quasi-arc is perhaps the thickest and gives the highest voltage, and with the same current would deposit at a greater speed in proportion to the greater voltage, except that the current has to be reduced on an electrode with this thickness of coating because of the density becoming too high and the radiation of the electrode being restricted.

The best result in speed and quality of deposition lies, in my opinion, in the thickness of coating which gives about 25 volts across the arc.

B. W. David (communicated after adjournment): In the papers on this subject the authors have set up various generators characteristics, which they assume to be necessary for the successful welding generator. It is interesting to note that those particular characteristics on which the merits of some particular machine are claimed, are the ones on which the greatest controversy of opinion is involved among their champions. In this manner our attention may be easily diverted from the real measure of the successful welding generator.

Without question such a machine must be easy to operate, both in itself, and at the welding electrode. Without question it must produce a good weld. Granting that all machines under discussion meet the latter condition, the real basis of merit is a practical one based on simplicity and reliability, both in manufacture and daily use. That equipment requiring the technical mind to trace the mazes of its circuits, and to comprehend its multitude of self, separate and combined and series excited fields, its nicely adjusted resistors, split poles, cross fluxes, and what not, is foredoomed to a troubled existence.

The split-pole self-excited generator was developed as a separately driven outfit, as a step-down a-c. converter, and also as an a-c. rotary converter, all used without balance resistance in the welding circuit. A large number of these machines are still in operation. This type of generator, while fairly simple in theory, is very undesirable from a manufacturers view-point. Maximum economy of space and material is difficult to obtain because of the increased number of poles. In simple units the necessity of using a two-pole armature winding is an obvious disadvantage. Furthermore the conditions for commutation are not ideal and require a very careful interpole design.

In 1914 the writer, himself, evolved the triple-field generator making use of the separately excited, self-excited, and the bucking series field, giving a constant current range throughout the unsaturated portion of the characteristic. Some fifteen single operator welding machines of this design, ranging fron 150 up to 600 amperes were constructed and are today still in operation. The added complication of field inter-connection is of doubtful worth, certainly unnecessary, and merely changes the design.

The active manufacture of the various types mentioned, has been set aside in favor of the plain, simple, and easily understood machine having a separately excited and bucking series field on the same pole. Technicalities, cubic differential equations, constant current, constant heat, or what not; to the contrary it has been proven by years of pioneering in the field of single operator machines with this simple apparatus that it does exactly what its operator requires of it, that it is as efficient, and is probably even easier to operate, and further because of its extreme simplicity can be kept in better working condition than any other of the machines of more complex design, which are now being offered for consideration.

It is, of course, certain that the highest development has not been reached as yet, but in conclusion the writer would urge that the only justification for a welding generator of complex design would be increased efficiency better use of materials, increased speed or ease of operation, or a better weld.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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POWER FACTOR IN POLYPHASE CIRCUITS

Preliminary Report of Special Joint Committee

THE subject of power factor in polyphase circuits has been the center of increasing discussion in recent years. No agreement has yet been reached upon a definition of the term as applied to polyphase circuits, nor even upon the underlying purpose which a definition should serve to express. In the absence of a practical commercial incentive to a universally accepted understanding as to the purpose and use of the term, little progress has been made toward such an understanding.

There has been no practical incentive for the reason that, until recent years, most polyphase loads were approximately balanced, while the differences between various possible definitions of power factor become of importance only in unbalanced loads, so that no refinement of definition has been needed. At present, however, there are increasingly important developments in types of industrial power loads which are attended by unbalanced conditions between the phases, unbalanced as to amount of loads and as to phase relations between current and voltage. In such cases the numerical value of power factor may vary widely with different definitions.

The increasing commercial importance of this character of load and the growing tendency toward such refinements in power contracts and rates as will reflect accurately the various elements entering into cost of service, have combined to render this power factor problem a matter of immediate and urgent practical importance.

In recognition of the importance of this need, the American Institute of Electrical Engineers, acting through its Standards Committee, and the National Electric Light Association, acting through its Technical Section, have united in the formation of a Committee to be known as the Special Joint Committee on Determination of Power Factor in Polyphase Circuits.

It was decided that this joint Committee should place the results of its labor before the parent organizations by which it had been appointed, namely, the Standards Committee of the A. I. E. E. and the Technical Section of the N. E. L. A. in a form which should indicate the conclusion already reached by the Committee but which would permit of further consideration and discussion by the two parent bodies before a definite solution is reached.

Two definitions covering two different forms of power factor in polyphase circuits were arrived at, together with some suggestions as to proper qualifying terms to apply to each definition. These definitions are as follows:

Definition 1. Power factor in a polyphase circuit is the ratio of the total watts to the (arithmetical) sum of the volt-amperes in the several phases, each measured to a non-inductive neutral point. This definition may be otherwise expressed as the weighted mean of the individual power factor in the phases (weighted according to the volt-amperes in each phase.)

Definition 2. Power factor in a polyphase circuit is the ratio of the total watts to the vector sum of the volt-amperes in the several phases.

A bibliography prepared by Dr. P. G. Agnew and Professor A. E. Kennelly is given as Appendix II to the report. Abstracts and translations of the work of the Italian, Gino Campos and of the German, Dr. F. Niethammer, have been prepared by Mr. W. H. Pratt and are attached to the report as Appendix III. These papers are of basic importance in considering the subject of polyphase power factor.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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POLYPHASE POWER FACTOR

BY F. C. HOLTZ

Chief Engineer Sangamo Electric Co.

IN a single-phase circuit the various factors relating to power and energy are very clearly defined. For example, power is defined as the rate of energy transfer; apparent power is defined as the product of the r. m. s. value of voltage across the circuit by the value of the current in the circuit.

An interesting development is to take the mathematical expressions which give proper interpretation to the single-phase problem and extend the investigation to include the three-phase problem.

In the single-phase problem we arrive at an expression for power in the form $P o = A + j B$ in which A is the true power and B is the reactive power or volts amperes. The apparent power is given by $\sqrt{A^2 + B^2}$ while the power factor is $\frac{A}{\sqrt{A^2 + B^2}}$ and the reactive factor is $\frac{B}{\sqrt{A^2 + B^2}}$.

Upon applying the same ideas to the investigation of the three-phase problem it develops that there is a one to one correspondence between the two. Power being expressed in the form $P_\phi = A_\phi + j B_\phi$ where A_ϕ is again the true power as measured by any of the well known methods and B_ϕ is the reactive power or volt amperes in the three-phase circuit. We also have that the apparent power is $\sqrt{A^2_\phi + B^2_\phi}$ while $\frac{A_\phi}{\sqrt{A^2_\phi + B^2_\phi}}$ and $\frac{B_\phi}{\sqrt{A^2_\phi + B^2_\phi}}$ is the three-phase power factor and reactive factor respectively.

It is shown that the power factor as defined by the above is in accord with the vector definition for power factor as outlined by the committee. The absurdities which result from using the arithmetic definition of power factor are shown.

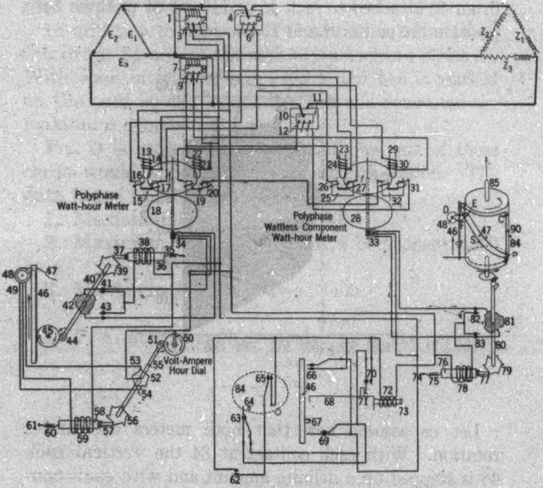
The problem of measuring the apparent energy in the three-phase circuit under the vector definition involves the necessity of integrating a quantity of the form $\sqrt{A^2_\phi + B^2_\phi}\, d\, t$. An instrument to do this must therefore integrate the square root of the sum of two squares. Such an instrument may be quite easily constructed.

For example referring to Fig. 7, we may assume two vector quantities in the same plane starting at a given instant t_o from some point O, one moving in the direction of $O\,Q$ and with a velocity which is a function of the time T, the other moving at an angle from $O\,Q$ and in the direction $O\,P$ with a velocity which is also a function of the time. At a time $\delta\, t$ after starting we have the two values $O\,P$ and $O\,Q$ and the difference $Q\,P$ which represents the approximate integral of the two vectors over the time $\delta\, t$. Suppose further that we construct two mechanically moving rods arranged at the proper angle to each other. To the point P is attached a flexible cord which passes over the point Q to a small drum located somewhere on the rod $O\,Q$ or its equivalent. Attached to the small drum is counter or other device which records accurately the length of $Q\,P$ removed from the drum and through auxiliary devices the points P and Q are returned to zero at definite intervals. It is so arranged that the mechanism of the drum shall record the sum of the lengths $Q\,P$.

$$\sum_1^n P\,Q = P_1\,Q_1 + P_2\,Q_2 + \ldots\ldots + P_n\,Q_n$$

where $P_1\,Q_1$ represents the value of $P\,Q$ during the first interval, $P_2\,Q_2$ represents that during the second interval of time etc. From this it is quite apparent that $\sum_1^n P\,Q$ will represent as close an approximation as is desired to the true integral of the two vector quantities. This can be regulated by regulating the time interval of reset dt.

The integration of the volt-amperes in an alternating-current circuit is a specific case requiring the application of the above. In this case the vectors are located at right angles to each other and the mechanical device is operated directly from the contacts of two watthour meters.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Polyphase Watt-hour Meter
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Polyphase Wattless Component Watt-hour Meter
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Volt-Ampere Hour Dial
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Fig. 8—Diagram of Connections for Kv-a-hr. Meter

Fig. 8 illustrates the case of a three-phase three-wire installation.

Here the two meters are shown connected to the instrument transformers. One is so connected as to register the watthours while the other integrates the reactive component. The contact devices are represented by 34 and 35. To the right of the figure is shown the assembled mechanism, the other part of the drawing is used to represent the electrical circuits involved. Instead of using two rods operating at right angles to each other one is replaced by a rotatable cylinder in order to economize on space and increase the accuracy of the instrument. One of the moving elements consists of a drum 48 attached to a vertical rack 46 and so arranged that with each contact of the wattless component meter the drum is rotated through a definite angle. The small drum 48 carries a silkcord 47 which passes through an eye *F* and is attached to the drum 84 at *P*. It is so arranged that with the drum 48 attached to rack 46 and drum 84 in their zero position the points *P* and *F* coincide.

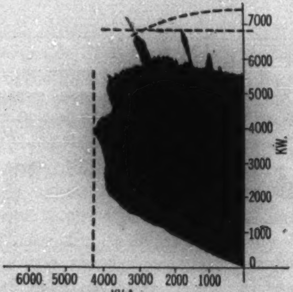

FIG. 9

Let us assume now that both meters are under rotation. With each contact at 34 the vertical rack 46 is stepped up a definite amount and with each contact at 35 the drum 84 is rotated through a definite angle. After a given time interval, say 10 minutes the positions of rack 46 and drum 84 are as shown. The cord removed from the drum 48 will be the length between *P* and *F*. Under conditions of design the rotation of 84 is proportional to $\int_{T_1}^{T_2} W\,dt$ and the height of *F* above its zero position is $\int_{T_1}^{T_2} R\,dt$ so that

$\overline{PF} = \sqrt{W^2 + R^2}\,d\,t$ which is the apparent energy during the time interval. At definite time intervals contact 62 is closed and through operation of solenoid 72 both rack 46 and drum 84 are returned to their zero position. This operation is continued so long as the system is in operation.

Drum 48 carries a three-lead contact 49 which is used to operate a distant dial mechanism so calibrated as to read directly in kv-a-hr.

In addition to the cord 47 a card 90 is mounted on the drum 84 and to the rack 46 is attached a stylus R'. With each interval of operation a new line is marked on the card 90 and length of which will determine the maximum demand in kv-a.

Fig. 9 is representative of about how one of these cards would appear after a month's operation. The data taken from this chart are as follows:

1. Maximum demand in kw., 6800.
2. Maximum demand in reactive component 4300 kv-a.
3. Maximum demand in kv-a., 7400.
4. P. F. at maximum demand $\frac{6700}{7400} = 90.6$ per cent.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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POWER FACTOR IMPROVEMENT DEPENDENT UPON ADEQUATE METERING

BY WILLIAM L. BROWN
Sales Service Manager, Electric Machinery Co., Minneapolis.

IT may be of interest at this time to consider what are the practical questions involved in the metering of loads of low power factor. What are the central stations actually doing?

During 1919 direct information was obtained from several hundred of the large power companies as to what was being done to regulate and improve power factor conditions on their lines. Many of these stated what means they were using to determine power factor of customer's loads and how they were applying the knowledge thus obtained. Companies who were incorporating power factor clauses in power contracts and meeting with a certain amount of success, related their experiences.

POWER FACTOR CLAUSES IN POWER CONTRACTS

To the question "Is there a power factor clause in your power contracts?" answers were as follows: Yes—73. No—113.

Many people will doubtless be surprised to know that there are so many large companies now imposing penalties on loads of low power factor. The list includes some of the largest central station companies on the continent, the generating capacities range in size from 5000 kw. to over 100,000 kw.

ARE POWER FACTOR CLAUSES ENFORCED?

Now the question naturally arises, are the power factor clauses being enforced? Up to the present time not many companies have been able to carry them out literally. Some of the Canadian companies have apparently been able to apply the rates automatically and base the energy charge of all customers (above 50 h. p.) on the average power factor maintained. Very few American companies have done this.

When asked the question, "Should a power factor clause be literally enforced?" 79 answered; 32 were of the opinion that the moral effect was the thing they were after; that under present conditions it was too difficult and expensive to enforce the clause literally.

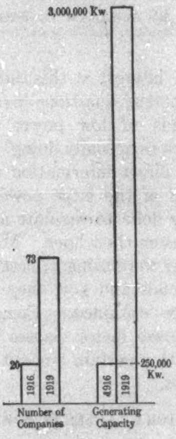

FIG. 1

Statements in this article are based on investigation of approximately 200 central station companies in U. S. and Canada.

The growth and importance of power factor regulation within the last three years are illustrated in Fig. 1.

In 1916 there were 20 companies with total generating capacity of 250,000 kw. imposing penalties for low power factor.

In 1919, 73 companies with total generating capacity of 3,000,000 kw. were imposing power factor penalties or making arrangements to do so.

We seem to have a variety of opinions as to how the power factor regulations should be carried out. It is worth noting that many of the companies who have carried no clause in the past are now of the opinion that such a clause should be incorporated and it should be literally enforced.

DO CENTRAL STATIONS HAVE ACCURATE DATA REGARDING CUSTOMER'S POWER FACTOR?

Out of forty companies that are now imposing a penalty for low power factor, thirty-two know the average power factor conditions of all their large customers—at least they claim to check up fairly close. The other eight admit they do not have accurate data to go by.

On further investigation of approximately 200 companies (with generating capacities ranging from 5000 kw. up) 127 had no practical knowledge of power factor maintained by any of their large customers while 63 claimed to keep a fairly accurate check.

NO STANDARD METHOD OF METERING POWER FACTOR

The thing which handicaps every company is the fact that there is no standardized method of power factor metering. It causes the greatest difficulty in basing rate schedules on power factor of loads. Many companies think they should keep power factor records on file especially in those cases where large numbers of motors are in operation with consequent variation of loads.

Over and over again the power companies send in this complaint, "We can find no practical method of metering power factor." One big western company writes, "We are strictly up against it and welcome suggestions which if followed would give us relief."

Several central station companies have designed graphic power factor meters and reactive kv-a. meters of their own which they claim to be using with considerable success. An Ohio central station company says, "We hope a commercial kv-a. meter will soon be on the market and believe we should sell kv-a. instead of kw."

When asked the question "Do you regard it as essential that there should be a definition of power factor of unbalanced phases?" 78 answered yes; 27 answered no.

Most companies would like to have continuous and complete records of power factor of the very large loads. For smaller customers they depend on periodic tests.

The places where power factor clauses are successfully enforced are the places where the metering question has been fairly well looked after. The great problem is to get an agreement with customers first as to what is average power factor, second relative time of maximum kv-a. referred to station demand.

It is interesting to note the various methods now being used by central stations to determine power factor of customers' loads:

First. Instantaneous readings of ammeter—voltmeter—wattmeter.

Second. Use of indicating wattmeter and three ammeters—for unbalanced loads.

Third. Use of indicating power factor meter either portable or switchboard type.

Fourth. Use of wattless component meters—registering kv-a. hours or reactive component in conjunction with watt-hour meters—for average power factor.

Fifth. Polyphase watt-hour meter specially connected to give the reactive instead of power component. Same principle as two-wattmeter method.

Sixth. Two single-phase watt-hour meters or wattmeters on balanced three-phase system.

Seventh. Power factor of maximum demand on three-phase circuit determined with one polyphase and two single-phase watt-hour demand meters. (Error to customer's advantage.)

Eighth. Periodic tests by the ratio of revolutions of polyphase watt-hour meter elements.

Ninth. Graphic power factor meters—portable and switchboard types.

In addition to the above, a number of companies are now using special instruments built according to their own design—such as special graphic meters which give a simultaneous graphic record on same chart of watts and volt amperes; also special makes of graphic kv-a. meters.

BENEFITS TO BE GAINED BY ACCURATE POWER METERING

The use of synchronous apparatus can be stimulated as in no other way. There can be no inducement made nor penalty imposed without it. Take as an example the experience of a West Virginia company which originally required only that the customer install synchronous motors or synchronous converters for the majority of his load and hoped thereby to maintain a comparitively high power factor at each customers installation. They found, however, during the years of 1917 and 1918 that due to careless operation of this apparatus they were not receiving the full benefit, and the company therefore put it up to the customer to maintain 95 per cent power factor or else be penalized in case his power factor fell below that. The previous method had been to give the customer a discount in case synchronous apparatus was installed.

It therefore became necessary for the company when putting the new tariff into effect to study carefully each customer's installations and also be well prepared to explain the reasons for putting these restrictions on his power factor.

If a power company can accurately meter power factor it will know pretty closely what low power factor is costing. On the peak load it means the installation of additional capacity all the way from the generator to the customer's meter. At off peak times it means carrying excessive generating capacity at the power plant and thereby wasting coal, it also means excessive copper losses in all electrical equipment.

It is the opinion of a number of authorities that power should be sold on a schedule made up of a rate for true energy used and another rate for the reactive taken. The first rate should cover cost of investment in steam equipment, cost of fuel, boiler maintenance, etc., while a secondary rate should take care of the costs in the electrical plant which are dependent upon power factor.

The one dominant desire among all power companies is to obtain sufficient synchronous equipment on their lines. The synchronous motor today can be applied to many loads, and customers are glad to install them on account of their high efficiency and reliability of operation. But there is absolutely no reason for a customer to buy anything but unity power factor synchronous motors unless his power company will make him a special inducement to run them at leading power factor. There is no cheaper or better way of getting power factor correction than by the use of over-excited, load-driving synchronous motors. It is a well-known fact that such motors can accomplish best results for power factor correction while carrying mechanical load. They can deliver 70 per cent of their full rating in mechanical kw. output while at the same time delivering 70 per cent of their full rating in creative kv-a. corrective.

It must be borne in mind that a synchronous motor running at unity power factor is a very efficient method of driving a load, while a synchronous motor driving the same load while operating at a low leading power factor, say 70 per cent, is not as efficient a piece of apparatus and it is more expensive in its first cost. The efficiency is from 2 per cent to 6 per cent lower in a motor of leading power factor. These motors are being used very extensively in isolated plants which generate their own power, and up until the present time, such plants have gone further in the matter of power factor correction by use of this type of motor than can be said of central station customers. The reason for this is obvious. The power companies have not made sufficient inducement nor gone sufficiently into the matter of educating their customers to the value of high power factor. The isolated plant, however, which controls both the generator end and the motor end of the business can readily see the advantage of installing oversize synchronous motors for power factor correction.

If the power company will take all these facts into proper consideration they can make inducements which in almost every case would get the customer to install a leading power factor motor.

Where a consumer has sufficient synchronous capacity to operate at a leading power factor of 90 per cent or better at all times, he is entitled to a discount of 5 per cent of his consumption, or in case the consumer installs an automatic voltage regulator, he is likewise entitled to this same discount of 5 per cent from his monthly kw-hr. consumption. This is quite an item to a consumer whose total monthly consumption amounts to over 1,000,000 kw-hr.

A power company man from Canada opposes the practise of allowing a rebate for leading power factor. He says, "If the customer has a leading power factor of 95 per cent and you allow him a rebate of 5 per cent you are practically paying him for power at the same price at which you are selling it to him. You are losing any profit on your business and you are not gaining any energy except in some capacity in your apparatus." He further states that he certainly would not accept this kind of contract. Now the point that he overlooked is that the customer, is employing a larger motor of lower efficiency in order to get power factor correction, necessarily increased his energy consumption. This extra power, however, did him no good and it is only fair that the company should practically buy it back at the same price at which they sell it to him, otherwise they are penalizing the customer for helping the company out.

Now if there were standardized a method for metering power factor, central station companies could well afford to make definite inducements for running synchronous motors at leading power factor.

They should have men on their staffs who understand the synchronous motor just as thoroughly as the factories that build them. Perhaps this is putting the case a little bit to forcibly but at any rate they ought to have men who will familiarize themselves with synchronous motors of all types and all makes. They should travel over the country and see what is now being done in isolated plants and on other power systems. In fact, a department could be created in which actual cost records were kept of the various methods of power factor correction.

If the power company would make an accurate record of what their corrective apparatus is costing them and then make a reasonable estimate per kv-a. as to what they could allow the customer, they would be doing something worth while. They would soon find out that they could make a very liberal inducement and still be money ahead. The lower efficiency and extra losses running at leading power factor would not be as expensive per kv-a. in the load driving motor as it is in the large condenser. Power factor is corrected at the point where it is needed most, thereby relieving the rest of the system. The reactive current flows back and forth between the synchronous motors and the induction motors. The $I^2 R$ losses in the feeders and on the customer's circuits can be reduced by this plan, whereas if the condensers are located at the power companies' substations no such saving is possible.

If after proper investigation it is clear that a synchronous motor is suitable for the application, cost comparisons can be made between it and an induction motor. It is always possible to obtain reactive kv-a. a great deal cheaper by substituting synchronous motors for induction motors and eliminating the need of separate condensers. The cost per kv-a. in the case of the synchronous motor will probably be from 25 per cent to not more than 50 per cent of the cost involved, when induction motors and separate condensers are required.

To sum up the case from the central station standpoint.

Within the last two years practically all power companies have come to the conclusion they must penalize for low power factor.

The greatest obstacle is the chaotic condition of metering.

There is nothing inherently impossible in the problem if meter engineers and manufacturers and central station engineers combine to give it the attention it deserves. The early history of wattmeters was a record of unsatisfactory performance. But the problem was solved, as we all know.

The need of the hour is a standardized method of metering power factor. It is surely a fitting time for the meter manufacturers and the central stations to get together and solve this question.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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POWER FACTOR IN POLYPHASE SYSTEMS

BY FRANCIS B. SILSBEE
Associate Physicist, U. S. Bureau of Standards

THE use of a number of different definitions of "power factor" as applied to unbalanced polyphase system has led to much confusion. The present paper points out the mutual relations of a number of possible definitions and their relative merits for specific types of circuits.

Power factor has two distinct applications for economic and technical purposes respectively, and the selection of the single property which any one definition of power factor can determine must be considered from two distinct points of view. The economic importance arises from the fact that it is more expensive to supply a given power to an actual load in which the currents are not in phase with the voltages or in which the currents in the several phases are not equal in magnitude than it is to supply a standard load in which some or all of these adverse conditions are absent. Three distinct types of standard load have been tacitly assumed in the development of the several definitions now in use: (A) a balanced load in which both the currents and the voltages are symmetrical and in phase; (B) a load in which the currents have the same magnitudes as in the actual load but in which the current system as a whole has been shifted in phase with respect to the voltage system, so as to make the total power a maximum (*i. e.*, such a shift as would be produced by a symmetrical polyphase synchronous condenser); and (C) a load in which each current has the same magnitude as in the actual load that has been individually shifted into phase with its corresponding voltage.

There are also two distinct items of cost which must be considered (1) the fixed charges on the additional generator and line capacity required to supply the actual low power factor load and (2) the cost of the additional power lost in the generating circuits. The ratio of the actual power to the power which might be supplied to a standard load either (1) by the same generator capacity or (2) with the same power loss, may be taken as the logical economic definition of power factor.

For a standard load of type A and a symmetrical polyphase generator the most logical power factor is the ratio of the watts to the "effective volt-amperes," this latter qunatity being defined as the product of the square root of the sum of the squares of the live currents multiplied by the square root of the sum of the squares of the voltages to neutral. For a standard load of type (B) the "vector power factor" (Def. 2 of the special joint committee) and for a standard load of type C the "arithmetical power factor" (Def. 1) are most logical.

For technical purposes it is highly desirable to separate the effects of phase displacement, unbalance and wave form since the causes and remedies for each are quite distinct. This can readily be accomplished by defining one or more additional quantities so that one quantity (such for example as "vector power factor") indicates the phase displacement as distinct from unbalance; and a second quantity (such as balance factor) shows the symmetry of the loading as distinct from any general phase shift. The first quantity thus shows the extent to which conditions might be improved by the installation of synchronous condensers, while the second indicates the possible gain from phase converters or by rearrangement of the load.

For the first quantity the vector power factor is undoubtedly most useful. It is independent of the potential of the neutral point to which the voltages are measured, and also it may be obtained by adding the real and reactive components of the individual branches of any network. It is directly related to the mean magnetic and electrostatic energies of the system and to the amount of excitation required.

"Balance factor" is useful principally to indicate (1) the pulsations in instantaneous power and (2) the increase in heating of the generators which results from any unbalance. If the voltages are symmetrical these two properties depend upon the same combination of variables and may both be covered by a single definition. If, however, the voltages are not symmetrical this relation no longer holds and a uniform flow of power may result from a very unsymmetrical system of currents and voltages which would produce considerably increased heating. A convenient definition for the heating aspect is "effective balance factor" equals $\dfrac{\text{vector volt-amperes}}{\text{effective volt-amperes}}$, since this leads to the relation: Effective balance factor × vector power factor = effective power factor and serves to reconcile the economic requirements for a single factor based on heating effects with the technical requirements for a separation of phase shift and unbalance.

In the complete paper the various possible "volt-amperes," pulsation in power and related quantities are expressed mathematically so that their interrelations may be noted, and a number of numerical examples are worked out. The generalization of the definitions to include cases where the wave forms are not sinusoidal, and the instrumental methods for measuring the various quantities involved, are also discussed.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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POWER FACTOR AND UNBALANCE ON A POLYPHASE SYSTEM

BY CARL J. FECHHEIMER

Designing Engineer, Westinghouse Electric & Mfg. Co.

IN a single-phase or balanced polyphase circuit, the two factors which are contributory to power factors less than unity are wave distortion and phase displacement between current and voltage. In the paper the influence of wave shape is not considered, since in most large modern systems the wave form is nearly sinusoidal. The paper covers balanced and unbalanced polyphase systems without neutral current.

Power factor and unbalance are two distinct phenomena; therefore any attempt to formulate a definition which combines them is certain to be meaningless and of very questionable value.

The general case of a polyphase system is treated from the standpoint of two polyphase systems of opposite phase sequence. A proof is given that the usual unbalanced system of currents in machines is physically made up of two balanced systems of opposite phase sequence, thereby showing that the method is more than a mathematical convenience. In Appendix A, a physical proof is given for the fact that in the rotor of a synchronously driven single-phase induction motor or synchronous machine, double frequency polyphase currents are induced which are productive of m. m. f. which is constant in magnitude and rotates at synchronous speed in the opposite direction to the rotor, and this m. m. f., when combined with the alternating m. m. f. due to the stator currents, gives a resultant m. m. f. which is constant in magnitude and revolves at synchronous speed in the same direction as the rotor. In the induction motor the flux is proportional to this latter m. m. f.; in the alternator it is the "armature reaction." Thus, the alternating m. m. f., due to the single-phase stator currents gives rise to two equal and opposite m. m. f., both traveling at synchronous speed but in opposite directions.

In Fig. 2, *O A* is the m. m. f. of the polyphase double frequency rotor currents, which rotates negatively at synchronous speed. *O D* is the alternating stator m. m. f. which is fixed in position; and *O C* is the resultant of the two, which rotates in a positive direction at synchronous speed. The same final results would have been obtained had *O D* been taken as the resultant of *O C*, of positive rotation, and *O B*, equal and opposite to *O A*, and of negative rotation. Then the alternating m. m. f., *O D*, is the resultant of two equal m. m. fs., constant in magnitude, both rotating synchronously in opposite directions. One m. m. f. is productive of flux (or armature reaction); the other is that which is needed to neutralize the rotor magnetizing or damper currents.

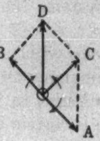

Fig. 2

Time angle = 30 deg.
Stator m. m. f. = 0.866 × 2 = 1.732
Rotor m. m. f. = 1, shifted clockwise 30 deg.
Resultant = 1, shifted counter-clockwise 30 deg.

In a polyphse machine, a rotating m. m. f. is produced by a polyphase system of currents, and therefore the two oppositely rotating m. m. f. systems in Fig. 2, may each be so represented, as in Figs. 3 and 4 where four-phase systems are shown. These combined give the single-phase system of Fig. 5.

Two equal three-phase systems of opposite phase sequence may be similarly combined to give a single-phase system. If the balanced negative sequence system is decreased in magnitude, or changed in phase relation to the positive sequence system the resultant is an unbalanced polyphase system. Conversely, the unbalance polyphase system may be broken into two balanced polyphase systems of opposite phase sequence; in appendices B and C are given illustrative methods, graphical and analytical, for doing this. This method of solving the unbalanced system is unique in that there *is but one solution* for any given system; on the other hand, the method of breaking into a balanced polyphase and superimposed single-phase systems permits of an infinite number of solutions.

The same general law for losses that applies to the in-phase and quadrature components of current, and the combination of currents of different frequencies, namely, the "sum of the squares" law, applies also when the unbalanced system of currents is broken into its constituent positive and negative sequence systems; this is proved in appendix D.

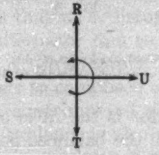

FIG. 3

Time angle = 45 deg.
Stator m. m. f. = 0.707 × 2 = 1.414
Rotor m. m. f. = 1, shifted clockwise 45 deg.
Resultant m. m. f. = 1, shifted counter-clockwise 45 deg.

The deleterious effects of a relatively small unbalance in voltage upon synchronous and induction machinery due to a not inconsiderable unbalance in currents are discussed; especially is this so with synchronous converters. The negative sequence currents in such apparatus, due to unbalance in line voltage, tend to correct the unbalance of the system, and these negative sequence currents are in consequence displaced 180 deg. from those negative sequence currents which produce the unbalance.

In any case, the true power in the polyphase system is the sum of the power in the positive and that in the negative sequence systems; similarly for reactive power the power factor is the ratio of the true power to the square root of the sum of the squares of the true and reactive powers. In general, the negative sequence voltage and power are small compared with the positive, so that the following definitions are proposed:

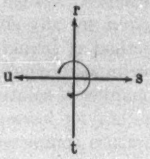

FIG. 4

Time angle = 60 deg.
Stator m. m. f. = 0.5 × 2 = 1
Rotor m. m. f. = 1, shifted clockwise 60 deg.
Resultant = 1, shifted counter-clockwise 60 deg.

"The power factor is the ratio of the true watts to the volt-amperes in the balanced positive sequence systems of volts and amperes."

"The unbalance factor is the ratio of the negative sequence amperes to the positive sequence amperes."

Thus, the proposed definition for power factor fits in with the usual idea: it is the cosine of the angle between current and voltage in the positive sequence system. The definition for unbalance disregards unbalance in voltage. Had it been expressed as "the ratio of the negative sequence volt-amperes to the positive sequence volt-amperes," it would have been undesir-

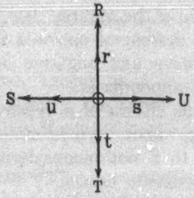

FIG. 5

Time angle = 90 deg.
Stator m. m. f. = 0.
Rotor m. m. f. = 1, shifted clockwise 90 deg.
Resultant m. m. f. = 1, shifted counter-clockwise 90 deg.

able because the negative sequence voltage is very largely dependent upon the synchronous and induction apparatus on the system, and consequently volt-amperes are no measure of the degree to which the system is unbalanced by a given customer.

The method of breaking the unbalanced systems into their constituent systems can be readily used in measurement, as is covered in another paper by Mr. R. D. Evans.

Parallel cases in which physical quantities are separated into their component parts are given. The most striking example is that of a distorted wave which is generated in the alternator due to the non-sinusoidal distribution of flux in the air gap. This wave is looked upon by all engineers as composed of one of fundamental frequency with superimposed waves of higher frequency. Physical evidence is also given by an oscillogram taken on the secondary of an induction motor when slightly unbalanced voltages were applied to the primary.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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POLYPHASE POWER FACTOR

BY H. L. WALLAU

Electrical Engineer, Cleveland Elec. Ill. Co.

THE Special Joint Committee on Determination of Power Factor in Polyphase Circuits has formulated two definitions for this factor.

As technicians we will doubtless all agree that Definition 2 which is mathematically accurate must be adopted to represent that quantity which today is referred to as power factor when applied to single-phase or to *balanced* polyphase circuits.

As commercial engineers we must recognize that Definition 1 has a broad field of usefulness, when applied to unbalanced circuits.

The central station industry today is compelled to take on single-phase loads of considerable magnitude.

These loads consist of welders of very poor power factor, (20 to 40 per cent) single-phase arc furnaces of relatively better (35 to 70 per cent) but nevertheless poor power factor, and in some cases single-phase railway loads also of poor characteristics, together with both inductive and non-inductive loads of other kinds.

If it were commercially praticable to compel the consumer to install phase-balancers or motor-generator sets in connection with such loads, the inherent difficulties due to out-of-balance loads would be done away with. Obviously, this is impossible. As a result the companies must operate under certain disadvantages, with these loads connected to their systems, such as reduced effective capacity of three-phase circuits, increased heating in cables, etc. In addition, if the out-of-balance becomes of sufficient magnitude at the power plant generator troubles will result.

Increased costs of supply result therefrom. Either the rates in our industrial schedules must reflect these added costs and proportion them over all consumers alike, or some means of penalizing the consumers causing the added burden must be incorporated therein.

This means is conveniently provided by a power factor clause. If, however, a strictly technical definition is adopted for incorporation into rate schedules, it takes no account of the effect of unbalanced currents yielding as it does a higher power factor than that obtained by the proposed commercial Definition 1.

To care for this condition a new factor, termed balance factor, has been suggested. After we have defined this factor in a manner satisfactory to all, what practical means will we have of obtaining it, and if obtained how shall we apply it? Its commercial use in rate schedules in addition to a power factor clause introduces a new stumbling block over which the prospective customer must be safely lifted. For that reason for commercial work Definition 1 is preferable.

This view is that of an engineer of a supply company interested in obtaining a definition that can be used in contracts for the sale of power, and is relatively simple and easily understood.

Definition 1 takes into account the maximum wattage obtainable based on the currents and voltages existing, and the ratio of this quantity to the actual watts measured is the power factor which, the author believes, should preferably be standardized for commercial work.

The name of the Committee suggests the next essential step. Not only must we define polyphase power factor, but we must determine a suitable, reliable, inexpensive and uniform method of measuring the quantity defined.

To date neither definitions nor methods are standard with us. There is a real need for both.

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POLYPHASE POWER FACTOR

BY P. M. LINCOLN
Lincoln Electric Co.

THE special joint committee has very wisely decided to submit the question of a definition of polyphase power factor to general discussion before making its final recommendations. To guide such discussion it has submitted two alternate definitions for polyphase power factor, *viz*: No. 1 which may be called the "arithmetical sum" definition and No. 2 which may be called the "vector sum" definition.

Let us consider the merits of these two definitions from a very elementary standpoint. Take for instance any load on a typical single-phase circuit, we may measure the watts, volts and amperes on such a load with standard instruments with which we are all familiar and there is no question concerning the accuracy and sufficiency of such measurements. If we call the volts so measured E and the amperes I, the volt-amperes of the circuit are the product of these two quantities $E\,I$. If we call the watts W, it has always been assumed that $W = E\,I \cos\phi$ and the power factor ($\cos\phi$) has always been taken as the ratio

$$\frac{W}{E\,I} = \frac{\cos\phi\, E\,I}{E\,I} = \cos\phi \qquad (1)$$

No challenge has ever been offered to this method of procedure, although it has long been recognized that where harmonics are present either in the voltage or current, the quantity $\cos\phi$ ceases to be a simple angular function. The above relation holds not only for single-phase circuits but it holds equally well for polyphase circuits *provided there is no unbalance in voltage or current.*

The above procedure does not represent the only method of arriving at the quantity which we call power factor or cos ϕ. Suppose we had a source of voltage which was always just 90 deg. in phase angle from that of the actual circuit under measurement. A wattmeter with such a voltage applied would then read $E\,I \sin \phi$ instead of $E\,I \cos \phi$. In single-phase circuits we have no method of measuring directly the quantity $E\,I \sin \phi$ but in polyphase circuits it is just as simple a matter to measure the quantity $E\,I \sin \phi$ or reactive component as the quantity $E\,I \cos \phi$ or watts. With these two quantities measured, it is then a simple process to arrive at power factor from the well known relationship $\sin^2 + \cos^2 = 1$. From this relationship it of course follows that $(E\,I \sin \phi)^2 + (E\,I \cos \phi)^2 = E^2\,I^2$ and

$$\cos \phi = \frac{E\,I \cos \phi}{\sqrt{(E\,I \sin \phi)^2 + (E\,I \cos \phi)^2}} \tag{2}$$

The fundamental difference between method (2) and method (1) for determining polyphase power factor is that method (2) treats a polyphase circuit as a *single* circuit while method (1) treats it as a combination of three separate single-phase circuits. Method (1) therefore involves the method of combining three separate values of I to get a single value as well as the question of power factor; that is, method (1) involves the question of unbalance as well as the question of power factor. This is essentially the same question that is involved between the two definitions of power factor that have been submitted to us by the special committee. Definition 1—the "arithmetical sum" definition—corresponds to method (1) for determining polyphase power factor while definition 2—the "vector sum" definition—corresponds with method (2) for determining polyphase power factor. Definition 2 and method (2) give results which are entirely independent of the question of unbalance. Since it is *power factor* that the committee has undertaken to define, it is my strong recommendation that definition 2 be the one adopted for polyphase power factor.

It has been urged that definition 1 embraces something more than power factor and should have another name. Various names have been suggested such as "mean power factor," "commercial power factor," "apparent power factor," "economy factor," "penalty factor." It is agreed that unbalance should be recognized as well as power factor; it is also agreed that it is logical to recognize still a third factor which shall be the combination—preferably the product—of power factor and unbalance factor. Such a third factor could logically be called by one of the names in the above suggested list. Since such a factor will have its principle use in dealing with questions of rates for electric service, the term "penalty factor" seems logical.

The question further arises, if we adopt definition 2 for polyphase power factor, is it not logical to adopt definition 1 as the definition of penalty factor or such other name as is finally agreed to? This question depends in turn on whether the ratio between the quantities given by definition 1 and definition 2 constitutes a logical definition for unbalance factor. Personally, I do not believe that this ratio does constitute such a logical definition. A definition should be chosen for unbalance factor which is logical in itself. The question of unbalance in polyphase circuits and of a factor which might logically be called "unbalance factor" is an abstruse one but it is one which should be considered and decided on its own merits. If it is outside the function of the existing committee to pass on the question of unbalance factor it is my opinion that they should ask for additional authority to include this question.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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POLYPHASE POWER REPRESENTATION BY MEANS OF SYMMETRICAL COORDINATES

BY C. L. FORTESCUE

Engineering Dept. Westinghouse Electric & Mfg. Co.

THIS paper is a more complete presentation of the subject of power representation by means of symmetrical coordinates, which was briefly outlined in the author's paper on Symmetrical Co-ordinates presented at the 30th Annual Convention, 1918.

A short treatment of the subject of single-phase power is given, and it is shown that the product $\check{E}\hat{I} = P + j\,Q$, where P is the true power and Q the reactive power, completely specifies all characteristics of the power when the phase of the e. m. f., $\check{E}$ is given. The phase of the double-frequency component $P_{\text{H}} + j\,Q_{\text{H}}$ is as much in advance of $\check{E}$ as $\check{E}$ is in advance of $P + j\,Q$.

The question of single-phase power measurements is touched on. It is shown that the wattmeter is normally a reactive power meter. It is only a true power meter when the current in the voltage coil is constrained to be proportional to, and in phase with, the impressed e. m. f. The watthour meter measures energy by reason of the fact that the current in the voltage coil is constrained to be in quadrature to the impressed e. m. f. If it were constrained to be in phase with the impressed e. m. f., which could be just as easily accomplished, this instrument would integrate the mean reactive power.

It is shown that four measurements are required to completely determine a single-phase load mathematically; the two complex variables $\check{E}$ and $\check{I}$, furnish all the necessary data.

In practical measurements, it is usual to specify the impressed e. m. f., the kilovolt-amperes, and the power factor; the true phase of the impressed e. m. f. being arbitrary.

A symmetrical polyphase system requires for its complete determination four quantities which are usually

1. The e. m. f. of one phase.
2. Its phase angle.
3. The current in one phase.
4. Its phase angle.

The particular phase under consideration may be termed the "principal phase."

Since an unbalanced n-phase system of currents and e. m. fs. may be resolved at most into n symmetrical systems of currents and e. m. fs., the maximum number of measurements required to completely define such a system is $4\,n$ and the minimum number may be from 4 up to $4\,n$ depending upon the nature of the dissymmetry.

The quantities expressed in symmetrical co-ordinates required to completely define a three-phase, four-tone system are:

$S^{\circ}\,\check{E}_{A0}$, $S^1\,E_{A1}$, $S^2\,\check{E}_{A2}$ the symmetrical components of e. m. f. and $S^{\circ}\,\check{I}_{A0}$, $S^1\,\check{I}_{A1}$ and $S^2\,\check{I}_{A2}$ the symmetrical components of current. The sequence symbols have the following significance. The S° system consists of three equal vectors of the same phase. The S^1 or positive phase sequence system consists of three equal vectors each of which lags $\frac{2\,\pi}{3}$ behind the preceding vector when taken in their alphabetical order. The S^2 or negative phase sequence system consists of three equal vectors each of which leads the preceding vector by $\frac{2\,\pi}{3}$ when taken in alphabetical order. These sequence symbols have the property of combining by the law of indices when products of two sequences are formed.

Instantaneous symmetrical systems are formed from the symbolic systems in exactly the same manner as for single-phase, and by following the same procedure for obtaining the power products as in the treatment of the single-phase system, we obtain similar expressions as in single-phase, and the zero frequency system of power vectors obtained completely specify the whole system of power vectors.

In an unbalanced three-phase, three-wire system there are three systems of zero frequency power vectors. The S° system consists of the true and reactive power due to the positive and negative sequence currents and e. m. f., and S^1 and S^2 systems whose instantaneous value is zero; they represent interchange of power among the phases, and therefore, are a measure of the degree of unbalance. It is shown that if the true and reactive power of each phase of a system are given and true power is taken as datum and the vectors so formed are resolved by the method of symmetrical co-ordinates, the systems of power vectors deduced above are obtained.

Three-phase power measurements are discussed: It is shown that in all the usual power measurements, the true and reactive power due to positive phase sequence and negative phase sequence appear as entities and may be separated. They are therefore not mathematical fictions but actual concrete quantities.

All true and reactive power measurements by symmetrical co-ordinates methods may be accomplished by a single movement. A comparison is made between the present method of measurements and those following the principle of symmetrical co-ordinates. The same number of instruments are required in each case but the symmetrical co-ordinates method completely determines all necessary factors without computation.

A method of graphically obtaining the complete solution of a system when measurements are given is shown.

The author holds that it is of the highest importance that definitions of the fundamental quantities and factors concerned with the measurements of e. m. f. current, power and energy be scientifically correct. It is not vitally important in practical work that all definitions shall be adhered to, but it is vitally important that our standard definitions be based on sound scientific principles.

Definitions of Power Factor. Power factor shall be determined from measurements of three-phase power by means of two or more wattmeters connected in the usual manner and from reactive power measured by a reactive power meter with the same connections as those used in measuring the true power. If P be the true power and Q the reactive power

$$\text{Power factor} = \frac{P}{\sqrt{P^2 + Q^2}}$$

The positive phase sequence power factor and negative phase sequence power factor are obtained in a similar manner.

Definition of Unbalance Factor. The ratio of one unit of the negative phase sequence component of a quantity to the corresponding unit of its positive phase sequence component is defined as the *unbalance factor* of the quantity under consideration. Usually the ratio of the magnitudes only need be considered.

Conclusion. The definitions proposed are easily measured and completely and uniquely determine all the quantities it is necessary to know in order to make proper charges for energy.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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MEASUREMENT OF POWER FACTOR ON UNBALANCED POLYPHASE CIRCUITS

BY R. D. EVANS

General Engineer, Westinghouse Electric & Mfg. Co.

THE object of this paper is two-fold, first, to give a scientific definition of power factor, and, second, to describe devices for measuring power factor and devices for measuring unbalance of a polyphase system.

ON THE DEFINITION OF POWER FACTOR

Power factor is a term which describes the flow of energy in an alternating-current circuit and which expresses a relation between the amount of energy converted into heat or mechanical work, and the amount of energy periodically absorbed and discharged in the circuit. Power factor is not the simplest factor related to true power and reactive power, but it is a convenient factor, as it expresses a ratio between the amount of power actually delivered in a circuit, to the amount of power which might be delivered without exceeding the same heating.

Power factor may be defined as the ratio of true power to the square root of the sum of the squares of the true power and the reactive power.

$$\text{P. F.} = \frac{P}{\sqrt{P^2 + Q^2}}$$

In the above expression, P is the algebraic sum of the watts, and Q is the algebraic sum of the reactive watts in all of the component parts of the circuit.

It is to be noted that the definition of power factor, from the standpoint of the flow of energy, is complete, and is independent of the presence of harmonics, plurality of phases, or unbalance, that is, the definition requires no extension to take care of these cases. On the other hand, the definition of power factor, from the standpoint of phase difference, or the ratio of watts to volt amperes, requires modification to take care of the complication resulting from the presence of harmonics, plurality of phases, or unbalance.

DEVICES FOR MEASURING UNBALANCE OF A POLYPHASE SYSTEM

Any system of unbalanced voltages may be resolved into two or more balanced systems of voltages of different phase sequences, each of which may be measured by stationary instruments. The meter for measuring positive or negative sequence voltage, on a three-phase, three-wire system, consists of a voltmeter with two windings and two external series impedances of different power factors. Another form of this instrument is described, and this consists of a standard voltmeter, two external impedances, and two standard potential transformers.

In a similar manner, the positive and negative sequence currents may be measured. Such an instrument is described and consists of a standard ammeter, two external impedances, and two standard current transformers.

The theory of these meters is explained from different viewpoints. One explanation is based on a physical interpretation of the mathematical solution. Another explanation is based on showing that the voltages of one sequence causes the meter to read and the voltages of the other sequence do not, and, therefore, in the presence of two sequences, the meter measures a quantity proportional to one of them. A third explanation is based on the idea of a network in connection with a meter element. From this point of view, it is only necessary to provide a network that permits only current of the desired sequence to flow in the branch in which the meter is connected.

POWER FACTOR METERS FOR UNBALANCED CIRCUITS

Having separated out the positive and negative sequence voltages and currents, the next step is to combine them for measuring power, reactive power or reactive volt-amperes and power factor. Standard instruments supplied with positive sequence voltage and current, measure positive sequence watts, positive sequence reactive watts or reactive volt-amperes, or positive sequence power factor. A meter is described which measures positive sequence power factor.

In an unbalanced system, as many separate meter elements are required to measure the total power, total reactive power, or the total power factor, as there are sequences of power present. A meter is described which measures total power factor on a three-phase three-wire system. This meter consists of two single-phase power factor meter elements, mounted on a common shaft, and one element supplied with positive sequence voltage and current, and the other element with negative sequence voltage and current.

CONCLUSION

Power factor, reactive factor, power and reactive power, must be defined so that their mutual relations are shown, and so that they correspond to the physical facts. A scientific or exact definition of power factor must first be formulated, and then on this foundation, a practical definition for use in rate making may be built.

The exact definition of power factor should be based on the flow of energy. Power factor is the ratio of true power to the square root of the sum of the squares of true power and reactive power.

It must be recognized that power and power factor do not completely describe the flow of energy, and that the use of some unbalance factor is necessary. Standard instruments may be employed to measure the unbalance of a system.

Power factor and unbalance are independent quantities and should be charged for separately.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920.

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POLYPHASE POWER FACTOR AND UNBALANCED LOADS

BY PHILIP TORCHIO

Chief Electrical Engineer, N. Y. Edison Co.

DEFINITION No. 1 in the Committee's report gives a numerical relation applicable to any polyphase circuit, which is also intended to give a measure, as a general average, of the greater outlay of apparatus and plant that the supply of unbalanced loads entails. Without attempting to state whether such numerical averaging gives a fair compromise measure of the added economical burden of carrying unbalanced loads, it is important that we definitely and clearly describe the results of the applications of this definition No. 1. For this purpose I submit in the table herewith comparative results of the amount of capacity of electrical equipment required for generating and transmitting a unit amount of power under different conditions of unbalancing, all giving the same average value of power factor definition No. 1:

Power factor by definition No. 1 per cent	Capacity of electrical equipment required for generating and transmitting 1000 kw. of		
	Balanced load	Unbalanced load	Unbalanced load (Single phase)
100–99.5	1000 kv-a.	1100 kv-a.	Not possible
86.6	1150 "	1250 "	1740 kv-a.
75.0	1330 "	1425 "	2000 "
50.0	2000 "	2175 "	3000 "

The above comparison shows that, with apparently the same value of polyphase power factor, the capacity of electrical equipment required for generating and transmitting the same amount of power may vary as much as from 1 to 1½. It is obvious that under such extreme conditions the cost of service would be quite different in the several cases.

On the other hand, the unbalanced load of one customer may be compensated by the contemporaneous unbalanced loads on other phases of other customers, so that, as a net result, on the supply company's equipment only a fraction of its equipment may be affected by the "apparent" low power factor due to unbalanced loads, this effect being perhaps limited only to the district where the power is distributed, but being balanced and equalized on the transmission line and the generators.

The power rate engineer may, therefore, have greater need of the "*comtemporaneous unbalanced loads and power factors of definition No. 2*" for the individual customer and for the district, than the average power factors of definition No. 1.

In this connection it would seem wise to make reference to the importance of defining the time intervals during which the power factors are to be taken. It will be of particular value to the power rate engineer to have *power factor diagrams* for the twenty-four hours for the customer and district under consideration, rather than the power factor for one minute or fifteen minute period of maximum demand or the average power factor for the twenty-four hours, which will be of small value to him in analyzing the cost of such service. The same comment may be made as to the recording of unbalanced loads.

Abstract of paper presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 1, 1920

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POWER FACTOR IN POLYPHASE CIRCUITS

BY W. H. PRATT
Electrical Engineer, General Electric Co., Lynn, Mass.

POWER factor in a single-phase circuit is defined as the ratio between the power and the volt-amperes. If the current and voltage have a sine wave form, the power factor is the cosine of an angle representing phase displacement. If one or both, current and voltage, are not of sine wave form, they may be replaced in many circumstances by equivalent sine quantities and the power factor may then be considered as the cosine of a correspondingly defined angle. It is probable that in the great majority of cases, when the expression "power factor" is used, it is thought of as a measure of phase displacement. However, fortunately or unfortunately, the definition is otherwise, and in certain important cases, the distinction is pronounced. Furthermore, the present definition is consistent with the continuous recognized use of the term since its first appearance.

A brief consideration of the scope of the problem at once brings to light difficulties which will presently be indicated. At the outset, it would seem very desirable, even necessary, that the definition to be selected should be consistent in every possible way with the single-phase definition, and at the same time, should express as nearly as possible the thought implied in the original use of the term—the ratio of the actual power to that that would be present were the currents and voltages interacting so as to produce the maximum power.

As has been remarked, the term power factor carries with it the idea of phase displacement, and consequently, is naturally associated with the idea of phase control, but in a polyphase circuit, if unbalanced, there are several phase displacements and consequently, no single number can uniquely indicate the nature of the several phase displacements that are responsible for the total results. Thus, while power factor as applied to a single-phase circuit is indefinite, in that it may depend on either phase displacement or difference of frequency, having reference to harmonics, or indeed a combination of both, in the polyphase circuit, there are the additional questions of dissimilar phase displacements and dissimilar wave distortions to be met.

There is another idea, which, however, is less consistent with the single-phase definition, which arises from the relation that in a sine wave single-phase circuit the power factor is the cosine of the angle of which the fraction $\frac{\text{reactive volt amperes}}{\text{power}}$ is the tangent. This result when extended to a polyphase sine wave circuit, may take the form: power factor = $\cos \tan^{-1} \frac{\Sigma \text{ reactive volt amperes,}}{\Sigma \text{ power}}$ or, further, if the wave forms are irregular, to a factor defined by the same expression but in which the angle involved has no physical significance.

The commercial aspect of the question tends to still further complicate the matter. It is now usual to introduce the idea of power factor into contracts for the purchase of power either as limits within which operations must be conducted, or as a measure of penalties or rates for such purchase of power. It is obviously unjust to penalize a purchaser for a condition beyond his control but the same remark cannot apply should a similar distribution of current and voltage arise by reason of apparatus within the control of the purchaser of power.

As an illustration of this idea; a given distribution of current and voltage may arise by connecting a perfectly symmetrical polyphase motor to a circuit having voltages unbalanced by some unsymmetrical load located elsewhere. In this case, the motor tends to restore the symmetry of the voltage of the system, but will draw from the system unsymmetrical currents. In contrast, an unsymmetrical grouping of single-phase apparatus may produce a similar unsymmetrical flow of current from a balanced system of voltages, and bring about an unbalance of voltages. To be sure, where the one would tend to restore the voltages to a balanced condition, the other would tend to affect them oppositely. We are thus presented with two cases which would be represented by a chosen factor by nearly the same number, yet should undoubtedly be treated quite differently.

This whole question was discussed very carefully about 1909 by the Italian Engineers, Campos[1], Lupi[2], Norsa[3], and also in a Vienna publication by Niethammer[4].

In the report of the Committee we find presented for discussion two proposed definitions, designated as Definition 1 and Definition 2.

These two quantities are very different in extreme cases but are exactly equal for balanced condition. Their relation is illustrated by the diagrams in the appendices to the Committee's report.

It is to be noted that the phase sequence may greatly affect the value obtained by Definition 1, but it is also to be noted that this also markedly affects the distribution of line currents. Further, a load which measured by Definition 1 may have a power factor much below unity, may by Definition 2 have power factor unity. As has been pointed out by Lupi, Definition 1 is a power factor of the lines, and Definition 2 gives a power factor of the load. But Definition 2 considers the load as a whole and so should make a proper measure of power factor for such cases as might be typified by a synchronous motor connected to a system in which the voltages are unbalanced. That is, Definition 2 should give a fair basis for apportioning charges as between a power company and a user of power, but it could not

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1. Campos, G. *Atti della Assoc. Ellettra, Ital.* 13, 197–210, 1909.

2. Lupi, D. *Atti della Assoc. Ital.* 14, 3–30, 1910.

3. Norsa, R. *Atti della Assoc. Ital.* 14, 31–45, 1910.

4. Neithammer, F., *Elektrotechnik und Maschenbau* 28, 1027–9, 1910.

be used together with a measure of power input to determine the capability of a motor for carrying load.

It seems to the writer that there is a real use for both definitions; that in cases of marked unbalance, where the one might be correctly employed, the other might lead to injustice, and vice versa. But Definition 1 most nearly contains the idea of the single-phase definition and so should take the name. On the other hand, Definition 2, which is in fact the characteristic of the load, obtained by summing the several vectors relating to its parts, might properly be designated by a name distinctly different from power factor.

The corresponding designations used by Campos are, Definition 1, apparent power factor, and Definition 2, real power factor.

There is another way of discussing the matter which is very interesting from the analytical point of view; namely, by making use of the conception of symmetrical components, as elaborated by Fortescue, a balance factor may be obtained from the ratio of the magnitudes of the positively and negatively rotating sets of vectors. The phase displacement between voltage vectors and the positively rotating set of current vectors may then be the basis of a power factor. The power factor thus obtained corresponds to Definition 2, and by using the two factors together, results fairly equivalent to Definition 1 may be obtained. The use of these two factors has the merit of giving a function directly related to the heating effects produced in the conductor carrying the unbalanced currents.

To summarize, it seems to the writer that Definition 1 most nearly extends the idea of the single-phase definition to polyphase conditions, and results in a quantity that is well suited for the evaluation of volt-amperes in the lines.

Definition 2 gives a quantity that has wide applicability and when the currents and voltages do not depart greatly from sine waves, is easily measured. It is, however, a characteristic of the load as a whole and is the result of the vector addition of its parts. It merits a distinctive name of its own.

Finally, there is no one factor that can describe all that it may be necessary to know under the general head of power factor, using the word in the very general sense that has been current up to now.

NOTE: It may be easily shown that except for conditions of extreme unbalance, slight variations in the position of the neutral point of Definition 1 result in insignificant changes in the value of the power factor so obtained.

DISCUSSION ON "REPORT OF JOINT COMMITTEE," "POLYPHASE POWER FACTOR" (HOLTZ), "POWER FACTOR IMPROVEMENT DEPENDENT UPON ADEQUATE METERING" (BROWN), "POWER FACTOR IN POLYPHASE SYSTEMS" (SILSBEE), "POWER FACTOR AND UNBALANCE ON A POLYPHASE SYSTEM" (FECHHEIMER), "POLYPHASE POWER FACTOR" (WALLAU), "POLYPHASE POWER FACTOR" (LINCOLN), "POLYPHASE POWER REPRESENTATION BY MEANS OF SYMMETRICAL COORDINATES" (FORTESCUE), "MEASUREMENT OF POWER FACTOR ON UNBALANCED POLYPHASE CIRCUITS" (EVANS), "POLYPHASE POWER FACTORS AND UNBALANCED LOADS" (TORCHIO), AND "POWER FACTOR IN POLYPHASE CIRCUIT" (PRATT), WHITE SULPHUR SPRINGS, W. VA., JULY 1, 1920.

F. C. Holtz: Single-phase power factor has been very simply defined as the ratio of the true watts to the volt-amperes in the circuit. In arriving at a satisfactory definition for the power factor in polyphase circuits, there are one or two points which I think should receive due consideration.

Since the single-phase circuit is only a special case of the polyphase circuit, power factor should be so defined that in the limiting case it should give results which are in agreement with the single-phase definition.

In discussing this paper I have tacitly assumed that the voltages and currents in the polyphase circuit were sinusoidal, but allowing any condition of unbalance whatsoever in the polyphase system. It is very interesting to take the simple mathematical expressions defining single-phase power factor and under such conditions apply them to the polyphase case, and then to interpret the results. In order to get away from the double frequency quantities I have introduced into the paper the quaternion notation in which the vector operators are represented by I, J, K. In applying these to the single-phase problem one readily arrives at the various expressions for power factor, reactive factor, volt-amperes, etc. By a simple extension the same notation is applied to the polyphase circuit with certain definite results. In the single-phase circuit, we find that the volt-amperes in the circuit are represented as the square root of the sum of the squares, that is, the square root of the watts squared plus the wattless component squared.

In the polyphase circuit we arrive at results which are very analogous. Take for the sake of simplicity, the balanced or symmetrical polyphase system, where we have three equal voltages placed symmetrically about the center. I then assume any three currents whatsoever in the circuit, having various values for their amplitude. Under such conditions, by continuation of our single-phase problem into the polyphase, we find that the watts are equal to the sum of the watts in the individual phases, that is we get W_1 plus W_2 plus W_3 as the watts in the three-phase circuit. The reactive component may be represented by R_1 plus R_2 plus R_3. If we have leading power factor in one and lagging power factor in the other, we represent one with the negative sign and the other by positive sign, and take the results.

We find readily that this particular idea leads to a definition of power factor which is in accordance with definition No. 2. The paper deals further with the possibility of measuring such quantities.

On the other hand, taking definition No. 1 in which the power factor is defined as the ratio of the watts to the arithmetic sum of the volt amperes in the several phases. Let us take the simplest case, that of a three-phase circuit under single-phase operation at unity power factor.

Upon applying the definition we obtain a value of 86.6 per cent for the power factor. Under such conditions you can see that this definition involves an idea of balance as well as an idea of power factor.

Now, then, the only way in which a consumer could better his power factor would be by balancing up the load, taking it off the one phase and equalizing it around the other two, and therefore obtain unity power factor by such a distribution of load.

Another example of the fallacy of definition No. 1 is shown by P. M. Lincoln, in which he takes a three-phase line and loads it equally on all sides, but putting on one side a load at unity power factor, say, 100 amperes, and on the other leg a load of 100 amperes at zero power factor leading, and on the opposite side a load of 100 amperes at zero power factor lagging, and he obtains the power factor which is 36.6. Then by a simple reversal of the phase rotation or interchanging of loads, he was able to bring the power factor up to 85 per cent by so doing, and therefore, under such conditions we would have two values of power factor, by simply changing the loads, which again shows it involves a question of balance, and therefore ideas other than those relating in a strict sense to the power factor in the single-phase circuit.

In my paper, I have taken up in no way the question of the entrance of harmonics into the equation. Such circuits are particularly difficult when viewed from one standpoint, and that is, suppose you have a sinusoidal e. m. f. and then, due to the characteristics of the circuits themselves, we find that the constants in these circuits are functions themselves of the current. Mr. Pratt brought out a very simple example of that in the case of the ordinary type *C* incandescent lamp, in which the tungsten at a low temperature has a low resistance, in fact, the starting current on the type *C* incandescent lamp is something about 5 times normal current. The resistance, evidently pulsates with the line potential, and although you may have a sinusoidal e. m. f. applied, the current itself will not be sinusoidal, but will be a fundamental plus some harmonics. In addition to this, there is the question of the magnetic circuits which with a sine wave applied e. m. f., call for a distorted wave form in the current. This is true in the case of arc furnaces, reactors, transformer cores highly saturated, etc. While we are making up a definition of power factor, we should attempt one which is the ideal—and that ideal will be a definition for polyphase power factor, which is so complete as to absolutely include every possible case—this will take care of the circuits which show a change in constants with a change in current.

V. Karapetoff: For practical purposes it is convenient to retain the right triangle, of which the horizontal side represents the true power, the vertical side the reactive power and the hypothenuse is what some of us please to call the apparent power or kv-a. The angle between the hypothenuse and the horizontal side is defined as the power factor. I am not sure that we are agreed upon the definition of reactive power or of apparent power even in a single-phase circuit with non-sinusoidal currents and voltages. The most convenient definition is the one which enables one to measure the quantity to be defined directly with an accurate available instrument. For this reason I am in favor of defining reactive power in a single-phase circuit as that indicated by a dynamometer-type wattmeter with its potential coil connected to a quadrature phase of the load voltage. This indication characterizes the circulating magnetic energy and can be obtained with a sufficient degree of accuracy on commercial power circuits. I also suggest the quantity so measured be named for the sake of brevity "Rewatts," instead of reactive watts or reactive volt-amperes.

A right triangle is completely determined by two of its elements, so that the watts, P, and the rewatts, Q, are all that is needed for the above mentioned triangle. The phase angle, ϕ, is determined by its trigonometrical tangent, equal to Q/P. The power factor is the cosine of this angle, though I believe that the tangent is a better quantity, and should be used more and more in place of the power factor. If your customer knows trigonometry, he will as easily understand the tangent as the cosine, and if he knows none at all, either term will serve the purpose. A tangent ϕ equal to 0.1 directly shows the customer that his reactive power is ten per cent of the true power. If he is penalized for reactive power, he can get a better idea from the tangent of the angle than from the cosine as to how near he is to the ideal condition of zero reactive power.

This tangent ϕ may be called the "idle power ratio," or whatever name the profession decides to adopt. I merely wish to emphasize the advantage of tangent ϕ over cosine ϕ for practical purposes, especially in polyphase circuits.

Similarly, the apparent power can be only defined as the hypothenuse of a triangle of which P and Q are the two sides. I do not see much advantage in measuring it directly or in trying to devise complicated recording instruments so as to get the value of the so-called apparent power over a month.

The above triangle and definitions are just as well applicable to an unbalanced polyphase circuit with non-sinusoidal currents and voltages. A polyphase wattmeter measures the total true power and a similar wattmeter with its potential coil "quadratured" measures the total rewatts. The triangle is thus complete and gives the equivalent phase angle and the apparent power. It is true that two equal polyphase loads of different degrees of unbalance may have the same equivalent phase angle and yet produce quite different results upon the generating and distributing equipment. This simply means that an unbalanced polyphase load cannot be completely determined by its power factor alone, or as a student once told me, "You cannot make one coefficient do all the dirty work." It is necessary to introduce some definition of the "unbalance factor." The two definitions may either be independent of one another or combined in two simultaneous equations. Thus a new construction material has to be characterized by at least two properties, say density and mechanical strength. In this connection I also wish to call your attention to my "Parable about the Mean Radius of the Ellipse," JOURNAL, A. I. E. E., V. 39, 732, 1920.

J. C. Lincoln: My brother desires the joint committee which is in charge of this subject to make several recommendations. The first is: "Recommend a definition for polyphase power factor." That, as I tried to outline is definite. The second recommendation is: "Recommend a definition for unbalance factor in the polyphase circuit." The unbalance factor and power factor are completely different from each other, and we will get into confusion if we try to put them together, and his third recommendation is: "Recommend a third factor both in name and character, which shall be the combination—in my opinion preferably the product, of power factor and unbalance factor."

C. L. Fortescue: A single-phase circuit in which current is flowing having a definite frequency requires four measurements for its complete determination. They are: Magnitude of impressed e. m. f.; phase of impressed e. m. f.; magnitude of current in circuit; and phase of current in circuit.

One of these quantities may be left out, because we are only concerned with relative phase positions, and are not concerned with the absolute phase position with respect to time.

The same number of measurements exactly is required to define a symmetrical polyphase system. You need to know the e. m. f. of one phase, and its phase angles; the current in one phase and its phase angle. These measurements completely define the symmetrical system. For that reason, so long as we were dealing with symmetrical systems, it was necessary only to define one quantity, the power factor, because its specifications were exactly the same as for a single-phase circuit, and we require nothing else.

When we have an unbalanced polyphase system we have a different story. As I pointed out two years ago in the paper on "Symmetrical Co-ordinates," an unbalanced system of current or e. m. f.—let us confine ourselves to three-phase—a three-phase unbalanced system of currents or e. m. f. may be resolved into three symmetrical systems of currents or e. m. f. When the three-wire system alone is considered, it may be resolved into two symmetrical component systems of currents or e. m. fs. If we are considering an unbalanced system of currents, one of those component systems of currents has the same phase sequence as the e. m. f. of the system. The other system has opposite phase sequence. In other words, if we take and define the voltages as *E-A*, *E-B*, and *E-C* and *E-B* follows *E-C* in time phase, and *E-A* follows *E-B* in time phase, then the currents can be resolved into systems of like character, and another system in which *I-B* leads *I-A* in time phase, and *I-C* leads *I-B* in time phase.

I introduced at that time a system of operating symbols which combined by the law of indices to give the proper phase sequence resulting from current flowing in unbalanced impedances and I appreciated, too, that these currents and e. m. fs. could be combined by the system of operators so as to give the power correctly, just exactly in the same way as we combined them in single-phase circuits. I will not go into the mathematical discussion of it, it will take too long. However, I have shown that when combining the currents and e. m.fs., so as to obtain power expressions, if we take an unbalanced three-wire system and measure the true and reactive power and taking the true power as datum lay off three vectors to represent the apparent power in each phase, these three vectors may be resolved by the method of symmetrical coordinates into components of zero, positive, and negative phase sequence.

We need to deal now only with the product which is of zero frequency. The double frequency product, as I have explained in the paper, is outside the question and does not need to be brought in at all. When the zero frequency products are given, all the rest are given, and do not need to be considered, they can be wiped off the slate.

The resolution is unique, and therefore the zero phase sequence power product will be the resultant of the two zero phase sequence systems obtained by considering the positive and negative phase sequence currents and e. m. fs. as referred to in equation (**26**) in the paper, and will represent the mean polyphase true and reactive power per phase. The positive and negative phase sequence power components will correspond with those in equation (**26**) and will represent the mean interchange of power among the phases. The power factor obtained in this way is that known as the vector definition, or as some of us call it, the Italian definition. It is the true definition of power factor in the polyphase system. There is no other definition that is the true definition.

It will also be evident that while the true and reactive power due to the negative phase sequence components of e. m. f. and current is a measure of the loss in power due to the flow of negative phase sequence current it is not a true measure of the detrimental effect due to the flow of this unbalancing system of currents. We have to consider the fact that this negative phase sequence system will not only heat the wires, but has other detrimental effects.

At the present time a method of measuring reactive power in polyphase circuits is used which does not measure the reactive power when the system is unbalanced. What should be done is to use a reactive power meter, a meter of the same type as the wattmeter, which when connected in exactly the same manner will measure reactive power.

There are other methods of measuring. We have these components which refer to the power due to the positive phase sequence systems, and power due to the negative phase sequence systems. We can define the power of each of these and measure the quantity separately and combine them vectorially and get the Italian definition. Each of these quantities may be measured by a single element meter with a proper training network. This at first sight may appear to be an artificial method of accomplishing these results, but, when it is remembered that the ordinary single-phase measurement of power is accomplished by means of a restraint in the form of non-inductive resistance in series with the voltage coil, and that the ordinary watthour meter for the measurement of single-phase energy depends upon a restraint in the form of inductance to obtain current in the voltage coil in quadrature to the impressed e. m. f., it will be realized that all schemes of measuring power and energy are more or less artificial. Because the proposed method happens to be a little different from what we are accustomed to, is no reason why it should not be attempted. The sole criterion should be the degree of accuracy that can be obtained.

R. D. Evans: Power factor is a term which describes the energy in an a-c. circuit. It expresses the ratio between the amount of energy converted into heat in mechanical work, and the amount of energy periodically absorbed and discharged in a circuit. Power factor is not the simplest factor relating to true power and reactive power, but it is a convenient factor, as it expresses the ratio between the amount of power actually delivered, and the amount of power that may be delivered without exceeding the same heating.

The definition of power factor for single-phase circuits adopted by the Institute and incorporated in the present rule cannot be readily extended to include that of polyphase circuits, and this is due, not to the fact that the power factor of polyphase circuits is different from that of the power factor of single circuits, but it is due to the particular way in which the power factor of the single-phase circuits has been found. In general it is to be noted that the definition of power factor from the standpoint of phase difference or the ratio of watts to volt-amperes requires modification to take care of the presence of harmonics, plurality of phases and of unbalancing. On the other hand, the definition of power factor from the standpoint of the pull energy is complete and is independent of the presence of harmonics, plurality of phase and unbalancing.

Considering next the device which separates and measures the components in an unbalanced system, as has been shown by Mr. Fortescue in the three-phase three-wire system, the unbalance components may be separated into positive and negative sequence of voltage, current and power. The theory of these instruments is given in the paper, but the significant thing in connection with these instruments, I believe to be, that these quantities are capable of measurement, because they have an actual existence. These quantities have been measured with considerable accuracy, and the apparatus required for measuring them is simple. The device for measuring positive sequence voltages consists of the meter element, and a descriptive diagram. The positive sequence ammeter consists of the meter element, two of different power factors, and two current transformers connecting the meter with the circuit. The negative sequence ammeter is obtained by the interchanging of the two meters, and similarly the negative sequence ammeter is obtained by the interchange of these two impedances.

Having separated out the positive and negative sequence components of voltage and current, the next step is to combine them to measure the power, the reactive power and the power factor. The instruments will measure positive sequence power, positive sequence reactive power and positive sequence power factor. In the three-phase three-wire system full power is the sum of the positive and negative sequence of the power. The total reactive power is the sum of the positive and negative sequence of reactive power. Hence, it is possible to measure the total reactive power of two phases by the use of two meter elements supplied with this positive and negative sequence component, voltage and current.

In conclusion, power factor, reactive factor, power and reactive power, must be defined so that their mutual relations are shown, and so that they correspond to the physical facts. A scientific or exact definition of power factor should be based on the flow of energy.

Power factor is the ratio of true power to the square root of the sum of the squares of true power and reactive power.

It must be recognized that power and power factor do not completely describe the flow of energy, and that the use of some unbalance factor is necessary.

A. Nyman: The selection of a suitable definition of power factor requires consideration from three main viewpoints: Technical, commercial, legal.

From the technical viewpoint, the power factor should give a measure of the losses in the system, the excitation of generator units, and the burden on capacity of power supply circuits.

In a single-phase system these values are entirely determined by the magnitude of the current.

In a polyphase system the sum of the squares of the currents determines the first factor. The second and the third factors are determined in addition by the voltage and current unbalance. A definition based on the sum of the squares of the currents, like one described by Mr. Silsbee, based on effective volt-amperes, would give a value satisfying the first factor and partly satisfying the second and third factors.

A definition based upon two arithmetic sums of volt amperes is an approximation simpler in form. It approaches very closely to satisfying the first factor and takes account of factors 2 and 3.

The vector sum of volt-amperes used as a basis for definition is claimed to be theoretically correct. It defines the power factor on the basis of energy components in the system. This definition, however, disregards the effect of the unbalanced loads.

An attempt has been made as outlined in Mr. Fechheimer's paper, to separate some of the objects of measurement by defining, separately, power factor, and unbalance. A large amount of mathematics is involved in presenting this method. The actual working, once mastered, does not appear to be difficult and might give a clearer conception of actual distribution of cost items. However, the two factors obtained do not give a direct measure of losses and the consequent expenses. They require additional mathematical manipulation in order to determine the total value of losses. For balanced voltage supply, the power factor determined by this method is the same as the one obtained by simpler means for the vector sum definition. The vector sum of volt-amperes will take into account voltage unbalance, while the power factor, as determined by this method, neglects it. This scheme of measuring does not cover all cases, but is probably the closest to exactness. Its main objections are the complexity and the need for special measuring instruments.

Chart 1 gives a number of concrete examples of balanced and unbalanced loads. It will be seen that Definition 1, as outlined by the committee, approaches closest to the one based on losses (Definition 4). Definition 2 disregards the additional losses, due to unbalanced current, and neglects also the effect of unbalance on excitation and on the capacity of lines. Definition 3 gives results equal to Definition 2. The values will differ, however, if any voltage unbalance exists.

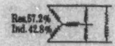
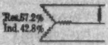
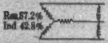
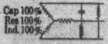
CHART NO. I.

1. Definition one of Committee. $\dfrac{\text{Watts}}{\text{Sum of Volt Amps.}}$

2. Definition two of Committee. $\dfrac{\text{Watts}}{\text{Vector Sum Volt Amps.}}$

3. Separate power factor & unbalance factor.

4. Loss basis. $\dfrac{\text{Watts}}{\sqrt{3\,(I_a^2 + I_b^2 + I_c^2)} \times \text{(average volts)}}$

Load	(1)	(2)	(3)	(4)	Error in (1)
					Compared to (4)
	.80	.80	.80	.80	0%
	.694	.80	.80	.566	22%
	.743	.80	.80	.716	4%
	.358	1.00	1.00	.346	3.4%

From the commercial viewpoint, the definition of power factor should be considered as affecting the following: The central station; the user of electric energy and the manufacturer of electric apparatus.

The central station is concerned in the accuracy of the term in order to get a reference to expenditures above the direct energy supplied, but determined by the nature of the load on the station. However, complication should be avoided. Introduction of two related terms like power factor and unbalance factor, each representing a certain phase of expenditure, would involve additional clerical work in determining charges. The changes in rates would be subject to legal action. Disputes with customers would require technical explanations of great complexity, yet even with such a definition, the charge would not correspond exactly to the cost of the load. A simpler definition, giving a fair measure of accuracy and involving only one term, appears as a desirable compromise.

CHART NO II.

$\frac{\text{Definition}}{\text{Aspect}}$		1. $\frac{\text{Watts}}{\text{Sum V. A.}}$	2. $\frac{\text{Watts}}{\text{Vector sum V. A.}}$	$\frac{\text{Pos. Seq. Watts}}{\text{Pos. Seq. V. A.}}$ 3. $\frac{\text{Neg. Seq. Cur.}}{\text{Pos. Seq. Cur.}}$
Technical		Takes account of phase lag and of current and voltage unbalance.	Takes account of nature of load only. Neglects unbalance.	Takes account of phase lag. Takes account of unbalance.
Commercial	Central Station	Simplicity & fair accuracy.	Additional measurements necessary for unbalance.	Special charge rates necessary.
	Customer	Simplicity	Unbalance must be charged separately.	Relatively complex and difficult to explain & measure.
	Manufacturer	Fair accuracy & simplicity. No special instrument necessary.	Incomplete	Very complete but complicated.
Legal		Simplicity	Incomplete and not so simple as (1).	Special rate complication in legal document.

The user of electric energy is concerned in getting a fair rate of charge for the energy used. The simpler the method of charging, the better, provided no particular load receives an excessive charge. However, no definition appears to discriminate between an unbalance inherent in the load or that created by an unbalance of the voltages of the system. Special measurements would be necessary to effect such discrimination. Meters of the type described by Mr. Evans, could be designed for this purpose. A far more serious effect of unbalance of voltages is the reduced illumination from the lighting fixtures on the lines of lower potential. Such an unbalance is really a mark of poor service and would be objected to by all concerned. It is really up to the power stations to distribute the unbalanced loads, or use apparatus for reestablishing the balance of voltages.

The manufacturer of electrical apparatus is interested in having a definition of power factor permitting proper rating of power apparatus. An exact definition is a good goal. A compromise for simplicity is, however, very desirable. The legal documents for ordering a piece of machinery are already heavily loaded with complicated technical matter.

The supply of instruments to measure whatever quantities may be decided upon is mostly a matter of design. Almost any definition can be directly measured by an instrument developed for the purpose. The advantage still lies with a definition which uses directly measureable quantities. Definition 1 of the Committee is of this nature.

Referring to the Chart 2, an outline is given of various factors affecting the definition of power factor and covered in this discussion.

R. Karl Honaman: Mr. Silsbee has taken up various suggestions for the definition of power factor and analyzed them with a view to determining the technical significance of each and also the economic significance of each, *i. e.*, the relation between the power factor and the increased equipment required to supply a certain distorted load or the increased losses with the same equipment. This is the important question in connection with the commercial application of any definition of power factor when it is used to determine rates.

The quantity in single-phase circuits that represents the ratio of kw. to kv-a. depends only on phase displacement between voltage and current. In polyphase circuits this ratio depends also on load distribution. The factor which combines these two effects and which has been called economy factor represents this ratio for polyphase circuits. Moreover this economy factor is the simple product of a factor determined by phase relation between voltage and current and a factor determined by the departure from symmetrical distribution of the load. It has been pointed out that ultimately it will be necessary to recognize that distortion in the polyphase system is really a combination of these two effects and that the situation will be clarified by avoiding trying to lump these two effects under one name. This paper has shown that this is sound economically as well as physically.

A. E. Kennelly: I think we are all agreed upon certain fundamental facts, first that if a polyphase system is balanced, we can determine the power factor with the same precision and the same degree of simplicity as in the case of a single-phase circuit, provided, of course, that we are dealing with sine waves. We also agree that if we have a polyphase system out of balance, and with harmonics, that the strict definition of power factor becomes so complicated and requires so much apparatus, that it becomes a matter of academic discussion rather than practical engineering operation. Therefore, it is necessary to agree, it seems to me, upon certain conventions which we can reasonably support, just as we have already come to certain conventions for the case of the single-phase circuit, where we have agreed to ignore the effects of harmonics in the ordinary measurement of power factor. When we have a balanced three-phase system, with one frequency and no harmonics we agree that we can determine the total active power of the system from the indication of a single polyphase wattmeter, or the total active energy by the reading of a single polyphase watthour meter.

We also agree, I think, that we can find the total reactive power in a polyphase system by the reading of a single polyphase reactive wattmeter. Our Committee was not asked to define the reactive factor for a polyphase system. It was asked to define the power factor. But, of course, as has been pointed out, if you can define the reactive factor, then you can define the power factor in terms of the same.

It is a question, of course, as to how much you are to depend upon slide-rule arithmetic and to what extent you want the apparatus to perform the operation for you automatically, but, at least, as engineers, we should be able to come to a conclusion, and record our conclusion, that in the case of the balanced polyphase system there is no dispute concerning what constitutes the active power by a polyphase wattmeter, and what constitutes the reactive power by a polyphase reactive wattmeter.

Assuming that we agree so far, then the question comes—What are we going to do about harmonics? The agreement we come to so far, judging from the papers and discussions, is that we can afford to neglect them. We can afford to say that harmonics will interfere with the strict mathematical statement of polyphase power factor, but harmonics are already neglected in the single-phase case, so that we can afford to neglect them in the polyphase case.

If we can agree, then, to ignore these harmonics, the sole remaining matter of discussion, and serious discussion among us, is the question of unbalance, and what should be done for unbalanced polyphase circuits. It is a serious question. To determine the power factor merely of a balanced polyphase system is well worth while; but it is not enough. We must seek for some agreement in regard to a quantitative definition of unbalance.

Five different ways of establishing an unbalance factor are proposed in the papers here before us. One of the five is the method which has been proposed by Mr. Fortescue, who has suggested a very beautiful way of resolving an unbalanced three-phase system into a forward balanced system plus a backward balanced system. That is a beautiful method, but requires at the present time, certain particular apparatus in order to effect this resolution, and it seems to me that we ought, at the present stage at least, to try to confine our operation to those which can be performed by the regular apparatus with which not only our customers are acquainted, but which all engineers are in the habit of using.

The Committee offered a suggestion in its first definition to combine a power factor, as obtained from the second definition, with an unbalance factor, to give what has been proposed as the "economic power factor," or "apparent power factor,"

Our difference of opinion comes there as to whether that particular method of adopting the unbalance factor should be the one we select or not. The unbalance factor can be referred back to the committee for further investigation and report, but it seems to me we can well agree on the power factor according to the second definition indicating that power factor does not recognize and is not concerned with unbalance.

Philip Torchio: Prof. Kennelly summarized the practical consensus of opinion of the speakers of the evening that the definition No. 2 of power factor being the ratio derived from the active power and the reactive power is a correct definition of polyphase power factor. I think on that we ought to agree.

My paper, on page 1490, points out that all the power engineer is concerned with is to have the power factor of definition No. 2, and the contemporaneous unbalanced load. The question of combining the unbalanced load with the power factor in one empirical formula, I think, is not a matter that should be decided by the Institute. I think it is a question of commercial application, and if we define the power factor according to the true logical definition No. 2, the power engineer can easily, from the readings of the instruments by which he will get the active power and the reactive power, secure sufficient data to define the unbalance. I do not think the Institute should at this time take up the question of defining the unbalance.

The definition of power factor is a technical and scientific subject, and I think the Institute will accomplish a thing which will be of great benefit to the industry by standardizing it.

In my paper I stated that the unbalance factor and the power factor should include time. I think the element of time should be incorporated in the ultimate definition.

This matter of *reactive power* has been discussed by our sister French organization, and they suggested that a special name should be given to it. It should be, probably, the province of this Institute to suggest a name. We call kilowatt the active power, and we ought to have some term to determine the name of the reactive power. I think it would facilitate the interpretation and the application of the definition to the industry if we should give it a definite name. Prof. Karapetoff has suggested rewatts—it may be that a better name could be used, but whatever name is adopted, I think it would be a great advantage to the industry.

Herbert Bristol Dwight: There are a number of engineers who wish to use definition No. 2 for power factor, and who wish to have a definition for balance factor. A definition for balance factor should be made as simple and easily understood as possible, and it should not be complicated by attempting to make it directly applicable to the computing of bills for energy. The present definition of power factor for balanced conditions is simple and is accepted by everyone, but it cannot be applied directly to the determination of an equitable price for energy, and so it seems too much to expect that a definition of balance factor can be applied directly in this way. The same is true of the definition for load factor. To state this more definitely the cost of energy per kilowatt hour cannot be said to have increased in the ratio of 80 to 100 because the power factor has changed from 100 per cent to 80 per cent. The cost has really changed by some smaller ratio than 80 to 100 and this smaller ratio can only be approximated roughly and varies greatly under differing conditions.

Therefore, the statement can be made that the numerical value given by a definition of power factor or balance factor is really of small importance, because it must be changed before it can be used in connection with the relations of sellers and purchasers of energy. The simplicity of the definition and its ease of explanation to non-technical customers are of far greater importance.

The following definition is suggested in the belief that it offers advantages of simplicity and ease of explanation. No attempt is made to separate the parts of the unbalance for which the customer and the supply company are responsible, nor is any attempt made to compute the losses in certain apparatus due to the unbalance. These are separate problems from the statement of a defintion.

Assuming that the voltage and current are sinusoidal and that Definition 2 of the Power Factor Committee is to be preferred, the volt-amperes in a polyphase circuit are equal to the vector sum of the volt-amperes in the several phases. I wish to suggest the following definition of the balance factor:

"The balance factor in an unbalanced circuit is the ratio of the volt-amperes in the unbalanced circuit (by the vector sum definition) to the volt-amperes in a balanced symmetrical circuit of the same kind, whose voltage between conductors adjacent in phase is equal to the highest voltage between any two conductors adjacent in phase in the unbalanced circuit, and whose current per conductor is equal to the greatest current in any of the conductors of the unbalanced circuit.

This applies to direct-current, single-phase and polyphase circuits.

N. B. This refers strictly only to cases where the voltage and current are both sinusoidal and to direct-current circuits."

The above definition can be explained to a non-technical purchaser of energy in the same way as the ordinary definition of power factor. In a bank of transformers, which may be used as an example, the iron magnetic circuit is of such size as to hold up a certain voltage, and the conductors are of such size as to carry a certain current. The provisions for voltage and current constitute a large part, though of course, not the whole, of the cost of delivering energy. If the voltage and current are in phase, a certain maximum power can be handled. If they are not in phase, the power is reduced, and the ratio of the actual power to the maximum power is called the power factor. So, also, if the voltages and currents are equal and symmetrical in phase, a certain maximum kv-a. can be handled. If conditions are unbalanced, the kv-a. are reduced, and the ratio of the actual kv-a. to the maximum kv-a. is called the balance factor, according to the definition which I have here suggested.

The above definition and explanation seem to me to be as simple and as easily understood as possible. If this definition is adopted by the Institute, the following simple statement could be made: Load factor, power factor and balance factor are all equal to the ratio of an electrical quantity as its exists, to the maximum value of the same quantity for which provision has been made.

Reference has been made to the request of the I. E. C. for a single syllable word to correspond with the word watt and to represent one reactive volt-ampere. Since reactive volt-amperes are for the most part caused by the magnetizing current of induction motors and transformers, I would suggest the word "mag." Thus, one reactive kv-a. would be equal to one "kilo-mag."

W. H. Pratt: I heartily sympathize with the views expressed by Prof. Karapetoff, and I may state further, they are much in line with some remarks I made in discussing the same matter with Dr. Kennelly some time ago, and that is, that we make a mistake by throwing things into the cosine form, when we are really not interested much in that cosine. There are many cases where we would greatly benefit our discussion if we at least went no further than to speak of the angle that corresponds to the determination which we made by the measurements.

The paper which I presented has a very restricted purpose—it had in view this thought—the power factor in its origin and as recognized by the definition which was established by the Institute, and which is still in force represents not the measure of phase displacement, but simply a factor to reconcile the readings of voltmeters and ammeters, with the wattmeter connected to measure the same circuit. As far as I can see, in spite of the fact that the term is used much more generally to describe a matter connected with phase displacement, it has never been definitely and correctly used except in this other manner.

For that reason it seems to me the Definition No. 1, which I grant has not a very extensive use, does give a quantity which practically exactly corresponds with the single-phase definition.

To be sure, a single-phase load applied to a three-phase circuit will give you a value of power factor, which, for a non-inductive load would be other than unity, but that is in the power factor of the single-phase load. The quantity described in definition No 2 is a much more useful quantity in general. It would seem to me, however, it does not fall in line with the definition used and recognized for single-phase work.

For this reason it would seem that if the quantity defined by No. 2 is recognized as the quantity we want to use, we should give it some distinctive name. It perhaps might be well, if we must get near to the term power factor, not to get any nearer than to call it vector power.

G. A. Sawin: In the Committee report it is stated: "The Committee now feels that its initial purpose was somewhat too ambitious and that in attempting a complete solution of a problem of this magnitude, did not fully realize the far reaching responsibility of attempting to make or advance the ultimate decision."

I think it would be most unfortunate if the Committee's functions were completed with this report, as it would not make for progress. I move that the Committee be continued in this work and instructed to revise its preliminary report in order to make to the Standards Committee a final report and further instructed in making that final report, the Committee pay particular attention to the consensus of opinion that has been expressed here tonight. That final report, should be made just as soon as the Committee can be called together, and the report prepared and submitted to the Standards Committee.

J. R. Craighead: In reference to the "conseusus of opinion" it seems to me that the Committee should be instructed to regard all the discussion that has been presented on this subject.

A. E. Kennelly: I am sure when the Committee convenes they will be a little in doubt as to just what the consensus of opinion may have been. If the meeting would be willing to formulate any expression of its consensus of opinion arrived at as a result of the discussion on this floor, for example, on the balance system, the Definition No. 2 might be taken as the factor, and I think it would be very gratefully received, I am sure, by the Committee as a whole.

The Chairman: There is no reason why you should not offer an amendment incorporating what you have in mind.

A. E. Kennelly: Will the amendment be acceptable in stating as the consensus of opinion namely, that Definition No. 2 be here accepted as the expression of the definition of the vector power factor of the balanced three-phase system, without regard to the question of unbalance?

George A. Sawin: I will accept the amendment.

Philip Torchio: Dr. Kennelly's amendment applies to a balanced load. The consensus of opinion of this meeting was that in defining power factor we should disregard the question of the effect of unbalance, and should adopt a definition of power factor as power factor alone, applied to a balanced or unbalanced load, and I would move a further amendment, if acceptable to Dr. Kennelly, that the balanced load be stricken out, and it should be made to apply generally to any polyphase load.

A. E. Kennelly: I accept the revision of Mr. Torchio, if that is acceptable to the original proposer.

George A. Sawin: If we could get the expression of opinion first that would help matters.

The Chairman: Suppose we divide it into two motions.

Philip Torchio: I will make the motion as follows:

That it is the consensus of opinion of this meeting that the polyphase power factor be defined as the ratio between the active power and the reactive power.

C. L. Fortescue: I suggest you express it as derived from the active and reactive power, because we still want to retain the cosine of the angle.

Philip Torchio: I will then revise the motion to read as follows:

The polyphase power factor be defined as being derived from the ratio of the active power to the reactive power.

The Chairman: You have heard Mr. Torchio's motion as he has reframed it. (The motion was duly seconded.)

W. H. Pratt: As I understand the motion it reads that it is the consensus of opinion of this meeting that the polyphase power factor be defined as being derived from the ratio of the active power to the reactive power. Now, my criticism is that it would be better to call that vector power factor, as related to polyphase circuits.

Philip Torchio: It would complicate the expression, as I understand it.

W. H. Pratt: I would rather not see the name of this thing tied down too definitely before the final discussion in the Committee.

C. L. Fortesque: Let us have one power factor—do not let us define it in any way. If we start to define that as vector power factor, we will have this and the other power factor. Call it power factor as derived from the ratio of the active or true power to the reactive power.

The Chairman: You have heard the discussion. All in favor say "aye". The motion is carried.

George A. Sawin: I will now renew my motion, that the Committee be instructed to formulate the final report to the Standards Committee, and do it immediately, including in this main motion the motion which has just been pased.

(The motion was duly seconded, put to vote and carried.)

W. V. Lyon (communicated after adjournment): In many commercial circuits the wave forms of voltage and current do not depart seriously from the sinusoidal. This fact might well have formed the basis for our definitions of the volt and ampere. While such definitions might have had practical value, their usefulness in scientific theory and research would have been slight. It would matter but little that the errors were generally small. Any unnecessary error in theory is not excusable. The responsible scientific bodies, however, have chosen definitions for these quantities which do not depend upon the manner of their cyclic variation. This stand is entirely right. Wherever possible, a definition should not have its application restricted to special or ideal conditions. The writer most heartily indorses Mr. Fortescue when he says:

"When new factors come up for definition, the proper procedure is to find out if there is some scientific basis on which to form a definition. A definition should not be based on mere surmise, or opinion, but should be scientifically derived, and those to whose duty it falls to frame such definitions should not be influenced by the state of the art, or by whether the requisite instruments are available or not, but should aim to make them as rigidly correct and as comprehensive as possible. The art will follow quickly enough. It is not vitally important in practical work that all definitions shall be rigidly adhered to, but it is vitally important that our standards and definitions are based on sound scientific principles. It is not of great importance whether a carpenter, in practising his trade, uses a wooden yard measure or a steel tape, but no one questions the importance of having accurate gages and standards of length."

Is it not best to incorporate this same breadth of view in the definition for power factor? The writer believes that if this is done, a simple, general and workable definition can be formulated.

In the term "power factor," we recognize that the rate at which a given alternating current does work is not entirely dependent upon its loss in electric pressure as it flows through the circuit. Both from our theoretical and practical experience, we have learned that, due to their inherent characteristics, some circuits absorb more power than others when a given current flows through them at a given loss in electric pressure. Usually, although not always, this may be explained as due to the cyclic storage and release of electric energy. When both the current and voltage vary sinusoidally, the maximum rate of this storage is generally known as the reactive power. For this ideal condition of voltage and current variation, the reactive power may be measured with a reactive wattmeter. Unfortunately, in the general case, when the voltage is non-sinusoidal or the circuit constants vary cyclicly, any rationally defined reactive power cannot be measured with a reactive wattmeter. For example, when a non-sinusoidal voltage of fixed r. m. s. value, but with a wave form which can be varied at will, is impressed on a circuit whose resistance and inductance are strictly constant, the conditions of energy storage can be altered to a considerable degree without changing the reading of either an active or a reactive wattmeter which is properly connected in the circuit. Again, any non-reactive circuit whose resistance varies cyclicly, as does that of an incandescent lamp, absorbs less power than if the resistance were constant and of such a value that the current would be the same as that through the lamp. A reactive wattmeter would, of course, fail to indicate, if connected in this circuit.*

The writer believes that no rational and general definition of power factor can be given which is based either on the idea of reactive power or of a reactive wattmeter reading. There is, however, a simple and general conception of power factor which might be used. Furthermore, it is equivalent to that already adopted by the A. I. E. E. for the case of a single-phase circuit. The scientific fact is this: there is one type of circuit, *viz.*, a constant non-reactive resistance, which will absorb more power than any other type when a

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*See "Reactive Power and Unbalanced Circuits," W. V. Lyon, *Electrical World*, June 19, 1920.

given current flows through it at a given loss in electric pressure. The commercial application of this is that a consumer who has any other type of load fails to utilize the power that is placed at his disposal. He demands the use of a certain capacity of electrical equipment, and then regularly in every half cycle (of voltage) he returns a portion of the energy he receives. He is in the same category as the merchant who regularly orders merchandise, and just as regularly returns a portion of it.

It may be of interest to some to prove that with fixed r. m. s. values of voltage and current, the greatest power is absorbed when the circuit is a constant non-reactive resistance,—constant in the sense that it does not vary in any way, even within the time of one cycle.

In general, the voltage and current may be analyzed into their Fourier components, thus:

$$e = \sum_1^\alpha \quad \sqrt{2}\, E_n \sin (n\, \omega\, t + \alpha_n)$$

and $i = \sum_1^\alpha \quad \sqrt{2}\, I_n \sin (n\, \omega\, t + \alpha_n - \theta_n)$

The mean-square voltage and current are:

$$(E^v)^2 = \sum_1^\alpha E_n^2$$

$$(I^a)^2 = \sum_1^\alpha I_n^2$$

The square of the average power is:

$$P^2 = \{ \sum_1^\alpha E_n\, I_n \operatorname{Cos} \theta_n \}^2$$

What we wish to prove is that for any value of frequency and with any harmonics present, the greatest value that the average power can have is the product of the current and voltage. A typical term in the square of the product of voltage and current is:

$$E_n^2\, I_n^2$$

The corresponding typical term in the square of the power is:

$$E_n^2\, I_n^2 \operatorname{Cos}^2 \theta_n$$

There are also typical pairs of terms in the square of the product of voltage and current, such as

$$E_n^2\, I_p^2 + E_p^2\, I_n^2$$

The corresponding typical product term in the square of the power, such as $2\, E_n\, I_n \operatorname{Cos} \theta_n \,.\, E_p\, I_p \operatorname{Cos} \theta_p$, may be written

$$2\, E_n\, I_p\, E_p\, I_n \cos \theta_n \cos \theta_p$$

There is a theorem in algebra that the greatest value of $(2\,a\,b)$ is $(a^2 + b^2)$ and furthermore, this occurs when a and b are equal. Thus the power is always less than the volt-amperes, except when the phase angle is zero (for every frequency) and when $E_n\,I_p = E_p\,I_n$, that is, when there is the same ratio between all of the corresponding harmonics of voltage and current. This is, of course, true only when the voltage and current have exactly the same wave form. The only conceivable kind of electric circuit which fulfills this condition is a constant non-reactive resistance. With such a circuit, the resistance, r, is

$$r = \frac{E^v}{I^a}$$

This relation holds, irrespective of the wave form of the voltage. This constant non-reactive resistance will absorb more power than any other type of circuit from a given root-mean-square current flowing through it at a given root-mean-square loss in pressure. If such be the case, it might be well to take this circuit as the standard and compare the power absorption quality of every other circuit with its own. This is equivalent to the present A. I. E. E. definition of power factor as applied to single-phase circuits. The power factor would be the ratio of the actual power taken by the circuit to that which the standard circuit would take under the same conditions of r. m. s. voltage and current. That is, it would be the ratio of the wattmeter reading to the product of the voltmeter and ammeter readings.

There is no difficulty in extending this conception of power factor to a circuit with any number of phases. All that is necessary is to imagine that the actual load is replaced by an arrangement of "standard" circuits, *i. e.*, constant non-reactive resistances—so adjusted that the root-mean-square line currents they take are equal to those taken by the actual load.* The r. m. s. line voltages are the same in each case. The writer has in mind a three-phase circuit, without a neutral conductor. In some other cases, the phraseology might need to be modified, but the underlying principle must be obvious. For any given values of r. m. s. line voltages and r. m. s. line currents, the power absorbed is never greater than when the load is a combination of constant non-reactive resistances. There are some special cases of extreme unbalance in which it is necessary to assume that one of the resistances has a negative significance. There are also cases in which the same power is taken by the actual load as by the resistance units, *i. e.*, those in which, assuming sine waves, the algebraic sum of the reactive volt-amperes is zero. The power factor is thus the ratio of the actual power to the greatest possible power that could be absorbed by any load taking the same r. m. s. line currents at the same r. m. s. line voltages. According to this conception, the power factor of any load would be the ratio of the measured power taken by the load to the measured power taken from the same system of voltages by an arrangement (Δ or Y) of "standard" circuits—the r. m. s. line currents being the same in each case. The standard circuit might, of course, be arranged in miniature, so that multiplying factors for voltage and current would be used.

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*Notice that this does not alter any degree of unbalance that may exist in the line currents.

This method might well be considered impractical in many cases, and it is desirable to develop others that will at least give a fair degree of accuracy. Fortunately, the power taken by the standard resistance units may be *calculated* in any case where the root-mean-square line currents and voltages are known.

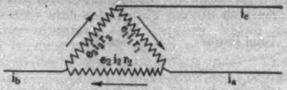

FIG. 1

Let Fig. 1 represent the "standard" circuits connected in delta, and adjusted so that the r. m. s. line currents I_a, I_b and I_c are respectively equal to the corresponding r. m. s. line currents taken by the actual load. Let the small letters represent instantaneous values of voltage and current. The directions of the arrows show the assumed positive directions of both voltage and current. Since the line current, i_a is equal to the difference of the adjacent delta currents, i_1 and i_2 and the sum of the line voltages is zero.* The following equation for the mean square line current I_a^2 may be written:

$$I_a^2 = \frac{1}{T}\int_0^T \left\{ \frac{e_1^2}{r_1^2} + \frac{e_2^2}{r_2^2} - \frac{2\, e_1\, e_2}{r_1\, r_2} \right\} d\, t$$

$$= \frac{1}{T}\int_0^T \left\{ \frac{e_1^2}{r_1}\left(\frac{1}{r_1} + \frac{1}{r_2} \right) + \right.$$

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*If $e_1 + e_2 + e_3 = 0$

$2\, e_1\, e_2 = e_3^2 - e_1^2 - e_2^2$

$$\frac{e_2^2}{r_2}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)-\frac{e_3^2}{r_1 r_2}\Bigg\}\,d\,t$$

Similar expressions for the mean square line currents I_b^2 and I_c^2 may be written.

The solution for the current distribution, and the power absorbed, involving as it does only the mean square line voltages, is independent of their wave form, and is thus the same as if the line voltages were actually sinusoidal. There is, however, a definite method of solution in the case of sinusoidal wave forms—*viz.*, the vector method. Thus a correct result is obtained for the solution of a certain problem involving an unbalanced system of non-sinusoidal voltages by employing vector analysis in exactly the same way it would be employed if the voltages and currents were sinusoidal. There is, then a simple graphical method for obtaining the power absorbed by the "standard" circuits.*

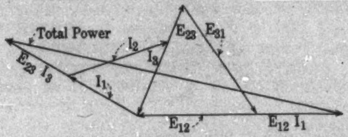

FIG. 2

The total power taken by the resistances is the vector sums of the volt-amperes of two wattmeters properly connected to measure the true power. The angle between the volt-amperes is the sum of the angle between the voltages E_{12} and E_{23} and the angle between the currents I_1 and I_3 when both voltages and currents form the sides of triangles as shown in Fig. 2.

In commercial practise, when it is not deemed necessary to actually measure the power factor as here defined, and when the harmonics present are relatively insignificant, an approximate value of the power factor might be calculated from the readings of active and reactive wattmeters in the proposed manner.

The definition that the writer proposes, being the ratio of the actual power to that absorbed by a "standard" circuit, is restricted in no way whatsoever by wave form, the number of phases, or the degree of their unbalance, and, thus being general, it seems to approach the ideal as set forth so admirably by Mr. Fortescue.

***Electrical World*, June 19, 1920.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 2, 1920.

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CONSIDERATIONS WHICH DETERMINE THE SELECTION AND GENERAL DESIGN OF AN EXCITER SYSTEM

BY J. T. BARRON
General Superintendent of Production, Public Service Electric Co.,
Newark, N. J.

AND ALEX. E. BAUHAN
Superintendent of Stations, Pennsylvania Water and Power Co.,
Baltimore, Md.

Excitation systems may be divided into (a) Central Systems with separately driven exciters, (b) Central Systems with direct-connected exciters, (c) Individual Systems with direct-connected exciters and (d) Individual Systems with separately driven exciters.

The paper discusses the factors which must be considered in determining which of the above systems is the best to use in any particular case. The topics discussed are:—Auxiliary Power Service, Voltage Regution, Storage Battery, Space Requirements, Initial Cost, Operating Cost, Effect of Power System Troubles, Effect of Exciter System Troubles, Simplicity and Reliability.

The paper also considers subjects related to the completion of an excitation design layout under the headings of Number and Size of Exciter Units, Kind of Drive, Shunt vs. Compound Exciters, Voltage, Bus Arrangements and Protection.

IT is the purpose of this paper (a) to discuss the factors which determine the selection of an exciter system for a generating station and (b) to discuss other general design features of excitation systems. No attempt is made to recommend any particular system for general application but rather to present a method of analyzing the problem for a particular case.

Excitation systems, as shown in the accompanying figure, may be roughly classified as:

A. Central system with separately driven exciters.
B. Control system with direct-connected exciters.
C. Individual system with direct-connected exciters.
D. Individual system with separately driven exciters.

Given a particular station, generator design and set of operating conditions, the question of which of the above systems is to be used can be determined mainly by consideration of the following topics:

1. Auxiliary power service.
2. Voltage regulation.
3. Storage battery.
4. Space requirements.
5. Initial cost.
6. Operating cost.
7. Effect of power system troubles.
8. Effect of exciter system troubles.
9. Simplicity.
10. Reliability.

When the particular system of excitation to be used has been determined, consideration must be given to the following matters before the general design of the excitation system can be laid out. These points in the main have little to do with determining the system of excitation.

1. Number and size of exciter units.
2. Kind of drive.
3. Shunt or compound wound exciters.
4. Voltage.
5. Bus arrangements.
6. Protection.

PART I

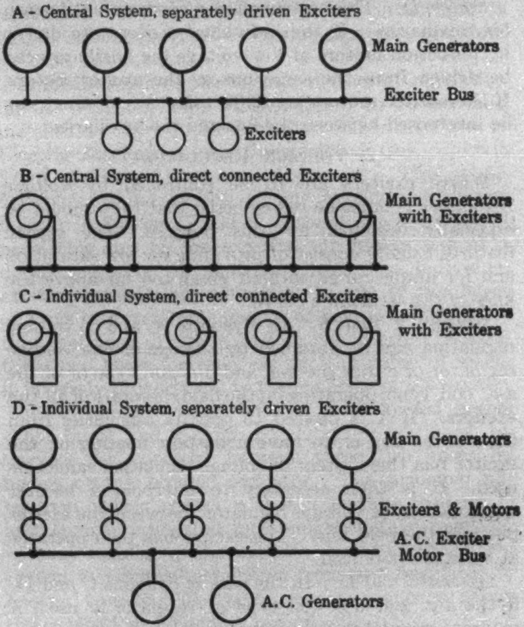
CONSIDERATION OF FACTORS WHICH DETERMINE THE SELECTION OF THE EXCITER SYSTEM

1. AUXILIARY POWER SERVICE

The practise of using the excitation system to supply auxiliaries is common in hydroelectric plants but is not the general practise in steam plants. It is not within the scope of this paper to discuss the merits of the different methods of driving auxiliaries but if it is decided that some of the auxiliaries must be fed by the excitation system, the choice of system is affected.

Systems A and B. These systems present no difficulties in the matter of power supply to auxiliaries. Auxiliaries may be supplied directly from the exciter bus or from a separate bus to which one of the exciters not needed for excitation purposes may be connected.

If the exciters are controlled by a voltage regulator it is not advisable to supply any auxiliaries from the excitation bus. In such a case it is necessary to have a separate bus for the auxiliaries. If enough exciters are provided to take care of the excitation on one bus and to take care of the auxiliaries on the other bus and still leave at least one spare unit, auxiliaries can be taken care of satisfactorily.


A - Central System, separately driven Exciters

B - Central System, direct connected Exciters

C - Individual System, direct connected Exciters

D - Individual System, separately driven Exciters

Main Generators
Exciter Bus
Exciters
Main Generators with Exciters
Main Generators with Exciters
Main Generators
Exciters & Motors
A.C. Exciter Motor Bus
A.C. Generators

Schematic Diagram of Excitation Systems

. If there are not enough exciter units to do this the only practical way of taking care of the auxiliaries is to feed them directly from the exciter bus and provide a booster for voltage regulating purposes, the exciter bus potential remaining constant.

It must be remembered that high voltages may be induced in the field circuits from main system troubles or by the loss of exciter supply and if provision is not made against this the high voltage may affect auxiliaries at least to the extent of blowing fuses.

System C. It is not practical to feed auxiliaries from the exciters in this system.

System D. This system offers a good source of power for auxiliaries. If the individual exciters are driven by induction motors at low voltage the auxiliaries can be driven from the same bus as the exciter motors. If the exciter motors are 2300 volts, transformers can be interposed between the bus and the auxiliaries.

2. VOLTAGE REGULATION

Where exciters are to be controlled by voltage regulators it must be remembered that they should be especially designed for this purpose. The special features usually consist of providing for low saturation and for ample range without going too far above the knee of the saturation curve.

Systems A and B. In Systems A and B voltage regulation can be obtained by the use of one voltage regulator of either the a-c. and d-c. coil type or of the a-c. coil type, operating on the field rheostats of all the exciters. If it is desired to operate auxiliaries from the exciter bus or to have a battery floating on the exciter bus this system of voltage regulation cannot be used. It is then necessary to interpose a booster controlled by a voltage regulator between the exciter bus and the field bus. The exciter bus then operates at constant potential.

Systems C and D. In the case of Systems C and D, if the a-c. and d-c. coil type of regulator is used, a separate regulator is usually used on each exciter. Each of these regulators must be equipped with a current connection which receives a current proportional to the generator current and at ninety degrees to the potential in the a-c. coil in order to stabilize reactive component distribution. When a regulator having only an a-c. coil is used, it is customary to operate exciters separately with but one regulator. The proper distribution of reactive component in this case is taken care of by designing or adjusting the exciters to have identical characteristics.

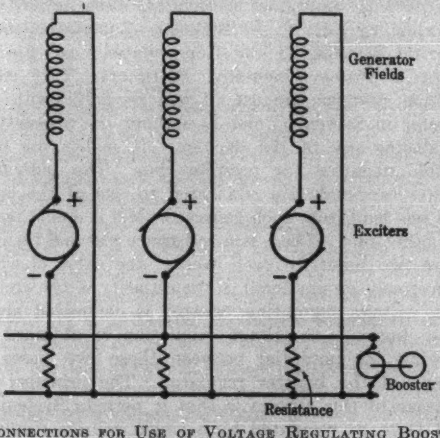
Using an individual regulator on each exciter makes it possible to regulate for two or three or more voltages on different sections of bus feeding different loads and fed by any combination of generators. If a-c. coil type regulators are used one will be required on each section of bus on which it is desired to have automatic regulation. This arrangement is of course not as flexible as that provided by having separate regulators on each generator.

A modification of the booster system of regulation for use with individual exciters has been devised and installed by Mr. F. E. Ricketts, Superintendent of Electric Stations of the Consolidated Gas, Electric Light & Power Company, Baltimore. This modification permits the use of one regulator with one booster in System C and D without the necessity of paralleling any of the exciters. It makes use of a double negative (or positive) bus. The individual exciter terminals are connected to one of the buses and one lead from each generator field is connected to the other bus. There is a resistance inserted between these two buses for each exciter, the value of which is inversely proportional to the capacity of the exciter. The voltage regulating booster is connected across these buses and voltage regulation is obtained by varying the potential between these two buses by means of the booster regulator. The principal advantage of this system is that it permits Systems C and D to be regulated by a single voltage regulator without paralleling the exciters and that the power system is practically immune from any regulator or booster trouble. The accompanying diagram shows the connections for such a system.

3. STORAGE BATTERY

A storage battery cannot be considered as taking the place of spare capacity. Its function is to immediately and automatically take over the load of an exciter which trips by relay action or ceases to generate, and to carry that load until the exciter is restored to service or a spare put in its place. It does away with the necessity of keeping spare exciters on the bus and thereby permits more efficient operation. A battery is justified only if the cost of operating spare exciters on the bus, or if the value of increased assurance of continuous service during times when no spare exciters are available, is greater than the fixed and operating costs of the battery. Large stations supplying lighting and commercial power load in the larger cities consider essential the installation of storage batteries.

The storage battery must be at least large enough to take over the entire load of one of the exciters for ten minutes. The battery in System A therefore must be larger than the battery for any of the other systems. It is not customary in Systems A and B to make the storage battery larger than what would be necessary to carry the entire load for more than one hour, seldom for more than one-half hour.



CONNECTIONS FOR USE OF VOLTAGE REGULATING BOOSTER WITH INDIVIDUAL EXCITERS

Systems A and B. Systems A and B offer the greatest opportunity for the advantageous use of a storage battery. If the exciters are controlled by an automatic voltage regulator it is not possible to float the battery on the bus without the use of complicated undesirable devices to control the position of the end cell switch. If automatic voltage regulation is desired a booster should be used for the purpose. The battery is then floated on the constant potential exciter bus.

Systems C and D. In connection with Systems C and D the battery cannot be operated in parallel with the exciters, but automatic devices can be installed to cut in the battery and to cut out a defective exciter. If such devices are installed the station attendants should be prepared to give such automatic devices the attention required to keep them in operating condition.

SPACE REQUIREMENTS

Space requirements will usually be a consideration only to the extent that they affect building and substructure costs. Definite space limitations may exist in a crowded city station.

System A. Exciters in System A will probably be fairly large units as compared with those in some of the other systems and will require that special consideration be given to providing space for them. The building and substructure will have to be made wider or longer to house them. For motor-driven exciters a space for transformers and switching structure may have to be specially provided. In the hydroelectric plant, turbines, governors, headgates, screens and other hydraulic equipment require space. In the steam station space for turbines, pressure and exhaust piping will be needed.

Systems B and C. These systems require very little otherwise unused space. In the vertical unit hydro plant the exciter will not require any additional head room over that needed for the assembly and dismantling of the units. Some space is needed for spare exciters if they are provided. In the plant with horizontal steam or hydro units some additional building space will be needed on account of the exciters, but not a great amount. If individual voltage regulators are used for each exciter this will take additional switchboard space.

System D. The space requirements for this system are about the same as those for System A. At each unit space must be provided for the individual exciter with its motor. Space needed for any spare exciters and their buses must also be kept in mind. As in System C individual voltage regulators call for additional switchboard space.

INITIAL COST

It has not been found possible to arrive at any definite comparison of costs that would be of value to anyone attempting to select a system of excitation for a certain station. Proper procedure in this case is to make a rough layout of each of the systems and to prepare cost estimates. System D will probably always cost more than any of the other systems. System A will cost more than Systems B and C in any except a very low head plant having a large number of units. A rough approximation of costs for a hydro plant having a head of 55 feet with eight 10,000-kw. generators operating at about 100 revolutions per minute shows that the cost of Systems B or C is 10 per cent to 30 per cent less and System D, 20 per cent to 40 per cent more than System A, the variations being due to variation in the details of design. The comparatively high costs in Systems A and D are due to the hydraulic equipment for driving the exciters, substructure and building requirements. For plants operating at still lower heads the advantage in favor of the direct-connected systems will be lessened by the rapidly increasing cost of the low-speed exciters.

For a large steam turbine plant Systems B and C will probably cost 30 per cent to 50 per cent less and System D, 20 per cent to 40 per cent more than System A. The figures for a large high-speed hydro plant will probably be similar to these.

6. OPERATING COST

Operating costs include fixed charges, attendance, supplies, maintenance material and labor, and energy.

Fixed charges are roughly proportional to initial costs. Attendance costs for the station are not ordinarily increased by the excitation system. A slightly better class of attendants may have to be provided on the main floor in large stations if they are expected to do any excitation switching. Supplies is a small item depending somewhat on the amount and class of equipment used.

Maintenance material and labor are dependent on the number of pieces and the class of equipment and the amount of complication rather than on the system used. Ten small exciters will require almost ten times as much maintenance as one large exciter. The principal maintenance costs in the large station will be in connection with circuit breakers, rheostats, control circuits, regulators, exciter commutators, batteries and automatic equipment.

COMPARISON OF EXCITATION EFFICIENCIES FOR HYDRO STATION

	Apparatus	System of Excitation				
		A	B	C	D	
		Central separate	Central direct	Individual direct	Individual separate	
Prime Mover Driven	Turbine	85% Eff.	90% Eff.	90% Eff.	85% Eff.	
	Exciter	93	90	90	A-C. Gen. 94 MotorGen. 85	
	Rheostats	80	80	..	..	
	Overall	63	65	81	67	
Motor Driven	Main Turbine	90	..	..	90	By emergency transformers
	Main Gen	96	..	..	96	
	Transformer	98	..	..	98	
	Motor-Gen. Set	87	..	..	85	
	Rheostats	80	..	..	..	
	Overall	59	..	..	72	

Energy costs may be compared in terms of efficiencies. In the following table are shown comparative overall efficiencies for the various systems in the case of a hydroelectric plant assuming the efficiencies of the component parts to be as indicated. Full-load efficiencies are assumed. No attempt has been made to make a comparison on the basis of all day efficiencies nor to take into account various operating considerations which might interfere with obtaining best efficiencies.

Rheostat losses may be reduced in Systems A and B if voltage regulators are used on the exciters. It is evident from the above table that the direct-connected systems without rheostats give the best efficiencies. The direct-connected systems will also probably have the best all day efficiencies.

A similar table for a steam plant cannot be conveniently drawn up on account of varying conditions. Efficiencies in this case will depend largely on the extent to which exhaust steam is utilized.

EFFECT OF POWER SYSTEM TROUBLE

(a) *Short Circuit on Feeder or on Bus Not Cleared Promptly.*

System A. Prime mover-driven exciters are unaffected. If only motor-driven exciters are used the exciter voltage will drop and cause reduced short-circuit current. This in some cases may be considered desirable from the standpoint of protection to oil switches and equipment and in other cases is considered undesirable for the reason that high short-circuit current is wanted for proper selective operation of relays and high excitation is desired in order to keep the machines in synchronism. If a motor-driven exciter is operating in parallel with prime mover-driven exciters, the motor-driven exciter may trip out by reverse current. If the other exciters on the bus are unable to take care of the excitation, a delay in restoration of service will result. This involves the attention of the operator at a time when he is busy with power system troubles. A storage battery or reserve prime mover-driven exciter capacity on the bus eliminates this trouble. If possible motor-driven exciters should not be used during times of impending power system trouble.

The potential induced in the generator fields will cause a fluctuation in the exciter bus voltage which will interfere with the operation of auxiliaries if they are fed from the same bus, at least to the extent of blowing fuses, and has in some cases caused a reversal of exciter polarity even though the exciters were shunt-wound.

Systems B and C. If the load due to a short circuit is greater than the pulling out point of the generator at the reduced value of terminal voltage and exciter current it will fall out of step, speed up and be brought back to normal speed by the governor. If the load is less than the pulling out point but greater than the prime mover capacity, the unit will slow down. If the short circuit is directly on the station bus all generators not separated by reactors will fall out of step with resultant variations in speed before governor operation takes place. If the generators speed up, exciter voltages will be increased. If the generators slow down, the exciter voltage will be reduced. Increased exciter voltage would be considered an advantage where higher short-circuit currents for proper relay action and greater synchronizing powers are desired, and would be considered a disadvantage where there is danger of mechanical damage to equipment or failure of oil switches. If it is decided that the worst possible variations in generator speed will not seriously affect the operation, then one of the principal disadvantages to the direct-connected systems has been eliminated. This is the decision usually reached in large steam plants using high-speed turbines, which are not permitted by their governors or emergency trips to attain any excessive speed.

Currents will be induced in the generator fields the same as in System A, which may result in reversal of exciters. In System C polarity reversal is usually of small consequence and merely results in the machine slipping a pole. Ordinarily in this system the exciters are not equipped with reverse-current relays and therefore cannot trip out by reverse current. If they are so equipped, as is the case when used with a storage battery, the reversal of polarity will cause the exciter to trip out and the battery to cut in.

System D. Exciters are not affected by generator speed fluctuations. There is a possibility of polarity reversal. This will cause the generator to slip a pole but will not otherwise interfere with operation.

General. Reversals, except in the case of compound exciters on central systems, will be of rare occurrence. Speed fluctuation with resultant variation of exciter voltage is the most important effect of short circuits and may be partially overcome by voltage regulators. System D is the most desirable. System A is the next best, being almost equal to System D if the exciters are prime mover-driven or if the motor-driven exciter load can be picked up by other exciters or battery. System C has a slight preference over System B, which is the least desirable.

(b) *Sudden Loss of Load.*

System A. Speed of motor-driven exciters is increased temporarily. If the compensation for voltage drop is considerable, induction motor-driven exciters may drop in speed and lose some of their load but probably not enough to trip out. Prime mover-driven exciters are unaffected.

System B. Generators will overspeed until governors catch them. If there are no regulators the main bus voltage may go to a high value as the effect of increased generator speed and exciter voltage is cumulative. Any feeders or load which may be left on the bus may be inconvenienced by this high voltage. Regulators will tend to prevent this increase in voltage.

If only some of the generators lose their load and thereby increase in speed, the shapes of the saturation curves on the exciters will be different and this may result in reversal or pumping between exciters if compound-wound.

System C. Same as System B except that exciters cannot reverse due to loss of load on only some of the generators as they are not in parallel.

System D. Increase in speed of generators will not affect exciter speeds and increase in voltage will only be proportional to increased generator speed which is easily taken care of by regulators.

(c) *Generators Falling Out of Step.*

System A. A transient will be caused in the exciter system. Bus voltage may be pulled down to a point where service to motor-driven exciters would be affected in which case reverse current relay may function. Prime mover-driven exciters are unaffected. If battery or sufficient spare exciter capacity is in service, it does not matter if one exciter trips out. Effect is not serious.

System B. Generator falling out of step causes temporary increased speed of that generator, equivalent in effect to loss of load on the generator. This causes distortion of saturation curves and if exciters are compound-wound may be the cause of pumping between exciters or reversal. If the speed of the other generators is pulled down, excitation will be decreased thereby. This decreases the synchronizing power and makes it less likely that the machines will restore themselves to synchronism. If voltage regulators are provided they will increase the excitation somewhat.

System C. There will be no interaction between exciters. If speed is pulled down by overload on generators still in step, excitation will be decreased with consequent less likelihood of the machines ever falling in step. Voltage regulators would tend to offset the effect of reduced speed. If bus voltage is not reduced voltage regulators will pump due to 90 deg. current coil.

System D. Unaffected. If bus voltage is not reduced voltage regulators will pump due to 90 deg. current coil.

General. With separately driven exciters of either the central or individual system there is more chance of the generators coming back into step than there is with the direct-connected exciters of either the central or individual systems as the better the excitation the more will be the synchronizing power between machines Induced currents from generator fields will flow in the exciter circuits. Their effects will not ordinarily be of consequence. Systems D and A are preferable to Systems B and C.

(d) *Automatic or Manual Opening of Fields to Clear Line Trouble.*

Each of the excitation systems will behave practically the same on this operation. The direct-connected systems have the disadvantage that the speed may in certain cases be low and when the field breakers are closed there is not as much synchronizing power in the machines as in the separately driven systems where the excitation has not been affected by the generator speeds.

(e) *Splitting of Station.*

This may occur intentionally, accidentally or automatically during trouble. Inasmuch as the potential and current to the regulator is likely to be forgotten during this operation, it is possible that voltage regulating difficulties may be caused thereby.

Systems A and B. If potential is taken from one set of generators and current for line drop compensation from the other the bus voltages of both sets of generators will fluctuate by an amount equal to twice the line drop compensation. This does not apply in System B if individual regulators are used on each generator.

Systems C and D. Unaffected if each generator has its own regulator with potential from its own potential transformers. Same as System A if only one regulator is used.

(f) *Accidental Shut-down of Generator Having a Direct-Connected Exciter Feeding other Fields.*

Systems A and D. Unaffected.

System B. Will affect operation to the extent that the exciter was depended on to supply excitation to other machines unless there is spare capacity.

System C. Will result in the loss of excitation to one of the generators in service. This generator may or may not fall out of step depending on the synchronizing power and its load. If the size of the generator is small in comparison with the size of the system no harm will be done to service,

8. EFFECT OF EXCITER SYSTEM TROUBLES

(a) *Ice and Trash Trouble.*

Systems A and D. Ice trouble is likely to effect waterwheel-driven exciters by cake ice, anchor ice or frazil accumulating on screens, in the turbines themselves or in exciter penstocks. It is important that the necessary precautions in the design and operation be taken to prevent interruption of exciter service by ice trouble. In a well designed layout and where facilities are provided for the prevention of ice trouble it should not be a serious hazard. Where there is any possibility of ice trouble, motor-driven exciters capable of taking care of all or a part of the excitation, depending upon how serious the trouble is likely to be and on the importance of the service, should be provided. An alternative is to equip the turbine-driven exciters with motors on the same shaft. In System D emergency service can be obtained from the main bus through transformers.

The effect of trash accumulating on screens is the same as ice trouble and the same considerations hold.

Systems C and D. Unaffected.

(b) *Short-Circuited Exciters.*

Systems A and B. The reverse-current relay on the damaged exciter will function. This will result in a disturbance to service depending upon the amount of load the exciter was carrying and the ability of the remaining exciters to pick up this load. If a storage battery is floating on the bus no serious disturbance will occur. If the short-circuited exciter happens to be the only one running, a total interruption will result. The exciter itself may be permanently damaged. If the reverse-current relay does not clear the exciter, the effect will be the same as a short circuit on the exciter bus. In addition if the short-circuited exciter is compound-wound reversal of its polarity may occur even if promptly cleared.

Systems C and D. If the size of the system as compared with the size of the generator is such that the bus voltage will not be materially reduced by the loss of excitation on one of the generators, no serious disturbance to service will result. If the generator is a considerable part of the system capacity and is heavily loaded the machine will fall out of step and thereby cause a more serious disturbance to service until the generator is cleared.

If a battery is automatically cut in, the generator will not fall out of step if the automatic equipment operates correctly and promptly.

(c) *Short Circuit on Exciter Bus.*

Systems A and B. If the short circuit is only momentary and clears itself, operation will be affected only by a slight disturbance to voltage. If the short circuit does not clear a total interruption to service will occur. Exciters not ordinarily being equipped with overload protection, the short circuit will have to be cleared manually. The damage to the bus may be such as to interfere with restoration of service on some parts of the house. If the battery feeds into the short circuit without overload protection, the battery plates will be badly distorted. The tendency will be on a large excitation system, especially if a battery is floating on the bus, for the short circuit to burn itself clear. A permanent bus short circuit is the most serious trouble that has to be contended with in the central system of excitation. Partial protection is obtained by operating the system in two parts. Fortunately this is a trouble that does not happen often especially if due care has been given to the location of buses and their mechanical protection.

System C. Unaffected.

System D. Unaffected as far as the d-c. side of the exciter is concerned. Considering the a-c. excitation bus the effects will be the same as in System A.

(d) *Accidental Shut-down of Exciter by Mistakes or by Accidental Operation of Headgates, Governor, Throttle or Interruption of Motor Drive.*

System A. The exciter will continue to operate as a motor. Serious disturbance to service will result unless the load which the exciter had been carrying can be picked up by other exciters operating in parallel or by a battery. If only one exciter was running, total interruption results. Reverse-current relay will cut out exciter if set low enough to do so.

System B. Same as System A if a generator whose exciter is needed for other fields is shut down. The exciter becomes a short circuit on the bus unless the reverse-current relay functions.

System C. Operation is not affected unless the exciter happens to be feeding another generator.

System D. The generator will fall out of step if it is large in comparison with the size of the system or if heavily loaded and will remain in step if small in comparison to the size of the system, causing little disturbance to service. A reversal of the exciter may be caused but this is of no consequence as long as the exciter operates individually.

If the turbine feeding one of the a-c. exciter generators shuts down the effect will be very similar to loss of one of the exciters in System A. If only one a-c. exciter is running the result will be a total interruption, if two a-c. exciters are running or if the a-c. exciter bus is in parallel through transformers with the main power bus, no interference with operation will result.

(e) *Open Circuit in Exciter Field.*

Systems A and B. The exciter will cease to generate and the disturbance to service will depend upon the ability of the other exciters or battery to pick up the load. If the reverse-current relay does not function the effect is that of a short circuit on the exciter bus and the exciter will be badly damaged. If the exciter is compound-wound its polarity is likely to be reversed. If the exciter is the only one running a total interruption results.

Systems C and D. The excitation of only one unit will be affected and the effects will be the same as loss of excitation on one unit from any other cause. If battery is provided the defective exciter may be automatically cut out and the field automatically put on the battery. There is no danger of the exciter being damaged by back-feed from other exciters.

(f) *Opening of Exciter Circuit Breaker Resulting in High Voltage.*

Systems A and B. If only one exciter is running and its circuit breaker is accidentally opened, the inductive action of the generator fields will cause high voltage on the excitation bus and on all circuits connected to it. If auxiliaries are operating from the bus their fuses or circuit breakers will be blown. Puncture of insulation and short circuits may result. Ground detectors will be burned out. This trouble constitutes one of the principal objections to using exciter buses as a source of power for control or auxiliary circuits.

Systems C and D. No high voltage can occur unless a circuit breaker without discharge resistance has been inserted in the circuit.

(g) *Regulator Troubles.* Among the common regulator troubles are open a-c. coil circuit, open d-c. coil circuit, contacts freezing, regulator switching mistakes, current for voltage drop compensation taken from wrong circuit or interrupted.

In the central systems A and B where it is customary to provide only one regulator, regulator troubles will naturally affect the entire excitation system.

In the individual systems C and D where it is customary to provide each unit with a regulator, the trouble will be isolated to one unit and will not be of any great consequence to the operation if several machines are operating in parallel.

Whenever a voltage-regulating booster is used, the effect is not likely to be serious because the worst that can happen is that the booster voltage will be lost. If the booster voltage is maintained at about zero, bus voltage will not be affected.

9. SIMPLICITY

From the standpoint of physical simplicity if the number of units is small System C is the simplest in that it has no special prime movers, governors, steam piping, penstocks, headgates, screens, etc. in that the volume of substructure and building occupied by excitation equipment is small and also in that ordinarily the system has no rheostats. If the number of units in the station is large, System C is complicated by the addition of so many more exciter units with their regulators. System B is not more complicated than System C when the latter uses a bus and generator field rheostats. System A can be almost as simple as System C, or considerably more complicated depending on the amount of equipment installed. If only two exciters are used and if these are steam-driven, it compares very favorably with System C for physical simplicity. If the exciters are waterwheel-driven, turbines, penstocks, governors with auxiliaries, headgates, screens, etc. add to the amount of equipment. If a motor-driven exciter and battery are added System A becomes more complicated than System C except for a large number of generators. System D is physically the most complicated.

From the operating standpoint, considering both normal and trouble operation, System A for a small station is the simplest provided the exciters are not compound-wound. If steam-driven exciters are used the complications of heat balance are introduced. If System A has more than two or three exciters, some of them motor-driven, it becomes more complicated with respect to normal operation than System C. With respect to operation during power system trouble System A is simpler than System C, especially if only prime mover-driven exciters are used or if a battery is provided. System B is more complicated from the operating standpoint than either Systems A or C. System D is the simplest of all to handle during power system trouble but for normal operation is not as simple as the other systems on account of the many operations involved in starting and stopping.

It should be noted that the simplest system fundamentally can be made the most complicated by attempts to make it too flexible, by the addition of an undue amount of remote control and automatic features or by awkward location of various parts of the system such as putting switches at points which are not easily accessible to the operators. Voltage regulating equipment introduces some complication especially if this equipment is of an unusual or special nature or if a large number of regulators are needed.

10. Reliability

Under the heading of reliability are usually discussed the various operating characteristics of the systems as outlined above. The topic in this discussion is intended to cover merely the inherent reliability of the equipment itself. From this standpoint reliability does not become an important factor in determining the system of excitation for the reason that any of the systems properly designed and laid out in accordance with good practise are very reliable. Many more troubles will be caused by the operating characteristics than by the breakdown of excitation equipment. The number of breakdowns however will be proportional to the number of units in use. For instance, a station having ten small exciters will roughly speaking have five times as much trouble as a station having two large exciters. Remote control, automatic protective devices and regulators introduce an element of unreliability, which is far greater than that of the apparatus proper. This is especially so if such equipment is used in a station where operating and maintenance attendants are not sufficientlv skilled.

PART II.

CONSIDERATION OF MATTERS RELATED TO THE LAYOUT OF AN EXCITATION SYSTEM BUT NOT MAINLY CONCERNED IN THE CHOICE OF SYSTEM

1. Number and Size of Exciters

The exciter capacity must be capable of exciting the entire station under rated load and power factor conditions on the generators. If auxiliaries are to be supplied from exciters this must be taken into account. Provision must also be made for rheostat losses. In every case the number and size of exciters should be such that the largest exciter can be taken out of service at any time without interfering with the operation of the station. In a few large important stations it may be advisable to provide for a second exciter being out of service.

The ultimate as well as the immediate exciter requirements should be kept in mind in laying out the equipment. The present exciter equipment should as far as possible be such that it can be used to best advantage in the ultimate layout. This may warrant substructure, hydraulic equipment, piping and even exciter sizes larger than at present needed.

System A. A minimum of two exciters, each capable of carrying the entire load, is necessary for a small or moderate size station. The probable maximum requirement for such stations is three exciters any two of which are capable of carrying the entire load. An overload rating on the exciters may permit shutdown of a second exciter for a larger part of a day than otherwise.

For the large station three exciters, any two of which can carry the entire load, with possibly overload ratings will be sufficient for most cases. Where the plant is solely depended on for important service, four exciters, any three of which can carry the entire load, is advisable. If in addition the service is at high load factor any two of the four exciters should be able to handle the entire load.

In the case of a certain projected large important hydro plant where there is a slight possibility of ice and trash trouble, four exciters are contemplated, two of them turbine-driven and two of them motor-driven, any two of the exciters being sufficient to carry the entire plant.

In the case of the steam-driven exciter, additional capacity may be provided for emergencies by making the exciter larger than normally required and designing the turbine for best economy at normal load with an overload valve for developing the maximum rating of the exciter.

It is sometimes stated that investment in idle spare exciter capacity decreases with the increase in the number of exciters. This is true for the exciter generator alone but is questionable if the total cost of the equipment, including prime mover or motor, switching equipment, steam piping or hydraulic works, substructure and building, etc., is considered. In any event this point does not in itself justify the use of more than three exciters in System A.

Where a booster is used for voltage regulating purposes with this system, the current-carrying capacity of the booster must be equal to the total excitation requirement. The voltage rating of the booster depends upon the required range of voltage regulation.

One booster is sufficient provided that hand regulation of voltage is satisfactory during times that the booster is out of service. A second booster can be added at a later date if the station is extended.

System B. Usually this system has one exciter on each unit of a size sufficient to supply the excitation for that particular unit.

In some cases the exciters can be reduced in numbers and made correspondingly larger. If exciters are not placed on all of the units a small or unimportant station should be provided with at least three exciters, any two of which should be large enough to carry the entire load. For a larger station with more units and of greater importance not less than four or five exciters should be provided. This system requires more exciter units than System A for the same class of service for the reason that there is more likelihood of the exciters being out of service on account of turbine or generator repairs.

With an exciter on each unit the spare capacity can be supplied by making the exciters larger than would otherwise be required. For a station with a small number of units the exciters should be double size. For a larger number of units the exciters need not be double size to provide the same amount of spare.

If the exciters are not oversized a separate motor-driven spare must be provided. Its size is determined by the size of the largest exciter. If insurance against the loss of a second exciter is desired, two spare motor generator sets or one spare sufficiently large to handle two units must be provided. Only in a large and important station with more than three or four units would it be necessary to provide for the breakdown of more than one exciter.

System C. Each exciter must be of a size sufficient to excite the field of its own unit. It is necessary to provide at least one spare motor-driven exciter capable of exciting the largest unit, with necessary switching facilities. If insurance against the loss of a second exciter is desired, two spare motor-generator sets or one spare made sufficiently large to handle two units must be provided. Only in a large and important station with more than three or four units would it be necessary to provide for the breakdown of more than one exciter. The reason that motor-driven spare is recommended is that it is idle most of the time, represents a smaller investment than a prime mover-driven set and satisfies operating considerations.

For a station containing only a small number of units the spare capacity may be supplied by making the direct-connected exciters large enough to excite two units.

System D. System D requires that each generator have its own exciter and it is not customary to make this exciter larger than what is required for one field. At least one spare exciter capable of exciting the largest unit with a bus running the length of the station so that any field can be connected to it should be provided. If the number of units is large or the service important, two spare sets should be provided. In one of the stations using this system at the present time no spare has been provided and whenever it is necessary to shut down an exciter the entire unit has to be shut down.

With regard to the a-c. exciters at least two, either of which can carry the entire load, must be provided. Both should be prime mover-driven. If provision for getting a-c. excitation power from the main buses is provided, not more than two a-c. exciters are needed. Transformer connection between the a-c. exciter bus and the main station bus large enough to carry the entire exciter load is advisable. There is no particular objection to putting all of this capacity in one bank of transformers but it is preferable to divide it up into at least two banks.

2. KIND OF DRIVE

Referring now only to System A, if two exciters are provided they should both be prime mover-driven. In hydro plants if these exciters are likely to be subject to interruption due to ice or trash, motor drive must be provided. The motors may be mounted on the same shaft as the turbines or a third or fourth motor-driven exciter may be provided. In the steam plant it may be desirable to equip the steam-driven exciters with motors also for the purpose of regulating heat balance. The governors should be set so as to maintain exciter speed in event of failure of the motors.

If three or four exciters are provided at least two of them must be prime mover-driven. There is no objection to a third exciter being prime mover-driven except in a hydro plant subject to trash or ice and in a steam plant where heat balance may be affected. In most plants the best arrangement will probably be to have two of the exciters prime mover-driven and the third or fourth motor-driven.

For motor-driven exciters the induction motor is satisfactory and preferable in almost every case. The induction motor will ride through most any kind of a disturbance on the main power system even though the voltage may remain low for a considerable length of time. Generally speaking, synchronous motors are more likely to fall out of step during main system troubles and having a direct-current field requires one additional operation on the part of the operator when starting. Where the normal voltage may fluctuate as much as twenty to thirty per cent due to line drop compensation, it may be preferable to install the synchronous motor drive because of being unaffected by variation in supply voltage.

In favor of the synchronous motor it may be said that it is possible to specify a synchronous motor specially designed so that it may be almost the equal of the induction motor in stability during power system trouble. The starting of the synchronous motor need be no more difficult than that of the induction motor except for the fact that the d-c. field has to be attended to. If a synchronous motor is used it is advisable to provide a double throw switch for the motor field so that it can be connected to either the exciter end of the set or to the exciter bus. The synchronous motor has the disadvantage of having slip rings which will require maintenance attention.

Combination motor and turbine drive, either hydraulic or steam, is in successful use and offers a very good way of making a system having two exciters independent of either prime mover trouble or power system trouble. Without the installation of any automatic equipment not ordinarily used, this system functions practically automatically. If the exciters are being driven by the motors, loss of voltage with resultant reduction of speed permits the prime mover governors to function and transfers the drive to the prime mover. When voltage is restored to the motors the increased speed automatically cuts out the prime movers. The exciters may be driven from the prime movers at such a speed that the motors while still connected to the supply voltage take no power unless the prime movers fail. This is an excellent system for a plant subject to ice or trash or where manipulation of heat balance with steam-driven exciters is desired.

For the purpose of driving voltage-regulating boosters frequently used in connection with System A, direct-current motors fed from the exciter bus should be used. The booster should not be induction motor-driven from the main power bus. If this is done the operation of the booster will be complicated by power system troubles. If the motor loses its voltage the set will slow down, come to rest, start up in the opposite direction and run away. This involves the attention of the operator at a time when he is busy with power system trouble. The booster should therefore be driven independent of the main power system.

3. SHUNT OR COMPOUND-WOUND EXCITERS

Wherever it is necessary that exciters be operated in parallel it is recommended that shunt-wound exciters be used, especially if the exciters are to be controlled by voltage regulators.

The instances of compound-wound exciters causing trouble are so many that in spite of all the explanations of causes that may be given, it is difficult to convince an operating man that has been through any of these experiences that compound-wound exciters should ever be used where the exciters are operated in parallel. If the saturation curves and voltage characteristics of the exciters being operated in parallel are identical, and if the equalizer conditions are perfect there is no difficulty. However, regardless of the care which may be taken in obtaining these conditions when the exciters are installed, there are disturbing influences which may occur later to interfere with the perfect conditions. For instance, equalizer switches may develop high contact resistance. Equalizer connections may be taken down and carelessly reconnected. Saturation curves of exciters may be changed by putting shims under field poles or by taking them out on account of some commutation trouble, the effect on parallel operation being forgotten. The speed of an exciter may be changed from that at which it was intended to be operated. This may be due to carelessness of attendants in not having machines operating at correct speed, or it may be due to some trouble in the turbine such as ice, or may occur from power system troubles in the case of motor-driven or direct-connected exciters. All of these conditions tend to change the shape of the saturation curves and if operating under the influence of a voltage regulator, reversal is likely to be caused. The compound-wound exciter is also subject to reversal due to internal short circuits. Loss of exciter shunt field may also cause reversal of compound-wound exciters. If the driving power of a compound exciter is lost the tendency will be for the exciter to speed up and draw a heavy current and it may even run away if not prevented from doing so by reverse-current relay or over-speed trip.

It is therefore recommended that only shunt-wound exciters be used where exciters are to be operated in parallel and that wherever compound-wound exciters are at present so used especially in connection with voltage regulators, that steps be taken to remove the series windings if the characteristics of the machines permit. The series windings should be cut out of the circuit but not short-circuited. A short-circuited series winding causes sluggish regulator action. Where exciters are to be used individually with voltage regulators, there is no objection to the series winding. The series winding is then an advantage from the standpoint of voltage regulation.

Shunt-wound exciters also are subject to reversal from induced voltages in generator fields, regardless of whether they are operated in parallel or not. The induced voltage may be caused by a short circuit on the generator or by sudden reduction of its field current. These two causes may work simultaneously in direct-connected exciters. Sudden reduction of field current may be caused by sudden reduction of exciter speed, by rapidly cutting in a generator field rheostat with too large steps, or by rapid reduction of exciter field current when operating on the steep part of the exciter saturation curve. If the resultant induced voltage in the generator field exceeds the exciter voltage, reversal will occur. The greater the resistance in the field circuit the less is the likelihood of reversal.

4. VOLTAGE

One hundred twenty-five volts is satisfactory for small plants but for moderate size and large plants, 250 volts is preferable from the standpoint of reduced cost and size of apparatus and cables. Where station lighting is at 250 volts a-c. the exciter bus can be used for emergency lighting service if it is at 250 volts.

It is suggested that for large stations consideration may be given to higher voltages than those at present in use, possibly 500 or 550 volts. Safety to men is the most important point to be taken care of should such a change be made. In stations where it is at present the practise to run station auxiliaries at 440 volts a-c. or 2300 volts a-c. there seems to be no reason why the higher voltage cannot be extended to the excitation system as far as safety is concerned. There may be some question as to the reliability of higher voltage on commutators and slip rings but comparison with 550-volt railway apparatus should answer these questions.

5. BUS ARRANGEMENTS

Bus connections and exciter and field connections should be made as simple as possible consistent with the required degree of flexibility. If complete flexibility is attempted it is almost certain to result in bus connections which are too complicated to be easily operated and which will be the cause of operating mistakes. A bus arrangement which permits of the exciters being paralleled by any more than one set of switches is likely to be dangerous.

System A. In the majority of stations where this system is used it will be sufficient to provide a single bus which can be split in the middle between two exciters. If three exciters are used two sets of sectionalizing switches may be installed, one on either side of the middle exciter which should be the motor-driven one. In a larger and more important station two buses should be used. A common negative bus with two positive buses will even then be sufficient in many cases although two negative buses as well as two positive buses make it possible to take care of repairs on the buses better.

Where a double bus arrangement is used it should be possible to put any exciter, any field and any auxiliary circuit on either bus. A double bus is an advantage particularly where generators may have to be operated at abnormally low field currents such as when drying out armatures or in making special tests which require low voltage. It will also be of assistance for battery charging purposes if special facilities are provided for this purpose. Switches should be provided for tying the buses together. The use of a double bus makes it possible to use exciters which have been reversed without taking the time to rectify the polarity. Before anything more complicated than the double bus arrangement is used, serious consideration should be given to the likelihood of any such arrangement being the cause of operating mistakes.

System B. Same as System A. If each generator is equipped with an exciter it may give additional flexibility to provide connections for throwing the exciter on its own field alone. A single bus will then probably be sufficient.

System C. It is commonly stated that in this system no bus is necessary. However if provision is to be made for breakdown of exciters and for the use of a spare exciter or a battery, it is necessary that a bus be provided although it may not be as heavy a bus as in System A. A second bus for this purpose in most stations is not necessary. With a bus the only difference between System B and System C is in the position in which the exciter switches are thrown. In either case it probably will be wise for the exciter switches to be double throw so that the exciter can be connected directly to its own field or to the bus.

System D. On the d-c. side the connections should be the same as for System C. On the a-c. side the same considerations hold as in the case of System A, a double bus being very desirable.

6. PROTECTION

The minimum amount of automatic protective equipment consistent with safety to service and apparatus should be installed. The class of attendance which the apparatus is to receive should be kept in mind if unusual or complicated devices are contemplated. Such equipment cannot be kept in operating condition without skilled operating and maintenance attendance and added maintenance cost.

Generator field breakers are always non-automatic unless some kind of field-destroying device is used.

Reverse-current relays should be used on exciters which are to operate in parallel with other exciters or with a battery, but no overload relays should be used. Reverse-current relays for this service are generally not very reliable, but as there is no adequate substitute it is necessary to install them. Low-voltage release on circuit breakers is not considered advisable.

It is recommended that all exciter equipment which is likely to run away, particularly motor-generator sets, be provided with speed-limit devices independent of any protection which might be expected from reverse-current relays.

Where there is a possibility of high voltage induced from generator fields causing serious trouble, electrolytic arresters should be installed on the bus or on the fields leads.

Ground detectors in the form of lamps should be installed on the bus, or on each exciter where isolated, with the lamps in such a position as to be always visible by attendants.

Where the main generators are protected by relays they should be connected to also trip the field breaker.

System A. Exciters should have reverse current but no overload protection. It is an advantage to set the reverse-current relays above the current required for simple motoring of the exciter. This eliminates the time required for reparalleling in case driving torque is momentarily lost.

Motor-driven exciters should be protected by overload relays on the motor side. The relay setting with regard to both current and time should be such that it will trip out only in case of a short circuit in the motor or on the leads to the motor. The overload setting should not interfere with the exciter motor pulling back into step after voltage has been completely interrupted and the motor has come almost to stand still, assuming that the motor will stand such treatment. The current transformers should be made sufficiently large and strong to stand the maximum possible short-circuit current without blowing apart. This will probably involve high ratio current transformers which will be unsuitable for metering purposes. Separate low ratio transformers may have to be provided for metering.

Battery circuit breakers should be non-automatic.

A voltage regulating booster used in connection with System A should have overspeed and reverse rotation devices tripping d-c. short circuiting breaker, booster field and motor oil switch. The booster positive, negative and short-circuiting knife switches should be equipped with contactors which will close the booster short-circuiting breakers, etc. in case the knife switches are operated in the wrong sequence.

The booster motor, assuming it to be direct current driven from the exciter bus, should have an over-load trip on its circuit breaker. If the booster is alternating current motor-driven, its overload relays should trip only in case of short circuit and not interfere with the motor pulling in after a-c. system trouble.

System B. Same as System A.

In the hydroelectric station particularly, the advisability of installing over-voltage relays or over-speed relays to cut down the exciter voltage in case of runaway, should be considered. They should not however be installed unless really necessary as they add complication and will be a source of some trouble in themselves.

System C. Ordinarily this system will have no automatic circuit breakers. The protection for motor-driven spares can be the same as mentioned for System A.

The use of over-voltage or over-speed relays to reduce exciter voltage in case of runaway should be considered but avoided if possible.

In certain installations where it is desired to have a battery automatically pick up a field in case of failure of one of the direct-connected exciters, special equipment consisting of a reverse-current relay on the exciter, a low-voltage relay, an over-load relay in the field and other auxiliary relays is necessary. This scheme involves complications which are not recommended in a station not prepared to properly maintain such equipment. It further introduces the possibility of induced voltages in the generator field cutting out the exciter and cutting in the battery. This might occur simultaneously on all generators.

System D. All d-c. circuit breakers are ordinarily non-automatic.

If overload relays are used on the individual exciter motors, a-c. exciters or emergency transformers they should be set to open for short-circuit conditions only and not so as to interfere with induction motors pulling back into step. It may be advisable to provide reverse-current relays on the emergency transformers to protect the exciter system in case of trouble on the main system. Any series transformers in the high-voltage leads of the emergency transformers must be strong enough to withstand maximum possible short-circuit current.

DISCUSSION ON "CONSIDERATIONS WHICH DETERMINE THE SELECTION AND GENERAL DESIGN OF AN EXCITER SYSTEM",((BARRON AND BAUHAN), WHITE SULPHUR SPRINGS, W. VA., JULY 2, 1920.

Roy C. Muir: The system described under "D", which involves separate a-c. auxiliary generators, solves the problem of excitation and auxiliary power at the same time. Although auxiliary power is not properly a part of the paper under discussion, it is so important, particularly in hydroelectric stations, that any excitation system which provides a reliable source for auxiliary power should be given preference. There has been no other adequate solution of this problem, particularly in starting up the station the first time or in emergencies, as it is necessary to start up the governor pumps first and this requires a source of power other than the main generating units.

In considering the cost of the excitation system, therefore, it should be borne in mind that the separate a-c. auxiliary generators also take care of the cost or part of the cost of the auxiliary power system and the difference in the combined cost of excitation and auxiliary power systems by this method and the other method described is not so large as it would appear.

Mr. Summerhayes states in his paper (beginning page 1575), that the separate a-c. auxiliary generators were most useful in the case of large stations with low-speed generators. The reason for this is that large low-speed generators require a large amount of excitation and unless the excitation required is sufficient to permit of auxiliary excitation units large enough to give satisfactory hydraulic units, this system would not be considered. The hydraulic unit in order to be reliable, must be large enough to pass the ordinary rubbish and ice without blocking it up. The cost of direct-connected exciters on large low-speed units is so high that the cost of separate auxiliary generators and high-speed motor driven exciters compares quite favorably.

There is one point which relates to excitation I wish to bring out in connection with Mr. Schuchardt's and Dr. Steinmetz' papers which were presented yesterday. Nothing was said in that discussion regarding the effect of the excitation system. I believe the Commonwealth Edison Company do not use automatic voltage regulators. The best way to consider the excitation of a system of this sort is that the system requires a definite excitation for any definite load, assuming a constant voltage. In the case of hand regulation, if some sudden load comes on the station, there must be a considerable voltage drop and there is no provision made to take care of it automatically. During the normal changes of load the adjustment can be made by hand. During this 15 minutes of shut down and low voltage, which has been described, we do not know what the operators were doing, whether they were trying to adjust the voltage in various stations by hand and whether that had any effect on the hunting. If they had had automatic voltage regulators, which would tend to hold the voltage up during the trouble, would the effect have been different?

W. F. Sims: In connection with this paper a statement regarding the development of the excitation system of the Commonwealth Edison Company, extending over a period of years, may be of interest.

1902, when the Fisk Street station was first laid out it was planned to have the excitation from individual separate exciters, and the first three units were operated that way, with a separate bus with a battery and a throw-over switch to be used in case of emergency. The original exciters were compound wound, but owing to the difficulty found in connection with battery charging, the series fields were removed and the exciters operated as shunt machines.

As the station was extended, it was found desirable to operate from a common exciter bus, with separate exciters, some steam driven and some motor driven, and at present time in the Fisk Street station the ten vertical units are fed from two sections of exciter bus.

The main bus is being operated in two sections, and each section of the exciter bus has three sources of supply,—some of the exciters being driven by prime movers, other motor driven from "A" section and the remainder motor driven from "B" section.

Separate exciters were used at the Quarry Street station, and also for the first two units in the Northwest station, but with the advent of the horizontal machines, with their direct-connected exciters, the company came to the conclusion that entire dependence could be placed on the reliability of shaft-driven exciters, and this system of direct-connected exciters has been adopted for all horizontal units, with a reserve in each station of one motor-driven exciter on a common bus, with provision for hand throw-over from any machine in case of emergency.

Provision has also been made for throw-over connections in the control circuit, so that once one of these machines is thrown to the reserve exciter, the operator uses the same control switches that he did when operating on the shaft-driven exciter, and we feel that the method answers all purposes in a large central station system.

E. G. Merrick: The aim in designing an excitation system is to obtain the greatest simplicity commensurate with reliability of service. This term reliability refers not only to emergency periods, but also to normal operation. Emergency cases are very rare, and, while they must be provided for, it must be borne in mind that the provisions made, must not be carried to excess otherwise normal operation becomes more difficult and therefore subject to more delays than those we are endeavoring to avoid.

It is difficult to make a tabulation which will cover the case completely. We should really have several tabulations similar to that given, one for large stations, one for small stations, and even for hydroelectric and steam stations separately.

I think that most of us will agree in general with the tabulation given on a theoretical basis. When we consider the matter practically, however, some of the statements made may be open to question. In the case of System C with individual exciters, the operation was considered "bad". Now, in practical operation it has not been found so, and the decision of the Southern California Company to operate the new Big Creek Power House No. 8 in this manner—as mentioned by Messrs. Cox and Michner—supports this statement.

The System "D" was considered "complicated". This has been used in the large plant at Keokuk and also at Cedar Rapids and is handled so easily and with so few operators that, from a practical viewpoint, it does not strike one as complicated.

System "C" with individual voltage regulators was also considered "complicated." As a matter of fact, if a regulator controls a number of exciters, it usually requires more attention than several regulators controlling individual machines. Another point, which is not often considered, is that synchronizing is very considerably simplified. The operator does not have to regulate the voltage and his entire attention can be given to the synchroscope.

There was a statement in Mr. Ross' paper that they had not used steam turbine and motor driven exciters because they found it impossible to obtain satisfactory exciters of larger capacity than 100 kw. I do not quite understand the statement, because there are many large turbine sets, such as those at Windsor, with 150 kw. exciters at 1800 revolutions, which are operating in a perfectly satisfactory manner.

William F. Dawson: I was rather surprised to find the general feeling that automatic regulators were undesirable or had been left out of service after having been installed. I can readily see, on a large central power plant how that could be, the enormous size of the system as compared with the fluctuations of the load would justify it, but we must not forget that there are also a great many steam driven units being operated in factories, and on other loads that cannot be regarded as central power stations with very large fluctuations of the load. There are considerable fluctuations on the load, and there is no question in such cases but what there is a distinct advantage in the automatic regulator and, where a single machine is operating on the load it may even be that the machine can be overloaded and "break down" or lose its voltage, and that throws the motors out, when hand operation is used, whereas, the automatic regulator is so quick acting that 30 or 35 per cent greater load can be obtained before the breakdown comes.

The general feeling seems to be towards shunt-wound exciters. Every designer of exciters will welcome the suggestion, particularly in respect to direct-connected exciters for small steam turbines. The machines are high speed and small, and space for all of these series connections is also small, it is often a very difficult task, to put in the extra windings. If you can get along with shunt-wound exciters in such cases, the manufacturer will be happy.

Another thing is with the small high-speed direct-connected exciters and compound winding, it is quite a difficult matter to make them run in parallel. The users are attempting to put them in parallel without realizing some of the technical details, and they usually have to appeal to the factory. Steadying resistances have to be put in so as to get equal drop between the equalized side of the machine and the switchboard, through the series fields. It is complicated and expensive, and I do not see what particular use it is to you.

Just a little suggestion I have to offer is that any commutating pole machine will be very much inclined to "hog" the load, even though it is shunt wound, if one attempts to connect it in parallel, while the brushes are back of the neutral. The best place for these brushes, if the designer has done his work right, and the adjustments are right, is about at the neutral, but to help stabilize a parallel operation, I recommend that they be placed just a fraction ahead of the neutral.

V. Karapetoff: Some of the speakers and authors of our papers have remarked that there is a gradual tendency in American practise away from common busbars for exciters and towards individual exciters, which is more in line with the earlier European practise. There is a possibility, then, that we may follow this European practise a little further, and provide a magnetic interlinkage between the alternator and its exciter, for the purpose of taking care automatically of changes in the alternator load and keeping the voltage approximately constant.

Some such practical schemes have been worked out in France by Prof. A. Blondel and his co-workers, and I should like to call attention to the principle of such automatic regulation of voltage of a direct-connected exciter, without any commutator for the rectification of the main alternating current.

Several articles on the subject will be found in the leading French electrical periodicals for the last ten or fifteen years. I shall mention in particular. *Bulletin de la Societe Internationale des Electriciens, La Lumiere Electrique*, and *Revue Generale d'Electricite.* The easiest way to find these articles would probably be through the "Science Abstracts," part B.

W. J. Foster: I would like to ask Prof. Karapetoff if he is familiar with a line of machines that were built by the General Electric Company about 15 years ago, known as the compensated or "Form D" line? If I am not mistaken, that line embodied precisely this idea. If so, there is plenty of literature on the subject, I would also say, for Mr. Dawson's information—Mr. Dawson was then an European engineer,—that while we could adjust the exciters for 80 per cent and unity power factor, from no load to full load, we could not compensate for all the intermediate power factors satisfactorily. The generator was rather a complicated thing, and consequently when a satisfactory field regulator was developed this line of machines was abandoned.

Now, I wish also to say in that connection that this idea occurred to an American engineer in the spring of 1893, on which the General Electric Company's design was based. It was in connection with our first work on rotary converters, and this engineer looking over what we were doing suddenly had the thought come to him if that machine were mechanically connected to an alternator, it would take care of the voltage automatically, and he mentioned it to me, and somewhat later, fully ten years later, we started in and made the application. I am very sure that that engineer was not familiar with what Prof. Blondel was doing—In fact, I am not sure whether the work of Prof. Blondel was as early as 1893.

V. Karapetoff: In the machines of your make I am familiar with, additional excitation is furnished through a synchronous commutator consisting of several segments and there is considerable sparking. We had at Cornell two or three of the old machines and we had to discontinue them. That was the arrangement without the commutator, just slip rings.

W. J. Foster: You say there are no commutators in this machine?

V. Karapetoff: Slip rings. In the early G. E. machine there was a synchronous commutator which sparked.

W. J. Foster: We may have had some commutators that sparked, but I do not recall them.

A. E. Bauhan: Mr. Muir points out that in the papers by Mr. Schuchardt and Dr. Steinmetz, presented yesterday, no mention was made of the relation of excitation to the troubles which they had. The relation between excitation system and generators falling out of synchronism is repeatedly touched on in our paper and is an important consideration.

In Mr. Sims' outline of the evolution of excitation practise in Chicago he mentions that the Fisk St. central system is split into two parts. This is an excellent modification of the central system and diminishes the effect of excitation and system trouble.

Mr. Merrick referred to a tabulation which is included as part of this closure. Mr. Merrick's criticisms are largely answered in this more detailed tabulation. The tabulation, however, necessarily sacrifices completeness of statement and is presented more for the purpose of giving a comprehensive view of the problem. The paper itself really needs to be studied in detail in order to apply the table in any important case.

The point that regulators assist the operator in synchronizing is true, of course, in many cases. In others, where the compensation for line drop is considerable, the regulator does not bring the generator voltage to the bus voltage immediately, and needs adjustment by the operator.

Mention was made by Mr. Dawson and others of the omission of the regulator. I wish to say that in Baltimore the regulator is not used, not because of any trouble with the regulator but simply because it is not needed. Very close regulation is not called for because of the nature of the load and the use of means of regulation in the distribution system. The regulators at the Holtwood hydro plant, 40 miles away, compensate for voltage drop in the transmission lines.

Prof. Karapetoff's contribution was very interesting. I thought at first the apparatus which he mentioned

COMPARISON OF EXCITATION SYSTEMS

	A Central Separate	B Central Direct	C Individual Direct	D Individual Separate
(1)—*Power Supply to Auxiliaries*	Good	Good	Not Suitable	Excellent
(2)—*Automatic Voltage Regulation*	If used for auxiliaries or with battery requires booster system of regulation		Permits use of individual regulators	
(3)—*Use of Storage Battery*	Most advantageous use of battery. Requires booster system of regulation		Battery cannot be floated. Can be used only with complications	
(4)—*Space Requirements*	Large	Small	Small	Large
(5)—*Initial Cost* (*A* = 100%) Low head hydro Steam or high head hydro	 100% 100%	 70% to 90% 50% to 70%	 70% to 90% 50% to 70%	 120% to 140% 120% to 140%
(6)—*Efficiency* (hydro, excluding rheostats)	Good	Excellent	Excellent	Poor
(7)—*Effect of Power System Troubles* (a) Sustained Short Circuit	Prime mover driven exciters unaffected. Speed of motor-driven exciters affected	If speed increases, higher short-circuit currents and better synchronizing power. If speed decreases, lower short-circuit currents and lower synchronizing power		Unaffected
(b) Sudden loss of load	Speed of motor-driven exciters affected	Speed of exciters affected		Unaffected
(c) Generators falling out of step	Speed of motor-driven exciters may be affected	Speed of some of exciters affected. Synchronizing power may be decreased		Unaffected
(d) Opening of fields to clear line trouble	Excitation unchanged	If speed is below normal generators are more likely to fall out of step due to reduced excitation		Excitation unchanged
(e) Splitting station with regulator potential on one section and compensating coil on other section	Voltage fluctuation equal to twice line drop compensation results		Unaffected with individual regulators. If only one regulator voltage fluctuation results	
(f) Shut-down of generator with direct-connected exciter needed for other units	Unaffected	Serious disturbance to service	Loss of excitation on a running generator which may fall out of step	Unaffected

COMPARISON OF EXCITATION SYSTEMS—*Continued*

	A Central Separate	B Central Direct	C Individual Direct	D Individual Separate
(8)—*Effect of Exciter System Troubles* (a) Ice and trash (hydro)	Serious reduction of service unless ample motor driven spare is available	Unaffected	Unaffected	Serious reduction of service unless ample motor driven or transformer spare is available
(b) Short-circuited exciter	Unaffected if exciter can be spared. Serious if exciter cannot be spared.		If generator is large in comparison with size of system it may fall out of step	
(c) Short circuit on exciter bus	Short circuit usually clears itself. If not total interruption results.		Unaffected	Short circuit usually clears itself. If not total interruption results
(d) Accidental shut-down of exciter	Serious disturbance unless exciter can be spared		Unaffected	If d-c., exciter, may cause generator to fall out of step. If a-c., exciter serious disturbance unless machine can be spared
(e) Open circuit in exciter field	Serious disturbance unless exciter can be spared		Generator may fall out of step	
(f) High voltage from opening last exciter breaker	Affects entire exciter system. May cause breakdown of insulation. Will blow fuses and breakers on auxiliary circuits		Ordinarily not possible. In any event affects only one unit.	
(g) Regulator troubles	Affects entire system		Affects entire system if only one regulator is used and only one unit if individual regulators are used	
(9)—*Simplicity* Physical	Simple for small or moderate size station. Complicated if many exciters and battery are used		Very simple for small number of generators. Complicated for large number of generators	Complicated
Normal Operation	Simplest	Simple	Simple	Complicated
Operation During Trouble	Simple	Complicated	Complicated	Simplest
(10)—*Reliability*	Depends on number of pieces of equipment, amount of remote control, automatic and regulating devices.			

was identical with the apparatus mentioned by Mr. Foster, which has been described in the Institute PROCEEDINGS in a paper by Rushmore in 1912.

Mr. Merrick seems to have inferred that because system *C* does not rank favorably from the standpoint of effect of power system troubles it is considered generally as a poor system. This is not the impression intended. Behavior during power system troubles is only one of several considerations and will in many cases be outweighed by other considerations, such as cost, simplicity, and behavior during exciter system trouble, especially if highly refined operation is not called for. The disadvantages of the individual direct-connected system are ameliorated if the size of the individual unit is small as compared with the capacity of the system if the prime mover speed can be closely regulated during and after short circuit, or if speed fluctuations can be compensated for by voltage regulators. Hence it is being found in many cases to be the best selection.

It has been frequently remarked and has been stated at this meeting that the cost of the excitation system is of no consequence. I hold that is not true and that in the majority of stations cost is very largely taken into account. It may not be in certain large metropolitan district stations, but we are considering not only the large stations but also moderate size and small size stations in our paper.

The argument used is that the cost of the excitation system, expressed as a percentage of the total cost of the plant, is very small and doubling the cost of the excitation system is of no consequence in the total cost. That is very true but the same argument can be and is applied to other items, which, in the aggregate, perhaps make up more than 50 per cent of the cost of the entire station. If the cost of the excitation system is increased on the strength of this argument it is logical to increase the expenditures for the other items as well. Failure to do so would result in an unbalanced design.

It is therefore evident that cost is an important consideration in selecting the system of excitation, and in my opinion accounts in part for the tendency in favor of the direct-connected system in places where the mechanical interlinkage of the power and exciter systems is not of great consequence.

In discussing this subject of excitation there is a tendency to use the word "reliability" in a too general sense. I believe the word should not be used as referring to the general operating results obtained from a system but rather to the inherent dependability of the apparatus itself. Using the word in this more specific sense, the reliability of the various systems is not an important consideration in the choice of system. The apparatus in all of the systems is very reliable and furthermore, reliability depends more upon the amount and character of equipment involved than the system used.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 2, 1920.

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FACTORS IN EXCITATION SYSTEMS OF LARGE CENTRAL STATION STEAM PLANTS

BY J. W. PARKER AND A. A. MEYER
Both of The Detroit Edison Co., Detroit, Mich.

THE purpose of this paper is: To point out the most essential requirements of excitation schemes; to outline two general methods followed in the design of such systems and from which a variety of schemes are built up; and to discuss briefly the merits as well as demerits of factors determining the success of various schemes.

A. General Requirements

In the installation of a generating unit, there are many component parts all of which must function properly for continuity of service. Of these, the excitation system is a very vital one deserving careful attention. Its rank of importance compared to other component parts is not to be argued, because successful operation of a generator unit depends on the proper functioning of all component parts. There seems to be a wide diversity of opinion, however, regarding the equipment and its assembly as required for furnishing excitation to the main generator units. Many of the schemes of course, are determined largely by local conditions and no attempt is made in this paper to argue for standardization, nor for a most ideal scheme. The intention is to point out some of the underlying principles affecting an excitation system and determining continuity as well as quality of service of the main generating units.

Simplicity is the keynote to successful operation. Some designers in laying out an exciter system, and in their anxiety to approach perfection, will consider many possibilities rather than the comparatively few probabilities which might interfere with continuous operation of a generator unit. As a consequence, a complication of safeguards are incorporated, sometimes at great expense, and all for good purpose, but not always with positive assurance that each will function as intended. Other engineers will carefully weigh the probabilities against the possibilities and risk the chance of some possibilities not occurring, rather than introduce complications equally apt to cause interference. It must not be forgotten that a multiplicity of safeguards multiplies the chance for trouble.

Reliability of excitation is of course most desirable, but its relation to the reliability of other component parts of the main generator unit must be carefully considered. Too much effort is sometimes expended to obtain a very high degree of reliability in the excitation system alone without much consideration of the reliability of other essential component parts of the main unit. The turbine and generator proper, as well as the accessories such as the excitation system, the governor, the oil pump, and various other auxiliaries, are all subject to failures. Each is a link in a chain whose overall strength depends upon the weakest link. Reliability should be provided for proportionately among the various links, rather than indiscriminately by biased attention to one component part or other, as *e. g.* the excitation system.

Of course, a high degree of reliability must be reasonably provided for. This can be attained by selecting simple equipment composed of few parts, each liberally designed mechanically as well as electrically. Further insurance of reliability can be obtained by making excitation an independent system, free from ties to other service auxiliaries. Going a step further and segregating the exciter equipment into separate independent units, each having equipment assembled which serves only its respective main generator unit, would afford not only immunity from external disturbances, but would also isolate excitation troubles within one unit.

Working in opposition to simplicity and reliability are commonly found provisions for flexibility in an excitation system. It is felt by some engineers that great flexibility is required and as a result, all sorts of switching arrangements and reserve apparatus of extra large capacity are provided for. It would be much more practicable to curtail flexibility in an excitation system to an absolute minimum in favor of simplicity. Adaptability of the exciter units to take energy from several sources of power depends upon the general scheme of excitation and will be referred to again below.

B. Two General Schemes

One very common practise has been to supply excitation energy to the fields of all main generators from a common exciter bus, the latter in turn served by several separately driven exciter generators operating in parallel. In some cases the exciters are motor-driven, in others turbine or engine-driven. In any case continuity of supply and voltage regulation of great accuracy are essential. If only motor-driven exciters are employed, taking their energy from the main station bus,—often the case in small stations—very bad regulation may result from the cumulative effects of system disturbances which might otherwise be quite unimportant. This would be especially characteristic of a system employing exciters driven by induction motors. If the exciters are steam-driven the governing characteristics of small turbines or engines must be contended with. A great many turbine-driven exciter sets of approximately 200-kw. size, built in the last six years, have performed very poorly in this respect. The important part to be considered in regard to a common exciter bus is, that the latter entails the maintenance of an additional energy system, secondary to the main energy system, but equally important and requiring safeguards and careful attention to insure continuity of service and the required degree of accuracy of voltage regulation.

Another method of supplying excitation, which is growing in popularity and avoids some of the undesirable features of the common bus system is the individual exciter scheme. In this, individual exciters are devoted solely to serving their respective main generator units. By this individual arrangement, simplification in many ways is easily attained. Moreover, disturbances of the system are limited to a single generator unit instead of endangering operation of the entire plant, *e.g.* a ground coming on the field circuit of one unit would permit continued operation until the opportunity arrived for shutting down that particular unit. Whereas, if the fields of all generators were served from a common exciter bus, it would not be good practise to continue operation of a unit with such a fault, because the appearance of a second ground would not only affect the one unit but endanger the operation of every exciter and main generator in the station. Another favorable feature of the individual scheme is seen in its adaptation to rapidly increasing station capacity. With increase in number of main generator units additional exciter equipment can be installed in like ratio and without disturbing existing systems. In the common excitation bus scheme, the exciter capacity is sometimes necessarily out of proportion to the main generator capacity.

Means of driving individual exciters is another phase of the problem. Recourse to any driver commonly employed with exciter units is permissible, but if motor drive is used, the same care must be taken to secure independence of plant electrical disturbances. If steam drive is used, performance is still dependent upon the speed characteristics of some small prime mover. Latterly, it has become rather common practise to supply the auxiliary power requirements of large power plants from several auxiliary turbines. Such turbines ensure continuous running of essential motor-driven plant auxiliaries, even in the event of a main a-c. system shut-down. A throwover connection from this auxiliary power bus to the main station bus gives additional reliability. It would seem good practise to connect motor-driven exciter sets to this auxiliary power bus, but taking advantage of the security thus afforded as far as continuity of drive is concerned has two distinct disadvantages. A very much more accurate standard of speed and voltage control would be required for the auxiliary energy system and even more important, the exciters would be subject to serious disturbances incident to the operation of numerous pieces of apparatus throughout the plant which are connected to the auxiliary power bus.

Direct connection of exciter generators to the main shafts of main turbines seems an excellently simple solution. Attention of the turbine room operator to an additional machine is not required. The exciter benefits by the good speed regulation of the most accurately governed prime mover in the plant, the main unit. The question of reliability of the prime mover driving the exciter is automatically eliminated, since shutting down the main turbine simultaneously removes the need for excitation of that unit.

There seems to be only one main objection by some engineers to the direct-connected exciter, that the loss of the exciter entails losing from commission the corresponding main unit. However, with the exciter very liberally designed both mechanically and electrically, the chance for trouble with this unit may be so minimized that sole dependence upon it is no more hazardous than a dozen other vital accessories of a main unit, to say nothing of the turbine and generator proper.

C. STANDBY EXCITATION

Standby equipment for every component part of a main turbo-generator unit might appear desirable on first sight, but every engineer realizes its impracticability. Within practicable limitations standby can be provided only for the more extensive components, such as *e.g.* the excitation system. By standby, is meant the substitution of excitation from another source for the normal excitation of a generator unit. The kind of reserve and method of applying same depend upon the general scheme of normal excitation.

Assuming that the common excitation bus has enough exciter capacity connected at all times to permit a sudden shut-down of one unit, additional standby is frequently provided by means of a storage battery. This is usually floated on the common bus and is intended to supply the bus automatically with sufficient current for complete excitation of all main generator units, at least for a reasonable period, in the event of a partial or complete failure of the exciter system. Incidentally, it might be mentioned that a battery so connected acts very efficaciously as a stabilizer for the load fluctuations as well as the vagaries of the several exciters paralleled upon the bus, and thus affords better voltage regulation of the exciter bus.

In the individual exciter scheme, provision for standby for normal excitation is not so clearly established. The reason for this is of course quite evident in that the excitation system belonging to each main generator unit is less extensive, more independent and less subject to troubles. This is especially so with direct-connected exciters.

An individual standby exciter for each main unit would certainly be unnecessary. A common standby bus however, to which the field of any main generator unit can be connected is more practicable and is very commonly employed. The importance of the common standby bus is more or less a matter of opinion. In some cases a special bus used exclusively for excitation is installed and connected to a floating battery. This hardly seems warranted for an occasional duty. If a continuous-current auxiliary-service bus is employed it is usually made reliable for various other reasons and would answer all requirements for standby excitation. In some installations of direct-connected exciters no standby whatever is provided, acting on the theory that although a chain is no stronger than its weakest link, on the other hand it is no weaker by the addition of another link built as strong or stronger than the other links. Here the generator with its auxiliaries is analogous to the chain and the separate exciter to the additional link. In at least one large plant employing direct-connected exciters, five years' sucessful operation has proved that no undue risks have been taken with direct-connected exciters. During this period, the plant increased from 40,000 to 100,000 kw. and only one emergency shut-down is on record, chargeable to the exciter—a preventable accident caused by a foreign agent short-circuiting an unprotected brush holder.

Throwover to the standby bus is accomplished usually manually, but sometimes automatically. Automatic throwover is somewhat questioned as to whether or not it is practicable or even possible with some types of turbo generators. The sensitiveness of the governor and steam valves and the synchronizing power of the generator when the load is suddenly dropped, as well as the speed of the relay and of the closing field breaker onto the throwover bus are very big factors apt to fall short sometimes. With a well and liberally designed excitation system the occasions for throwover would be so rare that the apparent gain by the automatic feature might be offset by the added complications giving rise to trouble.

D. CONTROL OF EXCITATION

The method of varying the exciting current to the main generating field for voltage regulation depends of course, upon the general scheme of exciter connections. With a common excitation bus, rheostats must be provided for both the exciter field and the main generating field circuit. The rheostat in the exciter field is usually small in range and used to maintain a predetermined and constant exciter bus voltage for good parallel operation of exciters. The rheostat in the main generating field has a wide range of small steps, and is employed for controlling the voltage regulation of the main generating unit. The exciter field rheostat is adjusted sometimes to assist the main field rheostat in meeting the excitation requirements of heavy load or other demands on the main generator, but demands within reasonable limitations are usually met by means of the main field rheostat alone, thus simplifying the routine of switchboard regulation.

With individual excitation the exciter field rheostat alone is usually employed to vary the main field current. The exciter field should be designed liberally and the rheostat made capable of adjusting the exciter terminal voltage through a wide range in order to meet occasions like the dropping of the exciter voltage due to abnormal slowing down of the main unit on account of heavy overload or low steam pressure. With exciter field control a comparatively small rheostat performs the duty which otherwise would be required of a large and inefficient main field rheostat. But where standby excitation from a common bus is provided for it is quite common practise to install also a main field rheostat for each unit, but which normally is inoperative. It is only in emergencies when normal excitation is shut off and excitation taken from a new source that this rheostat is used to regulate the main field current. It is evident from the foregoing that the main field rheostat is quite essential in a common bus excitation scheme, but may be eliminated in the individual excitation scheme, except for its need as a reserve in connection with some standby schemes. In the individual exciter scheme, the exciter field rheostat only is essential for normal operation.

In large central stations where the load fluctuations occur at a comparatively slow rate, these rheostats are regulated very satisfactorily by the station operators. In some generating stations supplying energy to a system or part of a system which fluctuates rapidly or to long transmission lines, automatic operation of field rheostats would perhaps be more desirable. Provisions are sometimes made to automatically compensate for line drop on long transmission lines through excitation of the main generators. Such cases, however, are in the minority and are impracticable except where the load on the generating plant is of such proportions and of such nature as to permit operation of the main generator buses in separate and distinct units. Such splitting up of the main generator buses except in special cases is very undesirable from several other considerations in plant operation. In most large central stations carrying a mixed load which is transmitted to substations from which it is distributed, the load fluctuations are comparatively slow and can be met by manual control. The latter method also permits better flexibility in dividing the energy itself as well as the reactive component among the various machines according to the best overall plant economy of the main units in service at any one time.

High quality of regulation and good control in excitation are obtained by means of exciters with shunt fields and interpole windings, a type which has come to be quite universally used. There is a tendency to increase the exciter voltage from 125 volts for the relatively smaller generator units, to 250 volts for the large units on account of the reduction in the size of the exciter as well as in the leads from the exciter to the main generator field. In a large unit the current values, even at 250 volts are not to be disregarded. The old objection to automatic tripping of the main generator field circuit seems to have been withdrawn in the present tendency to provide generators with protective relay schemes which automatically cut off the field excitation on occasion of internal trouble in the main generator circuit.

E. Economy

Energy for excitation, though relatively of small amount when compared with the main generator output, is nevertheless of considerable importance when measured in absolute terms, and the economy of its generation is accordingly deserving of some attention. Its importance, however, is only secondary to considerations of reliability and good regulation. An arrangement devised to obtain the maximum of reliability may, nevertheless, be likewise the most economical, as will be pointed out.

The steam-driven prime movers of the plant fall into two classes—those exhausting to condensers from which the latent heat of the exhaust steam is carried away by the circulating water and wasted—and those units which exhaust to condensers utilizing the exhaust heat. Obviously the energy that can be developed advantageously by the latter class of steam units is limited—in plants not complicated by central heating or process work, limited practically by the heat absorbing capacity of the feed water in being raised from the temperature of main unit condensate to the temperature desired for boiler feed. Every kilowatt-hour generated by the main unit, wasting its exhaust heat to the circulating water, costs the plant upwards of 19,000 B.t.u. chargeable to such generation, whereas every kilowatt-hour generated by prime movers of the second class, costs the system very little more than the heat equivalent of the electrical energy generated. This is all very common knowledge, and very many plants have availed themselves of this source of cheap energy by employing small turbines and engines to drive plant auxiliary machinery. What seems not to have been so clearly realized, however, is the fact that economy dictates the employment of the most efficient auxiliary turbine available, in order to skim the maximum of cheap electrical energy from the live steam devoted to feed water heating. Hence the employment of the extraction type of main turbine from which steam is bled from an intermediate stage in quantities sufficient for feed water heating. Here the early stages of the main unit perform the part of an auxiliary turbine. An alternative is the employment of several auxiliary turbine generators supplying energy to an auxiliary power bus, as described above. The amount of energy developed should be controlled at all times by the demand for exhaust steam to heat the feed water. The load thus carried will bear little or no relationship, however, to the momentary power demand of the motor-driven auxiliary machinery. Arrangement should be made to deliver all excess energy not demanded by plant auxiliaries to the main station bus. Similarly, the auxiliary bus should be able to derive energy from the system at certain periods of the day or in case of shut-down of an auxiliary turbine. These auxiliary turbines can be made of sufficient size to attain very respectable economy, approaching closely, if not equalling that of the early stages of the main unit.

Practically all station auxiliary machinery is then made motor-driven with consequent simplicity and flexibility of control. Every such piece of apparatus shares the reliability afforded by a twofold energy supply and, as already explained, excellent heat economy has been attained. Further economy can be gained by direct connection of auxiliary apparatus to either the main or auxiliary prime mover, and the consequent elimination of intervening generator and motor losses. This arrangement is hardly feasible for any machinery except exciter generators. The exciter direct-connected to the shaft of the main turbine may, therefore, be employed under certain conditions, with maximum economy, besides affording the major advantages of reliability and simplicity. In plants which do not employ auxiliary turbine generators supplying an auxiliary bus, on the other hand, it may be necessary to make exciters turbine-driven in order to utilize all of the available energy of the steam devoted to feed water heating. The heat economy of such small exciter sets is ordinarily rather low, however, and some of the advantages of the other method are forfeited.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 2, 1920

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EXCITERS AND SYSTEMS OF EXCITATION

BY H. R. SUMMERHAYES

Lighting Dept. General Electric Co.

IN laying out the excitation system for the generators of a central power station, the primary requirement is reliability, that is, continuity of service. First cost and economy in operation are secondary, but, nevertheless, must be given consideration.

To meet the first requirement:

1. The exciters should be machines of good design and liberal size.
2. The method of drive should be reliable.
3. All electrical connections and wiring should be as short and simple as possible, and located and supported to be safe from external injury.
4. Method of control should be simple and reliable, and operation convenient.
5. Reserve capacity should be supplied and reserve driving source.

The systems of excitation which have been used or proposed may be divided into two general classes:

1. Common excitation plant (exciters operating on parallel on a bus supplying excitation to all generators).
2. Individual exciters (not operating in parallel).

The first plan was for many years the standard American practise for both steam and hydroelectric plants, excepting in some small plants where belted individual exciters were commonly used.

European practise, on the other hand, has shown a preference for individual exciters, and in recent years American practise has tended toward the use of individual exciters, for reasons which will be discussed.

One reason for the American preference for common excitation plant may have been the use of large alternators driven by slow-speed Corliss engines, on which it was relatively expensive, in cost and floor space, to arrange for direct-connected exciters.

At the same period, European plants were installing high-speed vertical engines, for which the exciters on account of the speed were of small dimensions and weight and could readily be overhung on extended shafts.

When steam turbines came into general use manufacturers were somewhat unwilling to lengthen their shafts and complicate their problems of balance, expansion, etc., to add direct-connected exciters, and for vertical-shaft turbines there was the further objection that the exciter would be in an inaccessible location. There was also the conservation of power plant engineers and the general appreciation of the reliability of excitation afforded by having a battery floating on the common excitation bus.

It is interesting to note, however, that of the steam turbines, 7500 kv-a. and over, sold by one manufacturer during the last five years about 45 per cent were equipped with direct-connected exciters; and of the generators, 1000 kv-a. and over, for water wheel drive, by the same manufacturer, 75 per cent had direct-connected exciters.

Some of the hydroelectric generators of low speed and large size without direct-connected exciters were equipped with individual exciters driven by motors.

COMPARISON OF VARIOUS PLANS OF EXCITATION

Common Excitation Plants. Common excitation plants in which the exciters are operated in parallel on a common bus have the advantage as compared with individual exciters that the bus voltage is kept constant, so that a storage battery may be kept floating on the bus at all times ready to take up the excitation load in case of exciter trouble, also that the constant-voltage exciter bus offers a source for the supply of lighting, auxiliaries, and sometimes the control of electrically-operated switches. If automatic voltage regulators are used directly on the exciters this constant voltage is no longer maintained and this advantage disappears unless a regulator is used on a booster between the constant-voltage exciter bus and a varying-voltage bus to which the generator fields are connected. These common excitation plants have the disadvantage that any trouble on the main exciter bus may cause a shut down on the entire generating station.

Individual Exciters. In the case of individual exciters, where one exciter is supplied for each machine and the exciters are not normally operated in parallel, trouble on one exciter circuit will only affect one generator. The exciter circuits are short and simple and not liable to trouble.

Methods of Driving Exciters. Whether the common excitation plant or individual exciters are used, the method of drive is important.

For individual exciters usually only two methods of drive are used, namely, exciters directly connected to the generator shafts and exciters driven by motors.

In the latter case the motors may be connected to the main bus or preferably they should be connected to an auxiliary bus supplied by an a-c. generator driven by a prime mover.

Transformers are also furnished, so that the motors may be supplied from the main bus in emergency.

This method of driving individual exciters is used chiefly for large hydroelectric plants where on account of the low speed of the vertical shaft generators direct-connected exciters become too expensive. For individual exciters it may be said that direct-connected exciters are preferable on account of cost, reliability of drive and shortness and simplicity of wiring.

Direct-connected exciters large enough to excite two units are sometimes specified. For steam turbines, such large exciters may be undesirable on account of their weight and size, being too great to overhang on extended shaft. For turbines up to 1800 rev. per min. direct-connected exciters are reliable machines and have given good service records. For turbines of 3600 rev. per min. direct-connected exciters are often used, but in order to obtain the best results as to commutation, and make such machines as reliable as those of lower speed, great care must be exercised in manufacture.

Exciter Drive in Common Excitation Plants. In the case of common excitation plant, a number of arrangements for driving the exciters are in use. The most reliable and efficient arrangement is the direct-connected exciter, unless there are reasons, such as too high speed or too low speed, against using it. Belted units are widely used in small plants where the engine speeds are low and the use of a belt involves very little risk of trouble. On account of the low engine speed, a considerable saving of cost and space is made by using belted instead of direct-connected exciters. Geared exciters have been proposed for large low-head hydroelectric plants.

The plan most commonly used for common excitation plant is to have some of the exciters motor-driven through transformers from the main a-c. bus and some of them driven by separate prime movers.

Another plan which has been used in connection with some large steam plants is to have the exciters motor-driven from an auxiliary a-c. bus supplied by auxiliary generator units designated as "house turbines." Transformers connecting auxiliary bus to main bus are supplied for emergency use or for adjusting the power on the auxiliary bus for heat balance purposes. This auxiliary bus is used also for the supply of auxiliary power for the whole station, such as circulating water, air, and hot well pumps, stoker motors, economizer and draft fans, coal crushers and conveyors, etc. In very large stations an auxiliary bus and its generating unit may be supplied in connection with each main generating unit on the system.

An arrangement commonly used in hydroelectric plants and used occasionally in steam plants is to have each exciter connected to a prime mover and to an a-c. motor supplied from the main bus, so that the exciter may be driven by either or both.

This arrangement has been used in steam plants of moderate size and in hydroelectric plants for the following reasons:

1. The reason which applies to both cases is to have

two separate sources of power for the exciter drive. In hydroelectric plants for high head where the exciter water wheel nozzles, on account of their small sizes were likely to become blocked, it has been for many years the practise to have an induction motor connected to the bus mounted on the same shaft as the exciter and the water wheel, so that when the water wheel fails to carry the load the induction motor will take it up. In steam plants the chief reason for using this arrangement is to provide means of adjusting the amount of exhaust steam available to heat feed water, which is done by adjusting the governor of the exciter unit to take more or less power, the remainder being supplied from the motor.

This arrangement in steam plants has the disadvantage that to obtain an efficient turbine the speed must be high, possibly too high for the proper design of the direct-current generator, or of the motor, necessitating sometimes a geared connection which, of course is very disadvantageous for a high-speed continuous running unit.

The plan of direct-connected exciters on the main generator shaft, exciters not operating in parallel the voltage of each generator controlled by the exciter field, appears to be the most reliable and simple method of excitation for large plants wherever the speed requirements do not make a direct-connected exciter out of the question.

For all large stations using individual exciters it is desirable to have an emergency excitation bus with a reserve exciter driven by a separate steam turbine, water wheel or motor, so that any generator field may be thrown on this bus in case of trouble with one of the individual exciters. The question as to whether a storage battery is necessary will depend on the number of units in the plant and on the importance of the service.

VOLTAGE OF EXCITER PLANT

For many years the standard pressure has been 125 volts and this pressure is still standard for small and medium size plants. In recent years 250 volts has been coming into use and has now become standard for large plants for the following reasons:

Difficulty and expense of building high-speed commutators for 125 volts, especially turbine-driven exciters or water-wheel exciters which stand double speed. The space occupied by the commutator is reduced at 250 volts. Expense of bus bars, machine leads, circuit breakers, etc., to carry the large currents necessary at 125 volts, especially in large stations where the field currents are heavy and in long stations where the distances are great.

EXCITATION REQUIREMENTS OF ALTERNATORS

Voltage Range. In the steam turbine generators the armature reaction may be about equal to the no-load ampere turns. This means that with 100 amperes field current required to give full voltage at no-load, 200 amperes would be required to maintain full voltage at full load at the rated power factor. Since the alternator fields must be designed to take not over 125 volts at rated power factor full load and maximum temperature, and because a margin must be allowed in the design for variation in the material, etc., the actual machines may meet their requirements at 90 to 110 volts across the field, and this means that no-load full a-c. voltage the exciter pressure may go as low as 40 or 50 volts, or about 30 or 40 per cent of rated voltage.

The exciters and their rheostats and regulators must be designed with this range of voltage in mind.

For synchronous condensers the range of exciter volts 18 down to 10 per cent or less of full pressure and for synchronous motors the range depends on the range of power factor for which they are designed.

Kilowatts Required. The excitation requirements of alternators vary according to the design, but for modern standard lines may be summarized as follows in per cent of the kv-a. alternator rating:

Steam Turbo-Generators

1000 to 5000 kv-a.	0.5 to 0.3 per cent
7500 to 35,000 kv-a.	0.4 to 0.3 per cent

Water Wheel Driven Generators	
1000 to 5000 kw. slow-speed	1.5 to 0.8 per cent
1000 to 5000 kw. high-speed	1% to 0.5 per cent
7500 to 20,00 kv-a. slow-speed	0.7 to 0.5 per cent
7500 to 20,000 kv-a. high-speed	0.5 to 0.4 per cent
Motor Generaators	
1000 to 5000 kv-a.	1 to 5 per cent

Modern exciters are usually continuous-rated machines having no stated overload capacity, so that in order to allow liberal margin for emergencies of operation the full-load rating of the exciter should be 20 per cent greater than the stated excitation requirement.

RHEOSTATS FOR EXCITERS

For small exciters hand-operated rheostats mounted on the back of the switchboard, or operated by chain drive from a handwheel on the switchboard, are generally used.

For large plants the alternator field and the exciter field rheostats are nearly always electrically operated and this method of operation is recommended for any plant where electric control circuit for operating rheostats is available and where the main control board is on another floor or distant from the machines. For all such plants convenience of operation, location and wiring, as well as cost considerations, will usually give electrically-operated rheostats the preference.

For operation with automatic voltage regulators the exciter field rheostat is generally made three to four times the ohmic resistance of the exciter winding, which is from two to two and one-half times the resistance furnished with ordinary d-c. generators not used for exciter purposes.

For common excitation plant with hand voltage regulation, where the a-c. voltage is controlled by the alternator field rheostats, the exciter rheostats may be of ordinary design with resistance points closely graduated from 85 to 100 per cent of full exciter voltage and further apart for lower voltage ranges.

For individual exciters, non-automatic voltage control, where no generator field rheostats are used, the exciter rheostats should have closely graduated resistance steps all the way down to 30 per cent of the voltage and may have as many as 100 to 150 steps. When this method of regulation is used the alternator field rheostats may be dispensed with, but it is considered better practise to install them for emergency use in case some other source of excitation is resorted to, and it will usually be found that more stable operation at the lower ranges may be obtained by the use of these field rheostats to a certain extent to enable the exciters to work at somewhat higher voltage. If the alternator rheostats are used there is a slight sacrifice of efficiency.

CIRCUIT BREAKERS AND FIELD SWITCHES

As a general principle no automatic overload circuit-breaker or fuse should be installed in exciter circuits. When the alternator is short-circuited the alternator field current may rise to several times normal in the normal direction and an automatic circuit-breaker under such conditions might interrupt the exciter circuit, which must not be allowed. Short circuits in the generator field circuits, or on the exciter bus bars, are an infrequent occurrence and should be taken care of by the operator. It is considered better to risk injury to the exciter than to install overload devices which may operate at the wrong time.

With exciters operating in parallel it is desirable to have circuit-breakers between the exciters and the d-c. buses, operated by reverse current in case of trouble in an exciter or its prime mover.

Alternator field switches should be equipped with discharge resistance. Field switches and exciter switches should be electrically operated in all large plants and in other plants when dictated by convenience of operation, location and wiring. It is desirable, of course, to keep the field switches as near to the alternators, and the exciter switches as near to the exciters as possible, and to locate the exciter as near to the generator as possible in order to keep the exciter circuits short, since the shorter they are the less chance of trouble. Also for reasons of economy. Hence, hand operated field and exciter switches may be used even in large plants if, operated by the floor men, but it is generally desired to operate them from the central switchboard.

Field switches should never open on overload but may be made to open automatically when the main a-c. circuit breaker opens by the action of reverse-power or differential relays in the main alternator leads.

When individual exciters are installed automatic throw-over switches may be used to throw the alternator field over to a reserve exciter bus in case of failure of the individual exciter.

EXCITER BATTERIES

In most large steam stations, and in some large hydroelectric stations, a storage battery is provided capable of carrying the excitation requirements of the station for thirty minutes to an hour. This battery may be used as follows:

1. Floating on constant voltage exciter bus,
2. In reserve on emergency bus,
3. Battery separated into halves, each floating through a high resistance on variable voltage exciter bus with automatic switches to cut out resistance and throw the two halves of battery in series when required for excitation.

Charging a battery may be provided for by an exciter set designed for high voltage, by a special booster or charging set, or by separating battery in two halves and charging through resistance from the exciter bus.

In many stations the exciter bus is used to supply current for the control bus for the working of motor- and solenoid-operated circuit-breaker switches, field rheostats, indicating lamps, etc.

It is now considered better practise to provide a separate control bus, for the reason that during short circuits on the main generators, transient high pressure of the order of several hundred volts may exist in the generator field circuits and alternating currents of normal or double frequency are superimposed on the field currents.

It is difficult to insulate the multiplicity of switches, lamps, sockets, wiring, etc., in the control circuit for such high voltages, owing to the limited space requirements for control boards. For these reasons in large stations a separate control bus with a small battery and motor generator charging set is generally installed.

SHUNT VERSUS COMPOUND-WOUND EXCITERS

The relative advantages and disadvantages of shunt and compound wound exciters have been frequently discussed; and the selection has sometimes been dictated by the type of excitation plant used; sometimes by individual preference of engineers.

The matter may be summarized under the following headings:

A. Small belted exciters operating in parallel, no battery. Compound generally used, to keep exciter bus voltage constant with variation in excitation load due to either variation in a-c. load or to change in the number of alternators in service. With shunt exciters such variations would require adjustment of exciter field rheostats as well as generator field rheostats and the compound are usually preferred for convenience in operation.

If voltage regulators are used either shunt or compound exciters can be handled equally well.

For this class of exciters standard belted generators are used, which are manufactured and stocked in large numbers compound wound, so that reducing the varieties stocked is another reason for the choice of compound winding.

B. Motor-engine or water-wheel-driven exciters operating in parallel, no battery.

Compound preferred, for same reason as stated under A.

C. Exciters direct connected to main generating units, operating in parallel, no battery.

Compound preferred for above reasons, but stock requirements do not apply excepting in small units.

D. Exciters operating in parallel, with storage battery floating on exciter bus.

In this case either compound or shunt may be used.

Compound has the advantage of keeping the bus voltage constant with change in excitation load, and the disadvantage of possible reversal and motoring of an exciter in case of failure of its source of power. This contingency may be provided against by reverse-current relays, so far as motoring is concerned, but this may not prevent reversal of the polarity of the exciter due to the sudden reversal of the current in the series field.

Shunt exciters are safer, when in parallel with a floating battery, as regards possible overspeeding due to motoring, in case of failure of reverse-current relays. They are certainly less liable to be reversed in polarity when an exciter slows down due to trouble with its drive. They require more frequent adjustment of exciter field rheostats, but have the advantage of omitting the equalizer bus with its extra switches and connections. Commutating-pole shunt-wound exciters should be adjusted to have a drooping characteristic at all operating voltages, not only for proper parallel operation but in order to reduce the liability of reversal.

Individual Exciters. The shunt winding appears preferable for individual exciters, whether the a-c. voltage regulation is accomplished by the alternator field rheostat or the exciter field rheostat. In the former case there is no reason for compound winding unless for manufacturing or stock convenience in the smaller sizes. In the latter case, when the exciter has to operate down to a low voltage the shunt winding has more stability, particularly in exciters of the commutating-pole type. The shunt exciter is also less susceptible to reversal by discharge from the alternator field, or by residual magnetic effect from the alternator field.

Reversal. The reversal of polarity of exciter, due to failure of driving power, has been discussed above. It appears likely that in case of compound-wound exciters the chances of reversal due to this cause may be reduced by exciting the shunt field from the bus bars instead of across the exciter brushes.

The reversal of an exciter has been occasionally observed at the time of shutting down an alternator. Possibility of reversal at this time is apparent only on an individual exciter direct connected or otherwise driven from a main generating unit. In one case a steam turbine unit with individual direct-connected exciter was taken out of service, the field being left closed to bring the machine to rest quickly, and field opened after machine had reached a standstill. It appears probable that residual magnetism in the generator field structure would persist to a lower point of speed than that in the magnetic circuit of the exciter. The flux would be varied in passing the armature slots, causing weak alternating currents to flow in the field circuit, which may account for reversal of the exciter.

Assuming that the field current does alternate or reverse, it is apparent that a series field on the exciter would be effective in reversing the residual magnetism of the exciter.

In the case of a commutating-pole exciter the position of the brushes would determine the influence of the commutating field on reversal.

EXCITERS USED WITH REGULATORS

When a vibrating contact regulator is used to control the a-c. volts pressure through the exciter field there appears to be little choice whether the exciter shall be shunt or compound wound, so far as the action of the regulator is concerned. For shunt wound, the field current handled by the regulator is greater. For compound winding the field current is less, but a greater range of voltage must be applied to obtain the same quickness of regulation.

The shunt across the series field makes the latter a damper winding, which impedes sudden flux changes, but this is partially neutralized by the action of the series field with changes in current.

For sensitive regulation the shunt machine is undoubtedly better, since the entire field is controlled by the regulation, and on account of the absence of the damper formed by the series winding. For most plants the compound exciter is satisfactory with a regulator, since its period is usually faster than that of the alternator field.

COMMUTATING-POLE EXCITERS

Some years ago during the first period of experience with commutating-pole exciters some troubles were encountered in operating them in parallel. These were due to incorrect design or to incorrect adjustment of the commutating field strength or the position of the brushes.

Shunt generators to operate in parallel with proper division of load must have a drooping characteristic, and compound machines to operate in parallel must have a drooping characteristic without the series winding. Machines having a rising voltage characteristic without series winding in operation will be liable to give trouble in parallel operation, either as shunt or compound generators, unless the regulation of their prime movers is sufficiently poor to overcome the rising characteristic of the machine itself.

With commutating-pole exciters it was soon found that if compounded flat in test at 125 volts full load 125 volts no load, then when operated at lower voltages (as frequently happens under control of a regulator) they had a rising characteristic and were therefore unstable in parallel operation. This trouble was sometimes made much worse by a slight backward brush shift required for good commutation when the commutating field was too strong.

To take care of this the expedient adopted for a time in one manufacturing plant was to flat compound the exciters in test at 80 volts, thus ensuring a drooping characteristic at pressures above 80 volts, the division of load at lower pressures not being so important.

This expedient involved carrying in stock generators for ordinary purposes compounded flat at 125 volts and generators for exciter purposes flat compounded at 80 volts, or the delay of testing and adjusting shunt after receipt of an order.

With further knowledge of the characteristics of commutating-pole machines and more closely correct designs, the above expedient was abandoned, and it is the present practise to proportion properly the commutating field so that the machine is not over compensated, the design is such that some range of brush shifting is allowable, and the brushes are given a slight forward shift so as to obtain a drooping characteristic at all voltages within the range where parallel operation is required. With this arrangement, exciters when compound wound may be compounded flat at 125 volts and still operate properly in parallel at lower voltages.

The stability of commutating-pole machines depends greatly on the brush position, commutating-pole field strength and voltage at which it operates with regard to the saturation curve of the unit. These three factors affect equally the stability of the shunt and compound-wound unit. In addition, the compound-wound unit is affected by the amount of compounding, the nature of the compounding curve and the size of equalizer connection, also the proper resistance in the equalizer circuit.

Many engineers feel that brush position alone changes the characteristic of the commutating pole machine and, whenever a change is desired, the first resort is always to change the brush position. In many cases, the desired effects can be secured in this way, but nearly always a change in commutating field strength, together with a change in brush position, if necessary, will obtain the results required in a more satisfactory manner.

It is a well known fact that shifting the brushes back from direction of rotation on a commuating-pole generator improves the voltage regulation of the machine, and, on some machines, it is possible to obtain practically a flat voltage characteristic curve on a shunt-wound unit, and, in exceptional cases, it is possible to obtain a rising voltage characteristic curve. The latter obtained by a combination of over-compensated commutating field and backward shifting of brush. In fact, the two act together in that to hold commutation with a back shift of brushes it is necessary to use a stronger commutating field than would be required with the brushes on neutral with proper compensation.

The over-compensated commutating field tends to magnetize the main pole, and, therefore, has a compounding effect. The magnetizing effect is due to the short-circuit current in the coil undergoing commutation. The reverse is true for under-compensation, since the short-circuited current in the coil is reversed for this condition.

We have never found an instance where successful parallel operation could not be obtained after the brush position, as well as the commutating pole field strength, were properly adjusted for shunt-wound machines, and where these two conditions were met and the equalizer connections properly made on the compound-wound machine.

In this connection also it is desirable to arrange the switches of a machine so that the equalizer switch is closed with the line switches and not before. If closed before, the machine on the bus is operating with an additional shunt across its series field and the incoming machine is operating with series as well as shunt excitation, resulting in a lower shunt excitation and, therefore, a chance for instability when the series part of the excitation is changed in amount, and, in extreme cases, in direction.

VOLTAGE REGULATORS

Large city central stations supplying power from the generator bus bars at generator voltage through a multiplicity of feeders have seldom found it necessary to resort to automatic voltage regulation, because the sudden changes of load are small in proportion to the generator capacity.

Exceptions are noted, such as the plants at Philadelphia, Baltimore and Pittsburgh, where unusual requirements in intermittent loads exist, due to the supply of main railway electrification or steel mill loads.

In hydroelectric plants, on the other hand, the power is usually carried through a few large transmission lines, the interruption of any one of which means the loss of a large proportion of the station load, necessitating automatic voltage regulation of the generators. The vibrating contact forms of regulators devised by Tirrill are the only ones in wide use in this country, and no others will be discussed.

These regulators can be made to take care of exciters operating individually or in parallel in sizes up to the largest which it has been found necessary to use. The exciter field current is controlled through relay contacts; the relay coils themselves being actuated by direct current passing through the main regulator contact. Up to about four amperes at 125 volts a single relay handles the exciter field current; for larger exciters the field rheostat is divided into sections short-circuited by a number of relays, the general rule being that each relay will take care of two amperes field current; that is to say, for a total of ten amperes field current there will be at least five relays. At 250 volts half the current is handled. To obtain the best results the kilowatt output per exciter should not be more than 25 kilowatts per relay. When the field current is more than twenty amperes, it is generally desirable to split the field, so as to keep the actual field current handled by the relays below 20 amperes.

Since a 12-relay regulator will handle about a 300-kilowatts exciter, and regulators have been made with as many as 48 relays, which could handle four such exciters in multiple, it is evident that the regulator can be made to take care of very large excitation plants.

For still larger plants, if such should be contemplated, other methods of application of vibrating contact regulators may be used, so that we can say there is no limit to the size of plant which can be regulated on this principle.

A single regulator may be used to control a number of exciters operating in parallel, or to control a number of individual exciters not operating in parallel, when the alternators excited run in parallel. It is also possible to use in the latter case individual regulators, and in this case the proper division of reactive component among the alternators is accomplished by a compensating coil on the regulator supplied from a current-transformer connected in such a way that the current is at a right angle phase relation to the voltage which the regulator is maintaining. This last arrangement is favored for large hydroelectric plants having individual exciters, as the individual regulator may be mounted near each exciter.

Stops may be provided on regulators to limit the field current of an alternator and this is always done then regulator is used with a synchronous condensers keeping the voltage of the receiving and constant up to the limit of output, which is determined by a limiting field current.

Regulators for large plants are provided with accessories, such as over-voltage relays and over-current relays, which cut in an extra block of resistance in the exciter field, so that in case of over-speed on the generators or relay contacts sticking, unduly high voltage will be prevented, and in case of short-circuit the action is to prevent over-excitation of the fields.

DISCUSSION OF VARIOUS PLANS OF EXCITATION

In selecting a plan of excitation for any plant the local conditions will govern to some extent. In either steam or hydroelectric plants there will be a certain amount of auxiliary power about the station which which must be supplied. In a hydroelectric plant the requirements for auxiliary power are not very exacting as to continuous operation unless motor pumps are supplied for the step bearings. Most other motors about such plants are for intermittent operation and may be taken care of by an auxiliary bus supplied by step-down transformers from the main bus.

In a steam plant the continuous operation of many of the auxiliaries is of vital importance and since variable speed motors are supplied for many of the auxiliaries in order to obtain economical operation at part loads, direct-current motors are frequently used for part of the auxiliaries. At first sight, the best source of power for such auxiliaries would appear to be the exciter bus and, thus, the question of choice of excitation plans becomes involved with the other auxiliaries in the station.

The earlier practise in this country was to operate all of the auxiliaries, such as the circulating water pumps, hot well pumps, feed pumps, stokers, draft fans, etc., by steam power. Thus, insuring an ample supply of exhaust steam to heat the main feed water. In some cases this provided too much steam and some steam had to be wasted and in any event the driving of many of the auxiliaries by individual steam turbines or engines is somewhat wasteful, since the small steam machines consume steam per horse power output at a rate several times that of the main turbine. If a certain number of pounds of exhaust steam is required to heat feed water it is evidently more economical to use that steam first in a large and efficient turbine, so as to get as many horse power as possible out of it before passing it into the feed water heater. Modern practise is now tending toward the operation of as many of the auxiliaries as possible electrically, particularly on account of the convenience, reliability and freedom of repair of the electric motor itself, and partly on account of the high efficiency obtained by this method of operation, whether the electric power for the auxiliaries is derived from the main buses or from a separate auxiliary generating source of high efficiency.

The main boiler feed pumps are usually run by steam, but if all the other auxiliaries are electrically operated, as appears to be the modern tendency, there will not be sufficient exhaust steam from the feed pumps to heat the feed water. If the electrical auxiliaries are operated from the main bus feed water may be heated by bleeding the main turbine at an intermediate stage and drawing off sufficient steam to heat the feed water. This is an efficient method of operation, since the steam is used very efficiently in producing mechanical power in the main turbine before it is drawn off. Such an arrangement should be operated on the unit system; that is to say, with a certain bank of boilers supplying a certain turbine, the steam drawn from such a turbine should be used to heat the feed water for its own bank of boilers.

This method has the disadvantage that in case of trouble on the main bus many of the station auxiliaries may be interrupted, although in modern stations with current limiting reactors on the feeders the experience has been that the bus voltage is not sufficiently lowered by a feeder short circuit to interfere with the operation of the auxiliaries.

The house turbine arrangement in which a house turbine with its own boilers supplying power to an auxiliary bus is installed for two main generating units or one house turbine for each main generating unit in a very large plant, appears to possess many advantages. When the main units are 15,000 to 30,000 kw. each, each house turbine may be 1000 to 2000 kw., large enough to obtain efficiency in the use of steam. The auxiliary bus may be connected by transformers to the main bus with automatic relay arrangements so that in case of trouble on the main bus the auxiliary bus is cut off and supplied only by its own power.

This arrangement possesses the advantage that the supply of power for the auxiliaries, including the excitation, is independent of the main supply and also the advantage that the heat balance is readily adjustable by adjusting the amount of power supplied by auxiliary generating unit, so that the amount of steam exhausted by it is just sufficient to bring the feed water to the proper temperature, the remainder of the auxiliary power being supplied from the main bus, which is operating in parallel.

When such an auxiliary house plant is supplied, it is generally alternating current and the excitation for the main unit may be supplied from a motor-generator set run from the auxiliary bus. It would seem desirable that these exciter buses for separate units of the power house should not be operated in parallel, but provision may be made for connecting them in parallel, and to obtain the greatest safety, a reserve exciter unit and storage battery with emergency bus may be supplied, to which any generator field may be connected.

For the very largest steam station, such a plan appears desirable, but it is a still better plan, when auxiliary house plant is used, to supply the excitation for the main units from direct-connected exciters on the main units. There should still be installed a reserve exciter bus with battery and reserve exciter driven from the auxiliary bus, with automatic throw-over switches, so that in case of trouble with any direct-connected exciter the field of that alternator is disconnected from the exciter and thrown on the reserve excitation bus. With this plan the excitation is kept separate from other auxiliaries and the exciter bus is not liable to trouble resulting from motor trouble; also the exciter and field connections are kept short and simple as possible. This plan has the further advantage that the a-c. voltage may be controlled by the exciter fields and the losses in the main field rheostats eliminated. It is desirable, however, to supply main field rheostats for emergency use.

In connection with auxiliary house plant, generating alternating current for the supply of most of the auxiliaries, motor-generator sets have been installed fed from the auxiliary bus to produce direct current for the variable speed auxiliaries. This complication of an extra d-c. bus may be done away with if an alternating-current motor with good adjustable speed characteristics were available. Such a motor is now coming into use. It is a three-phase commutator motor with three sets of brushes on the commutator and the speed is varied by shifting the brushes, which may be done by distant control by a small motor geared to the brush shifting yoke. The motor has a series characteristic and is, therefore, well adapted for driving fans or centrifugal pumps, but is not as good as an adjustable-speed shunt-wound motor for applications where it is desired to adjust the speed through a wide range and keep the speed constant for varying load.

There has been some experience with such motors in continuous operation driving mine fans, which indicates that they are as good as direct current motors with a possible disadvantage of more brush wear. Further experience will undoubtedly justify their use for the exacting requirements for power house service for such purposes as fans and centrifugal pumps.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 2, 1920.

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THE APPLICATION OF D-C. GENERATORS TO EXCITER SERVICE

With Particular Reference to the Requirements of Automatic Voltage Regulators

By C. A. Boddie and F. L. Moon
Both of the Engineering Dept., Westinghouse Electric & Mfg. Co.

In the design of direct-current generators for exciter work, there are a number of problems peculiar to this kind of service, the variety and importance of which have been considerably increased by the use of automatic voltage regulators. It is the purpose of this paper chiefly to discuss the relation of exciter design to some of these problems, such as voltage range, responsiveness, stability, and type of drive, with especial reference to the requirements of automatic voltage regulators.

Choice of Exciter Voltage, Voltage Range, and Stability

THE choice of exciter voltage whether 250 or 125 volts, is not a matter of great importance in the majority of cases. There are some cases, however, in which it is more desirable to use one than the other. A given machine requires a certain kilowatt excitation regardless of the voltage, so that with the higher of the two voltages a lower current is required and consequently a smaller size of conductor must be used for the field coils. In machines of small capacity this may result in a wire of such small cross-section, and a coil of such large number of turns that a large amount of coil space is taken up by the insulation. Trouble may also be experienced in high-speed machines, in supporting the coil properly against centrifugal force. In such a case the use of 250 volts may be a real handicap. Small turbo-generators or other machines on which it is desired to use copper strap bent on edge for the field coils, also have a lower limit for 250-volt excitation on account of the difficulty experienced in bending the thin straps to the proper shape. In some of the very large turbo-generators a limit is reached in the other direction. That is, if 125-volt excitation is used the current becomes so large that difficulties in current collection are encountered. In order to accommodate a large current it becomes necessary to equip the machine with a heavy collector and a large number of brushes, which usually results in higher maintenance costs with the high collector speeds involved. In general, the larger alternators should have 250-volt exciters and the smaller ones 125-volt exciters.

The question of exciter voltage range does not arise when the a-c. generators are excited from a constant voltage bus, since in that case the exciters are required to operate at one voltage only. This condition also permits operating the exciters well above the bend in the saturation curve, so that good stability is obtained. But when the alternator excitation is controlled by varying the exciter voltage, the factors of voltage range and stability require consideration. In alternators of modern design the range of exciting current required at unity power factor is approximately 1½ to one; and about two to one at 80 per cent. power factor. This means a range in exciting voltage of about 2½ to one at 80 per cent power factor; and in more exceptional cases the range in exciting voltage may reach three to one. A still greater range in exciting voltage may be required by generators or synchronous condensers which have to supply charging currents to transmission lines at light load. Here a range in exciting voltage down to the residual voltage of the exciter may be required.

Standard regulators of the Tirril type are usually arranged for a voltage range of two to one or 2½ to one. This is the range over which the d-c. control magnet and the rheostat shunting relays function properly. If the regulator has an a-c. vibrating magnet in place of the d-c. control magnet, and a source other than the exciter is available for energizing the shunting relays, the range over which the regulator is operative extends down to residual voltage. Attention should be given to the performance of the exciter over a range somewhat exceeding that for which the regulator is designed. When operating near the "full field" or maximum voltage point of the exciter, the time during which the field rheostat is cut in is very small compared to the time during which it is short-circuited. This means that the shunting relays are open a very small proportion of the time, and unsatisfactory operation is the result. As a consequence it is the practise to leave a certain margin of maximum voltage over the highest voltage required for the main generator fields; this margin is usually 10 or 15 volts for 125-volt exciters and 20 or 30 for 250-volt exciters.

The principal factors upon which the voltage range of an exciter depends, may be seen from a study of Fig. 1. Let OB represent the relation between field current and voltage applied to the field circuit with the field rheostat short-circuited; and OC that with the field rheostat cut in. With the rheostat cut out, the voltage and field current will rise to values corresponding to the intersection P. When the rheostat is cut in the voltage drops to a small, definite value, but not to zero since the saturation curve really does not pass through the origin but crosses the vertical axis slightly above it owing to the remanent magnetism. When the machine is connected to a load, say a fixed resistance, the saturation curve is tipped to the right on account of the internal drop. The maximum voltage which can be obtained in this case, is the ordinate of the intersection P′. If the exciter has a differential series winding the shunt field current must be increased to counterbalance the bucking series ampere turns, so the load saturation curve takes a position still further to the right, say OE. The maximum exciter voltage obtainable in this case is still less. If the exciter has a cumulative series winding, the shunt field current for a given voltage is reduced so the saturation curve is moved to the left, and has some such position as OF. A greater maximum voltage and field current is obtained

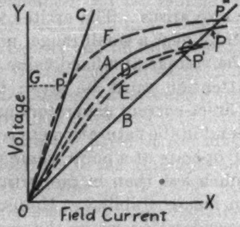

FIG. 1

in this case, depending upon the intersection P″. The curve OF may cross the resistance line OC at a considerable voltage, as at P″. In such a case the exciter voltage could not be reduced below the value OG. If the resistance of the field rheostat is increased sufficiently the angle YOC will be reduced, and OF and OC can be made to cross only at residual voltage. It is thus evident that the effect of load and differential series windings is to reduce the maximum voltage and field current; while cumulative series windings increase the maximum voltage and field current, but if strong enough may greatly increase the minimum voltage obtainable.

When regulator operation is used, no difficulty in operating on the straight part of the exciter saturation curve arises. The exciter voltage rises and falls periodically over a small range, no attempt being made to hold the voltage at a fixed value. When hand control is used, attention must be given to the question of stability. Direct-current generators usually are operated above the bend in the saturation curve, as it is considered necessary to operate at a point where the terminal voltage increases much less than in proportion to the excitation, in order to insure stability. Undoubtedly this is an important consideration where the generators are to supply power to constant potential circuits and at varying loads. But the conditions are more favorable when the load is an alternator field. Such a load is subject only to slow change due to heating, and sudden fluctuations are altogether absent. Besides, it is generally necessary to maintain low exciter voltage only for infrequent and brief intervals. Suppose OA, Fig. 2, to be the exciter saturation curve for constant resistance load, and OB the position of the volt-ampere characteristic of the shunt field. When permitted to build up, the voltage will rise to the point P and stop. It is evident that a moderate change in the position of the saturation curve, such as would

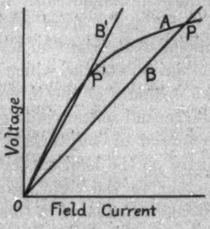

FIG. 2

result from a change in speed or a change in load, would have a small effect on the voltage. If the shunt field resistance is increased until OB assumes the position OB′, the new intersection is P′ and the terminal voltage assumes a value corresponding to this point. Under this condition a change in speed or load would make more difference in the terminal voltage than before, for at P′ the slopes of OB′ and OA are not very different. At this point also a slight change in the resistance of the shunt field circuit would make a larger difference in the terminal voltage, and for the same reason. This indicates that the exciter field rheostat should have rather fine stops in order to obtain closer adjustment of exciter voltage. The influence of load changes can be minimized by having a certain amount of series field in the exciter.

The voltage stability of an exciter can be improved to a certain extent if necessary by making the saturation curve bend away from the "air gap line" at a much lower voltage than normal. The principle employed is to bring about progressive saturation in some parts of the magnetic circuit. Suppose two portions of the magnetic circuit A and B (Fig. 3) are separated by a small distance, but are connected by a certain amount of magnetic material. At low inductions the reluctance of the path d, will be very low; but at high inductions the reluctance will approach that of an air gap of the length d. Thus a virtual increase in the air gap length may be produced at relatively small values of total flux. At a higher value of flux another part of the circuit may be made to saturate. In this way a saturation curve which bends continually from half normal voltage or less can be produced. It is impracticable to improve the stability by this means below one-half of normal voltage. This increased saturation necessitates a more expensive exciter; and if automatic voltage regulating equipment were used, it also would be larger and more expensive.

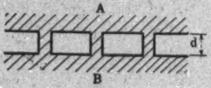

FIG. 3

RESPONSIVENESS

The matter of responsiveness is of interest only when automatic voltage regulators of the Tirril type are employed. By responsiveness is meant the quickness with which the machine voltage responds to changes in the field circuit resistance or exciting voltage. As a measure of this we might use the time-rate of change of exciter or alternator voltage when its field resistance is suddenly varied by a certain percentage; or for the complete unit of alternator and exciter, we might take the time-rate of change of alternator voltage (r.m.s. value) for a certain percentage change in the exciter field circuit resistance. Obviously no very definite values can be assigned to this quantity as it is affected by many variables such as the voltage, load, and power factor. Responsiveness is a matter of interest regardless of the size of the equipment and the character of the load. A prevailing opinion is that a large plant does not require such rapid response from an exciter as a small one, since the load changes are usually more gradual. In reality, a more responsive exciter is needed to make up for the inherently great sluggishness of large generating equipment. When a slow exciter is used in connection with such a plant, which is inherently slow on account of its size, it becomes necessary to damp down the main control of the regulator to prevent hunting, to such an extent that the general response of the system is but little better than with hand regulation. If the load changes are slow, this may impose little hardship except when handling the regulating equipment as when taking a machine off the line. The readiness with which changes can be made by the operator and the general reliability of the regulating system, are impaired by extreme sluggishness.

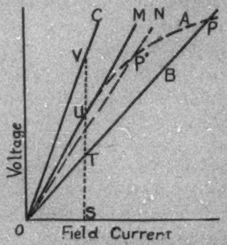

FIG. 4

To understand the influence of various factors upon the responsiveness of exciters, requires a consideration of the basis upon which an automatic voltage regulation system works. Referring to Fig. 4, let OA represent the no-load magnetization curve of a self-excited d-c. machine. Suppose OB to represent the relation between applied voltage and current in the field winding. If at a given instant the field current is OS, the resistance drop in the field winding is ST, which is less than the voltage developed in the armature by the amount TU. This excess voltage compels the current to increase, and this change in current results in an inductive counter e.m.f. which just balances the voltage TU. As the current increases the excess voltage measured by the separation between OB and OM (referred to as the "lower opening") also increases, and the rate of increase being proportional to the voltage compelling such increase, the current rises faster and faster. Assuming constant inductance, it is easily proved that the current varies in proportion to $\epsilon^{\frac{R}{L}t} - l$. The change of current with time is represented by OA of Fig. 5. Under the assumptions made, this also represents the variation in the machine terminal voltage. Suppose resistance is added in the field circuit so that the new volt-ampere characteristic coincides with OC of Fig. 4. When the field current is OS the resistance drop in the field is SV, which exceeds the voltage generated in the armature by the amount UV. Obviously the current cannot remain at the value OS, but must diminish, the rate of decrease being sufficient to cause an inductive counter e.m.f. which balances UV. The excess voltage (referred to as the "upper opening") becomes less and less as the current falls, so the rate of decrease becomes less. Accordingly the current variation in this case follows the law $\epsilon^{-\frac{R}{L}t}$ and may be represented by BC of Fig. 5.

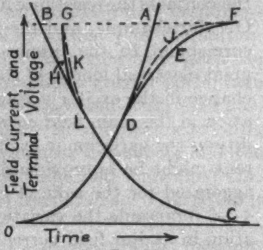

FIG. 5

The conditions in an actual machine would differ from the preceding in several respects. In reality there are two branches of the magnetization curve, owing to hysteresis. Also, the magnetization curve would not continue indefinitely as a straight line, but would bend over to the right and become flat as at A in Fig. 4. The effect of saturation is to lessen and finally to reduce to zero the margin of voltage tending to increase the field current, so that the rate of current increase becomes less and less, as indicated by DEF of Fig. 5, and reaches a steady value which is determined by the intersection P, of OA and OB in Fig. 4. And the voltage tending to reduce the current when additional resistance is inserted in the field circuit, is increased, so the field current would fall off faster as indicated by GHL of Fig. 5. Since the terminal voltage does not increase at the same rate as the field current when saturation is approached, the time rate of increase of terminal voltage would follow along DJF of Fig. 5; and since the voltage does not fall as fast as the field current when the latter is decreasing, the corresponding curve of e.m.f. variation would be near GKL. It should be observed that the time required for the current to fall from maximum to minimum may be quite different from that required for the current to build up from minimum to maximum, difference depending largely upon the relative difference in slopes between OB and OM, and between OM and OC.

When a vibrating type of regulator is used, a shunt path about the exciter field rheostat is opened and closed alternately as a result of which the exciter field current and voltage periodically rise and fall. When the shunting relay contacts are closed the field resistance line may coincide with OB (Fig. 4); and with OC when the contacts are open. For a given percentage time of closure of the contacts the resistance line may have the mean position ON, for example, and the resulting mean voltage would correspond to the intersection P′. On account of the great electrical inertia of the main field, a relatively large change in the exciter voltage produces only a small variation in the main field current. Under steady load conditions the variation in exciter voltage and main field current might be approximately as indicated in Fig. 6 (exaggerated for the sake of clearness). The curve, 1-2-3-4-5-6 represents the variation in exciter voltage; the variation in exciter field current would be somewhat greater owing to saturation, so it might vary as indicated by 1-2′-3-4′-5-6′. The change in main generator field current would be relatively much less, as indicated by curve AB. The IR drop in the main field circuit is proportional to the main field current, so it may be represented by curve CD whose ordinates are proportional to those of AB. The vertical separation between CD and 1-2-3-4-5-6 is the voltage at any instant tending to change the main field current. From a to 2 this excess voltage is

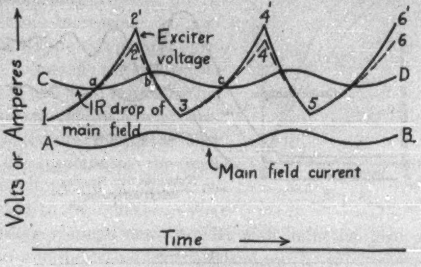

FIG. 6

increasing, so the main field current is increasing at an increasing rate; beyond 2 and extending to b the excess voltage still tends to make the main field current larger, but at a diminishing rate of increase which ceases when b is reached, since the excess voltage becomes zero at that point. Similarly, over the range b-3-c the exciter voltage is insufficient to maintain the main field current at its original value, so the latter diminishes. Instead of steady conditions suppose an increased load is suddenly thrown on the system. The tendency of the station voltage is to drop acts through the regulator to keep the exciter field rheostat short-circuited a greater percentage of the time, and the exciter voltage and main field current attain higher mean values. In attempting to bring the voltage to normal the regulator overshoots and the new mean values of exciter voltage and main field current are reached after several oscillations. This effect is indicated in Fig. 7.

The quickness with which the exciter voltage responds to the impulses of the regulator depends upon the interlinkages of flux and turns in the field circuit and upon the separation between the saturation curve and volt-ampere characteristic of the field circuit. In fact, the time-rate of change of the exciter terminal voltage in per cent of normal voltage can be proved equal to the instantaneous difference between the terminal voltage and the IR drop in the field circuit, divided by the interlinkages of flux and turns at rated voltage. The number of interlinkages is found by multiplying the field winding turns connected in series by the flux per pole including leakage flux. The effective "opening" between the magnetization curve and resistance line is influenced by several factors. The effect of hysteresis is to reduce the "opening." Cumulative series field windings increase the "lower opening" and decrease the "upper opening," differential series has the opposite effect. The result of armature reaction is to reduce the "lower opening" and increase the "upper opening." Usually some portion of the magnetic circuit such as the yoke, and perhaps the poles, consist of solid conducting material. When the field current varies, such parts act in a measure as a short-circuited secondary winding of a transformer. The effect of the induced currents amounts to a reduction in the upper and lower "openings."

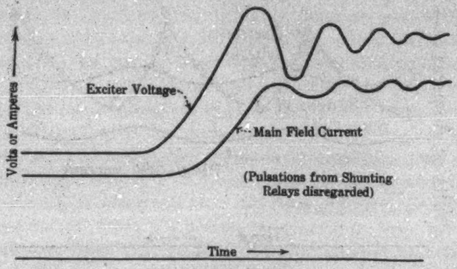

FIG. 7

The closeness with which the exciter load current follows changes in its terminal voltage, has an influence upon the responsiveness of the exciter. In Fig. 8, let OA represent the no-load magnetization curve; and OB the magnetization curve for constant resistance load such as a generator field. Owing to the magnetic inertia of the main field, considerable variations up or down in the exciter voltage can occur without material change in the field current of the main generator. As a result the saturation curve really applying is that for constant-current load, such as MN for a decreasing voltage, or RS for an increasing voltage. This results in a greater opening between the saturation curve and field resistance line than would otherwise exist with a pure resistance load, so the response to the regulator impulses would be expected to be quicker. That exciters respond more rapidly with inductive load than with non-inductive load is borne out by experience.

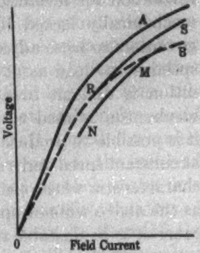

FIG. 8

PARALLEL OPERATION OF EXCITERS

In generating plants it is often the practise to operate exciters in parallel. When hand regulation is used, this does not involve any problems other than those met with in parallelling other d-c. generators; but if the exciters are under regulator control, more things are involved owing to the range of voltage over which the machines must operate. When two or more units, each having its own regulating device or under hand control, are run in parallel, a certain fundamental condition must be met; otherwise instability resulting in surges will result. This proposition is not limited to d-c. generators, but applies to all equipment operated in parallel, such as steam engine governors, waterwheel governors, a-c. generating units, and voltage regulators. This essential condition is that the quantity being regulated must have at the point where the parallel units are tied together, a drooping characteristic with reference to load. As this principle frequently is not understood, a simple illustration is given.

Assume two steam engines mechanically locked to the same line shafting, each having its own governor, and assume load to be taken from this common shaft. Engines driving alternators operated in parallel are locked together electrically, and the two engines must rotate revolution for revolution just as rigidly as if they were mechanically locked to the same shaft. Suppose the governors to be so adjusted that each unit has the same speed at no load as at full load. In practise, this condition is difficult to attain, unless a special restoring mechanism is used to prevent hunting. Nevertheless, it is possible to produce a governor which will maintain at constant speed under all conditions or even give a rising characteristic when operated alone. Such a combination as the above will not operate in parallel with another because it is in unstable equilibrium, there being no force to control the distribution of load between the units. That the system is unstable becomes apparent if we assume one governor to be adjusted for a slightly higher speed than the other. If the common shaft is revolving at a speed which is correct for the low-speed governor, it will be too slow for the high-speed governor which will accordingly cause its steam valve to open in an attempt to raise the speed. But any increase of speed will cause the low-speed governor to close its valve in its attempt to run at its own speed. This results in the low-speed governor shutting its steam valve entirely, and therefore, carrying no load while the high-speed governor throws its steam valve wide open and attempts to carry the entire load. This will be the result, no matter how slight the difference in governor settings. Even if the two governors could be adjusted for exactly the same speed, and each engine were carrying its proper share of the load, anything which disturbed this division of load would result in a redistribution which might be anything. For as long as the main shafts are rotating at the speed required by the governors, these governors would be in equilibrium independent of the opening of either steam valve, and, therefore, either engine might carry all or none of the load, or any proportion of it.

In practise the method of compelling two units to operate stably in parallel is to introduce an inherent droop in the regulation curve. It has been found necessary to adjust governors so that the drop in speed, from no load to full load, is 2 per cent or more. The more the inherent droop, the greater the tendency toward stability, and vice versa. That this inherent droop is a stabilizing influence can be seen by reference to our steam engine example. Suppose, in the foregoing, each governor were adjusted for a 4 per cent drop in speed, from no load to full load; then when they are locked together on the common shaft, if such a load is taken from this shaft as to lower the speed 4 per cent, each engine would be working at full load. If the shaft slowed down only 2 per cent the valves would both be open to the half load position. That is, the load carried by each engine is a direct function of its speed. Suppose also that No. 1 machine momentarily took more than its share of the load, due to a temporary opening of its steam valve. This would tend to speed up the shaft, which increase in speed would cause No. 2 governor to close its steam valve in proportion to the increase in speed, which would, therefore, throw still more load on No. 1 engine. This process of unloading No. 2 machine and loading No. 1, would continue until equilibrium is reached. The reaction differs fundamentally from that in which no inherent droop is assumed, in that there is a definite valve position for each engine for every speed, and stability of operation results; any tendency to shift load is checked by the mutual action of the governors.

In applying this theory to exciters, we see that the first essential is that the exciter tend to shirk its load; that is, when it takes more than its share of the load momentarily, this must result in a drop in the terminal voltage with a consequent tendency to drop the excess load. This condition is generally fulfilled in shunt machines by the internal voltage drop; this drop makes the terminal voltage fall off with increase in load. This condition is not inherently present in compound machines unless the so-called equalizer is used. Without the latter a change in load produces a change in voltage which accentuates the load change; hence it will not operate stably, but is likely to drop its load entirely or take an excessive load. When an equalizer of low resistance is provided, the series field strength is determined by the total load, and not by the load on the machine in question. If one machine robs another of load, the excitation of the former does not increase, but its voltage diminishes owing to the increased internal drop, and it tends to drop some of its load.

Suppose two compound generators are running in parallel, one of which is a low-speed non-commutating-pole machine with relatively high internal drop, and quite saturated, and the other a high-speed commutating-pole machine with relatively small internal drop and little saturated. The latter machine will have but little tendency to shirk its load; furthermore, owing to the impossibility of providing an equalizer of zero resistance, an increase in load on this generator may result in a slight increase in series field ampere-turns, and because of little saturation this may have sufficient effect on the voltage to overcome the internal drop. As a result its voltage may increase slightly if it takes load from the other machine; and it will be very liable to take all of the load. The unstable condition may also arise from overcompensated commutating poles or from a backward brush lead. To remedy such instability it is necessary to increase the drop in the machine (for constant series excitation) as by use of a series resistance, or a bucking series winding connected inside the equalizer, or a slight forward shift of the brushes if commutation permits. A similar unstable condition may arise with shunt machines of the high-speed commutating-pole type, especially when working low on the saturation curve as is sometimes necessary with exciters. The inherent internal droop being very small, any slight disturbance may cause one machine to drop its load and another to take it all. To ensure proper load division, remedies similar to those just mentioned must be applied.

Exciters arranged for parallel operation under regulator control, are adjusted to give the same maximum voltage under load. Then under short-circuit conditions when the regulating rheostats are shunted out, the exciters build up to the same maximum voltages, and share the load properly, cross-currents between them being avoided. In Fig. 9 are plotted two saturation curves of exciters in terms of terminal voltage and percentage of maximum ampere-turns. Having the same maximum voltages, the two saturation curves have a common intersection at P, and the lower resistance lines coincide. As these machines may have differently shaped saturation curves, the effective resistance lines ON and ON′, which give the same stable operating voltage E, may be some distance apart. Now, the two exciters being under control of the same regulator, the percentage time during which the regulating rheostats are shunted, must be the same. Hence for the effective value of the rheostat resistance to be different in the two cases, the total must be correspondingly different. This is taken care of in an approximate way by turning in enough of the regulating rheostat to make the time required for the voltage to drop from 125 to 30 volts about the same as that taken for the voltage to build up from 30 to 125 volts. Such marked differences in the shape of exciter saturation curves, however, are seldom found.

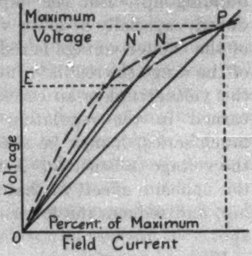

FIG. 9

Under steady conditions the question of exciter response has little to do with the matter of parallel operation. At times when swings in exciter voltage take place in response to regulator impulses, the load division may be affected by this factor. If the exciter bus voltage rises from 60 to 110 volts suddenly for example, and the voltage of one exciter rises much more rapidly than another, the former will momentarily take more than its share of the load. For this reason good armature stability is desirable when exciters in parallel are under regulator control.

COMPOUND VS. SHUNT FIELD WINDINGS

The question of series field windings for exciters arises because of their relation to the performance of the regulating equipment and their effect upon the stability and reliability of parallel operation.

The general effect of series windings upon the action of the regulating equipment may be inferred from their influence upon the saturation curves. From Fig. 1 it may be seen that cumulative series results in an increased opening between the saturation curve and lower field resistance line. It therefore tends to increase the rate of building up. But the "upper opening" is greatly reduced, especially at lower voltages, so the rate of decay of the exciter voltage is reduced; and on the lower part of the range the regulator may have very poor control of the voltage unless an excessive amount of resistance is turned in the regulating rheostat. And if there is much series, it may be absolutely impossible to reduce the voltage sufficiently. A differential series winding has the opposite effect, in that it increases the upper opening, and reduces the lower opening and maximum voltage.

With any considerable cumulative series windings, difficulty with polarity reversal may be experienced. If a compound exciter is thrown on the bus while its voltage is less than the bus voltage, or if some action takes place which reduces its voltage below that of the bus such as a momentary drop in speed, a sudden reversal of current through the machine may take place. This current rush is likely to buck out the field flux and reverse the residual, as well as kick out the breaker. If the breaker has no overload trip, the machine may be seriously damaged, or perhaps run away; even a shut-down of the station may result.

In general, any considerable amount of series field winding in an exciter is a detriment from the voltage regulator standpoint. Exception should be made in the case of non-commutating-pole exciters having the brushes shifted forward, which introduces a demagnetizing component of armature reaction. Sufficient cumulative series windings to balance this component of armature reaction are entirely permissible, and are in fact an advantage since less shunt field current is then required. In broad range regulating it is quite essential to have the demagnetizing component of the armature reaction neutralized by a series field winding, to avoid the possibility of polarity reversal. Suppose the exciter voltage is rapidly falling to zero owing to the regulator opening the contacts across the field rheostat. On account of the inertia of the main field the exciter load current does not fall off proportionately, but hangs on; so when residual voltage is reached, there is still considerable armature current, and the demagnetizing effect of this is liable to reverse the residual magnetism. This results in the voltage building up with the opposite polarity. Sufficient series field turns to counterbalance the demagnetizing component of armature reaction, will prevent such polarity reversal. In the smaller sizes of d-c. generators only compound-wound machines are kept in stock. Hence it may be desirable in the case of small exciters to purchase standard compound-wound machines because they can be obtained quickly; but even in such cases it is usually possible and is preferable to disconnect the series winding and operate the exciter shunt wound.

OMISSION OF MAIN FIELD RHEOSTATS

From several standpoints it would be desirable to omit the main field rheostats. They waste considerable energy; they may occupy considerable space, as much as 200 cubic feet in some instances, and the cost of a large rheostat may amount to several thousand dollars. In general terms, the main rheostat is required for the purpose of adapting a constant voltage source of excitation to the varying excitation requirements of the alternator. Obviously, if the rheostat is omitted, a suitable means of varying the exciting voltage must be provided in its stead; as for example a separate exciter for each alternator, or a boosting and bucking series-connected generator in the alternator field circuit.

The booster may form a desirable substitute for the rheostat, particularly when a broad range in exciting voltage is required. It has an advantage over the rheostat in that it can either buck or boost the voltage of the exciter bus so that it eliminates an appreciable part of the power loss, and occupies a small amount of space. This scheme is more likely to be suitable when the main units are very large and few in number, and the exciter is used to supply power for station lights or auxiliaries, or it is desired to float a storage battery on the exciter bus. Automatic voltage regulation could be obtained by providing a regulator to control the booster field.

Ordinarily the question of omitting the main field rheostats arises only in the case of a station having a separate exciter for each main unit. When separate exciters are provided, it is possible, and customary, to vary the excitation of the main generator, by adjusting the voltage-limiting rheostat of the exciter. The main field rheostat is ordinarily all turned out except when the voltage across the main field at no load, normal voltage, is less than the minimum for which the regulator was intended; in such a case a small amount of resistance must be turned in, and a main rheostat is thus necessary. There is some advantage in this condition in that the response of the alternator voltage to changes in the exciter voltage is quickened.

Regulators must be taken out of service occasionally, and then hand regulation must be resorted to unless spare regulators are provided. When synchronizing a machine or taking it off the line, and sometimes for other reasons, a low excitation voltage is required; if this low voltage is obtained by adjusting the exciter field rheostat, it means working the exciter at a low voltage. If the exciter is of normal design this involves working on the straight part of the saturation curve. For reasons already referred to, rather unsatisfactory control over the exciter voltage is obtained under this condition. Where the maximum reliability and ease of control is desired, sufficient adjustable resistance should be placed in series with the main field to make it unnecessary to operate on the straight part of the exciter saturation curve.

The matter of reserve excitation equipment should be considered. A storage battery or an auxiliary exciter set may be provided for use in emergency. The voltage of a storage battery cannot be varied over much of a range, and a rheostat in series with the alternator field is needed in order to adapt the voltage of this source to the needs of the alternator. If it must be possible to excite two or more generators at one time from this storage battery, the situation is similar to the central direct-current system, so far as need of field rheostats is concerned. It might be thought that the voltage of an auxiliary exciter could be adjusted to suit a particular generator, but if it must be possible to excite more than one generator from this source, at the same time, main rheostats must be provided just as in the case of exciters operated on common bus.

LIMITATIONS IMPOSED BY AUTOMATIC REGULATORS

It has been mentioned that a responsive exciter is essential to the satisfactory performance of automatic regulating equipment. A further important characteristic the exciter should possess is moderate field current. This arises from the fact that only a limited value of current can be properly handled by the shunting relay contacts. Owing to a slight amount of inductance in the section of resistance to which the contacts are connected, the current through the contacts does not instantly fall to zero when the contacts start to separate; and the result is that a small arc forms. A small amount of sparking of this sort normally takes place during operation, and causes no injury. If the section of the rheostat shunted by the relay contacts is relatively large, the increased amount of inductance has a tendency to make the arc more persistent. Also, with more resistance, the voltage across the latter builds up to a higher value as the contacts open, and tends to maintain the arc. With excessive current the arc does not diminish as the contacts open, but generates enough heat to maintain the arc. By dividing the rheostat into a greater number of sections and placing a pair of contacts across each section, the inductance per section and the voltage drop per section are reduced, and the tendency to spark thereby lessened. Some gain also may be had by winding the resistance "non-inductively." Condensers connected in shunt to the contacts, serve to absorb the current when the contacts start to open, and cause an appreciable lessening of the arcing; but beyond a certain point they do not improve conditions.

Generally no gain can be obtained by operating contacts in parallel. There is no action present to make the two pairs share the current upon opening, and one pair is likely to have to break the full current. If one pair of contacts takes more than its share of current the voltage drop in the arc would decrease, while the drop in the parallel arc would increase; so the first arc would take still more than its share of the current. Any resistance or inductance inserted in series with each of the contact pairs would promote proper load division, but would defeat the very purpose of the contacts which is to short-circuit a portion of the rheostat.

With the type of shunting relay contacts in use a maximum current of about 22 amperes cannot in general be exceeded without flashing of the contacts as a result. However, if the block of resistance to which the pair of contacts is connected is abnormally small, such as one-half ohm or less, the resistance of the leads to the contacts becomes appreciable by comparison, and the limit is somewhat higher. Flashing at the contacts is likely to burn them so severely that the shunting relays must be taken out of service. On this account in some plants having exciters with excessive field currents, much trouble must be taken to avoid flashing of the relay contacts. A glaring example is to be found in one plant where the exciters have such large field currents that the exciting voltage cannot be raised above 90 volts, while the regulator limits the minimum voltage to 70 volts. Hence considerable hand control or adjustment is required, which aims to keep the regulator operating at near 80 volts. In fact many of the supposedly successful regulator installations are so made that even moderate load changes require readjustment of the exciter or main field rheostats, and most of the regulation is hand regulation.

The shunting relay contacts are most likely to be overloaded and flash over at time of short circuits on the system. The low a-c. voltage at such a time causes the regulator to keep the exciter rheostat short-circuited continuously, so the exciter voltage and field current build up to the maximum of which the machine is capable. It is apparent from this that the maximum exciter field current should be kept within the safe capacity of the shunting relays.

It has already been shown that the responsiveness of an exciter depends upon the "opening" between saturation curve and field circuit resistance lines, the number of turns in the field circuit, and the flux linked with those turns. But little can be done to improve the response by increasing the opening or by reducing the amount of flux; that is, little can be gained by varying the design proportions. The chief possibility lies in a reduction of the number of turns in the shunt field winding. This is more apt to become necessary with exciters built on large frames. Yet, since a reduction in the number of turns means a corresponding increase in the field current, the limit imposed by the shunting relays may be exceeded; to avoid this difficulty the current may be divided up among two or more separate circuits. This may be accomplished by winding the field coils with several conductors in parallel; or a series parallel connection of the field coils may be used, which will give an equivalent result. Each circuit as thus obtained would have its own rheostat and shunting relays. Very satisfactory results can be secured in this way, as experience indicates.

DIRECT-CONNECTED EXCITERS

Driving the exciter by connecting it directly to the main unit, should be seriously considered in many cases. This method is particularly applicable when individual exciters are wanted, or when a very simple type of drive is desired. Where the main unit is isolated such as a synchronous motor or condenser or a small generator is likely to be, it has strong advantages. From the exciter standpoint the advisability of using this method of drive is mainly dependent upon the speed of the main unit.

Small low-speed engine-type or waterwheel units usually do not have direct-connected exciters on account of cost. There are several reasons for the relatively high cost. Small generators at such low speeds, require an exciter kw. capacity which is a considerable percentage of their own rating, and relatively much larger than a high-speed machine requires. Furthermore, the size of an exciter for a given kw. rating increases even faster than its speed is reduced; as a result the exciter becomes excessively large for its kw. rating if the speed is very low. It being usual for the exciter to be overhung on the main shaft when direct-connected, the mechanical parts of the main unit may have to be strengthened up considerably in some cases to provide the necessary support for the exciter; this of course increases the cost. For these reasons it is more usual to use a belted exciter, if the exciter is to be driven from the main unit.

For small belted alternators, or those driven by steam turbines through reduction gears, or coupled to high-speed waterwheels, direct-connected exciters are economical. As the capacity of the alternator is increased, the speed at which direct-connected exciters become economical, drops lower and lower. The generators used in high-head water power plants, synchronous condensers, and frequently synchronous motors, operate at speeds which are high relative to the rating, and direct-connection of exciters is very satisfactory from the cost standpoint.

For large generating units in low-head power plants, direct-connected exciters are quite economical. But another factor enters in this case owing to the relatively large physical dimensions of the exciter; this element is sluggishness toward the impulses of an automatic voltage regulator. There has been some feeling in the past that this was an inherent objection to the use of large exciters where automatic voltage regulation was desired. However, entirely satisfactory response can be obtained by parallel circuits in the shunt field though some increase in size of regulating equipment is required; the objection that such machines are too sluggish has thus been practically removed.

In the case of turbo generator units the high speed deters the use of direct-connected exciters. Exciters of this type for 3600 rev. per min. turbos are difficult to build, expensive, more difficult to maintain and less reliable than moderate speed generators. For very large units which have a lower speed, 1800, 1500 or 1200 rev. per min., direct-connected exciters are much more feasible, and are frequently used.

DISCUSSION ON "THE APPLICATION OF D-C. GENERATORS TO EXCITER SERVICE" (BODDIE AND MOON), WHITE SULPHUR SPRINGS, W. VA., JULY 2, 1920.

R. E. Doherty: I wish to mention a few points in connection with the authors comments upon the time that it takes an exciter to build up. I was unaware that there was such widespread confusion regarding the factors that make these exciters vary, some slow and some fast. If it is so, it cannot be charged to the lack of literature on the subject. The works by Steinmetz, Karapetoff, Berg and others explain that phenomenon very clearly.

There is one other point. In the matter of the equation given for the fundamental relation connecting the resistance and inductance and time. The authors arrived at the conclusion that to change the time it was necessary to attack L, and to do that, you must change the number of turns. I would just like to call attention to the fact that might be done also by putting resistance in series with the field circuit. It is the matter of the ratio of L/R and that can be increased by adding resistance in series with the field circuit.

Another point is that it is not a matter of number of turns, but rather one of pounds of copper. In the same magnetic circuit of roughly the same configuration that is, a coil of the same configuration, you can get the same L/R whether you have one turn or a thousand if you have the same number of pounds. If you wish to increase L/R you must add pounds.

I wish to bring out one point regarding the stability of exciters and the effect upon the stability of the generator. The authors' Fig. 2 shows the saturation curve, A, of the exciter and also the drop across the shunt field circuit B. The slope of "A" is a function of the speed whereas, the slope of "B" is independent of the speed and the function of the shunt field circuit resistance only. The intersection of these two curves is the point of stable operation. It is obvious that if by change of speed "A" is lowered, or by a change or resistance "B" is raised, an unstable condition is approached. When the slopes of the straight portions of these curves are the same, the exciter becomes unstable. It is obvious, therefore, that if the exciter normally operates at relatively low saturation, say at the point c, a momentary drop in speed with the accompanying drop in the curve "A" will cause the exciter to lose its voltage, and therefore, remove the excitation from the generator field. This, of course, applies particularly to non-automatic regulation.

In a normally designed exciter the curve "*A*" does not pass through zero, but at some appreciable value above zero voltage on account of the remnant magnetism. Curve "*B*" however always passes through zero. It therefore, follows that there is always an intersection between the two curves. But although there is still a stable point of operation, the drop in voltage for a given drop in speed is so great that the excitation is reduced on the generator to a critical value. Instances of this sort have occurred on rather low-speed water-wheel driven generators. I happen to remember the figures which were: A 10 per cent change in speed, say a drop from 60 to 54 cycles which occurred at a momentary short circuit, caused by a drop of 50 per cent in the exciter voltage and, therefore, practically removed the excitation from the generator causing the latter to pull out of step. It is, therefore, desirable in hand regulated stations of this sort to operate the exciter well above the knee in the saturation curve.

C. A. Boddie: Mr. Doherty states that he did not know that there was such widespread confusion regarding the factors that control the rate of building up of d-c. generators, but that there is plenty of literature on the subject, the whole phenomena being very clearly explained in the works of Steinmetz, Berg, Karapetoff and others.

He then proceeds to suggest that the time of building up depends on the ratio (L/R) which may be made shorter by increasing R and leaving L alone instead of taking measures to reduce L by reducing the number of shunt field turns as pointed out by the authors.

Mr. Doherty probably remembers the usual logarithmic curve showing the rate of building up of current in a circuit containing inductance and resistance of which the time constant is (L/R) and which has been exhaustively treated by many authors for the past 50 years.

The remaining comments regarding the stability of an exciter based on the intersection of the saturation curve and resistance line have been matters of general knowledge among engineers or over 20 years and have no real place in this discussion.

But on the major issue the author wishes to state that neither Steinmetz, Berg, Karapetoff, or anybody else, to the best of his knowledge has explained the phenomena of building up of a d-c. generator. In 1907, when Mr. Tirrill approached Steinmetz and Berg on the subject, neither displayed any clear understanding of the problem. At least an understanding of which practical use could be made. Further, in a paper read before the Pittsfield Section of the Institute in 1911, Mr. H. A. Lacock, then head of the Voltage Regulator Department of the General Electric Co. gave no indication of a real understanding of the phenomena of building up when he advances the then prevalent idea that a straight line saturation curve in an exciter is directly conductive to quickness of response.

During the period between 1907 and 1914, there were numerous instances of voltage regulator installations where poor results were traced directly to the inherent sluggishness of the exciters used. It was noticed, that as the exciters became larger in physical dimensions and slower in speed that sluggishness was more common, and this became so generally known that several noted engineering firms recommended strongly against the use of direct-connected exciters for low-speed hydroelectric installations.

This situation continued as late as 1914, when during the negotiations for a large hydroelectric generating station the consulting engineer insisted on having direct-connected exciters notwithstanding the objection of the manufacturer, on the score of responsiveness. The consulting engineer further stipulated, that the time of building up of these exciters over the operating range, should not exceed four seconds. These machines were built according to the best opinion available at that time. That is they had liberal dimensions, low flux densities and a fairly straight saturation curve which was thought to make the machines build up quickly. When the first machine was assembled, and tested, its time of building up was found to be approximately 20 seconds; the machine as it then stood being entirely unsuited for use with a voltage regulator.

The writer had penetrated the secret of building up years previously while studying characteristic curves as a student, but while the idea was often helpful, there was no important practical application since the rate of building up was a matter of no consequence in either the design or operation of d-c. generators except in the one case, where a machine was used as an exciter in the Tirrill system of voltage regulation.

Briefly the theory is this: Let Fig. 1 represent an armature of a d-c. generator with its shunt field circuit connected directly across the brushes. In Fig. 2 let $O E A$ represent the no load saturation curve *i. e.*, for any field current I, the voltage delivered at the armature terminals is $C E$. Let the straight line $O B$ represent the resistance of the field circuit *i. e.*, for any field current I the drop in the resistance of the field circuit is $C D$. Now, at the intersection P of the saturation curve and resistance line, the voltage generated by the armature and the drop in the field resistance are equal and has been recognized for many years as the stable operating point of the machine. This is the important point to consider, when designing machines for the usual class of service so that but little attention was directed to the state of things represented by an ordinate through some point *E* other than the stable operating point *P*. For the point *E* shown the field current corresponding is *I*. The drop in field resistance is *C D*, that is, in the entire field circuit measured from brush to brush. The generated voltage appearing at the armature terminals is *C E*. It will be observed that the generated voltage *C E* exceeds the ohmic drop in the field circuit by the amount *D E*.

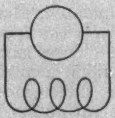

Fig. 1

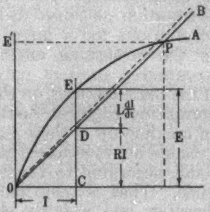

Fig. 2

A little reflection shows that such a condition is not possible unless the field circuit contains some impedance other than the simple ohmic resistance, otherwise the diagram would call for two different voltages existing across the brushes, at the same instant. Since the field circuit contains inductance it is at once obvious that the field current must be increasing at such a rate that the field flux cutting through the turns of the shunt field coils generates in the field circuit a voltage equal to *D E*, the inductive drop in the field circuit. The inductive drop at any instant may be expressed as $L\frac{dI}{dt}$ where *L* is coefficient of self induction of the field circuit current and $\frac{dI}{dt}$ is the rate of the change of field current expressed in amperes per second.

The general equation expressing the relations at any instant is:

$$E = R I + L \frac{d I}{d t}$$

which is immediately recognized as Helmholtz's original equation for building up of current in an inductive circuit.

If in the second term of the equation, we assume for simplicity that the coefficient of self induction L is a constant, we see that the distance $D E$ is a measure of the rate of building up of field current, *i. e.*, (amperes per second), or $D E$ is a measure of the rate at which the ordinate $C E$ is moving to the right. This rate diminishes as the ordinate approaches P where it finally becomes zero. If the voltage should by any means get beyond P the machine will build down at the rate measured by the vertical distances between the curves.

The practical points of interest are that for a given maximum voltage E' the rate of building up is directly proportional to the vertical distance between the saturation curve and the resistance line. A straight line saturation curve (shown dotted Fig. 2) gives very little opening between the curves and therefor makes a slow exciter, since there is but little voltage available for building up. This is just the reverse of the previously prevailing opinion. Further, for a given saturation curve and resistance line, the product $L \frac{d I}{d t}$ is at every instant equal to the opening $D E$ so that to increase $\frac{d I}{d t}$ we must reduce L. If we cut L in half, $\frac{d I}{d t}$ is exactly doubled that is the rate of building up is doubled (assuming of course that the saturation curve has not been altered). Hence, for a machine having a given saturation curve and set for a given maximum voltage the only way to quicken its rate of building up is to reduce the value of L. This conclusion is directly contrary to Mr. Doherty's statement that it is not necessary to reduce L. It is further apparent, that any increase in field resistance as suggested by Mr. Doherty in order to reduce the value of L/R will raise the resistance line $O B$; move the maximum voltage point P toward E; close up $E D$; thereby reducing the rate of building up until finally the machine refuses to build up at all and is therefore inoperative. This conclusion is apparent to any one who has ever watched the building up of a machine, with its field rheostat in various positions.

The above is pointed out merely to make it more clear that the phenomena has not been and still is not understood even by those who have accustomed themselves to think it is such a simple matter as to be obvious.

The foregoing theory was applied to correcting the excessive time of building up discovered in the previously mentioned exciters. The saturation curve was altered so that it remained fairly straight only within the operating range after which saturation set in rapidly. For a given maximum voltage setting, this change greatly increased the voltage available for building up. The inductance of the field circuit was cut down by reducing the number of turns in series. The machine in question had 8 poles and it was convenient to use the original field coils connected in two parallels, each circuit having 4 coils in series and therefore one-half the original value of inductance. A fixed resistance equal to that of 4 coils was placed in series with each circuit so that the resistance per circuit was the same as the original 8 coils in series. This change doubled the rate of building up and together with the previous change in the saturation curve brought the machine within the 4 second time required in the specifications.

Since that time the Westinghouse Company, has been building its exciters to definite time constant specifications when intended for Tirrill regulator use. Further analysis along the above lines shows that the only factors which the designing engineer must consider is the flux linkage of the field circuit, that is flux per pole × turns per pole × poles in series. With a 250-volt machine and a typical saturation curve this flux linkage should not exceed 15×10^9. This gives a theoretical time of building up of 3 seconds over the arbitrary range of 60 to 250 volts which is modified by eddy currents in the yoke, hysteresis, etc., so that the actual machine when set for a maximum of 290 or 300 volts will build up over the range of 60 to 250 volts in 4 seconds or less. This has been found to be the most desirable time constant for an exciter under Tirril regulator control.

It has also been found that Tirrill regulator contacts will flash over at currents exceeding 22 amperes regardless of the number of relays used. It is not safe to operate above 19 amperes and the exciters of the Westinghouse Co. are designed so that the current in no one circuit shall exceed 17 amperes under maximum conditions (*i. e.*, during short circuit on the a-c. generator calling for full exciter voltage) and the flux linkage of each circuit figured at normal voltage, no load, shall not exceed 15 × 10^9.

When machines are designed to these specifications there is no difficulty in making large low-speed direct-connected exciters just as responsive as a small motor-driven set so that responsiveness is no longer a factor when deciding on the question of direct-connected exciters vs. independent drive.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 2, 1920.

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EXCITER PRACTISE IN THE NORTHWEST

BY J. D. ROSS

Superintendent of Lighting, Member, Board of Public Works, Seattle, Wash.

The author gives in table form the generator installation and the exciter sets, with size, voltage, drive, regulators and system of exciter connections for the principal hydroelectric plants in operation in the Northwest. He analyzes their essential characteristics and the difficulties encountered with automatic voltage regulators in plants having generators of different sizes and makes. Simplicity of non-automatic apparatus at hydro power stations seems to be desired. In the Seattle plant, operated by the writer, the steam plant is depended upon for regulation. The details of the installation are given. The author advocates for an entirely new large hydroelectric or steam plant a system of excitation in which each generator is supplied with its own shunt-wound exciter driven by a prime mover and furnished with its own regulator. In an existing plant, not designed on the unit system, the use of one large exciter to operate the entire plant through field rheostats is the best compromise, the old exciter system being used as the duplicate for emergency.

THE practise in excitation of generators in the Northwest is about the same as that in all parts of the country. It is the outcome of an evolution from small to large plants necessitating the use of various sized generators excited by various sized machines connected to a single bus or in a few cases, to a double bus.

The following table of a number of the principal hydro plants shows the Western practise as it now is:

Most companies are alive to the importance of having the best system and realize the possibilities offered in regulating voltage from the power house. The fact is, however, that Tirrill regulators are little used at the Western hydro power houses and even where provided, they are in a number of cases lying idle. Larger modern machines may be working beside small machines of older type. Their characteristics

DATA ON EXCITERS—TYPICAL PACIFIC COAST HYDROELECTRIC STATIONS

Station	Company	Main Generators			Exciter Sets					Exciter Connection
		No.	Size kw.	Prime Mover	No.	Size kw.	Voltage	Drive	Regulator	
Washington										
White River	P. S. T. L. & P. Co.	3	15,000 15,000 16,000	Francis	2	225	240	1 Induction motor 1 Pelton wheel & Motor (same shaft)	Tirrill	Single d-c. bus
Electron	Do.	4	3,500	Pelton	2	150	125	Pelton-and-motor		Do.
Snoqualmie No. 1	Do.	4	1,500	Doble	3	75	..	2 Pelton,		
		1	5,000	Francis				1 Pelton & motor		Do.
Snoqualmie No. 2	Do.	1	8,750	Francis	1	200	..	Doble and motor		(Single unit)
Nooksack	Do.	1	1,500	Pelton	1	45	125	Belted to main unit.	None	" "
Elwha	N. W. Pwr. & Mfg. Co.	2	3,000	Francis	2 each large enough for whole plant			1 Belted 1 Motor		D-C. bus
Cedar Falls	City of Seattle	4	2–1,500	Pelton	3 2	75				
			2–5,000	Francis	1	150	125	Pelton wheels	"	Single Bus
Nisqually	City of Tacoma	4	4,750	Francis	2	300	125	Doble	Tirrill	" "
Little Falls	Wash. Water Power Co.	4	5,550	"	4	120	250	On generator shaft	"	" D-C. Bus
Long Lake	" " " "	2	14,000	"	2		250	Do.		" " "
Oregon										
White Salmon	Northwest Elec. Co.	2	6,000	"	2	..	..	1 Motor 1 Turbine		D-C. bus
Cazadero	Port.Ry.Lt.& Pwr.Co.	5	2,500	....	2	75	120	On generator shaft		
			3,000		1	60	120	" " "		Unit system & bus
			3,750		1	50	120	" " "		
					1	85	125	Motor and Turbine	Tirrill*	

DATA ON EXCITERS—TYPICAL PACIFIC COAST HYDROELECTRIC STATIONS—*Continued*

Station	Company	Main Generators			Exciter Sets					Exciter Connection
		No.	Size kw.	Prime Mover	No.	Size kw.	Voltage	Drive	Regulator	
Estacada	Port. Ry. Lt.&Pr. Co.	3	3,667		3	60	125	On generator shaft		Unit system & bus
					1	85	1 5	Motor & turbine	Tirrill	
Bull Run	" " " " "	3	3,750		2	150	250	Turbine	Tirrill*	D-C. bus
California										
Drum	Pac. Gas & Elec. Co.	2	12,500	Pelton	2	400	125	Pelton and motor	....	D-C. bus
Feather River	Great Western Power Co.	5	10,000	Francis	2	250	250	Waterwheel	....	" "
Stanislaus	Sierra & S. Francisco Pwr. Co.	4	8,500	Pelton	2 2	300	250	"	Tirrill	" "
San Francisquito No. 1	City of Los Angeles	6	9,375	Impulse	3	250	..	"	....	"
San Francisquito No. 2	" " " "				3	75	..	On generator shaft	....	
	(Under construction)	3	17,500	Francis	1	90		motor		
Big Creek No. 1	Pac. Light & Pwr. Co.	2	17,500	Pelton	2	150	..	Motor & wheel	Tirrill	" "
Big Creek No. 2	" " " " "	2	17,500	"	2	150	..	" " "	"	" "
Bishop Creek No. 1	Nevada-Cal. Power Co.	1	7,500	"	2	60	110	1 Wheel & motor	"	" "
Bishop Creek No. 2	" " " "	3	2,000	"	2	100	120	1 water wheel 1 motor & wheel 1 wheel	"	" "
Canada										
Lake Buntzen No. 2	B. C. Elec. Ry. Co.	3	8,900	Pelton	3	300	250	Wheel & motor	"	" "
Stave Falls	Western Power Co.	4	9,000	Francis	2 each large enough for whole plant			Water wheel	Tirrill*	" "

*Not used

are different yet they are all fed from the same exciter bus. The exciters are often of several sizes and types and this further complicates the system and makes automatic regulation difficult and sometimes practically impossible. In a number of cases, exciters are driven by induction motor and water-wheel on the same shaft. In these the regulator is used with the motor drive, probably because of the difficulty of governing the small waterwheel. A small waterwheel is about as difficult to govern as a large one, especially where rapid fluctuations are required. A flywheel on the exciter shaft would do much toward maintaining steady speed but does not appear to be used in any coast plant. There is a feeling that the higher voltage maintained by a Tirrill automatic regulator on short circuit is a hazard not to be disregarded and as it is common practise to tie all machines and transformers to the high-tension lines through non-automatic apparatus, there is a good deal in the contention. The simplicity of non-automatic apparatus at power stations except on duplicate transmission lines seems to be desired more than automatic regulation while the use of induction regulators in substations is depended on to give the proper regulation of voltage.

The writer is at present operating both hydro and steam plants furnishing current to 63,000 consumers as well as Seattle's buildings and street lights.

The hydro plant like most others is a product of evolution containing various sizes of generators and exciters. As in most plants, a single direct-current bus is used. The Tirrill regulator is at present idle. A 20,000-kw. steam plant is largely depended on for regulation. A new 15,000-kw. generator is being installed at the hydro plant and a new 15,000-kw. generator is also being installed in the steam plant.

In deciding the question of excitation for these new units, two systems appealed very strongly; first, the use of duplicate exciters, either one of which is able to handle the entire station. This allows of easy installation of a Tirrill automatic or other regulator and is the extreme of simplicity; second, the use of an exciter for each machine on a separate prime mover and operating on the unit system with such switching arrangement as would allow a spare exciter to replace any other exciter in case of emergency. Auxiliaries should not be run from an exciter bus in a large system but should have a separate source of power, preferably from a prime mover. There is too much at stake to tie both exciters and auxiliaries to one bus. In both hydro and steam plants in Seattle, auxiliaries are run from a bus entirely independent from the exciters.

There has been very great advancement in the science and art in recent years and apparatus is so much more reliable that in the hydro system of the City of Seattle it was decided to break away from previous standards and install the new 15,000-kw. unit entirely separate from the other four machines. Good automatic regulation with continuity of service must be accompanied by simplicity of design. In this new unit the pipe line is entirely separate. There are no busbars or oil switches between the generator and its transformers, all switching being done on the high-voltage side at 66,000 volts. The reactance of machine and transformers are made high to resist short circuits. Class B-mica insulation is being used on transformer coils instead of the standard Class A. The transformers are of the type using the winding entirely on the center leg of a three-legged core, the coils being wound on a drum. The round coil admits of winding with Class B insulation. The exciter was then chosen of a size ample for the full final development of the entire station, 36,000 kw., it being the desire to have only one exciter operating at a time. A Tirrill or Westinghouse regulator installation on this exciter now becomes simple and reliable. Operating in conjunction with good governors on the main turbines, this system should give the very best results.

A governor is also supplied for the exciter impulse wheel and the shaft is extended for an induction motor or flywheel or both if desired later.

In order to carry out as far as possible the idea of simplicity and closer regulation, the distribution system in Seattle is also being radically changed. The new method is described in detail in the *Electrical World* of January 10, 1920. Besides giving much better regulation, this method effects a considerable saving as well. An interesting paradox presents itself in this distribution system, for while better regulation is effected, the automatic induction regulator will be eliminated.

The regulation of the entire system will now fall on the Tirrill regulator at the hydro plant and a similar one at the city's steam plant.

It was found impossible to get a satisfactory steam turbine and motor-driven ecxiter over 100 kw. due to commutator design for high speed. This difficulty was removed by placing an exciter on each end of the shaft of the exciter turbine. This exciter set is also equipped with an induction motor floating on the line. This acts as a brake if the turbine governor fails and is also used to maintain the desired heat balance.

The new 15,000-kw. steam unit is also being connected direct to its transformers without switches as in the case of the hydro unit. The plant will then have a capacity of 35,000 kw. and will be controlled by one exciter.

Synchronous condensers automatically controlled are used to keep a steady voltage at the substations.

There seems to be no logical excuse for the compound-wound exciter. Its tendency when used with a regulator is to pile up a high voltage on short circuit and to give troubles in paralleling. The shunt-wound machine if given a very liberal voltage range is nearer the ideal. At the Seattle steam plant, two compound-wound exciters defied all efforts to parallel until the series winding was removed. The tendency was for one to reverse suddenly and shut down the plant during normal operation. In order to use one exciter with automatic regulation, it is, of course, necessary to have similar or nearly similar characteristics of excitation in the main generators. This is also true of any system of excitation and in some of the coast plants, the Tirrill automatic regulator has been abandoned due to a difference in the characteristics of the generator fields at various loads.

In general, the trend of the large coast concerns is away from a lot of small generators and exciters to large generating units.

There is a tendency to use a combined waterwheel and motor drive in each power house for at least one of the exciters installed. There are notable exceptions, however, that give excellent service.

The Washington Water Power Company use an exciter on each generator shaft, each exciter large enough to supply two generators. All exciters are connected to a direct-current bus. No extra exciter is considered necessary. The City of Los Angeles is planning a new plant using the same system but with an additional motor-driven exciter. These systems where used are giving practically 100 per cent service. In some quarters there has been a strong feeling against exciters on generator shafts for the reason that a sudden heavy load changes the speed and lowers the exciter voltage which in turn drops the generator voltage and the effect is cumulative. This trouble seems to be negligible on large mixed loads supplied from waterwheels with modern governors. Undoubtedly the best exciter system for an entirely new hydroelectric or steam plant of large size would be one in which each generator is supplied with its own shunt-wound exciter of ample size and quite ample voltage range. The exciter would be driven by a prime mover unless the load were a mixed one without undue fluctuations which the regulator could not easily take up, and each exciter would be furnished with its own regulator. Only such switching devices would be used on the exciters as would allow a spare exciter to take the place of any other during emergency.

This system is nearest to the ideal because the field characteristics of generators are so often different and when a new generator is installed the progress of generator design is often such that we must either allow a difference or take a step back by accepting an old design. With an individual exciter and regulator for each machine each new generator may be purchased with its proper exciter, regardless of the relative size or characteristics of the new and old machines.

In adding a new machine in a plant not designed on the unit system, but using various sizes and types of generators having various characteristics and a number of exciters also of odd sizes and makes, the use of one large exciter to operate the entire plant through field rheostats is the best compromise. The old exciter equipment may then be used as the duplicate for emergency.

Presented at the 36th Annual Convention of the American Institute of Electrical Engineers, White Sulphur Springs, W. Va., July 2, 1920.

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GENERATOR EXCITATION PRACTISE IN THE HYDROELECTRIC PLANTS OF THE SOUTHERN CALIFORNIA EDISON COMPANY

BY H. H. COX AND H. MICHENER
Both of the Southern California Edison Co., Los Angeles, Cal.

THE more recently constructed hydroelectric plants of the Southern California Edison Company are either directly connected to the two 240-mile, 150,000-volt Big Creek lines, or connected to it through 43 miles of 60,000-volt lines and step-up transformers. Probably the most extreme range of excitation of any system in operation is required to meet the conditions of voltage control imposed by 150,000-volt lines of this length.

The plants connected to these lines, and which will be considered in the following, are Big Creek Power Houses Nos. 1 and 2, each with 32,000-kw. installed capacity, Eagle Rock Substation with 54,000-kv-a. installed step-down transformer capacity, Kern River No. 3 with 32,000-kw. installed capacity which will be put into operation in the latter part of 1920, and Big Creek Power House No. 8 with 22,500-kw. installed capacity, which will be put into operation during 1921.

The present plants will be dealt with first as follows:

APPARATUS INSTALLED

One-half of the original equipment was furnished by the General Electric Company, and the other half by the Westinghouse Company.

P. H. No. 1. Two 17,500-kv-a. generators with two 150-kw. exciters, water-wheel driven. One has also an induction motor drive. Each exciter is of sufficient capacity for two generators, the other acting as a spare. Exciter fields are supplied from a motor-generator set, called an agitator, consisting of a 250-volt d-c., and a 125-volt d-c. machine connected in series opposition, driven by an a-c. motor. The 250-volt machine is under control of a standard range Tirrill regulator controlling it from 125 to 250 volts. The resultant voltage, 0 to 125 volts, of the two machines is taken to excite the fields of the main exciters, giving them a range from 0 to maximum. This equipment was furnished by the General Electric Company.

P. H. No. 2. Two 17,500-kv-a. generators with two 200-kw. exciters, driven same as P. H. No. 1. The exciters are under control of a broad range regulator controlling their voltage from residual to maximum. This equipment was furnished by the Westinghouse Company.

Substation. Two 15,000-kv-a. synchronous condensers with 65- and 100-kw. direct-connected exciters, General Electric and Westinghouse respectively. Each exciter is of sufficient capacity for its condenser only. One 100-kw. motor-driven exciter for spare, which for months at a time is not used. These exciters are controlled by broad range Westinghouse regulators which operate the exciters successfully from residual voltage, about 15 volts, to maximum, about 300 volts. This range is necessary to make the condensers operate at full load lagging or leading. One regulator is a spare.

OPERATING

Starting the system from dead is usually done by getting two generators together on a bus, then lowering the voltage to practically zero and closing in on a line with only a bank of transformers on at the receiving station, then building up with rheostat of 250-volt machine of agitator set till proper voltage is reached at substation for synchronizing. The condenser is usually in operation from other plants connected to distributing system with regulator in service.

Only about 25 per cent of normal field current is required to obtain this voltage, using two generators on one 240-mile line. This is done by hand. After synchronizing, the voltage at the power house is raised to normal, thus picking up considerable reactive current, and the regulator is put into service.

When using only one generator the line charging current causes the generator to become self-exciting, and with no field current, the General Electric generator will build up to 106 per cent voltage, and the Westinghouse generator will build up to 135 per cent.

By means of the agitator set the field, and consequently the armature voltage of the General Electric exciters at P. H. No. 1 can be reversed. This causes the exciting current in the field coils of the generator to flow in a direction opposite to that of the normal exciting current. A reversed field current of 20 per cent the magnitude of the normal field is sufficient to bring the generator voltage down to approximately 85 per cent normal which is the required voltage for synchronizing at the substation. The agitator set gives the sensitive control, which is very desirable for this purpose. No method is available for the control of the Westinghouse generators under the above conditions, hence it is always necessary to have two generators, or one generator and a condenser, when using these generators to build up on a line. It is very seldom necessary to build up on a line with one generator.

Normal conditions of operation are with voltage regulators in full control of the exciters at both the power houses and at the substation.

The regular daily cycle of operation of the condensers covers from full load lagging (one condenser) to full load leading (two condensers)

CHANGES BEING MADE

There is now being installed a third 17,500-kv-a. unit at Big Creek No. 2, but the excitation system is not being changed.

At Eagle Rock Substation the step-down transformer capacity is being increased to 96,000 kv-a. and a third condenser is being installed. This condenser has a capacity of 30,000 kv-a., and the direct-connected exciter has a capacity of 150 kw. No other changes are being made in the excitation system, except the substitution of a 160-kw. spare exciter set in place of the present 100-kw. spare exciter set.

At the Kern River No. 3 power plant, which will be put in operation during the latter part of this year, there will be two 17,500-kv-a., vertical shaft units. The generator and excitation system characteristics are such that one generator will be capable of handling a 140-mile section of the 150,000-volt line, and the 43 miles of 60,000-volt line, which connects Kern No. 3 and the Big Creek Line—the 150,000-volt line being open at the receiving end.

The excitation system will consist of a 95-kw. exciter direct-connected to each generator, and a 200-kw. induction motor-driven exciter for a spare. Each of the three exciters will be controlled by a separate broad range voltage regulator. This provision is necessary for the reason that during a part of each year one unit will be operating at 60 cycles and the other at 50 cycles.

The plans which are being developed for excitation at Big Creek Power House No. 8 are the result of experience with the plants described above. This experience has led to the belief that maximum reliability and simplicity, rather than flexibility are the characteristics to be striven for. The plan which seems most desirable and which will probably be put into effect is as described below.

The initial development will consist of one 22,500-kw. 25,000-kv-a., vertical shaft unit with an exciter of liberal design mounted on the top of the generator. A spare exciter the duplicate of the one mounted on the generator, will be provided, but it will not be connected to any motive power. In case the exciter on the generator burns out, it will be removed from the generator and the spare exciter will be mounted in its place. It is expected that nothing short of a burn-out in the exciter will necessitate the shutting down of the unit on account of the exciter. There will be no exciter driven by a motor or a water wheel, or by both, with its complications of electrical and hydraulic connections.

Future installations will increase the number of units in this plant to four. The spare exciter will be a spare for all the units in the plant. There will be no exciter bus for the paralleling of exciters. The exciters on all the units will be controlled by one voltage regulator.

In this way it is expected that operation difficulties will be reduced to a minimum.

DISCUSSION ON "EXCITER PRACTISE IN THE NORTHWEST" (ROSS) AND "GENERATOR EXCITATION PRACTISE IN THE HYDROELECTRIC PLANTS OF THE SOUTHERN CALIFORNIA EDISON CO," (COX AND MICHENER), WHITE SULPHUR SPRINGS, W. VA., JULY 2, 1920.

Carl J. Fechheimer: The transmission system of the Southern California Edison Company brings to our attention a condition which will probably be met frequently on future long lines with the high transmission voltages now being considered especially with frequencies of 50 or 60 cycles. On such systems the kv-a. rating of a very large generator may easily be exceeded with the transmission line open, due to the current required to charge the line. In the particular case of the Southern California Edison Company, with 150,000 volts at the substation at Eagle Rock, with the line open, about 24,000 kv-a. are required to charge the transmission line which is a considerable overload for one 17,500 kv-a. generator. As pointed out by the authors, the voltage actually exceeds normal, and the current from each generator is consequently proportionately increased.

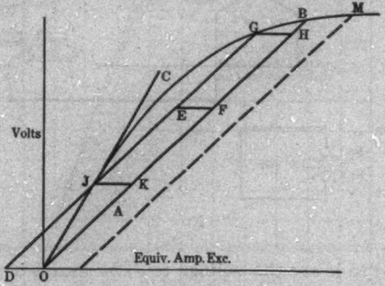

FIG. 1

With the extension to Kern River Power House Number 3, there is a large negative reactive drop in the transformers and in 43 miles of lines at the lower transmission voltage of about 60,000; so that, with 150,000 volts at Eagle Rock, with the line open, there is only about 60 per cent normal voltage at the generator terminals, assuming that all the charging current for the 240 miles line is furnished from Kern River. Aside from the question of overload, (if one generator of 17,500 kv-a. is used), the generator, if of normal design, must be worked on that part of the saturation curve which is substantially straight and therefore is unstable. Thus in Fig. 1, OAB is the condenser charging curve, OCB is the no load saturation curve, the field amperes equivalent of the armature amperes excitation being plotted as abscissas. Without any excitation in the fields, the voltage would reach the value B on open circuit, a point of stability. If now a certain negative excitation of value $DO = JK = EF = GH$ is applied to the fields, the voltage drops to G, which, due to saturation is also a point of stability. If, however, it is desired to lower the voltage to J, by means of negative excitation, a point of instability is reached, for the saturation curve being straight, the voltage continues to fall, then the polarity of the poles reverses, and the voltage, builds up to a value M, corresponding to an excitation of DO plus the excitation from the condensers. Even with a voltage regulator having a rapid time element, it is not possible to hold the voltage at the point.

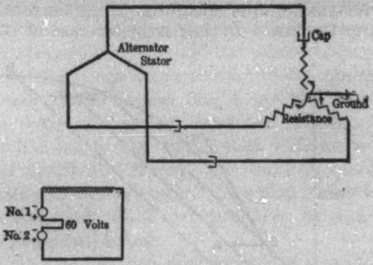

Fig. 2

In order to verify this point, tests were made with static condensers, to imitate the transmission line capacity, in series with adjustable resistors, connected to a three-phase 50-kv-a. six-pole alternator, normally rated at 2400 volts, 60 cycles, as shown in Fig. 2, (instruments not shown). Two small duplicate 125-volt exciters were connected in series opposition, the voltage of one being held constant at 60, and the other was altered automatically with a voltage regulator, which was set to maintain constant voltage at the alternator terminals. Changes in external load could not be affected so readily as changes in speed, and furthermore, since, with the regulator maintaining voltage, a comparatively small increase in frequency corresponded to a relatively large reduction in flux in the alternator, the speed was increased to obtain the point of instability.

Fig. 3 shows the no-load saturation and short-circuit curves; and the saturation curve taken with condensers without current in the field coils. The ordinates for the latter are the voltages read at the alternator terminals changed to correspond to 60 cycles; the abscissas are the field currents which correspond to the armature currents on sustained short circuit.

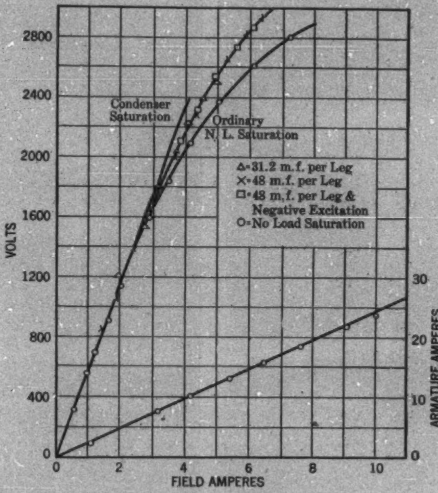

Fig. 3—Saturation and Short-Circuit Curves
Voltages are at 60 Cycles

Fig. 4* was taken with the regulator in circuit set to maintain the voltage constant at 1000, and with exciter No. 2 (Fig. 2) self-excited. (The beginning and end

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*It was found that with the low speed at which the oscillograph was operated, motor drive for the drum was not feasible; therefore, hand operation was employed, with the result that the speed was not uniform, and in consequence the period of one cycle changed materially.

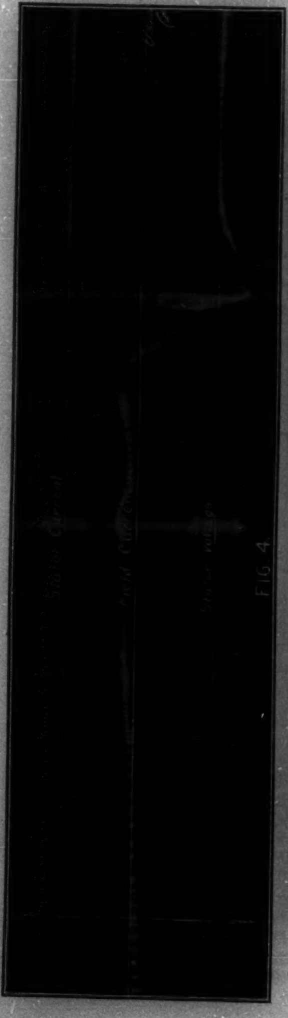
Stator Current
Field Current
Stator Voltage

FIG. 4

Stator Current
Field Current
Voltage

FIG. 5

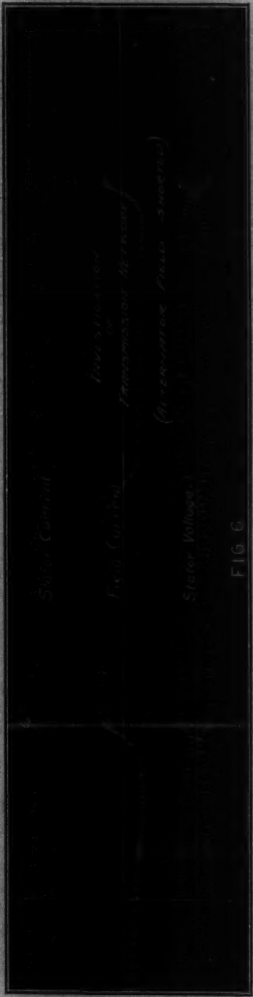
Stator Current
Field Current
Stator Voltage
Investigation of Transmission Network
(Alternator Field Shorted)

FIG 6

of oscillogram are not shown). The excitation was positive if the voltage of exciter No. 2 was greater than that of No. 1 as at the beginning of the oscillogram. The capacity was approximately 48 microfarads per leg. Instability was reached at about 47 cycles, and with —0.55 amperes in the field coils. The voltage reached so high a value after reversal of polarity that the relay opened the armature circuit at about 2000 volts.

Fig. 5 was taken under the same conditions as Fig. 4 except that exciter No. 2 was separately excited from 250 volts and a resistor of 1.2 times field resistance was placed in series with the field coils of No. 2. This materially changed the time constant, and the regulator was thus enabled to respond more rapidly. Reading taken just before the state of instability was reached was 1050 volts, 10 amperes alternating current, 51 cycles, — 0.6 ampere field. In this case, although slightly higher frequency corresponding to a little lower flux was obtained before the polarity reversed (52 per cent instead of 56 per cent of normal flux), it was still impossible to hold the voltage constant, and the voltage reached about double the value which the regulator was set to maintain when the relay opened the armature circuit. If the circuit had not been opened, the regulator would have reversed the excitation which would have caused the voltage to build up still higher. The influence of induced current in the field circuit is very marked.

If there be no direct excitation in the field coils, and a load be placed upon the alternator before exciting in order to reduce the voltage, then another phenomenon may occur; the machine may pass into the asynchronous state. That is, the power load is greater than the alternator can supply operating at synchronous speed, and it therefore "slips" ahead. There are then two frequencies, that of the line, and that of rotation, the slip frequency being the difference between the two. This is clearly shown in Fig. 6. The fields were short-circuited and the field current record gives a clear picture. Reading taken just before unstable condition was reached was 1900 volts, 15 amperes, 44 cycles, approximately 48 microfarads per leg. If on a given system, an alternator is used to charge a transmission line and the voltage rises above normal with zero excitation, the application of load may cause the alternator to become asynchronous, and there is a probability that phenomena may be introduced on the transmission system which have not hitherto been encountered.

Other tests made with negative excitation without the use of the regulator, showed that once the critical point was reached, and the straight part of the saturation curve approached, the voltage dropped to zero and then built up to a rather higher value with the polarity of the poles reversed.

It will therefore be seen that whenever zero or negative excitations are employed in conjunction with a system for which the leading reactive power is high, there is danger of reversal of polarity and therefore of building up to high voltages, or of passing into the asynchronous state. It is not feasible to make a voltage regulator so rapid that it will automatically maintain constant voltage on the straight part of the saturation curve; even when more sensitive than ordinarily, the regulator was unable to maintain constant voltage; and if made too rapid there would be a decided tendency for the regulator to hunt.

The obvious way to avoid the difficulty is so to design the alternator that more ampere turns are required to force the flux through the magnetic circuit at the point on the saturation curve considered than can be supplied by means of reactive power from the electrostatic field of the transmission line. This however, may not be feasible, as it may necessitate making the alternator much larger and more expensive than is required for normal load conditions. Another solution embodying alternators of normal design, is to connect two or more generators in parallel before connecting to the transmission line. This method is frequently used by the Southern California Edison Company.

A third solution consists in using reactors in parallel with the transmission circuit, so that they can absorb the surplus charging current. A fourth solution consists in winding into the poles of the alternator special coils in which the current can be controlled. Each coil is so placed as to include about one-half the pole section, there being about equal iron section beyond the coils. The m. m. f. due to the current in this auxiliary winding causes leakage flux to flow which saturates that portion of the magnetic circuit, and thereby the shape of the saturation curve is readily controlled.

James Lyman: It is interesting to see the unanimity of opinion that has been expressed here on the subject of excitation for large turbo generator stations. The manufacturers have found difficulty in making reliable exciters for direct connection to turbo generators of 2000 to 5000 kw. capacity running at 3600 rev. per min. We understand, however, that these difficulties are being overcome.

We have recently looked up the records of a number of large power stations we have designed having turbo generators with direct-connected exciters, and with one reserve turbo driven exciter for the station. We have found that the direct-connected exciters have been giving very reliable service, the reserve exciter being seldom, if ever, called on. In the case of one power station at Tocopilla, Chile, S. A., designed and built by European engineers, having four 10,000-kw. units, no provision had been made for reserve excitation, and we added a fifth 10,000-kw. American built turbine. We recommended a reserve turbo driven exciter. The exciters should be made of ample capacity and of rugged and reliable design so that the possibility of their giving trouble will be very remote.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Portland, Ore., July 21, 1920.

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FACTORS CONTROLLING THE DESIGN AND SELECTION OF SUSPENSION INSULATORS

BY W. D. A. PEASLEE
Electrical Engineer, Jeffery-Dewitt Insulator Co.

A discussion of the factors entering into the design and operating behavior of suspension insulators and the problems to be solved in designing a suspension insulator to overcome the objectionable features shown by experience to affect seriously the operation of the insulators in service.

Factors to be taken into consideration in the selection of suspension insulators for a given condition are given and a brief discussion of the general trend of future improvements is presented.

INTRODUCTION

IN the early days of electrical distribution of power the insulator problem was unimportant. The insulator gave more satisfactory service than the rest of the apparatus essential to the generation and distribution systems. As long as the voltages were low the dielectric field distribution was of relatively small importance. As the transmission distances and therefore the economic transmission line voltages increased the insulator problem became more acute. The first attempt to meet the insulation requirements of these higher voltage lines was an increase in the physical dimensions of the lower voltage type of unit. No attention was given at this time to the distribution of the dielectric field or its shape although the laws governing the dielectric flux distribution in such cases were well-known.

As a result with the increased voltages came an increasing amount of insulator trouble until when the transmission voltage passed the 30,000-volt mark, the insulator problem became of greatest importance. Improvement in design through rational study of the problems had brought the reliability of other parts of the transmission and generating systems to a very satisfactory point. The insulator, however, had not made a corresponding advance and failures were encountered at a rate that for a time threatened the success of high-voltage transmission of electrical energy.

The attention of the insulator manufacturers was turned at once to the problem and many new designs were brought out as suggested remedies for this situation. Practically none of these was based on a rational study of the insulator as a dielectric problem, most of the improvements being made from the narrow standpoint of the small experience then available. The problem was attacked by manufacturers and research men of the country, but unfortunately from widely different points of view. The manufacturers being limited by manufacturing difficulties and the great cost of a radical change in methods, clung to small changes in existing forms and processes, while the research man attacked the problem from a scientific standpoint, based on a careful study of the dielectric and mechanical problems involved, but too often handicapped by a lack of knowledge of manufacturing processes and their limitations. For these reasons many excellent ideas coming from both sources were laid aside from lack of coordination of the two lines of study.

In the early insulator types, at times the flash-over distance was much greater than warranted by the thickness of dielectric and many failures by electric puncture were encountered, also the design was such that corona was formed at different places on the insulator at low voltages.

Gradual improvements in the design eliminated many of these objectionable features one by one, and improvements in manufacturing methods brought forth constantly improving grades of porcelain.

When the pin type insulator reached a limit set by size, weight and cost, the suspension type unit was introduced. This was a decided step forward in insulator practise, but unfortunately the designers of the suspension type unit still neglected a thorough consideration of the dielectric field of flux in the designing of their units, making them simply mechanical modifications of the existing types.

Thus a great many faults of the early pin type insulators were repeated in the first suspension units. Due to its small size the flash-over voltage of the suspension unit was practically always below the puncture voltage, though, as will be shown later, the margin was not sufficient, and, with the introduction of the electric tests on assembled units in the factory, very few direct puncture failures were encountered when the insulators were first placed on the line.

At this point, however, a type of failure appeared which may be classified as a deterioration failure, the insulator passing successfully severe factory tests, but failing after a period of service on a transmission line under conditions less severe than those successfully resisted in the course of factory testing.

The study and analysis of this problem has filled the pages of engineering literature during the past ten years and many divergent theories regarding the causes of and remedies for the various types of failure have been advanced. At the present time insulators successfully passing factory tests deteriorate in service at rates varying up to 20 per cent per year. As stated by a prominent transmission engineer quite recently:

"All insulators at present on the market seem to be subject to a steady depreciation that is too large to be ignored or accepted as an operating necessity."

The conventional type suspension insulator unit, and also, to some extent, the multi-shell pin type unit, seem in general to be subjected to the types of failures indicated in the following table:

MECHANICAL FAILURES

a. Due to the use of materials having widely different coefficients of cubical expansion as in conventional cap and pin construction which causes enormous stress under temperature changes.

b. Due to mechanical overloading.

c. Due to shocks as shooting.

d. Due to lightning and power arcs.

ELECTRICAL FAILURES

a. Actual electrical puncture.

b. Leakage under adverse conditions followed by flash-over and heavy power arc.

c. Due to porosity.

In the conventional type of insulator three materials, porcelain, cement and steel, are tightly compressed in contact in an unyielding fashion. These materials have different coefficients of cubical expansion and the temperature variations, in many cases quite abrupt, met with in operation, seem to set up internal stresses which crack the porcelain, leading to electrical failure. Further the cement itself is subject to volumetric changes somewhat of cyclic nature and also of a crystalline growth character that contribute to these phenomena. Prominent engineers have expressed the opinion that 85 per cent of the failures of insulators of this type were preceded by mechanical failures of this class. The sun striking upon insulators on a frosty morning has in many cases been the signal for some rather startling exhibitions of such failures. In connection with this, the internal stresses existing in the porcelain parts due to improper manufacturing methods and firing, have doubtless contributed to this condition. That the manufacturers recognize this weakness is well shown by the elaborate precautions that have been taken to reduce this effect through the medium of felt washers, lead thimbles, etc., appearing more recently in their designs.

That the transmission engineers of the country have realized the importance of the deterioration type of failure is indicated by the extensive study, which has been made of the various methods of testing employed by most engineers responsible for large transmission systems today, such as the megger and buzz stick methods. Reliance is placed on these methods, to detect the beginning of this deterioration permitting the removal of the affected insulator before it has dangerously weakened the string. Many engineers are also advocating the deliberate addition of several units to an insulator string above the number required for actual insulation purposes as an insurance against this deterioration regarded by them as inevitable.

Failures due to mechanical overloading are rare in modern lines as the lines are usually designed with proper consideration of extreme loading conditions and ample mechanical safety factors. The same remark may be applied to failures from shock and shooting, and although at one time about the most popular outdoor sport, in certain localities, for irresponsible people, was the shooting off of the power company's insulators, fortunately, this condition is no longer of very great importance. The failures due to lightning and power arcs are, however, at the present time rather large. It is doubtful if we could define exactly what might be considered a direct stroke of lightning, and probably such strokes on transmission lines are rather rare. Lightning flash-over of an insulator string is usually in itself rather harmless, but the power arc that follows the static flash-over is extremely destructive to any but the most substantial types of insulator. The thinness of the porcelain part of the conventional type insulators, combined with the abrupt changes in form and surface directions renders them susceptible to destruction under the action of the intense heat of such an arc. Any insulator with thin petticoats is very likely to be considerably damaged by power arcs as the temperatures and the mechanical stresses involved are very high. The chief requisite for an insulator in this regard is strength, gradually increasing thickness from the edge of the skirt inwards, and a high thermal capacity. Insulators so designed will successfully resist severe power arcs and lightning surges, especially when the system is equipped with the proper kinds and numbers of relays, to a remarkable degree.

An actual electric puncture is probably rare on any modern insulator that has been properly fired, most electrical failures being the result of previous mechanical failures.

Leakage is a problem that is to a large extent dependent upon localities and specific conditions. Smelter fumes, salt fogs, dust storms and many other causes tend to make the leakage effect vary and it has been generally conceded that in bad localities the only remedy is a periodic cleaning of the insulators. A few extra units added to a string will postpone the inevitable cleaning, but it is probably safe to say that under bad conditions no insulator string could be used commercially that would not require cleaning after a time. In connection with this, however, as leakage always culminates in a flash-over, it is important that the insulator be able to withstand power arcs, especially in regions subject to bad leakage conditions.

Porous porcelain absorbs moisture from the atmosphere, thereby decreasing its electrical resistance. The leakage current flowing through the porcelain under electrical stress tends to heat localized portions to a very high temperature. This local heating causes mechanical failure followed by the passage of a power arc through the porcelain, or due to the negative temperature coefficient of electrical resistance of porcelain, the leakage current may under certain conditions gradually increase with a concomitant increase of temperature, this action being cumulative until the porcelain is punctured. A good many instances of failure of this kind both in laboratories and under field conditions have been encountered. Good glazing postpones the deterioration of porous porcelain but cannot eliminate it.

Until recently the progress in the manufacture of suspension type insulators has been rather largely along certain detailed attempts at the improvement of certain specific faults such as the utilization of felt washers, lead thimbles, etc. in the conventional cap and pin type design. The problem had not been attacked from a sufficiently scientific standpoint and there is still great need of a scientific study of this problem based on an analysis of the dielectric field of flux around insulator strings, and the electrical and mechanical requirements of the units in relation to the limitations imposed by ceramic and manufacturing conditions.

The following discussion of the analytical and experimental work undertaken along this line from the electrical and ceramic standpoint, the progress that has been made and the results that have been secured in the form of rationally designed insulators will, it is hoped, be of some interest to the operating engineer and stimulate further study and advance in this vital subject.

FACTORS GOVERNING RATIONAL INSULATOR DESIGN

The requirements to be met in the design of suspension insulators may be broadly classed under two headings:

1. The insulator must support the line mechani-

cally with adequate safety factors under the most adverse conditions.

2. The insulator must insulate the line with adequate safety factors under any electrical conditions not rendering other apparatus on the line inoperative.

It is obvious that any suspension insulator must be designed in the form of a unit that will meet widely divergent conditions. That is, from the manufacturing standpoint it is inadvisable to manufacture units of different mechanical strengths for different weights of conductors and climatic loadings. The design hinges then upon a unit that in the heaviest lines considered under the most adverse conditions of loading will give an adequate safety factor and will yet be cheap enough to be used on the less important lines.

The insulation afforded is obtained by building up strings of different lengths, but it is hardly advisable to attempt to insulate a line at great expense to withstand almost infinite voltages when, due to the limitations of other apparatus connected to the line, the system will be inoperative under extreme over-voltage conditions.

A study of existing lines and the probable limitations in conductor sizes and tower spacing of lines from 150,000 volts down, indicates that a mechanical strength of from 9000 to 10,000 pounds is adequate for a suspension unit, provided the unit is so designed that repeated stressing does not injure the unit electrically. The rational design herein discussed is, therefore, based on this mechanical strength requirement. The amount of discussion that has taken place recently regarding the use of porcelain in compression and tension makes it advisable at this point to discuss this matter a little in detail. The mechanical strength of ordinary porcelain in tension is in the neighborhood of 1500 lb. per sq. in., while the compressive strength is around 40,000 lb. in a porcelain having reasonable dielectric and temperature change resisting qualities. On account of the wide differences in these two figures many engineers have been dubious of the advisability of using porcelain in tension. The same argument might be used against the employment of cast iron in tension, and yet, although having very largely the same mechanical characteristics as porcelain, cast iron is consistently used in tension in the design of machines and structural members. As long as the unit stresses in the material are kept below the ultimate strength of the material with due regard to adequate safety factors, there is no rational objection to the employment of porcelain in tension any more than there is to a corresponding utilization of cast iron.

INSULATOR SHAPE AS AFFECTED BY THE DIELECTRIC FIELD OF FORCE

The dielectric field of force between similar electrodes is in general an ellipsoid of revolution though this is not strictly true, except between electrodes which are confocal hyperboloids of revolution, and no insulator electrodes are of this form. The agreement of the dielectric field with the ellipsoid is only approximate. However, the insulator should in general be symmetrical and conform as far as possible to the shape of the dielectric field. The placing of dielectrics of different specific inductive capacitances in series should be avoided, and therefore, the surface of the insulator should follow as closely as possible the lines of force in the field. In general the equipotential planes between the insulator electrodes should intersect for equal increments of potential, equal zone widths on the insulator surface. In connection with this point it is interesting to study Fig. 1. In this figure it will be noted that the conventional type unit does not conform to this requirement and the result of this lack of conformity is the appearance of corona on the unit at relatively low voltages, the corona appearing first where the equipotential planes are closest together. The requirement of a symmetrical shape introduces at once the problem of attaching the hardware to the porcelain in a different manner from that employed in the conventional insulator type. At the same time it becomes necessary to develop some form of hardware that will eliminate the terrific stresses imposed by the conventional type of hardware as previously discussed. Furthermore, in addition to the above requirements a large thermal capacity is necessary in a unit to enable it to resist power arcs and this demands a rather massive porcelain structure.

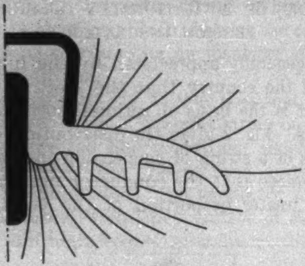


FIG. 1-a—POTENTIAL DISTRIBUTION ON THE SURFACE OF CONVENTIONAL INSULATOR, INDICATED BY THE INTERSECTION OF THE INSULATOR SURFACE WITH THE TRACES OF THE EQUIPOTENTIAL SURFACES

The voltage intervals between equipotential surfaces are equal.

The design of the hardware presents a further difficulty that is solved only by a compromise between ease of assembly and security of the connection against actual failure or uncoupling. Furthermore, the hardware and shape design of the porcelain structure must

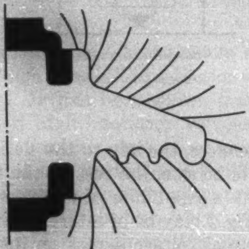


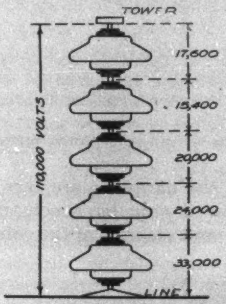
FIG. 1-b—POTENTIAL DISTRIBUTION ON THE SURFACE OF RATIONAL SUSPENSION INSULATOR, INDICATED BY THE INTERSECTION OF THE INSULATOR SURFACE WITH THE TRACES OF THE EQUIPOTENTIAL SURFACES.

The voltage intervals between equipotential surfaces are equal.

be such as to resist to the greatest possible degree abrupt temperature changes.

DEVELOPMENT OF A RATIONALLY DESIGNED SUSPENSION INSULATOR

It is not generally appreciated by the high-tension engineers of the country that the electrical duty of the end unit is the basis of rational suspension insulator designs. Fig. 2 gives the distribution of voltage on the units of a string of suspension insulators, and it will be noticed that the conductor unit is carrying by far the greater proportion of the voltage stress.



VOLTAGE DISTRIBUTION
ON STRING OF
SUSPENSION UNITS

FIG. 2

This unit is, therefore, the key to the design as, if it is so designed as to be safe, the rest of the units are obviously well within safe limits of engineering practise. The curves of Fig. 3 may be of interest, giving the percentage of the total voltage across an insulator string that is carried by the line and tower units respectively. These figures bring out to a marked extent the advantage of a proper distribution of the equipotential planes as previously discussed in that such distribution produces a unit in which the corona voltage is very high. The reason for this unequal distribution of voltage has been discussed in engineering literature of recent years and will not be commented on here. In this connection, however, Fig. 4 is illuminating in the light that it sheds upon the distribution of the equipotential planes on an insulator string. This method of illustration is most graphic in showing the actual physical conditions surrounding an insulator string under operating conditions.

Referring again to Fig. 2, it is seen that under the conditions given with a five-unit string on a 110,000-volt delta-connected line (conditions which are being successfully met at the present time by the rational design under discussion), the conductor unit is subjected to a normal-frequency voltage of 33,000 volts. The maximum high-frequency transient that has been reported by writers and investigators as likely to be met with in high-tension transmission lines is around 100,000 volts. It has been shown that the effect of the normal and high-frequency voltages combined in a circuit is to produce stresses which are the arithmetical sums of the normal and high-frequency voltages. This is readily understood as according to the law of probability, the high-frequency and normal voltage peaks will coincide in time relation a certain percentage of the time. The very high time-lag of such highly damped high-frequency transients as are encountered on transmission lines, renders possible the application of such combined voltage stresses to an insulator without flashing it over. In other words, the line unit in Fig. 2 might have impressed upon it a total stress of 133,000 volts and though the flash-over voltage of the unit is 100,000 volts this unit would not

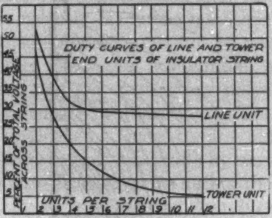


FIG. 3

flash over under these conditions due to the large time-lag just mentioned. As insulators should operate with an adequate safety factor, it is obvious that under such conditions a puncture value of around 300,000 volts at 60 cycles is necessary. In other words the puncture voltage of a rationally designed suspension unit should be in the neighborhood of three times the dry flash-over voltage at normal frequency and this is the fundamental basis of the design of rational suspension insulator units.

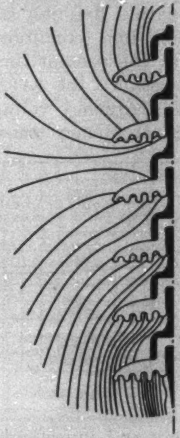


FIG. 4—DISTRIBUTION OF POTENTIAL ACROSS STRING OF INSULATORS

Taken from "Distribution of Potential about High-Voltage Insulators — Alcutt and Skolfield, *Journal of Electricity*—June 17, 1916.

While leakage is a question very largely hinging upon particular climatic or other conditions a rational shaped design has been found to improve the ability of a given length of surface leakage path to limit the leakage current. Instances are known of ideal shape designs wherein the flash-over of the insulator was the same when previously cleaned, as when covered with a considerable coating of dust. This, of course, is an extreme condition but the fact remains, and has been demonstrated in the laboratory, that proper surface shape is much more efficient in this respect than surfaces wherein the divergence from the direction of the lines of force is marked. The length of the leakage path of a suspension type insulator is rather limited. The units are 10 in. (25.4 cm) or 11 in. (27.9 cm) in diameter, and if many thin petticoats are added to the unit to increase the leakage distance they are rendered much more susceptible to destruction by power arcs on account of the thin porcelain necessarily involved.

Porous porcelain is undoubtedly the cause of a great deal of insulator depreciation. One large insulator manufacturer has recently made the published statement that non-porous porcelain could not be made, stating, "a low moisture absorption is desirable, but it must not be assumed that any satisfactory porcelain can be made which will have zero absorption." This statement is absolutely challenged. One of the first objections to a rational insulator design was made by ceramic people who stated some years ago to the author that there was no doubt that such a design was desirable but that it was impossible to make a porcelain insulator of the shape, volume and thickness necessary without having it very porous. After a great deal of factory and laboratory research this problem has been solved and insulators can be made in practically any size or shape of absolutely non-porous porcelain as determined either by the psychometer or impregnation tests. This matter will be further discussed later in the paper.

It is well-known that the efficiency of an insulator string is a function of the ratio of the capacitance of the metallic interconnecting parts between the disks to ground to the capacitance of the insulator itself as a condenser and that the string efficiency is improved as this ratio decreases in numerical value. This point must be carefully considered in any rational design and hardware with a large surface between the units avoided as much as possible.

The advisability of a high impulse ratio has been admitted only quite recently by engineers in general, and this feature is of importance because the impulse ratio is a measure of the ability of the insulator system to withstand lightning frequency flash-over. The impulse ratio of a unit and of the string built up from such units is a very important feature of insulator design, and one which has not received the attention that it should have received from most manufacturers.

The voltage at which corona appears on a unit is of great importance as a reference to Fig. 2 will show, and it is important to have this corona-forming voltage as high as possible. On a rationally designed insulator this voltage should be considerably above 30,000 volts while in many conventional type insulators at present on the market corona appears rather decidedly at voltages from 12,000 to 16,000 volts. Fortunately the conditions necessary for the attainment of the above features of insulators influences in the right direction the value of voltage at which corona will appear on the unit. Many lines are in operation with conventional type units wherein additional units have been added above the actual insulation requirements of the line to insure the operation of the string without corona on the conductor unit.

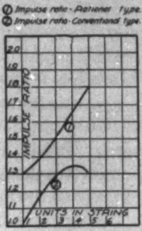


FIG. 5

The conditions discussed in general require conflicting features of design and render the design of any insulator more or less of a compromise, and the skill of the insulator designer is tested in producing the particular compromise giving the best solution under the limitations of ceramic and manufacturing possibilities. The following brief description of some of the experi-
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mental work undertaken in the research laboratories of Jeffery-Dewitt Insulator Company in the study of the design and manufacture of suspension insulators may be of interest in showing something of the amount of work involved in such studies and something of the tendency and possibilities of future development.

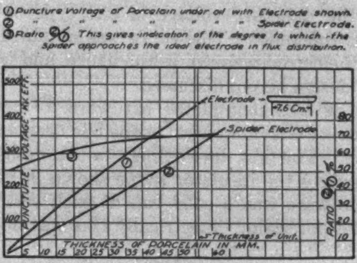

FIG. 6

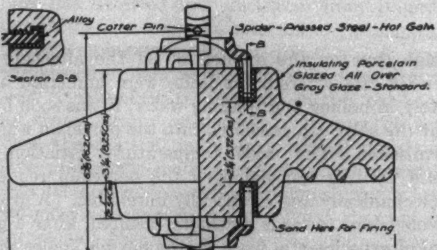
Alloy
Section B-B
Cotter Pin
Spider - Pressed Steel - Hot Galv.
B
B
Insulating Porcelain Glazed All Over Gray Glaze - Standard.
6⅜" (16.2 Cm)
3¼" (8.25 Cm)
2¼" (5.72 Cm)
Sand Here For Firing
Square Shank Link - Drop Forging
11" Diameter (27.94 Cms)

FIG. 7

DETERMINATION OF THE THICKNESS OF THE DIELECTRIC BETWEEN ELECTRODES

As previously discussed the puncture voltage of the high-tension insulator should be approximately three times the dry flash-over voltage. Having given then an acceptable dry flash-over voltage and mechanical strength, the first problem in the design of a rational insulator is the determination of the dielectric thickness between the electrodes. The curve 1 in Fig. 6 gives the puncture voltage of one type of porcelain against thickness. This curve was obtained with the form of electrodes shown in the figure, and is the basis of the design herein discussed.

The development of the hardware to meet the requirements of symmetrical shape has been an interesting one. The first development in this design was approximately the insulator shape shown in Fig. 7, and the hardware was a solid cap at each end cemented into the porcelain. Electrically this was an excellent design, but on the application of the alternate immersion test wherein the units were immersed alternately in boiling and freezing water, it was soon found that the solid cap cemented into the porcelain was not permissible. The wide temperature variation imposed by this test damaged the porcelain rendering it mechanically and electrically unreliable. A gradual development towards flexibility resulted, after a great deal of experimental work, in the flexible spider shown in the design of Fig. 7. One of the features governing the development of this spider was the requirement that the plane of dielectric stress be maintained as near as possible normal to the plane of mechanical rupture a condition which this method of attachment fulfilled admirably. The legs of this spider are fastened into the porcelain by an alloy having sensibly the same coefficient of cubical expansion as porcelain. By this means the well-known detrimental effects of cement are eliminated. The flexibility of the spider

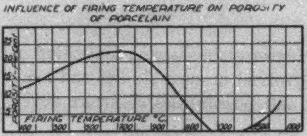

FIG. 8

legs combined with this alloy give a unit that will withstand the alternate immersion test an indefinite number of times without any detrimental effect on the insulator. Tests have been made of this character up to 100 alternate immersions followed by high-frequency flash-overs and final breaking in a tension machine. All of the tests indicate that the unit as designed is free from the detrimental effects of wide temperature variations. After the development of this spider, curve 2 of Fig. 6 was made to determine the efficiency of the spider as a flux distributor. This efficiency is given in curve 3 of the figure and shows in the thickness of the porcelain used in the unit (57 millimeters), 2¼ in., a value of 72 per cent. This value is the ratio of the puncture voltages in the same thickness with the two types of electrodes, but as the puncture voltage is that at which the dielectric flux concentration at its point of maximum intensity exceeds the critical value, it is also a measure of electrode efficiency as a means of obtaining a uniform flux distribution.

The difficulty of making a porcelain insulator of this thickness has deterred manufacturers from progress in this direction, and we can substantiate a manufacturer most emphatically in this difficulty. A great many thousand dollars were spent before we discovered how to manufacture porcelain in this thickness without firing strains or porosity. It may be said here that the solution involved a radical departure from the prehistoric methods of porcelain manufacture that have been followed continuously for a long time by most porcelain factories. These changes are met throughout the process from the original handling of raw materials through the final firing processes, and it is only upon the development of these special processes, utilization of special drying methods and the use of the tunnel kiln for firing control that the problem has finally been solved. Fig. 8 gives the firing temperature porosity curve for one porcelain body and illustrates the narrow range over which this body may be fired to produce non-porous porcelain. In securing this curve the porcelain test pieces were fired to various temperatures and cooled, the test being made at room temperature, that is, the abscissas on the curve represent maximum firing temperature of the sample while the ordinates are the porosity of the sample after firing and cooling. A discussion of this characteristic has been given.[1]

TESTS AND INVESTIGATION OF THE RATIONAL INSULATOR AS DEVELOPED

As mentioned before, the impulse ratio of a string of insulators is of very great importance and with an insulator of rational design the impulse ratio should be high. The curves in Fig. 5 are very interesting in this connection and show the excellent results secured by this rational design in impulse ratio in the individual unit and strings. The unit as designed will, therefore, for a given number of units in the string, have a very much higher flash-over to lightning disturbance than the conventional type unit with a lower impulse ratio. This, combined with the large mass of porcelain, thick petticoats and general substantial character, gives the unit a remarkable ability to withstand lightning conditions and their resultant power arcs.

The claim has been made that repeated mechanical stresses will weaken porcelain, and also that porcelain in tension is weakened electrically when under stress. In investigating this, units of the type described have been stressed to 9000 lb. (4100 kg.) in the tension machine and subjected to dry flash-over at 200,000 cycles while under stress. This test was continued until the units had been under stress for several days and at the application of high-frequency flash-over for as long periods as 100 hours there was no indication that this mechanical stressing affected in any way the dielectric quality of the insulator. This is not surprising as the plane of mechanical stress is normal to the plane of dielectric stress as before mentioned. To study the effect of repeated or continued mechanical stress, strings of insulators have been hung out in the weather with a dead load of 5000 lb. (2270 kg.) each, and periodic tests are being conducted to ascertain the condition of these insulators, and the results so far have not shown any indication that this fear is warranted. Further, repeated shocks and tension tests on porcelain samples under various conditions indicate that porcelain is not injured in any way by repeated stressing, unless the applied loads stress some part of the porcelain beyond its ultimate strength. If this is done, porcelain will fail quite naturally, as will cast iron or any other brittle material, but the results of practise and continued and careful laboratory tests indicate that the previous fear of fatigue due to continued working of the porcelain mechanically is ungrounded. These tests are being continued and some rather interesting reports will be made to the Society in the future as to the results secured from laboratory and practical tests of this nature.

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[1] "High-Tension Insulator Porcelain," W. D. A. Peaslee, A. I. E. E. White Sulphur Springs, June 29, 1920. "Test of Electrical Porcelain in Factory & Laboratory", W. D. A. Peaslee, American Society for Testing Materials, Asbury Park, N. J., June 22d–25th, 1920.

Factory Tests

It is doubtful if any developed tests at the present time that can be applied to an insulator without injuring it will prophesy its operation when on the line as to depreciation and for that reason we have made a rather radical departure in some respects from ordinary factory testing.

The fuchsine method of testing for porosity[2] is used in our factory, one unglazed unit being placed in each car of 70 insulators that pass through the tunnel kiln. This unit is broken up immediately on the removal of the car from the kiln and the pieces subjected to the fuchsine test. On the slighest penetration the entire carload is rejected and scrapped. This test gives a very satisfactory control test for porosity and guards effectively against any errors in raw materials or reading of the pyrometers that might cause porosity through firing outside of the permissible range as indicated in Fig. 8. It insures the scrapping of the small number of porous units inevitable in quantity manufacture of porcelain. After the inspection of fired porcelain the hardware is assembled and each unit is subjected to a 5000-lb. (2270-kg.) mechanical load. After this load each unit receives a two-minute dry flash-over at 200, 000 cycles. The units are then again inspected and turned over to the assembly department. It is believed that the best insurance the customer can be given as to the quality of the product he is buying is that the manufacturer started with a rationally designed product correctly proportioned and manufactured to fulfill the required conditions. A certain percentage of the finished product should then be tested to destruction to determine that the required standards of manufacture are being maintained. To this end a certain percentage of the product delivered to the shipping department is selected at random by the research laboratory and tested to destruction. The plotting of the data secured from these tests establishes a probability curve for the product considered, the study of which has revealed some very interesting things regarding manufacturing limitations.

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[2] American Society for Testing Materials, Asbury Park, N.J., June 22d–25th, 1920.

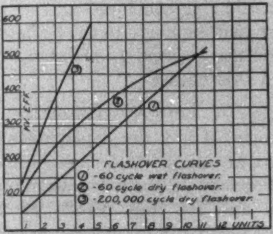


FIG. 9

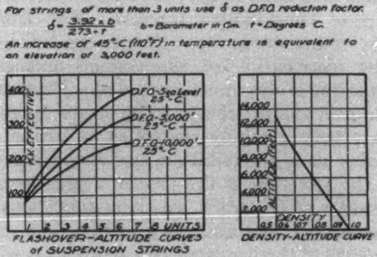
Furthermore, when this probability curve is once established with accuracy any test falling outside the determined limits on this curve is a danger signal and further tests are at once made. If these tests confirm the first results an immediate investigation is made to determine wherein the factory processes are not maintaining the required standards. It is believed that this method of testing is better than the imposition of very severe acceptance tests on all the units, as such tests, unless carried to destruction, tell very little regarding the future performance of the insulator.

OPERATING CHARACTERISTICS AND SELECTION OF UNITS

The selection of proper insulator strings for any given transmission line involves a rather careful study of a good many conditions. Given a rationally designed insulator, the individual characteristics of which are accurately known, the selection of the number of units for a string for given conditions is a matter of the development of the proper safety factor for the right conditions, using as a basis the worst line conditions liable to arise. In this connection the curves of Fig. 9 are of considerable interest. Wet flash-over values are rather deceptive. Due to the leakage currents, the distribution of the voltage amongst the units of a string is very much improved. Furthermore, wet flash-over values are apt to be erratic unless conditions are very carefully controlled as to the purity of the water used, precipitation, size of spray, etc. In general, the selection of the proper strings involves a determination of the worst electrical conditions likely to be met on the line and the selection from characteristic curves of the insulator of a string that will give the desired safety factor under these worst conditions. The curve 3 of Fig. 9 is very interesting as a measure of protection afforded to disturbance of lightning frequency. In Fig. 10 some rather interesting data are given, that is too often ignored in the selection of insulators for a transmission line, especially when it is remembered that an increase of 45 deg. cent. (110 deg. fahr.) in temperature is equivalent to an elevation of 3000 ft. (914.4 m.) of the line. After a preliminary selection of a string has been made, a study should be made of the duty on the conductor end unit to determine whether or not from the characteristics of the insulator this unit is working within safe limits. If not, the string should be readjusted to operate this unit under proper conditions, and then the results examined on the basis of the margin of safety afforded on the failure of one unit. The readjusted values of the voltage on the different units and the resulting safety factors will give a very good idea of the advisability of further insurance against trouble by the addition of end units. This question is, of course, an economic one, and the amount of money that it is permissible to pay for such protection is a question that each engineer must decide for himself.

FUTURE PROGRESS

The insulator situation today is in a state of constant development and considerable progress may be expected in the near future. Certain recent investigations indicate[3] that piezo-electric effects may be of considerable influence in porcelain depreciation and recent developments indicate that this situation will soon be met in a very satisfactory manner. Also some rather interesting work is being done at present on the solubility of porcelain in water under the conditions existing in the capillary passages connecting the voids of porous porcelain. Investigations are under way using pressures around 10,000 lb. per sq. in. with very high and very low temperatures to accelerate this action and, by means of the microscope, determine from samples of porous porcelain that have depreciated in the field compared with the porcelain subjected to



For strings of more than 3 units use δ as D.F.O. reduction factor.

$\delta = \frac{3.92 \times b}{273 + t}$ b = Barometer in Cm. t = Degrees C.

An increase of 45°-C (110°F) in temperature is equivalent to an elevation of 3,000 feet.

FIG. 10

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[3] W. D. A. Peaslee, "High-Tension Insulator Porcelain," A. I. E. E. White Sulphur Sorings, June 29, 1920.

accelerated tests in this manner, to what extent this solubility may be responsible for increasing porosity. The problem of very high-voltage transmission systems is being studied and some new types of insulators made up of rather special porcelain bodies are being developed that will meet this situation without difficulty and by the time there is money available to build any of the large projected extremely high-voltage lines, insulator manufacturers will be ready to meet the problem.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Portland, Ore., July 21, 1920.

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UNIT VOLTAGE DUTIES IN LONG SUSPENSION INSULATORS

BY HARRIS J. RYAN AND HENRY H. HENLINE
Both of Leland Stanford Jr. University

POTENTIOMETER measurements* were made of the maximum and average voltage duties occurring in strings made up of *10-inch cap and pin type units* wherein the numbers of the units were varied from 2 to 20. The numbers of units in the string and their corresponding maximum to average voltage duty-ratios thus obtained were used to locate curve II in Fig. 1. The increase in the duty-ratios is small for the shortest string of two units, it accelerates rapidly as the string is lengthened to 7 units and thereafter it remains constant at the rate of 0.09 per unit added from 7 to 20. Such increase is low and accelerating in *short strings* of from 2 to 7 units while it is high and constant in *long strings* of 7 or more units.

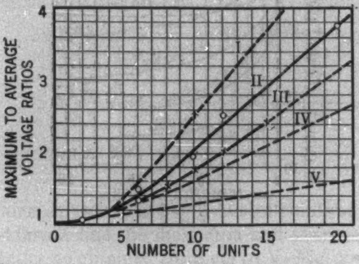

Fig. 1

In a string of 10-inch *bomb and link type units* the maximum to average voltage unit duty-ratio was found to be 2.5 thus locating approximately the steeper broken line curve I, Fig. 1.

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*The High-Voltage Potentiometer by Harris J. Ryan. Trans. A. I. E. E. Vol. 35, p. 1131, 1916.

The relation between the unit maximum voltage duty, number of units in the string and three-phase line voltage is given by the equation

$$e_{md} = \frac{d_r\, e}{1.73\, n}$$

wherein e = three-phase line voltage
e_{md} = maximum voltage unit duty
n = number of units in string
d_r = corresponding duty-ratio taken from sources in Fig. 2.

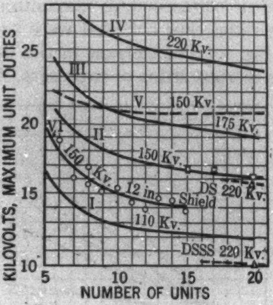


FIG. 2

By means of this equation, using values of d_r from curve II, Fig. 1, the maximum voltage unit-duties for corresponding numbers of cap and pin units in the string were determined for line voltages of 110, 150, 175 and 220 kilovolts and employed to locate curves I, II, III, and IV in Fig. 2. Again by using values of d_r for bomb and link units from curve I, Fig. 1, for a line voltage of 150 kilovolts, curve V, Fig. 2, was located.

An inspection of these curves reveals the fact that when the maximum and average voltage unit-duties are assumed to be limited to 18 and 10 kilovolts respectively for cap and pin type units the upper limit of line voltage will be 150 kilovolts and the length of the string will correspondingly be 9 units. Increasing the number of units from 9 to 20 at this fixed line voltage would lower the maximum voltage unit-duty from 18 to 16 kilovolts, only, permitting a corresponding rise of line voltage in the proportion of 16 to 18, *i. e.*, 11 per cent, or from 150 to 165 kilovolts, an amount that would hardly pay in whole or in part.

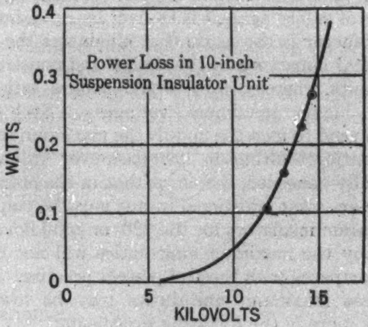


Fig. 3

For a nine-unit string of bomb and link units supporting a 150-kilovolt line the maximum and average voltage unit-duties as obtained from curve V, Fig. 2, are 20.9 and 9.6 kilovolts respectively. While these values are probably permissible, judgment in regard hereto does not appear to be as well defined as for the corresponding values of 18 and 10 assumed for the cap and pin type units.

The curves reveal further that an increase in string length from 10 to 20, cap and pin type units, will cause a corresponding increase of about 9 per cent in the line voltage at which the air about the unit adjacent to the line conductor must be broken down. They reveal likewise no increase in the corresponding line voltage for the units of the lower capacitance.

When used to support and insulate a 220-kilovolt line a 13-unit cap and pin string would operate at maximum and average unit voltage duties of 25 and 10 kilovolts respectively. The arrival of loss and corona formation with increase in voltage duty sustained by a 10-inch cap and pin type unit may be noted in the watts lost-kilovolts curve in Fig. 3. Many engineers feel that a duty of 25 kilovolts for a single 10-inch cap and pin type unit is too high because of corona formation and the injury to hardware and porcelain that may result, likewise because of the low factor of safety against flash-over by cascading. This latter factor is the cause that eliminates the value of a radical departure in the design and construction of the units whereby they would endure satisfactorily much higher maximum voltages. Such practise would tend to increase unduly the maximum to average duty-ratio resulting in low flash-over values. It is generally conceded, therefore, that in the present state of the art some additional means must be employed in suspension insulators for the 220- or 250-kilovolts lines whereby the maximum unit-duties will not be excessive compared with those of present practise.

These maximum unit-duties may be lowered by one or more of the following expedients:

I. Increase in size and capacitance of some or all of the units; grading.

II. Increase in the number of strings in the insulators.

III. Use of static shields.

The first of these, *i. e.*, larger units and grading, is regarded primarily as a manufacturer's problem and will not be taken up in the present paper.

As to what may be accomplished by means of the second and third of these expediencies the following is an illustration: The reductions in maximum unit voltage duty produced in a 20-unit, 10-inch cap and pin type, insulator, (1) by using two strings and (2) by adding static shields made of 2.5 inch well casing 4 feet in diameter, were observed in 1917 by Dr. Leonard F. Fuller and one of the authors in the development of an insulator for the temporary support of a heavy radio aerial operated at 100 undamped wave kilovolts to ground. The results obtained are targeted correspondingly for the 220-kilovolt power line in Fig. 2. The maximum unit voltage duty for the single string was found to be 23.5, for the double string, 15.5 and for the double string with static shields, 10 kilovolts.

Another set of measurements was made by the authors for the specific purpose of illustrating the effect that the use of a small static disk shield would have upon the maximum voltage unit-duty in a single string of cap and pin type units. A dimensioned sketch of the shield used in this case is given in Fig. 4. It is 12 inches in diameter and was developed by Mr. John A. Koontz. A single twenty-unit string without the shield was first set up and the maximum voltage duty carried by the end unit adjacent to the line conductor was determined. The string was then shortened first to 17, and then to 15 units and the corresponding maximum voltage unit-duties also determined. The values thereby obtained were targeted with little squares for a line voltage of 150 kilovolts in Fig. 2, and the values were found to be in agreement with the corresponding values in curve II as calculated from curve II, Fig.1.

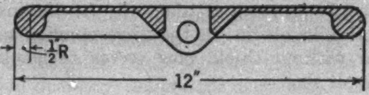


Fig. 4

The static shield was then mounted and as the string was further shortened, one unit at a time, the corresponding maximum voltage unit-duties were again observed and marked with small circles to locate curve VI, Fig. 2. The same set of values was used to locate the maximum-to-average voltage unit-duty ratio curve III, Fig. 1. For the string of 10 units supporting the 150-kilovolt line, the effect of the presence of this static shield was to lower the maximum voltage unit-duty from 17.5 to 15.0 kilovolts. This corresponds to an allowable upper limit of line voltage of 150 × 17.5 ÷ 15 = 175 kilovolts. The essential values for such higher voltage would stand as follows:

No. of units in string,	ten with 12-in. shield.
Line Voltage,	175 kilovolts
Voltage to neutral	101 "
Average voltage unit-duty,	10.1 "
Maximum " "	17.5 "

The corresponding values for the nine-unit insulator without static shield supporting the 150-kilovolt line are:

No. of units in string,	nine without shield.
Line Voltage,	150 kilovolts
Voltage to neutral,	86.6 "
Average voltage unit-duty,	9.6 "
Maximum " " "	18. "

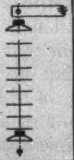
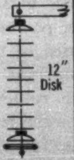
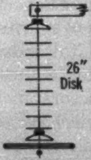
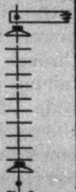
It follows that the *ten* 10-inch cap and pin unit suspension insulator equipped with the 12-inch static shield should serve as satisfactorily for the insulation of the 175-kilovolt line as the corresponding *nine*-unit insulator without shield now serves to insulate the 150-kilovolt line.

From these two illustrations it is seen that by the practicable expediency of increasing the capacitance of the units, of increasing the number of strings of units or of using static shields, or by a combination thereof, the maximum voltage unit-duty may be lowered in the long strings (15 units) from 30 to 11 kilovolts for a corresponding average voltage unit-duty of 10 kilovolts. Or more technically, the rate of increase of maximum to average voltage unit-duty ratio with unit increase of string-length can be decreased in long strings by such expedients from 0.2 to 0.02.

Many physical, economic and practical factors enter into the general problem of the extra high-voltage line insulator, *first* as to the make-up and service character of the single unit apart from the insulator as a whole, and *second* with respect to the number of units in the insulator string, the number of strings and the form and size of the static shields to be employed. The present units in use are due to a highly organized and experienced cooperation of ceramic, structural and electrical engineers. The integrity of the individual units is a matter of the utmost importance for which, because of the nature of things, the manufacturer must assume responsibility. Not until the manufacturer has amply demonstrated his immediate readiness to deliver by economic quantity production, durable units in which radical changes have been made in design, size and rated mechanical and electrical duty, can the transmission engineer count upon the use of such units for the support of extra high-voltage (220 kv.) power lines. Such lines will constitute channels through which enormous powers will flow for maintaining the vital industries of regions that are statewide. Avoidable interruptions cannot be tolerated. Uncertain factors in the reliability of these superpower lines will have to be reduced to a minimum. It has taken years to develop for economic quantity production the excellent ten-inch units that are now in extensive and dependable use. Unless the manufacturer can make an ample showing that he is ready to deliver in quantity, designs for extra high-voltage line insulators that have an aggregate superiority in essential qualities, *i. e.* form, durability and cost, the transmission engineer will assuredly undertake the insulation of the extra high-voltage lines with the present units because he knows the essential limits within which he can depend upon them.

The electrical factors in the extra high-voltage insulator that concern the transmission engineer, eventually reduce to three fundamentals, viz.: (1) type of unit, (2) maximum and (3) average voltage unit-duties.

The determination of these must rest upon a capable and experienced judgment for any particular requirement. It is the purpose of the authors to deal, only with the quantitative relations that exist between the second and third of such fundamentals, viz.: *the maximum and average voltage unit-duties in line suspension insulators made up of units in common use,* to constitute an aid to the judgment for those who must decide upon the make-up of the insulators to be used for the extra high-voltage lines for which they will be responsible. As yet reliable analytical methods have not been found for the determination of these maximum to average unit-duty relations when two or more strings and static shields are involved. The relations can be determined only by measurements. Such measurements must be undertaken indoors. It is work that requires large open spaces that are ordinarily difficult to provide. At no time in the present work did the authors have at their disposal as large a space as desired in order to be assured that the presence of walls or nearby laboratory equipment would not affect the value of the results. It was necessary, therefore, to test the integrity of the results by such means as were at hand. In so doing measurements were repeated with the insulators and their high-voltage line conductors inverted in position with respect to the ground and the potentiometer; by moving them away from nearby walls and model of tower to a posi-

	 Fig. 5		 12″ Disk Fig. 6		 26″ Disk Fig. 7		 Fig. 8	
Strings	Single 10		Single 10		Single 10		Single 12	
Line Kv.	173		173		173		208	
Kv. to Neut.	100		100		100		120	
Unit No.	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total
1	19.4	19.4	15.8	15.8	11.8	11.8	25.0	20.8
2	15.6	15.6	14.2	14.2	12.4	12.4	18.0	15.0
3	11.9	11.9	12.5	12.5	11.0	11.0	14.0	11.7
4	10.6	10.6	10.3	10.3	10.6	10.6	11.0	9.2
5	8.4	8.4	10.0	10.0	9.9	9.9	8.0	6.7
6	5.5	5.5	6.8	6.8	9.0	9.0	7.5	6.2
7	8.0	8.0	7.0	7.0	8.4	8.4	7.0	5.8
8	6.8	6.8	7.6	7.6	8.8	8.8	6.0	5.0
9	5.9	5.9	8.4	8.4	8.5	8.5	6.0	5.0
10	7.9	7.9	7.4	7.4	9.6	9.6	6.0	5.0
11							6.0	5.0
12							5.5	4.6

Figs. 5—6—7—8

tion of greater elevation and isolation in the center of the building. When substantially the same results were obtained for all such positions as specified, it was concluded that the nearby walls and low elevation of the insulator and high-voltage conductor did not produce results essentially different from those that would be obtained when the insulators would be mounted in the open from regular tower cross-arms.

The results obtained are expressed in the series of Figs. 5 to 21 and the accompanying tables. In the figures, sketches approximately to scale are given of the forms of long insulators of single and double strings with or without static shields. In the table found below each insulator sketch, the values of the voltage duties are given that are carried by the corresponding units in the string. Such values throughout for all of these cases have been adjusted to a uniform average unit-duty of 10 kilovolts. The actual voltages applied to the insulators when the measurements were made were always near to the value

$$\text{Kilovolts} = 10n$$

wherein n is the number of units in the insulator string. Thus one may note in the tables the ratios of any actual to average unit voltage duties by dividing the duty as given by 10. For the aid of those who are accustomed to think of the unit-duties in percentages of total applied voltages, parallel columns have been inserted in the tables giving such percentages. It is assumed that these sketches and tables are for the rest self-explanatory and that little further narative in regard to them is necessary.

From the upper end of the insulator under test metal tubes were mounted horizontally and vertically to the floor so as to affect the electric field about the insulator in the manner that would be done by the upper portion of a particular tower with its cross-arm that will be used in a new project for the support of a 150-kilovolt power line. The upper or cross-arm end of the unit was fixed at about 16 feet from the concrete floor and the vertical central axis of the insulator was about eight feet from the dry brick wall of the laboratory. Exceptions hereto occurred when the insulator was inverted or removed to the center of the laboratory for checking purposes. In the latter case the 20-foot, 1-inch line conductor, maker's standard line clamp and first unit were mounted approximately 16 feet from the floor while the upper grounded end-unit was higher by the length of the string, *i. e.* at a height of from 20 to 25 feet while the central axis of the insulator was remote from the vertical grounding wire about 10 feet and the nearest brick wall about 15 feet. Exception likewise occurred in the single case of the insulator mounted with jumper for strain duty. The strain insulator, one-inch diameter aluminum cable conductor and jumper were mounted in the vertical. The lower end unit was approximately eight feet from the ground. It is believed, however, that the results

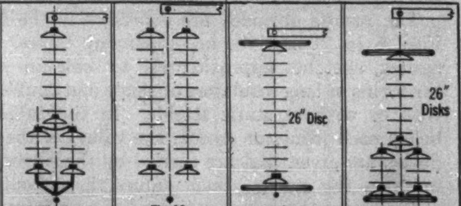


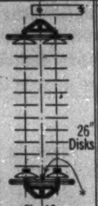
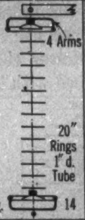
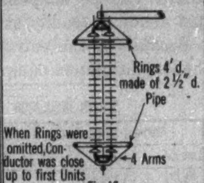
Fig. 9 — Fig. 10 — Fig. 11 — Fig. 12

Strings	Single-Double-12		Double-12		Single-12		Single-Double-12	
Line Kv.	208		208		208		208	
Kv. to Neut.	120		120		120		120	
Unit No.	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total
1	22.9	19.1	22.7	18.9	17.0	14.2	14.3	11.9
2	15.4	12.8	18.3	15.2	15.8	13.2	13.1	10.9
3	12.6	10.5	13.1	10.9	14.3	11.9	11.5	9.6
4	8.7	7.3	9.9	8.3	11.7	9.8	10.1	8.4
5	10.5	8.8	8.5	7.1	10.3	8.8	9.6	8.0
6	8.8	7.3	7.7	6.4	8.7	7.2	11.2	9.3
7	7.7	6.4	7.0	5.8	7.6	6.3	9.5	7.9
8	7.0	5.8	6.8	5.7	7.3	6.1	8.8	7.3
9	6.5	5.4	6.6	5.5	7.1	5.8	8.4	7.0
10	6.6	5.5	6.5	5.4	6.8	5.7	8.0	6.7
11	6.5	5.4	6.3	5.3	6.7	5.6	7.5	6.3
12	6.8	5.7	6.6	5.5	6.7	5.6	8.0	6.7

Figs. 9—10—11—12

obtained are near to those that would be obtained correspondingly under actual service conditions.

Unit voltage duty measurements were made with and without a single, circular, 26-inch diameter static shield mounted between the line conductor and the first unit. The results obtained are given in the tables below Fig. 16 which illustrates this laboratory set-up for the study of a long strain insulator with line jumper. From them the lesson is drawn that the jumper is

	 Fig. 13		 Fig. 14		 Rings 4' d. made of 2½" d. Pipe When Rings were omitted, Conductor was close up to first Units 4 Arms Fig. 15			
Strings	Double-12		Single-15		Double-20			
Line Kv.	208		260		346			
Kv. to Neut.	120		150		200			
					WITHOUT SHIELDS		WITH SHIELDS	
Unit No.	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total
1	15.3	12.7	19.9	13.3	24.5	12.2	15.8	7.9
2	14.3	11.9	20.6	13.7	20.5	10.3	15.2	7.6
3	12.7	10.6	15.0	10.0	16.0	8.0	14.4	7.2
4	10.8	9.0	14.3	9.5	14.3	7.2	13.8	6.9
5	9.4	7.8	12.7	8.5	12.2	6.1	12.8	6.4
6	Not	Observed	10.3	6.9	9.8	4.9	11.4	5.7
7	Not	Observed	8.9	5.9	8.0	4.0	10.4	5.2
8	Not	Observed	7.5	5.0	7.5	3.7	9.6	4.8
9	Not	Observed	6.9	4.6	7.7	3.8	9.2	4.6
10	Not	Observed	6.4	4.3	7.5	3.7	9.0	4.5
11	Not	Observed	5.9	3.9	6.7	3.4	8.6	4.3
12	Not	Observed	5.7	3.8	6.5	3.2	8.0	4.3
13			5.5	3.7	6.7	3.4	8.4	4.2
14	*	Mounting	5.3	3.5	7.3	3.6	8.4	4.2
15	Conductor	on Yoke	5.1	3.4	6.6	3.3	8.4	4.2
16	below Sh	ield made			6.0	3.0	7.7	3.8
17	no pract	ical differ-			5.8	2.9	7.4	3.7
18	ence				7.2	3.6	7.0	3.5
19					8.8	4.4	7.0	3.5
20					10.5	5.3	6.9	3.5

Figs. 13—14—15

effective in lowering the maximum unit voltage duty thereby offsetting the rise in such duty produced by mounting the line conductor in the axis of the insulator instead of at right angles thereto; and the further lesson that with the aid of a static shield the strain insulator can be adapted for extra high-voltage duty just about as readily as the suspension insulator.

Because of the extra tower clearance required for the long insulators equipped with large guards the idea naturally developed that perhaps if the "arcing horns" were used and sufficiently extended and maintained in the plane of the conductor they might serve to reduce the maximum unit voltage duties sufficiently without increasing the necessary tower clearances. Accordingly a cap and pin 15-unit suspension insulator equipped with great arcing horns to function as a static shield was arranged as in Fig. 17 and the unit voltage duties and corresponding percentages of line voltage were measured and determined and tabulated below the figure. The results were rather disappointing. They indicate the superiority of the shield which, to save

STRAIN INSULATOR

	Without Static Shield		With Static Shield	
Line Kv	208		208	
Kv. to Neutral	120		120	
	UNIT DUTY		UNIT DUTY	
Unit No.	Percent of Total Voltage	Kv.	Percent of Total Voltage	Kv.
1	16.	19.2	9.5	11.4
2	13.	15.6	9.5	11.4
3	11.	13.2	9.	10.8
4	9.	10.8	8.25	9.9
5	7.75	9.3	8.	9.6
6	6.75	8.1	7.5	9.
7	6.	7.2	7.5	9.
8	5.5	6.6	7.5	9.
9	5.5	6.6	7.75	9.3
10	5.75	6.9	8.	9.6
11	6.5	7.8	8.25	9.9
12	7.	8.4	9.	10.8

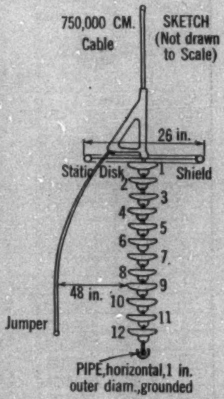
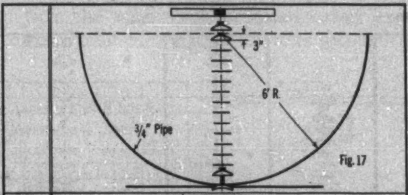


FIG. 16

tower clearance when necessary, may be made elliptical as proposed by Mr. Peek.

To aid one's judgment as to the effect of lessened insulator capacitance and of rain upon the values of the unit voltage duties, duty measurements were made upon three long strings of unit-types as follows: *cap and pin, bomb and link and core and tine, all dry, and core and tine, wet.* The results thus obtained are given in the tables below Figs. 18, 19 and 20. The "rain" introduced a strong grading effect, greatly reducing the maximum to average unit voltage duties. Such reduction in this particular case was made to occur from 2.2 to 1.4.



Strings	Single-15
Line Kv.	206
Kv. to Neut.	150

Unit No.	Unit Duty in Kilovolts	Unit Duty Percentage of Total
1	22.5	15.0
2	18.0	12.0
3	14.2	9.5
4	13.5	9.0
5	9.8	6.5
6	8.4	5.6
7	7.8	5.2
8	7.1	4.7
9	6.7	4.5
10	6.5	4.3
11	6.3	4.2
12	6.6	4.4
13	7.1	4.7
14	7.5	5.0
15	8.0	5.4

Fig. 17

At the suggestion of one of the engineers who has followed the progress of these studies, the unit voltage duties were determined in an insulator that was subjected to two synchronous in-phase voltages, 63.5 and 127 kilovolts, corresponding to a double line voltage of 110 and 220 kilovolts. The purpose hereof was to obtain knowledge of the corresponding reduction in unit-duty voltages that result from the use of the associated lines, one of which would serve for the flow of trunk line power and the other for power for more local purposes. A single 15-unit cap and pin type string without static shield was arranged to support two 1-inch diameter conductors as illustrated in Fig. 21. The conductor carrying the lower voltage was mounted in the horizontal between the seventh and eighth units. The companion conductor carrying the higher voltage was mounted parallel thereto

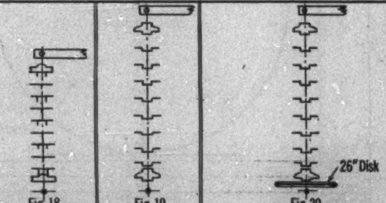


Fig. 18 Fig. 19 Fig. 20

Strings	Single-10		Single-10		Single-10			
Line Kv.	173		173		173			
Kv. to Neut.	100		100		100			
					DRY		WET	
Unit No.	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total	Unit Duty Kv.	Unit Duty Percentage of Total
1	24.8	24.8	31.0	31.0	21.7	21.7	8.4	8.4
2	19.6	19.6	19.2	19.2	17.0	17.0	14.0	14.0
3	12.2	12.2	11.0	11.0	13.2	13.2	13.2	13.2
4	9.7	9.7	8.8	8.8	10.1	10.1	10.1	10.1
5	7.1	7.1	Not	Observed	Not	Observed	Not	Observed
6	4.5	4.5	Not	Observed	Not	Observed	Not	Observed
7	4.0	4.0	Not	Observed	Not	Observed	Not	Observed
8	5.5	5.5	Not	Observed	Not	Observed	Not	Observed
9	6.2	6.2	Not	Observed	Not	Observed	Not	Observed
10	6.4	6.4	Not	Observed	Not	Observed	Not	Observed

Figs. 18—19—20

from the end unit by means of the maker's standard clamp.

With the insulator, conductors and voltages arranged as specified the unit voltages were then measured and the values obtained were used to locate curve 1, Fig. 21. This curve reveals the manner in which the unit voltage duties varied from unit to unit. For comparison the broken-line curve II was located with the corresponding unit voltage duties that were obtained from the same insulator arrangement except that the conductor carrying 110 kilovolts was omitted.

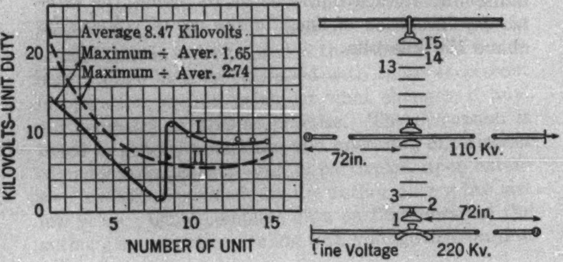


FIG. 21

It is of interest to note that the ratio of the maximum to average unit-duties was reduced from 2.74 to 1.65 by the presence of the additional conductor carrying one-half main-line voltage.

ACKNOWLEDGMENTS

In closing the authors desire herewith to express their appreciation of the helpful cooperation they have had in the promulgation of these laboratory studies of Messrs. F. G. Baum, S. Barfoed, L. F. Fuller, W. A. Hillebrand, J. P. Jollyman, J. A. Koontz, and J. Mini; for static shields, conductor clamps and other insulator hardware furnished by the Great Western Power Co. and the Pacific Gas and Electric Co.; for insulator units supplied by the Jeffrey-Dewitt Insulator Co., the Ohio Brass Co. and the R. Thomas and Sons

Co.; and for assistance in manning the high-voltage potentiometer and preparing figures and tables by F. F. Evenson.

CONCLUSIONS

1. Suspension insulator units in common use can be satisfactorily employed for the make-up of insulators for 250-kilovolt lines.

2. Increase in the number of strings in the suspension insulator will permit the use of a limited increase in line voltage.

3. Static shields in requisite forms will lower maximum unit voltage duties so as to permit the satisfactory insulation of lines for the use of voltages far above 150 kilovolts.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Portland, Ore., July 21, 1920.

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ELECTRICAL CHARACTERISTICS OF THE SUSPENSION INSULATOR-II

The Line Insulator at the Higher Voltages

BY F. W. PEEK, JR.
General Electric Co., Pittsfield, Mass.

IT is the purpose of this paper to review the duties of the line insulator at voltages above 100 kv. and compare them with the duties imposed by the lower voltages. It seems desirable to do this at the present time in order to predict the reliability of future high-voltage lines as compared with those at present in operation, and to point out what changes, if any, are necessary in present practise. The discussion is based in general upon data and operating experiences of many investigations and, in particular, upon extensive investigations made by the author during the last few years. Quite complete data on that phase of the author's investigation dealing with voltage distribution will be given.

PRESENT STATUS

At the present voltages the problem is primarily a mechanical one. Mechanically, porcelain would never be selected as a line support. It is unreliable in tension, subject to cracking, and if made in large pieces subject to porosity. The greatest care is necessary in manufacture to secure a uniform, tough, non-brittle material, free from porosity. Unfortunately it is the only material that we know of at the present time that will withstand the weather without the carbonization and deterioration of organic compounds under the electrical stress.

Generally after three to five years of more or less successful operation, insulators selected by the most careful electrical tests begin to fail rapidly. There are of course exceptions, but the experience is quite general and most operating companies anticipate breakdown by periodic tests designed to weed out faulty units.

The apparent deterioration of porcelain is generally due to one of the following causes:

1. Gradual mechanical cracking, due to expansion of cement or tight-fitting metal parts, or to internal firing strains or brittle porcelain.
2. Gradual absorption of moisture due to porosity.

The greater part of the trouble has been due to cracking under stresses caused by expansion of cement or of tight-fitting or cemented-in metal parts. Cracking may also be caused by uneven expansion in very thick porcelain parts of different shapes.

The foregoing causes of deterioration are well verified in practise, because the type of insulator in which the porcelain units are strung together by loose fitting metal parts or cables shows no deterioration after ten years or more of service. This was found to be so even when some of the earlier units were made of poor material. The absorption of moisture seems to be due to a considerable extent to breathing. The presence of a damp sponge of cement is thus also undesirable from this standpoint as it keeps moist the air breathed by the porcelain.

The solution of the deterioration problem seems to be to start with a design as free as possible from expansion troubles and the selection of a tough, non-porous porcelain. Years of service have been the best criterion of design. Regarding the selection of material, no present, practical electrical test will anticipate future cracking due to internal strains or brittle porcelain or will indicate porosity in dry porcelain. The desired results can probably best be attained by testing a small percentage of the product to destruction from day to day after the usual electrical and inspection tests have been made, the object being to determine if the product is up to the standard and of uniform quality. This idea is not new, but is used in the manufacture of lamps and in other industries. Electrical, mechanical and porosity uniformity tests are necessary. Extremely accurate tests are not necessary, but it is necessary to have tests that can be quickly made so that any fault can be at once detected and remedied.

In our investigation we made first the electrical tests, followed by mechanical impact tests to destruction. Samples were then taken from the head and thick parts of the units and subjected to a porosity test. In this test the samples were placed in a dye solution under pressure, after which the depth of penetration was noted. The porcelain was placed in three arbitrary mechanical and porosity grades and a graphical chart made indicating the percentage in each grade as shown in Fig. 1. We found this method of great use not only in checking the product and comparing different materials, but also in studying deterioration of insulators in service.

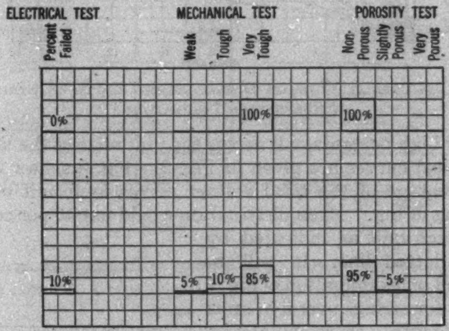

Fig. 1—Sample Sheet Showing Method of Plotting Electrical, Mechanical and Porosity Tests

INSULATORS FOR THE HIGHER VOLTAGES

The problem of deterioration discussed above is independent of the voltage. Its effect on operation should be less at the higher voltages because of the greater number of units used in a string. There are, however, certain factors unimportant at the lower voltages, which become of increasing importance as the voltage is increased.

Uneven Voltage Distribution. It has long been known that when insulators are placed in series in a string the arc-over voltage is not the sum of the arc-over voltages of the individual units, but less. This is due to the uneven division of voltage on the different units. A very high percentage is across the unit nearest the line. Typical curves are given in Fig. 2. Fig. 3 shows the percentage of the total voltage across each unit of a string of ten. Even distribution would put 10 per cent on each unit as indicated by the dotted line. It will be noted on examination of Fig. 2 that for strings over five units in length, there is always about 30 per cent of the total voltage across the insulator nearest the line. This will vary from 20 to 30 per cent with different types of insulators and with different hardware on the same type. The

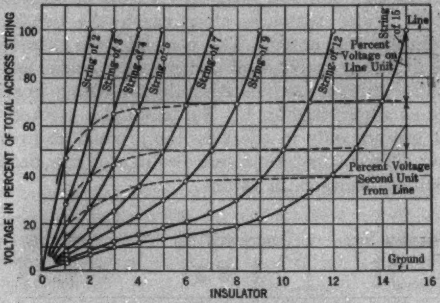

Fig. 2—Typical Voltage Distribution Curves on Strings of Suspension Insulators

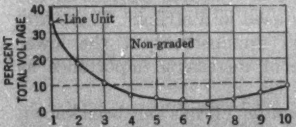

Fig. 3—Voltage Distribution on String of Ten Insulators

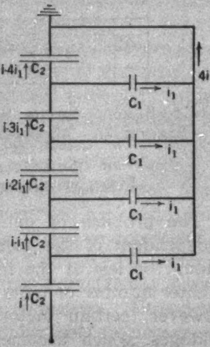

Fig. 4—Cause of Uneven Distribution

The capacities to ground C_1 cause an uneven distribution of current through the insulator capacity C_2.

importance of a consideration of this factor at the higher voltage is at once seen since the maximum unit stress increases directly with the voltage. At 100 kv. the operating stress on the line unit is 17.4 kv.; at 220 kv. it is 38 kv., which is higher than is desirable.[1] It is desirable to lower the operating stress on the units near the line.

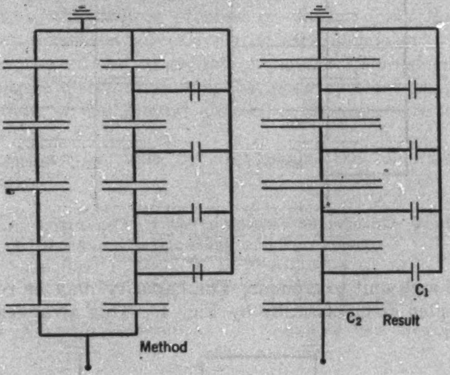

Fig. 5—Grading by Making Internal Capacity of Insulator Units C_2 Large Compared to the Capacity to Ground C_1.

A mathematical analysis of the cause of uneven distribution has already been given.[2] It seems best, however, to give here an elementary review of the causes of uneven distribution in order better to discuss the methods of remedying it.

A string of line insulators may be considered as being made up of a number of capacities in series. There is also capacity from the hardware and fittings

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[1] $\frac{220}{\sqrt{3}} \times 0.30 = 38$. In the case of a grounded line on a system with isolated neutral this voltage would be $220 \times 0.30 = 66$ kv.

[2] Peek, "Electrical Characteristics of the Suspension Insulator—I," A. I. E. E. Transactions 1912, Vol. XXXI, page 907.

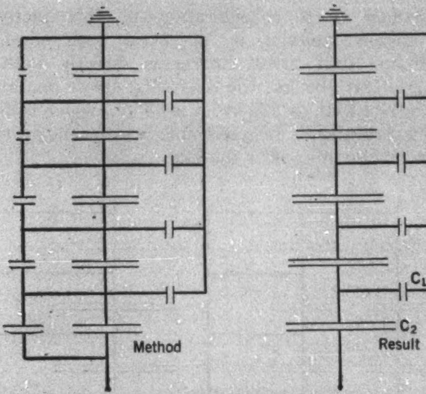
Method
C1
C2
Result

Fig. 6—Grading by Changing the C_2 Capacities in Proportion to Their Respective Currents

of each unit to ground. The capacity may be represented diagramatically by Fig. 4. This arrangement

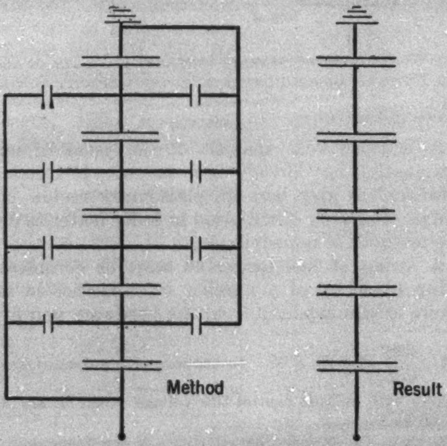
Method
Result

Fig. 7—Grading or Shielding by Eliminating the Effect of the Capacities to Ground

is approximate, but suitable for the purpose of illustration.

It is at once apparent that the capacity currents cause unequal voltage distribution. If i is the total current through the first insulator, the capacity current through the second one from the line is $i - i_1$ where i_1 is the capacity current to ground; and third from the line $i - 2\,i_1$; the fourth $i - 3\,i_1$, etc. The current and, therefore, the drop is greatest on the line unit; the current decreases successively on each unit from the line by the current to ground of one unit. Correction of voltage distribution may be made by eliminating the effect of the ground current or the capacities to ground, C_1.

This may be done by any one of the following methods:

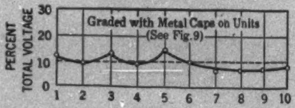

Fig. 8—Voltage Distribution on String of Ten Insulators

1. Increasing all of the C_2 capacities without increasing the C_1 capacities so that the effect of the current to ground is relatively less. See Fig. 5.
2. Increasing the C_2 capacities of the insulators along the string in proportion to the currents flowing through them. This means highest C_2 capacity on the line unit, less on the next unit, etc. See Fig. 6—This is generally called grading.
3. Elimination of the ground capacities by means of an antenna shield from the line. See Fig. 7. This may be called shielding.

In order to get appreciable results by method 1, it is necessary to add capacities with solid insulation, or plates in intimate contact with the porcelain. Plates along the string will not help materially, as the percentage increase in the C_2 capacities will generally be smaller than the percentage increase in the C_1 capacities.

Method 2 is practical but it must be accomplished by adding capacities with solid insulations. In practise it means the addition of metal caps or plates in intimate contact with the units, or varying the wall thickness of units, or placing a varying number of units in multiple along the string, or using different sizes of units. Very good results can be obtained by this method. Its undesirable feature is that it requires different kinds or sizes of units in the same string.

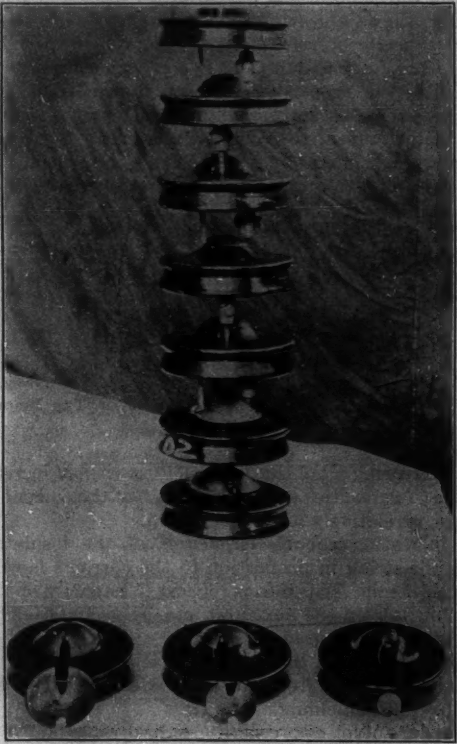

FIG. 9—GRADING BY METAL CAPS

Figs. 8 and 9 show the voltage distribution in a string graded by placing different sizes of caps along the string.

Very good voltage distribution can be easily obtained by method 3. The ground capacities may be thought of as being due to grounded antenna extending along the string. The effect of this antenna may be eliminated by a similar antenna connected to the line and extending along the string as in Fig. 10. In practise the antenna is the very simple shield shown in Fig. 11. The hardware shown in Fig. 11 is not standard but was selected for laboratory purposes to give the worst distribution. The shield readily corrects it. The maximum unit stress would be less on a 220-kv. shielded string than on present non-shielded strings operating sucessfully at 100 kv. and less. The distribution with and without the shield is shown in Fig. 12. A standard hardware giving better distribution with and without the shield is shown in Fig. 13.

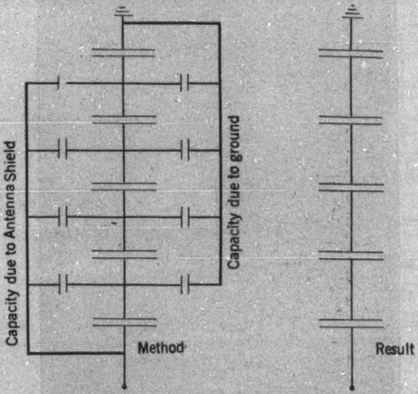

Fig. 10—Shielding by Eliminating the Effect of the Capacity to Ground

This method requires no special units. The antenna has an additional advantage of acting as a corona shield, eliminating corona from the string and tending to keep the power arc from the insulators and from burning the conductors. Even if the voltage were

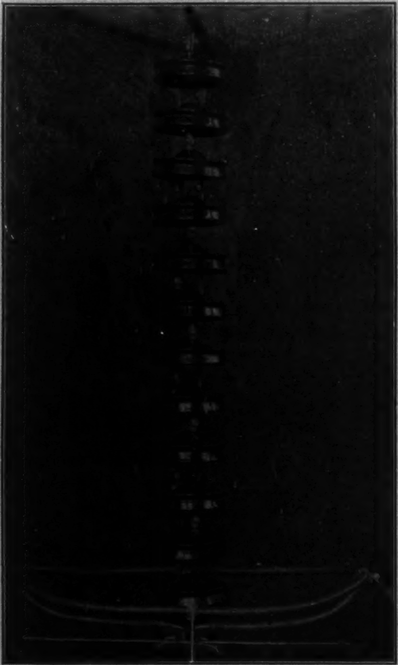

FIG. 11—ANTENNA SHIELD FOR EQUALIZING VOLTAGE ON INSULATORS
(Laboratory hardware)

evenly distributed a corona eliminating shield would be advisable at the higher voltages.

Heretofore the main argument given for equalizing voltage distribution has been to reduce the string length by increasing the arc-over voltage for a given number of units. It is much more important to reduce the operating stress on the line-end units.

With certain types of units the wet arc-over voltage may be higher than the dry arc-over voltage for long strings.[3] This is because rain may assist in grading a badly unbalanced string. As a general rule, a string cannot be very greatly shortened in practise by grading because of the effects of rain, dirt, etc. on the perfectly graded string. There would be little gain in increasing the dry arc-over voltage if the wet arc-over voltage were decreased. Rain would not increase the wet arc-over voltage of a graded string. The wet arc-over voltage would generally be lower if the string length were decreased.

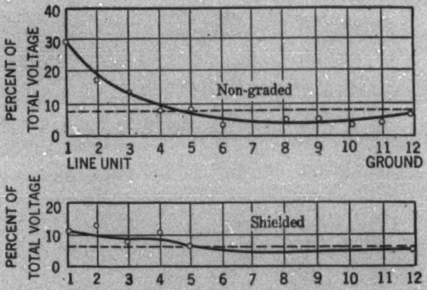

FIG. 12—VOLTAGE DISTRIBUTION ON A STRING OF TWELVE INSULATORS
(See Fig. 11—Laboratory hardware.)

There is an additional reason why grading will generally not make possible an increase in the arc-over voltage or a decrease in the string length. For any unbalanced string there is more or less complete automatic grading as the arc-over voltage is approached. Near arc-over excessive corona forms on the line unit, to a less extent on the next unit, etc. These sheets of corona act as capacity plates and grade the string, thus automatically raising the arc-over voltage. If this were not so, it would be expected that the arc-over voltage for the non-graded string above could for five

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[3] Peek, "Electrical Characteristics of the Suspension Insulator—I." A. I. E. E. TRANSACTIONS 1912. Vol. XXXI, page 907.

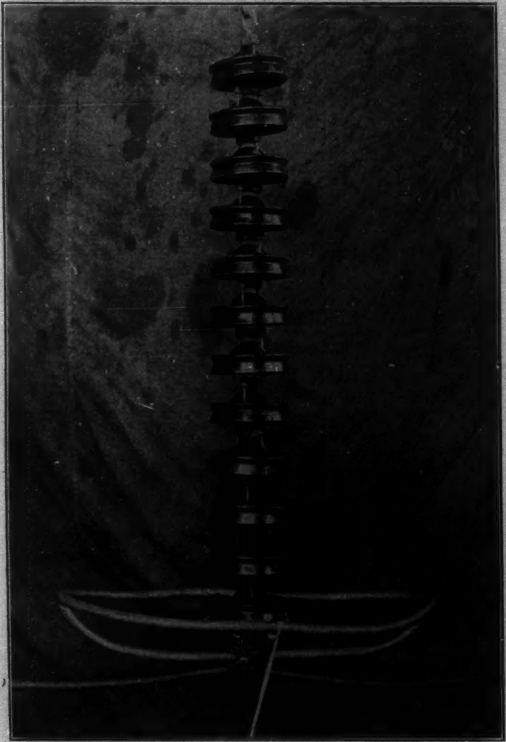

FIG. 13—ANTENNA SHIELD
Insulators strung with standard hardware.

or more units, never be higher than $\frac{1}{0.30} = 3.3$ times the arc-over voltage of a single unit. As a matter of fact, it is much higher, due to the automatic grading of corona.

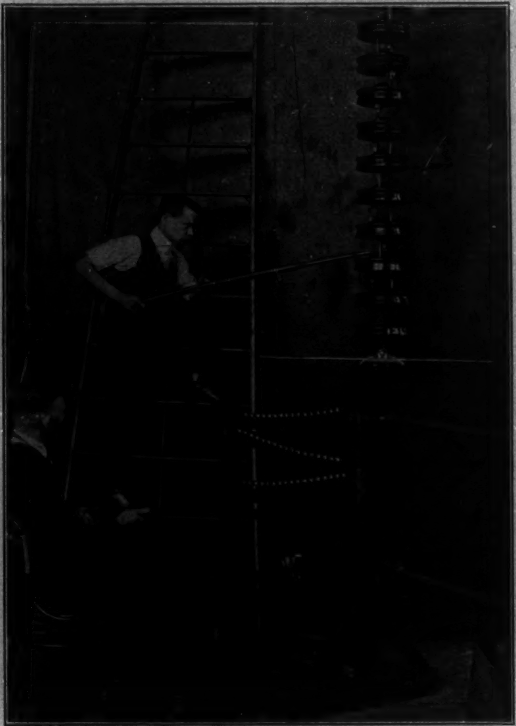

FIG. 14—METHOD OF MAKING DISTRIBUTION MEASUREMENTS

The wet and dry arc-over voltage of insulators decreases almost directly with barometric pressure. The insulator problem for the higher voltages is therefore more difficult at higher altitudes.

It has been found that the wet lightning arc-over voltage is usually as high as the dry lightning arc-over voltage.[4] The lightning arc-over voltage is, to a great extent, a question of string length. If the string length is decreased due to grading the lightning arc-over voltage will also be decreased.

TESTS

It may be of interest to describe the method of making tests. A water resistance with 50 taps was placed across the line in parallel with the insulators as shown in the illustration, Fig. 14. Each tap had a fixed and known potential above ground, *i. e.*, a given percentage of the applied voltage. Connection was made between the particular unit, the potential of which was being investigated and various taps on the resistance until a tap was found of the same potential. In making this measurement a wire was fastened to a tap and the other end of the wire fastened to a steel point at the end of a fibre rod. When the rod was brought up to the cap a spark would indicate a difference of potential. The connection on the tap was then changed, or the operation was repeated until no spark occurred when the point touched the insulator cap.

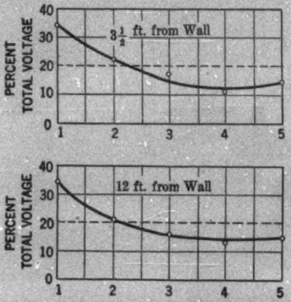

FIG. 15—THE EFFECT OF PROXIMITY TO WALLS
Voltage Distribution on String of Five Insulators.

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[4] Peek, "The Effect of Transient Voltages on Dielectrics," A. I. E. E. TRANSACTIONS, 1915, vol. XXXIV, page 1857.

It was found that the steel point, because of the color of the spark, indicated the smallest difference of potential. In making measurements it is necessary to prevent as far as possible any distortion of the field by the wire and point. A distortion is indicated by different potential readings when measurements are made at different points around the insulator. A stream of water was kept running through the tube in order to keep the temperature constant. Approximate potential readings can be obtained by placing a very small gap shunting a forked stick across the units. This method gives fairly accurate results on the two or three units near the line, but quite inaccurate results on the others.

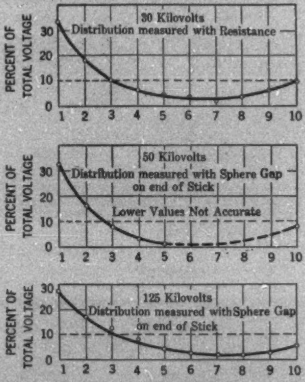

FIG. 16—THE EFFECT OF APPLIED VOLTAGE ON THE DISTRIBUTION OF A STRING OF TEN

The effect of proximity to tower, etc., is given in Fig. 15. It will be seen that it is not great. The effect of corona on distribution is not great at the operating voltage. For instance there is very little difference in the voltage distribution between 30 and 130 kv. 130 kv. is approximately the voltage to neutral of a 220-kv. system. Fig. 16 shows the distribution at 30, 50 and 125 kv. The 50 and 125-kv.

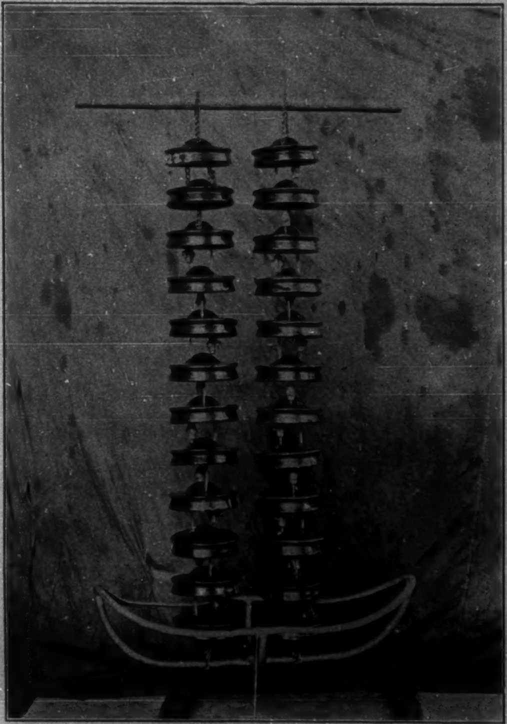

FIG. 17—ANTENNA SHIELD ON A DOUBLE STRING
Laboratory hardware—See Fig. 18.

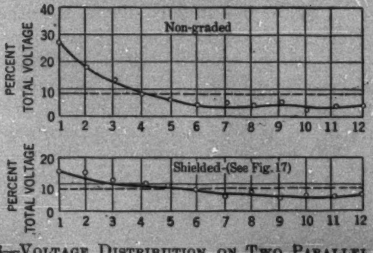

FIG. 18—VOLTAGE DISTRIBUTION ON TWO PARALLEL STRINGS OF TWELVE INSULATORS

Vol.
1920

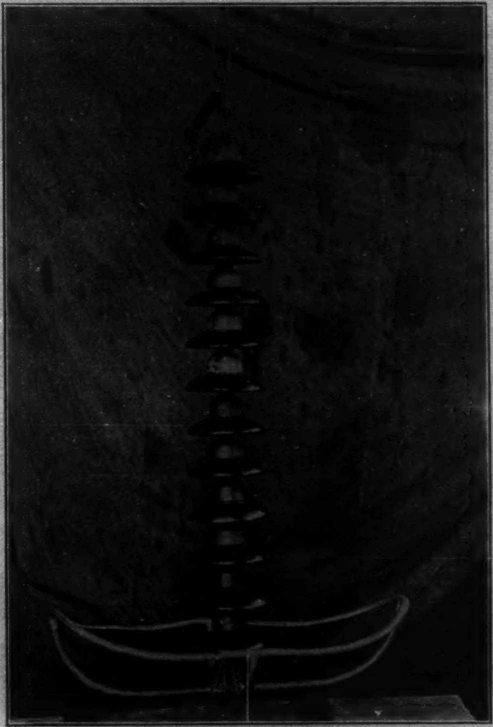

FIG. 19—ANTENNA SHIELD
(See Fig. 20.)

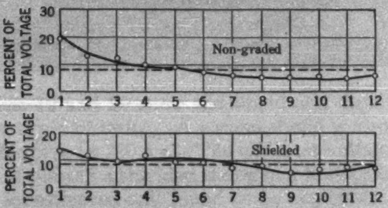

FIG. 20—VOLTAGE DISTRIBUTION ON STRING OF TWELVE INSULATORS
(See Fig. 19.)

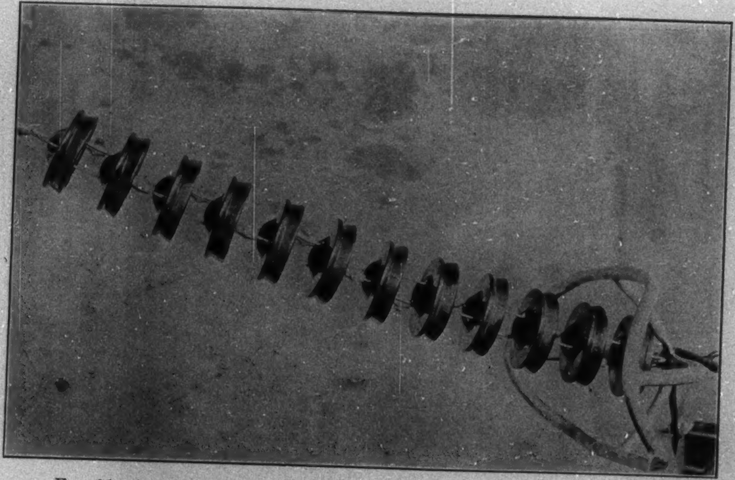

FIG. 21—ANTENNA SHIELD FOR EQUALIZING VOLTAGE ON INSULATORS

curves were made with a spark gap and are accurate for the three units near the line only. Fig. 17 shows the shield used on a double string. The distribution is very good as shown in Fig. 18.

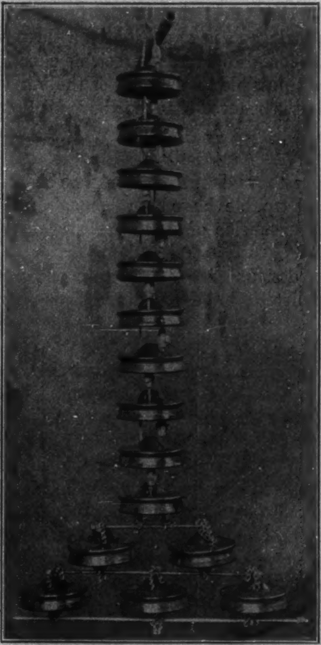

FIG. 22—TWO AND THREE UNITS IN PARALLEL AT LINE END OF STRING OF 12 UNITS AS A MEANS OF IMPROVING VOLTAGE DISTRIBUTION ON THE STRING

Figs. 19 and 20 show the shield and distribution on other types of units. A corona shield is also desirable on this type at 220 kv. The effect on voltage distribution of placing a shielded string in a horizontal position was investigated and found to be negligible. See Fig. 21.

The method of grading shown in Fig. 22 was not found to be very satisfactory.

Fig. 23 shows a form of shield that gave fairly good results.

Distribution tests were made to see if there would be any disturbing effect due to the proximity of other insulator strings as in operation on a three-phase line. There was found to be no appreciable effect.

It is not possible to show all of the various shields tried, but it is believed that the above fairly well covers the field.

SUMMARY

From the above discussion it may be concluded that:

Insulator troubles have been due mainly to cracking caused by expansion of metal parts, cement, etc. and to porosity.

Certain designs with loose-fitting parts have been free from deterioration. Other designs should be so modified as to relieve them as far as possible from expansion troubles.

Tough non-porous porcelain is desirable.

The old method of basing everything on electrical tests should be abandoned. Severe electrical tests are often harmful. The electrical strength is often secondary to other characteristics. An electrical, mechanical and porosity uniformity test should be established, in which a small percentage of the product is tested to destruction from day to day to ascertain if it is running brittle or porous or is weakened by firing strains.

For the very high voltages that are at present being considered, greater reliability may in many respects be anticipated than for the lower voltage lines. The lightning arc-over voltage and dielectric strength will be relatively higher and induced lightning voltages, sufficient to cause arc-over, will be less than on low-voltage lines. Increasing the number of units in series decreases the probability of complete string failure.

Uneven distribution of voltage on the string becomes more serious at the higher voltages because of the high stress on the unit near the line. For strings of more than four or five units the stress is practically a constant percentage of the operating voltage independent of the string length.

Uneven distribution can be corrected by shielding, shielding prevents excessive corona on the line end units and tends to direct the power arc away from the string. The maximum unit stress on a 220-kv. shield string can be made less than on a 100-kv. non-shielded string.

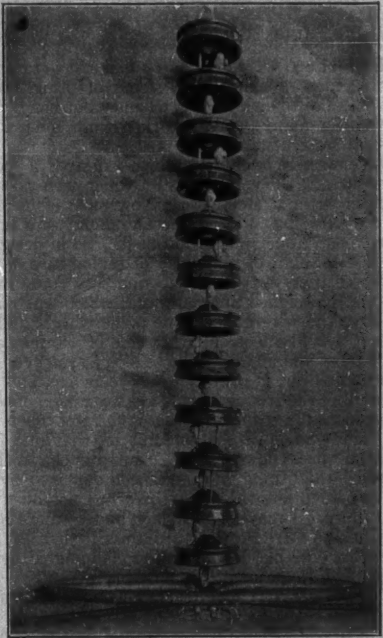

FIG. 23—ANTENNA SHIELD FOR EQUALIZING VOLTAGE ON INSULATORS

Briefly, outages due to insulator troubles will probably be less frequent at the higher voltages than at present.

The author acknowledges the assistance of Mr. W. L. Lloyd, Jr. in making this investigation.

DISCUSSION ON "FACTORS CONTROLLING THE DESIGN AND SELECTION OF SUSPENSION INSULATORS" (PEASLEE), "UNIT VOLTAGE DUTIES IN LONG SUSPENSION INSULATOR STRINGS" (RYAN AND HENLINE) AND "ELECTRICAL CHARACTERISTICS OF THE SUSPENSION INSULATOR AT THE HIGHER VOLTAGES" (PEEK), WHITE SULPHUR SPRINGS, W. VA., JULY 21, 1920.

J. B. Fisken: I am going to make the statement that this idea of shielding a string of insulators by means of an antenna shield is not new but has been in operation for many years, and then I am going to explain that statement.

Some eleven or twelve years ago I desiged a power line for 60 kv., the conductors on which were to be of aluminum without steel core. Now most of us know what the effect of an arc on an aluminum wire is. It is disastrous; and I was afraid of it. So in designing this line I thought of some way of taking care of that arc, and as there was little experience to guide me at that time, I designed an arcing rod, which was placed just above the wire in the same plane with it and extended it out five feet on either side of the insulator. It is the most abominable thing to look at you ever saw. That line was put in operation ten years ago, and notwithstanding the troubles that have been had generally with suspension insulators there has not been one single failure of a suspension unit on that line from a natural breakdown. The insulators have been damaged by lightning and have been damaged by the small boy with the rifle, but there has not been one single failure. I may be all wrong, and I want Professor Ryan and Mr. Peaslee to discuss it, if they will, and tell me whether I am right in claiming to be unwittingly the inventor of the antenna shield.

M. T. Crawford: The development of a suspension insulator that will be reliable and efficient is not only pertinent to the successful operation of present 100-kv. and future 200-kv. lines, but is of importance in connection with lower voltage lines. The cost of good pin type insulators has more than quadrupled, and we now find that the cost per mile of 55-kv. lines having heavy conductors is less with the suspension insulator form of construction than with pin type insulators. This is due to the limited mechanical strength of the pin insulator screwed on a threaded iron pin, necessitating fairly short spans. Pin insulators cemented on iron pins have additional mechanical strength but deterioration is marked due to thermal expansions and electrolytic corrosion of the iron pins. With the added mechanical strength of the suspension insulator, span lengths may be safely increased from 50 to 100 per cent and considerable saving per mile effected by the reduction in number of line supports and insulators.

A considerable number of the Hewlett bomb-and-link type three-unit strings have been in continuous service for ten years on the 55-kv. non-grounded neutral system of the Puget Sound Power and Light Company, installed at strain points where the screwed on pin insulator did not possess adequate mechanical strength. A few years ago a number of four-unit, cap-and-pin type strings were installed, and recently a number of core-and-tine type three-unit strings have been installed, both for strain purposes and suspension construction.

In the ten years operation of the bomb-and-link type units there has been no sign whatever of deterioration and the only weak point in evidence has been the loss from straight puncture of a few strings now and then where the line was subjected to a direct lightning stroke in the immediate vicinity. It is believed that the very small point of contact between the link hardware and the surface of the porcelain is a weak point in this design, although not a serious one, as it results in a concentration of potential that renders it liable to puncture under abnormal electrical conditions. The cap-and-pin type units were subject to such rapid deterioration that their use was discontinued. The core-and-tine type units have only been in service a year, and their satisfactory operation over this period is of course not conclusive.

It seems that the principal questions about the core-and-tine type are in regard to the practical possibility of making such thick pieces of porcelain non-porous and homogeneous, and the possibility of the higher voltage rating of the individual units tending to increase unduly the maximum to average duty-ratio, resulting in low flash over values from cascading.

It is of some interest in connection with the possibility of making satisfactory thick porcelain, to call attention to an installation of over 10,000 pin insulators made in 1898. They were approximately 6 in. in diameter of solid porcelain exceeding 2 in. in thickness in places. After ten years operation on 30-kv. lines some of them were placed on 13-kv. lines and as far as is known all have operated continuously without any indication of deterioration or porosity. Another batch of about 5,000 exactly similar units made several years later deteriorated so rapidly that it was necessary to replace them within a few years. This indicates that fairly thick porcelain was made non-porous and homogeneous as far back as 1898 and it should be possible now.

In Professor Ryan's paper reference is made to the undue increase in maximum to average duty-ratio and consequent cascading by the increase in rating of individual units. His investigations seem to show that the most efficient string is about 7 units on 100-kv. duty, additional units being of little value. I would like to ask why then would not a similar 5 to 7-unit string be efficient on 200-kv. duty if the size and rating of the units were correspondingly increased, and why should the tendency to cascading be any greater, assuming that proper static shields were employed in each case? Other things being equal, a reasonably short string is a decided advantage, in that it reduces the extra vertical spacing needed between wires to avoid their coming too close together when snow drops off of different wires in long spans.

E. R. Stauffacher: The Southern California Edison Company is very serious in its intention of converting its 150-kv. Big Creek line to 220 kv. within the next two or three years.

For, with the further development of our construction program in the vicinity of our present Big Creek plants, it will be necessary either to go to 220 kv. on our present transmission line, or to build new lines between the plants and Los Angeles.

Accordingly we are planning to make some investigations in the field. We have available a bank of transformers of 150,000 volts on the high side, which if connected star, will give us the potentials we would like to have for our test. The question of the ability of the transformer terminal bushings and the terminals of the windings to withstand potentials in the neighborhood of 250 kv. when grounded has been taken up with the manufacturers and we are now waiting their reply. We expect to utilize a number of miles of one of the Big Creek lines and make all kinds of field tests on the insulators that any one may be able to suggest to us.

We have made some preliminary studies in regard to the length of the insulator strings and the size of the shields which it will be possible to use on the present Big Creek Line towers when these lines are converted to 220 kv. operation.

The following notes are the results of these preliminary studies made by Mr. Michener on Mr. Barre's staff:

The string lengths used are all based on our present units which are 5.75 in. long, and our present hardware which makes a 9-unit suspension string 63 $^9/_{16}$ in. long from the center of the pin in the tower brackets to the center of the conductor. The minimum allowable clearance from tower is assumed as 4 ft. The distance between the conductors is approximately 17 ft.

1. *Dead Ends*

Any length of double strings with any size of shield may be used.

2. *Suspension*

a. On Outside Wires:

1. Any length of single string can be used where clearance from ground in center of span will permit, provided that the tie-down insulators or sufficient holding down weight is used to maintain proper clearance from tower in times of high winds.

Ten units with a 12 in. shield will reach the minimum clearance of 4 ft. from tower when insulators swing at 45 deg. from vertical.

A similar string with 15 units will reach the minimum clearance when swinging at an angle of 30 deg. from the vertical.

2. Any length of double strings arranged in an inverted V can be used where the clearance of the center of the span cannot be reduced below that which now obtains. As the string length is increased the angle of the V will increase to maintain the conductor at the same height.

With this arrangement the 10 and 11-unit strings can be equipped with 12 in. shields and still not reduce the minimum clearance below 4 ft. when, due to wind, the insulators hang at 45 deg. out of the vertical plane.

For strings of 12 or more units the shield can be increased to almost any desired dimensions without reducing the minimum clearance below 4 ft. when, due to wind, the insulators hang 45 deg. out of the vertical.

3. Two strings of units arranged in a V in a plane perpendicular to the length of the line, with the spread at the top and with the apex at the conductor. With 11 units in each string the conductor will be at the same height as at present, the angle at the apex will be about 70 deg. and a shield approximately 18-in. in diameter can be used since the insulators will not swing in the wind.

The length of these strings can be increased with a corresponding decrease in the angle at the apex. When the strings are 16 units in length, the angle at the apex will be approximately 50 deg. and a 12-in. shield can be used.

This arrangement will decrease the distance between conductors by 3.5 to 4 ft.

It may be necessary to put an extension on the end of the cross arm in order to make the angle at the apex great enough to hold the conductor from swinging under high winds.

b. On Middle Wire.

1. Any length of single string up to and including 20 units can be used, provided the conductor is rigidly tied down. In this a 24 in. shield can be used. With a 20-unit string the conductor will be lowered about 2.5 ft. and the point of support for the insulator string will be raised about 3 ft. from the positions now occupied.

2. The conductor can be maintained at the same height and the string length increased to 15 units (possibly a few more) by raising the point of support of the insulator string. In the case of the 15-unit string the top four units of the string will be between the steel members of the top of the tower with slightly more than one foot clearance between each edge of these units and the steel in a direction parallel to the length of the line. With the 15-unit string and a 12-in. shield, the insulator string can swing 20 to 30 deg. out of the vertical without reducing the clearance to less than the allowable minimum of 4 ft. This means that tie-down insulators or a holding down weight are required.

3. The conductor can be held at the same height and rigidly so it cannot swing by using two strings of insulators arranged in a V with the spread at the top and the apex at the conductor. The strings can be of 15 or 16 units in length without lowering the conductor and 22 or more units in length by lowering the conductor. With this arrangement shields of almost any size can be used.

From the above it seems certain that we shall be able to get by with 220 kv. without making any great changes in the towers. At present the V arrangement with the apex at the conductor for all three conductors seems most favorable to me.

If it would be possible for Professor Ryan to make some investigations of the voltage distribution when insulators are arranged in V or an inverted V it certainly would be of interest to us at this time.

L. C. Williams: I think it would be interesting to the members of the Institute who are not familiar with the Big Creek line to have Mr. Stauffacher give some idea of the type of construction. Will you give us an idea of the construction of that line?

E. R. Stauffacher: The Big Creek line consists of two duplicate transmission lines, approximately 240 miles long, running between our Big Creek plants No. 1 and No. 2 to the substation at Eagle Rock. The conductor size is 683,000 cir. mils of aluminum, with a steel core. There are three conductors per line approximately 17 ft. apart. They are arranged on a horizontal plane. There are nine units on our suspension towers and two strings of eleven units at each end of our dead end towers. As far as possible, we tried to design the line so it would keep out of the valleys and away from any chance of floods. We have had some disastrous experiences with what few floods that we have had in Los Angeles. The longest span I believe is approximately 3000 ft., near Sunland.

G. E. Quinan: It has been suggested in Mr. Peek's paper that the increase of porosity in porcelain is due to freezing. It has also been suggested, not in these papers but in the technical press, that the Piezo-electric effect may have something to do with the deterioration of the porcelain. The rapid, alternating electrical stress, tending to deform the micro-crystals, may very possibly be responsible for some of the deterioration that has been heretofore laid to expansion as the result of temperature. There is a great deal undoubtedly yet to be done in understanding just what is going on inside of porcelain under the effect of the electrical stress. We have also, all of us, I guess, had a good deal of experience with the effect of cement and the tendency to expand when used in pin type insulators, causing hair cracks in the insulator and ultimate puncture.

Now if the electrical men will study ceramics, or the ceramist will study electrical phenomena, we may hope to get enough of a working knowledge of the characteristics of porcelain to get a very satisfactory insulator.

W. A. Hillebrand: The fundamental characteristic of porcelain, the governing characteristic, is that the work must be formed before it is fired. In comparison with other ceramic products, for instance with glass, the whole is melted, fused to a fluid mass and then formed. With porcelain all work must keep its shape in the kiln. Now that means at once that you must have a body composition or it must be composed of materials with different melting points, some of which will not melt with the maximum firing temperature, so that your pieces will not be deformed by the time you get them out. As a result of experience we have porcelain composed of three constituents: Quartz, feldspar, clay. The quartz will melt around 1710 deg.

cent. The clay is decomposed during the firing, the water of crystallization driven off, and some of the quartz dissolved, leaving for the most part refractory sillimanite. The feldspar is the substance which melts, runs throughout the whole as a binder, as the vitrifying agent, forming a glass, holding the rest together very much after the manner of concrete, wherein the cement acts as a binder and the rock and sand as the aggregate. So it is that your molten feldspar after it is cooled is the binder and the other solid particles of quartz and metamorphosed clay, then probably sillimanite, are the aggregate and complete vitrification means that the interstices between all the aggregate, just as in a dense concrete, must be completely filled or very nearly completely filled by the feldspar glass.

Now the curve which Mr. Peaslee shows of temperature and porosity indicates that below, say about 1300 deg. cent. your ware is porous with generally intercommunicating pores, because you have not carried your temperature high enough and held it long enough for the molten feldspar to run throughout the entire mass and completely seal it together. You can carry your temperature a little bit higher and gas begins to be given off, the principal gas dissolved in the quartz being probably nitrogen, and under those conditions your ware comes out bloated or blistered with actual faults running through it. So that you have a comparatively narrow temperature range in which you will get satisfactory vitrification of the piece.

Now there is one point that has not been dwelt upon. That is, that your satisfactory vitrification depends not alone upon a proper firing temperature for the body composition taken as a whole, but also you must have and must maintain throughout the entire process of handling the clay uniformity throughout the piece. Bear in mind you have the three constituents: The quartz, feldspar and clay, of different weight, different specific gravity. They are ground together in a mill, about the size of flour; that is, a fine, powdery mass, thoroughly stirred together in the various forms of the mill, with water taken out as a fluid slip. That is run into a bag press, taken out and left in the clay cellar to soak.

Now it is extremely important during all this mixing that the whole mass be kept continually agitated, that is, as it goes through the bag press that you do not have segregations, a dropping out at some point and allowing it to settle and getting to a different grade of composition. So that it is not alone necessary to have accurate kiln control but it is most necessary to have most accurate control first of all in the selection and the production of your raw materials, and then in the handling of them in every step down to the kiln.

Then the curve which Mr. Peaslee shows is absolutely controlling in the ceramic process.

Furthermore, there is an additional factor which he did not mention. That is, up to the point of about 800 deg. cent., during which time the water crystallization in the clay is given off, and there is approximately 10 per cent. linear shrinkage in the ware, the temperature must be very uniform.

There are two vital points in the firing process: The first, that during which the water of crystallization is driven off at about 800 deg. cent. and the second the approach to the maximum firing temperature.

Referring to porosity, which is vital, I will say that with the leading procelain manufacturers porosity ceased to be a matter of importance to the engineer five years ago. I think not for five years has any appreciable amount of porous ware been put out; and while all the stress which has been laid upon the matter of porosity is probably correct, weight should nevertheless be given to methods which have been tried and proven by experience for governing this matter, and I will say that there is probably no one thing which is watched and must be watched more carefully in an insulator plant than this matter of porosity.

Coming now to the other matter discussed in the three papers under question, there are two things in the use of long insulator strings today which weigh very heavily upon the mind of the operating engineer. One is the initial corona point or the point at which corona will form around the unit next to the conductor. The other is the potential carried by that unit with the possibility of cascade action. Now personally I think that danger from cascade action with a close coupled cap and pin type union has been very much exaggerated. Many of you I know will disagree with me very strongly, but on the other hand I think there is comparatively little evidence to show that the danger from cascade action in a long string is a real one. It may be under certain atmospheric conditions. We know that there are unexplainable things taking place on lines in certain sections of the country, but until that is definitely laid to a failure by cascade action with the surface of the insulator wetted down, say by dew or fog and not by rain, I personally hold to the opinion as stated.

Now for example Professor Ryan makes a statement on page 6671 to the effect that, "The curves reveal further that an increase in string length from ten to twenty units will cause a corresponding increase in steep wave front flash-over voltage." Now I understand that he modified that to apply to 60-cycle voltage.

Now I don't know whether that statement is based upon tests or not, but figures that I have for cap and pin type units, in, for example, a 10-unit string, with dry flash-over of 480,000 and a 15-unit string, 50 per cent increase, the dry flash-over is 670,000; that is a 42 per cent increase. And the point is that as Mr. Peek has brought out, particularly the corona forming in the bottom unit, the unit next above it, will automatically grade the string so that your flash-over finally occurs at a very much relatively higher value. Furthermore, particularly in certain sections of the country, you have lines subjected to lightning. I know one case where the lightning hit in the middle of the span did not flash over the insulators on either side, although it burned down the conductor. You may have potentials up to four or five hundred thousand volts and unquestionably on that basis, those units should have gone over and yet they did not. Now for example a moderately dispersed field is I think desirable, intending to throw the arc out away from the insulator; that is, in its initial stage of formation. Mr. Peek says that a shield will in itself throw the arc away from the insulator. Under certain conditions that is possibly true. On the other hand, I think it is equally possible that due to the change in the form of the field, the fact that under certain conditions you will get the greatest stress concentration on the second unit, possibly above that, that you may actually have the arc thrown into the string.

Now I am not arguing either for or against a shielded string. The point that I do want to make is that so far as I am personally concerned, as a result of my own experience, I am not yet convinced that even for 220 kv. a shield is necessary.

On page 1671 a curve is given showing the watts lost at sixty cycles on an insulator. That curve would be rather disturbing but for the fact that I think that loss is primarily in the air about the unit, and with a very much better chance for heat dissipation. If that loss were taking place in the unit itself, it would be a matter that I would consider very serious, but under the circumstances today, under normal conditions where the heat generated can be removed, I think that is applied at the surface and can readily be taken away by air currents and that is something that I would like to hear a little bit further about from Professor Ryan.

Now as to the various types of unit that are under discussion. For example, we will take the three: The cap and pin type, the Jeffery-Dewitt and the Hewlett. I am going to rate these. That is, these are taken with just single units alone, the characteristics of the unit. I rate the cap and pin type at one hundred for corona flash-over, wet and dry. For initial corona, and that is the point just about where the corona voltage begins to be seen in the dark, the cap and pin type at one hundred; the Jeffery-Dewitt a little over two hundred; the Hewlett sixty-two; flash-over cap and pin type one hundred, Jeffery-Dewitt one hundred sixteen, Hewlett one hundred three. Wet, cap and pin type one hundred; J. D. one hundred eighty-nine, eighty-nine per cent increase in wet, the Hewlett one hundred fourteen. Dry, one hundred, one hundred ten and one hundred.

As to puncture value, the Jeffery-Dewitt is probably at least two and maybe three times that of either of the others. Now at 220 kv., with a 12-unit string of cap and pin type units, according to Professor Ryan's curves the unit next to the conductor would carry 24,400 volts. At about 21,500 sea level and on a day when the air is comparatively bad as a conductor, the first visible corona will begin to show. The stress on this unit will be 24,400. There are ten-unit strings, cap and pin type units, operating very satisfactorily at 165,000 volts on the line of the Aluminum Company in Tennessee, at elevations running from 1500 to 2500 ft. That line has been in operation for nearly a year and a half now and no trouble whatsoever reported. They have not, as far as I know, had a single flash-over.

Now 24,400 I personally do not consider a voltage at or near sea level as too high to put upon a single unit, and, as stated, I think there is little likelihood under ordinary conditions of such a string failing by cascading.

In the matter of insulator design, if you will refer to Mr. Peaslee's paper on page 1653. Now the initial corona with this unit appears, as is to be expected, in the air immediately around the pin. It spreads out until it meets the petticoat, is deflected and stopped. That is an expedient that is many years old, was called attention to at least twenty years ago. It was one of the most valuable ones in insulator designs of affording a deflecting barrier for your corona streamer, in order to prevent it running out. Now with the use of flanges, of about the proportion shown, adding something like 17 per cent dead weight over a smooth disk, you increase your dry flash-over on that account as over a smooth disk approximately 50 per cent; you increase your leakage distance about 40 per cent. Furthermore, with light thin barriers, an arc is held down away from the body of the porcelain itself, and I have seen such a unit with the flanges completely stripped off, stripped perfectly clean as though knocked off with a hammer and the body of the flange itself absolutely uninjured and sound.

Now with a unit of this description I will ask you to note that it is extremely important to have a high corona point, for the reason that the moment you get streamers started to a point like this they begin to run together. If you have them running together the distance is shortening and there is the tendency immediately to run out and grow after the familiar manner of such streamers under a 60-cycle current. Now with this spider leg as an equi-potential surface, you will note that there can be no potential difference between these two points. That means an extremely low potential in here, and the lines of field are well thrown out in that direction (indicating with reference to illustration). The maximum concentration is well down in the porcelain, away from the air and has the weaker dielectric. The result of that is you have a high corona value, but on the other hand when you do break down it floods just like that, the whole under surface of the insulator, and this section in here will be completely dark. The characteristic is entirely different from this unit, for instance (indicating). The whole portion under here is apparently quite uniformly stressed. It all goes down at once, and with the result that you have a flash-over value for this unit with approximately ninety per cent increase in weight, with about sixteen per cent or eighteen per cent in volts. Now furthermore, by reference to the curves that are presented you will find that for say a ten unit string, with about nineteen to twenty per cent of the line potential carried by the first unit, with units of the J. D. and of the Hewlett type you have approximately thirty per cent; that is about fifty per cent more voltage carried by the line unit than the other; and it would be correspondingly desirable to have a high corona point. That is secured in the J. D. unit by the design of the insulator itself. It is absolutely necessary if you are going to use a Hewlett insulator, with corona coming at about fourteen thousand volts, if you are going to use that at high voltage, to use some sort of shield as proposed by Mr. Peek or Professor Ryan.

With regard to the matter of depreciation, Mr. Peaslee mentions current rates as high as twenty per cent. For units manufactured within the past five years the records show that over a four year period the rate is about one thousandth part of a maximum; that is about a fiftieth of one per cent a year. That rate will not continue. Just what it will do we don't know.

The statement is made that porcelain, cement and metal are held together in an unyielding assembly. That also needs modification, because it is characteristic of cap and pin type units today that they are set together with a yielding medium between them at all points, and that has made a difference in comparison with units built five or six years ago, of at least three hundred to one in the rate of failure.

As to the importance of three hundred thousand volt or better puncture value, that is a matter largely for opinion. Punctures of suspension units today, that is of sound units, are extremely rare, even under lightning potentials. Providing you have a string that is free from arcing, that is, that will not fail by cascading, wherein you pile up a full line voltage upon a single unit, experience shows you do not get punctures, and, as in everything else, it is a compromise. If you insist upon a high puncture value per unit, then you have got to go to a certain design which has inherent weaknesses that will be brought out later.

Now on the matter of impulse ratio, while I will not pretend to argue either one way or the other, I think that is a matter that should be clearly understood, particularly by anyone buying or using insulators.

Now my understanding of impulse ratio is this: For example, if you take a spark gap, a needle gap, you have an extremely high dielectric flux concentration at the needle points. On sixty cycles you will have a brush or corona around the points that will grow and your voltage is raised, gradually narrowing the gap, and finally it will spill over. That is, you have a dispersed field. As you get away from it the field very rapidly drops off in intensity. Now under a steep wave front impulse coming on such a gap, you ionize the air in the immediate vicinity of the needle points. Your streamer runs out; it is an extension of the electrode that comes now at a point; you have an ionizing potential at the end of the electrode which is enabled to ionize some more, and so it grows just as it would on the sixty cycle, and on the sixty cycle there is time enough for this action to take place, for it to grow across. With a high-frequency impulse there may not be time enough for that to take place. The result is you have very much higher peak voltage required to spill over your needle gap than you do for an equal distance where the field is uniform; as, for instance, in a gap between large spheres. That leads to this general conclusion, which I think will hold: That is, that the poorer the stress of distribution the higher impulse ratio. That is, I think in general you will gain a high impulse ratio at the expense of poorer stress of distribution. I myself have made one or two experiments which tend to bear that out; and that I think is also shown by the curves which Mr. Peaslee shows, where he shows a higher impulse ratio for the Jeffery-Dewitt over the cap and pin type unit. He also shows about a fifty per cent higher stress distribution on the unit next to the conductor.

One other feature with regard to the Jeffery-Dewitt unit which should be taken into account by anyone who is considering the complete insulator situation, as the titles of these papers would lead us to infer; that is that the porcelain is used in tension. There is no objection to that whatsoever so long as your porcelain is sound. The only objection comes due to the liability of porcelain cracking with time in accordance with its inherent characteristic.

Now as Mr. Crawford has shown, and as every insulator man who is familiar with insulators to any considerable extent knows, your porcelains manufactured at different times have been extremely varied. That is true of porcelain put out by a single manufacturer. Even with the most careful process and control you will have certain variations within the product of any one manufacturer running along from year to year. Now with an insulator of this type, if the porcelain does crack your line probably comes down. With an insulator of the cap and pin type, where you have an overlap, or with the Hewlett type where you have an interlink, a crack in the porcelain does not mean a line failure, and experience has shown that in that respect, that is for the same number of units under equal conditions of service, under conditions such as obtained in lines of say one hundred thousand volts, one hundred miles or more in length, that the liability of failure from that cause must be in the ratio of about two thousand to one in favor of the Jeffery-Dewitt over the cap and pin type for the same number of service interruptions.

Now as to the possibility of failure. Mr. Peek has made the statement that after ten years or more of service, insulators of the Hewlett type show no depreciation from cracking. Mr. Crawford from his experience, has supported that very definitely. On the other hand, I know of one line in which the Hewlett insulators were installed about ten years ago, wherein they are cracking today at the rate of about one per cent a year. That is, the porcelain is falling off from the central bomb. Now that again is very likely, or quite possibly due to the different characteristics of the ware as put out in the two lots of insulators, but until the thing has been demonstrated over a sufficiently long period of years to settle absolutely, there is always that question of uncertainty. A very large number of tests may be made upon a single piece, that show externally no depreciation whatever; on the hundred and twenty-fifth it may go to pieces like that (speaker snapping his finger); go like a pistol shot, without any signs of failure whatsoever; and that is a matter of experience and a matter also of laboratorial experiment. And so while the tests given by Mr. Peaslee with regard to the temperature cycles on page 1662 are interesting and are all very valuable, I personally could not consider them conclusive until sufficient time has elapsed, as he himself has said, to settle this question.

And as to the insulator situation, as to what type should be used, that, no man at present is in a position to say. We are extremely fortunate in having had practically all the data with regard to voltage distribution obtained for us after very long, laborious and painstaking effort, because it settles a vexatious question, settles once for all just what the values are, what to expect, and enables us to talk in concrete terms and to go ahead upon some definite, rational basis, instead of one of conjecture; and there are enough units of the various types now in service so that within a reasonable time, much less than the span of life that most of us look forward to, why we will have very definite information.

J. B. Fisken: There is one question that I would like to see eliminated if it is possible to eliminate it. It is the question of the depreciation of porcelain. I don't know how many years experience are necessary to convince us that porcelain can be made that does not depreciate. Personally I believe it can and has been done. My experience is gained from a line that has been in operation now for seventeen years. The insulators on that line are standing up. It was operated for about six months or a year at forty-five kilovolts, and since then is being operated at sixty kilovolts. Of course these insulators are such that none of us would accept them today, because they are narled and blistered and uneven and they would not pass a physical inspection, but they are functioning. That is the main thing. Some of those insulators a few years ago were sent east to be tested, they were put through the regular tests and stood up as high as when they were originally tested back about—I think they were purchased in 1901. Now it is a question of the length of time that must elapse before it is possible to say that there is no such thing as depreciation of porcelain, and I would like to hear the authors of the three papers, or at least the two that are present discuss that a little in their reply. Personally I am satisfied in my own mind that such a thing as depreciation or deterioration of porcelain does not exist, and that that can be eliminated from the discussion entirely.

K. A. Hawley: I am going to talk both as a user of insulators and then as a manufacturer's representative. In 1913, I helped to buy the insulators for the Norfolk and Western, and for the Pennsylvania electrifications. At that time, we had trouble to get the manufacturers to offer a satisfactory puncture value for the insulators; that is, punctured under oil. The specification written then called for an instantaneous test of 120 kv. Late in 1914, we purchased a third lot, and at that time we had to change our specification to read only 115 kv. of puncture value. Now at the Locke Insulator Factory, we are using in our standard specification, a test for puncture starting at ten per cent below flash-over, 70,000 volts. We apply the voltage for thirty seconds; then we raise the voltage ten thousand volts and continue that at thirty seconds, and so on by steps of ten thousand volts every thirty seconds until it punctures. We are now regularly reaching 160 kv., and a fair proportion of the porcelain goes to 170 kv. Now as against the old specification made in 1913, this test can be considered equivalent to over 100 kv. volts instantaneous application, very nearly a one hundred per cent increase in porcelain puncture value.

As to porosity, we have subjected samples that our inspectors have picked out as being along the lower limits of commercially practical porcelain to 13,000 lb. per sq. in. pressure in fuchsine dye and alcohol without any penetration that we can observe under the microscope. This indicates the improvement that has been made in porcelain within our factory in the past five years.

We all know of lots of porcelain, such as Mr. Fisken just told us about, that have stood almost with one hundred per cent record over a great number of years. It would be a very sorry comment on the modern scientists if they could not find what that old porcelain is. We feel very confident that we know what it is and can regularly commercially supply that porcelain. In the assembly of a cap and pin type insulator, expansion troubles and so on, we believe are thoroughly cared for by the proper expansion joints. So that we have no fear in the future of a modern cap and pin type insulator.

L. C. Williams: There is one point in the manufacture of porcelain, which is purely a ceramic process, and which I believe to be of a great deal of value, and which the operating engineers particularly may not appreciate and that is the point of drying the clay before it is fired; and that point in itself will set up stresses in the porcelain which are only aggravated by the firing. In watching the firing of porcelain the greatest care is taken to watch the uniformity of fire, not only from its start but at the point of soaking, which is of vital importance, and, as I said, increases or aggravates any stresses which may be set up in the clay in the drying rooms. That is purely a ceramic consideration, and I think that possibly Mr. Fisken's remarks as to the operation of the insulators which he had in service can be accounted for very much on the same theory as he accounts for his arcing horns. Manufacturers ten and fifteen years ago made porcelain which has undoubtedly shown remarkable operating success. On the contrary, from operating experience, I have known manufacturers ten years ago which made porcelain which would fly into a million pieces if you gave them a hard look. The obvious conclusion to come to in our present day practise is that we have to find out and standardize those practises which made good porcelain and eliminate those causes which made bad porcelain. In other words, the performance of insulators ten years ago is no criterion of the performance of the present insulator which we are going to get today, regardless of whether it is a bomb-and-link type, a cap-and-pin type, or the core-and-tine-type. It is perhaps fortunate, that we have more operating data on the cap-and-pin type insulator than we have on the other types, and as a result we are able to forecast more accurately the probable operating results of the cap-and-pin type insulator than we are of the other types. Time alone is going to tell whether or not the cap-and-pin type insulator as it is manufactured today is going to do equal work or better work or worse work than the other types. It resolves itself merely into a question of whether the operating engineer is going to take the newer types and carry on his own experiments or whether he is going to benefit by the experience of the operating engineers who have had long experience in using the cap-and-pin type insulator.

S. C. Lindsay: There is one thing that seems to me to need investigation, and that is the effect of vibration of these insulators under tension. I am of the opinion that you are going to have a great deal of difficulty in getting anything in the nature of porcelain to stand up over a long term of years when it is used in the form of a cap-and-pin type insulator and is subjected to mechanical stresses and vibrations at the same time. I think there will be a definite length of life to those insulators with any improvement in the manufacture of porcelain that can be made, on account of vibrations. I believe the next most fruitful field for investigation is that of vibration under operating conditions.

L. Lauridsen: One of the 60-kv., 60-cycle transmission lines of the Portland Railway, Light and Power Company is of steel tower construction and equipped with suspension insulators. The line is about thirty miles in length and was put in operation about 1912. Within a year after the line was put in operation, line failures began to occur; and by the early part of 1914, the failures occurred with a frequency of about one a month. It was then decided to test the insulators of the line. This was done by means of a megger with a 1000-volt generator and two thousand megohms as its highest finite scale reading. Each insulator unit was tested from pin to cap. About fifteen per cent of the entire number of insulator units on the line was found to test below 2000 megohms. After the completion of the test, these defective units were all replaced. During the following year, 1915, the line was again tested, and about five per cent of the number of units was found to test below 2000 megohms, but none were replaced that year. A third test of the line was made during 1916, which showed that the number of defective units had increased from five per cent to eight per cent. About three per cent of the total number of units tested below 100 megohms, and these were replaced. At the beginning of 1917, there were, therefore, about five per cent of the total number of units on the line which tested below 2000 megohms. Due to war conditions and increased line loads during 1917, 1918 and 1919, the line could not be spared for making insulator tests during that period; and although no insulator replacements had been made since 1916, line failures did not begin to occur until the latter part of 1919. During the early part of 1920, the line failures became more numerous, and it became absolutely necessary to have the defective units replaced. A megger test of the entire line was therefore made which showed that the number of insulator units testing below 2000 megohms had increased from about 5 per cent in 1917 to 11.4 per cent in 1920. Only the units testing below 100 megohms were removed. These amounted to 7.7 per cent, leaving 3.7 per cent defective units, testing below 2000 megohms, at present in service on the line. Our tests seem to show that there is a gradual deterioration taking place in the insulators. An insulator unit, which one year would test a definite value between 2000 and 100 megohms, and which was permitted to remain on the line, would invariably show a lower resistance at the following test. Whether this apparent depreciation is due to original defects in the porcelain, or to defects acquired in service is a matter for investigation. In our experience we find that the number of failures is about five per cent greater in the units in strain position than those in suspension.

The gentleman who just spoke mentioned vibration as a possible cause of insulator failures. The only vibration on our line is that due to windage, and I doubt that this is of sufficient severity to be a factor in the number of failures on our line.

We consider the megger a useful instrument in weeding out defective insulator units in service on the line, and it is our plan to test our lines each year and replace all units testing less than 100 megohms.

A. C. Pratt: We have had the usual experience I think that all companies have had. Our early insulators ran six per cent a year. Our insulators since 1914 are showing very good, probably one half per cent up to date. We have 10,000-volt insulators twenty years old that are still failing. Answering Mr. Fisken's question to some extent, they are now failing faster than ever.

C. P. Osborne: I notice that a lot of insulators have small hair cracks around the cap where the metal joins the porcelain. This is not noticeable except under a microscope. But in breaking the insulators for inspection and putting ink on them, you can find these little hair cracks through the glaze. I am wondering if that is caused from corona or is it from poor porcelain or from deterioration. We are very anxious to find out what the reason for that is. I thought perhaps by getting some information from Mr. Peaslee and Professor Ryan, who are able to tell us these things, we could be enlightened on that point.

D. W. Proebstel: In Alaska, in 1912, I placed in operation a 22 kv. transmission line, only 15 miles in length, and within less than a year after the line was in operation I had no less than 20 failures of insulators; not so much from the insulator itself as from the pin supporting the insulator. The pin that was used was an oak pin; and upon examining the pins that had failed I found that the top of each had been disintegrated until it was about the consistency of wet chalk. I did not know what the cause was at that time but I supposed it was due to the salt air. Since that time I have come to a different conclusion. My theory is that the corona set up around the insulator, liberated ozone, which is a molecule of three parts of oxygen, combined with the nitrogen, forming nitrogen tetroxide. This was absorbed by the moisture which was on the insulator pin itself, or in the insulator pin and formed nitric acid which disintegrated the pin, causing the failures. Now the question I am going to ask the authors is this: Isn't there a probable source of mechanical failure of insulators due to a forming of nitric acid in and about the metal part of insulators, causing electrolytic action and corrosion and increasing the strains and stresses about the mechanical contact of the metal with the porcelain? And further, is it not likely that the chemical action there would cause absorption of water by the porcelain, resulting in failures?

J. C. Clark: Are Figs. 1-a and 1-b of Mr. Peaslee's paper drawn from data actually obtained in tests, or are they made up from a knowledge of the relative permittivities of porcelain and air?

I find it difficult to reconcile the curves shown in Fig. 5 for "conventional" and "rational" designs of insulator with the voltage distribution given for these designs in Figs. 1-a and 1-b respectively. The improved distribution shown in Fig. 1-b would lead me to expect a *lower* impulse ratio for the "rational" design rather than the *higher* one which is given in Fig. 5.

Is it really desirable or not to have a high impulse ratio on an insulator string? I believe that this question cannot be answered without a full appreciation of all the phenomena arising on a high-voltage transmission system. Involved in the proper answer are the subjects of the ability of the porcelain to withstand arc-overs, and the design of line hardware and accessories such as arcing-horns and shields.

Reference has been made to the use of the megger for periodically weeding out defective units. I believe the megger commonly used is the Evershed instrument containing a small 1000-volt generator and a scale reading a maximum of either 2000 megohms or 5000 megohms. I wish to point out that there is great need for an improved megger to be used in this work. Let us remember that the resistance of the conventional cap-and-pin type porcelain suspension insulator in good condition is of the order of 500,000 to 1,000,000 megohms. Assuming that a 5000-megohm megger is used, we see, therefore, that a unit which is bad enough to give a reading at the upper end of the scale is already down in resistance to say, ½ of 1 per cent of its normal value. Thus the amazing fact is developed that if it is not quite so bad, but perhaps down to 7500 megohms or ¾ of 1 per cent of its normal resistance, the megger operator accepts it as a unit of "infinite" resistance. This means that every periodic weeding-out employing this instrument probably passes many very bad insulator units which may require only a few more weeks time to become potential sources of serious line trouble. Furthermore, it has become fairly well established that there is a "coherer effect" in such insulating materials as porcelain which manifests itself in resistance measurements by making the resistance a function of the voltage used, so that the resistance decreases greatly with increasing voltages up to a point beyond which the resistance is constant during further increase of voltage up to the point of actual failure of the dielectric. I believe it has not yet been established whether or not the voltage employed in the Evershed megger is high enough to give resistance values of the order of those generally obtaining during the normal operation of the insulator on the line.

I believe that the time is again opportune for the Institute to develop a new set of insulator specifications in the way in which such a set was attempted about seven years ago. I think that the repetition of this attempt would bring at least the valuable result of eliciting important contributions from the operating engineers. The resulting discussion would be most helpful in clarifying the ideas now prevailing regarding proper insulator specifications and testing.

In this connection, it is important to note that there are apparently coming into existence two schools of thought regarding the conduct of high-voltage insulator testing. One of these schools seems to believe that there is grave danger of using too high a testing voltage, or of applying a high testing voltage for too long a time, with the result that good material suffers permanent injury. It is alleged that, if a test of such nature be repeated again and again, a certain small percentage of failures will appear *each time,* so that the ultimate result would be the destruction of *all* the insulators. The other school apparently believes that weeding out of defective material can be most successfully accomplished by the application of a voltage limited only by the dry flash-over of the units under test. This flash-over voltage would be applied to all units for an indefinitely long time (with intervals frequent enough and of sufficient length to prevent much heating of the porcelain due to flash-over) until no more failures within a certain definite time just before the close of the test period. This second school would contend that all of the material passing the test would withstand its repetition indefinitely, and would deny that good porcelain is ever injured by strenuous high-voltage testing.

I think that all are agreed that the first requisite of a suspension insulator is that it support the line under all conditions of operation. It should not part by pulling or breaking or burning in any case. This is of great importance.

It is evident that there is still room for improvement in the so-called bomb-and-link type. Mr. Hillebrand mentioned units of this type which are failing regularly by spalling off. Can you state whether a copper, or a steel link is used, Mr. Hillebrand?

Mr. Hillebrand: That I cannot tell. The operators of the line gave me the information about the porcelain, but that particular thing I did not get.

Mr. Clark: That point seems important because it was soon found when steel link connectors were used that the porcelain spalled off, but this trouble ceased immediately when insulators of identical make were linked up with copper.

Mr. Hillebrand: One of the most interesting and significant things in regard to that experience is that the trouble did not begin until that insulator had been in use about eight years.

Mr. Clark: Such experience as I had learned of with these units developed that when the steel link was used, the insulators developed either mechanical failure, *i. e.*, spalling, at a point where the relatively unyielding steel connector developed highly concentrated stress on the porcelain surface, or else the unit punctured electrically there. This trouble ceased immediately, however, when copper was substituted for steel.

Concerning the relation of electrical stress and porcelain deterioration, I believe Professor Ryan can tell us of some very illuminating facts which he has gathered as the result of testing of insulators which had been stored under differing conditions.

Of course, all recognize the difficulty of designing a successful cemented-type porcelain suspension unit. Great care must be taken to avoid injury to the porcelain due to difference in temperature coefficients of expansion, or to the improper arrangement of parts so that heavy loads cannot be safely carried, or to the use of improper cement. Some engineers are pessimistic regarding even the possibility of attainment of a successful design. I have hitherto felt that there can be no such thing as deterioration of porcelain considered by itself. If it is true that deterioration takes place in the porcelain itself more or less independently of the other parts of the insulator, it would be of the greatest value to possess a fuller fundamental knowledge of the material as soon as possible.

H. H. Schoolfield: In line with Mr. Clark's remarks I think as we have representatives of the insulator manufacturers with us today in such numbers and such a variety, it would be a good time to put in a word regarding uniform rating of insulators. In looking over some of the catalogs of the different manufacturers we find that two insulators, especially of the pin type, which are practically identical in design, have entirely different ratings of wet flash-over dry flash-over and recommended line voltage. The question was brought to my attention the other day, in looking over the table of wet and dry flash-over values given the proposed electrical safety code by the Bureau of Standards. I found that pin type insulators we have been using on our 66-kv. lines for a number of years, and which have given us practically no trouble, would not come up to the Bureau of Standards' requirements in regard to wet flash-over and dry flash-over, if we took the manufacturers' catalog rating as a basis. Another manufacturer with an identical insulator, using catalog figures would comply with the Bureau of Standards. I understand this difference is due principally to the method of making the tests. Some manufacturers use the needle gap, some the sphere gap, and I think it would be well for the Institute to work along lines toward having the manufacturers get together and adopt some uniform method of testing and rating of their insulators.

W. D. A. Peaslee: With regard to Professor Ryan's remarks in his paper as to the advisability or possibility of an insulator unit having a higher corona forming voltage than at present in general practise, it must not be forgotten that there are other features of design which must be taken into consideration. The production of a unit with a higher corona forming voltage would not be of such great value unless the flash-over voltage of the unit were raised at the same time. In the production of such a unit the capacitance of the unit and the capacitance of the hardware connecting the units together to ground, is another feature that comes in, and a variation in those capacitances will do things to the shape of your flash-over and efficiency curves that are rather surprising. That is one of the fields that is under careful investigation at the present time, to discover just exactly what effect certain changes in those capacitances will have, and to what extent they can be controlled and reduced to analytical laws.

The statement came up several times in the discussion as to the unreliability of porcelain in tension. Porcelain in tension, or porcelain in general has somewhat the same characteristics in a stress-strain diagram as cast iron. A somewhat distinct lack of yield points; that is, the yield point and rupture point coincide, and the tensile strength is very much lower than the compression strength. At the same time, in structural work we find the cast iron employed quite commonly in tension.

We have made a great many tests of porcelain in compression and tension, and I will introduce at this point the question of vibration that was brought up, and we have found no evidences, nor have I seen test results that give us evidence that porcelain, either in tension or compression, is injured by either repeated stressing or by vibration, which is the same thing, provided that at no point in the porcelain do your stresses exceed the critical value at which rupture occurs. We have had pieces of porcelain under tension and under compression with shocks applied to them at a rate of several per second, totaling into the millions of shocks, in which as near as could be determined the stresses were within eighty per cent of the strength of the procelain; and we have been able to find no difference in the porcelain after that treatment than before. But it must be borne in mind that the total stress in porcelain, whether in tension or compression, is the vector sum of the initial stresses in your porcelain and the applied stresses. Now that subject has been investigated I know in more than one laboratory very carefully.

Mr. Williams brought out very well the question of drying. We claim that practically no defects or cracks occur in the firing of porcelain that have not been borne somewhere else; and that a piece of clay or a porcelain body brought to the furnace, without initial stresses, laminations or strata, and of a shape that will shrink—and remember that all porcelain in firing changes in volume— and change in volume without producing stresses, will go through the furnace properly, and while it is extremely necessary to control very closely your furnace operations it is just as necessary to control every other operation of your plant. In showing you that, I am going to tell you a little bit of the process we employ. From the time the raw material comes into the plant it is under the control of two distinct departments, one of which is production control test, and the other of which is manufacture; and no part of the raw materials, from the time they come to the plant, can leave a process until they have received the approval of the production control test. Samples are taken at every part of the processes from one end to the other, and before the body is permitted to be fired it is fired through a shortened cycle in our laboratory kilns; if it passes those tests and is approved by the laboratory it is allowed to go to production. Now we have a great deal of trouble in our experience with this question of drying, and we have obtained some rather interesting data. Porcelain used to be dried from the outside in, by being placed in a warm atmosphere, practically zero humidity and heated up and dried. If you dried the outside skin on this body, then you dried another one, and another one, and another one, those layers contracting in different ways and at different times, bringing a body to the kiln that had stresses set up in it. That can be shown by taking a drill and cutting little cylinders out of the body in different places. If you fire those cylinders and they bend, you have initial stresses. When you take a piece of porcelain and dry it so no matter where you drill a cylinder out of it and fire that cylinder you can't get a distortion of the cylinder, you have pretty nearly eliminated the drying stresses.

Drying is now done I think in most porcelain factories by drying from the inside out. The wet body is placed in a drier in atmosphere practically one hundred per cent humidity, the temperature brought up and the humidity is gradually reduced automatically, and we find a rather startling result. With 20 per cent humidity you can get a bone dry clay. And not only that, you don't get a drying in laminations and layers, but it dries so far as we can determine from the inside out and uniformly. And I will say for ourselves that the bending and cracking trouble in our kiln was reduced about 50 per cent immediately upon placing that drier in operation. And that was before we learned to use the drier.

Mr. Williams' remarks were very much in order. There is not a process in the manufacture of porcelain that can be neglected. You have to watch it all the time, and you are confronted with operating a factory with labor and the human element entering into the situation. An amusing little incident occurred on our night shift on the controlling of our kilns. We were having a lot of trouble. Everything was right; the written records came in fine. We put some automatic recorders on various places and found out that we busted up several fine crap games about two o'clock in the morning; so we saved lot a of insulators we had been losing. You have to watch, not only your raw material and the processes, but the human people in your factory; and the consequences of a very slight slip are very, very serious to the manufacturer of porcelain. The raw material must be watched. It is rather a startling thing to see the variation in iron, that your magnetic separator takes out of your raw materials, even when they come from the same mine at different times.

Now in regard to testing, it is true there are two schools of thought arising in testing, and I believe that both are right. I believe the testing of a certain proportion of your product to destruction at all times to keep your probability curve in a factory, to establish it and know it, is necessary; but I also believe emphatically that every unit that comes out of a factory should receive a mechanical test, and after that should receive an electrical flash-over test. A unit that will not stand an electrical flash-over should not be put on any transmission line, I don't care where it is. Furthermore, I do not agree with the people who say that testing may injure a good insulator. I venture to say that any insulator that is good originally can stand almost an infinite amount of testing without injury. I will lay the cards on the table. I have taken one of Mr. Hillebrand's units and put it for 100 hours under 200,000 cycle flash-over, and it is a good unit, Mr. Hillebrand, yet. I have done the same thing with others. And I maintain that that does not injure the insulator if it is made right. But if it is not made right it will injure it.

In our test, after the mechanical load is applied they go to the electrical testing machine to receive a two-minute flash-over at 200,000 cycles; and we have found that every unit that punctured on that 200,000 cycle test, if we broke it open carefully we found a bleb in it somewhere, and that 200,000-cycle test will get that; and if it is right, no matter whether the cap and pin type, Jeffery-Dewitt, or what it is, it will stand that electrical test, and I don't agree with the people who say it will hurt it at all. If it is made right in the first place it will be right.

The same question comes up with deterioration. We have had units under hundreds of hours of 200-cycle test; we have had them under vibration, and we have had them under combined tension and high-frequency tests, until we are perfectly satisfied that if the factory can produce in the first place the right porcelain, and if the porcelain and hardware are assembled in such a way that the differences of expansion do not make any difference, I don't believe that the combined mechanical and electrical test has any harmful effect upon the unit. I have been unable to find any cases of that. We have eliminated the 60-cycle test, for the simple reason that if we take 50,000 insulators, test them to flash-over, we will have a certain amount of rejections, and we go over to the 200,000 cycle lastly and get a few more. But we have been unable, with any uniformity, to reverse that situation. If we test clear through with the 200,000 cycle first, in very, very rare instances have we been able to puncture one under flash-over with a 60-cycle test. I am emphatically in favor of a test, mechanical and electrical, on every unit that goes out of a factory, to insure that it is right. For our own purposes, we take the tests to destruction on a certain number of units to keep control of the general trend of our factory. We keep control as closely as we can on our processes, but the destruction test will give you a general over all view of what your factory is doing and how well you are living up to your standard.

In regard to the piezo-electric effect, I think I started something when I made that suggestion some time ago. We have been making some investigations on that, and also on the fact that porcelain is slightly soluble in water, and we are carrying out some tests at the present time, using 10,000 pounds pressure and porcelain that we know is porous, forcing water into and out of the pores of that porcelain a great many times and trying to discover if there is any apparent increase in the porosity. The tests so far have been rather disappointing, but it is a very difficult thing to get hold of. The establishment of a quantitative porosity test in porosities that are in very small fractions of one per cent, very small decimals of one per cent, is a rather difficult proposition. Qualitatively you can show it; quantitatively I have not yet been able to do it.

With regard to the cascading, I agree with Mr. Hillebrand to a large extent on that subject. I don't believe we are going to run into a great deal of difficulty with cascading. An insulator on the line, as Mr. Peek has said, is automatically graded when the voltage comes up, and I have yet to see a string of insulators brought up to voltage flash-over as a string that appeared to start as a cascade. I have seen them brought up, have brought them up myself and have had the end unit flash-over without flashing over the whole string. And I am simply unconvinced at the present time that the cascading is a dangerous thing.

With regard to the question of the design of the Jeffery-Dewitt insulator to have a high corona forming voltage, the design of that insulator is a compromise on a dozen or so different requirements. The weight and thickness of the insulator are not used directly to secure a high corona forming voltage; they are used to secure a high puncture value, a rugged strength to resist the heat of power arcs and the general shape that is desirable between electrodes.

Now the point was brought up as to why this unit shows a high impulse ratio when it should show a low, and my opinion on that is, that it is a function of two effects, and I have made tests that have indicated to a certain extent that that is the case. The hardware on those units is galvanized and covered with small needle points to a certain extent, as is all galvanized hardware, and although the spider is large and at first sight looks like a large sphere, it has sharp edges on the sides of the legs and sharp points of galvanizing. Furthermore, the electrodes are so far apart that the flux density in the air at the surface of the electrode is different from what it is in the concentrated type, and I have been able to take a spider leg, fill it in between the spider legs with alloy and polish it carefully and reduce the impulse ratio to a very small decimal over one, as would be expected from a sphere gap. But with the construction used, with the fine points and the galvanizing in existence and the sharp edges of the legs, we have the impulse ratio shown. That impulse ratio curve was a test curve and was not made by our laboratory but by an outside laboratory to confirm our own figures. So we know that we do get it, and when I found we did get it I had something of the feeling of Mr. Hillebrand and wanted to know why, and began experimenting to find out, and I believe that is the answer.

With regard to the depreciation of porcelain, I agree with Mr. Fisken. There is no depreciation in good porcelain. I have some microscope slides of some of the first porcelain I guess that was made in history. It is awfully good porcelain. An insulator is not only porcelain, and I maintain there can be depreciation of insulators even though there is not a depreciation of porcelain; and I think one of our difficulties in getting together on it is that we are not thinking about the same thing. When we talk of insulator depreciation we are speaking in general of an insulator unit, and when we speak of porcelain depreciation as porcelain depreciation, I agree with Mr. Fisken, there is no such animal if it is properly made.

Now with regard to the meggar testing, I have done some work with the electron tube as a meggar, using around 80,000 volts direct current. I had a bright idea that I was going to measure the electrical resistance of insulators and throw out everything below a certain resistance. I had it fine. But the trouble was I took the insulators that would measure five million megohms when they came out of a kiln, throw them out in the back yard for three weeks and they would measure about thirty megohms; under proper treatment they would come up around five million megohms, and you didn't know where you were; whereas a unit that only measured about a half million megohms would measure a half million megohms under all conditions, no matter what was done to it. I don't believe that the meggar test is good for anything except to show that a thing is so bad it ought to be taken out and scrapped immediately. If the insulator shows a reading on the meggar I would not want it on a line of mine, because it is so bad that it is in danger.

The mechanism of failure of a porous insulator is something that I don't believe is very well known. The negative temperature characteristic of porcelain I think enters into it to a considerable degree. You will have a small conducting path in your porcelain; it will become water-logged and begin to conduct; the temperature will rise, and as soon as the temperature of the porcelain rises a little bit locally you have a much better conductor and more current flows and your heating effect is a function of the square of the current, and your current is a function of the resistance of the path, and it does not take very long after your porcelain gets to a certain temperature to go pretty quickly. We have tested that out pretty thoroughly in electric furnaces with porcelain puncturing at different temperatures in electric furnaces, and the knee of the curve is pretty steep. When you get with a given body to about a certain temperature it goes pretty quick. The porous porcelain that is in existence today will be weeded out in time; and, as Mr. Hillebrand says with reference to the porcelain made in the last four years, I believe it is within probable human ability to produce a uniform product, non-porous;

but we have a great many hundred thousand insulators out yet that will have to go through a cycle to get rid of that.

I was very glad to hear Mr. Hawley's statement as to the improvement that they have been able to make in the puncture voltage. The question of how great a puncture voltage a unit should have is a question largely of the judgment of the engineer. We don't believe that a unit should have less than 250,000 volts puncture. A great many people don't agree with us. The only thing that will answer that question is the next twenty years of operation on transmission lines. I think practically all the porcelain manufacturers today are making good non-porous porcelain, practically the same quality, and from now on, excluding a lot of these old insulators that Mr. Fisken speaks of that are out on the line, we have probably an even start. The next twenty years ought to tell pretty closely whether it is a question of porcelain or design. The porcelain manufacture has developed from a black art or science, in which everything was mystery and secret process, to a scientifically controlled process, and I think that is true of every factory in the United States of any size. Therefore we are all on an even basis now in the manufacture of good porcelain. The next few years are going to begin to give us some information as to whether or not those that have perfectly honest differences of opinion regarding the design are all wrong or all right, or part of us are wrong, or where we stand, and while the laboratory tests are of great value and help us a lot, it is pretty generally admitted that the one thing that will answer the question is service out on the line and what happens.

With regard to the failure of insulators in a strained position, I believe that the results are a little confusing in that respect, for the reason that they refer to old insulators. I do not believe, and Mr. Hillebrand may have some information on this subject, that the deterioration or failure of insulators in strained positions is any greater than the suspension, when we consider the insulators made in the last four years. They are making good porcelain now. The cap and pin type insulator have a yielding layer between their cement, steel and porcelain, and they will stand expansion and contraction. With your string in a horizontal position none of the insulators are shadowed by the insulators above them. On a frosty morning when the sun hits the top unit of the suspension string the units below that unit are to a certain extent in the shadow and are allowed to come up to temperature more slowly, but in the horizontal position the sum will strike them all, and I believe it is simply a question of the theory of probability, that there are more exposed to being hit and there will be more that will show up with failure. But I have noticed pretty carefully the reports of operating men, and so far as I can see from the meager returns yet in, the insulators made in the last four years are not showing that wide difference in failure between the suspension and strain stress.

With regard to the immersion test, I will be perfectly frank with you. I don't regard that as in any way conclusive except as the megger test is. An insulator that will not stand the immersion test from freezing and boiling water I don't think is much good, but I don't know many made up in the last three years that won't stand it. If they don't they should not be considered. But if they should stand it, there is not much of any way of saying which is the best of all. Now I know, to be perfectly frank with you I have had cap and pin type insulators under fifty or sixty immersions in boiling and freezing water, and I have had many of our own. A few have failed and the others have gone on. But it is an indication that they will stand forty or fifty of those immersions, which are more severe generally than the climatic changes; and if an insulator appears that won't stand that I would be pretty suspicious of the insulator.

I don't believe that the corona forming nitric acid in the air is anything we have to worry ourselves very badly about. I have never seen any indications on an insulator yet of that action. We have in certain places some cases of insulators on railway construction wherein the locomotives stop under the insulator string and sulphuric acid forms on the cold insulator, which has given some trouble due to corrosion in galvanized fittings. If there was very much nitric acid formed by corona you would get that pretty positive. The fact that we haven't run into that very much makes me believe that it is not an important feature in the design.

Regarding the mortality tables of insulators—that study is being made for a rather different purpose, I believe—I don't think I brought out very clearly just what the purpose of that is; but what we are trying to find out is how long an insulator stays up on a line regardless of why it comes off. Now we have some very valuable data acquired already regarding poles, and the data were prepared in this way; we wanted to know how long a pole stayed up no matter whether it was taken down by the local councilmen or an automobile ran into it, or they changed the pavement and had to move it, or whether they got a joint pole committee and had to put up a bigger pole. The data we want are, as follows: When an insulator fails we want to know two things,—how long that insulator has been out and how many insulators there are on the line of the same age as that insulator. That is the fundamental data we will ask of you. Eventually those tables will give you, as the data becomes more complete, how long an insulator will live, regardless of why; and in the question of financing new projects and studying the cost of maintaining lines, that information is of a great value. In fact, it is quite a serious thought among large corporations at the present time to write insurance on certain features of their line construction. They haven't got the insulator yet on the basis of these mortality tables which have already been developed to a rather large degree. As insulator manufacturers we are very much interested n the operating man's troubles, but one difficulty we find with the operating man is that he tells us his troubles without the history. He will take you into his office and will say, "Look at that insulator. It has got a hole in it you can stick your fist through." "Fine! How did it happen?" "I don't know. It was taken off of the line." "How long had it been up?" "I don't know." "When did you get it?" "I don't know. It was bought some time ago." You don't know where it was, you don't know the exposure to the sun, you don't know the temperature ranges it was subjected to, and he doesn't. He has not been keeping those records, and as a consequence we get vast volumes of data which we do not dare to use for the simple reason we can't form any conclusion because we don't know the history. If the operating men will only watch their insulators and the history of those insulators, and when an insulator fails, or any of them for any reason fail, if we can know when it failed and the history of the insulator it will be of the greatest value to us as manufacturers. I want to urge on all the operating men here to get their records in that kind of shape and let us have them, and the more we get of that kind of information the faster we can bring this insulator situation to a point where it is satisfactory and the greater progress that we will make.

S. C. Lindsay: Mr. Peaslee, before you close I would like to ask a couple of questions regarding those vibration tests Now in the first place on what types of insulators did you make those tests?

W. D. Peaslee: I made those tests on three makes the cap and pin units, our own units, and the Hewlett.

S. C. Lindsay: You said the vibration tests ran into the millions. Did those millions approximate the number of millions of vibrations they would be likely to experience over a period of eight or ten years of service?

W. D. Peaslee: Well, that is a little hard to say. We had a rig set up with a string of insulators on it and a cam raising and dropping a weight on it, and some of the tests we ran as high as eleven to twelve million repetitions, shown by a revolution counter; some were only four or five millions; and after those tests we could find no change in the average tensile strength of the units. Not only that, we have taken porcelain and made up special test pieces, placed them in tension and placed them in compression and run up into ten and twelve and thirteen million impacts on those, and in that case we had the stresses absolutely controlled and we knew to what value we were stressing the porcelain. Naturally in the insulator we had no idea of exactly the value of the maximum stress in the porcelain, and in all of our cases we have found an absence of results that indicate that there is. And furthermore, from the standpoint of theory, I am unable to see any reason why it should be so. A piece of porcelain has the strength, we will say, of 5000 pounds in a given area. If you stress that porcelain to 2000 pounds and bring it back, you can stress it as many times as you wish, you won't get any permanent set. There seems to be no molecular or other kind of flow or motion in it. So as long as you keep the stresses below the rupturing stress of the porcelain, how can there be any deterioration?

S. C. Lindsay: There can't. That is just what I was bringing out. After that it went through several hundred thousand million vibrations probably, over a period of ten years that those stresses were set up by the vibration beyond the limit of the porcelain to stand up.

W. D. Peaslee: Well, it will break at the first vibration that goes above the ultimate strength. Those stresses are not cumulative. The porcelain is elastic within its strength and those stresses don't add up. If you have got a permanent set you may expect an addition of stresses, but without permanent set and with porcelain of the vitreous character that it is—and I have never seen melted feldspar crystallized by vibration; and under the microscope this porcelain that had been through several million cycles showed no difference in structure from what it had before it was subjected to test. So I can't see any reason that, as long as the stresses are kept within the critical strength, there would be any weakening in strength due to repeated load. And that is all the vibration is anyway: It is the repeated load, very rapidly applied.

S. C. Lindsay: Your conclusion then is that there is nothing to this theory that vibration ultimately breaks down the insulator?

W. D. Peaslee: No, I don't believe there is. I can't accept that as a theory at all.

There is one question that was asked which I forgot to answer. That was whether the figures shown there were tests or made up from a knowledge of the shape of the field. Those figures are made up from a series of a great many hundred tests on different shapes and forms of dielectric boundaries between the air and dielectric, and are one of a great number of tests that we have made in that study.

F. W. Peek, Jr.: The Hewlett type suspension insulator is the original suspension insulator. The first suspension insulators put into practical operation were of this type. We have, therefore, a longer practical experience with this type than any other. After a considerable number of these insulators had been put into operation, the cemented type insulator came into extensive use. This followed because given electrical characteristics could be obtained at less cost with the cemented type than with the Hewlett type. It was soon found, however, that in general a very large percentage of the cemented type units failed after several years of operation. The great importance of the mechanical design is now well understood and undoubtedly great improvements have been made in the mechanical design of the cemented unit. I have made a study of Hewlett insulators located in different parts of the country, and have not found any evidence of appreciable deterioration either in operating records or in electrical and mechanical tests made on the many units returned to the laboratory. The recorded cases of trouble were confined to failures of the more or less crude early types of hardware.

Mr. Crawford's experience with the Hewlett insulator is practically the universal experience with this type. Mr. Hillebrand states that he has found a case where the Hewlett type of insulator has depreciated 1 per cent after ten years of service. This is unusual. Although I have investigated practically all systems using Hewlett insulators, this case has not come to my attention. It may be due to steel hardware as Professor Clark points out. It is of interest, however, that this case cited against the Hewlett would be considered an unattainable record for the older cemented types, where the depreciation is sometimes 25 per cent after three to five years.

Regarding the discussion on transient arc-over voltages, it is my belief that an insulator should have high impulse ratio. It is possible to obtain this without a sacrifice in the characteristics at normal frequency.[1]

While it is true that the maximum unit voltage stress for a long string is less on the cemented types than on the other types, the same shield reduces the stress to practically the same value on all types. As I have already pointed out in my paper, the ring or antenna type of shield corrects uneven voltage distribution, prevents excessive corona on the line end units and tends to direct the power arc away from the string. The maximum unit stress on the 200-kv. shielded string can be made less than on the 100-kv. non-shielded string.

In conclusion I wish to state that I have faith in the successful operation of transmission lines at 220 kv.

Harris J. Ryan: In regard to the original application of the insulator shield by Mr. Fisken: The purpose of the shield brought forward in the present papers has particular reference to the problem of the suspension insulator when used for line voltages in excess of 150 kv. Its purpose is twofold: (1) To limit the maximum voltage duty to be carried by any unit in the string; (2) To lessen the punishment of the string units when flash-over occurs. Undoubtedly Mr. Fisken's design was in effect, an early application of the static and flash-over shield to pin type insulators. Probably the excellent durability noted is due to the improved flash-over characteristic caused by the shield and good fortune in securing insulators in which the porcelains are so made and assembled that cracking is virtually absent.

Mr. Crawford is right in bringing up the possible value of holding to *seven* units in a string and of making them larger than at present for line voltages in excess of 110 kv. Two problems are involved in this proposition: (1) The quantity production of such larger units; (2) Their electrical characteristics, which in any event would have to be checked or adjusted by actual test.

In regard to Mr. Stauffacher's remarks concerning the proposition to elevate the voltage on the Big Creek-Los Angeles transmission from 150 to 220 kv. We will gladly do all in our power to cooperate in the undertaking to determine the values of the insulator factors that will enter into such undertaking.

Too little is known at present (July 1920) of the details of flash-over formation to deal with the subject in the manner presented by Mr. Hillebrand. What he has said in this instance is to be taken as an excellent effort to define the problem of flash-over with and without cascading rather than to present factors which are definitely understood in reference thereto. Regarding his reference to porcelain as "cracking with time in accordance with its inherent characteristic": I can not agree that this has been demonstrated, as yet, for good grade high-voltage porcelain. It is an open question today as to whether good grade porcelain has such liability to crack when exercised by thermal or mechanical cycles to a reasonable extent. Herein, I share Mr. Fisken's impression that there are many evidences that porcelain can be made which will not deteriorate. Laboratory tests and observations in the field should be undertaken to settle this question as quickly as possible.

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[1] Peek, The Effect of Transient Voltages on Dielectrics, A. I. E. E. TRANSACTIONS 1915.

The suspension insulator when used as such can not be subjected to severe mechanical vibrations. Mr. Lindsay, however, is right in emphasizing the importance of knowing just what the effect of such vibrations is upon the durability of the units. It must be that in dead-end duty these effects are occasionally severe due to longitudinal resonant mechanical vibrations set up by the wind, particularly in long spans.

Regarding the "watts lost" curve on page 1671: Mr. Hillebrand is entirely right in his surmise that the loss occurs for the most part in the air surrounding the unit and not in the unit proper. It is well to remember that the air is there and necessarily a part of the insulator assembly; that no good comes from operating it overstressed or ionized.

"Impulse ratio" considered by Mr. Peaslee and discussed by Mr. Hillebrand must be a highly uncertain factor in flash-over characteristics of insulators. For example the results of measurements made since the date of this discussion, of the maximum r. m. s. values of 55,000-cycle voltage transients required to break down air columns between a one-inch tube terminated with a three-inch sphere as the main electrode and a grounded square plate having an area of 25 square feet are given in the following table:

Gap in inches	Required for flash-over		Impulse ratio
	R. m. s. max. 55,000 cycles transient, kilovolts	Appx. estimate for 60 cycles, kilovolts	
10	250	140	1.78
25	300	290	1.03
50	370	540	.69
75	440	790	.56
100	500	1040	.48

The values here given are only approximate and may be subject to some revision when finally reported.

Piezo-electric effect in the undissolved quartz crystals in the porcelain has been brought up as a possible cause of cracking of porcelain. Now it is true of the quartz crystal that in a plane parallel to the face of the crystal and at right angles to the optic axis the crystal will expand or contract on direct or reversed electrification. In so doing it also expands or contracts under such electrification in a direction that is at right angles to the face of the crystal and to the optic axis. It should be remembered that these expansions and contractions are enormously small due to electric fields set up by 10,000 volts per inch; and that the quartz crystal remaining in good electrical porcelain is a body so small that its matrix of elastic amorphous material easily accommodates such dimensional changes without mechanical overstress. The radio engineer has used porcelain extensively in intense high-frequency fields without noting a tendency to deteriorate.

Professor Clark has referred to our studies of insulator durability in relation to the daily variation of the elements. The results obtained through these studies have been published in: Discussion, TRANS. A. I. E. E. Vol. XXXVI, p. 563, 1917 and the report of the Insulator Committee, John A. Koontz, Chairman, of the National Electric Light Association, Pacific Coast Division, 1920.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Portland, Oregon, July 22, 1920.

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BRIDGE METHODS FOR ALTERNATING-CURRENT MEASUREMENTS

BY D. I. CONE

The Pacific Telephone & Telegraph Company, San Francisco, Cal.

THE extension of the field of application of electric energy to human service requires more and more of measurements of electrical quantities, of varying grades of precision. Very prominent in the history of electrical measurements is the so-called "bridge" method, the fundamental principle of which is the equalizing of the potentials of two chosen points in a network of electric circuits. The original application of this principle was made by S. H. Christie in 1833 to the measurement of resistance to direct current in the arrangement long familiarly known as the Wheatstone Bridge. Numerous forms of Wheatstone Bridge for direct-current measurement have been developed. The conditions for its use have been investigated thoroughly and are well-known.

Of later development and less known and understood are the applications of the bridge principle to measuring impedances of alternating-current circuits. However, the knowledge of the Wheatstone Bridge as used with direct current can immediately be made use of for alternating-current testing by applying the principle stated by Kennelly in the following words.[1]

> It is however, a seemingly universal and a wonderful law, that all the numerical formulas and rules of quantitative behavior for continuous-current circuits, or conductors, are exactly the same for single-frequency alternating-current circuits or conductors, in respect to potentials and currents as also (with minor reservations) to power and energy if these formulas and rules are interpreted as relating to complex numbers.

The purpose of this paper is to present a resumé of simple methods of utilizing "bridge" networks in alternating-current measurements of impedances and their components of effective resistance, self and mutual inductance and capacitance, and in frequency measurement.

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[1] Kennelly—Hyperbolic Functions Applied to Electrical Engineering

A great variety of arrangements have been described by numerous writers. It is not attempted to include all of these, but to present such a group as will give latitude of choice to suit the apparatus that is available, or permit the use of several methods to check against each other. By the methods shown approximate measurements over wide ranges of values can be made with very simple apparatus. It is hoped that others will add from their experience to the value of the collection.

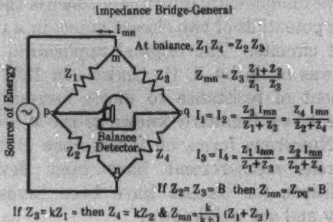
Impedance Bridge-General
I_{mn}
At balance, $Z_1 Z_4 = Z_2 Z_3$
$Z_{mn} = Z_3 \frac{Z_1 + Z_2}{Z_1 \; Z_3}$
$I_1 = I_2 = \frac{Z_3 I_{mn}}{Z_1 + Z_3} = \frac{Z_4 I_{mn}}{Z_2 + Z_4}$
$I_3 = I_4 = \frac{Z_1 I_{mn}}{Z_1 + Z_3} = \frac{Z_2 I_{mn}}{Z_2 + Z_4}$
If $Z_2 = Z_3 = B$ then $Z_{mn} = Z_{pq} = B$
If $Z_3 = kZ_1$ = then $Z_4 = kZ_2$ & $Z_{mn} = \frac{k}{k+1}(Z_1 + Z_2)$
Source of Energy
m
n
p
q
Z_1
Z_2
Z_3
Z_4
Balance Detector

FIG. 1

Before taking up the various practical forms of bridge network, it will be helpful to consider the conditions for balance and other characteristics of a "Wheatstone bridge" wherein the four resistances are replaced by impedances. Thus generalizing the Wheatstone bridge network and utilizing the ordinary formulas in accordance with the principle quoted from Kennelly's statement given above, the relations shown in Fig. 1 are determined. In place of resistance is written the impedance.

$$Z = \sqrt{R^2 + X^2}\,\underline{/\theta} = R + jX$$

Where R = effective resistance

X = effective reactance (+ if inductive, − if capacitive)

$$\theta = \tan^{-1}\frac{X}{R}$$

The standard rules for combining complex numbers must of course be observed in using the formulas. Besides the formula connecting the several impedances at balance, the impedance of the whole bridge between the corners m and n is given, also the division of current among the branches. These are useful when it is necessary to determine the current input to the circuit whose impedance is being measured. It is of interest that when two opposite impedances of the network, as Z_2 and Z_3 or Z_1 and Z_4, are equal, the impedances Z_{mn} and Z_{pq} are alike equal to that same value.

Interchanging the energy source and balance detector connections, (energizing at p, q, and equating the potentials of m and n for balance) thesame impedance relation prevails for balance, but the impedance, Z_{pq}, is in general different from Z_{mn}. Thus,

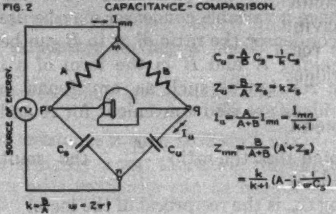
FIG. 2
CAPACITANCE - COMPARISON.
I_mn
m
A
B
SOURCE OF ENERGY.
P
q
I_u
C_s
C_u
n
C_u = A/B C_s = 1/k C_s
Z_u = B/A Z_s = k Z_s
I_u = A/(A+B) I_mn = I_mn/(k+1)
Z_mn = B/(A+B) (A+Z_s)
= k/(k+1) (A − j 1/(ωC_s))
k = B/A
ω = 2πf

FIG. 2

$$\frac{Z_{pq}}{Z_{mn}} = \frac{(Z_1 + Z_3)\,(Z_2 + Z_4)}{(Z_1 + Z_2)\,(Z_3 + Z_4)}$$

$$= \frac{Z_2\,(Z_1 + Z_3)^2}{Z_3\,(Z_1 + Z_2)^2}$$

When $Z_3 = kZ_1$

$$\frac{Z_{pq}}{Z_{mn}} = \frac{Z_1 Z_2}{(Z_1 + Z_2)^2}\;\frac{(1 + k)^2}{k}$$

The arrangement to be used in practise to secure greatest sensitivity, for a given departure from the balanced condition, depends on the impedances of the branches, including the balance detector and energy source, and on the kind of circuit being measured. For example, where a telephone receiver is used, the disturbing extraneous noises may be less with one connection than the other. If the balance detector has higher impedance than the energy source, it should be connected between the two opposite points of the network having the higher impedance.

Specific applications of these general formulas, and examples, will be given below. It is to be observed that the bridge methods here described provide for comparisons among resistances, inductances, capacitances, etc. and not for absolute measurements of any of them in terms of the fundamental units.

Fig. 2 shows the arrangement (due to De Sauty) for comparison of two capacitances. For the standard capacitance a variable condenser may be used, or with a fixed condenser the ratio of A to B can be adjusted, as by having A and B in the form of a slide wire. Variable resistances such as are found in Wheatstone bridges are also convenient for A and B. It is to be noted that the ratio $\frac{A}{B}$, the multiplier for obtaining C_u, is the reciprocal of the factor $\frac{B}{A}$ for Z_u and R_u since the impedance of a pure condenser is inversely proportional to its capacitance. C_s should be as nearly as convenient of the same magnitude as C_u. The method allows of precise adjustment for balance only when the two capacitances have very closely the same power factor, as for example the case of comparing good mica condensers.

Where the power factors of the standard and unknown condensers are different, and in general for measurement of capacitive impedances, including both resistance and capacitive reactance components, the form shown in Fig. 3 (due to Wien) can be used. A variable resistance r is arranged so it can be made part of either R_s, the effective resistance of the "standard" arm, or R_u, the effective resistance of the "unknown" arm. If the standard condenser C_s has a very small energy loss or effective resistance component, as in the case of a good mica condenser or an air condenser, R_s will be practically identical with its component r, external to the condenser. Mica condensers have phase angles of from one or two minutes up to six or seven minutes or more, depending on their quality. At 1000 cycles per second and with a 0.1 microfarad condenser having a phase angle of five minutes the reactance is $\dfrac{10^6}{2\,\pi\,1000 \times 0.1}$ or 1592 ohms. The resistance is then 1592 sin 5 minutes or 2.3 ohms. For ordinary work, therefore, the effective resistance of a mica condenser can be neglected. With paper condensers much greater effective resistances are encountered.

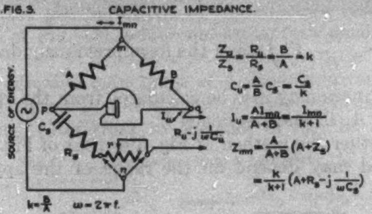
FIG. 3.
CAPACITIVE IMPEDANCE.
I_{mn}
m
A
B
SOURCE OF ENERGY.
p
q
C_s
I_u
$R_u - j\frac{1}{\omega C_u}$
R_s
r
n
$\frac{Z_u}{Z_s} = \frac{R_u}{R_s} = \frac{B}{A} = k$
$C_u = \frac{A}{B} C_s = \frac{C_s}{k}$
$I_u = \frac{A I_{mn}}{A+B} = \frac{I_{mn}}{k+1}$
$Z_{mn} = \frac{A}{A+B}(A+Z_s)$
$= \frac{k}{k+1}\left(A+R_s-j\frac{1}{\omega C_s}\right)$
$k = \frac{B}{A}$
$\omega = 2\pi f$

FIG. 3

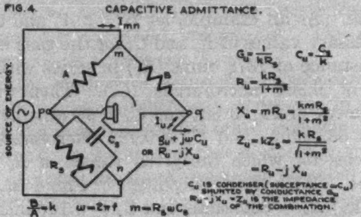
FIG. 4.
CAPACITIVE ADMITTANCE.
I_{mn}
m
A
B
SOURCE OF ENERGY.
p
q
C_s
I_u
$g_u + j\omega C_u$
OR $R_u - jX_u$
R_s
n
$G_u = \frac{1}{kR_s}$
$C_u = \frac{C_s}{k}$
$R_u = \frac{kR_s}{1+m^2}$
$X_u = mR_u = \frac{kmR_s}{1+m^2}$
$Z_u = kZ_s = \frac{kR_s}{\sqrt{1+m^2}}$
$= R_u - jX_u$
C_u IS CONDENSER (SUSCEPTANCE ωC_u) SHUNTED BY CONDUCTANCE G_u
$R_u - jX_u = Z_u$ IS THE IMPEDANCE OF THE COMBINATION.
$\frac{B}{A} = k$
$\omega = 2\pi f$
$m = R_s \omega C_s$

FIG. 4

The arrangement of Fig. 4, where the standard resistance and condenser are in parallel, may be designated as an admittance bridge, since it is adapted to determine more directly conductance $\left(g_u = \dfrac{A}{B R_s}\right)$, susceptance $\left(b_u = \omega C_u = \dfrac{A}{B}\ \omega C_s\right)$ and admittance $\left(Y_u = \dfrac{A}{B}\ Y_s\right)$ than the form previously described. Impedances may also be calculated from the values of R_s, C_s, A, B, and frequency f.

Whether the method of Fig. 3 or that of Fig. 4 shall be used may depend on the range of the apparatus

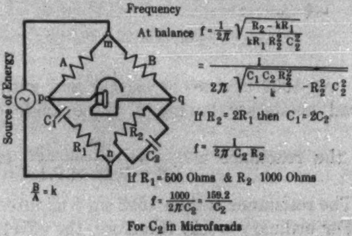
Frequency
At balance $f = \frac{1}{2\pi}\sqrt{\frac{R_2 - kR_1}{kR_1 R_2^2 C_2^2}}$
$= \frac{1}{2\pi\sqrt{\frac{C_1 C_2 R_2^2}{k} - R_2^2 C_2^2}}$
If $R_2 = 2R_1$ then $C_1 = 2C_2$
$f = \frac{1}{2\pi C_2 R_2}$
If $R_1 = 500$ Ohms & R_2 1000 Ohms
$f = \frac{1000}{2\pi C_2} = \frac{159.2}{C_2}$
For C_2 in Microfarads
Source of Energy
p
q
m
n
A
B
C_1
R_1
R_2
C_2
$\frac{B}{A} = k$

FIG. 5

available. As an example, let $B = 2\,A$, and consider the necessary values of R_s and C_s for the case where Z_u is a resistance of 1378 ohms (R_u) in series with a condenser of 0.362 microfarads (C_u). Then with the arrangement of Fig. 3 (R_s and C_s in series) the setting of R_s is 689 ohms and C_s is 0.724 microfarad for all frequencies for which the given values of R_u and C_u hold true. With the arrangement of Fig. 4 (R_s and C_s in parallel) $R_s = 800$ ohms and $C_s = 0.100$ microfarad at $\omega = 5000$ ($f = 796$ cycles per second). For other frequencies the Fig. 4 bridge values would differ.

The above example illustrates the fact that the shunting of a condenser by a resistance affords a means of increasing its effective capacitance. This is shown more fully in the curves of Fig. 18 for a capacitance $C_2 = 0.1$ microfarad shunted by a resistance R_2

varied over a wide range, the frequency chosen being $\omega = 2\pi f = 5000$. The effective capacitance of the combination is C_1, and the resistance, R_1.

A method is thus afforded for using a fixed condenser as C_s for measurements with the Fig. 4 bridge. On account of the variations of the effective resistance, (see R_1 of Fig. 18), additional series resistance (like r in Fig. 3 and 6) is necessary to obtain balance.

The effective inductance of a fixed inductance coil can be varied in like fashion to the condenser just described. Referring to the case of Fig. 18, for the chosen frequency, a self-inductance L_2 of 0.4 henry (of low resistance) shunted by resistance R_2, exhibits the same effective reactance (X_1) and resistance (R_1) as the condenser C_2 of 0.1 microfarad. The effective inductance L_1 is always *less than* L_2.

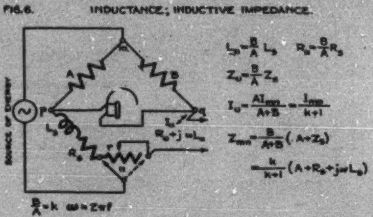
FIG. 6.
INDUCTANCE; INDUCTIVE IMPEDANCE.
SOURCE OF CURRENT
m
A
B
P
q
L_s
R_s
r
n
L_u
$R_u+j\omega L_u$
$L_u=\frac{B}{A}L_s$ $R_u=\frac{B}{A}R_s$
$Z_u=\frac{B}{A}Z_s$
$I_u=\frac{AI_{mn}}{A+B}=\frac{I_{mn}}{k+1}$
$Z_{mn}=\frac{B}{A+B}(A+Z_s)$
$=\frac{k}{k+1}(A+R_s+j\omega L_s)$
$\frac{B}{A}=k$ $\omega=2\pi f$

FIG. 6

A bridge method of determining frequency, using resistance and capacitance only, connected in series on one side and in parallel on the other, is shown in Fig. 5. The formula for the general case is rather complicated, but by choosing the shunt resistance R_2 equal to twice the series resistance R_1, the condition for balance is $C_1 = 2\,C_2$ and the frequency is

$$f = \frac{1}{2\pi C_2 R_2}.$$

By setting $R = 1000$ ohms

$$f = \frac{1000}{2\pi C_2},$$

C being expressed in microfarads. A corresponding method might be used employing inductances of low effective resistance.

A bridge for the direct comparison of inductances and measurement of inductive impedances (due to Maxwell) is shown in Fig. 6. As in the case of the bridges already described, a slide wire may be used to form *A* and *B* arms, with a fixed inductance L_s. A shunted inductance can also be used for values below the range of L_s. To measure inductances higher than $k L_s$, a condenser may be connected in series with Z_u. A condenser of capacitance C' can be considered as a negative inductance of size $L' = \frac{1}{\omega^2 C'}$. The inductance L_u will then be $k L_s + L'$.

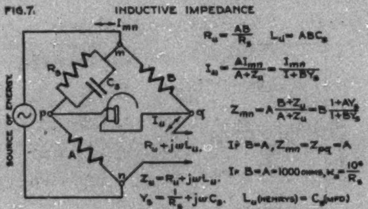
FIG.7.
INDUCTIVE IMPEDANCE
I_{mn}
SOURCE OF ENERGY
P
m
q
n
R_s
C_s
B
A
I_u
$R_u+j\omega L_u$.
$R_u=\frac{AB}{R_s}$ $L_u=ABC_s$
$I_u=\frac{AI_{mn}}{A+Z_u}=\frac{I_{mn}}{1+BY_s}$
$Z_{mn}=A\frac{B+Z_u}{A+Z_u}=B\frac{1+AY_s}{1+BY_s}$
If $B=A$, $Z_{mn}=Z_{pq}=A$
If $B=A=1000$ ohms, $K_s=\frac{10^6}{R_s}$
$Z_u=R_u+j\omega L_u$.
$Y_s=\frac{1}{R_s}+j\omega C_s$.
$\omega=2\pi f$.
L_u(HENRYS) = C_s(MFD)

Fig. 7

Methods will next be described for comparing self-inductance with capacitance, or stated more generally, of determining inductive impedances by comparison with capacitive impedances, or vice versa.

Fig. 7 shows an arrangement (due to Maxwell) much used for determining inductive impedance in terms of a resistance and condenser. It can, of course be used also for the converse process. A series resistance and condenser can be used in place of R_s and C_s, but the formulas are less simple. This form is not shown. Especially simple calculations are afforded for the Fig. 7 bridge by making *A B* (each in ohms) equal one million. Ordinarily R_s and C_s are the variable elements, but balance is obtainable by varying others such as *A*, *B* or L_u. A vector diagram of currents and potential differences of this bridge is shown in Fig. 17, which gives data of a numerical example.

In Fig. 8 is presented a modification of the bridge just described, known as the Anderson bridge. A resistance *r* is connected between condenser C_s and corner *p*, and the detector is connected to the junction of *r* and C_s. By varying *r*, a fixed condenser C_s can be used for inductive impedance measurements over a wide range. Where *r* is large it has been found advantageous to interchange the energy source and the detector.

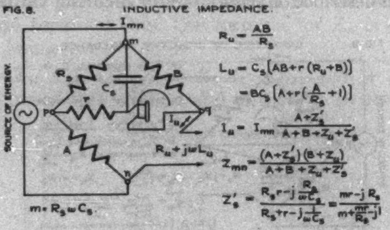
FIG. 8.
INDUCTIVE IMPEDANCE.
I_{mn}
m
R_s
C_s
B
r
p
q
I_u
SOURCE OF ENERGY.
A
$R_u + j\omega L_u$
n
$m = R_s \omega C_s$.
$R_u = \frac{AB}{R_s}$
$L_u = C_s [AB + r(R_u + B)] - BC_s [A + r(\frac{A}{R_s} + 1)]$
$I_u = I_{mn} \frac{A + Z'_s}{A + B + Z_u + Z'_s}$
$Z_{mn} = \frac{(A + Z'_s)(B + Z_u)}{A + B + Z_u + Z'_s}$
$Z'_s = \frac{R_s r - j \frac{R_s}{\omega C_s}}{R_s + r - j \frac{1}{\omega C_s}} = \frac{mr - j R_s}{m + \frac{mr}{R_s} - j1}$

Fig. 8

Fig. 9 presents a bridge set-up suitable for making measurements in quick succession of capacitive and inductive impedances. For measuring capacitive impedances, it is the same arrangement as that presented in Fig. 4. By switching the condenser C_s so that it is connected from *n* to *q*, instead of to *p*, balance is obtained for an inductive impedance Z_u. The same formulas apply in each case, changing sign of the reactance X_u, when ratio arms *A* and *B* are equal. This bridge can also be used to determine frequency.

The bridge shown in Fig. 9 may be described as a resonance bridge, when X_u is inductive, since the arm *n q* is made non-reactive at balance. A well-known and widely useful resonance bridge is shown in Fig. 10, which can be used to measure either inductive impedance ($R_u + j\ \omega\ L_u$) or capacitive impedance

$$\left(R_u - j\,\frac{1}{\omega\,C_s}\right)$$

by using variable series capacitance C_s or inductance L_u to resonate the branch $n\,q$. It is also a convenient means of determining the frequency, using fixed condenser and variable self-inductor or vice versa. With a variable standard inductance this arrangment can be used like that of Fig. 9 for measuring inductive and capacitive impedances alternatively by simply placing the variable inductor in the $n\,q$ branch for measuring capacitive impedance, or in the $n\,p$ branch for inductive impedance, as was shown in Fig. 6. The methods described on page 1748 for securing variation of effective reactance of fixed condensers or inductances can be applied to this bridge.

FIG. 9. CAPACITIVE OR INDUCTIVE IMPEDANCE.

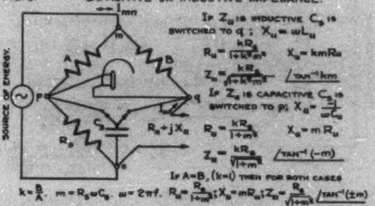
I_{mn}
m
A
B
P
q
n
C_s
R_s
$R_u + jX_u$
SOURCE OF ENERGY.
IF Z_u IS INDUCTIVE C_s IS SWITCHED TO q ; $X_u = \omega L_u$
$R_u = \frac{kR_s}{1+k^2m^2}$ $X_u = kmR_u$
$Z_u = \frac{kR_s}{\sqrt{1+k^2m^2}}$ $\angle \tan^{-1} km$
IF Z_u IS CAPACITIVE C_s IS SWITCHED TO p; $X_u = \frac{-1}{\omega C_u}$
$R_u = \frac{kR_s}{1+m^2}$ $X_u = mR_u$
$Z_u = \frac{kR_s}{\sqrt{1+m^2}}$ $\angle \tan^{-1}(-m)$
IF A=B, (k=1) THEN FOR BOTH CASES
$k = \frac{B}{A}$. $m = R_s \omega C_s$. $\omega = 2\pi f$. $R_u = \frac{R_s}{1+m^2}$; $X_u = mR_u$; $Z_u = \frac{R_s}{\sqrt{1+m^2}}$ $\angle \tan^{-1}(\pm m)$

FIG. 9

Next to be considered is the measurement of mutual inductance. A method which is also applicable to mutual capacitance and resistance, or stated generally, to mutual impedance determinations, makes use of bridges such as those already described. Consider two circuits, of impedances Z_1 and Z_2 respectively, and mutual impedance Z_m[1]. If connected in series the resultant impedance is $Z_a = Z_1 + Z_2 - 2\,Z_m$. By reversing the connections of one of the two, the sign of the Z_m term is reversed and the impedance is

$$Z_b = Z_1 + Z_2 + 2\,Z_m$$

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[1] See Bibliography No. 5.

Z_1 and Z_2 can be eliminated from these two equations leaving

$$Z_m = \frac{Z_b - Z_a}{4}$$

Where one terminal of Z_1 and Z_2 is common, for example the case of two earth-return wire circuits, the above procedure for obtaining Z_b is not possible. However by measuring Z_1, Z_2 and Z_a, Z_m can be evaluated, for

$$2\,Z_m = Z_1 + Z_2 - Z_a.$$

A check may be had by measuring the impedance Z_{12-0} from the two free terminals in parallel to the common terminal, but the formula for Z_m is complicated, being

$$Z_m = Z_{12-0} \pm \sqrt{Z_{12-0} + Z_1\,Z_2 - Z_{12-0}\,(Z_1 + Z_2)}$$

The use of this method and of the Fig. 10 bridge will be illustrated by measurements on a retardation coil having two windings on an iron-wire core.

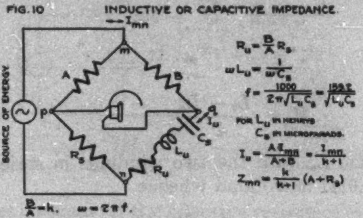
FIG. 10
INDUCTIVE OR CAPACITIVE IMPEDANCE.
I_{mn}
m
A
B
SOURCE OF ENERGY
P
q
I_u
C_s
L_u
R_s
n
R_u
$R_u = \frac{B}{A} R_s$
$\omega L_u = \frac{1}{\omega C_s}$
$f = \frac{1000}{2\pi\sqrt{L_u C_s}} = \frac{159.2}{\sqrt{L_u C_s}}$
FOR L_u IN HENRYS
C_s IN MICROFARADS.
$I_u = \frac{A I_{mn}}{A+B} = \frac{I_{mn}}{k+1}$
$Z_{mn} = \frac{k}{k+1}(A+R_s)$
$\frac{B}{A} = k.$ $\omega = 2\pi f.$

Fig. 10

The frequency was first determined by balancing a 0.1004 microfarad condenser (C_s) with an inductance L_u = 0.379 henry, (using an inductometer), R_s being adjusted but its value not required. A and B were each 1000 ohms resistance. Their values likewise are not required for finding the frequency

$$f = \frac{159.2}{0.379 \times 0.1004} = 816 \text{ cycles per second.}$$

$$\omega = 5130. \quad \omega^2 = 26.3 \times 10^6$$

The resistance components, being small compared to the reactance components will for brevity be omitted from consideration, leaving only the reactances. The factor ω then cancels out, thus

$$Z_m = \frac{Z_b - Z_a}{4}$$

becomes $\omega M = \dfrac{\omega L_b - \omega L_a}{4}$

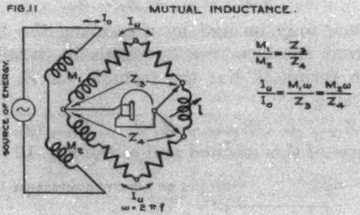
FIG.11
MUTUAL INDUCTANCE.
I_o
I_u
SOURCE OF ENERGY.
M_1
M_2
Z_3
Z_4
l
I_u
$\omega = 2\pi f$
$\frac{M_1}{M_2} = \frac{Z_3}{Z_4}$
$\frac{I_u}{I_o} = \frac{M_1\omega}{Z_3} = \frac{M_2\omega}{Z_4}$

Fig. 11

or $$M = \frac{L_b - L_a}{4}$$

To find L_a, with the two windings in series aiding, $C_s = 0.0141$ microfarad whence

$$L_u \;(= L_a) = \frac{10^6}{\omega^2 \times 0.0141} = 2.70 \text{ henrys.}$$

Reversing connections, placing the two windings in series opposing; to find L_b, $C_s = 0.0232$ microfarad,

whence $L_b \;(= L_u) = \dfrac{10^6}{\omega^2 \times 0.0232}$

Thus $L_u = 1.64$ henrys.

$M = 1/4\,(2.70 - 1.64) = 0.265$ henry.

The impedance of one winding, Z_1 the other being open-circuited, was as follows:

$C_s = 0.0348$ microfarad. $R_s = 254$ ohms $= R_u$

$$L_u = \frac{10^6}{\omega^2 C_s} = 1.096 \text{ henrys } X_u = \omega L_u = 5610 \text{ ohms}$$

whence
$Z_u \quad (= Z_1) = 254 + j\,5610$ ohms.

The direct comparison of two mutual inductances is effected by the bridge of Fig. 11 (due to Maxwell). The impedances Z_3 and Z_4 include the inductive reactances and resistances of the secondaries of the mutual inductance coils M_1 and M_2, besides variable resistance. The small variable inductance l is made part of either Z_3 or Z_4 and is necessary to equate the phase angles of Z_3 and Z_4. With M_1 and M_2 known this bridge is available for ordinary impedance measurements, also.

The circuit of Fig. 12 is a very convenient method of determining frequency in terms of mutual inductance and capacitance, or for measuring either of the latter, if the other and the frequency are known. It depends for its effectiveness on the effective resistance components in the branch C and in the mutual impedance of the coil M being very small. If, due to eddy-current effects, the secondary induced e. m. f. of the coil M is not in quadrature with the primary current, or if there is energy loss in the condenser so that the potential drop across it is not in quadrature with the current, precise balance is not obtained. As an example, it was found impracticable to make a very close determination by this method of the mutual inductance of the iron-cored coil of which measurements were described above.

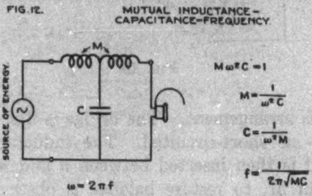

FIG. 12

Another bridge, (due to Carey Foster) for comparing mutual inductance with capacitance, is given in Fig. 13. The frequency is not involved in the equation. The method also permits the determination of the resistance of the arm containing the capacitance, so that the effective resistance of a condenser can be measured, and thus its power factor.

A bridge for comparing self and mutual inductances, (due to Heaviside) is shown in Fig. 14. A method of differences, (due to Campbell), which can also be applied in principle to other bridge networks, has been developed for the measurement of small self-inductances using this arrangement. The bridge is first balanced, with $n - n'$ short-circuited. The inductance to be measured is then inserted between n and n' and the change in M_2 to restore balance is observed. It is necessary that the self-inductance L_2 of the secondary winding of the mutual inductance coil remain constant.

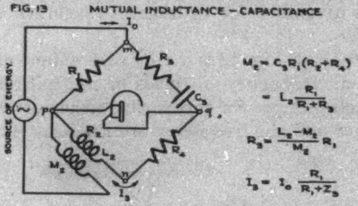
FIG. 13
MUTUAL INDUCTANCE – CAPACITANCE
SOURCE OF ENERGY
I_0
m
R_1
R_3
C_3
p
q
R_2
L_2
M_2
R_4
n
I_3
$M_2 = C_3 R_1 (R_2 + R_4)$
$= L_2 \frac{R_1}{R_1 + R_3}$
$R_3 = \frac{L_2 - M_2}{M_2} R_1$
$I_3 = I_0 \frac{R_1}{R_1 + Z_3}$

Fig. 13

For the comparison of low impedances, for example in determining the conductivity of electrolytes, and cases where contact or leading in resistances are of importance, the Kelvin double bridge is used. The arrangement is shown in Fig. 15. Where, as in electrolytes, there is capacitive reactance effect, it may be necessary to shunt the standard resistance with a small variable condenser, to obtain exact balance.

An important use of bridge networks is the determination of unbalances, or the difference between two impedances or admittances; often of two sides of a circuit with regard to some common reference, usually "ground." The most obvious way is to measure each impedance separately. However, it is frequently the ratio of values that is desired and not the actual values.

For this purpose, if the phase angles of the impedances being thus compared are nearly alike, the type of bridge shown in Fig. 2, 3, etc. having ratio arms *A* and *B* in adjacent branches, permits direct determination of the ratio of the impedances in the two other branches, by adjustment of the ratio $\frac{B}{A}$ to obtain balance. This method becomes unsatisfactory if the phase angles of the impedances being compared are unlike.

A bridge arranged to measure both conductance and capacitance components of the unbalance between two admittances Y_a and Y_b is shown in Fig. 16. As illustrated, a special condenser is so arranged that capacitance *C* is added to one side and subtracted from the other simultaneously as the condenser is shifted from its neutral position. The effect can also be secured by an ordinary condenser connected alternatively from *p* or *q* to *n*. A similar arrangement balances the conductances. At balance the readings of *C* and *r* show the difference between the two admittances Y_a and Y_b. To obtain the ratio or percentage of unbalance it is necessary to measure Y_a and Y_b.

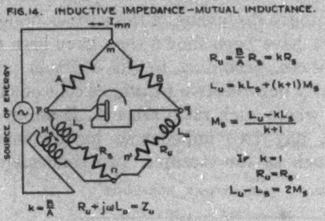
FIG. 14. INDUCTIVE IMPEDANCE—MUTUAL INDUCTANCE.
I_{mn}
SOURCE OF ENERGY
m
A
B
p
q
L_s
M_s
L_u
R_s
R_u
n'
n
$R_u = \frac{B}{A} R_s = kR_s$
$L_u = kL_s + (k+1)M_s$
$M_s = \frac{L_u - kL_s}{k+1}$
If k = 1
$R_u = R_s$
$L_u - L_s = 2M_s$
$k = \frac{B}{A}$
$R_u + j\omega L_u = Z_u$

FIG. 14

The admittances of conductors sometimes depend on the condition of neighboring conductors. In the study of such cases the "direct admittances" are made use of.[2] The direct admittance between two conductors *a* and *b* is that admittance which obtains when all neighboring conductors are at the same potential as one of them. The dotted structure in Fig. 16 shows a method of adjusting the potential of conductors *X* to that of *a* and *b*, so that direct admittances or direct admittance unbalances can be measured.

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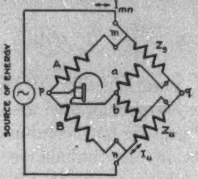
[2] *See Bibliography No. 5.

FIG. 15. LOW IMPEDANCE-KELVIN DOUBLE BRIDGE.



AT BALANCE

$$\frac{Z_u}{Z_s} = \frac{B}{A} = \frac{b}{a}$$

$$I_u \doteq I_{mn}$$

$$= \frac{A}{A+Z_s} I_{mn}$$

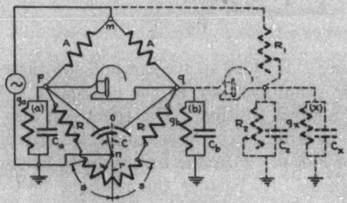
$$Z_{mn} \doteq Z_s + Z_u$$

$$= Z_s \frac{A+B}{A+Z_s}$$

Fig. 15

In the above discussion of the various bridges where ratio arms have been shown they have been indicated as resistances, according to the common practise of using nearly non-reactive resistance coils or slide wires. In cases where such ratio arms *A*, *B* are in adjacent branches, any two impedances having equal phase angles, such as for example, two condensers or two highly reactive coils can be substituted for such resistances, without altering the formulas.

FIG. 16 ADMITTANCE UNBALANCE.



$$C_b - C_a = 2C \cdot \quad g_b - g_a = \frac{2r}{(R+S)^2 - r^2}$$

APPARATUS SHOWN DOTTED CAN BE ADJUSTED TO EQUATE POTENTIAL OF OTHER CONDUCTORS, X, TO THE POTENTIAL OF a AND b, BEING MEASURED.

Fig. 16

While the use of unequal ratio arms provides a flexible bridge and greatly increases its range, simplicity in calculation and the minimizing of errors due to inexact realization of assumed conditions (for example, residual inductance or capacitance in assumed pure resistance coils) dictate the use of equal ratio arms wherever practicable.

Several different types of sources of energy at frequencies over the range from direct current up to radio frequencies, or over parts of this range, are available. It is not attempted here to discuss these. Among such are the various forms of alternator with rotating armature or field, or of the inductor type, the singing telephone or "howler," the microphone hummer, the Vreeland sine wave oscillator, and, most recently developed, the vacuum-tube oscillator. For a full discussion the reader is directed to references of the bibliography.

FIG. 17 VECTOR DIAGRAM FOR INDUCTIVE IMPEDANCE BRIDGE. (SEE FIG. 6.)

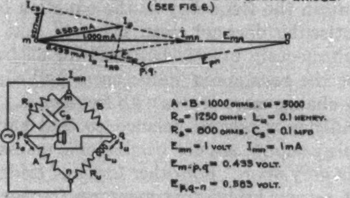

I_{cs}
I_s
0.583 mA
1.000 mA
I_{mn}
E_{mn}
m
n
0.433 mA
I_u
I_{ns}
E_{mp}
E_{pn}
p.q.
I_{mn}
R_s
C_s
B
A
I_s
I_u
L_u
R_u
p
q
$A = B = 1000$ OHMS. $\omega = 5000$
$R_u = 1250$ OHMS. $L_u = 0.1$ HENRY.
$R_s = 800$ OHMS. $C_s = 0.1$ MFD
$E_{mn} = 1$ VOLT $I_{mn} = 1$ mA
$E_{m-p.q} = 0.433$ VOLT.
$E_{p.q-n} = 0.583$ VOLT.

Fig. 17

The most commonly used balance detector is the telephone which has the advantages of cheapness, simplicity, and ample sensitivity for all ordinary testing. It is most convenient at frequencies of from about 300 to a few thousand cycles per second, though measurements can be made at somewhat lower frequencies with it directly. However, small higher harmonics in the wave from the energy source have so much greater a factor of audibility that a pure wave shape is needed.

A method was developed many years ago to adapt the telephone to the direct-current Wheatstone bridge. This can be employed for measurements at very low frequencies. It consists of varying at an audible frequency the impedance of the balance detector circuit, so that whenever any current of low frequency exists in this circuit it will be broken up into audible impulses. The early device was a toothed wheel interrupter. A modern method of achieving the same result is to connect a telephone transmitter in series with the balance detector telephone[3]. A telephone receiver, excited at an audible frequency, say 800 cycles is then used to operate the transmitter, thus varying the impedance of the detector branch.

For frequencies above a few thousand cycles, the heterodyne principle common in radio practise can be employed, the telephone receiver in the balance detector position being energized also by an auxiliary source of such frequency that the beats between the measuring frequency and this auxiliary frequency are audible.

Some forms of bridge, such as Figs. 2, 3, 7, etc. do not contain the frequency in the equation of balance. In practise this does not mean, however, that the bridge is simultaneously exactly balanced for all frequencies, because the resistances, inductances and capacitances usually change more or less with frequency. However, the tendency is toward balance, so that harmonics in the energy supply are not troublesome unless the wave shape is very poor. For other forms of bridge, such as Fig. 10, or any bridge that measures frequency, where balance is obtained by resonance, the bridge is definitely unbalanced at all frequencies other than the one for which the bridge is set. With such bridges purity of wave form of the energy source is important, as observation of the silence point in a telephone receiver for one particular frequency becomes difficult or impossible as the noise from other harmonics is increased.

To purify the wave-form supplied to the bridge a filter or wave screen is often of advantage. The simplest form consists of an inductance coil and condenser in series with the energy source. It is also possible to resonate the detector circuit for the frequency under observation.

For precise work and especially at low frequencies the vibration galvanometer has been very successfully used. It is a much more complicated and delicate apparatus and is sensitively tuned to the frequency of the energy supply.[4]

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[3] See Bibliography No. 11.

As the frequency employed is increased the problem of guarding against current in the detector circuit, due to unbalances among the bridge arms and to ground, becomes more acute. To quote from Campbell:[5]

> The difficulty encountered lies mainly in the direct capacity between the different parts of the system. Since the ether permits the flow of alternating currents in all directions, the attempt to employ an ordinary balance for alternating measurements is much the same as the attempt to measure resistance with a Wheatstone bridge immersed in a conducting fluid, such as acidulated water.

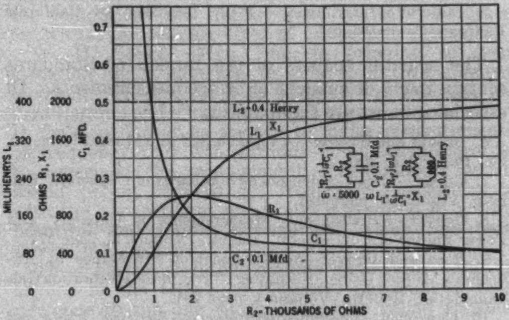

FIG. 18—EFFECTIVE RESISTANCE, REACTANCE, AND CAPACITANCE OR INDUCTANCE

Obtained by shunting condenser of 0.1 microfarad or inductance of 0.4 henry by a variable resistance, R_2.

For measurements at 1000 cycles and less this problem is only important in the more precise measurements. A simple device which is of aid in this connection is to connect the energy source to the bridge through a transformer having its neutral point grounded. Similarly, the balance detector may be connected to the bridge through a transformer. More elaborate arrangements are described in the literature.

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[4] See Bibliography No. 1, 2 and Bureau of Standards Publication.

[5] See Bibliography No. 10.

A number of references to books and periodicals are given below for the guidance of those seeking further information.

BIBLIOGRAPHY

1. Laws—Electrical Measurements—(1917), contains mathematical discussions of numerous bridges, of alternating-current energy sources, and balance detectors. It also gives many references to the literature of the subject.

2. Malcolm—The Theory of the Submarine Telegraph and Telephone Cable—(1917). Chapter VII—Alternating-Current Measurements.

3. Fleming—Handbook for the Electrical Laboratory and Testing Room.

4. Smith—Electric and Magnetic Measurements—(1917).

5. Inductive Interference—(1919), published by California Railroad Commission.

The scientific papers of the Bureau of Standards contain many discussions of bridge measurements. Of especial interest are the following:

6. Brooks and Weaver—A Variable Self and Mutual Inductor. Sci. Paper No. 290.

7. Curtis—Mica Condensers as Standards of Capacity. Bulletin, Volume 6, page 431.

8. Grover—Simultaneous Measurements of Capacity and Power Factor of Condensers. Bulletin, Volume 3, page 371.

9. Rosa and Grover—Measurement of Inductance by Anderson's Method Using Alternating Currents and a Vibration Galvanometer; Bulletin, Volume 1, page 291.

10. Campbell—The Shielded Balance—*Electrical World and Engineer*, April 2, 1904.

11. Kennelly, Laws and Pierce—Experimental Researches on Skin Effect in Conductors. TRANS. A. I. E. E., Volume 34, (1915.)

12. Drysdale—A Universal Inductance and Capacity Testing Bridge—Electrician (London) January 23rd, 30th, 1920.

DISCUSSION ON "BRIDGE METHODS FOR ALTERNATING-CURRENT MEASUREMENTS" (CONE), PORTLAND, ORE., JULY 22, 1920.

W. A. Hillebrand: In any laboratory the problem continually arises of accomplishing results with limited apparatus available, which Mr. Cone has carefully recognized in his presentation. In one instance the writer was faced with the necessity of calibrating a millivolt meter with full scale reading of one tenth volt. The nearest available standard of reference was a 15-volt voltmeter, giving a ratio between full scale readings of one hundred fifty to one, a single division on the larger meter being full scale reading on the other. However, by means of a bridge circuit, using two resistance boxes of a type found in most laboratories and a small portable galvanometer, the comparison was quickly made with an accuracy far within the reading error of either instrument.

W. D. Scott: Mr. Cone, in bringing to your attention Fig. 5, gave way to some of his own inherent modesty. He neglected to state it was through his investigation and effort that Fig. 5 became a possibility. It developed that he wanted to make measurements of frequency and was without adequate apparatus, so conceived the arrangement of equipment which would permit him to get frequency measurements with the apparatus at hand, and it resulted in a very beautiful and simple arrangement which has been shown in his paper.

These particular bridge circuits have great value to anyone in the telephone industry, particularly with reference to the one which Mr. Cone called to your attention as being generally used by telephone people. With the aid of this equipment we are able to make accurate locations of small irregularities in our long circuits; such, for instance, as short lengths of intermediate cable or insulated wire in open wire circuits, so that if by any chance any of this insulated wire has been put in these open-wire circuits we are able to locate the geographical distance from the sending end to this particular piece of open wire and determine how much difficulty we experience from the insertion of such things in our long lines. The equipment has generally become more useful in recent years through the commercial development. This is particularly true with respect to the form of a-c. generator. Before, the devices which we had were not particularly portable, but now, through the use of the vacuum tube, we have a portable type oscillator of a comparatively generous output that can be carried around by two men and by using a tube as a generator and two tubes, as for amplification, it makes a very satisfactory device for taking this sort of measurement.

It is rather of interest to note generally that in the use of the a-c. bridge, particularly with the admittance measurement, that one has to be rather careful in what you have. Unless the bridge is rather carefully shielded and carefully screened, the position of the observer or the position of the observer's hands will affect the balance; so that it is necessary in a good many instances to take particular pains with arms of the bridge in shielding them, as mentioned in Mr. Cone's paper.

H. V. Carpenter: I would like to ask Mr. Cone just what the situation is now in regard to getting resistances which are strictly resistances. Under most testing conditions a resistance is called a resistance if it is wound in the ordinary non-inductive fashion; but I imagine that when we get into these new frequencies, of a few hundred thousand cycles, that the resistance will have to be made with a very great deal of care.

If you notice the curve for R_1 in Fig. 18 you will see that as you continue to add resistance to the circuit, the effective resistance rises to a maximum and then decreases again. That is not very difficult to explain, but when you first strike that, it is rather startling.

D. I. Cone: A question has been raised as to the dependability of resistances at high frequencies. The manufacturers of high-grade apparatus supply resistances which show very little of either residual inductance or capacitance, at least for frequencies up to some tens of thousands of cycles per second. The Bureau of Standards has made a thorough study of the problem and has prepared specifications covering the manner of winding the resistance coils so that these residual effects will be small. Description of this work can be found in their publications referred to in the paper.

With respect to the bridge of Fig. 5, referred to by Mr. Scott, it should be stated that the development of this arrangement is primarily due to Mr. R. C. Mathes, with whom the writer was working at the time.

Presented at the Pacific Coast Convention of the American Institute of Electrical Engineers, Portland, Oregon, July 23, 1920.

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POWER FACTOR CORRECTION ON DISTRIBUTION SYSTEMS

BY D. M. JONES

Lighting Engineering Dept., General Electric Co., Schenectady, N. Y.

POWER FACTOR CORRECTION ON DISTRIBUTION SYSTEMS

POWER factor is a characteristic of electrical energy which is being transmitted by an impressed alternating voltage and resultant alternating current whereby it is possible for the current to rise and fall in magnitude before, with or after, the equivalent rise or fall in the alternating electromotive force. Tied up with this relative timing of voltage and current waves (which make up one pair of the cause and effect twins of the electrical family) is much of the hope and grief of the distribution game.

DEVELOPMENT OF DISTRIBUTION

The development of the art of distribution of electrical energy has been practically dictated by the developments in electrical energy producing and consuming devices. The demand that distribution systems cover over larger areas has forced higher distribution voltages. The safety of the consumer and lower relative cost of manufacture in case of most small electrical power consuming devices has kept the mass of consumers' services at a relatively low voltage. This conflict of requirements could only be adjusted by a practical pressure charging apparatus.

The advent of the transformer as a device which very neatly did this work in a-c. circuits, easily gave this type of service a big lead in the distribution field. This lead was accentuated by the discovery of the principle underlying the induction motor and subsequent production of this motor for general a-c. commercial motor service.

The ruggedness and simplicity of the electrical and mechanical design of both the wound-rotor and squirrel-cage types of this motor early caused it to be considered by the consumer a most satisfactory appliance for converting electric energy to energy of rotating motion, and this attitude has resulted in practical limitation of d-c. motors in industrial service to either those conditions where a-c. service was not available or to special applications where large speed variation was desired with relatively high torque at low speeds. In this latter field the d-c. motor is still pretty well entrenched on account of its inherent ability, not only to meet the above conditions, but even to do so with relatively high efficiencies.

Two notable exceptions to the above general tendency towards the use of alternating current, however, exist: The d-c. distribution networks covering the dense business sections of some of our largest cities; and the use of direct current in electrical traction activities.

The d-c. network generally has a basic historical reason for existence. The natural place for first electrical lighting plants to appear was in the heart of the largest cities and as these first plants were all d-c., the distribution systems in these spots started as d-c. systems. When the a-c. generating station arrived there was so much d-c. equipment in operation in the section, that it was less of a convulsion to substitute converting devices for changing the a-c. power received from new central stations to d-c. power to fit the old equipment, than to change the whole system to a-c., that this solution was generally adopted. Another potent argument for the d-c. service in these congested sections was the fact that it could be backed up with a storage battery; (generally of capacity to carry full load for 15 minutes) which did away with the possibility of short failures of power with attendant danger to life and property in these dense sections. Now, however, (a) the improvement in reliability of a-c. distribution due to the improvement of general equipment and construction practises, and also to the development of automatic control features; (b) the relatively lower cost of a-c. distribution due to absence of synchronous converters and storage batteries and reduction in quantity of copper required due to the fact that with a-c. distribution the power is generally brought much nearer to consumer at the higher voltage; (c) the reduced upkeep and attention, due to absence of converters and batteries above mentioned; (d) and finally the habit a d-c. network has, when it once does go dead beyond the ministering aid of the storage battery, of going dead all over; have not left it in undisputed sway even here, but rather in the position where it must fight harder for its grasp on the situation with passing years; particularly, when it is a matter of determining the type of new installations.

In the traction field the incomparable excellence of the d-c. motor, as compared with the a-c. type, has always kept most of the trolley wires of the world carrying d-c. power. This has not been without aggresive competition, especially in the realm of heavy haulage where the a-c. motor bids fair to divide the honors for a time. At present, however, the ultimate distribution of the electrical energy for traction purposes is increasingly done in d-c. form.

BAD POWER FACTOR A DANGEROUS CONDITION

It is thus evident from the foregoing that power factor, being a characteristic of a-c. distribution, is a phenomenon, whose field of influence already is widespread; and it is broadening at almost a cumulative rate. The fact that the electrical energy delivery team (current and voltage) in an a-c. system have the capacity of doing more or less work for their size, as one might say, according to whether they pull together or not; suggests at once the advisability of seeing what can be done to get the most out of them. This advisability is accentuated by an appreciation of the fact that the capacity of the electrical end of generating equipments and distribution systems is based upon the size of "current" the "off-horse" of this team, and not upon the work the team accomplishes. And again, since the electrical losses throughout are dependent, not only upon the magnitude of the current value alone, but even increase with the square of the current value, it is evident that in a large measure this animal also eats according to the size of his frame and not according to his productive activities.

Its Relation to Capital Invested

The fact that the load of power systems varies cyclically with the hours of the day and that the maximum current demand for the day, in the past, has generally come with the evening lighting peak, which is an inherently good power factor load, has made many careless as to what the power factor of the lesser loads through out the day was, as the lighting peak had dictated the current-carrying capacity of the distribution system.

When this low power factor industrial load, which had generally been welcomed at a reduced rate as a filler of load curve valleys, began to demand the expenditure of money for distribution capacity due to its presence, beyond that which was demanded by the evening peak, it challenged attention to the seriousness of the evils of bad power factor from the economic standpoint. For example; the capital cost of a central station system beyond the generating station bus per kilowatt-hour delivered per year will vary over a range of approximately 2 cents for large dense installations to 12 cents for small extended distribution systems. Assuming 4 cents capital investment per kilowatt-hour delivered per year and 15 per cent interest and depreciation charge on basis of 90 per cent power factor at maximum current demand as a concrete case, the interest and depreciation on transmission and distribution systems per kilowatt-hour delivered will be 6/10 of a cent.

If the power factor of maximum current peak were now arbitrarily lowered to 60 per cent and load remained the same, and assuming general transmission and distribution investment were increased proportionally with current increase, the interest and depreciation per kilowatt-hour delivered for the year would be increased 50 per cent. The capital charge would now be 9/10 cent instead of 6/10 cent for each kilowatt-hour delivered; or an increase of 3/10 of a cent in delivered cost of every kilowatt-hour, due to increased capital charge occasioned by the drop in power factor from 90 per cent to 60 per cent lagging. Such a saving would buy the coal under average conditions for some plants.

EFFECT ON LOSSES

The increased I^2R losses with the change for the worse of power factor are often considered negligible and are admittedly difficult to reduce to the concrete.

Let us consider however the copper losses in a three-conductor 250,000 cir. mil distribution cable carrying 60-cycle 15,000-volt three-phase power, which is a fairly representative condition. Assuming an average daily load curve on the cable with load factor of 53½ per cent and current of 200 amperes at evening peak, as shown on Fig. 1 with power factor varying from 90 per cent at peak to 60 per cent throughout day as shown by power factor curve on above mentioned figure we have the following results:

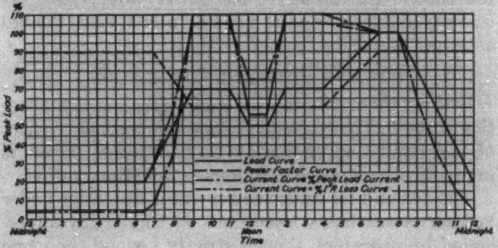

FIG. 1

Total kw-hr. per year delivered over cable 21,900,000.

Total I^2R losses for year per mile 144,000 kw-hr. Or about 66/100 of one per cent of power delivered disappear in I^2R losses per mile of cable. If now the power factor of above assumed load were brought up to a constant value of 90 per cent throughout 24 hr. the above results are changed as follows:

The I^2R losses per mile per year are reduced, to 88,500 kw-hr. or about 4/10 of one per cent with a saving in losses of about ¼ of one per cent of power delivered per mile of cable carrying load under conditions as assumed.

In these days when demand for electric energy is in excess of the available supply, the elimination of losses means additional power sold. On this basis the raising of the power factor of the load up to a constant value of 90 per cent and resultant reduction in losses of ¼ of one per cent per mile of cable would make available for consumers 55,500 kw-hr. per year for every mile of distribution cable in service under equivalent conditions.

If this power were sold at 3 cents per kw-hr. a revenue of $1665.00 per year would be received per mile of cable for what under the first condition went into heating the cable. Furthermore, the power would have become available during the day load period when wanted as shown by shaded area on Fig. 2.

POWER FACTOR AND REGULATION

Besides these primary evils of excessive capitalization costs and unwarranted losses, the secondary effect of bad regulation chargeable to bad power factor is in practical experience as serious as the primary evils.

The demand for excessive generator excitation under lagging power factor conditions so thoroughly appreciated by operators, though strictly beyond the scope of this discussion, cannot be passed unmentioned.

The excessive voltage variation produced in distribution wires and cables and especially on transformers by conditions of bad power factor are facts both within our field and worthy of consideration. This voltage variation is aggravated by conditions of power factor not only by the fact of the unnecessary magnitude of current for given power transmitted but in the case of the average condition, where the power factor is lagging, the delay of the current wave behind the voltage wave tends to make the voltage induced in the reactance of the lines and transformers directly subtractable from the generated voltage, while in case of unity power factor condition, this voltage is in quadrature and has small effect on magnitude of line voltage.

As an instance of this consider the average 60-cycle 2300 to 220/110-volt type of distribution transformer. Such transformers are commonly produced with resistance of from 1 to 1¼ per cent and with reactance of 2¾ to 4½ per cent over range of capacity from 1 to 250 kv-a. or more. Full load on transformer of this type with say 1¾ per cent resistance and 3½ reactance means a drop in voltage at unity power factor of about 1.8 per cent of line voltage which at 0.7 power factor lagging becomes 3.7 per cent drop or a little over double what it was with the same current flowing in transformer as at unity power factor or nearly 3 times the drop that would have occurred in delivering same power at unity power factor.

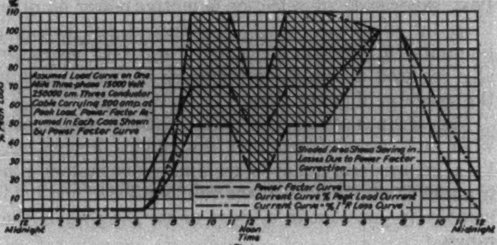

FIG. 2

In the case of the substation transformer working between voltages from 6600 to 15,000 volts or more down to voltages of 4600 volts or less, a representative example may be considered to have 1.3 per cent resistance and 6 per cent reactance. Here the full-load voltage drop with unity power factor would be about 1⅛ per cent of normal voltage, while with the same kv-a. load on transformer and power factor of 70 per cent lagging the drop would become about 5.2 per cent or nearly 4 times its previous value, and about 5½ times more than would have occurred on basis of equal power transferred. It is fairly evident therefore, that transformers become vicious voltage "ir-regulators" with power factor variation even at constant load.

SOURCES OF BAD POWER FACTOR

The review of these evils of bad power factor bring at once to mind the question of probable causes and available remedies for this condition.

THE INDUCTION MOTOR AND POWER FACTOR

Probably the most prevalent source of the trouble is found associated with the use of the induction motor.

The use of too large a motor, resulting, it is regrettable to say, from a misapplication of good intentions, is a practise which has been one of the chief offenders. Past experience in selection of motive power of other than the electrical variety has taught the lesson of liberality in selection of capacity and this has naturally resulted in the same tendency in many cases in the selection of motor sizes.

The seriousness of the result will be appreciated when it is remembered that the lagging magnetizing kv-a. on a general line of 60-cycle induction motors, runs from 30 to 60 per cent of the full load kv-a. and holds practically constant in a given motor over whole load range.

For example, consider an induction motor which at full load has a power factor of 85 per cent, at ¾ load it will drop to 77 per cent power factor, at ½ load it will drop to 62 per cent power factor, at ¼ load it will drop to 38 per cent power factor. Inasmuch, as the per cent magnetizing current generally increases with decrease of rated speed and with decrease of h. p. rating, it is at once evident that the small induction motor which forms such a large proportion of total connected motor load, is probably one of the most vicious of the species. This is due not only to the fact of the inherent low power factor at full load; but more particularly to the fact that due to its small size; its installation is given very little engineering consideration and capacities are frequently poorly selected.

This general condition has proved of sufficient practical importance to power companies to have resulted in certain of them in the cotton and woolen mill districts of New England accepting the responsibility of checking the customer's motor installations in this respect. The result of these investigations has generally been a more or less extensive overhauling by the customer of his motor installation for the purpose of operating all motors at better load factors, with resultant improvement in power factor highly satisfactory to both parties.

The power company is able, by the change, to furnish better voltage regulation at the same load, and with diminished losses, or handle an increased power load over the transformer and distribution lines at no increased cost for generating equipment, transformers or distribution copper; while the customer finds himself with an improved drive and generally a few idle motors on hand, for emergency service or extension purposes, which were previously tied up on his lines.

Activities in the industrial field which result as plainly in mutual good are highly inviting in these days of intensified group strife.

THE TRANSFORMER AND POWER FACTOR

The same bill of complaint can be brought against the transformer though in a lesser degree, for this appliance also takes a constant lagging magnetizing kv-a. at a given voltage; but its magnitude is smaller than with the induction motor, varying from 1 to 15 per cent of full load kv-a. as the higher percentage values of magnetizing current in general appear in power transformers, this larger lagging kv-a. in general is added to the over-abundance already being taken by the induction motors comprising the power load.

That this magnetizing kv-a. of transformers is not entirely negligible in extreme cases may be seen from the following:

A transformer with 15 per cent magnetizing current, carrying normal secondary load at unity power factor would take primary power at approximately 99 per cent power factor which would drop to 95½ per cent power factor if its secondary load were reduced to one-half normal and to 85½ per cent power factor if it were again reduced to one quarter normal.

Effect of Over-Voltage on Power Factor of Induction Motor

Even the values of magnetizing current just given for induction motors and transformers may be materially increased with the raising of the applied voltage above that for which the appliance is rated.

Since induction motors when run at 10 per cent over-voltage may easily take a magnetizing current 20 to 25 per cent in excess of that required, at normal voltage, it is evident that it is possible to find conditions where the disturbing effect of induction motors on power factor may even be much in excess of that previously delineated.

Over-Voltage on Transformer in Relation to Power Factor

With the transformer, the application of over-voltage is even more serious than with the induction motor, in this respect, due to the lack of any air gap in its magnetic circuit.

The pole type variety of this appliance, which is generally forgotten after once installed, and considered to be operating satisfactorily unless emitting smoke, though designed for low iron losses, will often sustain a 50 per cent increase of magnetizing current at 10 per cent over-voltage which amounts to 55 per cent increase in lagging kv-a. taken by the appliance.

The power transformer of larger size such as typically used for handling industrial customers who are supplied on separate feeders, or such as are installed in the power company's substation for reducing the higher distribution voltage to that of the substation's feeder bus, is a much more serious offender than the pole type in matters of the above nature. Not only do they take a larger percentage exciting current at normal voltage due to the fact that they are expected to operate at better load factor than transformers of the pole type, and, therefore, iron and copper losses are more nearly equalized; but also sustain a greater increase in this current with increase of voltage. In fact the magnetizing current may frequently double with 10 per cent increase in voltage above normal, making the statements previously given on basis of normal voltage conditions much too conservative. It becomes apparent, therefore, that in some cases, transformers also can make themselves distinctly felt as destroyers of good power factor conditions.

MISCELLANEOUS SOURCES OF BAD POWER FACTOR

Besides these causes of trouble, there must also be mentioned certain more specialized types of electrical appliances which inherently produce a low power factor load of which the bad power factor can only be corrected for, not provided against. Common examples of these types are electric welding machines, arc furnaces and some types of heating appliances.

The problem here, however, is a localized one of special nature, rather than a general condition and should be handled on that basis.

A more unusual and unique condition is that occasionally found in systems where the transmission network is very highly extended in proportion to load carried by it, with the result that the capacity current taken by the lines is so large in proportion to the load current that bad power factor means a leading power factor and an induction motor becomes a corrective device. Such cases are not hypothetical, for there comes to mind such a system in the Middle West adjacent to one of the standard lagging power factor characteristics where considerable consideration has been given to interconnection whereby each would act as a power factor corrective device for the other.

SYNCHRONOUS MOTOR IN GENERAL AS A REMEDY FOR BAD POWER FACTOR

In eliminating bad power factor at its source, a most valuable ally is the synchronous motor. This device has been on the market for twenty years or more; but until recently was considered so limited in its application, both on account of its torque characteristics and greater trouble in starting, due to the necessity of switching on field and synchronizing, which were wont to be considered as delicate operations; that it is probable that less than 5 per cent of the motor horse power in this country today is of synchronous variety.

Recently this motor has begun to come into its own due to steady improvement in its design characteristics and simplification of starting equipment, and also to increasing familiarity of operating public with this type of device. Slowly this motor has enlarged its usefulness until it now may properly be given consideration on any constant speed application which can be handled by an induction motor.

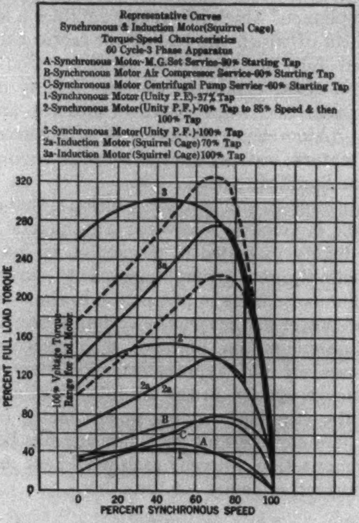

FIG. 3

The fact that the synchronous motor has actually been applied with good results to the drive of compressors, grinders, stone crushers, pumps, both centrifugal and plunger blowers, steel and copper rolls, cement and rubber mills, flour mill rolls and line shafting in general, is concrete evidence of the fact that this motor has already covered a wide field in its application.

UNITY POWER FACTOR SYNCHRONOUS MOTOR

When operated without capacity for over-excitation of its field, this device takes energy at unity power factor and even then, in a real sense, is a power factor corrective device, for it relieves the system of the lagging kv-a. that would have been added by the induction motor and thus by the substitution, really furnishes corrective kv-a. to the system of probably 30 per cent of its normal rating.

Synchronous motors of this particular type have recently become available in a consecutive line over approximate range from 50 h.p. at 1200 rev. per min. to 450 h.p. at 600 rev. per min. specifically designed to invade as far as possible, the field of motor application now held by the induction motor, with representative results for particular ratings which are very encouraging, as to the future of the device.

Referring to Fig. 3 and 4 giving comparative representative torque-speed and kv-a. input-speed curves respectively for standard synchronous motors, induction motors and unity power factor synchronous motors above mentioned, it may be interesting to note that with about 60 per cent as much kv-a. the recently designed unity power factor synchronous motor will easily duplicate the starting torque characteristics of the standard synchronous machine for motor-generator set service. Comparative inspection of curves 2 and 2*A*, Fig. 3, giving relative torque characteristics of unity power factor synchronous and squirrel-cage induction motors when operated from 70 per cent voltage tap reveals the fact that the synchronous motor up to about 85 per cent speed delivers well above full-load torque and from kv-a. input curve 2, Fig. 4, it is evident that maximum input is approximately only 3½ times normal kv-a. At speeds above 85 per cent the torque characteristics of these two types are very similar. It is also entirely feasible at the 100 per cent torque, 85 per cent speed condition to throw full voltage on the synchronous motor when it would pass over to full-voltage characteristic and again take a maximum of about 3½ times normal kv-a. and develop a torque which would not drop to the normal full-load value until 95 per cent speed were reached. From 95 per cent speed and full-torque condition it would then pull into step, by application of the field. In other words, this last fact means that with motor at full-torque load and synchronous speed the field could be removed and motor would drop to 95 per cent speed only and again pull into step with reapplication of the field. It is clear also from full-voltage curves, that the maximum torque of this type of synchronous motor is entirely comparable with that of squirrel cage induction types.

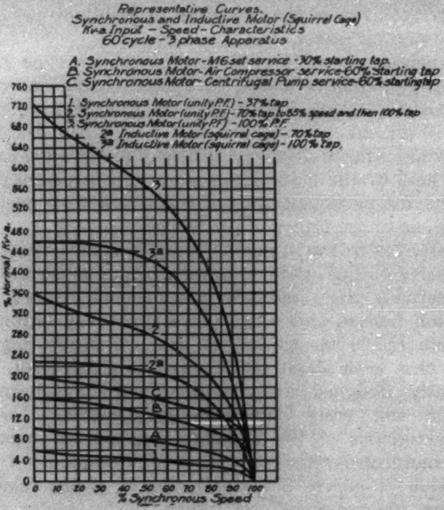

FIG. 4

Nevertheless it must not be considered that such a motor will handle any application which the induction motor, even of squirrel-cage design, can; for as a general thing the squirrel-cage induction motor will develop the comparative starting torque for a much longer time with the same temperature rise in the squirrel cage winding, than will the synchronous competitor. Therefore, when the total amount of momentum to be stored up in rotating mass of the driven appliance, during starting cycle, is very large, it is possible to exceed the safe temperature limits of the starting winding in the synchronous machine.

This class of load is apt to trouble even the squirrel-cage motor and, by the conservative, would be assigned to the field of the wound-rotor induction motor, which by proper selection of external resistance can be arranged to give maximum torque at any speed up to full-torque speed and, due to considerable heating being in grids, external to machine, has capacity for maintaining large torque for comparatively long periods of time.

Since the wound-rotor induction motor, due to higher cost, is generally reserved for installations with exceptional starting duty requirements, or speed variation capacity, it is not so general in its use and comparative in its duty, as is the squirrel cage type. Due to this, in conjunction with the fact that the torque-speed and input-speed characteristics are functions of the starting resistance and result in an infinite series of curves to cover all possibilities, no attempt has been made to include the wound-rotor induction motor in the detailed comparison, though in some applications, even this device can be supplanted by the synchronous machine.

It is evident, therefore, that considerable relief from the extension of bad power factor in the future, should be obtained by the increased use of the unity power factor synchronous at the expense of the induction motor. This method of treatment is especially advantageous as it eliminates the lagging kv-a. of the induction motor that would have existed instead of correcting it after it has been added.

PHASE MODIFIER AS A POWER FACTOR CORRECTIVE DEVICE

Another method of bad power factor elimination which is better known in European practises than with us, is the use of the phase modifier. This scheme is practically that of furnishing an exciting voltage of proper magnitude, frequency and phase relation to the rings of the wound rotor induction motor, the power factor of which it is purposed to raise. The limitations of this device from an engineering standpoint are due to the facts:

(1) That it is impractical to use with a motor which is frequently started.

(2) That the amount of correction is dependent upon the design and also load on the induction motor, is unsusceptible of variation at will, is practically limited by relative cost of modifier to corrective necessary to bring the induction motor load to unity or less.

(3) That it is not operative without the induction motor.

(4) That it is applicable only to constant speed, non-reversable, wound-rotor induction motor.

(5) Finally that the scheme necessitates an additional machine either driven separately or by the induction motor.

The commercial possibilities of the device lie in offsetting the cost of the additional machine, by the saving in cost of the induction motor designed for high iron densities and resultant high magnetizing current, and by the capitalization of the increase in power factor obtained by modifier over what would have been obtained from standard designed induction motor. The fact that the standardized factory production methods of the country make it very difficult to lower induction motor costs by deviation from standard design, may possibly account for the limited exploitation of this method of power factor correction within our own field.

OVER-EXCITED SYNCHRONOUS MOTOR AS CORRECTIVE AGENCY

With the application of the synchronous motor, with ability for over-excitation to a system, corrective capacity can be obtained for bad power factor conditions already existing in amounts variable at will up to the limits of the machine and in a very economical way. This fact is evident when it is remembered that the additional corrective kv-a. of the motor does not add directly to the load kv-a. with the result for example, that a motor having a rating of 80 kv-a. at unity power factor would have a rating of only 100 kv-a. at 80 per cent leading and yet would add to itself a power factor corrective capacity of 60 kv-a. That is to say, by increasing the machine rating 25 per cent above unity power factor rating, a corrective capacity is obtained equal to 75 per cent of the original rating of the machine.

Wherever, therefore, it can be arranged to use a synchronous motor for power purposes, it can be selected to furnish a considerable amount of corrective kv-a. at a very low first cost and operating expense, as the major portion of both is chargeable to the power load.

A notable example of the application of this principle is found in the use of the synchronous motor-generator set in the substation. Where there is a combined demand for direct current, generally for traction purposes, and for a-c. energy localized around one substation, it is very convenient to obtain the direct current from the generator of a motor-generator set and power factor correction for a-c. load from synchronous motor of the same set.

Although the 60-cycle railway converter both as a normally and automatically operated device is on firm ground as a satisfactory piece of electrical equipment, the motor-generator set on account of relatively large capacity for power factor correction is a very active competitor of the synchronous converter in many cases and lends itself admirably to any demand for automatic control.

SYNCHRONOUS CONDENSER AS A POWER FACTOR CORRECTIVE APPLIANCE

When the demand for over-excitation on a synchronous motor is pushed to the limit and becomes entirely dominant in the design, we have the synchronous condenser, which is a motor to the extent only of revolving itself and a provider of leading corrective kv-a. to practically its total rating.

The adaptability of this machine is very high as can be illustrated by the fact that it is made in voltages from 220 or below to 13,000 volts or better in sizes from approximately 100 to above 30,000 kv-a.

The secondary effect of the variation of voltage drop, due to changes in power factor, has so often been made use of in connection with the synchronous condenser, that it is often used primarily as a voltage regulator. It lends itself admirably to this purpose as the excitation of the condenser can be automatically controlled so that by thus varying the power factor of the line, constant voltage can be held within the limits of the capacity of the machine. The condenser may even be arranged to place itself on the line and take itself off again as need for it is evidenced by the value of the line voltage; or even as determined by the dictates of a time clock.

The disadvantage of synchronous condensers lie in the fact that there is no power output which can be called upon to share the cost and operating expense with the power factor corrective capacity and these factors therefore, mount rapidly as compared with the synchronous motor situation. As a power factor corrective device only, it is, however, easily the leader in general usefulness and without it large problems of a-c. distribution and transmission would be difficult of solution.

STATIC CONDENSER

One of the newer agencies for power factor correction which overlaps in its field of application, the low capacity end of synchronous condenser territory, is the static condenser which is nothing more than the commercial application of the "Leyden Jar" to the problem of power factor correction.

As compared with the synchronous condenser, it has the following disadvantages:

(1) Is practically of fixed capacity as operated.

(2) Corrective capacity is leading only, not leading or lagging as desired, as with synchronous condenser.

(3) Takes more floor space per kv-a.

(4) In large sizes costs more per kv-a.

These shortcomings far from remove its usefulness, as it has the following outstanding qualities of merit.

(1) Very low losses (1 to 2½ per cent, while synchronous condenser losses vary from approximately 10 per cent to 3 per cent with increase of size).

(2) No rotating parts, therefore, no lubrication costs. no bearing wear and no noise.

(3) Less attendance.

(4) Easier to put on or take off line—accomplished simply by closing or opening an oil circuit breaker.

(5) Operates at low temperature rise and, therefore, does not heat up operating room as much as a synchronous condenser.

The limits of the field of application of this type of apparatus in competing with the synchronous condenser, are for practical purpose those of economic balance and not those of design, for the static condenser can be furnished in any capacity by continued multiplying of condenser sections. The practical usefulness of the static condenser can be best exemplified by the fact that there is today in process of manufacture a total kv-a. capacity, equal to the total capacity produced over the whole eight or nine-year period of its manufacture.

POWER FACTOR CORRECTIVE ENGINEERING

Having arrived at the conviction that power factor correction is generally a thing to be attempted it might be worth while to point out that the sane way to go about it is:

First—To locate its causes.

Second—Remove the causes insofar as practical.

Third—If correction is still needed, install corrective devices as near the source of bad power factor as possible, limited by the economic fact that power factor corrective devices cost more per kv-a. in small units.

The limit to which correction should be carried may be arbitrarily set:

(1) By voltage regulation requirements.

(2) By balance of yearly value of savings against yearly cost of correction.

(3) By economic balance as described under (2) supplemented by power factor penalties and bonuses (one or both) in power rate.

(4) By necessity of carrying more power over available distribution lines irrespective of economic balance.

(5) Or even, by condition that has recently risen in the East, when for lack of available power, customers were limited by power companies to a certain kv-a. demand. In such a case, it was found advisable for certain customers to correct the power factor absolutely up to unity, so critical was the need for the additional power. The urgency of the case can be better appreciated when it is remembered that for addition of say 10 per cent of load kv-a. in power factor corrective kv-a. to load, with an initial power factor of 70 per cent the reduction in load kv-a. would be about 7½ per cent of original kv-a. load. By addition of successive amounts of corrective kv-a., 10 per cent addition at 80 per cent power factor condition would reduce kv-a. by about 5½ per cent of original kv-a., and last 10 per cent necessary to bring load up to unity power factor

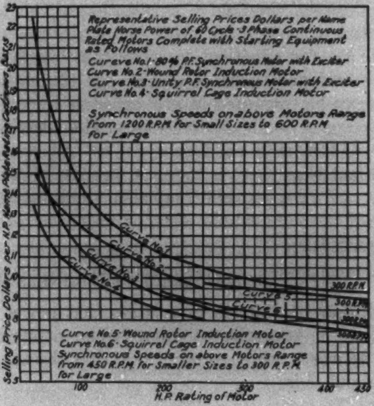

FIG. 5

would effect a reduction in load kv-a. of only 8/10 of one per cent of original kv-a.

ANALYSIS OF THE COST OF DIFFERENT POWER FACTOR CORRECTIVE DEVICES

The final question in this situation is sure to be, "what will it cost?" and some general facts on this may be fittingly considered.

Referring to Fig. 5 for approximate information on 60-cycle motor prices—curve No. 1 gives price per horse power of 80 per cent power factor synchronous motor complete with exciter and starting equipment in capacities ranging from 50 h.p., 1200 rev. per min. to 450 h.p., 300 rev. per min.

Curve No. 2 furnishes equivalent information on wound-rotor type of induction motor complete with starting equipment in sizes 50 h.p., 1200 rev. per min. to 250 h.p., 600 rev. per min. and continues in curve No. 5 over a range from 250 h.p., 450 rev. per min. to 450 h.p., 300 rev. per min.

Curve No. 3 covers the unity power factor synchronous motor complete with exciter and starting equipment, over range from 50 h.p. and 1200 rev. per min. to 450 h.p. and 300 rev. per min.

Curve No. 4 represents prices of induction motor, of squirrel cage variety complete with starting equipment, in ratings from 50 h.p., 1200 rev. per min. to 250 h.p., 600 rev. per min. which continues in curve No. 6 over range from 200 h.p., 450 rev. per min to 450 h.p., 300 rev. per min.

Using the above price curves as a basis supplemented by assumption of 30 per cent exciting kv-a. for induction motors throughout, we have Fig. 6 on which curve No. 1 represents the power factor corrective kv-a. that would be added to systems by substitution of a unity power factor synchronous motor for induction motor of same rating, plotted against the motor rating.

Curve No. 3 represents the power factor corrective kv-a. obtained by similar substitution of 80 per cent leading power factor synchronous motor.

Curve No. 2 shows price per kv-a. of power factor corrective capacity obtained against the size of each of the two motors wherein substitution was made in case where unity power factor synchronous motor was used as a substitute for squirrel cage induction motor, while curve No. 4 gives equivalent information in case 80 per cent power factor leading synchronous motor were substituted.

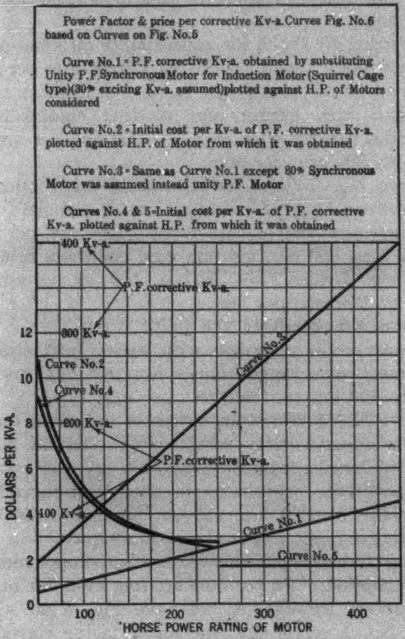

FIG. 6

Curve No. 5 continues the comparison over a range from 250 h.p. at 450 rev. per min. to 450 h.p. at 300 rev. per min.

The comparison fails in this range for unity power factor size motor, as it probably will not cost more than the induction motor of squirrel cage type.

The interesting fact becomes evident that in case either substitution is practised the price per power factor corrective kv-a. obtained is about the same for a given motor size although of course the total amount obtained is approximately three times as much with 80 per cent power factor motor as with unity power factor variety. The choice of type of synchronous motor will therefore be made on the basis of relative characteristics and amount of corrective kv-a. desired.

In Fig. 7, we have a comparison of price per kv-a. of obtaining power factor corrective kv-a. by the several methods.

Curves 1, 2, 3 and 4 give price per kv-a. of static condensers complete for voltages of 220, 440, 550 and 2200 respectively; the variation in price being accounted for by the fact that the condenser voltage in 200, 440 and 550-volt instances is reduced by an auto-transformer to meet rated voltage, and the greater the reduction, the greater the auto-transformer price per kv-a. of static condenser.

Curve No. 5 represents the price per kv-a. for 60-cycle, three-phase size condenser complete with exciter and starting equipment and it should be noted that for small sizes even the initial investment is less for the static condenser.

Curve No. 6 shows price per power factor corrective kv-a. obtained by substituting unity power factor synchronous for squirrel cage induction motor, plotted against the amount obtained over the range of motors 50 h.p. to 250 h. p. giving a maximum of approximately 65 kv-a.

Curve No. 7 brings out the cost per power factor corrective kv-a. obtained by substitution of 80 per cent power factor synchronous motor for squirrel cage induction motor over range 50 h.p., 1200 rev. per min. to 250 h. p., 600 rev. per min. giving a maximum of about 223 kv-a., and curve No. 8 continues this information over motor range from 200 h.p., 450 rev. per min. to 450 h. p., 300 rev. per min. reaching a maximum available power factor corrective kv-a. of approximately 400.

A casual observation of these curves should convince one of the advisability of using the substitution method of obtaining power factor corrective kv-a. to the extent that it is possible of application.

The limitations of the substitution method are obviously due to necessity of disposing of replaced induction motor at no loss to comply with the assumptions. In case of new installations, this is entirely applicable; in other cases, the prices given must be increased to cover loss, if any, on replaced motor.

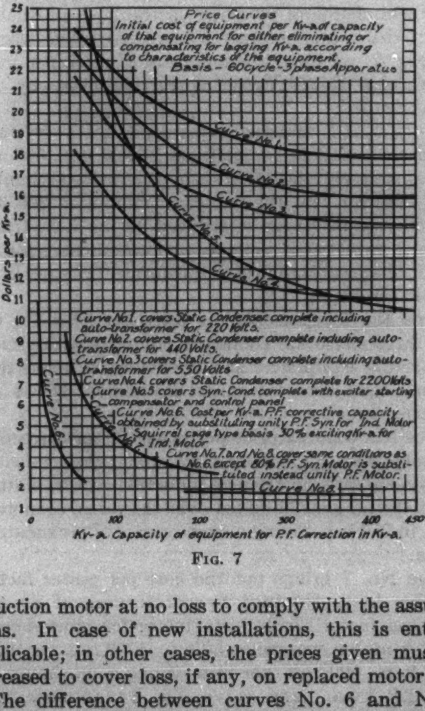

FIG. 7

The difference between curves No. 6 and No. 7 should not be misconstrued. From curves it would seem that a cheaper way to obtain given corrective kv-a. can be found with the use of unity power factor motor in substitution game, than by the use of 80 per cent power factor type. It should be remembered that to get the same amount of correction as can be obtained by substituting a 100-h.p. 80 per cent power factor synchronous motor for a squirrel cage induction motor, it would be necessary to substitute a unity power factor synchronous motor for an induction motor of about 350-h.p. rating. As the cost per corrective kv-a. goes down with increase in size of motors considered, it is seen that the conditions above stated are not justly comparable. It is much better to remember that for a certain motor in question to be substituted the price per corrective kv-a. obtained with either type of synchronous motor is about the same but the amount is greater with the 80 per cent power factor type.

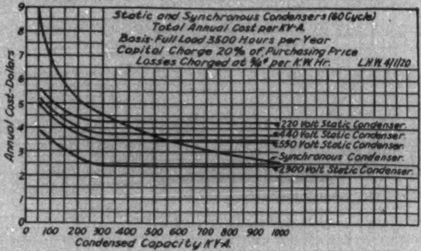

FIG. 8

There has been no attempt to include the capitalization of the difference in efficiencies of the two general types, *viz.* synchronous and induction motors. It is probable that the synchronous variety would have the better of it by 1 to 2 per cent at full load and add something more to the reasons for the change.

When no more relief can be obtained in the battle against bad power factor by procedure above outlined, the synchronous and static condenser become the logical subjects for consideration.

Here the relative efficiencies are at so great a variance that an analysis on the basis of yearly expense composed of capital charge, to cover interest on purchase price of apparatus and depreciation of same, plus losses at a price per kw-hr. must be made.

Referring to Fig. 8 will be found such a comparison for synchronous and static condensers over rating from 60 to 1000 kv-a. and standard voltages of 220, 440, 550 and 2200 volts. Each equipment is charged in each respective capacity with 20 per cent of first cost, as capital charge per year and to this has been added the losses on basis of full-load operation for 3500 hours per year at ¾ cents per kw-hr.

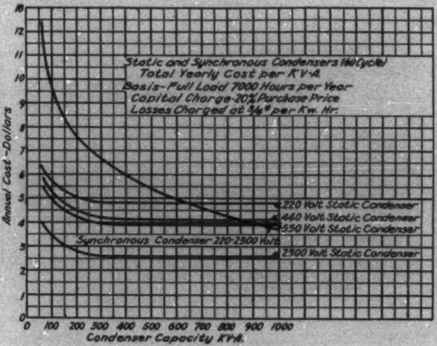

FIG. 9

On this basis it will be noticed that the static condenser in lower voltages is the cheaper device to own and operate in sizes approximately 500 kv-a. and below, and in case of 2200-volt apparatus the static device maintains its superiority up to capacities of about 1000 kv-a. If it were more reasonable to assume 24 hours per day operation on all days except Sundays and holidays as an equivalent basis of comparison and leave other assumptions untouched, results will be obtained as shown in Fig. 9 when the range of superiority of the static condensers may be said to have been increased 50 per cent making thereby the static device cheaper, in lower voltages in capacities of 750 kv-a. and below, and at 2200 volts in capacities well above the 1000-kv-a. rating.

Thus for cases where the condenser capacity is needed in amounts of approximately 750 kv-a. or less with a certainty of pretty constant operation and no demand for automatic voltage control, it is well to remember the static condenser as a worthy competitor of the synchronous device.

CONCLUSION

There is right now a peculiar pertinence to a review of a field so full of possibilities of reduction in delivered cost of electric power. Civilization's hope of the future lies in man's growing ability to produce per unit of time with resultant reduction of cost in terms of unit wage. Man's greatest aids in this battle are machines and power; which is growingly electric power.

Thus the delivered cost of electric power is becoming incorporated more and more widely into the cost of the necessities of life which is making the cost of electric energy a question of universal interest, and the engineer, if he maintains his expected measure of usefulness, must analyse his power producing and distributing plant today as never before, with the hope of new savings not only for his own and employer's good, but for general good as well.

It is hoped that the foregoing general facts may be fruitful to you in ideas which may aid in the broad problem before us all.

DISCUSSION ON "POWER FACTOR CORRECTION ON DISTRIBUTION SYSTEMS" (JONES), PORTLAND, ORE., JULY 23, 1920.

J. E. Woodbridge: When you read Mr. Jones' summary of the various things that will reduce power factor, it is a wonder that we have any power factor left, especially as he has not really told the whole story. Consider a transmission system. We generate our power in hydroelectric plants, if we are lucky; and then we proceed to step it up in one bank of transformers, and we transmit it somewhere and step it down to a secondary transmission voltage—two banks of transformers. Then we take it along to another substation and transform it down to a distribution voltage—three banks of transformers. Then we take it further and transform it again to the consumer's voltage—four banks of transformers, each with the magnetizing current Mr. Jones has told us about and each with another element that he has not told us about that comes in under load, and that is the drop due to imperfect interlinking of the magnetic circuits of primary and secondary, giving a series reactance. Our lines also, as soon as we load them, introduce a reactive element, which piles up still more lagging kv-a.; and of course the same is true of the motors as Mr. Jones has pointed out.

It is unfortunate that we have gotten in the habit of looking at this matter from the point of view of power factor. If we looked at it from the point of view of reactive factor we would see the serious side of it much more clearly. It takes a lot of reactive factor to give an appreciable effect on the power factor. For example it takes 14 per cent of lagging load to reduce the power factor one per cent from unity to 99. We are in the habit of thinking that 99 is about as good as unity. Well, it is pretty good, but it means a 14 per cent reactive load which, though it does not affect the heating very much, has a very serious effect on regulation.

W. J. Davis, Jr.: There is one point that Mr. Jones might have emphasized, namely—the important part that the synchronous condenser has played in the increased use of 60 cycles as a standard frequency for power transmission. When we investigate the feasibility of transmitting large quantities of power over long distances, requiring a transmission potential of 100,000 or 150,000 volts, we find that in many cases, the line regulation at frequencies as high as 60 cycles becomes inherently too poor to prove acceptable or satisfactory for power and lighting requirements. This condition is due to the high line reactance and heavy charging current at this frequency and the poor power factor of the load obtaining on systems supplying power for industrial purposes. By the proper use of synchronous condensers the line regulation of these large high-voltage systems may be made to successfully meet the most stringent requirements. Indeed, the experience with high-voltage long distance transmission lines using a frequency of 60 cycles with synchronous condensers at the receiving end has been that the service given to the consumer has been very much improved.

When the conditions of quantity of power and distances of transmission are such as to require the use of very high voltages, say 150,000 or greater, the use of synchronous condensers also permits the use of smaller units in the power plants than would otherwise be required. In the proposed Pitt River development of the Pacific Gas & Electric Company for example, the charging current of the transmission line is calculated to be about 45,000 kv-a., which is beyond the economical capacity of a water wheel and its generator in the power plant, necessitating the use of a generator very much larger in capacity than the prime mover, or requiring a complicated arrangement for throwing two units together in getting on the line. By the use of a suitable synchronous condenser at the receiving end of the line, or some other suitable location, the size of the units at the power house may be reduced to that which will prove most economical as fixed by the quantity of water available and the head under which the water wheel must operate.

L. T. Mervin: The paper points to a matter which I am sure is of extreme concern to all utilities. All of us have low power factor, and most of us of a distressingly low value, and we are all greatly concerned as to how to correct it. Low power factor requires increased investment over and above that necessary for the production of the energy actually delivered, and the question in the minds of all of us is, not merely shall we pass the burden of this increased investment on to the consumer, but how shall it be done. I had hoped that the paper would dwell somewhat on suggested methods of charges, tariff rates, etc. Occasionally articles do appear in the technical journals bearing on this point, and there have been recently a number of valuable suggestions, but so far no proposition has been advanced that is broadly applicable to the problem so far as I know. If there are any here who have, in their own judgment, arrived at some satisfactory power factor clause in their power contracts, I, for one, would be very grateful if they would expand somewhat on their method and tell us how satisfactorily it has worked. There are two broad phases of the power factor question that must be faced by each utility, one the purely engineering aspect of construction and operation, the other concerns the economics of the problem.

On the operating end our experience shows, in the Company with which I am connected, that we get more intelligent system operation and voltage control by the operators themselves if we have no power factor meters at all on our switchboards, but substitute instead reactive component meters, which we find much more valuable from an operative standpoint. Using the rather loose technical term of "magnetizing current" for the reactive component, and marking upon the double scale of the instrument "Incoming" and "Outgoing," the operator is not bothered with the thought of whether the power factor, on a tie-line for instance, is "leading" or "lagging." His main concern is the direction of the flow of this magentizing current and the method of placement throughout the system by voltage control.

The operator soon comes to realize that under fixed load conditions on the system the quantity of magnetizing current required is also fixed and all that he can do through voltage control is to shift the burden of magnetizing current supply as between the various synchronous units on the system. I frequently use the term "system power factor." In using this term I realize I run the risk of criticism by the technical purist. It is a convenient term for the lack of a better, and merely means the arrival at the well known ratio of the power triangle by using the total kilowatts generated on the system, and the total magnetizing current also on the system, as shown by wattmeters and reactive component meters. The shifting of the burden of magnetizing current is very readily handled over transmission and tie lines by the station operators on observing and studying their reactive component meters.

On the economic side the broad problem is to shift as much of the burden of poor power factor that would otherwise be imposed upon the power system onto the consumer by requiring him to supply the magnetizing current of his own load or paying the power company for carrying the burden through properly designed tariffs. I feel that even were the power company adequately paid under his tariffs for the investment required to carry magnetizing current, sound economics of the question are not then totally satisfied. It is an economic waste to the utility itself to have to sacrifice generating capacity for the production of reactive power.

H. J. Ryan: I want to join with others in emphasizing the importance of this paper and the suggestion made by Mr. Woodbridge. *Power factor* is the ratio of *real power* to the total volt-amperes; *inductive factor* is the ratio of the volt-amperes consumed inductively to the total volt-amperes; correspondingly *condensive factor* might mean the ratio of the volt-amperes consumed condensively, *i. e.* by a static or synchronous condenser, to the total volt-amperes. These things are of the highest importance in transmission and distribution. In most respects as a practical proposition they are simple and not complex. We should know them by a few simple terms. We should form and maintain the habit of using such terms often. In delivering 600 kw. to an induction motor driven factory at a power factor of 60 per cent it is necessary to mobilize along therewith 800 induction-reactive kilovolt-amperes. There is herein a psychological matter of real importance. The habit of quoting only the power factor was formed years ago to meet the need of keeping in mind the kilovolt-ampere loading capacity of machinery and auxiliaries in relation to the kilowatts employed. Now-a-days, as the present paper abundantly demonstrates, voltage regulation and distribution require due attention with respect to the reactive volt-amperes in circulation. Frequently in these respects such reactive volt-amperes are more controlling than the accompanying kilowatts. If we bear in mind that in certain essentials the induction factor is of greater importance than the power factor and if we use the factor as often as we should those who control the disbursement of funds will be as willing to authorize the purchase of inductor factor as of power factor meters.

R. F. Hayward: With reference to the question whether we should talk about power factor or reactance factor, I want to see a meter that throws on a screen in sight of the operator the actual vector diagram showing the angles of phase displacement. It is easier to train operators through geometrical pictures than in any other way; and if you could only produce an instrument that would flash vector diagrams on a screen, I think it would be a tremendous help, in regulating generators.

We have a case in our power plant at Stove Falls, where we are supplying one line of transmission to the Puget Sound Traction Light & Power Company at Bellingham and the other line goes into Vancouver. Bellingham sometimes wants a voltage which varies differently to the voltage requirements at the Vancouver end, and we have no means at that station of independently regulating the voltage without putting the two lines on different generators, which would be uneconomical. Consequently we occasionally have, simply through means of excitation, to create an otherwise unnecessary voltage drop on the Vancouver transmission line in order to keep the voltage correct on the Bellingham line. The simple correction of course would be a regulator placed on the Bellingham line, but the regulator itself would cost between $10,000 and $12,000 installed, which is one third of the original cost of the generators. That is only one case of power factor correction.

I admit that I have needed education in the matter of power factor, for this reason: Starting very early in supplying power it was always impressed upon me that the power company's business was to give the service to the customer in the way in which the customer wanted to use it. Now those were early days. Our first motor installation in Salt Lake City was for the cement works, which we operated with a synchronous motor, and that being an old time synchronous motor; was possible only because we had a large clutch on the pulley for starting purposes.

Now there are very many mechanical installations where it is not at all necessary to have a heavy load for starting, and where a clutch is perfectly satisfactory; and therefore the synchronous motor can be used very much more than we usually realize.

To revert to my ideas of the power company providing for all of these things, at first it used to seem to me it was up to the power company to put in transformers that were big enough, and lines that were big enough to provide for low power factor and starting current in order to be sure that the power user got his power in good shape. That was in the days when we didn't know much about power costs. But as one studies power costs, the other phase of the situation becomes apparent and shows how vitally important it is to keep power factor corrected at every possible point. As a practical point, all air or ammonia compressors are a most valuable help, because they can be driven by synchronous motors, and usually run continuously.

Contracts made in Canada with the big grain elevators generally provide that the elevator-operators shall correct their own power factor. The grain elevators are always heavily over motored, and the power factor is consequently low.

With respect to Mr. Merwin's question as to contracts, I can only state what is our practise in Vancouver. For any power factor between 80 and 100 per cent the customer pays only on kilowatts; if he goes below that his demand is computed as 80 per cent of the maximum kilovolt amperes.

H. T. Plumb: In many localities the power companies have been very negligent in the matter of giving their customers encouragement in keeping their power factor high. They have been in fact nearsighted in the matter; and now they are reaping the reward, and are trying to get out from under what is really their own fault. Some of their customers have been far more long headed than they have, for which these can be thankful now that the power companies are waking up and are beginning to penalize low power factors with higher rates. On the other hand, most consumers have been small consumers. They have not taken the time or thought to inquire into matters of power factor. They have been ignorant, and they have over motored, in other words, used motors that were too large. That is true especially of small users. Some of the larger consumers who have had electrical engineers on the job have realized the errors of using motors that were too large for the applications and have kept the size of the motors down and overloaded the motors, and they have done well except that they have also gone to extremes. In many cases the people who were careful to keep their motors not too large, have used too high a voltage on the motor, and if they would take pains to put a power factor measurement on that motor, they would find that they had a big magnetizing current due to the extremely high voltage. So they made an error which they were not looking for in trying to avoid the other.

In the intermountain region there are a number of applications made of synchronous motors. In the first place, we have many irrigation districts where much water is to be handled with low head pumps, low-speed motors therefore. We have been putting in, in every case where we could, synchronous motors, and they do the work most beautifully. In very low speeds they are even cheaper to buy than induction motors, and except for a little fear on the part of operators they are easily handled as induction motors. In fact, we have some synchronous motors installed on pumps where there are no meters, no field rheostat handles, and the outfit is started by an ordinary compensator the same as an induction motor. The operator himself may not know the difference when he starts up one of these synchronous motors and throws it on the line. Excitation is fixed at the proper point where it will give him the proper power factor.

I am sure besides low head pumping there are very many other places where synchronous motors could be introduced to very good advantage, not only to the consumer but especially to the power company. We have tried in connection with mine haulage to install motor-generator sets with a synchronous motor for that reason. The coal cutting machines are driven by induction motors; the tipples are driven by induction motors; and all those loads of course, have lagging power factor. By putting in synchronous motors on the mine haulage motor-generator sets we are able to correct the power factor and give each mine a very good power factor on the average. That applies especially to the coal mining districts, but also to metal mining localities. On the power companies' long distance transmission lines synchronous condensers are used for regulation. The scheme used by the Utah Power & Light Company in transmitting over their 130 kv. line from Idaho to Salt Lake is to maintain a constant generator voltage at the generators in Idaho and deliver a constant voltage at the bus bars in Salt Lake by means of synchronous condensers whose excitations are controlled automatically by Tirrill regulators. This is found to be the most flexible way to work and the generator voltages practically never are changed except on the fourth of July or at some such a time when the big metal mills drop their loads. In passing I might state that some of these metal mill loads have an average yearly load factor of ninety per cent.

Other instances might be of interest. A large lumber mill in Idaho had a water power plant for driving their motors. Their center of distribution was about a thousand feet away from the power plant and they had gotten to the limit of voltage; they had 440-volt motors and they could not get more than about 370 volts on any of these motors because this thousand feet of transmission line was built of bare wire spaced so far apart that the reactance was excessive. They asked me to come up and look it over. I suggested putting the wires closer together, but found they could not do this because they were in a canyon where the wind blew hard and the bare copper would swing together and make short circuits. Finally, we decided that a synchronous condenser was the thing to use.

They had a blower which ran twenty-four hours a day, and we replaced its induction motor with a synchronous motor a little larger and with some excess exciting capacity. Now they are able to get full voltage over the same transmission line. This is a case where they had reached the limit of the transmission line; they could not add more notors, and they were not properly operating the motors which they had; low voltage slowed down the whole process of the sawmill. Now they are up to speed again and up to voltage and everybody is happy.

There is another instance where capacity had been reached, but this time differently. I was telling that story about the sawmill to a friend who operates a factory where they take potash out of the waters from Salt Lake. He believed we could do something of the sort at his place, and brought in a little diagram of connections. I found that he could not prevent line drop because his center of distribution was the power house. He was disappointed. He came back in a day or two and said "We are up against it for power at our place. This year we must have another generator." I found that he had two engine-driven generators loaded to full capacity, and they were adding more motors. What we did for him was to recommend that he buy two kinds of condensers, first, that he buy a steam condenser for his engines so that they could carry the increased power load, and then that he also buy a synchronous condenser to carry the wattless current of his motor loads. He did so, saving the buying of additional boiler capacity and additional engine and generator capacity, simply by installing a small synchronous motor. The synchronous motor is replacing one or two induction motors at the same time; so it is killing two birds with one stone. This is a case where the limit of generator capacity had been reached and a small synchronous motor saved the day and saved considerable money, perhaps twelve times as much as the motors cost.

Another place where synchronous motors are coming into use, is in the large metal mills. Those mills have induction motor drives throughout. In one mill there is a load of about 20,000 kw. with a load factor of about ninety per cent; and it is all induction motors. In order to reduce the enormous copper losses and to keep down the investments in copper, which were still more enormous, they sought some application of synchronous motors and found they could drive air compressors, vacuum pumps and machines of that class very nicely by synchronous motors. They have installed about twenty fairly large motors, and these give a very much better power factor. They are able to add very considerably to the number of motors in their mill without increasing their transformer capacity and without increasing the size of transmission lines.

H. V. Carpenter: I wonder if it is commonly appreciated that the reading of the reactive factor meter as compared with the reading of the watt meter is a direct measure of the extra losses that you are throwing away in your lines? We take the vector diagram, add up the useful current in one phase and the wattless current at ninety degrees to get the total current used; but to get the losses in the copper due to those currents we must add the two losses arithmetically. Each current causes loss independently of the other, as we see by examination of the formulas,

$I^2 = i^2_r + i^2_x$, so $R I^2 = R i_r^2 + R i_x^2$.

Mr. Hayward's notion of a little vector diagram right before the operator is rather startling, but I think he is absolutely right in saying that you can tell the operator, "this vector must not be here; so adjust your instruments until it disappears." You can tell your operator that and get away with it, whereas if you talk power factor to him very much he will not appreciate it.

There is one point in connection with the application of static condensers which I think should be mentioned. That is the different fashion in which they are affected by a drop in voltage as compared with synchronous condensers. Suppose we have two lines just alike, both operating with very bad power factor, and we correct one with synchronous condensers and the other with static condensers; suppose something comes on which pulls the voltage down ten per cent; the static condensers will balk just to the extent of that ten per cent, while the synchronous condenser will give an increased correction. Lowering the applied voltage on a synchronous condenser gives you the same effect as raising its excitation, in that it makes it give you a larger corrective action. Probably Mr. Jones can give us some further light as to the extent of that effect; at any rate we should remember that the synchronous machine will automatically adjust its capacitive correction in the right direction while the static condenser inherently corrects most when least needed.

S. C. Lindsay: This matter of power factor correction is a real live issue at this time with the company with which I am connected. We are making a very careful study of it and one of the most amazing things is that power companies all over the country, including our own have let this situation grow to the extent that they have before commencing an analysis of what it was costing them to have these large reactive components on their system. In the analysis we are making we are trying to figure the thing right down to the last point and find out just what it is costing us, we can afford to spend to correct it, and what will be the net gain after we make our investment.

We have in our system a rather peculiar condition. We have three water power stations tied together. We have to operate with practically fixed voltages at each of those stations. We have some radial lines running from one of those stations, on which there is a heavy low power factor load. The load on the radial lines frequently calls for a higher voltage at the step-down substations than we are able to furnish, by reason of the base voltage at the generating station. If we raise the voltage of one station a little it throws a very bad power factor on the tie-lines. If we raise the voltage at say our White River station we still try to hold a fixed voltage at the Snoqualmie station. At the White River station we can generate in the neighborhood of 50,000 kw. when operating around unity power factor, but if the power factor is eighty-five per cent or lower we can only generate about 42,000 kw. So we must devise some means of making sufficient correction to enable us to operate those generators to the full capacity of the water wheels. As an illustration of what it is likely to cost, if we let that condition continue and put in more water power capacity it will cost us in the neighborhood of anywhere from $200 to $300 per kilowatt of installed capacity. If we put in synchronous condenser capacity, it will cost about $15 per kv-a. We will gain from that about 8000 kw. of generator capacity which we could sell. Those are approximate figures illustrating a condition. They are not to be considered final.

We will gain then, the additional 8000 kw. or about $40 or $50 per kilowatt against a cost of $200 or $250 for a new development. But even that does not correct the power factor at a large number of other points on the system. If the analysis is carried out completely for the entire system and the gain in line and transformer capacity added to the generator capacity, I think the net financial gain will be quite large.

C. A. Whipple: I believe in the light of this welcome guest, improved power factor, we have lost sight of one of our great friends, the leading and lagging factor in our high-tension transmission systems. Scarcely anything has been said today regarding this feature of transmission problems.

The question of power service is a matter of giving satisfaction and service to the customer. We can all give that service. But the question is how to accomplish that at the lowest cost. We find that there are a great many cases where by taking, as it might seem, the viewpoint so far presented today, unity power factor, that we would have the ideal condition. I feel we are far from facts in that respect. In the high-tension transmissions such as cover the whole Pacific Coast, one of the cheapest methods of giving regulation, which is our prime factor, is accomplished by varying the power factor. This is done by introducing low power factor at points where voltage is high, as at light loads. Possibly large consumers are at extreme ends of a transmission line; at intermediate points other consumers are supplied. The loads at these different points vary both in power factor and in the quantity of demand. Therefore the voltage regulation is a problem of considerable concern.

I have had occasion in a number of instances where looking into the question of securing regulation, to combat efforts to increase the power factor in certain districts, finding that by introducing a lagging element in the load, I could thereby improve their operating conditions. Such an instance is where a small load is taken by a branch line at a point comparatively near the generating point. By introducing a heavy lagging current at the time of light load, voltage can be retained at a point which would be satisfactory, whereas at heavy periods of load the power factor is increased by standard synchronous apparatus as may be necessary. The problem to my mind seems to be to introduce synchronous and condensing equipment at these points, which are most in need of the improved power factor. And a method which has seemed to be the most feasible, particularly during the period of the war when we had to take what we could get and not what we wanted, was to stimulate the customer to the purchase of synchronous equipment where it was feasible, by the power company assuming a certain portion of the increased cost of that synchronous equipment, and also by inducing certain customers to keep their induction load off the lines at certain periods of peak load; that is under conditions where the lines were heavily loaded.

Another method that has worked out economically in two instances has been to purchase larger generating equipment than required for service at reserve steam power plants. This was necessary to carry the lagging component of the district, the increased cost for which in one case amounted to about one quarter of what it would have cost to put in a synchronous condenser to perform the same service. In other words, I doubled the cost of the generator purchased for the purpose of the corrective power factor thereby introduced.

I feel that under conditions mentioned by Mr. Hayward and existing with the Puget Sound Traction, Light & Power Company, and with some of the other power companies in this district, that in these power systems the increasing of power factor in all cases is not desirable. We wish to keep our lines under voltage control, reducing at times the power factor in certain sections to lower voltage, for we live in a district where water power is going to waste a large portion of the year. Where regulation is secured without cost we do not care if the consumption of power is somewhat greater.

D. M. Jones: I don't know right now of any other problem in which extensive amount of engineering work on the part of the operating people will get them so much for their money. We sit back rather behind a screen as you might say, and take the problems that are pushed up to us on paper, are asked for answers very often and give the best we can on the basis of the facts given. We would like more often to encourage people to tackle this problem of power efficiency by detailed engineering examination of their own premises, which we cannot give. I think there are a number of operating companies where it would pay them to hire a young engineer and just turn him loose on that one job. They will get more for their money than I think they really appreciate. It is such an easy way to correct power factor by selling them a big condenser; that is a good deal like giving a man patent medicine for his ills. It may seem queer that a person in my position would ever complain about that method of procedure, but we like to see a job well done.

I wish also to thank Mr. Carpenter for calling my attention to a thing that should have been mentioned in regard to the static condenser, which he has so well brought out. The corrective capacity of a condenser drops with the voltage applied. The current demand by the condenser will drop practically with the drop in the voltage. On the face of it it would look as if you were going to get disturbing effects, theoretically at least, which would bother you considerably in the voltage regulation of your line with the use of the static condenser, but under normal conditions of operation of this appliance, however, where it is used in relatively small capacities necessarily from economic reasons, the voltage variation actually encountered has never been serious. Practically it has not bothered. That is the only answer I can give to it.

Presented at the 364th Meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., October 8, 1920.

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ECONOMIC STUDY OF SECONDARY DISTRIBUTION

BY P. O. REYNEAU AND HOWARD P. SEELYE
Both of the Detroit Edison Company.

INTRODUCTION

IN working toward the efficient and economical design of the central station system as a whole no link in the chain connecting the consumer with the coal pile may be overlooked. The ultimate purpose of all study in this direction is to enable energy to be delivered to the customer at the least possible cost per unit, while at the same time good service is maintained. To this purpose considerable attention has been paid to generating plant, transmission lines and substations, but on the final link before reaching the customer—the distribution lines—the tendency has been to apply "rule of thumb" methods and "experience" only to the layouts. When it is considered that even in a well designed system the investment in distribution lines will often be from one-fifth to one-fourth of the total investment on the system and that the energy losses on these lines will be equal to or somewhat more than one-half of the total loss between the generator and the customer, it may be expected that a study of the economical design of distribution lines will be found of great profit. Such has been found to be the case, and the results of the study of secondary distribution described in this paper have been already applied to good advantage in the layout of such lines.

There are several conditions pertaining to the secondary system which make the careful layout of such a system especially important. The number of transformer installations is so large and the lines spread over so great an area that constant or even frequent inspection is impossible. The load is subject to irregular increases. In districts which are newly built up, new services are constantly being added. In old districts, new appliances are being purchased and the load on old services thereby increased. On this account any design must be made to cover a period of years and the increase in load for that period estimated from past experience. On the other hand, care must be taken not to install too much capacity and thereby increase the cost beyond the limits of economy. The problem must be carefully studied to obtain the balance between low cost and good service for any particular case.

In general the problems most often encountered are of the following types:

1. New lines in thinly populated districts where the load will probably build up rapidly.
2. Old lines in residence districts well built up where revision is necessary to care for a slowly increasing load.
3. Old lines in districts with heavy, increasing load such as business districts.
4. Exceptional installations such as for permanent loads with no increase or for a decreasing load.

In attacking such a problem we can often determine from tests and from past experience what the density of the loading is and how it will increase for some years in advance. We are usually limited to certain stock sizes of transformers and of wire, on any system, due to practical considerations of manufacturing and stock keeping. The problem then is to determine the proper combination of wire, transformer and transformer spacing in order to give good conditions of operation and also to show the least cost per year for the load densities expected during the period of time under consideration.

The purpose of this paper then is to study from an economical viewpoint the conditions generally met with in secondary distribution and to furnish as far as possible guidance for the designer to aid him in any particular problem with which he has to deal. It is clearly understood that no definite rules can be established which will fit all conditions. The variations in the problem are too many. The most that can be done is to furnish means for readily discovering the limitations of any problem and of proceeding within these limitations to the most economical design.

The study has been carried forward from three different angles. First from the theoretical; second from a semi-practical, that is by adopting certain standards and studying their behavior; third from a purely practical, giving the designer data on the operation of various transformers and wire sizes under the conditions ordinarily encountered in practise.

In all this discussion it has been assumed that the loading is such that it may be considered as uniformly distributed along the line. The unit used is called load density, given in kilowatts per thousand feet. The line is assumed to be three-wire secondary, spaced 42 in. between outside wires. The cost of right-of-way, poles, cross arms and insulators is not included in any of the computations as it is assumed this would be the same under any given condition. Also the difference in length of primary for different transformer spacings is not considered. In actual design under known conditions a correction should be made for this. The loading conditions are taken as those of residence lighting districts although the same methods could be adapted to any other conditions of loading if its characteristics were known. Transformers are assumed to be in the center of the secondary served, feeding both ways. The symbols used and all the mathematical calculations will be found in Appendix A and B. Only the discussion will be presented here.

DISCUSSION OF METHODS USED IN DERIVING EQUATIONS AND THEIR APPLICATION

THEORETICAL. In the theoretical discussion ideal conditions are assumed which will rarely if ever be met with in practise, but it can be shown by a study of the results how they may be applied to practical conditions. These assumptions are that the line is indefinite in length so that the transformers may be placed at any exactly determined spacing and that the spacing will change with the load; that the transformer is always of a size just equal to the load to be carrried, that is equal to the load density at peak load times the spacing; that the wire may be of any cross-sectional area and vary with the load. Such a condition could only be obtained in a case where the load showed only seasonal variations and no yearly increase. However, in practise we usually design for a certain period at the end of which it is assumed the load density will be a certain amount.

The general method has been to obtain an expression for the annual cost per 1000 feet of line and to determine, by finding the first derivative and setting it equal to 0, the condition under which this annual cost is a minimum. This is the most economical condition.

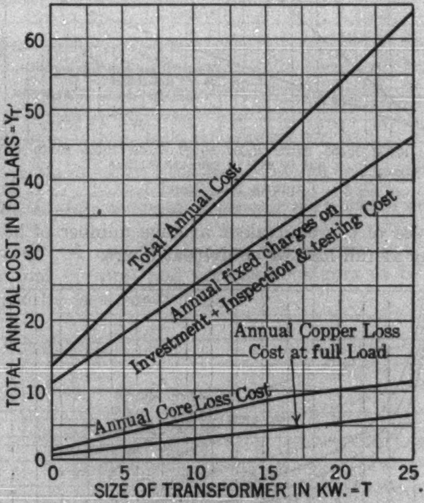
Annual Cost of Secondary Distribution. The general equation for the annual cost is first obtained as follows:

(Total annual cost per 1000 ft.) = (Annual Cost on transformers per 1000 ft.) + (Annual Cost on line per 1000 ft.)

$$Y = Y_t + Y_L$$

The annual cost on transformers must include interest, depreciation, insurance and taxes on the investment represented by the transformer in place. This investment is made up of the purchase price, plus freight and warehouse charges, plus cost of installation including lightning arresters, etc. It also includes the cost per year of inspecting and testing and the cost of energy losses. The core loss is practically a constant quantity for 24 hours per day throughout the year. The copper loss on the other hand depends on the load. If the characteristic variation of this load from hour to hour, day to day and month to month is known, the average loss per day can be determined in terms of the year's peak load. In this case the peak load is assumed to be just equal to the capacity of the transformer. The cost of energy at the transformer must also be carefully determined. The cost for copper loss will be considerably higher than that for core loss on account of the lower load factor. The sum of all these items makes up the annual cost on a transformer.

It was found that if the value of the transformer annual cost is plotted against the transformer size that the curve for values between 0 and 25 kw. may be approximated by a straight line of the formula $Y_{T'}, = K_1 + K_2 T$, T being the transformer size and K_1 and K_2 constants to be determined for any particular combination of transformer cost, energy cost, etc. (See curve No. 1.) This becomes $Y_T = \frac{1000}{S}(K_1 + K_2 T)$ per 1000 ft., where S is the length of secondary belonging to any one transformer or the distance between transformers where banked.

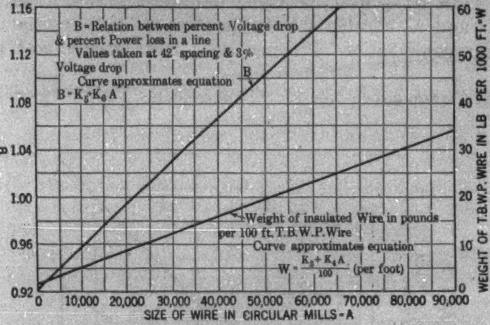


CURVE NO. 1—ANNUAL CHARGES ON TRANSFORMERS

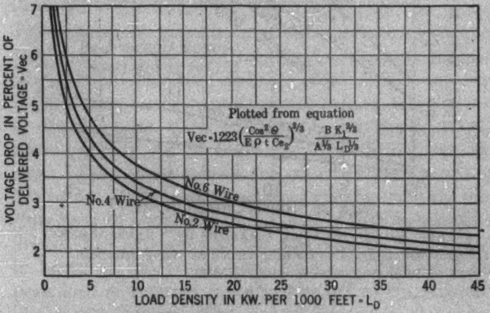
Transformers assumed to be fully loaded at yearly peak load.
Curve of total cost approximates equation

$$Y_{T'} = K_1 + K_2 T$$

The annual cost on the line includes interest, depreciation and taxes on the investment cost of the wire in place, including purchase price and cost of installation, also the cost of annual energy loss due to resistance. The copper loss is arrived at by the same method as the copper loss on the transformer, that is by use of the equivalent average number of hours per day at full load or equivalent hours.



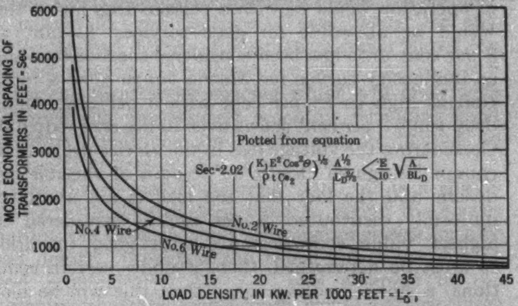
Curves No. 2 and 3



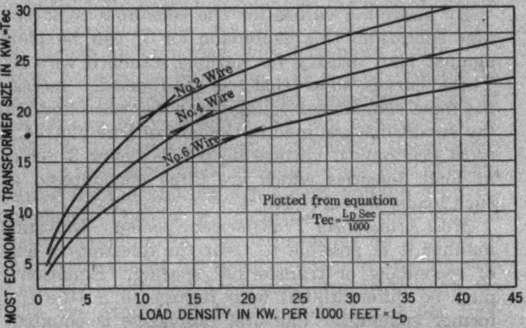
Curve No. 4—Most Economical Voltage Drop in Per Cent of Delivered Voltage

Load Uniformly distributed—Transformer size just equal to load.

The total annual cost per 1000 ft. of installation is now obtained by adding these two quantities, annual cost of transformers and annual cost on line, and the equation obtained as shown in equation (4)—(Appendix B) for *Y* in terms of *S* (length of secondary), L_D (load density), *A* and *W* (cross-sectional area and weight per foot of wire) and various constants to be determined by local conditions.



Curve No. 5—Most Economical Spacing of Transformers
Limited by a maximum allowable voltage drop of 3 per cent.
Load uniformly distributed—Transformer size just equal to load



Curve No. 6—Most Economical Transformer Size
Being just equal to the load at the most economical spacing

Most Economical Voltage Drop. One of the most important controlling factors in determining the length of a secondary or the spacing of transformers is the maximum allowable voltage drop. It has usually been considered that the most economical condition of operation would be with a voltage drop higher than would be allowable for good service. In our case 3 per cent drop has been considered the limiting value, as luminosity curves for Mazda lamps show a reduction as high as 18 per cent with 5 per cent voltage drop while 3 per cent shows over 10 per cent reduction. Considering the voltage loss in the service drops which cannot be figured closely on account of variable conditions and the fact that the load is not absolutely uniformly distributed, 3 per cent is considered the highest value commensurate with good operation. It must be determined, then, if under certain conditions, a smaller voltage drop than this will be more economical.

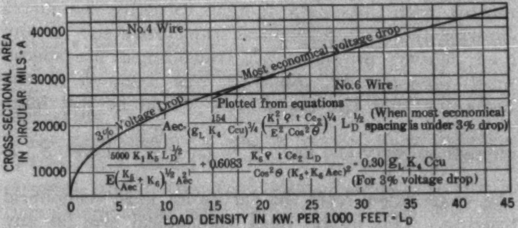
The expression for Y can be reduced to an equation in terms of L_D (load density), A (cross-sectional area of wire), and V (per cent voltage drop). If this is then differentiated with respect to V and the first derivative set equal to 0, an expression is obtained for the most economical voltage drop in terms of wire size and load density. Equation (**6**). (Appendix B).

By assuming values for the constants to fit particular conditions this expression for V can be plotted against load density for various standard wire sizes. These curves show that, as load density increases, the most economical voltage drop decreases and under the conditions assumed in the curves here plotted, the most economical voltage drop falls below 3 per cent at load densities which are often encountered with such loads, see curve No. 4.

Most Economical Transformer Spacing. In a similar manner it is possible to treat the question of length of secondary or transformer spacing. Equation (**4**) may be differentiated with respect to S (the transformer spacing). The first derivative is equated to 0 and we have the expression for the most economical spacing, equation (**7**). The transformer spacing however must be governed, for the smaller load densities, by the limiting voltage drop—taken here as 3 per cent. Hence equation (8) is derived which gives the spacing for a maximum voltage drop of 3 per cent. If now the constants are evaluated these curves may be plotted for various sizes of wire, using, for any particular load density, the equation which shows the shortest spacing. We obtain the set of curves No. 5 giving the transformer spacing which will give, with any wire size, the greatest economy, providing good operating conditions are maintained by having no voltage drop greater than 3 per cent.

Most Economical Transformer Size. It is a simple matter with these data at hand to derive the curves showing the most economical transformer size for any load density, providing the transformers are spaced most economically. Since it was assumed in the beginning that the transformer would be just large enough to carry the load, $T = L_D \frac{S}{1000}$ where S is the value taken from the curves for most economical spacing. This is the most economical size for any load since the annual charge on the investment represented by the transformer is a much greater proportion of the total annual charge than the cost of energy losses. Therefore the use of a larger transformer, even though under-loaded, would be more costly.

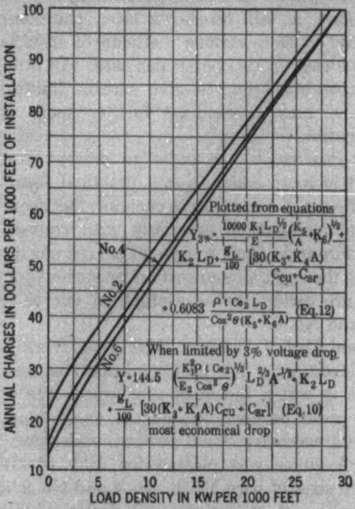
Most Economical Wire Size. It is now possible to attack the problem of the most economical size of wire for any load density. We will assume that it is feasible to use the most economical transformer size at its most economical spacing for any load density, modified by the limiting 3 per cent voltage drop requirement. Then if we substitute in our original equation, equation (4), the expressions for S used in plotting the curves for most economical spacing and for spacing limited by 3 per cent drop in voltage, we obtain two expressions for the annual cost per 1000 ft. in terms of load density and cross-sectional area of wire (A). Here it is necessary to introduce two approximations. The weight per foot of wire (w) enters the equation, also the quantity B which is the constant relation between per cent voltage drop and per cent power loss for any size of wire. It is found by plotting values of w for standard sizes of wire of the range of sizes which would be used in secondaries that the expression $w = K_3 + K_4 A$ is a very close approximation, K_3 and K_4 being constants (see curve No. 2). Also it is found that the value of B for any size of wire may be approximated very closely by the straight line function $B = K_5 + K_6 A$, where K_5 and K_6 are constants (see curve No. 3). These must be derived from the particular values of B which apply to the conditions being studied since these values vary for different spacings between wires. Substituting these expressions in the equations referred to above we obtain the two general expressions for annual cost per 1000 ft. in terms of wire size for maximum economy of transformer spacing and for 3 per cent voltage drop equations (**10**) and (**12**). These are now differentiated with respect to A and the equations (**11**) and (**13**) are obtained between the most economical wire size and the load density for most economical spacing and for 3 per cent voltage drop. The constants were evaluated and these curves plotted (No. 7), the 3 per cent curve for low load densities and the maximum economy curve for high loading. They furnish a graphic representation of the most economical size of wire to use under any load density providing ideal conditions obtain in the way of transformer size and spacing.



CURVE NO. 7—MOST ECONOMICAL WIRE SIZE

Transformer size assumed just equal to load at most economical spacing limited by 3 per cent voltage drop

Purpose of Theoretical Curves. At first glance it may appear as if these curves being obtained on the basis of such theoretical assumptions could have very little practical value. However, when attacking a practical problem of this nature the data from these curves may be used as the basis upon which to start the calculations of annual costs under operating conditions. If for example the present load density and the load density which is to be expected at some certain future time are known, by going to the theoretical curves we may determine (a) whether the voltage drop is to be limited by the 3 per cent maximum, (b) what would be the most economical conditions of transformer size and spacing for present operation and for operation at that future time, and (c) what standard



CURVE NO. 8—SHOWING COMPARATIVE ECONOMY OF VARIOUS WIRE SIZES IN SECONDARY INSTALLATIONS

Transformer sizes and spacings assumed to be those most theoretically economical, limited by 3 per cent maximum voltage drop.

Annual cost includes line and transformers.

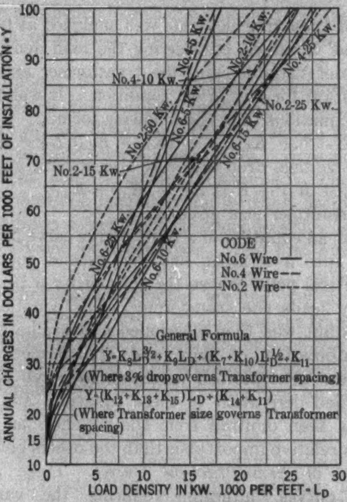
size of wire will be most economical over the period. The curve for the most economical wire size covers, for each standard size, such a range of load densities that we should be able at once to select our wire size without further computation. Having determined this and knowing what stock sizes of transformers and practical spacings come the nearest to fitting the ideal conditions over the period under consideration, we can then investigate, as will be shown later, the comparative economy of such various methods of installation as could be used in this particular case. In other words these theoretical curves give certain limitations on which we may proceed to further more practical investigation.

SEMI-PRACTICAL. In order to present our results in a little more concrete and practical form and to show the exact comparative economy between various types of installation, especially with respect to the size of wire to be used, a series of curves was developed showing the exact annual cost under various conditions. These are called the semi-practical curves. Curves No. 8 and No. 9.

Annual Cost of Standard Wire Sizes Working under Ideal Conditions. The first condition was assumed to be that in which the most economical size of transformer could be used, spaced the most economically or, where necessary, for 3 per cent maximum voltage drop. A curve was plotted for each of the three standard sizes of wire No. 6, No. 4 and No. 2, showing the annual cost at various load densities, (see curve No. 8). This is, in reality, simply plotting equations (**10**) and (**12**) as developed above.

Annual Costs per 1000 ft. of Installation for any Combination of Standard Sizes of Wire and Transformers. As the next step in proceeding from the general problem to the concrete example various combinations of standard sizes of transformers with standard sizes of wire were assumed and curves developed showing the annual cost of each of these combinations at various load densities. The transformer spacing was still assumed to be always the theoretically best spacing for each particular load. This enables us to compare for example the economy of a 10-kw. transformer and No. 4 wire with that of a 15-kw. and No. 6 wire at any load density.

The method of developing these curves has some points of interest although the equations are merely variations of our general equation for annual cost per 1000 ft. It is seen that for any size of transformer, as the load density increases a certain point is reached where the spacing is no longer governed by the allowable voltage drop but by the size of the transformer itself. Hence each curve will consist of two parts, the lower where the voltage drop governs the spacing, the excess transformer capacity is provided, the upper where the transformer size governs the spacing and the voltage



Curve No. 9—Showing Comparative Economy of Various Combinations of Secondary Installations

Load uniformly distributed.
Voltage drop most economical—maximum 3 per cent.

drop is less than the allowable. The total annual cost is made up of five items.

1. Transformer core loss.
2. Transformer copper loss.
3. Copper loss on the line itself.
4. Fixed charges on the transformer (interest, depreciation, taxes, inspecting, tests, etc.)
5. Fixed charges on the line. (Interest and depreciation).

Each of these five elements was analyzed as to constants and variables, considering the load density L_D as the chief variable, and the transformer and wire sizes constant for any given condition. It was found that the equations took the following form:

$Y = K_8 L_D^{3/2} + K_9 L_D + (K_7 + K_{10}) L_D^{1/2} + K_{11}$

when the voltage drop and wire size govern, and

$Y = (K_{12} + K_{13} + K_{15}) L_D + (K_{11} + K_{14})$

when transformer size governs.

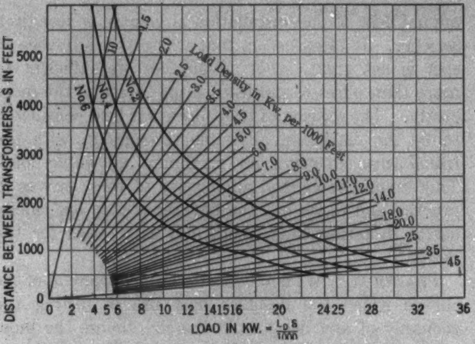
The first is an equation of a third degree curve in $L_D^{1/2}$ breaking into a straight line (the second equation) at the critical point where the spacing for 3 per cent drop fully loads the transformer. The equation for each constant was then developed and evaluated for each combination of wire and transformer. The results were then plotted as shown in curve No. 9. The detailed derivation of these equations and constants has been omitted from the appendix as these are of less relative importance than the others given.

Purpose of Semi-Practical Curves. These semi-practical curves, although reducing the variable elements, still retain enough of the ideal condition so that they cannot be used as an absolute criterion but merely as a guide. They do show however, concretely, the relative economy of the various standard sizes of wire when used under the most favorable conditions and this may be taken as a guide to their comparative behavior under all conditions. The second set of curves also shows concretely the relative economy of the various transformer sizes with any one size of wire as well as the relative economy of various sizes of wire with any size of transformer. This comparison of economy is valuable in showing the exact amount by which the annual cost of one installation is greater or less than another. It often occurs that where the difference in cost is not great, other advantages are sufficient to more than offset it and lead to the choice of the more costly. The spacing of transformers is here considered to be the maximum allowable throughout, with the transformer carrying its maximum allowable load. This limits the general application of these curves in practise and hence like the first series they are chiefly useful in establishing limits and as a basis for the design.

PRACTICAL. We now come to the development of the curves which the designer may use in testing the economy of any design and thereby choose the most economical from several alternatives. Here no "most economical" conditions need be assumed. The curves simply represent annual costs as they occur under any condition which may be encountered.

Load Curves for Secondaries. The first curve is a development from the two theoretical curves, the most economical transformer spacing and most economical transformer size. By plotting the transformer size against the spacing we obtain for each size of wire a curve showing the most economical spacing or the spacing limited by 3 per cent voltage drop for any total load on the transformer (see curve No. 10). By drawing diagonal lines, one for each load density desired, we can now show for any particular load density, the maximum economical spacing, and the minimum transformer size with that density and spacing. This curve merely simplifies the former two and serves the same purpose, not introducing any new principles. It is evident that any point below the curve will indicate a drop less than the value used on the curve. This curve is of use in determining what alternative designs may be feasible with any load and standard equipment and what changes may be made to care for an increase.

Line Cost Curves. The equation for annual charges on the line, (equation (3) Appendix B) was next developed. All constants were evaluated and a curve plotted for each desired spacing—100-ft. in-

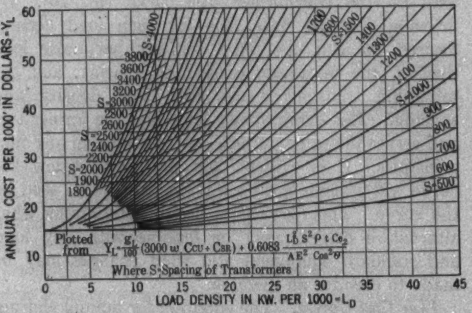


Curve No. 10—Load Curves for Secondaries

Three-wire secondaries—244/122 volts at customer.
Most economical transformer spacing limited by 3 per cent voltage drop.
Power factor 95 per cent —Load uniformly distributed—42-in. spacing between outside wires.

tervals were used—showing the annual charges per 1000 ft. in terms of the load density for each standard size of wire. (See curve No. 11 for No. 4 wire.)

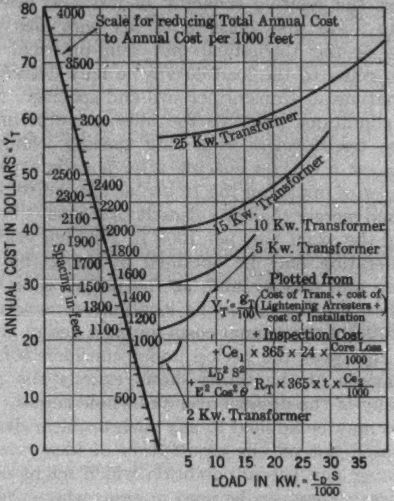
Transformer Cost Curves. The third set of curves shows the annual cost on the transformer for any



Curve No. 11—Line Cost Curves

Annual cost per 1000 ft. of three No. 4 secondaries including fixed charges and cost of lost energy.

loading, (see curve No. 12). This of course is equal to the fixed charges, plus the core loss cost which is constant for all loads, plus the copper loss cost which varies as the square of the load. The equation for each of the standard sizes of transformers, 2, 5, 10, 15 and 25 was developed and plotted. Since this curve shows total annual cost on a transformer and not cost per 1000 ft. of installation a scale was added on the diagonal at the left by use of which, with a pair of triangles, the cost per 1000 ft. may be obtained by the principle of similar triangles. Draw a line from the total cost obtained to the spacing as shown on the diagonal scale and a parallel through 1000 ft. will indicate the annual cost per 1000 ft. on the vertical scale. (See Appendix B for illustration.)



CURVE NO. 12—TRANSFORMER COST CURVES

Total annual cost on various sizes of transformers under various loads. Includes fixed charges and cost of energy losses.

Cost of Replacing Transformers. Two more items of cost are of interest to the designer and these are arbitrarily fixed by local conditions, the cost of changing the size of transformers in the same location and the cost of changing the location of a transformer. These will be practically constant for all sizes and may be determined in any case from local cost records.

Application of Practical Curves. We are now ready to furnish the designer with the information necessary to test the relative economy of any two alternative designs. He first determines his wire size from a study of the theoretical and semi-practical curves. Then, going to the load curves he may determine his alternatives in transformer size and spacing. Assume that conditions point to the alternative of installing 10-kw. transformers at a long spacing, changing to 15 kw. at a shorter spacing after a certain period of years, or of installing the 10 kw. at the shorter spacing now and merely changing sizes at that time. From our curves the exact cost per 1000 ft. for each year under consideration may be obtained by using the correct loading and spacing and, at the proper time, adding the cost of either changing location or changing size. The total of the annual costs for each design gives the total cost over the period under consideration and a comparison of the totals shows exactly the relative economy of the designs over the whole period. These curves may be applied to any such problem since they are based not on the assumption of ideal conditions but cover any actual condition which might occur in practise. They can be used in cases where the transformer spacing cannot be uniform on account of local conditions of pole spacing, secondary length, and street and alley arrangement, a very usual case. When there is doubt about the wire size a study of the various possible combinations making use of these curves will soon determine the size for greatest economy. Similar curves can also be developed to suit other classes of problems such as concentrated loads, loads with characteristic variations different from those of the residence load used here, as in business districts, power loads, etc.

Example of Application of Practical Curves. A concrete example of the use of the above curves may be helpful. Assume that tests on a district show a load density of 8 kw. per 1000 feet, with No. 4 secondary wire already in place. Our load curves show for that loading and size of wire, 12.8 kw. load at 1800-ft. spacing to keep within 3 per cent drop in voltage. We wish to provide for an increase in load which we will estimate may go to 15 kw. per 1000 ft. in six years. For the present a 10-kw. transformer spaced at 1400 ft. would care for the load while at 15 kw. per 1000 feet there would be required a 15-kw. transformer at 1000-ft. spacing or a 25-kw. at 1200 ft. In order to avoid too many changes we may space 10-kw. transformers at 1000 ft., changing in three years to 15 kw. or we may put in 15 kw. transformers now at 1500 ft., changing the location in two years to 1000 feet. Other alternatives might be considered but these two will serve as an example.

For the first alternative, assuming uniform increase in load density of 1 2/5 kw. per year.

	Per 1000 ft. installation	
1st year.		
Line Cost — $L_D = 8$ kw., $S = 1000$......	$16.00	
Transformer Cost—10 kw. at 8-kw. load ..	32.00	
For year		$48.00
2nd year.		
Line Cost — $L_D = 9\ 2/5$ kw., $S = 1000$...	16.30	
Transformer Cost — 10 kw. at 9 2/5.......	32.70	
For year		$49.00
3rd year.		
Line Cost — $L_D = 10\ 4/5$ kw., $S = 1000$.	16.80	
Transformer Cost — 10 kw. at 10 4/5.....	33.70	
For year		$50.50
4th year.		
Line Cost — $L_D = 12\ 1/5$ kw., $S = 1000$.	17.30	
Transformer Cost — 15 kw. at 12 1/5.....	42.90	
Cost of Changing Size (10 kw. to 15 kw. on same pole).........................	7.00	
For year		$67.20
5th year.		
Line Cost — $L_D = 13\ 3/5$ kw., $S = 1000$.	17.80	
Transformer cost 15 kw. at 13 3/5........	43.60	
For year		$61.40

6th year.		
Line Cost — L_D = 15 kw., S = 1000.....	18.40	
Transformer cost — 15 kw., at 15.........	44.40	
For year		$62.80
Total for 6 years		$338.90
Second Alternative.		
1st year.		
Line Cost — L_D = 8 kw., S = 1500......	17.20	
Transformer Cost — 15 kw. at 12-kw. load.	35.50	
For year		$52.70
2nd year.		
Line cost — L_D = 9 2/5 kw., S = 1000.....	$18.00	
Transformer Cost —15 kw. at 15 4/5......	37.20	
For year		$55.20
3rd year.		
Line Cost —L_D = 10 4/5 kw., S = 1000...	16.80	
Transformer Cost — 15 kw. at 10 4/5.....	42.30	
Cost of changing location...............	20.50	
For year		$79.60
4th year—same as 1st alternative (less charge for changing size).....................	60.20	
5th year—same as 1st alternative...........	61.40	
6th year " " " "	62.80	
Total for 6 years		$371.90

A saving of $33.00 per 1000 ft. of installation, or approximately 10 per cent of the total cost over a period of six years by the first method thus demonstrating its economy. It is well to note that a large part of the difference in cost is due to the fact that in the first case the size of transformer is changed while in the other the location but not the size is changed. If a further refinement of the comparison is desired, interest may be considered on the yearly items up to the end of the period under consideration. Usually such refinement is not necessary however.

CONCLUSIONS

A study of all these curves gives considerable aid in determining certain standards of design as well as the final particulars for any special problem. There also may be obtained a definite knowledge of the behavior of secondaries under various conditions of loading and operation. It is purposed here to take up each curve in detail, to bring out its characteristics and its possible use.

Most Economical Voltage Drop. The curves on voltage drop show that the most economical condition varies inversely as the cube root of the wire size also inversely as the cube root of the load density.

For low-load densities the economical drop is high but decreases rapidly, while at high loading the decrease is comparatively slow. It is clearly shown that the most economical voltage drop may be well under that allowable for good service for loads which may be often encountered in practise. Under the conditions and prices assumed in the present case the 3 per cent limit seems to have some justification by economy for ordinary loads.

Two conditions must be considered which might affect these curves, *i. e.*, the price of materials and cost of energy and the fact that here the transformer was considered just sufficient to carry the load while ordinarily, when designing for an increasing load, the transformers are underloaded. It is seen from the equation of the curves for economical voltage drop that the cost of copper does not affect this discussion. This is due to the fact that the annual cost is based on a unit of 1000 feet, hence for any given price of copper the annual cost per 1000 feet of three-wire line is a constant no matter what the load. The cost of energy enters as an inverse factor to the 2/3 power. Also it is a small element in the factor K_1, which is also to the 2/3 power but in the direct ratio. Hence an increase in the cost of energy would increase both the numerator and the denominator but the latter slightly more than the numerator, hence all the curves would be raised slightly. This effect would be small however for ordinary fluctuations. In the case of an increase in the transformer price there would be no change in the curves providing the increase were proportional to the size since the factor K_1 would not be affected by such an increase.

In ordinary design for an increasing load the transformer would be made larger than sufficient to carry the present load to allow for the anticipated increase. A study of the curves for the various components of the annual charge on a transformer and the equation resulting therefrom, $Y_{T'} = K_1 + K_2 T$, will show that if they are developed with the transformer working below its rated loading, and if the percentage of underloading is kept the same for all sizes, the factor K_1 will be very little affected, the effect being similar to an increase in price proportional to size. Since this is the only part of this equation that enters into the equation for most economical voltage drop it follows that if a design could be limited to any given percentage of underloading throughout, the curves would still show the most economical condition of voltage drop.

Most Economical Transformer Spacing. These curves for the most economical transformer spacing (curve No. 5) are derived from the same general expression for annual cost per 1000 feet as those for economical voltage drop. Hence the same results might be expected from the use of either of these sets of curves with the exception that where the most economical spacing would give a maximum voltage drop of more than 3 per cent we have corrected it for that value making it such as to give 3 per cent.

These curves show for very low load densities, extremely high spacings which are probably much greater than it would be practicable to use since for such a distance and such light loads the effect of the non-uniform loading would be considerable. As is shown by the equation, the spacing for 3 per cent drop varies as the square root of the wire size while for greatest economy it varies as the cube root. It also varies inversely with the load density, to the square root in the first case, the 2/3 power in the second. For ordinary loadings encountered in practise and the usual range of wire sizes it is seen that spacing of from 1000 to 2000 feet is the most economical and practicable. For the higher loadings the most economical spacing decreases very slowly, remaining over 500 feet up to high values of L_D.

Changed conditions would have a similar effect on these curves as on those for economical voltage drop, in the range of values for which the most economical voltage drop governs the spacing. That is, a rise in the price of energy would lower the curves slightly; the prices of wire and transformer would not have a noticeable effect. For the condition of underloaded transformers, if the proportion of underloading were fixed there would be slight change. In practical designing, however, when considering the amount of this margin in transformer capacity to be used, it might be found relatively more economical to use a transformer size somewhere near the theoretically most economical and obtain the margin in capacity by using a spacing less than the most economical spacing. This may have some advantage over using the most economical spacing, as shown by the curves, and a larger size of transformer than the most economical, when the design is to cover several years and the cost of changing sizes and locations is taken into account. Hence, care must be used in placing too much dependence on the strictly theoretical values in practical design. The choice must be tested by the actual year costs as shown by the cost curves.

Most Economical Transformer Size. The curves for the most economical transformer size simply show the size of transformer which will carry the load when the spacing is the most economical or just enough to give 3 per cent voltage drop. They have relatively less practical value excepting that it is from these and the spacing curves combined that the practical load curves are obtained.

Most Economical Wire Size. The wire size is the first thing to determine in a design and must be chosen to cover long periods of increase in load as replacement of secondary wire is very costly. Hence for secondaries a standard must be chosen for installation in new work which will show good economy through the greatest range of conditions to be encountered. The curves seem to indicate clearly that under the conditions and prices assumed No. 6 wire should be used as a standard in all new work, in districts where ordinary residence lighting load is expected. The economy curve rises very rapidly at low densities up to about 20,000 cir. mils or nearly to No. 7 at about 7 kw. per 1000 feet. From here the rise is less rapid but still considerable until it crosses the value of No. 6 wire at 15-kw. load density. The load density of 15 is a normal loading. It would not be advisable to use any size less than a No. 7 since the loadings at the smaller values are subject to such rapid increase. Even at No. 7 the economical load is fairly small (7 kw. per 1000 feet). On the other hand, the curve rises slowly after passing No. 6 and only reaches No. 5 at a loading of about 31 kw. and No. 4 at 40 kw. which are high densities and to be encountered only in special cases. It is interesting to note that for all values below a No. 6 wire the economical size is governed by 3 per cent voltage drop while above that the most economical drop governs, the curves crossing at 19-kw. load density.

Since the curves were figured at a low copper price, in case of an increase in price, the curves would be lowered, *i. e.*, a smaller wire size would be indicated for any particular loading. An increase in energy cost would slightly raise the curve, an increase in transformer price if proportional to size would not affect the discussion. Since the curves were figured on the assumption that the transformer spacing was the most economical and the size just equal to the load, a change in these conditions might affect the most economical wire size somewhat. A fixed proportion of underloading as above shown would have little effect but if different conditions of spacing were assumed, the design should be tested by use of the cost curves for various sizes of the wire.

Semi-Practical Curves. The curves which we call semi-practical show a little more concretely the relative economy of installations with the various sizes of wire, in dollars per year per 1000 feet. They show the actual annual cost for different types. The excessive cost of No. 2 wire for ordinary loads is clearly demonstrated being from $3.50 to $6.00 per year more than No. 4 for loadings up to 15 kw. per 1000 feet.

When we go from the ideal size of transformer to practical stock sizes, still assuming the best spacings to be used, there are some conditions in which the relative wire economy is somewhat different. These curves also give an indication of transformer economy. It seems to be quite clearly shown that, under the assumed conditions, the use of large transformers such as 25-kw. is not justified except with very heavy loading, even considering the possible reduction in the number of transformers and hence in the core loss. The increase in the investment cost more than equalizes such saving.

Practical Curves. The use of the cost curves in designing has already been explained. It may now be readily seen how a study of the theoretical and semi-practical curves applied to any problem will give a basis upon which to formulate a design which can then be tested for actual economy by application of the exact costs to be expected. We can determine from this, in case of a new line, the size of wire and then the spacing and size of transformers which will care for several years of increase. The exact number of years will be determined by the rate of increase together with the economy of the design, including cost of changing sizes and locations. Or, in case of remodeling an old district, we start with a given size of wire which although perhaps not the most economical, will not justify the cost of change. We can then choose and space our transformers most economically with regard to that size of wire. In a special case where no increase in load is expected the theoretical curves will give exactly the design to use. In other cases where the loading, voltage, etc., are somewhat different, by proper substitution in the theoretical formulas, curves could be plotted which would apply to that particular condition.

General. The curves given here should not be accepted for general application to design problems. The costs and conditions of loading used were of local derivation and apply only to the organization and the time for which they were obtained. Similar curves should be developed for the study of conditions in any other locality and they should be revised from time to time to meet changing conditions. These examples are given here merely to indicate the characteristics of such curves.

It is evident that no very simple means of correctly designing a distribution system in regard to transformers and secondary wire can be made available due to the many varying conditions encountered and the large number of factors to be taken into account.

The elements of good judgment and experience are as necessary in the solution of these problems as in any other problem of engineering. The object of this study has been to analyze and evaluate the factors of the design of the secondary system that lend themselves to such definite analysis and to present the results as aids in the application of good judgment and experience to the best possible solution of the problem.

In conclusion the authors acknowledge gratefully the assistance of Mr. Harold Cole and Mr. Lansing W. Thoms in the preparation of this paper.

APPENDIX A

TABLE OF SYMBOLS

The following is a list of the principal symbols used, with their general definitions. In Appendix B, the significance of each symbol will be explained for the individual case under consideration.

A = cross-sectional area of wire in circular mils.
A_{ec} = most economical cross-section of wire in circular mils.
B = constant relation between per cent voltage drop and per cent power loss.
C_{cu} = cost of insulated copper wire per lb.
C_{e1} = " " core loss per kw-hr. at transformer.
C_{e2} = " " copper loss per kw-hr. at transformer and secondary.
C_{sr} = cost of stringing wire per 1000 ft. of line.
E = Secondary receiver voltage between outside wires.
g = per cent interest, depreciation and taxes (g_t on transformer, g_L on line).
I = total current on secondary.
$K_1, K_2 ' ' ' ' ' K_n$ = the numerical constants.

L_D = load density in kw. per 1000 ft.
R_T = equivalent resistance of transformer in ohms
R = resistance in ohms.
S = spacing of transformers in feet.
S_{ec} = most economical spacing of transformer.
T = size of transformers in kilowatts.
T_{ec} = most economical size of transformer in kilowatts.
t = equivalent hours per day in terms of yearly peak load to give a total energy loss equal to the actual.
V = per cent voltage drop.
V_{ec} = most economical per cent voltage drop.
W = total load on secondary in watts.
w = weight per foot of insulated wire in lb.
Y = total yearly cost per 1000 feet of installation.
Y_T = total yearly cost of transformer per 1000 ft. of installation.
Y_T' = total yearly cost per transformer.
Y_L = total yearly cost of line per 1000 ft. of installation.
Y_L' = total yearly cost of line for given spacing.
$\cos\theta$ = power factor of load.
ρ = resistivity of wire in ohms per mil-foot.

APPENDIX B

DERIVATION OF FORMULAS

List of Fundamental Assumptions:

1. Continuous three-wire secondary.

2. Uniformly distributed load.

3. Difference in cost of primary wire and copper loss in primary for different spacing of transformers neglected.

4. Cost of poles, cross arms, pins, insulators and right-of-way same for all cases and hence omitted from discussion.

These four assumptions are carried through the entire study. Any additional assumptions are stated in the derivation of the individual formula.

A. DEVELOPMENT OF THE THEORETICAL CURVES

1. *Annual Cost of Secondary Distribution.*

Annual cost per 1000 feet of installation = Y

Y = (Total annual charges on transformers per 1000 ft. of line) + (total annual charges on line per 1000 ft. of secondary)

$$= Y_T + Y_L \qquad (1)$$

Where $Y_T = \Bigg(\frac{g_T}{100}$ (purchase price + cost of handling + Cost of installation + cost of lightning arresters and equipment) + cost of core and copper loss + cost of inspection $\Bigg)$

$$\frac{1000}{S}$$

g_T = per cent interest + depreciation + taxes on transformer.

S = spacing of transformers in feet.

Y_T — can be expressed as a function of the transformer size, T, as follows:

$$Y_T = \frac{1000}{S}\,(K_1 + K_2 T)$$

Where K_1 and K_2 are constants and are found by plotting total annual charges on transformer against transformer size. (See curve No. 1.)

Assuming a transformer size just sufficient to carry the load, then $T = L_D \frac{S}{1000}$.

Where L_D = load density in kw. per 1000 ft.

Then $$Y_T = \frac{1000}{S}\left(K_1 + K_2 \frac{L_D S}{1000}\right) \qquad (2)$$

Y_L = investment cost of material per 1000 ft. of line + installation charges per 1000 ft. of line + $\frac{1000}{S}$ × cost of copper loss in secondary.

$$Y_L = \frac{g_L}{100}(3 \times 1000 \times w \times C_{cu} + C_{sr}) + \frac{1000}{S}$$

$$\left[2\left(\frac{I}{2}\right)^2 \times \frac{\rho}{A} \times \frac{S}{6} \times 2 \times t \right.$$

$$\left. \times\, 365 \times \frac{C_{e2}}{1000} \right]$$

Where

g_L = per cent interest + depreciation + taxes on line.
w = weight of insulated wire in lb. per foot.
C_{cu} = cost of insulated wire per lb.
C_{sr} = cost of stringing 1000 ft. of line.
I = total current in secondary at transformer.
ρ = resistivity of wire per mil-foot.
C_{e2} = cost of copper loss in secondary per kw-hr.
t = equivalent hours per day which yearly peak load should continue in order to give an $I^2 R$ loss equal to the total actual $I^2 R$ loss for the year.
A = cross-sectional area of wire in circular mils.
E = voltage between outside wires of secondary.
$\text{Cos}\ \theta$ = power factor of load.

$$I = \frac{L_D S}{E \text{ Cos } \theta}$$

and

$$Y_L = \frac{g_L}{100}(3000\, w\, C_{cu} + S_{sr})$$

$$+ S^2 \left[\frac{L^2{}_D}{E^2 \cos^2 \theta} \times \frac{\rho}{A} \right.$$

$$\left. \times\, t \times 60.83\, C_{e2} \right] \quad (3)$$

$$\text{Then} \quad Y = \frac{1000}{S}\left(K_1 + \frac{K_2 L_D S)}{1000}\right. + \frac{g_L}{100}$$

$$(3000\, w\, C_{cu} + C_{sr})$$

$$+ S^2 \frac{(60.83\, L_D{}^2\, \rho\, t\, C_{e2})}{A\, E^2 \text{ Cos } \theta} \quad (4)$$

Equation (**4**) gives the total annual cost per 1000 ft. of installation as a function of the spacing and load density.

2. *Most Economical Voltage Drop.*

In order to obtain the most economical per cent voltage drop it is necessary to obtain Y as a function of the per cent voltage drop.

This is done as follows:

W = total load on secondary in watts.

V = voltage drop on secondary in per cent of delivered voltage.

B = constant relation between per cent voltage drop and the per cent power loss.

Then $$W = L_D S = \frac{A E^2 V}{300 B S}$$

Whence $$S = \frac{E}{17.32} \sqrt{\frac{A V}{B L_D}}$$

Substituting this value for S in equation (**4**,)

Then $$Y = \frac{1000 K_1}{\frac{E}{17.32}\sqrt{\frac{A V}{B L_D}}} + K_2 L_D$$

$$+ \frac{g_L}{100} (3000 \, w \, C_{cu} + C_{sr})$$

$$+ \frac{E^2}{(17.32)^2} \frac{A V}{B L_D} \left[60.83 \frac{L_D^2 \rho t C_{e2}}{A E^2 \cos^2\theta} \right]$$

$$= 17{,}320 \frac{K_1}{E} \sqrt{\frac{B L_D}{A}} V^{-1/2}$$

$$+ 0.2028 \frac{\rho t C_{e2} L_D}{B \operatorname{Cos}^2 \theta} V + K_2 L_D$$

$$+ \frac{g_L}{100} (3000 \times C_{cu} + C_{sr}) \qquad (\mathbf{5})$$

The most economical per cent voltage drop is obtained when the first derivative of Y with respect to V equals 0.

$$\frac{d\,Y}{d\,V} = 0$$

$$= -\,1/2 \times 17320 \left(\frac{K_1}{E} \sqrt{\frac{B\,L_D}{A}} \right) V_{ec}^{-3/2}$$

$$+\,0.2028\,\frac{\rho\,t\,C_{e2}\,L_D}{B\,\mathrm{Cos}^2\,\theta} = 0$$

Solving for V_{ec}

$$V_{ec} = 1223 \left[\frac{\mathrm{Cos}^2\,\theta}{E\,\rho\,t\,C_{e2}} \right]^{2/3} \frac{B\,K_1^{2/3}}{A^{1/3}\,L_D^{1/3}} \qquad (6)$$

Equation (**6**) gives the most economical per cent voltage drop as a function of the load density. For fixed values of the constants this equation may be plotted as shown on curve No. 4.

3. *Most Economical Transformer Spacing.*

In order to obtain the most economical spacing of transformers it is necessary to have Y as a function of S. This is obtained from equation (**4**).

$$Y = \frac{1000\,K_1}{S} + K_2\,L_D + \frac{g_L}{100}\,(3000\,w\,C_{cu} + C_{sr})$$

$$+\,S^2 \left(60.83\,\frac{L_D^2\,\rho\,t\,C_{e2}}{A\,E^2\,\mathrm{Cos}^2\,\theta} \right)$$

The most economical spacing is obtained when the first derivative of V with respect to S equals 0.

$$\frac{d\,Y}{d\,S} = 0$$

$$= -\,\frac{1000\,K_1}{S_{ec}^2} + 2 \times 60.83\,\frac{L_D^2\,\rho\,t\,C_{e2}}{A\,E^2\,\mathrm{Cos}^2\,\theta}\,\mathrm{S}_{ec} = 0$$

Solving for S_{ec}

$$S_{ec} = 2.02 \left[\frac{K_1\,E^2\,\mathrm{Cos}^2\,\theta}{\rho\,t\,C_{e2}} \right]^{1/3} \frac{A^{1/3}}{L_D^{2/3}} \qquad (7)$$

Equation (**7**) gives the most economical spacing of transformers as a function of the load density.

It is necessary to limit the range of application of equation (**7**) to conditions where the voltage drop is less than 3 per cent. A second equation must be developed for 3 per cent drop to apply where the most economical drop would be greater than 3 per cent.

Practical considerations limit the drop to that value.

From above

$$S = E\sqrt{\frac{A\,V}{300\,B\,L_D}}$$

Using $V = 3$ per cent

$$S = \frac{E}{10}\sqrt{\frac{A}{B\,L_D}} \qquad \textbf{(8)}$$

= transformer spacing for 3 per cent drop.

Then, summarizing,

$$S_{ec} = 2.02\left[\frac{K_1\,E^2\,\mathrm{Cos}^2\,\theta}{\rho\,t\,C_{e2}}\right]^{\frac{1}{3}} \frac{A^{\frac{1}{3}}}{L_D^{\frac{2}{3}}}$$

$$< \frac{E}{10}\sqrt{\frac{A}{B\,L_D}} \qquad \textbf{(7a)}$$

which is general for all cases. (See curve No. 5.)

4. *Most Economical Transformer Size.*

The most economical size of transformer will follow directly from equation (**7a**) since it would be that size which would just carry the load at the most economical spacing.

or,

$$T_{ec} = \frac{L_D\,S_{ec}}{1000} \qquad \textbf{(9)}$$

(See curve No. 6.)

5. *Most Economical Wire Size.*

The most economical cross-section of wire is the cross-section which will give the minimum total annual cost with any particular type of transformer installation. Here, the condition of most economical spacing and size of transformers will be assumed.

Thus, substituting the value S_{ec} (equation **7**) for S in equation (**4**,)

$$Y = \frac{1000\,K_1}{2.02\left(\dfrac{K_1\,E^2\,\mathrm{Cos}^2\,\theta}{\rho\,t\,C_{e2}}\right)^{\frac{1}{3}} \dfrac{A^{\frac{1}{3}}}{L_D^{\frac{2}{3}}}}$$

$$+ K_2\,L_D + \frac{g_L}{100}\,(3000\,w\,C_{cu} + C_{er})$$

$$+ 60.83 \frac{L_D^2 \rho t C_{e2}}{A E^2 \operatorname{Cos}\theta^2} \left[2.02^2 \left(\frac{K_1 E^2 \operatorname{Cos}^2 \theta}{\rho t C_{e2}} \right)^{\frac{2}{3}} \frac{A^{2/3}}{L_D^{4}} \right]$$

It is possible to express w as a function of A as follows:

$$w = \frac{K_3 + K_4 A}{100}$$

Where K_3 and K_4 are found by plotting the weight of installed wire against its cross-sectional area. (See curve No. 2.)

The equation for annual costs per 1000 ft. now becomes

$$Y = \frac{496 (K_1^2 \rho t C_{e2})^{1/3} L_D^{2/3}}{(E^2 \operatorname{Cos}^2 \theta)^{1/3}} A^{-1/3} + K_2 L_D$$

$$+ \frac{g_L}{100} [30 (K_3 + K_4 A) C_{cu} + C_{sr}]$$

$$+ 248.5 \frac{(K_1^2 \rho t C_{e2})^{1/3}}{(E^2 \operatorname{Cos}^2 \theta)^{1/3}} L_D^{\frac{2}{3}} \ A^{-1/3}$$

Simplifying

$$Y = 744.5 \left(\frac{K_1^2 \rho t C_{e2}}{E^2 \operatorname{Cos}^2 \theta} \right)^{\frac{1}{3}} L_D^{\frac{2}{3}} A^{-1/3} + K_2 L_D$$

$$+ \frac{g_L}{100} [30 (K_3 + K_4 A) C_{cu} + C_{sr}] \quad \mathbf{(10)}$$

Equation (**10**) gives the annual cost per 1000 ft. of line, using the most economical spacing of transformers.

The most economical cross-section of wire is obtained when the first derivative of Y with respect to A equals 0 or

$$\frac{dY}{dA} = 0$$

$$= - 1/3 \times 744.5 \left[\frac{K_1^2 \rho t C_{e2}}{E^2 \operatorname{Cos}^2 \theta} \right]^{\frac{1}{3}}$$

$$L_D^{\frac{1}{2}} A_{ec}^{-4/3} + \frac{g_L}{100} \times 30\, K_4\, C_{cu}$$

Solving for A_{ec}

$$A_{ec} = \frac{154}{(g_L\, K_4\, C_{cu}^{3/4})} \left[\frac{K_1^2\, \rho\, t\, C_{e2}}{E^2 \operatorname{Cos}^2 \theta} \right]^{\frac{1}{4}} L_D^{\frac{1}{2}} \tag{11}$$

Equation (**11**) gives the most economical cross-section of wire using the most economical spacing of transformers.

It is necessary to limit the application of equation (**11**) to less than 3 per cent voltage drop and develop the equation for most economical wire size *with* 3 per cent drop. This is done as follows:

From the equation (**8**), the spacing which will give a 3 per cent voltage drop is,

$$S = \frac{E}{10} \sqrt{\frac{A}{B\, L_D}}$$

B may be expressed as a function of A as follows,

$$B = K_5 + K_6\, A$$

Where K_5 and K_6 are found by plotting B against the cross-sectional area, (see curve No. 3). Then:

$$S = \frac{E}{10} \sqrt{\frac{A}{(K_5 + K_6\, A)\, L_D}}$$

Substituting the value of S in equation (**4**), the expression for annual costs per 1000 ft. of line—(the spacing being limited for a 3 per cent voltage drop) becomes

$$Y_{3\%} = \frac{1000\, K_1}{\dfrac{E}{10} \sqrt{\dfrac{A}{(K_5 + K_6\, A)\, L_D}}} + K_2\, L_D$$

$$+ \frac{g_L}{100} [30\, (K_3 + K_4\, A)\, C_{cu} + C_{sr}]$$

$$+ 60.83\, \frac{L_D^2\, \rho\, t\, C_{e2}}{A\, E^2 \operatorname{Cos}^2 \theta}$$

$$\times \frac{E^2}{100}\, \frac{A}{(K_5 + K_6\, A)\, L_D}$$

Simplifying

$$Y_{3\%} = \frac{10{,}000\, K_1\, L_D^{\frac{1}{2}}}{E} \left(\frac{K_5}{A} + K_6 \right)^{\frac{1}{2}}$$

$$+ K_2\, L_D + \frac{g_L}{100} [30\, (K_3 + K_4\, A)\, C_{cu}$$

$$+ C_{sr}] + 0.6083 \frac{\rho\, t\, C_{e2}\, L_D}{\mathrm{Cos}^2\, \theta\, (K_5 + K_6\, A)} \tag{12}$$

Equation (**12**) gives annual cost per 1000 ft. of line using a spacing which limits the voltage drop to 3 per cent at full load.

The most economical cross-sectional area is obtained when the first derivative of $Y_{3\%}$ with respect to A is equal to 0.

$$\frac{d\, Y_{3\%}}{d\, A} = 0$$

$$= 1/2\, \frac{10{,}000\, K_1\, L_D{}^{1/2}}{E} \left(\frac{K_5}{A_{ec}} + K_6 \right)^{-\frac{1}{2}}$$

$$\left(- \frac{K_5}{A^2{}_{ec}} \right) + \frac{g_L}{100} \times 30\, K_4\, C_{cu}$$

$$- 0.6083\, \frac{\rho\, t\, C_{e2}\, L_D}{\mathrm{Cos}^2\, \theta\, (K_5 + K_6\, A_{ec})^2}\, K_6$$

From which

$$\frac{5000\, K_1\, K_5\, L_D{}^{1/2}}{E \left(\frac{K_5}{A_{ec}} + K_6 \right)^{\frac{1}{2}} \quad A^2{}_{ec}}$$

$$+ 0.6083\, \frac{K_6\, \rho\, t\, C_{e2}\, L_D}{\mathrm{Cos}^2\, \theta\, (K_5 + K_6\, A_{ec})^2}$$

$$= 0.30\, g_L\, K_4\, C_{cu} \tag{13}$$

Equation (**13**) gives the most economical cross-sectional area of wire when the spacing is limited by a 3 per cent voltage drop. (See curve No. 7.)

B. DERIVATION OF THE PRACTICAL CURVES

1. *Load Curves for Secondaries.*

From equation (**7a**)

$$S_{sc} = 2.02 \left(\frac{K_1 E^2 \operatorname{Cos}^2 \theta}{\rho\, t\, C_{e2}} \right)^{\frac{1}{3}} \quad \frac{A^{\frac{1}{3}}}{L_D^{\frac{1}{3}}}$$

$$< \frac{E}{10} \sqrt{\frac{A}{B\, L_D}}$$

From the combination of these two formulas the load curves for secondaries were obtained.

The lines showing the various load densities are obtained as follows:

$$\text{Total load in kw.} = \frac{L_D}{1000} \times S_{sc} \tag{17}$$

(See curve No. 10.)

2. *Line Cost Curves.*

From equation (3),

$$Y_L = \frac{g_L}{100} [(3000\, w\, C_{cu} + C_{sr})$$

$$+ 60.83 \frac{L_D^2 S^2 \rho\, t\, C_{e2}}{A\, E^2 \operatorname{Cos}^2 \theta}$$

The line cost curves are obtained by substituting the actual values of the various constants into this equation. (See curve No. 11.)

3. *Transformer Cost Curves.*

Annual cost of the transformer =

$T_{T1} = \frac{g_T}{100}$ (Cost of transformer + cost of light. arrest. + cost of installation) + inspection
+ cost of core loss
+ cost of copper loss

$= \frac{g_T}{100}$ (Cost of transformer + cost of lightning arresters + cost of installation)
+ cost of inspection

$$+ C_{e1} \times 365 \times 24 \times \frac{\text{core loss}}{1000}$$

$$+ \frac{L_D^2 S^2}{E^2 \operatorname{Cos}^2 \theta} R_T \times 365 \times t \times \frac{C_{e2}}{1000} \tag{18}$$

The method for determining Y_T (the cost per 1000 ft. of installation) graphically from $Y_{T'}$ (the total cost on one transformer) is as follows,

$$\frac{S}{1000} = \frac{Y_{T'}}{Y_T}$$

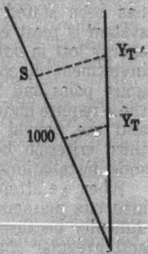
$$\therefore \; Y_T = Y_{T'} \times \frac{1000}{S}$$

($Y_{T'}$ = total annual cost on a transformer)

Y_T = annual cost of transformers per 1000 ft. of installation)

(See curve No. 12.)

Hence by adding the diagonal scale at the left, Y_T may be obtained from $Y_{T'}$, as follows by the method of similar triangles. Draw a line from the value of $Y_{T'}$, obtained on the vertical scale to the value of S used, on the diagonal scale. Draw a parallel line through 1000 ft. on the diagonal scale and where it intersects the vertical scale will be found the desired value of Y_T.



DISCUSSION ON "ECONOMIC STUDY OF SECONDARY DISTRIBUTION" (REYNEAU AND SEELYE), PHILADELPHIA, PA., OCTOBER 8, 1920.

Walter I. Slichter: In spite of the fact that the general fundamentals underlying the question of distribution are quite simple, there is a very complicated problem facing engineers which is not appreciated by the average person. There are three different criteria underlying the problem: first, a certain limitation in the variation of the voltage; second, a definite limit to the amount of money that may be lost in energy; third, a definite limitation to the heating of the conductors in order to avoid danger from fire.

But these limitations are subject to discussion. The first, as to how great a variation of voltage will still give the customer satisfactory service, particularly in connection with illumination. The second point has been stated simply by Lord Kelvin, that that system is most economical in which the value of the energy lost in one year is equal to the interest on the investment in copper. But present day systems require poles and insulators as well as copper and all these require maintenance, so that both the investment and the annual expenditure are less definite. The third criterion becomes of importance only in very short installations.

When we attempt to combine these three variables into one mathematical equation we meet with great difficulty because the variables take certain arbitrary and irregular changes in value. Thus the variation in voltage may reach a definite arbitrary limit when the density in the wire makes the heating dangerous. Thus even in a simple series distribution we find the relations somewhat complex.

When we introduce the transformer we have the new consideration as to how much energy may be lost on the primary side and how much in the secondary distribution, which, speaking mathematically, means that there are an infinite number of solutions to the problem. It is therefore a very interesting and valuable contribution to the art to combine these widely varying factors into equations and curves and by the introduction of commercial values and costs make them applicable to commercial problems.

There is one rather astonishing point made in the paper. The authors state as a general and definite finding that the maximum variation in voltage which conforms with reasonable values for the other quantities is 3 per cent. It would be deduced from this that many operating companies would save money by giving their customers better voltage regulation.

D. W. Roper: The authors in their paper apparently assume that the transformers are to be connected in multiple and their calculations appear to be based on this assumption. It might be proper to inquire, therefore, why should the transformers in an urban distribution system be connected in multiple? In residence districts, for example, it will ordinarily be found that the demand in several consecutive blocks varies over quite wide limits and under such circumstances it would not be desirable to install in each block the average size of transformer instead of the size determined by the loads in that block.

Also as a practical consideration, it is found in Chicago that there are hardly any service connections from the pole at the end of the alley so that by omitting this span entirely as well as the span crossing the street, there will be a saving of three spans of the secondary in each block, amounting to nearly 40 per cent of the total length. With this system the size of transformer is selected in accordance with the load in the block which is supplied and in our experience it is a more satisfactory arrangement than operating transformers in multiple with the difficulties which occur with such a system in case of over loads and burnouts.

It is interesting to note, however, that the relation between the size of transformers and the size of the secondary circuits as used in Chicago are quite similar to those shown by the authors, that is, in the outlying districts where the center of the load is not very great No. 6 wire is the most economical for secondaries and that as the transformer size increases, a larger size of wire for the secondary mains is more economical. It appears, therefore, that while the calculations of Messrs. Reyneau and Seelye are based upon somewhat different assumptions from those used in Chicago, their results correspond very closely to Chicago's practise and their results should serve as a very useful guide in designing distribution systems.

H. P. Seelye: Mr. Slichter's statement about 3 per cent voltage drop is quite true. I think it is largely due to the fact that the economical considerations have not been sufficiently studied in the design of secondary distribution that large voltage drops are thought to be economical.

An interesting point in this regard is the fact that the economical drop is shown in many cases to be less than 3 per cent, especially as it occurs with load densities which are found very often in the residential districts. A 15-kw. load density is quite a common occurrence in well built up sections, and the most economical voltage drop at that load is about the practical limit set, that is, a three per cent drop. With load densities higher than that, the voltage drop is even less than three per cent.

As to Mr. Roper's remarks in discussing the banking of transformers, I do not think the discussion as we have brought it out is based essentially on banked transformers. In other words, we have considered a theoretical condition of a transformer feeding equal distances in both directions, so that the fact that they are assumed to be banked is not essential to the theoretical results. Practically of course, there is a great deal to be said on both sides in the matter of banked transformers. The practise of making the secondaries continuous is advantageous in the case where a design is made with a given spacing of transformers, with the idea that when the load increases that spacing can be changed. This very often leads to economy in stringing secondary wire, even in cases where strain balls are inserted to separate the transformers. The excess cost of stringing more wire later on is saved and in many cases poles, guys, cross arms, etc., are also eliminated.

From the operating point of view, where transformers are banked, if one transformer goes out, the load can be carried and operation continued until the transformer can be replaced. This is applicable to places where frequent inspections of transformers are made, which is the condition assumed in this paper. In outlying districts we do not bank the transformers. It is obviously poor policy to interconnect large transformers with small ones, since if the large ones go out, the small ones will follow, as they cannot carry the heavy load. Where transformers are of nearly uniform size, we have found it advantageous to bank them in well-built up sections.

Presented at the 364th meeting of the American Institute of Electrical Engineers, Philadelphia, Pa., October 8, 1920.

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THE MEASUREMENT OF MAXIMUM DEMAND and THE DETERMINATION OF LOAD FACTOR

BY PERRY A. BORDEN
Hydroelectric Power Commission of Ontario.

SINCE Hopkinson in 1892 first suggested the idea that the distribution of a customer's load throughout the day should have some bearing upon the amount which that customer should pay to the Central Station for his power, the subject of demand and load-factor measurement in its various aspects has been many times discussed. In the determinination of watts, amperes or any similar electrical quantity we have definite units upon which to base our measurements, and there is little room for controversy. But, when the quantity "demand", being a more or less mathematical concept, embodying the combination of electrical units with time in a rather indefinite way, comes under consideration, very divergent views as to its nature, measurement and true significance may be and have been expressed.

The object of the present paper is not to introduce any radically new ideas, nor is it to advocate any particular policy as a panacea for all difficulties which beset the rate maker in his work. It is rather to give a bird's-eye view of the situation as it exists today; and, in an endeavor to reconcile some of the different opinions on the subject to show an actual comparison of the performances of a number of demand-measuring devices. And from this comparison have been deduced some interesting facts which would seem to have an important bearing upon the present day status of industrial load measurement.

For the purpose of a systematic study of the measurement of electrical demands the subject may be divided into three natural sections as follows:

I. The electrical quantity upon which the measurement is performed.

II. The method by which the measurement is accomplished.

III. The results obtained by the several methods upon a variety of types of loads.

Although these aspects of the subject are closely related, they will be considered as independently as possible, one of another.

I. THE QUANTITY MEASURED

In the establishment of a scale of charges for electrical energy the rate-maker at once recognizes the fact that he has the choice of a number of electrical quantities upon which to base his calculations. These quantities are affected in various ways by the nature and magnitude of the load under consideration. If the value received by a consumer from a certain amount of electricity were in direct proportion to the expense incurred in placing that electricity at his disposal, the problem of establishing charges for electrical power would be immensely simplified. Unfortunately, however, such is not the case; and the problem becomes one of finding a middle way between an absolute flat rate, based only upon the installed capacity of the load, and a straight energy charge established upon the readings of a watt-hour meter.

In hydroelectric power plants, where the fuel costs are nil, and the apparatus may be run at its ultimate capacity at practically no more outlay than at very light loads, the limitation of output is the *capacity of the equipment.* In fuel-consuming plants, where the combustion of coal or of oil is the source of energy, the cost of fuel, (the only item of cost whose value bears any direct ratio to the energy output), is seldom over half the annual expense of the undertaking, the remaining charges being practically dependent upon the capacity of the plant. In either case, therefore, the relationship between the watt-hours supplied and the cost of operation is a very indefinite one. The main object to be attained is that of keeping up the load factor; and the logical way to produce this result is to encourage each customer to do his share. It has of course, been demonstrated that cases may exist where it is desirable to have on the system a customer with a low load factor, who, by placing his peaks at the discretion of the Central Station, may serve to fill up the valleys in the load curve caused by another customer who cannot so well control his demands; but practically all such cases are of a particular nature; and as such, need not be considered under the general head. The most natural way to encourage a high load factor is by the production of a direct reaction upon the customer's pocketbook, and hence the justification of the use of demand in the establishment of rates.

The "capacity of the equipment" referred to in the former paragraph is a term which in itself might furnish material for much discussion, and its very uncertainty adds much to the intricacies of the problem of demand measurement. To the mechanical engineer such a term would present little difficulty, as it would merely signify a power value beyond which the weakest link in the system would fail. But to the electrical engineer the matter of heating of equipment is usually fully as important as any purely mechanical feature; and this heating is not in direct proportion to the amount of energy delivered to the consumers.

The temperature rise of a piece of apparatus is governed by the relation of the energy losses therein to the facilities for dissipating the heat generated, and to the time allowed for the heat to distribute itself away from its source. All of these factors are subject to great variation. If then, we wish to base our charge upon the capacity of the plant or investment necessary to provide power for the customer in question, we must pay some consideration to the temperature rise of our equipment due to his load. This means that, to obtain a fair basis we should endeavor to determine the energy loss produced in our equipment by the load under consideration and to incorporate this value in his power bills.

The direct measurement of loss, while possible on a single unit by use of a differential wattmeter, or by other means, is not practicable where the one bus supplies a number of independent loads. It is desirable, therefore to install upon each circuit that type of measuring instrument whose indications bear the closest relationship to the enrgy losses in the supply system. Hence the question, "What electrical quantity shall we measure?"

As possible answers to this question, consideration is given to the follwing quantities:

(a) Watts ($E\ I\ \mathrm{Cos}\,\phi$)

The majority of the present day demand meters operate as wattmeters and give the maximum demand of the load in watts consumed. This method has the advantage that it is universally applicable to all classes of loads,—direct and alternating, single-phase and polyphase, balanced and unbalanced. This value while representing the demands made upon the mechanical portions of the system, is not of necessity proportional to the heating effects.

(b) Volt-Amperes ($E\ I$)

Since for a given energy load, the heating of equipment is manifestly greater at low power factors than at high, a consideration of volt-amperes without regard to the actual energy supplied will have a tendency to encourage operation at high power factor, and thus reduce energy losses. This method is already used to a considerable extent by the employment of a watt-demand meter whose readings are coupled with those of a power factor meter at the time of the maximum demand. This scheme, however, or the use of a reactive-volt-ampere-meter in conjunction with a watt-demand meter involves the use of two instruments, one at least, of the curve-drawing variety; and is, therefor, suitable only for comparatively large loads from which the revenue would be sufficient to justify the expense of the equipment.

The volt-ampere meter has not yet been developed in a form suitable for direct determination of maximum demand, and does not at this time appear capable of such development, at a price to compete with the watt-demand meter on ordinary commercial measurements.

(c) Amperes (I)

The earliest types of demand indicators were dependent for their indications upon current alone. And, since it is the current which is responsible for the heating of conductors, there would seem to be justification for their use. On polyphase circuits, however, they are likely to introduce confusion and uncertainty.

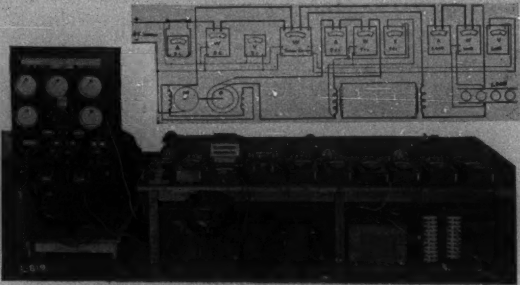

FIG. 1—TEST ON TRANSMISSION LOSSES

(d) Amperes Squared (I^2)

As the heating of a conductor is directly proportional to the square of the current flowing, it would seem that an instrument measuring demand upon this basis would find a certain sphere of usefulness. Such a meter would be identical in its principles with an ampere-demand meter, but would have an inherently uniform scale. Its construction, to operate upon either a thermal or an electro-mechanical principle should be quite practicable. It would, however, on unbalanced loads, be subject to the same limitations as the ampere-demand meter.

(e) Actual Energy Losses ($K E^m + R I^n$)

As stated above, while it is possible to measure the actual energy losses in a simple system, where but one load is fed from the portion of the plant under consideration; it is not practicable, when several loads are fed from one source, to apportion the responsibility for the energy losses without the most intricate mathematical operations and complicated metering equipment. The accurate measurement of energy losses under ordinary working conditions may be looked upon as a matter of purely academic interest and here laid aside in favor of some more practical if less precise method.

As a matter of interest, a number of tests were made upon a load fed through a series of circuits forming an artificial transmission system. The arrangement, as shown in Fig. 1, consisted of a direct-current motor driving a small alternator whose output was stepped up to the line, and down again to the load. Complete measuring equipment was installed at several points along the system, and a differential wattmeter connected in so as to totalize the losses. One set of curves was made with non-inductive loading, and another with considerable reactance in the circuit to produce a low power factor.

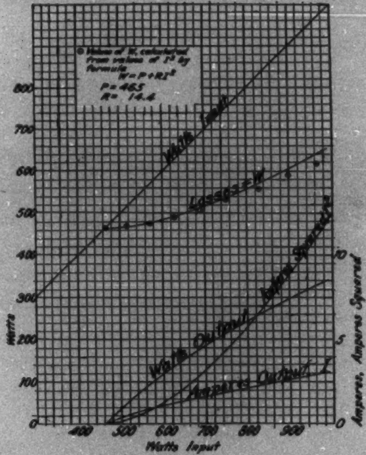

Fig. 2—Loss Measurements on Unity Power-Factor Load

The values obtained in these tests are shown in

Figs. 2 and 3 where the various quantities are expressed as curves on a base of watts input to the system. The curve of total loss in the system manifestly represents only the difference between the total input and the total output, and the same, of course, holds good for any portion of the system which it is desired to investigate. The total losses of the system, including those of the motor-generator set, the transformers and the line, are in this case taken, as being representative of the greatest variety and magnitude of energy losses, and therefore approaching most closely the average power system.

A glance at the curve sheets suffices to show that, whereas the watt and volt-ampere curves bear but little resemblance to that of the energy loss; a strong similarity exists between the loss curve and the curve of amperes squared. It is a simple matter to develop an expression connecting the two quantities, which, from the values of amperes squared will enable us to construct the curve of losses.[1] This, while quite applicable to those portions of the system which supply only one customer, is subject to the same handicap as other methods of total energy loss determination, where several consumers are fed from the one bus.

Summary—Part I.

1. A fair and equitable charge for electrical energy cannot be made on a basis of energy consumption alone. Some consideration must be made of the customers' demand upon the capacity of the plant.

2. In the determination of demand, cognizance should be taken of the heat produced in the equipment by the load under consideration, as well as of the mechanical limitations of the plant.

3. While the mechanical limitations of the plant bear a direct relationship to the energy output, the heating limitations are in direct proportion to the energy losses in the electrical equipment.

4. Energy losses cannot generally be measured, and are usually very difficult to estimate accurately.

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[1] See Appendix No. 1.

5. Energy loss may be expressed as the sum of a function of the voltage and a function of the current; and considering the complicated nature of the expression necessary to give a precise representation of this value, the voltage may usually be considered as constant, and the loss said, with little sacrifice of accuracy, to vary as the square of the current,

6. An exact apportionment of the I^2R losses among several loads fed from one system is made difficult by the fact that, while the total loss varies as the square of the total current, the ratio of the losses due to the respective currents is an indefinite quantity, depending upon a number of variables whose values it is not practicable to obtain.

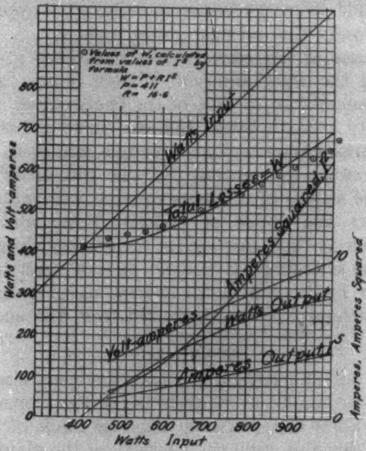

Fig. 3—Loss Measurements on Low Power-Factor Load

7. Since the true basis for demand measurement is an exceedingly evasive quantity, not capable of determination under practical working conditions, it is necessary that some compromise be made, and the nature of this compromise must be subject to

local conditions and to personal opinions on the economic questions concerned.

8. Tests would seem to indicate that, though volt-amperes cannot be said to be an exact representation of the quantity upon which demand would be based, they furnish us with a definite quantity whose value approaches nearest to the desired approximation.*

II. THE METHOD OF MEASUREMENT

In the present state of the art, there are recognized several methods of determining demand: These include measurements from the charts of graphic recording instruments, and readings obtained from specially designed demand meters. While from an analysis of the graphic meter chart it should be theoretically possible to determine the average power for any period and to select the greatest of these averages as the maximum demand, any person who has tried to do this with the chart of a load having any considerable degree of variation realizes that a positive determination of the true average over a chosen period is seldom practicable.

The only workable method of measuring maximum demand from graphic meter charts is by selecting that portion of the curve during which the indication remains at its highest value for the duration of the required time period. This is known as the "Sustained Peak." While the use of this quantity, in that it cannot exceed the true maximum integrated demand, might well meet with the customers' approval, it may be very unfair to the Central Station, in that the value obtained cannot be higher than the minimum point of any depression which may occur during the period of maximum demand. Moreover, the depth of any instantaneous minima of the load curve as drawn by the graphic instrument will depend greatly upon the amount by which the meter movement is damped. In Fig. 4 is shown a reproduction of charts taken simultaneously on the one load, by curve-drawing meters of different types. One of these instruments was of the well-known relay type; while the other, operating upon the induction principle, had almost critical damping. From these charts it will be seen that, by a slight difference in the operating characteristics of the recording meters, the load is shown, not only as having sustained peaks of differing values, but as having its peaks at different times of the day. Thus, on a fluctuating load, the values of maximum demand, as determined from the sustained peak are of little significance.

Since the ordinary curve-drawing meter does not justify its use for the determination of demand, it becomes necessary to investigate the possibilities of the various types of demand meters which, from time to time, have been developed; and to examine their principles of operation.

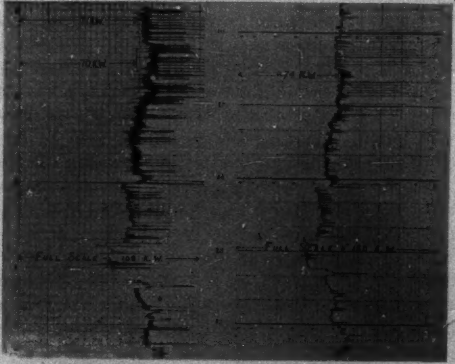

Fig. 4—Simultaneous Charts From Different Types of Curve-Drawing Meters Showing Sustained Peaks

Among the numerous types of demand instruments now known to engineers there are recognized two groups, which are classified according to the principle of operation rather than the quantity measured. These classes are as follows:

1. Those from which an integrated value of the load is obtained.

2. Devices which are time-lagged and cannot, in

the ordinary sense of the word, be said to give an integrated value.

Integrated Demand Instruments. The quantity determined by this class is usually the ratio of the total integrated value of the load consumed in the time interval to the time of the interval. This class naturally falls into five subdivisions of which, as we proceed, I shall endeavor to briefly describe one or more types to be found in each.

(a) Curve-drawing meters include all meters which give a continuous line record or chart of the quantity measured. From the charts so obtained it is possible by measurement or by integration to gain very complete information as to the load, its peaks, its total value and its demand over any desired period. But as shown in a previous paragraph, the systematic use of such instruments for the accurate measurement of demand is seldom practicable.

(b) Instruments which graphically record the demand for each successive time interval as fixed by a clock or other timing device, time also being recorded. The demand in each clock interval, and therefore the maximum demand, can be obtained from the record. This type and those in the two following classes operate upon what is known as the "Merz" principle, and are subject to the disadvantage that, the time periods being selected by a clock, no cognizance is taken of load conditions immediately previous or subsequent to each individual demand. The clock being unable to select the period of maximum power, the maximum demand as obtained from such an instrument is only the greatest of those demands which were measured by the individual meter in question. Two similar meters installed upon the same load, unless the tripping mechanisms operate in synchronism, need not indicate the same value. As examples of meters in this class may be named the General Electric type *G* meter, the Westinghouse type *R A* and the Piek or the N. E. I. C. meter.

The General Electric type *G* meter has essentially two parts, a registering element and a timing element, both of which are mounted within the same case.

In addition to the device proper, a complete outfit includes a contact device consisting of cam and contact brushes for mounting in the register of the watt-hour meter with which the demand meter is to be used. The registering element consists of a marking stylus, the electromagnet, ratchet and pawl mechanism, and gearing to transmit motion from the armature of the electromagnet to the stylus. The timing element consists of an eight day clock which drives the chart, and, at the end of each time interval, resets the stylus to its zero position. The charts are circular and arranged to cover a period of one week. They are made of a special coated paper similar to that used for steam engine indicator cards; and the record is made by a steel stylus.

The Westinghouse type *R. A.* meter combines the demand mechanism and the integrating mechanism in one case and the connection between the two is mechanical. The record paper is in the form of a strip and is of sufficient length to last for thirty-six days. The chart does not travel continuously but is advanced a short distance just before the pen is released and reset. Thus each demand indication is given a square top that makes it distinctly readable.

The Piek demand meter is probably the first graphic demand meter to make its appearance in this country. It was introduced about 1910. The measurement of energy is accomplished by a Westinghouse type *C* watt-hour meter bearing the ordinary integrating dial. Geared to, and operated by the meter is a parallel motion carrying a pen or stylus which travels in a straight line across the scale. The position of the pen is at all times recorded upon a paper chart driven forward by a self-winding clock. This clock serves also to drive a cam which, operating an electric tripping device, periodically releases the pen, and resets the demand mechanism to zero. These meters are built for periods ranging from one minute upwards.

(c) Instruments which, at the expiration of each time period, make upon a tape or chart, a record of the reading of an integrating meter. These meters are very similar to the foregoing class, except that whereas in the former, the record is graphic, in this case it is in figures. The "Printometer," now known as the General Electric type *P* demand meter is the best known example. The instrument contains a set of cyclometer type-wheels which are electrically interlocked with the register of a watt-hour meter. They are moved forward at a rate representative of the flow of power through the meter; and will, therefore, at any instant give an indication which is equivalent to the reading of the dial. Through the agency of a rubber platen and a copying ribbon, this reading is printed on a paper tape. The outfit is not self-contained, the demand indicator being separate from the clock and the meter, and requiring, like the type *G*, a contact-making device to be fitted to the register of the meter.

(d) Instruments which indicate, but do not record the maximum demand, the time intervals being fixed by a clock or other timing device. In meters of this class, the reading is obtained from a pointer, which must be manually reset to zero, and which gives only the maximum demand since the last previous resetting. As examples of this class may be named the General Electric type *M* (formerly the Maxicator) and the Siemens demand meter.

In its general principles the type *M* is similar to the type *G*. As the stylus is replaced by a pointer, however, it is not automatically returned to zero. The mechanism which forces the pointer across the scale returns periodically to zero but the pointer remains at a maximum deflection, and is reset by hand when the meter reading is taken. Instead of a clock, the *M* 4, for alternating currents, has for a timing element a constant-speed motor similar in its construction and operation to the moving element of an ordinary watt-hour meter.

The Siemens meter consists of a demand attachment mounted in the same case with a watt-hour meter of the standard type. The clock is a small, electrically-driven unit which, by means of a cam, mechanically actuates the tripping mechanism. A driving dog periodically geared to the watt-hour mechanism, impels a pointer around a graduated scale, and the position of the pointer indicates the maximum deflection of the dog, and hence, the maximum demand, since the pointer was last reset. At the time of reading the meter, this pointer is manually reset to zero. The Siemens meter, being of the purely indicating type, requires no attention other than the periodic reading and setting of the demand pointer.

A considerable number of meters using the Merz principle have been developed by British and European manufacturers but these differ from the Siemens meter only in details of construction.

(e) Instruments which make a record on a tape or chart when a certain fixed and predetermined amount of energy has been consumed, time also being recorded. This class differs from class (c) in that the demand is shown for any time interval, irrespective of when the interval began and ended. In other words, when a predetermined amount of energy has been consumed, a record is made, while in class (c) instruments a record is made when the predetermined time interval has elapsed. It differs from class (a) in that the record is not a continuous one, but periodic. Example: Ingalls demand recorder. In this meter the general scheme of operation is similar to that of the General Electric type *P* but the functions of the timing and recording elements are interchanged. A dot is made on the paper tape after the consumption of each predetermined block of energy. The rate of consumption is obviously greatest where the dots are most numerous.

Numerous attempts have been made to develop a meter which would overcome the selective characteristic of instruments of the Merz type; but owing to their mechanical complications, these cannot be said to have found sufficient favor to justify their production in quantities. A special instrument developed for this purpose by the writer is described in a later section of this paper.

Lagged Demand Instruments. A lagged demand instrument really comprises an indicating meter in which a retarding device is used, so that a definite time period must elapse before the full value is indicated. The time interval is independent of clock time.

The value measured by instruments in this general class differs from the integrated value by an amount which depends upon the particular type of instrument and the character of the load curve. With a perfectly steady load, all instruments of this class would indicate practically the same value as integrating instruments, but with a fluctuating load each type may give a different result. Hence, when referring to demands measured with instruments of this class, the particular type of meters employed should be mentioned. The class may be subdivided into the following two types:

(a) Instruments in which the rate of motion of the indicator over the scale is always proportional to the load. Example: Westinghouse type *R. O.* meter.

The Westinghouse type *R. O.* watt-hour demand meter, which made its appearance six years ago, embodies some distinctly original features. It combines in the one mechanism a wattmeter and watt-hour meter, both actuated by the same electromagnetic elements. The wattmeter is prevented from registering the instantaneous value of the load by an escapement operated from the watt-hour meter shaft. The indicating mechanism, therefore, constantly endeavors to indicate the load; but can only approach that value (as long as the indication is less than the load) at a rate proportional to the load. Since the speed of travel is proportional to the load, it will be seen that an indication of any sustained load will be attained at the end of a definite and constant time interval. The time period is thus established independently of any clock. Engaged with the indicating mechanism is a maximum pointer, which, by indicating the greatest deflection of the wattmeter element, determines the maximum demand.

Several instruments measuring substantially the same quantities as the *R. O.* meter have recently been patented, but as they have not made their appearance on the market it is not necessary to refer to them here.

(b) Instruments in which the rate of motion of the indicator decreases with the time of deflection. Examples: Wright, Reason, Lincoln type *R. H.*, General Electric types *W* and *H*.

The Wright meter is in principle a differential recording thermometer which measures the heat produced by an electric current. It consists of two bulbs of approximately the same size, connected by a U-tube filled with liquid and provided with a third tube; in close proximity to which the scale is fixed. The current is carried through a heating coil wound on one of the bulbs. The heat produced causes the air to expand and to depress the column of liquid in one side of the U-tube, causing it to rise in the other and overflow into the reading tube. The height to which it finally rises is an indication of the maximum value of the current which has passed through the coil.

The Reason demand meter is a similar piece of apparatus in which the glass tube is tilted by the measured current flowing in a solenoid, thus allowing the liquid to flow into the reading tube. Both this and the preceding type, being unaffected by the voltage of the circuit are purely ampere-demand meters.

The Lincoln type *R. H.* meter having already been described both in theory and in mechanical construction to the A. I. E. E.,[2] it is assumed that a very brief description will suffice to recall to mind its operating principles.

This demand indicator operates upon the principle of heat storage. By an ingenious, though simple arrangement of transformers and resistors it has been practicable to obtain a thermal quantity which is directly proportional to the energy in the circuit. An expanding spring, connected to a deflecting pointer enables an indication to be obtained of the watts passing through the metering element, and the instrument constitutes a true thermal wattmeter. The time element is introduced by loading the heaters with metallic masses, which do not instantly respond to temperature changes. Upon any change in load, the temperature of the mass begins to change and continues to do so until the rate of heat loss balances the rate at which energy is being supplied. This change in temperature takes place according to what is known as a logarithmic law.

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[2] TRANS. A. I. E. E. Vol. XXXIV, p. 2297

The General Electric type W demand meter is essentially a polyphase watt-hour meter with both electrical elements acting upon one disk, and a very strong damping system acting on the other disk to produce the necessary time lag. The rotating element is opposed by a series of springs which permit the disks to make three complete revolutions while the pointer makes one revolution. The instrument thus becomes an indicating wattmeter, very much over damped. This meter is usually constructed to have a time period of five minutes.

The type *H* is a thermal instrument and records the maximum current. The instrument works on a differential thermometer principle. Two similar thermostatic springs are mounted on studs, their free ends being connected by a cord which passes over a spindle, the latter carrying an arm which engages the pointer. The air temperature affects both springs to the same extent, thus causing no deflection of the pointer for its variations. In one of the two spring-supporting studs is contained a heating element through which the main current is passed; thus when current is being used, the temperature of one spring is raised above that of the other, causing the shaft to turn, and so produce a reading. This instrument combines in its principles the idea of the logarithmic increment of temperature, with that of the flow of heat along conductors.

The demand meters referred to above include only types which are now, or have been in actual commercial use. It is not the intention to convey the impression that these are all the types that have ever been developed. To obtain an adequate idea of the multitude of methods and principles which have been proposed for the measurement of this elusive quantity, it is necessary only to refer to the records of the Patent Offices for the past fifteen years.

A considerable number of "Excess Meters" have from time to time made their appearance but as these instruments cannot properly be considered as coming under the head of "Demand Meters," they are not given consideration in this paper.

III. OPERATION ON VARIOUS LOADS

Provided the laws governing the mechanical operation of any type of demand meter are known, it is possible to predetermine the indications of an instrument in which the quantity measured undergoes any prearranged cycle of variations. In comparisons of demand indicators this has frequently been done, and an opportunity thus given to study the inherent characteristics of various types of demand measuring devices, particularly at the start and the finish of the duty cycle.

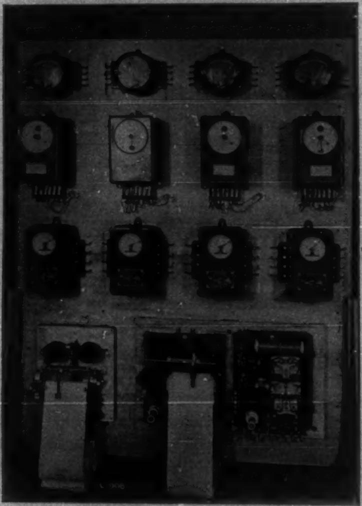

FIG. 5—COMPARATIVE DEMAND METER OUTFIT

Under actual service conditions, however, the duty cycle is seldom exactly duplicated; and as a slight change in this cycle might produce varying effects upon the operation of different instruments, the element of uncertainty so introduced would seem to justify service tests on actual typical loads as being the only fair comparison of demand meter types.

With the object of carrying out such tests, the writer collected a number of typical instruments, all of which were installed upon an easily portable panel, so arranged as to be cut in on the metering circuits of such typical installations as were accessible. This outfit is shown in Fig. 5, there being three distinct types of meters and, of each type, four time periods. The types are as follows: Top row, Lincoln thermal storage (logarithmic); second row, Siemens (Merz type, or block-interval); third row, Westinghouse type *R. O.* (mechanically lagged).

The time periods were 10, 20, 30 and 60 minutes for the two latter types, and 10, 15, 30 and 40 for the first, it being borne in mind that the time period of a logarithmic meter is arbitrarily defined as the time required for the instrument to reach ninety per cent of its ultimate indication on a steady load. Each meter was adjusted to its maximum possible accuracy and frequently checked during the progress of the tests. The 10-minute and 30-minute logarithmic meters were supplied by the manufacturer from standard stock parts; the other periods being made up from such material as was at hand. These latter, when tested, were found to have time periods of 15 and 40 minutes respectively; which values were adopted as satisfactory for the tests.

The block-interval meters being of a pre-war European type, it was not practicable to obtain suitable timing gearings; and special methods were used. Instead of cutting new gears for the timing elements, the clocks were removed, and replaced by small solenoids operated from a contactor carried by the master clock in the continuous recording meter.

The mechanically lagged meters were stock instruments, which were fitted with suitable timing gears specially ordered from the factory.

As it is a well-known fact that none of the types of meters commercially used will infallibly measure demand upon an arithmetical basis; and as it was very desirable to have some form of instrument which could be considered as a standard of comparison, it was found necessary to develop and construct a meter for this purpose. As this meter embodies some rather unique features which may be of interest, the following description of its working principles and operation is given.

The instrument itself was built of such odds and ends as could be found among the accumulation upon the shelves of the meter shop; and, as may be readily seen by reference to the illustration, (Fig. 6), does not partake of the general appearance of the apparatus produced by any of the well-known manufacturers of electrical equipment. However, as this device was constructed solely for experimental purposes, the use of such a nondescript instrument may be pardoned on the grounds of the interesting and satisfactory results obtained.

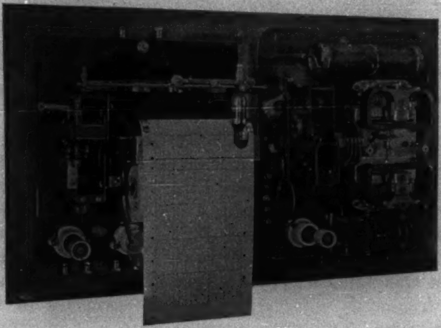

FIG. 6—CONTINUOUS RECORDING DEMAND METER

To give a clear understanding of the operation, a diagrammatic sketch is shown in Fig. 7. It will be observed that the working principle is exceedingly simple. A paper chart is caused to advance at a speed proportional to the load on the circuit, while a pen is propelled across the paper at a constant velocity in a direction perpendicular to the travel of the chart. The first motion is accomplished by means of a watt-hour meter, controlling the speed of the paper travel; while the second motion is produced by a clock, winding a cord about a friction-driven drum, so that the pen makes its transit of the paper in exactly ten minutes. At the end of each ten-minute period, a contactor driven by the clock energizes an electrical device, which forcibly returns the pen to its zero position, the cord unwinding from its drum, which slips on the arbor of the clock.

As the return of the pen is practically instantaneous it will be seen that the chart takes the nature of a series of curves one above the other, the corresponding ordinates on successive curves being separated by intervals of ten minutes. It will be seen that the slope of any of the curves at any instant is proportional to the load at that instant; and the series of curves may be considered as the translated portions of one continuous curve whose height from the point of starting represents the integrated value of the load.

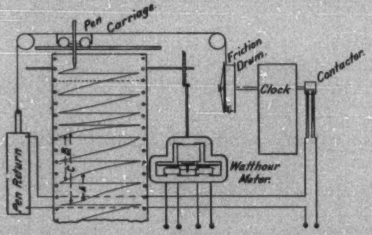

FIG. 7—DIAGRAM OF CONTINUOUS RECORDING DEMAND METER

Since measurement along any ordinate represents the advance of the paper, *i. e.* the integrated load, during the time period between two crossings of that ordinate by the pen, it manifestly represents the average load or demand during that period. It only remains, then, for the measurement of maximum ten-minute demand, to select the ordinate whose length between two successive curves is greatest and to express its length in terms of the units measured by the integrating meter.

Similarly, demands for twice the normal time period of the meter may be scaled off by measuring the distance along the greatest ordinate between alternate curves; and for greater multiples of the normal period, by selecting the desired number of curves to be spanned. Thus, from the one chart it is possible to determine, not only the true maximum demand for the fundamental period, but the demand for any multiple of that period.

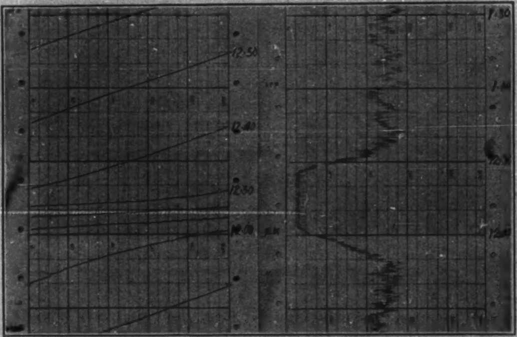

Fig. 8—Simultaneous Charts From Continuous Recording Demand Meter and Curve-Drawing Wattmeter

Referring to the chart shown in Fig. 7, the maximum ten-minute demand is indicated by the distance *A*, the twenty-minute demand by $B/2$ and the thirty-minute by $C/3$.

Fig. 8 shows a portion of chart from this instrument, side by side with the simultaneous record of a graphic wattmeter upon the same load. These curves illustrate the operation of the demand meter at the time of the noon valley on an actual factory load.

The portable panel carrying the demand meters and a curve-drawing wattmeter, was wired up to the metering circuits of several types of load, and careful records were taken of the indications. These records were averaged for each meter over several days, and the averaged readings expressed in the form of curves, as shown in Figs. 9 to 14.

Since demand is one of the elements entering into the computation of load factor, and since the values of demand may vary widely, according to the method of determination, it must necessarily follow that the value of load factor is subject to the same wide variations. But the latter quantity, being a ratio, and independent of the magnitude of the load, may be used in comparisons with much greater felicity than may the actual values of demand. The curves, therefore, are expressed in terms of load factor plotted against time period, a separate curve for each type of meter and a separate set of curves for each class of load. The numbering of the curves is the same as the vertical arrangement of the meters on the panel, *viz.*—1, logarithmic; 2, block-interval; 3, mechanically lagged; 4, true arithmetical average. With each set of curves is shown a sample of graphic wattmeter chart from the same load. In the load factor

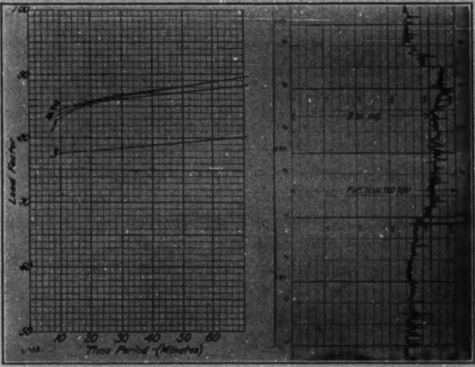

Fig. 9—Load of Electrical Testing Laboratory

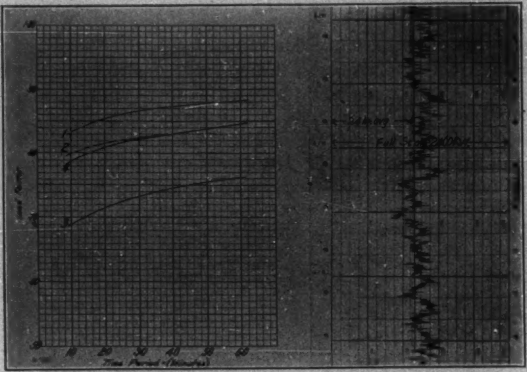

FIG. 10—LOAD OF RUBBER MANUFACTURING PLANT

calculations, the average power for the day is taken as the average over only that time during which the load curve maintained its peculiar characteristic nature.

Fig. 9 shows the values obtained with the meters

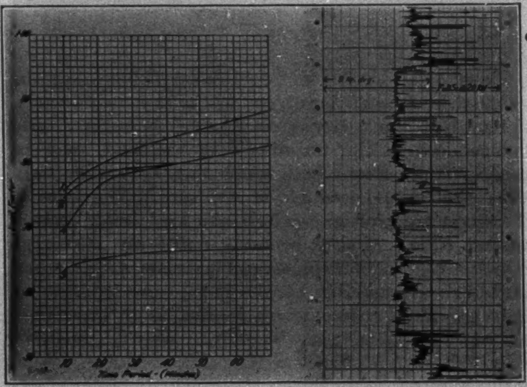

FIG. 11—LOAD OF SMALL MACHINE SHOP

installed upon the total load of an electrical testing laboratory. In this case the load was fairly steady all day, the peak occurring late in the afternoon, due to the additional power required by the lighting circuits.

Fig. 10 illustrates the curves obtained from the load of a large rubber manufacturing plant. This was a continuous 24-hour load with a deep noon valley. The peak, which was only slightly in excess of the average, usually occurred late in the afternoon.

The curves in Fig. 11 were obtained upon the feeder to a small machine shop, whose load included two heavy freight elevators. These are responsible for the severe swings above the days average. This load is included in the load referred to in Fig. 9.

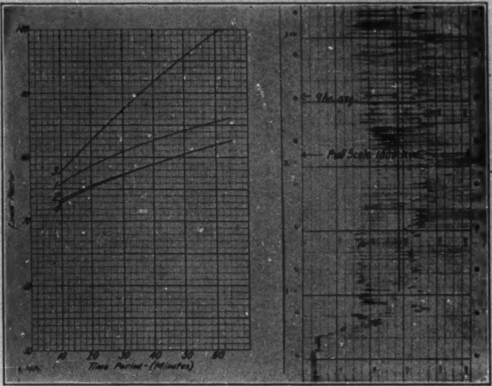

Fig. 12—Load of Brass Rolling Mill

In Fig. 12, are seen the results obtained with the load of a heavy brass-rolling mill. Here are found numerous and heavy swings both above and below the average for the day.

Fig. 13 shows the curves from a city tramway load; the section investigated operating from fifteen to twenty cars. The load curve shows, even during the hours of peak load, frequent depressions approaching the zero value.

The curves illustrated in Fig. 14, are presented more as a matter of interest than for any real technical value. They are plotted from values obtained by averaging the data upon which the other five sets of curves were based. It will be observed that the several curves lie in closer proximity to one another than in any of the individual tests.

It will readily be understood that these investigations might have been carried out indefinitely upon a great variety of typical load curves; but the results obtained on the five loads which were examined, indicate that a particular case must be made of almost every installation, and that attempts to generalize or to classify would be productive of little valuable information. Owing to the diverse nature of the results of the several tests, such facts as could be considered typical of the load curves or of the meter types can be presented only in a more or less disconnected way; and, as such, appear in the following summary.

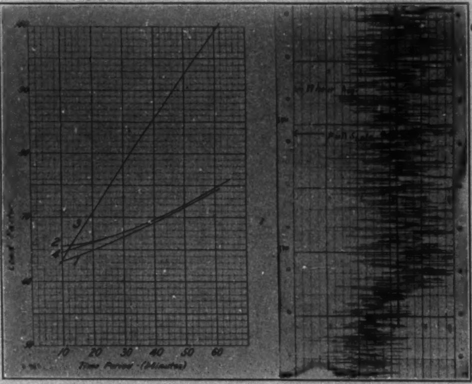

Fig. 13—City Tramway Load

Summary—Part III.

1. Except upon the most fluctuating loads, the demand meters of one type, and of differing time periods gave indications which, generally, within the limits of accuracy of the individual instruments, were in close agreement with one another.

2. The load factors as determined from day to day upon the one connected load, by a meter of one time period, differed more widely than did the values determined by the several meters of that type, upon one day's run.

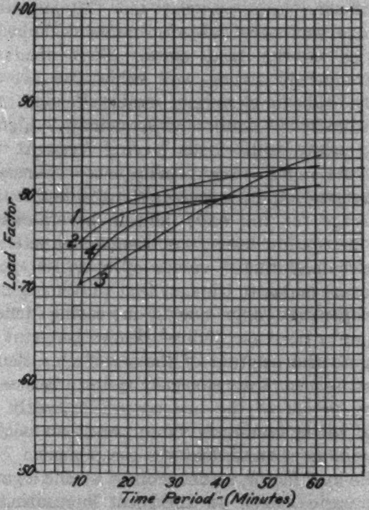

FIG. 14—AVERAGE OF FIVE TYPICAL LOADS

3. The values of load factor as determined from readings of meters of the Merz, (or block-interval) type were found to be very erratic; and even when averaged for the one load over several days, very difficult to reconcile into a smooth curve. A higher value of load factor (*i. e.* a lower peak) was usually shown

for the short periods, coming into approximate agreement with the true value of the arithmetical average, as determined by the continuous recording meter, as the time periods became longer. This phenomenon is doubtless due to the frequency with which these meters would trip in the middle of a peak, thus missing the period during which the demand was a maximum.

4. The mechanically lagged type of meters showed, generally, a comparatively wide range of indications with differing time periods; but the magnitude of this range was very subject to the secondary characteristics of the load curve. With loads whose value frequently swung well below the average for the day, the long period meters tended to give low indications, and, consequently, high load factors; while, with loads having frequent upward swings, the reverse was usually true. Consequently these meters would sometimes show indications below the average for the day; and in several cases, on widely fluctuating loads, the 60-minute meter of this type gave load factor values of over 100 per cent. This feature is due to the fact that the timing mechanism is operative only during increasing deflections; and, upon a decrease of load below the point corresponding to the position of the wattmeter element of the instrument, this element, quite irrespective of the time period of the instrument, instantly follows the load to its minimum value; and can only climb back again as permitted by the escapement.

5. The load factor, as determined by the logarithmic meters, varied, even on the most fluctuating loads, only a few per cent, between the shortest and the longest time periods.

6. With load curves wherein the peak occurred early in the day, the power having been off, or very low, during the night, the logarithmic meters tended to give readings lower than the true arithmetical peak, giving a higher value of load factor, the discrepancy approximating the difference in reading which might be expected from the "90 per cent" clause in the definition of their time period. But when the peak came late in the day, after a long-continued run, the

load factor as given by the logarithmic meter approached the arithmetical load factor, and in some cases acquired a lower average value. This is consistent with the heating of electrical apparatus.

CONCLUSIONS

It is manifestly impossible to arrive at any set rules governing the selection or operation of methods or apparatus for the determination of an empirical quantity. Consideration must be given to a variety of local conditions, which, in all probability, would not be duplicated in any two power systems. It is practicable, therefore, to give as conclusions to this study only a number of deductions which will be quite evident to a student of the subject; together with suggestions as to ways and means of overcoming some of the difficulties which have heretofore presented themselves.

1. In the determination of demand it is not feasible to obtain a quantity which will fairly represent all the factors to be taken into consideration. It is probable that the most logical quantity will be found in the volt-amperes of the load.

2. Values of "sustained peak," as determined from the charts of graphic meters are frequently misleading and of little significance.

3. Values of averged or integrated peaks are difficult to measure on graphic meter charts, and are subject to a large personal error.

4. Readings of the Merz or "block interval" type of meters, though in themselves very erratic, will, in the long run, average more closely to the arithmetical average than the indications obtained by any other method.

5. Individual readings of the logarithmic meters are more consistent, and usually closer to the true value of the arithmetical averages than are the individual readings of the Merz meters.

6. Since heating follows a logarithmic, rather than an arithmetical law, it would seem, where heating is being taken into consideration, that the logarithmic

average is fully as justifiable as the arithmetical for a basis of demand measurement.

APPENDIX I.

As the tests were made with a constant voltage on the generator bus during each run it was deemed permissible for purposes of demonstration, to neglect in each case the change in voltage at the receiving end, and to consider the losses due to potential alone as being of a constant value. The expression $K\,E^m$ then becomes a constant, and may be conveniently represented by P.

The expression for the total losses then becomes $W = P + R\,I^2$, and we have only to find values for P and R. Those are easily obtained as follows: By running the system at no load, normal voltage, a value of P is at once obtained by reading the input. Subtracting this from the total power consumption at any desired value of current there is left $W - P = R\,I^2$ in which R represents the equivalent resistance of the circuit, and should be an approximately constant quantity for all values of the load.

The application of the formula with these values for P and R to the curve of amperes should give a curve representing the total losses.

A typical calculation is given below:

Watts at normal voltage, no load, $= P = 411$
Losses at chosen point $= W = 605$
Current at same point $= I = 3.31$

$$\text{Then } R = \frac{W - P}{I^2} = \frac{194}{11.0} = 17.6$$

Under ideal conditions this value, representing the equivalent resistance of the circuit, should, of course, remain nearly constant at all points of the test; but as a number of losses of secondary order have been neglected, and as the voltage was assumed to be constant at all points in the system, a variation of several per cent was, in the actual tests, noted in the value of R.

In obtaining the calculated values for the losses, there was assumed a constant value for R, this value being computed from near the middle point of the original curve, and no consideration taken of possible variations due to changes in the temperature of the copper.

An examination of the curves will show that, considering the assumptions which have been made, the calculated values of total losses check remarkably closely with the actual readings throughout the length of the curve, both on the high and the low power factor loads.

To further demonstrate the applicability of this method, a number of readings were taken at random on different types of loadings with results as shown in the following table.

Condition	W_1	W_2	$W_1 - W_2$	$R I^2 = (W_1 - W_2) -$ Load A	I	I^2	$= RI^2/I^2$
A	400	000	400	000	000	000	....
B	1400	508	892	492	5.36	28.8	17.1
C	820	292	528	128	2.90	8.4	15.2
D	635	46	589	189	3.16	10.0	18.9
E	882	182	700	300	4.23	17.9	16.8
F	1064	296	768	368	4.66	21.8	16.9

W_1 = Watts input
W_2 = Watts output
$W_1 - W_2$ = Total losses
$R I^2$ = Copper losses
I = Secondary amperes
I^2 = Secondary amperes, squared
R = Equivalent resistance of circuit.

Following are the loads:

A:—No load,—excitation only
B —Lamp load only
C —Lamp load only
D —Choke coil alone
E —Choke coil and lamps in series.
F —Choke coil and lamps in multiple.

Taking the constancy of the calculated value of the equivalent resistance as the criterion by which to judge the fitness of the rule, we here find a maximum variation of 12½ per cent. A reversal of the formula and the use of resistance values having even this discrepancy would enable the values of energy loss to be computed from the current values with a far greater degree of accuracy than they could be determined from any series of readings of watts or volt-amperes of the load.

The application of such a scheme of demand measurement to determine heating effects in those portions of the system which supply only one consumer should not present any great difficulty. The two constants used in the formula could be determined for each installation; either by measurement or by calculation from the known characteristics of the lines and transformers. The application of these values to the readings of the ampere demand meter or the ampere-squared demand meter, as the case may be, would then give a check on the energy losses, and therefore, on the heating of the equipment feeding the load under consideration, thus establishing a basis upon which to compute the proportion of the overhead to be borne by that particular customer.

APPENDIX II.

In the following tables will be found the values from which were derived the curves in Figs. 9 to 13. These figures are expressed as watts in the metering circuits, no regard being given to the ratios of the instrument transformers. The values of average watts were obtained by dividing the watt-hours consumed in each day's run by the number of hours constituting the run. The maximum demands are the indications of the respective demand meters, multiplied by the proper constants and corrected for all known sources of error. Load factors were obtained by dividing the average watts for each day by the demand value for that day, as indicated by the meter under consideration. The several load factors determined by a meter of one time period and type for the several day's run were then averaged and the mean of these taken as one of the points upon the final curve.

Attention is here called to the indication of the Siemens meters occasionally being higher than the corresponding reading of the standard instrument, which would not seem in accord with the theories of that instrument. This is due to the uncertainty of the meshing of the gears, which introduced a possible error of about 1 per cent of the full scale of 300 degrees. On some occasions, when the reading was well down on the scale, this error would be greatly magnified, and increase or decrease the reading by several per cent. As all meters of this class are to a greater or less extent subject to this fault, it was taken as one of the characteristics of the type, and no effort made to correct for the inconsistency.

MAXIMUM DEMAND AND LOAD-FACTOR MEASUREMENTS ON TESTING LABORATORY

Type of meter.	Period. (mins.)		Day 1	Day 2	Day 3		Average
			Avg. watts. 671	Avg. watts. 824	Avg. watts. 660		
Logarithmic	10	Max. demand.	879	972	742		
		Load factor	76.5	84.9	88.9		83.4
	15	M. D.	858	965	730		
		L. F.	78.2	85.5	90.4		84.7
	30	M. D.	845	947	720		
		L. F.	79.4	87.1	91.6		86.0
	40	M. D.	848	949	712		
		L. F.	79.1	87.0	92.6		86.2
Block Interval	10	M. D.	792	1013	771		
		L. F.	86.7	81.3	85.5		84.5
	20	M. D	766	990	764		
		L. F.	87.6	83.4	86.4		85.8
	30	M. D.	750	989	754		
		L. F.	89.4	83.4	87.6		86.8
	60	M. D.	758	934	745		
		L. F.	88.5	88.3	88.6		88.5
Mechanically lagged	10	M D.	945	1040	810		
		L. F.	71.0	79.1	81.5		77.2
	20	M. D.	935	1040	800		
		L. F.	71.7	79.1	82.5		77.7
	30	M. D.	915	1030	795		
		L. F.	73.4	80.0	83.0		78.8
	60	M. D.	920	1000	790		
		L. F.	72.9	82.4	83.6		79.6
Continuous recording	10	M. D.	788	1020	770		
		L. F.	85.1	80.6	85.7		83.8
	20	M. D.	779	989	756		
		L. F.	86.0	83.4	87.4		85.6
	30	M. D.	760	89	750		
		L. F.	88.0	983.4	87.9		86.5
	60	M. D.	736	950	745		
		L. F.	91.1	86.8	88.5		88.8

MAXIMUM DEMAND AND LOAD-FACTOR MEASUREMENTS ON RUBBER MANUFACTURING PLANT

Type of meter.	Period. (mins.)		Day 1 Avg. watts. 445	Day 2 Avg. watts. 455	Day 3 Avg. watts. 440		Average
Logarithmic	10	Max. demand.	513	545	558		
		Load factor	86.7	83.5	78.7		83.0
	15	M. D.	514	534	545		
		L. F.	86.5	85.1	80.6		84.1
	30	M. D.	510	520	530		
		L. F.	87.2	87.5	83.0		85.9
	40	M. D.	496	504	532		
		L. F	89.6	90.3	82.6		37.5
Block Interval	10	M. D.	551	553	570		
		L. F.	80.8	82.2	77.2		80.1
	20	M. D.	530	547	563		
		L. F.	84.0	83.3	78.2		81.5
	30	M. D.	536	530	550		
		L. F.	83.0	85.8	80.0		82.9
	60	M. D.	513	526	539		
		L. F.	86.8	86.5	81.7		85.0
Mechanically lagged	10	M. D.	615	630	706		
		L. F.	72.4	72.3	62.3		69.0
	20	M. D.	618	624	650		
		L. F.	71.9	72.8	67.7		70.8
	30	M. D.	605	605	624		
		L. F.	73.5	75.2	70.5		73.1
	60	M. D.	588	578	587		
		L. F.	75.7	78.7	75.0		76.5
Continuous recording	10	M. D.	538	570	544		
		L. F.	82.7	79.8	74.0		78.8.
	20	M. D.	533	550	578		
		L. F.	83.5	82.6	76.1		80.7
	30	M. D.	530	538	556		
		L. F.	84.0	84.5	79.1		82.5
	60	M. D.	520	514	556		
		L. F.	85.5	88.5	79.1		84.4

MAXIMUM DEMAND AND LOAD-FACTOR MEASUREMENTS ON MACHINE SHOP

Type of meter.	Period. (mins.)		Day 1 Avg. watts. 600	Day 2 Avg. watts. 777			Average
Logarithmic	10	Max. demand.	795	1000			76.5
		Load factor	75.3	77.7			76.5
	15	M. D.	805	955			
		L. F.	74.5	81.4			77.9
	30	M. D.	776	900			
		L. F.	77.3	86.4			81.9
	40	M. D.	765	855			
		L. F.	78.4	91.0			84.7
Block Interval	10	M. D.	866	960			
		L. F.	69.3	80.9			75.1
	20	M. D.	842	906			
		L. F.	71.3	86.7			78.5
	30	M. D.	824	908			
		L. F.	72.8	85.5			79.2
	60	M. D.	802	873			
		L. F.	74.7	89.0			81.9
Mechanically lagged	10	M. D.	989	1152			
		L. F.	60.6	67.3			63.9
	20	M. D.	958	1130			
		L. F.	62.6	68.7			65.7
	30	M. D.	945	1130			
		L. F.	63.5	68.7			66.1
	60	M. D.	910	1120			
		L. F.	65.8	69.2			67.5
Continuous recording	10	M. D.	895	1060			
		L. F.	66.9	73.2			70.0
	20	M. D.	852	937			
		L. F.	70.4	82.9			76.7
	30	M. D.	834	913			
		L. F.	72.0	85.0			78.5
	60	M. D.	821	861			
		L. F.	73.0	90.2			81.6

MAXIMUM DEMAND AND LOAD-FACTOR MEASUREMENTS ON BRASS ROLLING MILL

Type of meter.	Period. (mins.)		Day 1 Avg. watts. 572	Day 2 Avg. watts. 622	Day 3 Avg. watts. 580	Day 4 Avg. watts. 590	Average
Logarithmic	10	Max. demand.	750	800	750	810	
		Load factor	76.2	77.7	77.3	72.8	76.0
	15	M. D.	730	780	745	790	
		L. F.	78.4	79.6	77.8	74.6	77.6
	30	M. D.	695	740	720	765	
		L. F.	81.2	84.0	80.6	77.1	80.7
	40	M. D.	675	748	690	750	
		L. F.	84.8	83.0	84.0	78.6	82.6
Block Interval	10	M. D.	756	832	780	837	
		L. F.	75.5	74.7	74.3	70.4	73.7
	20	M. D.	740	803	761	817	
		L. F.	77.2	77.0	76.1	72.1	75.6
	30	M. D.	730	790	746	789	
		L. F.	78.3	78.7	77.7	74.8	77.4
	60	M. D.	679	750	682	756	
		L. F.	84.2	82.9	85.0	78.0	82.5
Mechanically lagged	10	M. D.	683	750	780	835	
		L. F.	83.7	82.9	74.3	70.7	77.9
	20	M. D.	655	710	710	800	
		L. F.	87.2	87.5	81.6	73.7	82.5
	30	M. D.	600	700	680	750	
		L. F.	95.2	88.8	85.3	78.6	87.0
	60	M. D.	540	620	610	610	
		L. F.	105.8	100.2	95.0	96.6	99.4
Continuous recording	10	M. D.	788	840	780	825	
		L. F.	72.5	74.0	74.3	71.5	73.1
	20	M D.	750	807	773	802	
		L. F.	76.2	77.0	75.0	73.9	75.5
	30	M. D.	729	775	765	795	
		L. F.	78.5	80.2	75.8	74.2	77.2
	60	M. D.	675	734	727	750	
		L. F.	84.6	84.7	79.7	78.6	81.9

MAXIMUM DEMAND AND LOAD-FACTOR MEASUREMENTS ON CITY TRAMWAY

Type of meter.	Period. (mins.)		Day 1 Avg. watts. 297	Day 2 Avg. watts. 310	Day 3 Avg. watts. 491	Day 4 Avg. watts. 455	Average
Logarithmic	10	Max. demand.	482	435	809	752	
		Load factor	61.5	71.2	60.7	60.4	63.5
	15	M D.	497	446	772	715	
		L. F.	59.7	69.6	63.6	63.6	64.1
	30	M. D.	463	428	744	701	
		L. F.	64.0	72.5	65.9	64.9	66.8
	40	M. D.	455	426	703	663	
		L. F.	66.2	72.8	69.7	68.6	69.3
Block Interval	10	M. D.	500	422	746	700	
		L. F.	59.4	73.5	65.0	64.9	65.7
	20	M. D.	490	420	746	704	
		L. F.	60.6	73.8	65.7	64.6	66.2
	30	M. D	450	410	740	695	
		L. F.	66.0	75.7	66.3	65.4	68.3
	60	M. D.	402	387	669	633	
		L. F.	73.8	80.0	73.5	71.8	74 8
Mechanically lagged	10	M. D.	530	445	790	700	
		L. F.	56.0	69.7	62.2	65.0	63.2
	20	M. D.	495	400	750	585	
		L. F.	60.0	77.5	65.5	77.7	70.0
	30	M. D.	435	350	658	545	
		L. F.	68.5	88.5	74.7	83.4	78.7
	60	M. D.	320	290	550	423	
		L. F.	92.8	106.8	89.3	107.5	99.1
Continuous recording	10	M. D.	500	424	782	724	
		L. F.	59.4	73.1	62.8	62.8	64.5
	20	M. D.	479	415	765	710	
		L. F.	62.0	74.6	64.2	64.1	66.2
	30	M. D.	466	408	735	697	
		L. F.	63.8	75.9	66.9	65.2	67.9
	60	M. D.	461	402	708	677	
		L. F.	64.4	77.1	69.4	67.1	69.5

DISCUSSION ON "THE MEASUREMENT OF MAXIMUM DEMAND AND THE DETERMINATION OF LOAD FACTOR", (BORDEN), PHILADELPHIA, PA., OCTOBER 8, 1920.

Paul M. Lincoln: It is rather surprising to see the results which Mr. Borden has obtained. For instance, to make a comparison of the results shown by the tables pages 1881 and 1883. In the table on 1883 on one character of load, one of the types of demand meter shows somewhere from 110 to 130 per cent of the arithmetical average, while exactly the same comparison in table on 1881 shows somewhere between 60 and 80 per cent. That is there is a variation all the way from 60 per cent to 130 per cent,—for this particular type of meter as compared with the arithmetical average.

There is a reason for that, which reason Mr. Borden has not given clearly in his paper, and I want to go into these comparisons a little more in detail, and to accompany this with a plea for the kind of average given by the logarithmic or thermal storage type of demand meter.

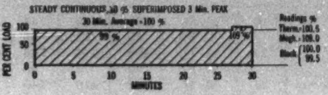

FIG. 1

It has always been my argument that the logarithmic average, which is the type of average measured by a thermal meter—gives a more equitable measurement of demand than does the arithmetical average or the average measured by any other type of meter, and I want to give you a few examples of theoretical load conditions, which to my mind prove that.

Unfortunately, Mr. Borden's results, given in his tables and the section of load curve which he gives are not necessarily conclusive; that is, the particular section of the curve which he shows, is not, I believe, the section which gives the maximum demand of the day.

The illustrations in this discussion show certain characters of load, and show the amount by which the various types of demand meters will vary when measuring certain characteristics loads.

For instance, Fig. 1 shows a steady load upon which a peak is superposed for a short time. That probably is most characteristic type of load we will meet in practise. It shows a load supposed to have continued steady indefinitely at a value of 99 per cent, and then for three minutes it increases to 109 per cent or a 10 per cent increment for 3 minutes, and then drops back to the original amount. The arithmetical average of these quantities over a thirty minute period is just

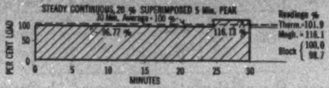

FIG. 2

100 per cent, so that if that load is measured on an arithmetical average meter it will measure just 100 per cent.

If you measure the same load with a thermal storage meter, it will show 100.6 or 0.6 above the arithmetical average. The mechanically lagged meter

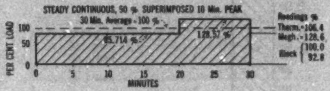

FIG. 3

will show 109 per cent, and the block interval meter, that is the arithmetical average meter, which may split the load at any point, will show a maximum of 100 per cent, and a minimum of 99.5.

If the magnitude of the peak is somewhat enlarged both in time and amount as shown in Fig. 2, making

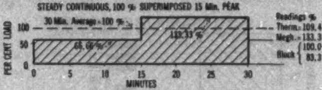

FIG. 4

a condition where we have practically 97 per cent of load for 25 minutes, followed by a 5 minute interval of 116 per cent, or 20 per cent peak on top of the steady load; the comparison of the various types is given in Fig. 2; the thermal storage meter registers 101.9, the mechanically lagged meter 116.1 and the block interval a minimum of 98.7 and a maximum of 100, depending on the location of the time split.

In Fig. 3 is another case where the increment of load is 50 per cent superposed for 10 minutes. In this case the thermal storage meter will register 106.4 or 6. 4 above the arithmetical average, the mechanically lagged meter 128, and the block interval meter will register 92.8 as a minimum and 100 as a maximum depending on the location of the time split.

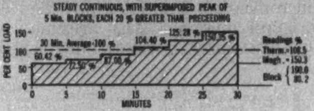

Fig. 5

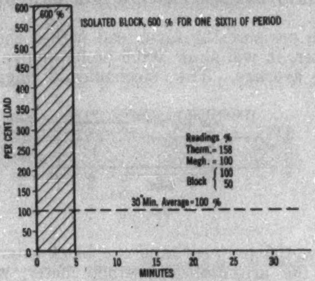

Fig. 6

Fig. 4 is another case where there is 100 per cent peak superposed on top of a steady load, the duration of the peak being 15 minutes. The thermal storage meter reads 109.4, or 9.4 per cent above the arithmetical average, the mechanically lagged 33.3 per cent above, and the block interval either 100 or 83.3, depending on the location of the time split.

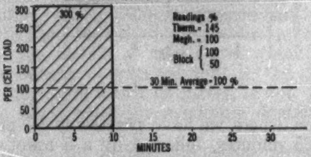

Fig. 7

In Fig. 5 is another load where there is a series of increments each of 5 minutes duration, each 5-minute period being 20 per cent in excess of the previous period. Here again the load is adjusted so that the arithmetical average over the whole 30 minutes is 100 per cent with that type of load, the thermal storage meter will indicate 108.5, the mechanically lagged 150.3, and the block meter 100, as the maximum, and the 80.2 as the minimum depending on the location of the time split.

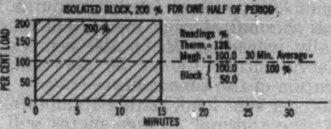

Fig. 8

In another class of load as shown in Fig. 6, where

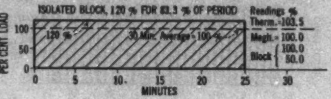

Fig. 9

we assume that the maximum load comes as one isolated block. A load that runs to 600 per cent of normal, and lasts for only five minutes. The arithmetical average of such a load is 100 per cent and the arithmetical average meter will read 100 per cent, while the thermal storage meter will read 158, and the block interval meter either 100 or 50, depending on the location of the time split.

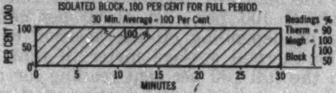

Fig. 10

In such a load, we all recognize that the heating effect of a load which goes to six times normal and stays there for five minutes, is much greater than one which goes to normal and stays for 30 minutes.

I believe that the customer who insists on taking his entire 30 minutes quota of load in five minutes and then is idle for 25 minutes should be penalized for taking his load in that manner and the thermal storage meter does this automatically.

Fig. 7 is a condition of an isolated block of load, consisting of 300 per cent for ten minutes. On such a load the thermal storage meter will read 145, the mechanically lagged or *R. O.* 100 per cent, and the block interval either 100 or 50 per cent depending on the location of the time split.

For a 200 per cent load, lasting 15 minutes, as shown in Fig. 8, the thermal storage meter will read 128, and the mechanically lagged or *R. O.*, 100, and the block interval either 100, or 50, depending on the location of the time split. There again, we have characters of load which give us much more heating than they would if the load were evenly spread throughout the entire time period.

In Fig. 9 we have 120 per cent load for 25 minutes. A load of that kind will give us on the thermal storage meter 103.5, on the mechanically lagged meter or *R. O.* meter 100 per cent, and block interval meter, either 100 per cent, or 50 per cent, depending on the location of the time split.

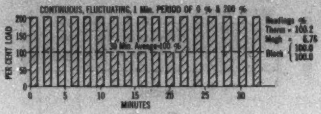

FIG. 11

Fig. 10 is a block of 100 per cent load of just 30 minutes, and the thermal storage meter gives 90 per cent, the mechanically lagged 100 per cent, and the block interval 100 per cent of 50 per cent, as the case may be depending on the location of the time split. The thermal storage meter in that case does not reach the full 100 per cent reading for the reason that it always indicates the *heating effect* of a given load independent of how that load is taken. If this load is continued steady, the thermal storage meter will indicate 99 at the end of the second 30 minute period, 99.9 at the end of the third, 99.99 at the end of the fourth, etc. Thus its indication follows the true thermal or logarithmic law.

We have another class of load, the continuously fluctuating load. In Fig. 11 I have shown a continuously fluctuating load consisting of one minute periods, of alternatingly zero load and 200 per cent load. If we put such a load on the various types of meters, they will register as follows: thermal storage meter 100.2; the mechanically lagged, 6.67 and the block interval 100. The mechanically lagged meter will read only a small fraction of the load due to its property of following a descending load instantly.

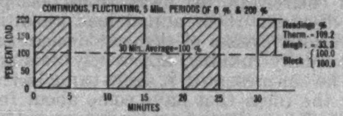

Fig. 12

Fig. 12 shows the effect of spreading the time period to 5 minutes instead of one minute, and again, in Fig. 13 to ten minutes. The comparisons of the various types of meter are given in the figures.

I think it will be seen that a study of these theoretical charts will indicate the reason for the vast variation in the indication of the various types of demand meters. I think Mr. Borden has done a service of great value in showing just what the comparative operation of these various classes of meters is, not only on theoretical loads such as I have shown here, but upon actual loads in the conditions of regular service.

C. I. Hall: To my mind, the valuable portion of this paper is concerned with the actual comparison of various kinds of demand meters on usual and practical kinds of loads. The attempt to analyze the measurement of demand theoretically is a good deal like a problem in psychology. It is not definitely capable of being determined under fixed conditions, on account of the fact that the conditions are always variable and are always varying. Therefore, it is necessary to my mind, rather than to attempt to consider it from the theoretical point of view, to consider it prac-

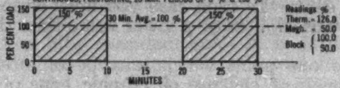

Fig. 13

tically and to amass a large amount of data. Any problem of this character requires a very large amount of data in order to reach proper conclusions rather than a mathematical discussion.

It is an old adage that figures can never lie, and yet they can be very misleading. An analysis of freak load conditions can lead to extremely erroneous conclusions. However, the thing I am trying to bring out in connection with Mr. Borden's paper is the fact that it has entirely gotten away from that point of view, and discusses these actual load conditions by installing various classes of demand devices on loads that were actually running, apparently, in Canada.

The data he has presented, it seems to me, have carried out exactly the ideas that have come about from the presentation of data in the past. Integrated demand has been defined in the Code for Electricity Meters as the standard. First we must determine how the various devices vary from that fixed standard, and second, the extent to which the time interval may affect the record. We have the old bugbear of the split interval. It begins to sound like a split infinitive.

The variation under theoretical conditions is not what is of interest to us, but the variation under practical conditions of rate-making. The data presented indicate that the amount of that variation in the block interval as compared with standard or elapsed interval, is negligible on the long time intervals and increases as the interval becomes shorter. That increase is to be expected. These variations, however, at the very short intervals as indicated in Mr. Borden's paper, I believe to be of interest. The maximum of these on the 10-minute interval, as shown on the load of the small machine shop, is approximately 7 per cent. The minimum is something less than one per cent and the average variation at the 10 per cent point is 2.1 per cent. In each case you must keep in mind that this variation is of a lower value than the standard.

There is one point which may be of interest. In the discussion of the use of graphic type of demand meter in recording the demand, I differ somewhat from Mr. Borden's point of view in looking at the amount of the variation. I always liked to consider that a graphic meter, which is of course always damped, is a lagged type of demand meter in which the record will, of course, vary with the time interval of lag. That is precisely the condition that exists in the two curves the author shows in Fig. 4. The one to the left is a very lightly lagged demand meter, and the one to the right much more heavily lagged, and therefore the indications of the heavily lagged are lower than those of the less heavily lagged.

I think it is well to lay additional stress on Mr. Borden's point that the variation of the time interval is not greatly important in the variations of demand. He states that "except upon the most fluctuating loads, the demand meters of one type, and of different time periods gave indications which generally, within the limits of accuracy of the individual instruments, were in close agreement with one another." That statement has been made heretofore many times, and supported by a great mass of data, although it still is the point which comes up constantly for discussion.

I wish to criticise, to a certain extent, the conclusions under Summary—Part III, Section 3 and Section 5; after reading them, it is found that they are statements entirely unsupported with data. It is usual in making a bald statement of that character to support it with data, so that the exact amounts of variations may be obtained. He states again, at the end of conclusion 6—"But when the peak came late in the day after a long-continued run, the load factor as given by the logarithmic meter approached the arithmetical load factor, and in some cases acquired a lower average value. This is consistent with the heating of electrical apparatus." It was my impression that this paper was a discussion of data, very valuable data, which have been actually obtained, rather than a discussion of rate-making. This verges closely on the rate-making problem, and we are concerned largely with the fact that the device varies from the standard, rather than what that varaiation may take into account.

Section 1 of the main conclusions reads: "In the determination of demand it is not feasible to obtain a quantity which will fairly represent all of the factors to be taken into consideration. It is probable that the most logical quantity will be found in the volt-amperes of the load." It seems to me personally that that is not a proper method of attacking the solution of the problem we have before us. The measuring of a thing undesirable both to the customer and to the central station is not a method of arriving at a final solution of the problem, but is merely a penalization of both the customer and central station for a condition which exists. The method of attack should be the elimination of these undesirable functions in central station operation rather than the metering of tically and to amass a large amount of data. Any problem of this character requires a very large amount of data in order to reach proper conclusions rather than a mathematical discussion.

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The important point is, how do these devices perform under normal conditions of operation, and without respect to the rate-making conditions involved—do they measure the quantity properly? That is the point we are interested in.

Rather marked attention is called in Mr. Borden's paper to the apparent discrepancy in results due to the increased readings of the block interval type of device over the lapse time interval arrangement. The point which Mr. Borden has laid stress on in that connection is that engagement of the mesh is apparently the only thing which can lead to this discrepancy; that is, the engagement of the mesh in the gears of the Siemens meter, which is used. It must be pointed out that there are observational errors which may have crept into the record taken from the so-called "scrap heap" meter which he has built up.

W. H. Pratt: The author has mentioned the measurement of volt-amperes, and he has also stressed the point in his presentation, and I think it is proper to state that there are at present available meters that will measure volt-ampere hours.

The principle on which these meters are constructed is this: a wattmeter measures the product of volt-amperes and the cosine of an angle which is the difference between the angle of lag of the current in a circuit and a characteristic angle of the instrument itself. If this characteristic angle of the instrument is made zero, the instrument measures watts, and thus we have the wattmeter similarly the watthour meter.

If, however, the characteristic angle is given some value other than zero, it is possible to make the instrument read a quantity which over a moderate range of angle is volt-amperes.

For power factors of 0.90 and below a considerable range may be covered. I think if you picture in your mind the form of the cosine curve, and remember that thirty degrees displacement corresponds to a power factor of about 87 per cent and that a cosine of 22 degrees is 0.98, you can see that by adjusting an instrument with an angle displaced by 11 degrees, and calibrating it one per cent fast, you could record, with a maximum error not exceeding one per cent, over the range of 22 degrees.

Twenty-two degrees plus another 22 degrees will cover a large range, and the second 22 degrees can be arranged by using two meters registering through a ratchet device; that shaft running the more rapidly would be the only one producing a record, and in this way it is possible to cover a range from about 0.90 down to 0.3. Other ranges can be provided for.

The author also mentioned the measurement of ampere squared hours, and I will state that meters for measuring this quantity have been produced and these meters have been put to a practical use.

P. A. Borden: You will observe that Mr. Lincoln, in discussing the subject, has viewed the matter of demand measurement soley upon the basis of heating of equipment. Now, while heating is an important factor, we must also take into consideration the matters of regulation and demands upon the mechanical capacity of the system, which will include turbines, penstocks and water or fuel supply. We cannot, therefore, look upon heating as the sole consideration, however important it may be.

Mr. Hall, in his reference to the graphic meter, stated that he considers the graphic wattmeter as a demand meter lagged through short time periods. It is doubtless a lagged meter of a short time period, but, unfortunately, we do not, in computing demands from the charts of graphic meters, consider it as that. My experience has been that in estimating demands from graphic wattmeter charts, we consider the chart as being an absolute record of all fluctuations,—that we look upon it as true "oscillogram" of the load curve and take little or no consideration of the damping of the meter. If we are going to consider the damping of the meter this damping must have a definite value, and in very few curve-drawing meters is this so. While an effort is made to have some meters deadbeat, most types provide an adjustment for differing classes of load, and in many types the degree of damping is noticeably susceptible to temperature changes. It is certainly not usual when reading the charts to consider damping characteristics of the meter.

I regret that I omitted to present certain data which formed the bases of my conclusions. The tests which I made resulted, of course, in an immense volume of figures. These I compiled and condensed to the degree in which they appear in the Appendix. The actual meter readings, which would appear to have no value which would justify their preservation, have been destroyed.

If I have trespassed upon the field of the rate maker, I can assure Mr. Hall that I have not done so with design. As it was, I found it necessary to revise parts of my paper many times in my endeavor to avoid this subject, and considering how closely associated are the two subjects, "demand measurement" and "rate making", I think Mr. Hall should have congratulated me upon segregating them as thoroughly as has been done.

Mr. Hall is of the opinion that a consideration of volt-amperes amounts to a penalization for the use of volt-amperes. What should be done is to provide something that would eliminate the necessity of using volt-amperes. When we begin to penalize people for low power factor, that is one of surest ways to bring about a condition which will tend to the elimination of low power factor. As to the meshing of the gears in the instruments, I may say that when I found meters reading higher than they theoretically should have done, I made tests, which satisfied me that the condition actually existed. The manner of the meshing of the gears would sometimes introduce discrepancies of as much as five per cent of the indication.

Mr. Pratt has made reference to the volt-ampere-hour meter. The idea of giving a wattmeter a characteristic angle other than zero so as to make it read approximately the volt-amperes in the circuit is not entirely new. I have studied the scheme to a certain extent in connection with integrating meters. There might be cases where the system would be desirable if it could be introduced without further confusing the issue of trying to explain power factor to a non-technical customer.

In the distribution of charges upon the basis of the ampere-squared meter, so called, we must consider the fact that though the total losses due to resistance are in direct proportion to the square of the total current in the circuit, the losses may not be distributed in proportion to squares of the several currents which go to make up the total. If *A* takes 25 amperes and *B* takes 30 amperes from the one transformer, it does not follow that the copper losses due to the currents will be in direct proportion to their squares. The ratio will depend not only upon the actual magnitude of the currents but upon their phase angles in respect to the line voltage and to each other.

Presented at the 365th Meeting of the American Institute of Electrical Engineers, Chicago, Ill., November 12, 1920.

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STUDIES IN LIGHTNING PROTECTION ON 4000-VOLT CIRCUITS—II.

BY D. W. ROPER
Commonwealth Edison Co., Chicago.

The paper continues the investigations described by the author in another paper on the same subject presented before the Institute in June, 1916 which had for its primary object the reconciliation of the differences between laboratory experiments and experience in service.

In the paper an endeavor is made to list the several factors which affect lightning arrester performance and to describe the methods of eliminating these several variables so as to permit the presentation of curves which show the relative merits of the arresters under investigation. The elimination of the several variable factors was accomplished by placing the arresters under practically identical conditions so that no type was at any material advantage or disadvantage on account of the preponderance of the factors which would affect its performance. This was found to be possible for all of the factors except the one relating to density of the arresters, that is, the number per square mile, so that the final curves show the performance of the several types of arresters as affected by their density. These results show that four of the types of arresters are practically identical in their protective value and about 60 per cent as efficient as a fifth type, while the results for the sixth type, being limited to a much smaller number of arresters, and a shorter period of time, do not appear to be conclusive.

1. INTRODUCTION

THE investigations forming the basis of this paper as well as the previous paper[1] on the same subject had as their primary object the determination of the relative merits of the several types of lightning arresters which were installed on the 60-cycle distribution system of the Commonwealth Edison Company in Chicago. The previous investigations had indicated in a general way the several factors which affected lightning arrester performance and also the extreme variability of the distribution and intensity of the lightning storms, from which it appeared that in order to get reasonably accurate results, it would be necessary to accumulate the experience with a large number of arresters over a period of several years. The keeping of systematic records of lightning arrester performance as outlined in the previous paper was, therefore, continued and the manufacturers of the arresters were advised from time to time of the results. These data showed some slight variations from year to year, but the order of merit of the lightning arresters as shown by the figures given in the previous paper was not altered. In the meantime the manufacturers of the arresters had on their part been making investigations and experiments for the purpose of improving their designs, and one of them, as a result of such laboratory experiments, presented a table of figures which he stated represented the relative merit of the various types of arresters. The experience obtained in Chicago, however, did not substantiate the results of the laboratory experiments and placed the arresters in quite a different order. In the hope of reconciling these differences and reaching conclusions, acceptable to all interested parties, regarding the relative merits of the several types of arresters, an investigation was started about a year ago in which the manufacturers cooperated. In the following, the methods used in this investigation and the final results secured are set forth.

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[1] TRANS. A. I. E. E., 1916, Vol. XXXV, p. 655.

2. DESCRIPTION OF THE SYSTEM

The system of distribution on which these investigations were made is a four-wire three-phase system, with the neutral grounded only at the substations. The normal potential on the distributing mains is 2080 volts between phase and neutral wires. The distribution pole lines are in the alleys, or along the rear lot lines in the center of the block where alleys are missing. Single-phase transformers are used exclusively and are connected between the phase and neutral wires except in the case of three-transformer three-phase installations in which case the common point of the transformer primaries is not connected to the neutral wire. Secondaries of power transformers are connected in delta. Power and lighting customers are supplied from the same primary mains, but the very large customers are connected to a 12,000-volt system. The feeders are all No. 0 wire and the mains No. 6 A. W. G. About 85 per cent of the feeders and 15 per cent of the mains are underground. About 99 per cent of the transformers are on poles and the rest in manholes or in vaults on customers' premises. At single-transformer installations a 2400-volt arrester is connected to the same phase wire as the transformer

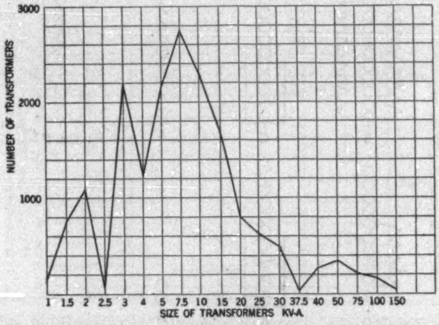

FIG. 1—DIAGRAM SHOWING THE NUMBER OF EACH SIZE OF TRANSFORMER IN SERVICE ON AUGUST 1ST, 1918

and a 300-volt arrester to the neutral wire. Where three transformers are installed for a power service there are three 2400-volt arresters, one connected to each of the phase wires; and one 300-volt arrester is connected to the neutral wire. Arresters are installed in this manner on the same pole with all transformers. The lightning arrester ground consists of one-half inch galvanized iron pipe ten feet long, driven into the ground at the base of the transformer pole. Secondary circuits are usually less than one block long and the secondary ground is similar to the lightning arrester ground, but is installed on the next pole. On long secondaries there are at least two such ground connections and in addition the neutral wire on the customer's premises in many recent installations is grounded to the water pipes inside of the building. Where there are four primary wires in an alley they are installed on the top arm and the secondaries on the second arm. Where there are only two primary wires the lighting secondaries may be installed on the same arm. More than 80 per cent of our poles are owned jointly with the Telephone Company, and there is a minimum clearance of four feet between the wires of the two companies. In order to regulate the voltage properly, the area supplied by each feeder is divided into three portions corresponding to the three phases, and the single-phase lighting load in each portion is all connected to one phase. Arresters similar to those

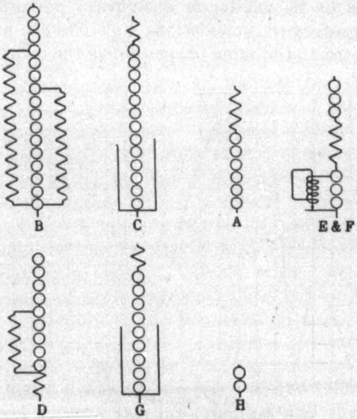

Fig. 2—Electrical Diagrams of the Lightning Arresters Used in These Investigations

The gaps are conventional and do not show the actual shape of the gaps on all of the arresters.

Arresters *E* and *F* have identical diagrams and differ principally in mechanical details, the amount of resistance and the length of the resistance rod.

Diagram *G* shows a type which was installed in 1920.

Diagram *H* represents the neutral 300-volt arrester installed on the neutral.

installed for the protection of transformers are installed on the cable poles where the underground feeders or mains connect with the overhead wires. The distribution system at this time includes about 100,000 poles, 20,000 transformers with a total capacity of about 270,000 kv-a., 6500 conductor miles of overhead primary line wire, 2200 conductor miles of underground cable and 2500 cable poles. The system serves about 400,000 customers.

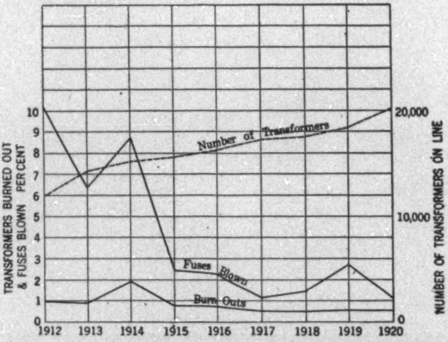

FIG. 3—GRAPHICAL RECORD SHOWING THE NUMBER OF TRANSFORMERS IN THE DISTRIBUTION SYSTEM AND THE PER CENT OF TRANSFORMER TROUBLES EACH YEAR OVER A PERIOD OF YEARS

In Fig. 1 is shown the number of each size of transformers on the line as of August 1st, 1918. This date was selected for the purpose of the calculations, as the number of transformers in service on that date was about the average of the number in the five-year period under investigation. Fig. 2 shows electrical diagrams of all of the lightning arresters used in the investigations. The letters shown on this diagram are consistently used throughout the several tables, diagrams and curves. Fig. 3 shows graphically the number of transformers on the distribution system over a period of years, as well as the percentage of transformer primary fuses blown and transformers burned out by lightning each year. The increase in the percentage of fuses blown during the years 1918 and 1919 was due to causes definitely known to be entirely distinct from lightning, but as some of these fuses were blown during the same day as lightning

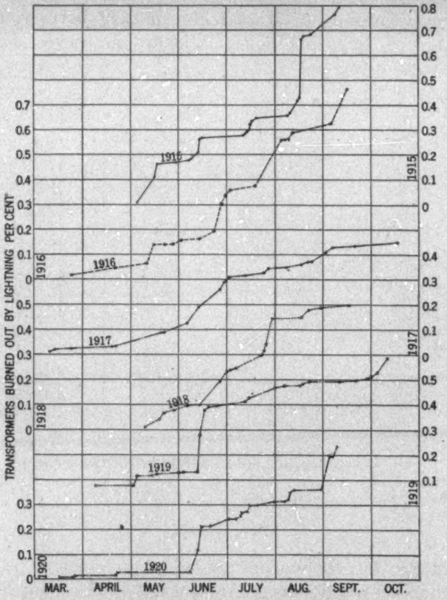

Fig. 4—Diagram Showing the Percentage of Transformers Burned out in Each Storm for the Years 1915-1919 Inclusive

storms, they were included with fuses blown by lightning because of the impossibility of accurately determining just which fuses were blown by lightning and which by other causes.

In Fig. 4 is shown the percentage of burn-outs of transformers for each storm during the five-year period and also for the year 1920, the percentages being plotted cumulatively. From these records it will be noted that it is not unusual to have over one-third of the total trouble in any one year due to lightning occur in one or two days. A composite of these curves for the five-year period is shown in Fig. 5, from which it will be noted that on the average, the lightning is quite uniformly distributed throughout the 4½ months from May 1st to September 15th, and that there is comparatively little trouble outside of this period.

Fig. 6 is an outline map of the portion of the city covered by the distribution system on August 1st, 1918, showing the section lines and the lightning arrester area numbers. These areas will be found to differ from those shown in the previous paper as some changes were made in 1917 for the purpose of trying another type of arrester, a new scheme of protection and incidentally securing a little better distribution of the various types of arresters over the different portions of the city. The shaded areas on this diagram will be referred to later in the paper.

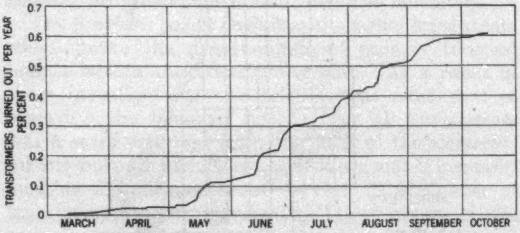

Fig. 5—A Composite Diagram of the Transformer Burn-Outs for Years 1915-1919 Inclusive

3. PRELIMINARY INVESTIGATIONS

From the previous paper and subsequent studies it appears that the factors which might affect lightning arrester performance are as follows:

1. The system of distribution and the grounding of the neutral.

2. Primary terminal boards.

3. The shielding effect of trees, buildings or wires of other companies.

4. The resistance of the lightning arrester ground connection.

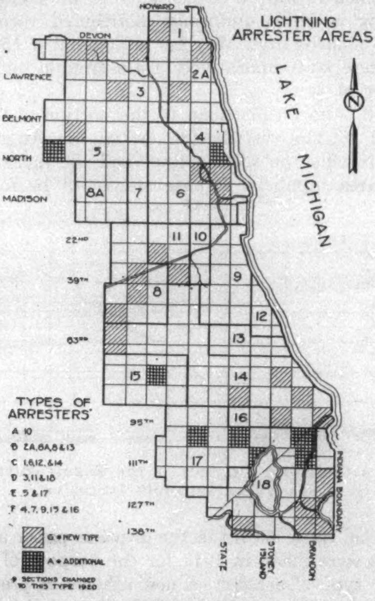
LIGHTNING ARRESTER AREAS
LAKE MICHIGAN
N
HOWARD
DEVON
LAWRENCE
BELMONT
NORTH
MADISON
22ND
39TH
63RD
95TH
111TH
127TH
138TH
STATE
STONEY ISLAND
BRANDON
INDIANA BOUNDARY
1
2A
3
4
5
6
7
8A
8
9
10
11
12
13
14
15
16
17
18
TYPES OF ARRESTERS*
A 10
B 2A,8A,8 & 13
C 1,6,12, & 14
D 3,11 & 18
E 5 & 17
F 4,7,9,15 & 16
G • NEW TYPE
A • ADDITIONAL
* SECTIONS CHANGED TO THIS TYPE 1920

FIG. 6—OUTLINE MAP OF THE CITY, SHOWING SECTION LINES AND LIGHTNING ARRESTER AREA NUMBERS

The lightly shaded areas show the sections in which arrester *G* shown in Fig. 2 was installed in 1920. The heavily shaded portions show the sections in which additional arresters of type *A* were installed in 1920 for the purpose of getting more conclusive information regarding this type.

5. The maker of the transformer.

6. The size of the transformer.

7. The age and previous service record of the transformer.

8. Variation in the distribution and intensity of the lightning.

9. The density of lightning arresters, that is, the number per square mile.

10. The design of the arrester.

Records were available for the five-year period 1915 to 1919 inclusive and these records were carefully compiled in great detail and arranged systematically so as to set forth all facts which might influence the results. The history of each burnt-out transformer from the date of its purchase to the time of its burnout was also assembled and tabulated. In Chicago the same system of distribution is used throughout the entire city and the neutral is grounded at each of the substations from which the circuits emanate, so that all types of arresters, as far as this point is concerned, are installed under identical conditions.

The previous paper describes the earlier experiments which prove the disadvantage of primary terminal boards within the transformer case. As a result of these investigations the practise was established of removing the terminal boards from all transformers which were returned from the lines to the storeroom for any purpose such as changes on account of increased load or discontinuance of service. The number of transformers which returned to the storeroom in this manner during a year was on the average about equal to the number of new transformers added to the system so that the percentage of transformers in service with primary terminal boards was rapidly reduced. It appeared from this investigation that the percentage of transformers with primary terminal boards was so low and the transformers so well distributed over the system that they had no appreciable effect on the relative performance of the lightning arresters.

In laying out the lightning arrester areas which were given in the previous paper, and which are also shown in Fig. 6 in this paper, it was the intention to arrange the boundaries of the areas and to distribute the several types of lightning arresters over the city so as to eliminate variables 3 to 8 inclusive as given in the above list. In order to determine whether this result had actually been secured, the records of the transformer burn-outs due to lightning were carefully investigated in cooperation with the manufacturers of the arresters. The previous paper had indicated the difference in the results as effected by the maker and the size of the transformer. In investigating these points we assumed that if the percentage of burn-outs of any one maker or any size of transformer which had been protected by any type of arrester was not seriously different from the percentage of such transformers protected by that type of arrester to the total on the system, then no type of arrester was at any disadvantage due to unequal distribution of such transformers on the system.

An investigation of the records demonstrated beyond question that the shielding effect of trees or buildings immediately adjacent to the lines considerably reduced the amount of damage on our lines from lightning. This was shown by the following facts:

(a) The percentage of poles in the distribution system shattered by direct strokes is extremely small, being of the order of 1/400 of 1 per cent. This is very much smaller than the corresponding percentage for transmission line poles belonging to the same company in the flat open country in the southeastern portion of the city and is also smaller than experienced in general by companies having transmission lines crossing open country. That there are many direct strokes in every severe lightning storm is shown by the newspaper reports on the day following lightning storms, which record the most severe or unusual cases of damage to trees, church steeples, chimneys, or other portions of buildings and structures.

(b) An investigation of the conditions surrounding the installation of 97 out of 529 cases covered by these investigations where transformers were burned out by lightning failed to reveal a single case in which the primary wires adjacent to the transformer were overshadowed by high trees or buildings immediately adjacent.

By "spot checking" selected portions of each of the lightning arrester areas in cooperation with the representatives of the manufacturers, it appeared, although the shielding effect of trees and buildings was considerable, that as far as could be determined without making a detailed survey and record of the conditions in each block throughout the city, no type of arrester was at any serious advantage or disadvantage on this account.

With the idea that some of the transformer burnouts might have been due to ground connections having a resistance so high as to render the arrester ineffective, the resistance was measured at 97 locations where the surrounding conditions were also noted in detail, and the results are shown in Fig. 7. On the same figure is shown, in a similar way, the resistance of 2770 grounds, both lightning arrester and secondary grounds, which were tested in 1916. It will be noted from this figure that the resistance of the lightning arrester grounds at the points where the 97 transformers were burned out by lightning averaged slightly above the resistance of the much larger number tested three years earlier. It therefore, appears that the burn-outs were not due to particularly bad ground connections for the lightning arresters at the locations where the transformers burned out.

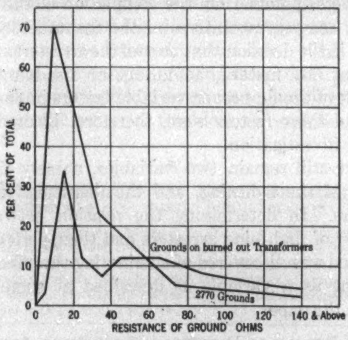

Fig. 7—Results of Tests of Resistance of Grounds

The line showing grounds on burned out transformers was made up from tests on arrester grounds at locations where 97 transformers burned out in 1919. The curve marked 2770 grounds shows the results of tests made in 1916 in cooperation with representatives of the Bureau of Standards.

In the same manner the records and the conditions surrounding the transformer installations were carefully and thoroughly examined to determine the effect of the other points 5, 6 and 7. These investigations included the assembling of the complete history of each transformer that had burned out during the five-year period and the compiling and assembling of all data which might serve to add to the information on the several points. On the completion of the investigation, the representatives of the manufacturers concurred in the decision that none of the arresters appeared to be at any material advantage or disadvantage on account of the first seven variable factors in the above list, and these factors were, therefore, ignored in the further investigation.

There still remain two variables, namely the variability of the lightning, and the density of lightning arresters. In determining the relation between the density of lightning arresters and their performance, a method was discovered of eliminating the effect of the lightning as a variable as described at some length later in the paper.

4. THE EFFECT OF DENSITY OF LIGHTNING ARRESTERS ON THEIR PERFORMANCE

A preliminary investigation of the effect of density was made by plotting the density of arresters in each original lightning arrester area against the percentage of burn-outs in that area. The points plotted in this manner were so irregular that they did not permit the drawing of any curve which might be considered as representing the results, but the method appeared to indicate that there was a very marked decrease in the percentage of burn-outs with increase in density which would warrant further investigation along this line. The results also indicated that some further subdivisions of the original lightning arrester areas would be necessary in order to eliminate the lightning as a variable. The manner in which the records were kept enabled this change to be made very readily by using the section (that is, the square mile) as the unit, resulting in an increase in the number of areas from 19 to

192. For each one of these sections there was determined from the records the number of transformers in the section as of August 1st, 1918. As there is an arrester on the same pole with each transformer, and comparatively few cases where there were two transformers connected to the same phase wire on the same pole, the number of transformers in each section was taken as the number of arresters. There was also determined for each section the number of transformer burn-outs and primary fuses blown by lightning during the five-year period and the actual area covered by the line. This latter quantity was determined by going over the large scale maps of the distribution system and assuming that a line through the center of the block covered the width

TABLE I

DATA FOR DETERMINING THE EFFECT OF DENSITY OF ARRESTERS ON THEIR PERFORMANCE

The types of arresters are indicated in Fig. 2, the area numbers are shown in Fig. 6, the section numbers in Fig. 11. The stars (*) indicate the sections in which the type of arrester was changed in 1917. Figures for the average curve are taken from this table but due allowance for this change was made in plotting the curves for the individual arresters, using the additional data given in Table IA.

Present type of arresters	Area number	Section number	Number of transformers in section	Number of burn-outs in section	Number of fuses blown in section	Area actually covered by lines	Density of arresters per square mile	Average per cent burn-outs per year
C	6	325	4	0	0	0.45	9	0.
D	18 *	987	1	0	0	0.09	11	0.
D	18 *	1067	4	0	1	0.13	31	0.
D	18 *	1107	1	1	0	0.03	33	20.
D	3 *	103	3	0	0	0.09	33	0.
D	18 *	1025	7	0	0	0.18	39	0.
D	3 *	99	7	0	1	0.17	41	0.
C	1 *	61	1	0	0	0.02	50	0.
D	18 *	1059	3	0	2	0.05	60	0.
D	3 *	97	5	0	2	0.08	62	0.
F	15 *	751	8	1	2	0.13	62	2.50
D	3 *	237	6	0	0	0.09	67	0.
E	5 *	271	20	2	11	0.30	67	2.
D	18 *	1065	19	2	0	0.27	70	2.11
F	9	447	5	0	0	0.07	71	0.
C	14	697	6	1	1	0.08	75	3.33
F	15 *	725	7	1	4	0.09	78	2.86
E	17	899	30	4	9	0.38	79	2.67
C	14	815	15	0	2	0.18	83	0.
F	15 *	753	26	3	4	0.31	84	2.31

TABLE I—*Continued*

Present type of arresters	Area number	Section number	Number of transformers in section	Number of burn-outs in section	Number of fuses blown in section	Area actually covered by lines	Density of arresters per square mile	Average per cent burn-outs per year
C	14	651	63	5	10	0.74	85	1.59
D	3 *	95	44	4	5	0.52	85	1.82
C	6	365	18	1	0	0.21	86	1.11
B	8	517	26	0	3	0.29	89	0.
E	17 *	945	17	3	2	0.19	90	3.53
D	3 *	105	22	3	3	0.24	91	2.73
D	3 *	136	23	4	2	0.23	100	3.48
B	8 *	559	10	3	6	0.10	100	6.00
F	15 *	809	3	0	0	0.03	100	0.
E	17	901	9	0	2	0.09	100	0.
D	18 *	1105	4	0	1	0.04	100	0.
C	14	819	47	9	14	0.46	102	3.83
E	5 *	273	32	2	12	0.31	103	1.25
E	17 *	947	33	1	5	0.32	103	0.61
D	18 *	1017	60	6	14	0.55	109	2.
E	17	893	65	12	29	0.59	110	3.69
B	8 *	601	33	2	7	0.30	110	1.21
D	3 *	168	22	2	6	0.20	110	1.82
C	14	817	31	1	2	0.28	111	0.65
C	14	865	20	2	11	0.18	111	2.
D	3 *	235	48	4	5	0.43	112	1.67
C	1 *	41	47	1	9	0.42	112	0.42
C	14	653	86	4	13	0.77	112	0.93
D	3 *	94	9	2	0	0.08	113	0.44
D	3 *	139	25	4	3	0.22	114	3.20
D	3 *	239	24	0	1	0.21	114	0.
D	3 *	137	62	5	11	0.54	115	1.61
E	17	857	51	6	16	0.44	116	2.35
F	15 *	727	52	5	6	0.45	116	1.92
F	16	863	29	4	4	0.25	116	2.76
E	17	935	22	2	6	0.19	116	1.82
D	3 *	199	41	4	4	0.35	117	1.95
C	14	649	41	8	8	0.35	117	3.90
D	18 *	1019	25	1	6	0.21	119	0.80
C	14	701	42	2	5	0.35	120	0.95
E	17	897	36	6	31	0.30	120	3.33
B	8	515	12	1	0	0.10	120	1.66
F	15 *	723	6	1	2	0.05	120	3.33
E	17	933	35	2	7	0.29	121	1.14
D	3 *	203	88	7	13	0.73	121	1.59
C	14	699	45	2	9	0.37	122	0.89
D	3 *	201	53	4	4	0.43	123	1.51
B	8A	394	32	0	8	0.27	123	0.
E	17	975	14	2	5	0.11	127	2.86
E	17	855	35	6	22	0.27	130	3.43

TABLE 1—*Continued*

Present type of arresters	Area number	Section number	Number of transformers in section	Number of burn-outs in section	Number of fuses blown in section	Area actually covered by lines	Density of arresters per square mile	Average per cent burn-outs per year
C	14	811	92	4	11	0.71	130	0.87
E	17	895	63	7	14	0.48	131	2.22
C	14	763	105	4	13	0.80	131	0.76
D	3 *	171	124	7	14	0.94	132	1.13
C	14	759	104	5	10	0.79	132	0.96
D	3 *	241	62	5	8	0.47	132	1.61
C	6	405	16	0	1	0.12	133	0.
B	8 *	561	109	4	6	0.83	133	0.73
C	14	813	44	0	9	0.33	133	0.
B	8 *	603	39	3	19	0.29	134	1.54
E	5 *	275	80	2	12	0.59	136	0.50
E	17	937	68	3	28	0.50	136	0.88
D	3 *	169	109	6	11	0.81	136	1.08
E	17	859	22	1	3	0.16	137	0.91
B	8A	314	49	1	7	0.38	137	0.35
E	17	977	107	13	24	0.78	137	2.43
F	16	861	36	3	3	0.26	138	1.66
C	1	65	88	2	3	0.63	140	0.46
B	8A	605	71	3	10	0.49	145	0.86
D	3 *	141	76	1	12	0.52	146	0.26
C	12	571	115	2	7	0.78	148	0.35
B	8	519	75	3	8	0.49	153	0.80
D	3 *	205	137	6	26	0.89	154	0.88
E	17	939	72	6	22	0.46	156	1.66
C	14	755	128	2	18	0.81	158	0.31
D	3 *	143	62	0	7	0.39	159	0.
C	14	757	119	2	6	0.75	159	0.34
C	1 *	63	106	4	4	0.66	160	0.75
C	14	765	70	3	9	0.43	163	0.86
C	1	43	30	0	2	0.19	163	0.
C	14	733	74	1	6	0.45	164	0.27
C	14	703	71	3	8	0.43	166	0.85
D	3 *	207	139	1	25	0.84	167	0.14
D	3 *	173	161	7	19	0.96	167	0.86
B	8A	315	116	2	10	0.72	167	0.33
B	2A	145	120	8	16	0.73	167	1.29
B	2A	109	148	2	8	0.86	172	0.27
C	14	821	85	3	11	0.49	174	0.71
D	3 *	243	171	2	15	0.98	174	0.23
C	14	737	54	1	5	0.31	175	0.37
D	18 *	1099	14	3	1	0.08	175	4.28
F	4	287	132	0	5	0.75	176	0.
E	17	907	74	3	8	0.42	176	0.81
C	14	823	16	0	3	0.09	177	0.
F	9	445	25	1	2	0.14	179	0.80

TABLE I—*Continued*

Present type of arresters	Area number	Section number	Number of transformers in section	Number of burn-outs in section	Number of fuses blown in section	Area actually covered by lines	Density of arresters per square mile	Average per cent burn-outs per year
B	2A	107	47	2	7	0.29	179	0.77
B	8	475	106	2	15	0.59	180	0.38
C	6	367	9	0	1	0.05	180	0.
B	8	435	85	4	10	0.47	180	0.84
E	17 *	905	45	1	14	0.25	180	0.44
F	7	357	140	0	8	0.77	181	0.
D	3 *	175	161	7	16	0.88	183	0.87
C	14	655	132	11	17	0.72	184	1.67
C	14	735	32	2	1	0.17	188	1.25
F	7	317	176	2	10	0.93	190	0.23
E	5 *	277	166	4	15	0.87	191	0.48
C	14	705	176	2	8	0.91	193	0.23
C	12	663	70	1	3	0.36	195	0.29
C	6	403	166	1	8	0.83	199	0.12
B	8	661	123	0	7	0.62	199	0.
D	18 *	1015	18	0	4	0.09	199	0.
C	6	407	2	0	2	0.01	200	0.
B	2A	215	77	2	7	0.38	202	0.52
B	8A	395	185	2	12	0.94	202	0.21
B	13	709	65	1	1	0.32	203	0.31
F	4	247	120	5	3	0.59	204	0.83
D	11 *	521	156	5	30	0.76	205	0.64
C	12	615	172	1	7	0.84	207	0.12
C	6	319	156	6	10	0.76	208	0.77
C	14	607	165	4	30	0.79	208	0.48
C	14	657	203	6	12	0.97	209	0.59
E	5 *	279	201	5	27	0.96	210	0.50
B	8A	354	105	1	3	0.50	210	0.19
C	14	867	146	8	21	0.69	212	1.10
F	9	485	119	3	9	0.55	214	0.50
F	4	245	205	2	10	0.95	215	0.20
C	14	761	170	7	19	0.79	216	0.82
B	8	437	216	1	13	0.99	218	0.09
B	13	659	196	4	14	0.89	221	0.41
B	2A	211	218	2	7	0.99	221	0.18
C	14	609	84	2	11	0.38	221	0.48
B	8A	355	122	0	4	0.55	221	0.
B	2A	177	209	4	11	0.97	221	0.37
F	9	525	152	9	17	0.68	224	1.18
D	18 *	979	142	6	15	0.63	226	0.85
F	9	613	159	0	5	0.79	227	0.
F	9	610	98	6	9	0.43	228	1.23
C	1 *	39	7	1	1	0.03	231	2.86
F	7	397	188	2	5	0.80	235	0.21
B	8	477	221	2	19	0.94	235	0.18

TABLE 1—*Continued*

Present type of arresters	Area number	Section number	Number of transformers in section	Number of burn-outs in section	Number of fuses blown in section	Area actually covered by lines	Density of arresters per square mile	Average per cent burn-outs per year
B	13	711	112	1	5	0.47	238	0.18
D	11 *	441	179	4	9	0.75	238	0.45
B	2A	147	193	0	9	0.82	238	0.
F	9	569	235	1	8	0.97	242	0.09
C	14	563	68	3	10	0.28	243	0.88
C	12	641	32	0	2	0.13	246	0.
A	10 *	523	118	3	14	0.47	249	0.51
B	2A	213	227	1	10	0.94	250	0.09
F	4	281	238	5	13	0.95	250	0.42
C	14	869	53	1	4	0.21	252	0.39
F	4	251	129	2	4	0.51	253	0.31
F	9	611	129	4	19	0.51	253	0.62
B	2A	179	204	3	4	0.80	255	0.29
D	3 *	209	161	2	11	0.63	256	0.25
A	10 *	443	203	3	11	0.78	259	0.30
D	11 *	479	129	3	9	0.49	263	0.47
A	10 *	483	153	1	13	0.57	269	0.13
C	14	731	108	1	4	0.39	277	0.19
C	6	399	263	3	16	0.95	277	0.23
F	4	283	267	1	18	0.96	278	0.08
F	9	567	224	6	22	0.79	286	0.54
D	11 *	439	183	0	11	0.64	286	0.
F	4	249	287	7	17	0.99	289	0.49
C	6	323	183	2	9	0.63	291	0.22
C	6	321	284	10	23	0.95	298	0.70
C	12	665	6	0	0	0.02	300	0.
C	6	359	226	2	15	0.75	303	0.18
B	13	707	119	1	6	0.39	305	0.17
F	4	285	243	7	15	0.77	316	0.58
C	6	363	264	7	12	0.82	322	0.53
B	11 *	481	113	0	8	0.35	323	0.
C	6	401	315	5	20	0.95	325	0.32
C	6	361	318	5	27	0.95	335	0.31
B	13	687	98	0	2	0.29	338	0.
C	14	565	21	0	3	0.06	350	0.
E	17	941	4	0	1	0.01	400	0.
F	9	527	32	2	0	0.05	640	1.25
Total & Averages			17,529	529	1,702	92.88	190	0.60
Total number of Section 192								

of the block. (This width varies in the different portions of the city from about 250 ft. to over 600 ft. and averages approximately 400 ft.) From these figures can be calculated the percentage of burn-outs in any section or group of sections. The data with the sections arranged in the order of density of arresters are shown in Table I.

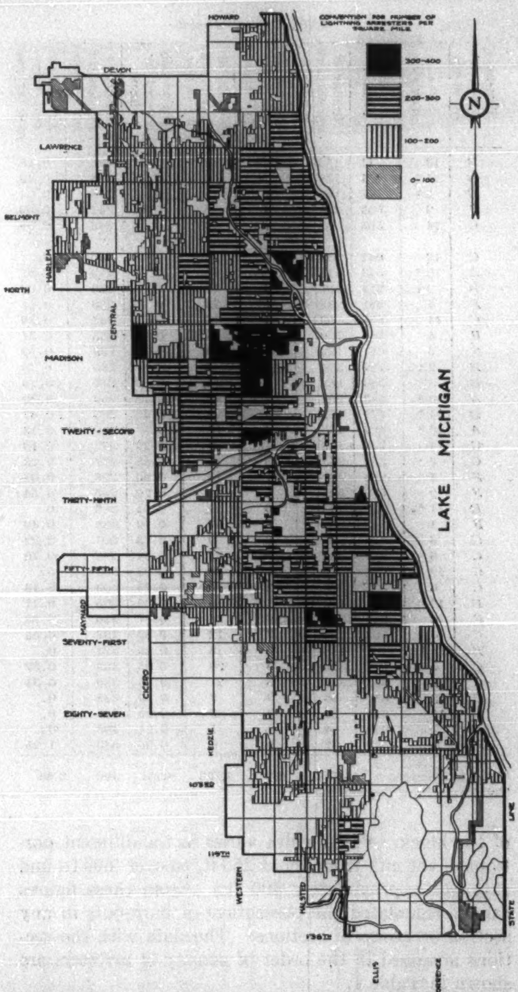
CONVENTION FOR NUMBER OF LIGHTNING ARRESTERS PER SQUARE MILE
300-400
200-300
100-200
0-100
N
HOWARD
DEVON
LAWRENCE
BELMONT
HARLEM
NORTH
CENTRAL
MADISON
TWENTY-SECOND
THIRTY-NINTH
FIFTY-FIFTH
MAYNARD
SEVENTY-FIRST
CICERO
EIGHTY-SEVEN
KEDZIE
103RD
119TH
WESTERN
HALSTED
138TH
ELLIS
TORRENCE
STATE
LINE
LAKE MICHIGAN

Fig. 8—Outline Map of Chicago, Showing Section Lines and Area Actually Covered by Distribution System

Density of shading indicates the density of lightning arresters.

The data in this table and other data regarding the system are shown graphically in several drawings which give a better idea of the conditions than can be obtained from tables of statistics. In Fig. 8 is shown an outline map of the city on which are shaded the areas actually covered by the lines, the number of arresters per square mile being indicated by the density of the shading. The distribution system extends into 192 sections covering 163.25 square miles within the city, while the area actually covered by the lines, determined in the manner above described is 93.49 square miles. As there were 17,529 transformers on the lines on August 1st, 1918, the average density of arresters is thus 187 per square mile.

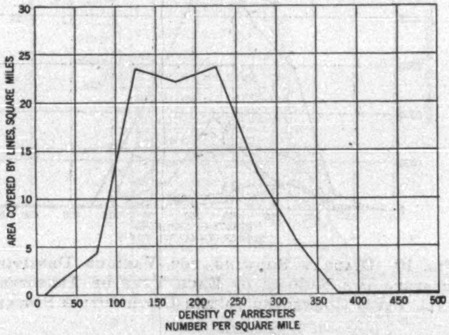

Fig. 9—Diagram Showing the Area Actually Covered by the Distribution System for Various Densities of Arresters

Fig. 9 shows these data in another manner, from which figure it will be noted that in the larger portion of the area covered by the distribution system, the density of arresters ranges between 100 and 300 per square mile. The number of arresters for various densities and for each type of arrester is shown in Fig. 10. In this drawing it will be noted that arrester *A* was installed in sections with a very narrow range in density. The section numbers given in the third column in Table I are shown in Fig. 11. The stars preceding the section numbers in Table I indicate the sections in which a change in the type of lightning arrester was made preceding the lightning season of 1917 for the purpose of permitting the installation of an additional type of arrester and securing a better distribution of the several types of arresters in different portions of the city.

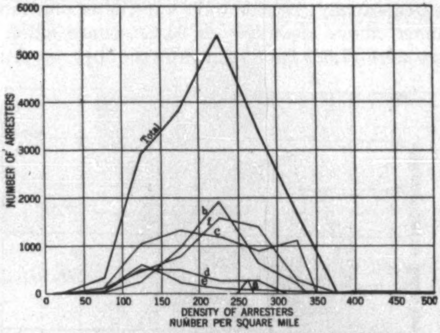

Fig. 10—Diagram Showing for Various Densities of Arresters the Number of Each Type of Arrester and of All Types Connected to the Distribution System

In Fig. 12 there has been plotted for each section the density of arresters as shown in the eighth column in Table I and the average per cent of burn-outs as shown in the last column. The final curve for all arresters showing the variation in the performance of arresters with their density is also shown in the same figure, but the curve cannot be drawn directly from the points shown in this figure because these points, representing different areas and different numbers of transformers, are not of equal weight. Nothing in the tables or records shows the wide variation in the distribution and intensity of the lightning quite so well as the plotting of these points in Fig. 12. Out of the 192 sections it will be noted that in about one-sixth of them the points are on the line of zero burn-outs, showing that there were no burn-outs whatever in these sections during the five-year period.

In order to secure points of equal weight for the purpose of drawing the curve, it was decided to have each point represent the experience with the same

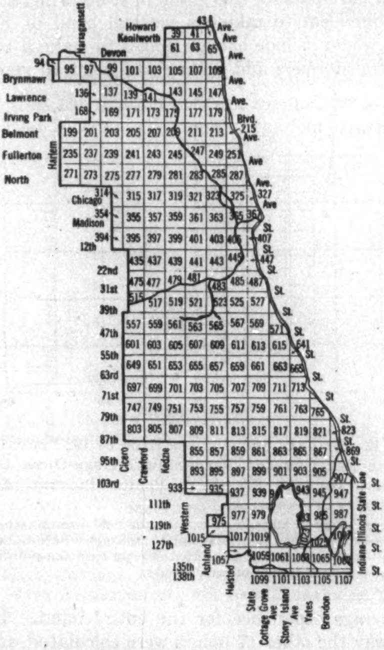
Howard
Kenilworth
Devon
Naragansett
Brynmawr
Lawrence
Irving Park
Belmont
Fullerton
North
Harlem
Chicago
Madison
12th
22nd
31st
39th
47th
55th
63rd
71st
79th
87th
95th
103rd
111th
119th
127th
135th
138th
Cicero
Crawford
Kedzie
Western
Ashland
Halsted
State
Cottage Grove Ave
Stony Island Ave
Yates
Brandon
Indiana-Illinois State Line
Ave.
Blvd.
St.
43
39 41
61 63 65
94 95 97 99 101 103 105 107 109
136 137 139 141 143 145 147
168 169 171 173 175 177 179
199 201 203 205 207 209 211 213 215
235 237 239 241 243 245 247 249 251
271 273 275 277 279 281 283 285 287
314 315 317 319 321 323 325 327
354 355 357 359 361 363 365 367
394 395 397 399 401 403 405 407
435 437 439 441 443 445 447
475 477 479 481 483 485 487
515 517 519 521 523 525 527
557 559 561 563 565 567 569 571
601 603 605 607 609 611 613 615 641
649 651 653 655 657 659 661 663 665
697 699 701 703 705 707 709 711 713
747 749 751 753 755 757 759 761 763 765
803 805 807 809 811 813 815 817 819 821 823
855 857 859 861 863 865 867 869
893 895 897 899 901 903 905 907
933 935 937 939 941 943 945 947
975 977 979 983 985 987
1015 1017 1019 1023 1025 1027
1057 1059 1061 1063 1065 1067
1099 1101 1103 1105 1107

FIG. 11—OUTLINE MAP OF CHICAGO, GIVING THE NUMBER ASSIGNED TO EACH OF THE SECTIONS AS SHOWN IN THE THIRD COLUMN OF TABLE I

number of transformers. At first trial it was agreed to assemble the data so as to get 18 points, each of which would therefore include the data from approximately 1000 transformers. The data for the first point were obtained by starting at the top of Table I and including enough sections to get a total of about 1000 transformers. Then the figures showing the area covered and the number of burn-outs was totaled for these sections, from which could be determined the average density of the arresters and the average per cent of burn-outs for this group of transformers. This was equivalent to taking a vertical band of Fig. 12 which would include enough points to make a total of 1000 transformers and finding one point to represent

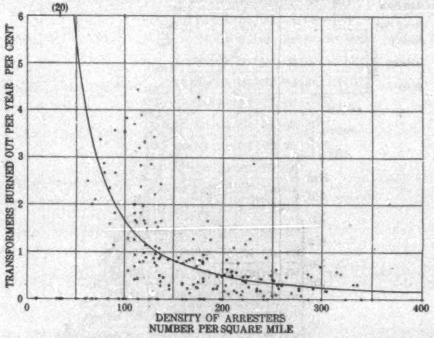

Fig. 12—Diagram Showing for Each of 192 Sections the Average Per Cent of Transformer Burn-Outs Due to Lightning for the Five-Year Period Plotted Against the Density of Arresters

The curve shows for all types of arresters the final determination of the relation between density of arresters and transformer burn-outs due to lightning. The curve cannot be plotted directly from the points shown in the figure as they are not of equal weight.

the average experience for the entire band. In the same way the other 17 points were calculated and are shown plotted to logarithmic coordinates in Fig. 13. The use of logarithmic coordinate paper was adopted for the purpose as it was found to greatly facilitate the work. There was some question as to whether the number of points selected for assembling the data in this manner had any effect on the resulting curve, but it appeared that if practically the same line were obtained by using a different number of points then there would be no serious error in the method. The same data were therefore assembled in a similar manner in 7 points, 4 points and 2 points and the results are shown respectively in Figs. 14, 15 16. After a number of attempts to draw curves through these points in the several figures, it was found that a straight line would properly represent the results just as well as any curve which might be drawn; and it was, therefore, assumed that the curve when drawn on logarithmic coordinate paper was a straight line, which is equivalent to assuming that the relation between the quantities is an exponential function. In each of the four figures the full line is determined by the points in that figure and the dashed line is the average of all of the four. It will be noted that the variation of the points through the straight line decreases as the number of points decreases, or in other words, as the number of transformers represented by one point increases. The average curve represented by the dashed line in these four figures transferred to arithmetical coordinates is shown in Fig. 12.

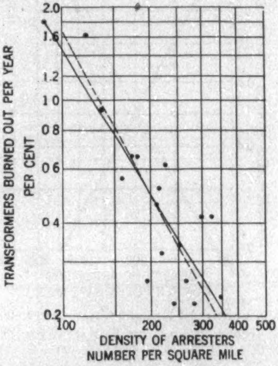

Fig. 13—Diagram to Logarithmic Scale Showing the Data in Table I and Fig. 12, Assembled into Eighteen Points Each Covering the Experience for the Five-Year Period With Approximately the Same Number of Transformers

While one engineer was engaged in the task of assembling the data and drawing the lines on logarithmic coordinate paper as above described, another engineer was given the task of assembling the data in a similar manner except that he used for each point the experience from an equal area covered by the lines as given in column seven of Table I, instead of an equal number of transformers. This was done with the idea that any serious personal errors or any error due to the assumptions made in drawing the curves or in transferring them to arithmetical coordinate paper would be indicated by differences in the final curves. After these two engineers had independently drawn final curves similar to the one shown in Fig. 12 the two curves were then transferred to the same sheet and found to be practically superposed. The equation of the curve in Fig. 12 is:

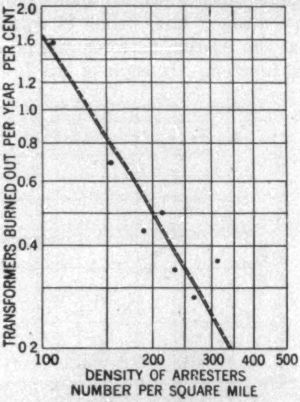

FIG. 14—SAME AS FIG. 13 EXCEPT THAT DATA ARE ASSEMBLED INTO SEVEN POINTS OF EQUAL WEIGHT

$$Y = \frac{5450}{X^{1.75}}$$

where X = the number of arresters per square mile, and

Y = The average per cent of transformers burnt out by lightning per year during the five-year period.

This equation means that the density of arresters has a very important influence on the results secured by lightning arresters. If we assume for example, that there are 1000 transformers installed in an area of 10 square miles each protected by an arrester on the same pole, and that later the number of transformers in this area is doubled and at the same time uniformly distributed, the results of the change are shown in Table II. From this table it will be noted that although the number of transformers in the area has been doubled, the percentage of burn-outs has decreased from 1.67 per cent to 0.5 per cent and that the actual number of burn-outs has decreased from 17 to 10 per annum. In other words, the doubling of the number of transformers and arresters in a given area will not result in more transformers being burnt out by lightning per year as might be supposed, but will result in an actual reduction of about 40 per cent in the number of such burn-outs per year.

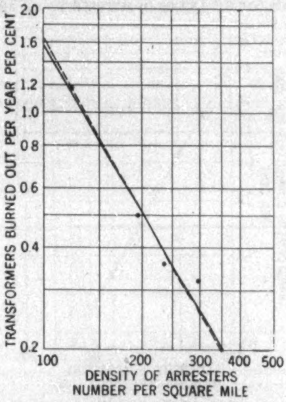

Fig. 15—Same as Fig. 13 Except That Data are Assembled Into Four Points of Equal Weight

The data for each type of arrester were then plotted in a similar manner, a set of four curves similar to Figs. 13, 14, 15 and 16 being drawn for each type of arrester. As might be expected with a smaller number of observations, the variation of the points from a straight line when plotted on logarithmic paper and the variation of the four lines from their average was somewhat greater than in the case of the corresponding lines for all types of arresters. These straight lines for the different types of arresters were not parallel, and when transferred to arithmetical coordinate paper as shown in Fig. 17, the curves cross each other in a confusing manner. While the curves thus drawn may be mathematically accurate, they appear to be physically impossible as there seems to be no sufficient reason why one type of arrester should be better than another at one density and poorer at another density. It seems more reasonable to suppose that if one arrester is better than another at any particular density of arresters, it will be better throughout the entire range of densities. After giving this subject considerable study it was decided to assume that the straight line representing the

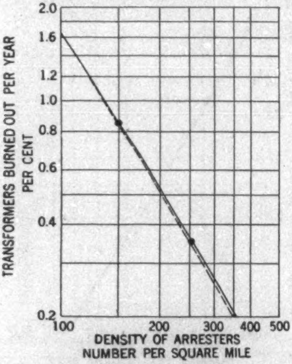

FIG. 16—SAME AS FIG. 13 EXCEPT THAT THE DATA ARE ASSEMBLED INTO TWO POINTS OF EQUAL WEIGHT

experience with any type of arrester, when plotted on logarithmic coordinate paper should be parallel to the line showing the results for all types of arresters, that is, the dashed line in Fig. 13. To make this change: the midpoint of the line for each arrester was found,

TABLE II

CALCULATIONS FROM ASSUMED DATA SHOWING THE EFFECT OF DOUBLING THE DENSITY OF ARRESTERS AND TRANSFORMERS

No. of arresters and transformers	Area square miles	Density of arrester	Average annual transformer burn-outs due to lightning	
			Per cent	No.
1000	10	100	1.67	17
2000	10	200	0.5	10

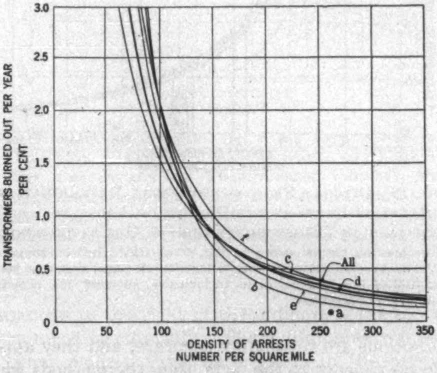

FIG. 17—DIAGRAM SHOWING THE FIRST APPROXIMATION OF THE RELATION BETWEEN THE DENSITY OF ARRESTERS AND THE PERCENTAGE OF TRANSFORMERS BURNED OUT BY LIGHTNING FOR THE FIVE-YEAR PERIOD, 1915-1919 INCLUSIVE

The curve for all arresters is plotted from the dashed line in Figs. 13 to 16 inclusive. The curves for the individual types of arresters were derived in a similar manner from logarithmic diagrams which are not reproduced.

A point instead of a line is shown for arrester *A* as the records for this type include only one transformer burn-out in the three years in which the arresters have been in service.

which is a point so located that there is an equal number of arresters represented by the line on either side of the point. The line which was finally taken as representing the experience with this type of arrester was then drawn through this midpoint and parallel to the dashed line in Fig. 13. The results of this assumption when transferred to arithmetical coordinate paper are shown in Fig. 18. If these several assumptions are reasonably accurate, and they appear to do no violence to the facts, then the methods which have been used result in curves which can be taken as representing the performance of each of the arresters with varying densities, and the most troublesome variable, that is, the variation in the distribution and intensity of the lightning has been eliminated by the method of assembling the data and drawing the curves. From these curves it will be noted that four of the arresters designated as *C*, *D*, *E* and *F* are so

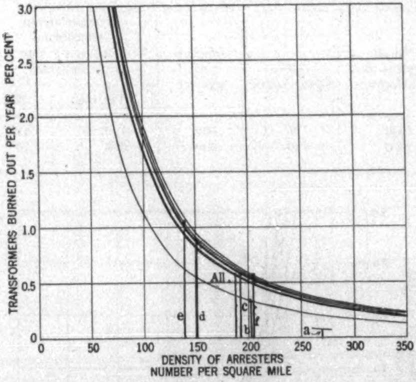

FIG. 18—DIAGRAM SHOWING THE FINAL DETERMINATION OF THE RELATION BETWEEN THE DENSITY OF ARRESTERS AND THE PERCENTAGE OF TRANSFORMERS BURNED OUT BY LIGHTNING

These are the curves shown in Fig. 17 modified by the assumption that the lines representing the data on logarithmic paper should be parallel to the dashed line in Figs. 13 to 16 inclusive, showing the experience with all types of arresters.

close together that the differences may be considered as well within the possible errors of observation.

In Fig. 18 an ordinate has been drawn to the midpoint of each of the curves as above defined or at the position corresponding to the average density for that curve, that is, for each type of arrester the number of arresters to the right of the ordinate is the same as the number to the left. These ordinates represent the same values that were given in the previous paper as showing the average experience for each type of arrester, but it is now seen that in the case of the four arresters *C*, *D*, *E* and *F*, the curves are so close together that the ordinates for these curves, instead of correctly representing the relative merits of the four arresters, are practically four different ordinates of the same curve. The four arresters are therefore of practically equal protective value.

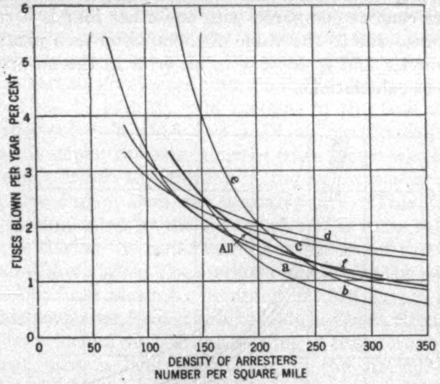

FIG. 19—DIAGRAM SHOWING THE FIRST APPROXIMATION OF THE RELATION BETWEEN THE DENSITY OF ARRESTERS AND THE PERCENTAGE OF TRANSFORMER PRIMARY FUSES BLOWN BY LIGHTNING, PLOTTED IN THE SAME MANNER AS THE CURVES IN FIG. 17

The curve for arrester *A* in this and the following figure was secured by using the records by quarter sections so as to secure an increased number of points.

It will be noted that the ordinates for curve *B* in Fig. 18 are about 40 per cent of the corresponding ordinates of the average of curves *C*, *D*, *E* and *F*. Arrester *B* is one of the oldest types on the lines and the arresters are fairly well distributed over a wide range of density as shown in Fig. 10. It is, therefore, considered that this difference of about 40 per cent as compared with the other four is a real difference due to the value of the arrester as a protective device and is not due to an error in the observations or calculations.

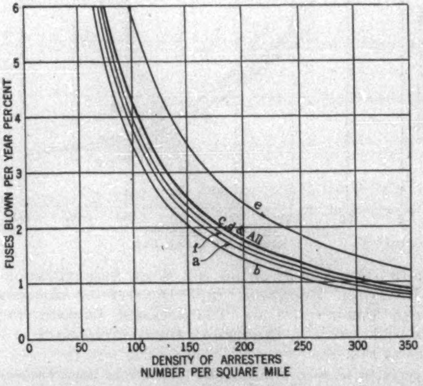

Fig. 20—Diagram Showing the Final Determination Between the Relation of the Density of Arresters and the Percentage of Transformer Primary Fuses Blown by Lightning During the Five-Year Period 1915-1919 Inclusive

Plotted in the same manner as the curves in Fig. 18.

In the case of arrester *A*, Fig. 6 shows that this arrester was installed in only three contiguous sections and Fig. 10 shows that these sections had a narrow range in arrester density. In addition the arresters had been in our service for only three years and in view of all of these circumstances, it appears that the data regarding this particular type of arrester are not conclusive. For the purpose of securing more conclusive data regarding this type of arrester, additional arresters were installed early in 1920 in the areas shown by the heavy shading in Fig. 6. The light shading in the same figure shows the areas in which an additional type of arrester was installed early in 1920.

The data showing the relation between density of arresters and the per cent of primary fuses blown by lightning were treated in the same manner as the data for the transformer burn-outs, and the results are shown in Figs. 19 and 20. On account of the fuse trouble experienced in 1918 and 1919 as mentioned earlier in the paper, the percentage of fuses blown which have been ascribed to lightning in the two years is known to be higher than the actual figure. This trouble was eliminated by the change of all the fuses to a new type during the latter months of 1919 and the earlier months of 1920. The marked reduction in the percentage of fuses blown by lightning in 1920 indicates that this particular form of fuse trouble has been eliminated. The principal cause of the blowing of transformer primary fuses in lightning storms is the arcing across the bushings where the primary wires enter the transformer case, and the scars from these arcs can generally be located. Some of the transformer manufacturers have increased the size of these bushings in their later designs and our records indicate a considerable reduction in the amount of trouble from this cause with the enlarged bushings.

On account of the nature of this trouble and on account of the several factors which modify the results, it is thought that the curves in Fig. 20 are less accurate than the corresponding curves for transformer burn-outs as shown in Fig. 18. The curves are, however, of the same general shape and show that there is a marked decrease in the percentage of fuses blown to the increase in density. The equation of the curve for all arresters in Fig. 20 is

$$Z = \frac{1300}{X^{1.76}}$$

where X = the number of arresters per square mile

and Z = the per cent of fuses burnt out due to lightning.

The investigation discloses a remarkable decrease in the percentage of transformer troubles caused by lightning with the increase in the number of arresters per square mile. The curves show further that there is a rather high percentage of troubles for the very low densities, and in those systems where the transformers, each of which is protected by an arrester on the same pole, are located several blocks apart, it is possible that although the arresters are actually of considerable benefit, the percentage of troubles is still so high that there may be some doubt in the minds of those in charge of such systems as to the value or even the advisability of installing lightning arresters. Where such conditions exist, the installation of additional arresters would in all probability reduce the percentage of trouble caused by lightning to such an extent that their installation would be entirely warranted by a reduction in operating expenses.

5. Lightning Arresters on Cable Poles

As indicated in the description of the system, about 25 per cent of the primary feeders and mains are underground, and there are about 2500 cable poles where the underground feeders or mains are connected to the overhead wires. At all of these cable poles each of the cables is protected by an arrester installed on the cable pole in a manner similar to those installed on transformer poles. These arresters have not been included in figuring the density of arresters for the protection of transformers.

It is a matter of common knowledge in Chicago that some years ago when the number of substations supplying the system was only one-fifth of the present number and when the average distance between transformers was considerably greater than at present, there was a large percentage of cable troubles due to lightning. This was particularly true where a short length of underground cable required for the crossing of a boulevard or a branch of the Chicago River or by other local conditions was of necessity inserted in a long stretch of overhead line. The records also show that although the load has increased more than tenfold since that time and the number of cable poles has probably increased in an even greater ratio, the annual number of burn-outs of such cables due to lightning has not increased and has probably been reduced. In 1918, for example, when the amount of damage to transformers by lightning was about the average for the five-year period as shown by Fig. 4, there were only three burn-outs of cables due to lightning, all of which occurred either at or in the vicinity of the cable poles. This number was about 1/15 of 1 per cent of the total number in service. This information, together with the curve in Fig. 18, leads to the conclusion that the arresters which have been installed on our lines for the protection of transformers must also assist in the protection of cables connected to the overhead lines. It also appears to be equally true that the arresters installed on the cable poles must assist in the protection of the neighboring transformers. These cable poles are fairly well scattered over the entire system and an examination of the maps and a count of the arresters on cable poles in a few sections indicates that the arresters on cable poles bear a fairly constant ratio to the arresters on transformer poles. If these arresters on the cable poles were included in calculating the density of arresters in Table I and in Figs. 17, 18, 19 and 20, it would probably be found that the curves in these figures would be slightly altered in their position without materially altering their form. The omission of the lightning arresters on cable poles in connection with Table I and the curves which follow does not appear to be a serious factor as the exact values of the percentage of burn-outs or of fuses blown for any given density refer specifically to Chicago conditions, while the form of the curves probably represents a fundamental principle in lightning protection.

If the density of the arresters in the vicinity of a cable pole is an important feature of lightning protection for cables connected to 4000-volt lines, it should be equally important in connection with arresters for the protection of cables connected to higher voltage lines. In and near many of the larger cities it is a quite common practise to have a number of lines partly overhead and partly underground operating at voltages from about 6000 to 25,000. There is a wide difference in the practise of various companies in the protection of such cables from lightning; some companies claim that so little benefit has been derived from the installation of lightning arresters on their 13,000 or 25,000-volt cable poles that they are no longer installed; other companies consider that the probability of interruption due to lightning is so great that they do not connect overhead lines to underground cables except in substation buildings where there are also installed transformers for changing the voltage, together with the usual equipment of arresters for the protection of the transformers. The transformers installed in this manner serve to isolate the overhead line from the underground cable and thus effectively protect the cable from lightning.

In such cases where lightning arresters are installed at points where high-voltage cables are connected to overhead lines, no company installs more than a single arrester. In Chicago there are a number of such cable poles on 12,000 and on 20,000-volt cables, and the percentage of burn-outs of these cables due to lightning is of the order of 10 per cent per annum, while in the 4000-volt cables it is a small fraction of 1 per cent. In view of this experience and of the curve showing the effect of density of arresters on the efficiency of the protection of line transformers, it appears that the high percentage of burn-outs experienced with the high-voltage cables is largely due to the low density of arresters in the immediate vicinity of the cable pole, and that a very considerable reduction in the percentage of such burn-outs could be secured by the installation of additional arresters in the vicinity of the cable pole.

6. Comments on the Designs of Lightning Arresters Covered by this Investigation

The earlier types of lightning arresters were all made with the metal parts enclosed in a wooden case so that the arresters could be periodically inspected, adjusted and repaired. The arcing parts were so designed that they could be readily renewed. The practise was to make an annual inspection followed by the necessary adjustment, repairs and replacements, and this annual overhauling was a serious item of expense.

Early in the period covered by these investigations, one of the manufacturers brought out a type of arrester which was self-contained, that is, the metal and other current-carrying portions of the arrester were assembled in a porcelain casing which did not require an external wooden box for its protection. In this type of arrester the metal parts and resistance rod are assembled in a porcelain housing, the several parts being fastened together by means of a sealing compound so that it is not feasible to inspect or repair these arresters while they are in position on the poles. The gaps in this new type, instead of being between brass points or spheres, are between round parallel plates so that a number of successive heavy discharges will not seriously alter the total length of the gaps. After several years' experience with this type of arrester, the conclusion was reached that such arresters had many advantages over the wooden box type besides a considerably reduced annual maintenance charge and it appeared entirely possible to make such arresters so that they would have a protective value equal to the earlier wooden box type. On reaching this decision the manufacturers were advised that no further purchases would be made of arresters which required a wooden box for their protection. Several types of arresters have been produced by the manufacturers since that time and several modifications in the design of the arresters have been made as a result of the information from these investigations which were communicated to the manufacturers from time to time. It is possible that the experience with the several types of arresters covered by these investigations, as well as the earlier types which they replaced, may be best summarized in the form of a tentative specification for lightning arresters, which would state some of the important points to be included and to be avoided in such design. Such a specification would read about as follows:

1. The arrester must consist of a number of gaps in series with a resistance, with the number of gaps and the amount of resistance properly adjusted to the line voltage so that the dynamic arc following a lightning discharge will be quickly broken without damage to the arrester.

2. The resistance rod must have the resistance uniformly distributed throughout its length, so as to prevent the progressive short-circuiting of the rod with heavy lightning discharges and the destruction of the arrester which will follow.

3. The amount of resistance in the resistance rod should not be seriously affected by repeated heavy discharges.

4. The leads for connecting the arrester to the line should leave the arrester so that they will form drip loops, and the leads should be so arranged that the arrester can be connected to a line wire on either side of the arrester.

For low maintenance cost the following features are desirable:

5. The enclosing case should be of fireproof insulating material that is not affected by the weather; and it should be constructed so as to protect effectually the metal parts from the weather, and to prevent accumulation of dust on the gaps.

6. The gaps in the arrester should be between parallel plates, disks, or rings instead of between cylinders or spheres so as to permit repeated heavy discharges without seriously altering the length of the gaps.

7. The arrester should be constructed so that in the event of the failure of the arrester to interrupt the dynamic arc the enclosing case will be shattered by the heat so as to give some visual evidence of the trouble and result in the opening of the circuit.

8. The arrester should be without moving parts or parts which require inspection, renewal or adjustment and should preferably be made in the form which cannot be inspected or repaired without removing it from the pole.

The experience with the arresters covered by this investigation indicates that several types of arresters are now available which comply with all of these specifications. The annual maintenance cost has up to the present time averaged well below 1 per cent per annum and is practically confined to the replacing of damaged arresters. It is possible that the present forms of gaps will not permit of an indefinite number of discharges without affecting the protective value of the arresters, but if such a condition should arise it would probably be manifested by an increase in the percentage of transformers burned out by lightning. The condition of the gaps could be determined by removing from the line and examining and testing the arresters which were protecting transformers that had burned out. The maximum number of arresters to be treated in this manner would be less than 1 per cent of the total number installed and this number in all probability could be considerably reduced by careful investigation of the conditions surrounding their installation along the lines described in the earlier part of this paper, and this would serve to still further reduce the cost.

7. SUGGESTED FUTURE PLANS

In Chicago it is the standard practise when making line extensions to install in the center of each block one pole larger and stronger than the others for a future transformer pole. By going over the map of the distribution system it has been found that if additional arresters were installed on these future transformer poles, so that each transformer will be protected not only by an arrester on the same pole but also by additional arresters one block away in each direction along the lines, then the additional arresters required will be only 2 per cent of the total number at present installed for the protection of transformers.

Calculations made with the aid of the curves show that these 2 per cent additional arresters will result in a saving of at least 7 per cent of the present number of burn-outs. The installation of these additional arresters would, therefore, appear to be warranted by the reduction in the operating and maintenance charges.

There appears to be a number of locations where submarine cables crossing the river, or lead-covered cables at other locations, connect with the overhead lines at points which are several blocks removed from the nearest transformer and they are, therefore, protected only by an arrester on the cable pole. In such cases it appears that additional arresters would be warranted in the vicinity of such cable terminals, and particularly so in the case of submarine cables and perhaps other special cases where the cost of repairs would be very heavy. In some cases where an overhead line is supplied from a single underground cable, and where there are no arresters on transformers within several blocks of the cable pole, it may be possible by the installation of additional arresters in the vicinity of the cable pole, to save the cost of an emergency supply.

8. Differences between Laboratory Tests and Service Experience

The investigation and the calculations herein described indicate that four types of arresters differing in such details of design as the number of gaps between line and ground, the amount of series resistance, and other features, are so nearly alike that they can be considered as having identical protective value under the conditions of service. The results of the laboratory tests, however, indicate that the arresters are quite appreciably different in their protective value, and up to the present time it has been found impossible to reconcile the results of the laboratory tests with the experience in service. It is suggested, however, that the impedance of the ground wire and of the pipe used as the lightning arrester ground may be sufficiently high to overshadow the differences in the amount of resistance and the number of gaps, and in this way tend to equalize the behavior of different types of arresters under the conditions of service. In view of these differences and of the expense and the time required to determine the relative merits of different types of arresters from the experience in service, it is suggested that it would be a very interesting and valuable development in the art if some form of laboratory tests for lightning arresters could be devised whose results would more nearly agree with the results of experience in actual service.

9. CONCLUSIONS

The conclusions from the investigations described in this paper, together with the more important conclusions from the previous paper, some of which have been modified and extended by these investigations, may be summarized as follows:

1. Transformer troubles during lightning storms may be reduced (a) by the removal of transformer primary terminal boards, (b) by the installation of lightning arresters, (c) by the use of larger bushings on the primary leads of transformers where they enter the case.

2. Lightning arresters installed on transformer poles are considerably more effective than if installed on the line poles.

3. Even in the most severe lightning storms, which apparently cover the given territory quite completely, there will be numerous extended areas within this territory which will be entirely free from lightning disturbances. Careful records extending over a period of several years are, therefore, necessary in order to determine definitely whether immunity from troubles due to lightning is due to the efficiency of the lightning protection or to the absence of lightning.

4. There is a very marked improvement in the effect of lightning arrester protection with an increase in density, that is, the number per square mile, and this effect is such an important factor in their performance that no accurate comparison of the relative merits of various types of arresters can be made without giving this point proper consideration.

5. Where the number of transformers, each of which is protected by an arrester on the same pole, is large per square mile so that the transformers and arresters are on the average only a few hundred feet apart, the total combined effect of all of the adjacent arresters is greater than that of the arrester on the same pole with the transformer.

6. In districts where transformers are widely scattered, that is, where the local density is materially below 100 per square mile and where continuous service is important, it will probably be found desirable to install arresters on line poles in addition to an arrester on the same pole with each transformer; where the local density is of the order of 50 per square mile, or lower, the installation of such additional arresters will probably be found to be warranted solely by the reduction in operating expenses.

7. The increase in the density of lightning arresters also results in a marked decrease in the percentage of burn-outs due to lightning of underground cables connected to overhead distribution circuits, and while the exact figures for the early years are not available, the percentage has been reduced from several per cent per annum with a very low density to a figure running well below one-tenth of one per cent per annum with the density averaging about 200 per square mile.

8. In the case of high-voltage cables, that is cables operating at voltages ranging up to 25,000, and where the present practise in this country calls for a maximum of one arrester at the point where the underground cable connects with the overhead line, the installation of additional arresters in the vicinity of the cable pole would in all probability cause a marked reduction in the percentage of burn-outs of such cables due to lightning.

9. The density of arresters, the shielding of high buildings, trees, etc., and perhaps also other features, have such an important effect on the amount of trouble from lightning that no accurate comparisons of the results secured in different cities can be made without giving due consideration to all

such features of the conditions under which the lightning arresters are installed.

10. For use in the protection of transformers in districts where each transformer is protected by an arrester on the same pole and where the density of arresters ranges above 200 per square mile, the most economical arrester of the several types covered by this investigation is probably the cheapest arrester. It is entirely possible and even probable that the local conditions will have an important bearing in determining the best type of arrester to be used in any given locality, and that where the amount of shielding from buildings, trees, wires of other companies, etc., is very slight and where the securing of adequate ground connections for the arresters is expensive it would be preferable, even in areas of low density, to use arresters whose discharge capacity is considerably greater and whose discharge potential is considerably lower than the arresters covered by these investigations and to confine the installation of the arresters to the transformer poles.

11. It is possible, by carefully distributing the various types of lightning arresters over a large area and by securing the results of the performance of arresters over a period of years, to place the several types of lightning arresters used for the protection of transformers under conditions that are practically identical as regards the features which would affect the relative performance of the various types of lightning arresters, and to secure data which will permit a comparison of the relative merits of the several types of lightning arresters as protective devices.

12. It is entirely possible to make lightning arresters of the self-contained type, that is, of a type not requiring an external protecting box and so constructed as not to require or permit inspection. The annual maintenance cost of such arresters is practically limited to the replacing of damaged arresters, and the total annual maintenance cost as indicated by an experience of five years with several thousand such arresters is well below 1 per cent of their original cost of installation. The adoption of such types of arresters will

result in a material reduction in the annual maintenance cost as compared with the older types.

13. A change in the form of lightning arrester gap from a cylindrical or spherical shape to parallel flat surfaces which was adopted by the manufacturers when changing from the wooden box type to the self-contained type of arrester, appears to result in a form of design which allows repeated heavy discharges without requiring renewal or adjustment of the parts, and has been an important factor in changing the design from a type requiring annual inspection, renewal and adjustment to a type which does not permit or require such annual attention.

14. The four types of arresters which have been designated by the letters *C, D, E* and *F* and which consist essentially of a resistance in series with a number of gaps, together with such additional features as antenna, compression chambers, expulsion chambers, and solenoids to vary the length of the gap following dynamic discharge, all appear to be practically identical in their value as devices to protect line transformers.

15. The type of arrester designated by *B*, which consists of a large number of gaps in series without any resistance, in addition to two other paths through a high and a low resistance shunting a large and a small number of gaps, appears to be a considerably better protective device than arresters designated by *C, D, E* and *F*, and as far as can be determined from present information, this difference in its value as a protective device appears to be due to features of its design.

16. With the aid of the data contained in this paper it should be possible to make estimates of the cost and results of lightning protection in Chicago with the same degree of accuracy as the estimates of cost of construction or maintenance of overhead lines, when the figures are averaged over a period of years.

17. The shielding effect of high buildings, trees and other similar features which might be considered as determining the exposure of the lines to lightning have an important bearing on the amount of damage that will be caused by lightning. In local areas of a distri-

bution system, which have for years shown a high percentage of troubles caused by lightning and where the troubles have been allowed to persist because of the thought that some mysterious influence local to the neighborhood attracted the lightning, it will probably be found that a large percentage of the trouble is due to the lack of shielding from the surroundings or a low density of arresters, and that the trouble can be materially reduced by increasing the density of the arresters in the locality.

18. Great caution should be used in attempting to compare the results secured by lightning arrester protection in Chicago with results secured in other localities without giving due consideration to all of the factors which might affect lightning arrester performance.

In conclusion the author desires to express his appreciation to the General Electric Company and the Electric Service Supplies Company for their many helpful suggestions and hearty cooperation during the progress of the investigations.

TABLE IA

DATA FOR DETERMINING THE EFFECT OF DENSITY OF INDIVIDUAL TYPES OF ARRESTERS ON THEIR PERFORMANCE

This table gives the data for all sections where a star is shown opposite the section number in Table I. These are the sections in which the type of arrester was changed during the years 1915 to 1919 inclusive, and this data should be substituted for the corresponding data in Table I in plotting the curves for the individual types of arresters.

Type	Years	Area No.	Section No.	No. of trans.	No. of B. O.	Area covered	Density of arresters
C	1	18	987	1	0	.09	11
D	4	"	"		0		
C	1	"	1067	4	0	.13	31
D	4	"	"		0		
C	1	18	1107	1	0	.03	33
D	4	"	"	"	1	"	"
F	1	3	103	3	0	.09	33
D	4		"		0	"	"
C	1	18	1025	7	0	.18	39
D	4		"		0		
..	0	3	99	7	0	.17	41
D	4				0		
B	1	1	61	1	0	.02	50
C	4				0		
F	1	18	1059	3	0	.05	60

TABLE IA—*Continued*

Type	Years	Area No.	Section No.	No. of trans.	No. of B. O.	Area covered	Densty of arresters
D	4		1059		0		
..	0	3	97	5	0	.08	62
D	4				0		
C	1	15	751	8	0	.13	62
F	4	"	"		1		
..	0	3	237	6	0	.09	67
D	4				0		
C	1	5	271	20	2	.30	67
E	4				0		
C	2	18	1065	19	2	.27	70
D	3		"		0		
C	1	15	725	7	1	.09	78
F	4		"		0		
C	1	15	753	26	1	.31	84
F	4		"		2		
C	1	3	95	44	2	.52	85
D	4		"		2		
C	1	17	945	17	3	.19	90
E	4		"		0		
B	1	3	105	22	3	.24	91
D	4		105		0		
..	0	3	136	23	0	.23	100
D	4				3		
C	2	8	559	10	2	.10	100
B	3		"		1		
C	1	15	809	3	0	.03	100
F	4		"		0		
..	0	18	1105	4	0	.04	100
D	4		"		0		
..	0	5	273	32	0	.31	103
E	4		"		2		
C	1	17	947	33	1	.32	103
E	4		"		0		
F	1½	18	1017	60	3	.55	109
D	3½		"		3		
C	2	8	601	33	1	.30	110
B	3		"		1		
..	0	3	168	22	0	.20	110
D	4		"		2		
C	1½	3	235	48	2	.43	112
D	3½		"		2		
B	1	1	41	47	0	.42	112
C	4		"		1		
..	0	3	94	9	0	.08	113
D	4		"		2		

TABLE IA—*Continued*

Type	Years	Area No.	Section No.	No. of trans.	No. of B. O.	Area covered	Density of arresters
F	2	3	139	25	1	.22	114
D	3		"		3		
..	0	3	239	24	0	.21	114
D	4				0		
C	1	3	137	62	2	.54	115
D	4				3		
C	2	15	727	52	1	.45	116
F	3		"		4		
C	1	3	199	41	1	.35	117
D	4		"		3		
F	1	18	1019	25	1	.21	119
D	4		"		0		
C		15	723	6	0	.05	120
F	4		"		1		
C	1	3	203	88	1	.73	121
D	4		"		6		
F	1½	3	201	53	2	.43	123
D	3½		"		2		
C	1	3	171	124	1	.94	132
D	4		"		6		
C	1	3	241	62	2	.47	132
F	1		"		2		
D	3		"		1		
C	2	8	561	109	1	.83	183
B	3		"		3		
C	2	8	603	39	2	.29	134
B	3		"		1		
..	0	5	275	80	0	.59	136
E	4				2		
C	1	3	169	109	1	.81	136
D	4		"		5	"	"
..	0	3	141	76	0	.52	146
D	4		"		1		"
F	½	3	205	137	3	.89	154
D	3		"		1		
B	½		"		1		
C	1		"		1		
B	1	3	143	62	0	.39	159
D	4		"		0		
C	1	3	207	139	1	.84	167
D	4		"		0		
C	1	3	173	161	1	.96	167
D	4		"		6	"	"
C	1	3	243	171	1	.98	174
D	4		"		1		"

TABLE IA—*Continued*

Type	Years	Area No.	Section No.	No. of trans.	No. of B. O.	Area covered	Density of arresters
F	1	18	1099	14	2	.08	175
D	3		“		1		
F	½	3	175	161	1	.88	183
D	3		“		3		
B	1½		“		3		
C	1	5	277	166	2	.87	191
E	4		“		2		
F	1	18	1015	18	0	.09	199
D	4		“		0		
F	1	11	521	156	0	.76	205
D	4		“		5		
C	1	5	279	201	2	.96	210
E	4		“		3		
F	2	18	979	142	3	.63	226
D	3		“		3		
F	1	11	441	179	0	.75	238
D	4		“		2		
B	1	10	523	118	1	.47	249
A	3		“		0		
F	1		“		2		
F	2	3	209	161	1	.63	256
D	3		“		1		
B	2	11	479	129	3	.49	263
D	3		“		0		
F	2	10	483	153	1	.57	269
A	3		“		0		
B	1	11	439	183	0	.64	286
D	4		“		0		
F	2	11	481	113	1	.35	323
B	3		“				

DISCUSSION ON "STUDIES IN LIGHTNING PROTECTION ON 4000-VOLT CIRCUITS—II (ROPER), CHICAGO, ILL., NOVEMBER 12, 1920.

Dr. Charles P. Steinmetz (read by H. R. Summerhayes): For some years we have realized that the conditions of lightning protection in primary distribution networks are in some respects materially different from those in high-voltage transmission lines. Many of the phenomena, which are of serious danger in the high-potential transmission line, such as steep wave front impulses, high-frequency traveling or standing waves, recurrent and cumulative oscillations, etc., can not develop to a dangerous magnitude in the primary distribution circuits. Dissipation due to leakage and the low-voltage character of the insulation, and interference within the network of circuits and apparatus dampen oscillations; due to the relatively low circuit voltage, the electrostatic energy is small and the most serious source or aggravating cause of lightning trouble in high-potential circuits, the arcing ground or oscillatory spark, cannot develop. On the other hand, due to the low circuit voltage, the insulation strength is low compared with the disruptive strength of lightning voltages, and the transformers distributed all over the circuits, make the system vulnerable throughout its entire extent.

The material given by Mr. Roper's paper therefore is the most valuable contribution ever made to the study of lightning disturbances in primary distribution networks, as it contains the exact performance records of nearly 90,000 lightning arrester years, comprising 529 apparatus failures, that is, an amount of data greater than has ever before been collected on lightning disturbances in primary distribution systems.

I wish to say that all the phenomena observed by Mr. Roper are in complete agreement with, and all the conclusions which he drew from his experimental observations, follow as theoretical conclusions from the statement:

In primary distribution circuits, lightning is the discharge of a very high voltage (of the magnitude of hundred thousand volts) and correspondingly high electrostatic charge, instantaneously produced over a large part of the distribution system.

These voltages are far higher than the insulation of the transformers can stand for any appreciable time. It thus is a race between the time lag of the transformer insulation, and the rate at which the lightning arresters can discharge the excessive voltage.

Herefrom immediately follows the all dominant character of the lightning arrester density, that is, the number of lightning arresters per square mile or per lineal mile of circuit. The rate at which the excess voltage decreases is directly proportional to the number of discharge paths, that is, the number of arresters; and the time, during which the transformer is exposed to excess voltage therefore inversely proportional to the number of arresters.

It also follows why transformer terminal boards and transformer bushings, though standing a higher sustained voltage than the transformer windings, are more vulnerable, since their insulation is air, which does not have the high time lag of the oil and solid insulation of transformer windings.

With 100,000 volts instantaneously impressed upon a 2300-volt lightning arrester, differences in the number, length and shape of the spark gaps, in discharge voltage or equivalent sphere gap, within the range which may be expected between different types of such arresters, can have little effect, as due to the excessive over-voltage the discharge begins instantly. An appreciable difference in the protective value however may be expected from the discharge rate of the arrester. It is interesting to note that the arrester, which shows a superiority sufficiently great not to be over-shadowed by the effect of the arrester density—a 40 per cent decrease in transformer losses—type B, is the only one, in which the discharge capacity is not limited by a series resistance.

An arrester not at the transformer, but at a small distance from it, would have the same effect in discharging the excessive voltage of the circuit, as an arrester at the transformer, and could thus differ in protective value only by the time lag given by the charge to travel the distance from the transformer to the arrester; about one-ten-millionth of a second per 100 feet. Aside from this, all the arresters within the area covered by the instantaneously produced excessive voltage, would equally share in protective value.

The question which then arises, is that of the origin of such a very high-voltage instantaneously produced over a considerable part of the distribution system.

I have given the phenomena of the thunder storm and the origin of the lightning flash considerable study for a number of years and find that such voltages must be produced on lines as result of the equalization of cloud potential by the lightning flash.

Let, in Fig. 1, *L* represent a wire of the primary distribution circuit, 6 meters above the ground *G*.

Let *C* be a thunder cloud, at an elevation of 1000 meters above ground *G*, having a potential difference of 20 megavolts against ground. There is thus an electrostatic field between cloud and ground, of a gradient of 20 kilovolts per meter. If the line *L* were perfectly insulated, by its position in the electrostatic field, 6 meters above ground, it would have a potential difference of 120 kilovolts against ground. It is however not insulated for such voltages, and while the cloud gradually builds up to 20 megavolts, a bound charge accumulates on the line *L*, by leakage through the insulation, corona, static spark over the arresters, etc., and so keeps the line substantially at ground potential. The cloud discharges by a lightning flash, its voltage disappears and the electrostatic field between ground and cloud collapses. The bound charge on the line *L* then becomes a free charge. Since as bound charge it kept *L* at ground potential though by its position in the electrostatic field it would have had a potential difference of 120 kilovolts, as free charge now it raises the line *L* to 120 kilovolts above ground. Hence, instantaneously, that is, with the rapidity with which the lightning flash discharges the cloud, a voltage of 120,000 volts is produced over that part of the distribution system, which was in the electrostatic field of the thunder cloud.

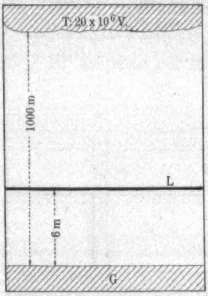

Fig. 1

This is the origin of the very high voltage instantaneously produced over a large part of the distribution system.

In reality, the phenomena in the cloud are not as simple. As the result of rain formation, potential differences against ground build up in the cloud, varying in magnitude probably between 10 to 100 megavolts in the various parts of the cloud, depending on the moisture content and thus the rate of rain formation. These potential differences between different areas of the cloud are equalized by the lightning flash, so that in some parts of the cloud the potential difference against ground is instantaneously lowered, in others probably raised. Thus if in some part of the cloud the potential difference against ground is lowered by the equalizing lightning flash from 60 megavolts to 40 megavolts, the bound charge on the line under this part of the cloud decreases from that corresponding to 60 megavolts to that corresponding to 40 megavolts, and a free charge corresponding to 20 megavolts thus appears. In other parts of the cloud, by the same lightning flash, the potential difference against ground may be raised from 20 to 40 megavolts, setting free on the lines under this part of the cloud a change of opposite polarity.

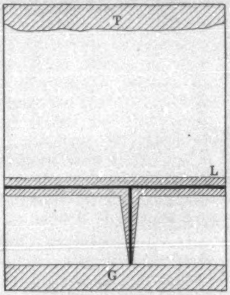

Fig. 2

This also explains why the impedance of the ground wire—which should be extremely high at the extreme rapidity of the discharge—seems to have so little effect, while even a small series resistance in the lightning arrester—small compared with the surge impedance of the line—has a marked effect. The ground wire also is in the electrostatic field between cloud and ground, and thus accumulates a bound charge, which becomes a free charge by the lightning flash. This, however, is a tapering charge, increasing from zero, or rather equality with the bound charge of the ground surface, at the bottom, to equality with the charge of the line, at the top. This charge, and the voltage produced by it, is shown by the shaded area in Fig. 2. This, however, is the distribution of voltage and thus electrostatic charge (or dielectric field) existing on the ground wire during the discharge of the lightning arrester. That is, there is no transient retarding the starting of the discharge current in the ground wire, since the energy, which the transient stores, is already there in the free charge left on the wire. The discharge current thus starts simultaneously throughout the length of the ground wire, at a rate depending on the initial potential gradient—20 kilovolts per meter. Its rate of rise is given by:

$$e = L \frac{d\,i}{d\,t}$$

or, with $L = 1.34 \times 10^9$; $e = 200$ volts per cm., this gives:

$$\frac{d\,i}{d\,t} = 150 \times 10^9 \text{ amperes per second.}$$

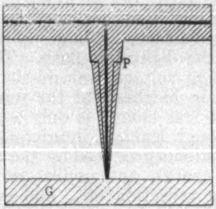

Fig. 3

With a surge impedance of the distribution lines of 400 to 500 ohms, and the lightning arrester connected into the line so that the discharge current can reach it from two wires, giving a surge impedance of 200 to 250 ohms, a voltage of 120 kilovolts would give a discharge current of 480 to 600 amperes. It would thus require about one three hundred millionths of a second for the current in the ground wire to build up. That is, the time lag of the ground wire would be of the magnitude of one three hundred-millionths of a second.

Suppose, however, a series resistance is used in the lightning arrester. The distribution of the bound charge—which is set free by the lightning flash—along the ground wire would still be the same as shown in

Fig. 2 or by the shaded area in Fig. 3. The distribution of voltage during the discharge of the lightning arrester however would be as shown by the heavy drawn line in Fig. 3, having a break, equal to the voltage drop across the series resistance, at the point *P*, where the arrester is located. That is, a rearrangement of the charge and voltage distribution in the ground wire becomes necessary, resulting in a transient retarding the discharge, that is, a time lag, which limits the protective value in this case, though the resistance may be far below the surge impedance of the lines.

From this explanation of the phenomena, we can realize the limitations within which the conclusions of Mr. Roper's paper apply:

They probably apply to all extended primary distribution systems, that is, networks of circuits of relatively low voltage, with about the same magnitude, and numerical values modified only by the climatic conditions, that is, by the frequency and severity of thunder storms, and in this respect Mr. Roper's statement is rather too modest. They would not, however, apply to circuits of materially higher voltage, in which the insulation strength of the circuits, and the discharge voltage of the arresters, are not negligible compared with the instantaneous voltage of the free charge produced by the lightning flash. They also would not apply to high-voltage transmission lines, in which the apparatus is localized at the terminals, the area affected by the free charge is only a part of the line, dissipation through leakage, interference, etc., is small, and secondary effects produced by the charge predominate, such as sparks, and oscillatory waves piling up the voltage by reflection etc., and secondary effects produced by the discharge, such as oscillatory arcs, make available for destructive action the engine power back of the generators.

J. L. R. Hayden (read by N. A. Lougee): The large amount of data given in Mr. Roper's paper enables us to investigate some further features. Some information on the protective screening effects of buildings, trees, etc., may be expected from the following reasoning: Column 7 of the table in Mr. Roper's paper gives the area covered by the lines in each of the 192 sections. As most of the sections are one square mile, it gives the part of the section covered by the lines (except in a few smaller sections, where correction is easily made). In general, where all or a large part of the section is covered by the lines, it may be expected that the section is well built up, and the screening effect of buildings, etc., therefore a maximum. Inversely, sections of which only a small part is covered by the lines, probably are sparsely built up, and the screening effect therefore a minimum. By dividing the data into two parts, for small and for large area of the section covered by the lines, and working up the two separately, a difference in the results should indicate the difference between low and high screening.

The material was divided into eight groups, by the arrester density, so that each group contains about the same number of failures. Then each group was divided into two sub-groups, of about the same number of failures, the one comprising the sections of small area covered by the lines, that is, probably low screening, the other the sections of large area covered by the lines, that is, probably high screening. The total material, and the two subgroups separately, were then worked up into empirical curves of the form proposed by Mr. Roper, by the Σ Δ method (Steinmetz, Engineering Mathematics, Chapter VI. C.), and gave the three curves shown in Fig. 4 of the respective equations:

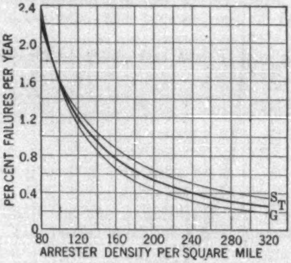

Fig. 4

T: Total data $y = \frac{1.62}{x^{1.6}}$

S: Small part of sections covered by the lines; probably low screening: $y = \frac{1.64}{x^{1.35}}$

C: Great part of sections covered by the lines; probably high screening: $y = \frac{1.59}{x^{1.85}}$

where y = percentage of failures per year;
x = arrester density, hundreds per square mile.

It is interesting to note the difference of the exponents, which means, that curve *G* is much steeper than *S*. At low arrester densities, the three curves come together but increasingly separate with increasing arrester density, so that at 120 arresters per square mile, there is a difference of 12 per cent; 38 per cent at 200 arresters, and 59 per cent difference in the percentage of failures at 300 density.

We may account for the increase of screening with increasing arrester density thus: At low arrester density, each arrester has to drain a considerable length of line, and the freedom from charge of its immediate neighborhood, due to the screening, has little effect on the total charge which the arrester has to carry off. With high density of arresters, however, each arrester drains only a small area, and the reduction of the volume of the discharge by the screened area is much more appreciable.

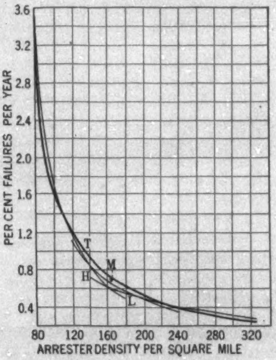

Fig. 5

The exponent 1.6 differs slightly from the value 1.75 found by Mr. Roper, probably due to the different grouping of the data here used. This suggests a change of the curve shape between high and low arrester density. Therefore, we worked the data up separately, for the range of low density, of medium and of high density. This gives the three curves shown in Fig. 5, together with the average curve *T*, of the respective equations:

L: Low arrester density $y = \frac{1.71}{x^{2.12}}$

M: Medium arrester density $y = \frac{1.48}{x^{1.6}}$

H: High arrester density $y = \frac{1.04}{x^{1.07}}$

As seen, the low density curve is very much steeper, about twice as steep, as the high density curve. In other words, increasing the number of arresters has much more effect at low than at high arrester density: At low arrester density, a 1 per cent increase of arresters decreases the failures by 2 per cent, while at high density it reduces the percentage of failures by 1 per cent only.

Using all the data, gives for Mr. Roper's equation of failures:

$$y = \frac{A}{x^{a}}$$

the constants: $a = 1.6$: $A = 1.62$

When using the exponent 1.6, but using only a group of the data for the calculation *A*, the value of *A* so derived, compared with the average $A = 1.62$, shows how the failures of this group compare with the average.

In Table I thus are given the values of *A* for the 9 conditions: low density, high density and total; low screening, high screening and total. While the numerical values themselves have little meaning, their general trend seems to me decidedly significant, in indicating the relative increase of failures with decreasing arrester density and with increasing screening, and the increased effect of screening at higher arrester density, as shown by the percentage difference given in the table.

At high arrester density, the exponent *a* in Mr. Roper's equation approaches 1. That is, the percentage of failures decreases inversely proportional to the number of arresters, in other words, the total number of failures approaches constancy.

This suggests plotting, not the percentage of failures, but the total number of failures, as function of the number of transformers or arresters per square mile. This is done in Fig. 6. Approach to constancy suggests the exponential function. This is the more indicated, as the phenomenon is one of probability, and the probability function is exponential.

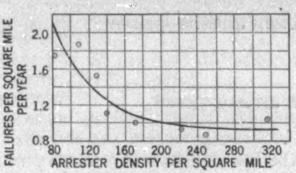

FIG. 6

If then t = number of transformers lost per square mile per year, by the Σ Δ method the equation is derived:

$$t = 6.8 \times 10^{0.94x} - 0.92$$

that is, for extremely high lightning arrester densities the average failures approach a minimum of 0.92 transformers per square mile per year; for very low arrester densities, they approach 7.7 transformers.

This equation is given by the curve in Fig. 6, and the eight groups of data marked by the circles.

TABLE 1

$$v = \frac{A}{1.6}$$

×

	A = Low Density	Total	High Density
Small area	1.69	1.86	2.05
Total	1.61	1.62	1.63
Great area	1.54	1.41	1.29
Max. dif.	0.15	0.45	0.76
Per cent	9.3	27.8	46.7

E. E. F. Creighton: When we come to a mass of useful data of the magnitude that Mr. Roper has presented it becomes a matter for careful study and thought for days. Speaking from nearly two decades of interest in the development of protective apparatus, I know of no other such labor expended in gathering valuable operating data and correlating it in a form to give useful conclusions. The process of collection of these data implies, in itself, a high degree of organization in the operation of this department in the Commonwealth Edison Co.

There is much that could be said on many of the points brought out by the correlated data. A number of these points may be more profitably discussed by those whose experiences come nearer to distribution practise than mine. The operating engineers, even if they do not favor us with their comments, must answer to themselves the question—How do these data and results bear on my own problems of protection? Time limits my discussion to one or two phases of the subject. These phases may be found in the answer to the question—What value are these data to an engineer occupied in researches and development of lightning arresters?

My first comments relate to the interpretation of data. Fig. 12 is a shot-gun diagram of the data in which the relation between the number of arresters per square mile as abscissas is compared to the percentage of transformer burnouts per year as ordinates. Mr. Roper has pointed out the efforts to make these data comparative. There are many factors involved, some of which are the same, on an average, in the different areas, but there are a few factors which not only vary considerably but their exact weight cannot be determined at the present time.

However, acceptable methods are followed which give, in the final step in Fig. 18, a direct comparison of the relative value of the arresters in giving protection. Mr. Roper has shown in Fig. 17 an intermediate step, assuming that each type of arrester gives a logarithmic curve, in comparing the density of arresters to the percentage of transformers burned out per year, and has pointed out the inconsistencies of these overlapping curves. In words, the logarithmic curve says that the phenomenon varies at any point of the curve in proportion to its value at that point. This statement seems to give only an approximation of the truth.

If the matter is looked at from the standpoint of mathemetical law, there enters the hyperbolic law which is a first-cousin to the parabolic law. It will be seen by a statement of this parabolic law that the data have elements very closely related. In its simplest form suppose one arrester in a given territory gives a certain number of burnouts. If two arresters are used there are two paths to ground and, neglecting all other factors except the resistance, there is half the ohmic resistance and therefore the number of transformers burned out might be somewhat proportional to one-half. When three arresters are used the resistance is one-third, when four are used the resistance is one-fourth, etc. These values of one, one-half, one-third, one-fourth, one-fifth for the ordinates, with equal units as the abscissas, give the familiar hyperbola. It should be noted that the arresters are not concentrated at a point but on the other hand neither is the charge that has to be dissipated by the arrester. From these general considerations I am inclined to think that both the logarithmic and the hyperbolic laws are involved and that the consideration of the combination of the two may give a consistent curve for each arrester, comparable with the others throughout the entire length of the curve. Also further study of all the elements involved in each case may give some change in the grouping of points which might clarify the shotgun diagram of Fig. 12. Mr. Roper has already done the most difficult part of the work in bringing order out of chaos. The difficulties he met can best be appreciated by those who have had to consider a mass of data which include so many variables, known and unknown.

Turning next to the question of design, the ultimate aim in all of this work is to get, as an ideal, one hundred per cent continuity of service. While it is not always economical to make such an installation there is still the desirability of having data which will allow the operating engineer to form a judgment as to the percentage of service he may reach with definite types of arresters and methods of installing them. Furthermore, from the development standpoint it is desirable to aim at 100 per cent efficiency even if the initial financial undertaking is impracticable because as soon as a thing becomes possible it is usual that the factors that make it possible can be adjusted to bring the cost down to a reasonable value.

The designer familiar with the characteristics of arresters involved in Mr. Roper's data is immediately given information on the character of the lightning discharges. It may not be generally known that it is possible to design arresters with as great precision as is attained by a designer of motors and the like. Laboratory methods of accomplishing this were developed years ago. It isn't a lack of knowledge of the characteristics of arresters that we have to contend with but a lack of knowledge of the nature of the discharges that are imposed upon them. I shall make an endeavor to interpret, to the best of my ability, the bearing that Mr. Roper's data have on explaining the nature of the lightning discharges. The arrester shown in Fig. 2 over the letter *B* is the first one of the several types developed for low-voltage distribution circuits. Although expense of construction is always an important factor it was considered of minor importance in this development as compared to being able to meet the unknown conditions of lightning discharges on a distribution circuit. One known factor was that the lightning discharge is of short duration. It was not know, however, whether all lightning was of high frequency, medium frequency, or low frequency. We had no way of knowing whether it was always of steep wave front or of how slanting a wave front. We could not tell whether the quantity was relatively great or small, which is only another way of stating that we did not know how many miles of line would be charged to a high potential by induction from thunder clouds.

We did know, however, that the dielectric of the transformers, the arrester was designed to protect, was tested at 10,000 volts for a minute between primary and secondary and that there were liberal factors of safety of insulation between turns and between layers. This gave the criterion of spark voltage of the arrester. The spark voltage should be made less than 10,000 volts if possible. On the other hand, the spark voltage must be above the value generated by accidental arcing grounds unaccompanied by resonance because such voltages are continual and will cause the destruction of this type of arrester. As to the laboratory tests—it required of the arrester that the equivalent sphere gap on high frequency, medium frequency, and low frequency under small and large quantities of discharge should be kept within the dielectric strength of the transformers so far as they were known by test.

To make a long story short, it resulted in the final design of this arrester in placing three gaps in series and arranging the electrostatic conditions between gaps such as to give a voltage breakdown of 6400 volts on 60 cycles and less voltage on high frequency. Tests were also made with single uni-directed impulses and also on direct current to make sure that no gradually accumulated charge would damage apparatus by not being able to spark over these gaps. It was necessary in this circuit to keep the series resistance very high because of the frequency of discharge and the weakness of the arc-interrupting power of three gaps. A great gain, however, was obtained by the fact that the line and lightning voltage had been led three gaps down the string of gaps. The sparks oscillating in these gaps are good conductors of electricity. The natural frequency of these sparks is of the order of a billion cycles per second. I am speaking now, not of the lightning discharge but of a local discharge between the brass cylinders which make up the three gaps.

With this tremendously high frequency and consequently short time of operation three of the series gaps have been bridged and a small discharge started to earth and the same lightning voltage may now jump the next three gaps with the same ease. There is here the evident advantage of bridging six gaps by making it in two lower voltage jumps of three gaps each. The excess voltage to jump the second group of three gaps is sometimes as small as 200 volts, although it required 6400 volts to spark through the first three. The ohmic resistance of this rod is not fixed—it does not follow Ohm's law. The resistance decreases as the lightning voltage increases. The relation between voltage and resistance follows a logarithmic law—at 500 volts applied the resistance is of the order of 100,000 ohms.

This brings the connection of the line through six gaps to the low resistance, and lightning charges too great for the high resistance find their way to earth through the low resistance rod (of the order of 25 ohms).

In the laboratory development, not knowing whether the quantity of electricity and the current of the lightning discharge would give an unreasonably high-voltage drop across this low resistance, the assumption was made that it might, and a shunt path of nine gaps was provided in parallel with the low resistance to meet this contingency. In the first conception of this arrester three resistance paths were laid out by the inventors, but it was found unnecessary to introduce the medium value of resistance.

Here, then, was an arrester with its several lightning paths which responded in the laboratory satisfactorily to high frequency, medium frequency, low frequency, single impulse, steep wave front, slanting wave front, small quantity of electricity, and large quantity of electricity.

In designing the compression chamber arrester I had in mind a more compact form of cheaper construction and based fundamentally on the assumption that lightning discharges were of fairly high frequency and of considerable quantity of electricity. Since the compression chamber arrester, as then designed, does not give a degree of protection equal to the graded shunt arrester one must conclude that there are either or both of the following factors in the induced lightning on distribution circuits. One of these factors is an occasional very heavy discharge. The other factor is an occasional single uni-directed impulse or a discharge of slanting wave front. Laboratory tests have shown that this arrester is not equal to the other in taking these discharges but have shown that the antennae make the compression chamber arrester extremely sensitive to high-frequency discharges.

The conclusions then are that occasionally there occurs extremely high voltage induced on the line involving a correspondingly high quantity of electricity and also that lightning discharges are not always at high frequency.

At the Washington meeting of the Institute in 1914 the work of Mr. L. A. De Blois in taking oscillograms of induction from clouds showed that strokes with slanting wave fronts occur from time to time. However, it was impossible to infer from these tests that there were no high-frequency oscillations superposed on these slanting wave fronts. The natural frequency of the oscillograph is only 5000 to 10,000 cycles per second and therefore the vibrator could not respond to a higher frequency in the clouds even if it existed. By inference the data that Mr. Roper presents indicate that such slanting wave fronts do exist without the presence of high frequency.

One of the characteristics of the distributed resistance arrester is its equivalent sphere gap under a discharge having a frequency of a million cycles per second. These are shown in curves in Fig. 7 of their discussion. The abscissas represent the d-c. voltage as measured by the sphere gap which starts the surge. The ordinates are the equivalent sphere gaps measured by the gap setting in parallel with the arrester. The curves are shown in pairs—the lower curve of each pair representing nine discharges over the sphere gap to one over the arrester, and the upper curve of each pair representing one discharge over the sphere gap to nine over the arrester. The lower pair of curves are the equivalent sphere gaps of the arrester under normal conditions of connection. The upper pair of curves are the equivalent sphere gaps of the fifteen gaps in series without the use of the resistance rods. Commenting on the normal equivalent sphere gap curves, an application of 10 kv. gives an average equivalent sphere gap of 10 kv. and the equivalent sphere gap increases gradually up to 16 kv. as the applied potential increases. In all this part of the curve the discharge passes over the six series gaps and through the low resistance but does not bridge the nine gaps in parallel with the low resistance rod. However, for an application of more than 26 kv. of lightning potential the spark takes a parallel path through the nine gaps and from there on up to an unlimited high potential the equivalent sphere gap remains constant at 16 kv. Herein lies the fundamental advantage of this type of arrester, namely the automatic limitation of the lightning voltage of the transformer terminals to a definite value which is within the dielectric strength of the modern transformers. All arresters with nine series resistances have their terminal voltage gradually increased proportionally as the severity of the lightning discharge increases.

Incidentally it should be noted that the equivalent sphere gap of this arrester is, in some cases, greater than the applied voltage. For example, at 10 kv. applied the equivalent sphere gap is 12.5 kv. when one discharge in ten is passing through the sphere gap. This datum, when the resultant voltage is higher than the impressed voltage, may seem erroneous but it is simply because the intrinsic conditions are not fully considered. The impressed voltage is d-c. and is measured by a sphere gap. When this discharge is turned into an impulse the voltage may easily double. As a very simple illustration of this condition—the application to an electrostatic condenser of 2 volts from a battery of low internal resistance will produce momentarily 4 volts at the terminals of the condenser. The equivalent effect is obtained in these equivalent sphere gap tests.

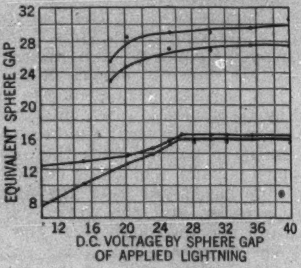

FIG. 7

The equivalent sphere gap of the gaps without resistance begins with an application of 18 kv. before they will spark. With an equivalent sphere gap of 24 kv. the curves, however, show the characteristic flattening down like the saturation curve of transformer iron. Dr. Steinmetz gave the underlying theory of the multigap arrester in a disucssion in 1906 at the Milwaukee meeting of the A. I. E. E.

What is the answer and what is the next step? Naturally it is the readjustment of the compression chamber arrester to respond better to low frequency impulses and higher lightning voltage which we are now convinced exist.

Another step is the housing of the distributed resistance arrester in a porcelain tube rather than in a wooden box.

The laboratory researches and design work on these arresters were very active ten to sixteen years ago. These arresters were sufficiently satisfactory to be lost to consideration during the great war when new problems of utmost importance were pressing for solution. I have to thank Mr. Roper for reviving my interest in this work and furnishing the incentive for further efforts toward improvement. Already a new factor has been discovered and utilized.

V. E. Goodwin: When Mr. Roper was giving his paper I thought of a definition of lightning which was given some years ago. I think it was originally given either by George Ade or by George Fitch. He said "That lightning is a stupendous demonstration of Ohm's law; in other words, millions of volts complicated by ohms and amperes." I think that is about as good a definition as I ever heard of lightning.

Referring to Mr. Roper's paper, it is pleasing to note that the art of protecting electrical apparatus against voltage disturbances has made material progress during the past ten years. This progress has been due, not only to the development of new protection methods, but also to a wider knowledge of the nature and character of the effects of lightning on electric circuits. We have had a good conception of these effects on high-voltage circuits, but until recently, have had little accurate data on low-voltage distribution circuits. Low-voltage arresters have, therefore, been designed to handle a wide range of impulse and high-frequency conditions. These arresters must have low cost and reliability; hence, it is difficult to incorporate all the best protection features for this entire range of conditions and still have an arrester which is cheap enough for the service.

In this paper, Mr. Roper has given the Institute the most complete operating record which has ever been collected. This paper is of greatest value since it shows the failure of transformers and fuses blown during lightning storms covering a period of five years and includes an average of some 17,000 installations. This paper clearly shows the futility of trying to draw conclusions on the relative merits of protective schemes, without the most careful study of operating data, including several thousand installations and comparing each year's operation with each successive year. A study of this report shows that with high density of arresters, transformer failures are reduced to a fraction of a per cent per year. This and other data show the prevalence of a certain class of disturbance having high rates of change of potential and large destructive charges. Such disturbances as these require the use of either a few arresters having high discharge rates or the use of a larger number of low discharge rate arresters in parallel. This point is further brought out by the fact that the type *B* arrester is superior to all the other types.

As these tests progressed, we have been able to better understand the nature of disturbances on these circuits and to work on the development of a protector which will have even greater discharge rates and at the same time incorporate the best features of the all-porcelain enclosed type.

By studying the transformer failures by storms and by years, it is noted that the greater losses seem to be confined to certain storms and that these losses for a given storm are grouped into a few square miles. The thought naturally comes to mind as to the possibility of many of these failures having been caused simultaneously by one unusually heavy lightning discharge. Such a discharge would release a very large bound charge on a system as large as the Commonwealth Edison Company's. Such a condition would suggest the application of a few additional arresters having a high discharge rate, as, for example, the aluminum or oxide film types, these arresters to be distributed about the city in the most important points. The same result could be obtained by the use of a greater number of arresters having a discharge rate intermediate between the aluminum and the multi-gap types.

The data presented in this paper, while collected on a four-wire grounded neutral system, probably represent conditions common to most low-voltage distribution circuits. However, non-grounded circuits may present slightly different results and it would be most interesting if one or more of the large companies operating non-grounded systems would tabulate their results and present the information to the Institute.

H. B. Gear: The results which have been tabulated in this paper seem to have settled one of the moot questions about the protection of distributing circuits, that is, whether or not there was one type of lightning arrester very greatly different from another. We seem to get reasonably good protection from all of these types, and the percentage of difference is so small as not to be at all serious.

However, the question remains as to how far we shall go from an economic standpoint in spending money for lightning protection in order to save ourselves damage to the transformer equipment and cables.

It is apparent from the law which has been worked out in connection with these figures that as more arresters are used per square mile, the cost in fixed charges and maintenance for the arrester equipment will increase while the cost of transformer burn-outs and the expense of replacing fuses becomes less. We may therefore strike an economic mean where the sum of the cost of transformer burn-outs and fixed charges on lightning arresters is a minimum.

Taking the figures for the Commonwealth Edison System for the past five years, and using the figures of 1917 as a basis for an average, the fixed charges on arrester equipment is approximately five times the cost of transformer burn-outs. It is $36,000 approximately for fixed charges on arresters and $7,200 for the repair of transformers, a total annual cost of $43,500. This represents the condition where the average arrester density for the entire city is about 190 per square mile. If an arrester density of 100 were taken the percentage of burn-out would be 1.7 according to the curve. This would represent transformer burnouts of the value of about $22,000 and fixed charges on arresters of about $19,000 or a total of $41,000. There is a point between at 125 where the total cost is about $39,000 which is about the economic minimum. These relations are apparent from the accompanying Fig. 8.

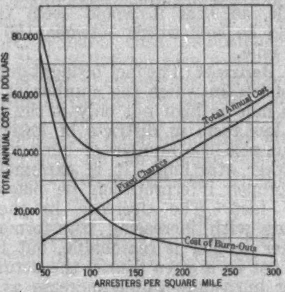

Fig. 8

Now what does this mean? As the City of Chicago is laid out, an arrester density of 128 arresters per square mile means an arrester for one transformer in each block. The higher densities occur only where there are separate transformers in the same block for power and light service. There are many three-phase installations in the same block with lighting transformers and in other cases single-phase units for power. This means that putting arresters on additional power transformers in the block is less effective economically after a density of about 125 is exceeded than it is for the lower densities.

From the point of view of the transformer burn-outs, there seems to be nothing gained under conditions in Chicago, in providing arresters much above the equivalent of 125 arresters per square mile. However, the blowing of transformer fuses may justify some additional lightning arrester investment.

The interruptions which were experienced before a lightning arrester had been put on every transformer, as shown, by the curve, produced a grade of service which was not satisfactory. The curves show a very decided flattening out above about 200 arresters per square mile, so that it would seem that practical limit should lie somewhere between 150 and 200 arresters per mile.

Edward Bennett: The fact brought out in the paper, that in districts in which the number of transformers per square mile is high not only is the percentage of burn-outs less than in districts of lower transformer density, but also the number of burn-outs is actually less, is rather startling. It is extremely important that the proper interpretation be placed upon this undoubted fact. The decrease in the percentage of burn-outs with increasing transformer and lightning arrester density is reasonable, but I would have predicted that *in a given district* the actual number of transformer burn-outs would increase as the number of transformers is increased from 100 to 300 per square mile.

The potential against ground to which the transformers in different districts are subjected during lightning disturbances is affected by two things which vary from district to district. The first is the impedance of the discharge paths from the lines through the lightning arresters to ground; the second is the electrostatic shielding effect of the buildings and trees adjoining the power lines and of other conductors, as telephone lines, on the same poles as the power lines. This gives rise to two ways of accounting for the decreased number of burn-outs in the districts of high transformer density. There is the explanation which Mr. Roper seems to favor, namely, that the increased number of lightning arresters per square mile which go with the increased number of transformers afford a freer discharge path to ground, and so lower the magnitude and the duration of the potential which is still maintained between the winding and ground after the arrester gaps break down. The second possible explanation is that the districts of high transformer density are also districts of high building density, and that the buildings and telephone wires in such districts more effectually "shield" the power wires. It should be recognized that the variation in the shielding effect from district to district may cause a far greater variation in the quantity of electricity induced upon the power lines (which must be discharged to ground) and consequently a greater variation in the magnitude and duration of the potentials to which the transformers are subjected than the variation caused by increasing the number of discharge paths from 100 to 300 per square mile.

That the author fully recognizes the necessity of taking into account the shielding effect of buildings and trees is evidenced by the ninth item of the conclusions. It is, therefore, probable, that the statements in items 4, 5 and 6 of the conclusions were made after a careful consideration of, and allowance for, the difference in the shielding effects in districts of high and of low transformer density. The paper, however, contains no statement which gives those unfamiliar with the local conditions any light on the average relative shielding conditions in the different districts. In the absence of such data, it would seem that the unquestioning acceptance of item 6 of the conclusions is not warranted. Undoubtedly Mr. Roper will throw additional light on this question in his reply to the discussion of the paper.

H. R. Woodrow: The arrester shown in Fig. 3 of the paper—without the external resistance, has apparently given slightly better protection. The elimination of the resistance makes it possible to get a greater discharge, and I should like to inquire if there has been any greater burning or other disadvantage in this type of arrester which has come to your notice.

The results of this investigation indicate that the major portion of our lightning troubles are due to voltages induced in the circuits rather than the trouble from direct strokes, as the advantages of multiplicity of lightning arresters apparently follow the inverse laws, which would be expected with the induced conditions, whereas a direct stroke of lightning would be localized more, and would not cover a wide number of arresters.

H. R. Summerhayes: Do the figures Mr. Gear has given include all of the costs due to transformer burn-outs—for instance, the loss of revenue during the time the transformer is out of service, the installation cost of replacing it, and whether any value is set on the lack of service during that time other than the loss of current. It seems to me that while it is difficult to evaluate, yet there is a certain value for giving good service which might justify the use of a greater number of arresters than the number which simply reduces the total cost of transformer burn-outs, and of arrester maintenance, to a minimum.

H. B. Gear: With regard to the cost of handling the transformers, that is included and only that part of the cost of the burn-outs which is due to repairing is included; it doesn't include the entire cost of transformers burned out except in such cases as the transformer is a total loss. You are quite right that it is quite impossible to evaluate the loss in service. The amount of revenue lost from burned out transformers is so small that it is not a factor, but the loss of reputation and the losses to customers does amount to an appreciable item where the number of burn-outs is two or three times as many as it is under a well-protected system.

I agree with Mr. Summerhayes that we should probably tend, on that account, because of the service features, to work above the minimum point of the curve.

D. W. Roper: It is very gratifying to the author to have Dr. Steinmetz confirm his deductions and to learn that these deductions have a somewhat wider scope than set forth in the paper. It is particularly interesting to know that Dr. Steinmetz confirms the deduction that all types of arresters which consist of a resistance in series with a number of gaps, and in some cases with additional features, are all practically equal in protective value; and also that in order to secure the best lightning protection for distribution circuits we must use a type of arrester in which one of the paths is through a number of gaps without any resistance in series. It is to be hoped that the manufacturers will respond by the production of such arresters which for permanance and low annual maintenance, should be contained in a porcelain case so designed as to prevent the entrance of moisture.

Dr. Steinmetz also points out that there is no theoretical advantage in having the arrester on the same pole with the transformer and that it would probably be just as effective if installed on the next adjacent pole, or in the same block. These deductions may in some cases result in a simplification of the construction on transformer poles and also reduce the number of arresters required as it has heretofore been thought necessary to install an arrester on each of the transformer installations in any one block.

It is interesting to note that the most effective arrester indicated by *B* in Fig. 2 is one of the oldest designs and was the subject of a paper before the Institute in 1907 (Protection Against Lightning and Multi-Gap Lightning Arrester by Rushmore & DuBois, TRANSACTIONS A. I. E. E., Volume XXVI, page 425). During the annual inspection of all of the arresters of this type on our lines for several years it was noted that the shunted gaps very rarely showed any signs of arcing, that is the percentage of arresters showing arcing on the shunted gaps was of the order of one-fifth of one per cent per annum. It was at first thought that this indicated that these gaps were of very little service in the performance of the arrester but it seems more proper to consider that it is due to the fact the arc across the shunted gaps is probably extinguished at the end of the next half cycle following the discharge. The frequent burning of the gaps in series with the resistance further indicates that the arcs across such gaps are extinguished more slowly than is the case with the shunted gaps.

The discussion by Mr. Creighton leads the author to hope that it may be possible to design an arrester which will have a maximum voltage across the terminals of the arrester that will be below the arcing potential across the transformer bushings and the dielectric strength of the transformer windings. The improvements in lightning protection during the ten years covered by these investigations have already eliminated nearly 90 per cent of the troubles from lightning and if such arrester can be produced then it should be possible to eliminate the other 10 per cent.

Mr. Woodrow inquired about the disadvantages of arrester *B*. The principal disadvantages of the present design of this type of arrester are the wooden box, which is required for protecting the arrester from the weather, and the occasional burning of the series gaps, so that the arresters must be inspected every year or two and the cylinders turned so as to expose a new portion for the gaps. The later designs of lightning arresters have the gaps in the form of a ring exposing a much larger area of the metal in the gap and it appears possible by using this form of the gap and by placing this arrester in a porcelain casing to retain all of the good features of this type of arrester and eliminate its principal disadvantages. With these changes in the design the annual inspection and adjustment will be eliminated, causing a reduction of about 25 per cent in the total annual charges for this type.

Mr. Hayden has made the assumption that the screening effect of neighboring trees and buildings is proportional to the density of the transformers and on the basis of this assumption has drawn some curves of lightning arrester performance which seem to indicate that a maximum effectiveness is reached at a rather low density. Following a careful study of selected portions of the city since the discussion it has been found impossible to check Mr. Hayden's assumptions. While it may be true that in general the greatest screening by overshadowing tree or buildings is in the areas of the greatest transformer density, it is equally true that in some sparsely settled outlying districts with comparatively few transformers per mile there are some sections in which the screening is equal to the best found in the thickly populated areas. As we are unable to check Mr. Hayden's assumptions we cannot confirm his conclusions based on these assumptions.

Referring to Professor Bennett's remarks on the subject of shielding and to his criticism of conclusion 6, it is very difficult, if not impossible, to draw any accurate conclusions regarding the effects of shielding without an extensive and expensive survey of the city to rate the shielding in the various sections and to make proper allowance therefor in the tables and curves. Instances are very rare in Chicago where the shielding is anywhere near uniform for an entire section or square mile and in the locations where the shielding is fairly uniform over a considerable area these areas may lie in two or three of the sections as shown on the table because of the non-coincidence of the boundaries. We do know that the shielding effect of trees and buildings may be considerable and we also know that in the newer districts which are being built up within the city, in general, they are areas which are entirely barren of trees, so that a few years after such real estate subdivisions are opened the pole lines are the most conspicuous feature of the landscape. Instances of transformers being burned out by lightning and located where they were considerably overshadowed by trees and buildings are comparatively few, but per contra we occasionally find a transformer fuse blown by lightning and located under the shadow of a tall church steeple. In a recent storm also the primary fuse on a subway transformer was blown, apparently by lightning, as it occurred during a lightning storm and no other trouble was found.

We have attempted, however, to get a rough quantitative idea of the effect of shielding by selecting 32 sections, that is about one-sixth of the total number, in which the shielding was known to be considerable, and then comparing the percentage of burnouts from these sections with the corresponding points on the curve in Fig. 12. This is the equivalent of attempting to draw a curve like Fig. 12 with only one-sixth of the points and so the results obtained in this manner would at best be only a very rough approximation. The ordinates of these 32 points, however, averaged about 82 per cent of the corresponding ordinates on the curve. It is obvious that if the curve for the shielded sections is below the curve in Fig. 12 the curve for the unshielded sections would be above that curve and the difference between the two new curves would show the effect of the shielding. The results from the 32 selected sections where the shielding is known to be rather high appear to warrant the conclusion that the effect of the shielding may account for as much as one-half of the protection, according to the degree of shielding, and that the remainder is due to the density of lightning arresters. Conclusion 6 appears to be entirely warranted by Chicago conditions because the outlying districts with the scattering transformers are in general entirely without protection except such as they receive from the arresters. It would not apply however, in some of the suburban districts beyond the city limits of Chicago, notably on the North Shore, where new real estate subdivision are being opened up in hardwood forests and where the clearing of the land for the purpose of providing

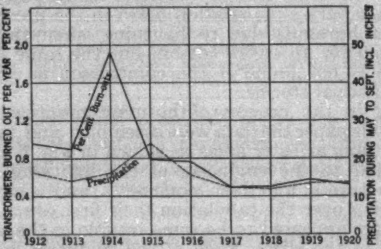

FIG. 9—DIAGRAM SHOWING FOR A PERIOD OF YEARS THE RELATION BETWEEN THE PERCENTAGE OF BURNOUTS DUE TO LIGHTNING AND THE PRECIPITATION DURING THE LIGHTNING SEASON

space for the building and the garden will supply the owner with enough cord-wood to last a year or two.

Dr. Steinmetz mentions the connection between rain formation and the potential differences which result in lightning. This phase of the subject was discussed at considerable length by Professor W. J. Humphries in a paper published in the *Journal* of the Franklin Institute for November, 1914. In this paper are given some data and curves showing the intimate relation between precipitation and damage by lightning. In Fig. 9 is plotted the percentage of burnouts in Chicago due to lightning and also the precipitation during the lightning season as obtained from the Weather Bureau records. This diagram appears to indicate a very close relation between the precipitation and the burnouts due to lightning when the figures are taken for an entire season, but the same intimate relation is not found if the comparison is carried to the individual storms.

Early in the progress of the investigations which preceded this paper the data were assembled and tabulated by lightning arrester areas as shown in Fig. 6 and then submitted to the engineers of the lightning arrester manufacturers for their comments and suggestions. In looking over the tabulation their first comment was that there appeared to be some trouble in the southern portion of the city. Each of the engineers desired to visit this section of the city having in mind that the cause of the trouble could probably be ascertained upon inspection. About the only result of the inspection was to note that the transformers were rather widely scattered and that there was very little shielding of the lines from overshadowing trees and buildings. It now appears that the reason that the lightning troubles are so numerous in this portion of the city is the low density of lightning arresters and the absence of any protection from trees and buildings, and that the amount of trouble is entirely explained by these two factors without recourse to the imaginary effect of such items as river valleys, hills or other geographical features of the surrounding country.

Presented at the 365th meeting of the American Institute of Electrical Engineers, Chicago, Ill., November 12, 1920.

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LIGHTNING ARRESTER SPARK GAPS—II

BY CHESTER T. ALLCUTT

Westinghouse Elec. & Mfg. Co., E. Pittsburgh, Pa.

This paper presents data giving the discharge characteristics of a commerical type of impulse gap under different conditions. Test data are also presented giving the characteristics of certain experimental gap structures designed to minimize the effects of adverse weather conditions, such as rain, fog, etc. A brief discussion of some of the factors that determine the degree of protection afforded by a lightning arrester spark gap is included in the paper. The term "protection factor" is defined and curves giving the protection factor of certain types of gap are presented.

IN a former paper presented before the American Institute of Electrical Engineers in June 1918,[1] the writer described a selective gap for lightning arresters and gave the results of a number of tests on an experimental form of such a gap. Since the presentation of that paper, this selective gap, now commonly known as the impulse gap, has gone into rather extensive commercial use and it is thought that some data concerning the electrical characteristics of the commercial form of gap might be of interest. The object of this paper is to present such data and to discuss some of the features upon which the protective value of the gap depends.

It is well-known that the danger to electrical apparatus due to line disturbances of steep wave-front may be all out of proportion to the actual voltage of the disturbances. This is because a voltage of high frequency or of steep wave-front does not distribute itself uniformly through an electrical winding, but tends to "pile up" on the end turns of the winding and thus greatly endanger the insulation between turns. Due to its high electrostatic capacity the electrolytic lightning arrester is particularly well adapted to discharge high-frequency disturbances. Unfortunately, the electrolytic arrester must be connected to a line through a spark gap, as permanent connection to the line will result in over-heating and ultimate destruction of the arrester. It is necessary that the spark gap which connects the arrester to the line be so adjusted that it will not discharge normal line voltage. It is highly desirable, however, that the spark gap should discharge high-frequency disturbances at the lowest possible voltage in order to obtain a high degree of protection against such dangerous disturbances. The value of the impulse gap lies in the fact that it is selective in its action, that is, it will discharge a high-frequency disturbance at a voltage considerably less than its 60-cycle discharge voltage. By means of this gap, an electrolytic arrester may be isolated from the line under normal conditions and yet is automatically connected to the line upon the occurrence of a high-frequency impulse even if the magnitude of this impulse is less than the 60-cycle setting of the gap.

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[1] C. T. Allcutt, "Lightning Arrester Spark Gaps." TRANSACTIONS, A. I. E. E., Volume XXXVII.

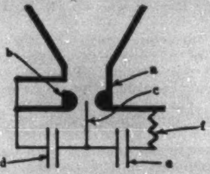

Fig. 1—Diagram of Impulse Gap

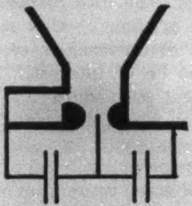

Fig. 2a—Impulse Gap on 60 Cycles

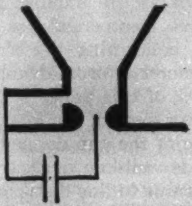

Fig. 2b—Impulse Gap with High Frequency Applied

Fig. 1 shows the electrical circuits involved in the simplest form of impulse gap. The gap proper consists of two sphere-horn electrodes *a* and *b*, which are connected respectively to the line and to an electrolytic arrester in the usual manner. In addition to these two main electrodes, an auxiliary electrode *c* is provided which is connected to one of the horns through a condenser *d* and to the other horn through a similar condenser *e* and a high resistance *f*. When a 60-cycle e. m. f. is applied across the gap the impedance of the resistance is negligible compared with that of the condensers, so the circuit is equivalent to that shown in Fig. 2A. In this figure, it will be seen that if the two condensers are equal, the potential of the auxiliary electrode will be half-way between the potentials of the two main electrodes. If the auxiliary electrode is placed half-way between the main electrodes, it will have practically no influence on the distribution of the electrostatic field between the two main electrodes, and consequently, the voltage required to produce a discharge across the gap will be substantially unaltered by the presence of the auxiliary electrode. If a high-frequency e. m. f., such as a line disturbance of steep wave-front, is impressed on the gap, conditions will be quite different from those existing when a 60-cycle e. m. f. is applied. In this latter case, the impedance offered by the resistance element is very much greater than the impedance of the condensers, so that the gap becomes very nearly equivalent to that shown in Fig. 2B. In this case it will be seen that if the capacity of the condenser is large compared with the capacity existing between the auxiliary electrode and the main electrode, practically the entire voltage will appear across half the gap, that is, between the auxiliary electrode and one of the main electrodes. A discharge across this half gap will immediately charge up the condenser to full voltage and a discharge across the other half of the gap will immediately follow. It becomes apparent from the foregoing that the discharge voltage of the gap should be much lower when a high-frequency e. m. f. is applied than when 60-cycle e. m. f. is applied. For example, if we assume that the separation between the main electrodes is, say, six cm., the gap will behave on a 60-cycle line in the same manner as an ordinary sphere horn gap having the same gap setting. When a high-frequency e. m. f. is applied to the gap, it will be very nearly equivalent to a needle gap having a setting of but three cm. As pointed out in the writer's former paper, it has actually been demonstrated by experiment that this construction does result in a much lower discharge voltage for high frequency than for 60 cycles.

Fig. 3 shows a standard 44,000-volt impulse gap. A rather exhaustive series of experiments was carried out on the gap shown in this photograph. The experiments involved tests on frequencies ranging from 60 cycles up to the highest frequencies that could be produced in the laboratory. Inasmuch as the other sizes of impulse gaps have characteristics very similar to those of the one shown, the writer is presenting here only the data obtained on this one size of gap, because the most complete series of tests was made on this size.

The gap shown in Fig. 3 differs somewhat from the experimental type described in the previous paper. The capacitances for maintaining the auxiliary electrode at the proper potentials are furnished by pin type insulators *a* and *b*. These insulators have an electrostatic capacity of approximately 2×10^{-11} farad. Due to the effect of capacity to ground, the auxiliary electrode is not maintained at a potential exactly half-way between the potentials of the main electrodes. It has been found by experiment that the proper position of the auxiliary electrode is such that its distance from the sphere horn *m* is approximately six-tenths of the gap length. In this position it does not materially alter the 60-cycle discharge voltage of the gap. The unbalancing resistance is furnished by the resistors *c* and *d* which are enclosed in porcelain tubes mounted on the porcelain pillar *e*. These resistors have about 250,000 ohms resistance each and are of a special composition that retains its high resistivity under the most severe conditions of high voltage and high frequency. The resistors are about 30 cm. in length. The auxiliary electrode *f* is a pointed brass rod 0.08 in. (0.2 cm.) in diameter held in a ⅜-in. (0.95-cm.) diameter brass support member *g*, which is in turn mounted on an arm *h* of 1-in. angle iron. The hemispheres *j* mounted on horns are made of brass and are 12.5 cm. in diameter. In practise the charging resistance of the arrester is connected between the horn members *k* and *l*. For the purpose of test, *k* and *l* were connected and grounded to the frame of the gap structure.

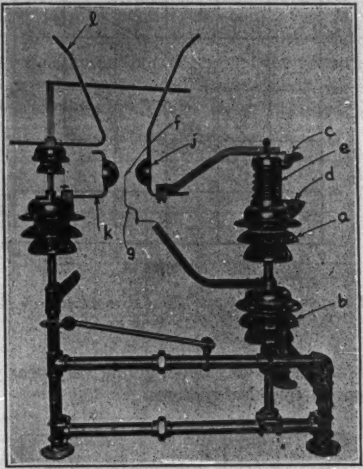

FIG. 3—44,000-VOLT IMPULSE GAP

The impulse discharge voltage of the gap was determined by a direct comparison with a 12.5-cm. sphere gap connected in parallel with it. The high-tension electrodes of the gap under test and of the sphere gap were connected by a straight lead and the connection

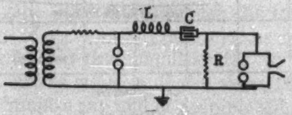

FIG. 4—IMPULSE GENERATOR CONNECTIONS

to the impulse generator was brought out from the middle point of this lead. The impulse generator itself was connected as shown in Fig. 4. The following circuit constants were employed, giving very nearly critically damped impulse voltages.

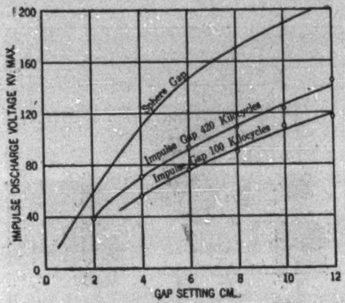

FIG. 5—IMPULSE DISCHARGE CHARACTERISTICS OF IMPULSE GAP COMPARED WITH SPHERE GAP

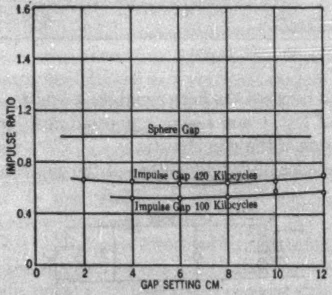

FIG. 6—IMPULSE RATIO OF IMPULSE GAP COMPARED WITH SPHERE GAP

For 100 kilocycle impulse:

$C = 5 \times 10^{-9}$ farad

$L = 1.25 \times 10^{-3}$ henry

$R = 1{,}000$ ohms.

For 420 kilocycles impulse:

$C = 5 \times 10^{-9}$ farad

$L = 8.3 \times 10^{-5}$ henry

$R = 320$ ohms

The condenser employed consisted of a stack of 100 impregnated paper condensers connected in series. Each condenser had a capacity of 0.5 microfarad. Single-layer inductances and water-tube resistors were used to complete the oscillating circuit. The test methods were identical with those outlined in the writer's previous paper.

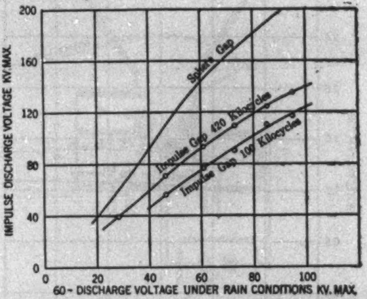

Fig. 7—Impulse Discharge—the 60-Cycle Discharge

Fig. 5 shows the impulse discharge voltage of the impulse gap plotted as a function of the gap setting. For purpose of comparison, a curve giving the impulse discharge voltage of a sphere gap is also shown. This curve also represents the 60-cycle discharge voltage of the impulse gap. The ratio between the impulse discharge voltage of a gap and its 60-cycle discharge voltage has been termed the "impulse ratio" by Mr. F. W. Peek.[2] The curves given in Fig. 6 show the impulse ratios of the impulse gap and of a simple sphere gap.

When a lightning arrester spark gap is placed out of doors, the impulse ratio is not the proper criterion of the protective value of the gap. The spark gap must be so adjusted that it will withstand normal line voltage under the most unfavorable conditions. The gap setting necessary in service is, therefore, determined by the 60-cycle discharge voltage under rain conditions, since, in general, the 60-cycle discharge voltage of a gap is considerably reduced by moisture on the surface of the electrodes, such as is caused by rain or fog. The impulse discharge voltage, on the other hand, is practically unaffected by rainfall. The degree of protection against high-frequency disturbances afforded by the gap therefore depends on the ratio of the impulse discharge voltage of the gap to the 60-cycle discharge voltage *under rain conditions.* The term "protection factor" has been proposed for this ratio.[3] For a spark gap located indoors or otherwise completely protected from the weather, the protection factor will be the same as the impulse ratio.

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[2] F. W. Peek, Jr., "The Effect of Transient Voltages on Dielectrics." Trans. A. I. E. E., Vol. XXXIV, 1915.

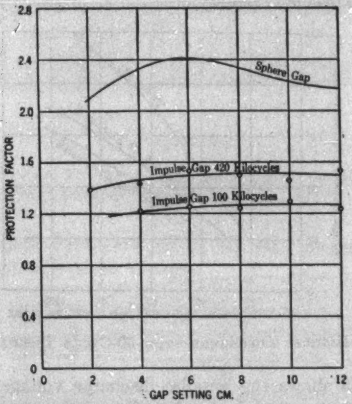

FIG. 8—PROTECTION FACTOR OF IMPULSE GAP COMPARED WITH SPHERE GAP

Fig. 7 shows the impulse discharge voltage of the impulse gap plotted as a function of the 60-cycle discharge voltage under rain conditions, while Fig. 8

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[3] C. T. Allcutt, Discussion of Mr. Peek's paper on "The Effect of Transient Voltages on Dielectrics—II." TRANS. A. I. E. E., Vol. XXXVIII, p. 1165.

gives the protection factor as a function of gap setting. A comparison of Figs. 6 and 8 shows that the degree of protection afforded by a gap is considerably lessened where the gap is exposed to rain or dew. It is at once apparent that it would be highly desirable to provide some means for protecting a lightning arrester gap from the objectionable effects of adverse weather conditions without going to the great expense of constructing a shelter sufficiently large to cover the entire horn gap structure.

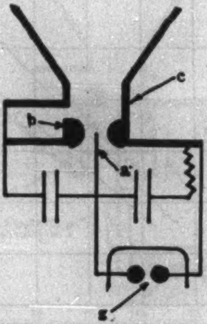

FIG. 9—PROTECTED IMPULSE GAP

In a recent paper presented before the Institute[4], Mr. F. W. Peek described a very ingenious structure designed to obviate the difficulties due to rainfall without the necessity of providing a shelter over the arc-breaking horns of the gap. This "double balanced sphere gap," as Mr. Peek called his structure, consists of two sphere gaps in series. One of these gaps is sheltered from the weather and the design is such that the discharge voltage of the two gaps in series is determined largely by the discharge voltage of this sheltered gap. The arc-breaking horns are located outside the shelter. For a better description, reference is made to Mr. Peek's paper.

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[4] F. W. Peek, Jr., "The Effect of Transient Voltages on Dielectrics—II." TRANS. A. I. E. E., Vol. XXXVIII p. 1137.

In his discussion of Mr. Peek's paper, the writer described a form of protected impulse gap that had previously been experimented with. This protected impulse gap is shown diagrammatically in Fig. 9. The gap structure shown in Fig. 3 was used for the main gap and a 6.25-cm. sphere gap *g* was connected between the auxiliary electrode and one of the main electrodes. The sphere gap *g*, shunting one-half of the main gap, may be sheltered from the weather while the main gap is exposed. The gap *g* is so set that it will discharge when the total voltage applied to the main gap equals the discharge voltage of the main gap under rain conditions. A discharge across *g* is followed immediately by a discharge between the auxiliary electrode *a* and the horn *b*. If *g* is so sheltered that its discharge voltage is independent of weather conditions, then the discharge voltage of the gap, as a whole, will also be substantially the same under all weather conditions. After a discharge has started across *g* and between *a* and *b* as described above, the rising arc quickly transfers from the auxiliary electrode *a* to the horn *c*, short-circuiting out the arc across *g* and effectively extinguishing it. This construction has been found to secure a very quick and

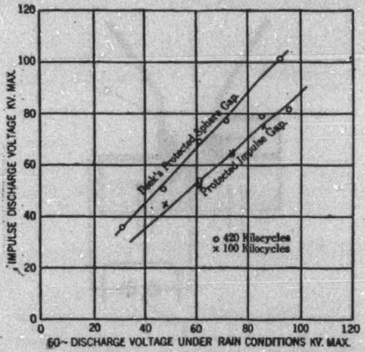

FIG. 10—DISCHARGE CHARACTERISTICS OF PROTECTED GAPS

effective suppression of the arc across *g*, thus permitting the use of a relatively small shelter.

Fig. 10 gives the impulse discharge voltage of Mr. Peek's double balanced sphere gap in comparison with the protected impulse gap described above. It is assumed in these curves that the shelter over the protected gap keeps the electrodes dry under all conditions. In the present state of development, however, there is some doubt as to the efficacy of these protected gap structures. It has been found that merely wetting the surfaces of the electrodes of a gap will reduce the discharge voltage to nearly as low a value as will actual rainfall. It is obvious that a shelter which merely protects a gap from direct rainfall does not necessarily prevent the formation of moisture on the electrodes in damp or foggy weather, and, unless the electrodes are kept dry under all conditions, the purpose of the shelter will be defeated.

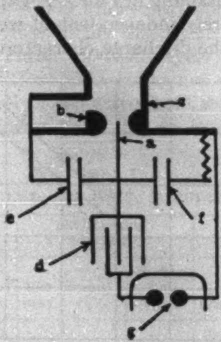

Fig. 11—Protected Impulse Gap

Another form of protected impulse gap that has been experimented with by the writer is shown in Fig. 11. In this case, the gap *g*, shunting one-half the main gap, is in series with a condenser *d* that is of somewhat higher capacity than the condensers *e* and *f*. In this case, a discharge across *g* will throw a relatively large capacity in shunt with the gap between *a* and *c* so that most of the voltage will appear between *a* and *b*. A discharge between *a* and *b* is, of course, immediately followed by a discharge between *a* and *c* and a free discharge of the disturbances is permitted between the main electrodes *b* and *c*. In this case, no current flows across the gap *g* except the charging current of the condenser *d*. The discharge across *g* is only a fine "pin spark" so that it may be found possible to enclose *g* in a hermetically sealed chamber of small dimensions containing dry air. A gap of this character should be entirely independent of weather conditions. Fig. 12 gives the discharge characteristics of this form of protected gap. In obtaining the data for this curve, the main gap structure shown in Fig. 2 was employed. One-half the gap was shunted by a 6.25-cm. sphere gap in series with a capacity of about 10^{-10} farad. At present, it is impossible to say whether a protected gap of this type will be commercially practicable. The writer hopes to have more information concerning the possibilities of this device available for presentation at some future date.

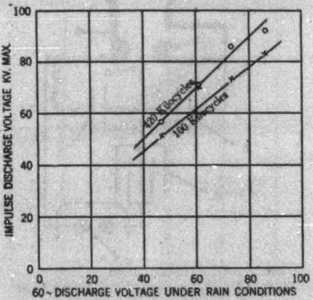

FIG. 12—PROTECTED IMPULSE GAP
See Fig. 11.

It will be noted that the test data presented in this paper give only the results of tests up to 420 kilocycles. Tests have been made in the laboratory at far higher frequencies, but it is not believed that these higher frequencies represent conditions that actually occur in a transmission line. In a recent paper presented before the Institute, Dr. C. P. Steinmetz[5] showed that an infinitely steep wave-front traveling along a transmission line will be reduced to a frequency of 1000 kilocycles after traveling about 300 yards. The 420-kilocycle wave-front used in these tests represents an infinitely steep wave after having traveled about one mile. Some curves showing the discharge characteristics of the impulse gap with extremely steep wave-fronts (3000 kilocycles) applied were presented by the writer in his discussion of Mr. Peek's recent paper. These 3000-kilocycle tests, however, represented conditions that could not occur in practise, as an infinitely steep wave-front would be reduced to a frequency lower than this in less than 100 feet.

TABLE I

STANDARD 44-KV. IMPULSE GAP

Gap setting cm.	60 ~ Discharge		100-kilocycle impulse			420-kilocycle impulse		
	Dry kv. max.	Rain kv. max.	Kv. max.	Impulse ratio	Protection factor	Kv. max.	Impulse ratio	Protection factor
2	59.5	28.5				39.5	0.66	1.39
4	109	46.5	56.5	0.52	1.22	71	0.65	1.43
6	147	61	76.5	0.52	1.25	93.5	0.635	1.53
8	170	73	90.5	0.53	1.24	109	0.64	1.50
10	190	85.5	110	0.58	1.29	124	0.65	1.45
12	207	95	117	0.565	1.23	145	0.7	1.53

TABLE II

PROTECTED IMPULSE GAP SHOWN IN FIG. 9.

Gap setting cm.	Aux. gap setting cm.	60-cycle discharge rain	100-kilocycle impulse		420-kilocycle impulse	
			Kv. max	Protection factor	Kv. max.	Protection factor
4	0.87	46.5	45	0.97		
6	1.15	61	52	0.85	54	0.88
8	1.4	73	65	0.89	64	0.88
10	1.7	85.5	75.5	0.88	79	0.92
12	2.0	95			81.5	0.86

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[5] C. P. Steimetz, "General Equations of the Electric Circuit." Trans., A. I. E. E., Vol. XXXVIII, 1919, Part I, page 191.

TABLE III

PROTECTED IMPULSE GAP SHOWN IN FIG. 11.

Gap setting cm.	Aux. gap setting cm.	60-cycle discharge rain	100-kilocycle impulse		420-kilocycle impulse	
			Kv. max	Protection factor	Kv. max.	Protection factor
4	0.87	46.5	51	1.09	56.5	1.21
6	1.15	61	68	1.12	71	1.16
8	1.4	73	73.5	1.01	86	1.18
10	1.7	85.5	83	0.97	92	1.08

Presented at the 365th meeting of the American Institute of Electrical Engineers, Chicago, Ill., November 12, 1920.

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LIFE AND PERFORMANCE TESTS OF O F LIGHTNING ARRESTERS

BY N. A. LOUGEE
General Electric Co.

I—LIFE RUN TESTS OF *O F* ARRESTERS

SINCE the first papers on the oxide film (*O F*) lightning arrester were given a little over two years ago,[1] the arrester has proved itself to be a worthy piece of apparatus by performance in regular service. Several hundred arresters up to 73-kv. rating, are now installed on both indoor and outdoor circuits, and higher voltage units will soon be in service. Figs. 1, 2, 3 and 4 show the typical designs used.

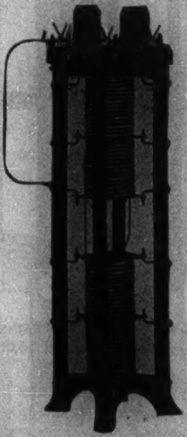

FIG. 1—OXIDE FILM LIGHTNING ARRESTER FOR INDOOR SERVICE ON THREE-PHASE CIRCUITS, 5000-7500 VOLTS

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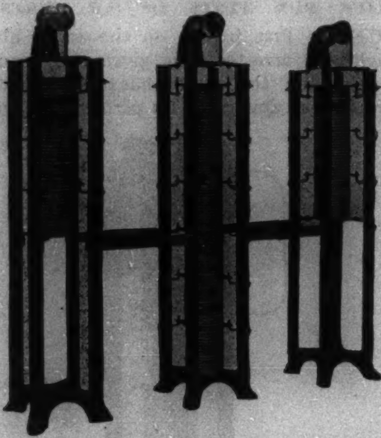
[1] *The O F Lightning Arrester*, TRANS., A. I. E. E., Vol. XXXVII, 1918.

In Fig. 1, the three phase legs and the ground leg are all arranged in one stack, the bottom section being the ground leg. In Fig. 2, the three phase legs are the upper sections and the ground leg is the lower section. In Figs. 3 and 4, the three phase legs and ground leg are set up parallel to one another. Fig. 5 shows the covered sphere gap used with the outdoor design, which permits of an indoor setting. Due to the small leakage current of these arresters (about 0.010 ampere), it is not necessary to use horn gaps to aid in breaking the arc, and it is, therefore, possible to use the covered sphere gap which has previously been described in the Transactions of the Institute.[2] Fig. 6 shows the testing device used and its method of operation, about which more will be said a little later.



IG. 2—Oxide Film Lightning Arrester for Indoor Service on Three-Phase Circuits, 15,000-25,000 Volts

The life of a lightning arrester is a very important factor, and one that has to be estimated from both operating and laboratory data. Operating data ob-

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[2] *The Effect of Transient Voltages on Dielectrics— II*, F. W. Peek, Jr., Trans., A. I. E. E., Vol. XXXVIII.

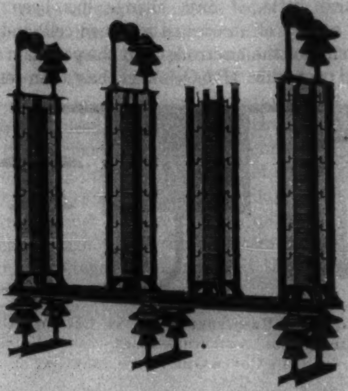

Fig. 3—Oxide Film Lightning Arrester for Indoor Service on Three-Phase Circuits, 37,000-50,000 Volts

tained during the past five years show that little deterioration has occurred to the *O F* cells. Cells have been returned from typical installations and

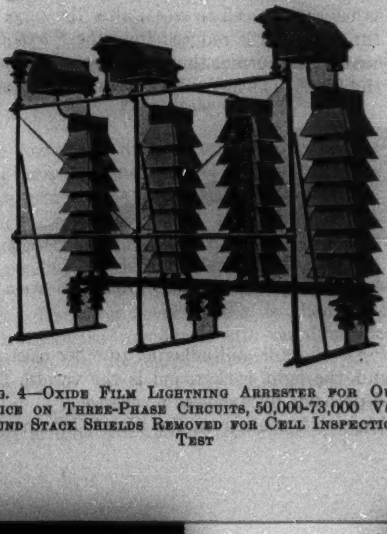

Fig. 4—Oxide Film Lightning Arrester for Outdoor Service on Three-Phase Circuits, 50,000-73,000 Volts.—Ground Stack Shields Removed for Cell Inspection and Test

tested and little, if any, change has been found. Fig. 7 is a view of an opened returned cell, and shows the film side of the electrodes and the porcelain spacer. The lead peroxide (PbO_2) filler has been removed.

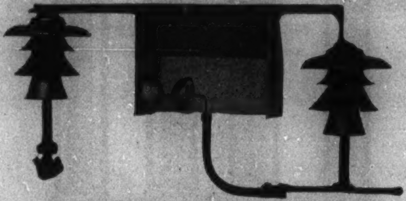

Fig. 5—Covered Hemisphere Gap as Used on Outdoor Type Oxide Film Arrester—50,000-73,000 Volts—Section of Cover Omitted to Show Gap

This cell was returned recently from a 13,000-volt arrester installed early in 1916, and which has been subjected to much more than average service, due to its location and surroundings. The few white spots in the illustration are discharge areas covered with yellow litharge (PbO). This PbO area or plug is what has caused the cell to reseal after the surge has passed through, and is reduced from the PbO_2 filler by the heat of the current through the small discharge spot in the film. The larger dark areas are where some of the PbO_2 filler is still adhering to other discharge areas, and the light background is the varnish film. The lead peroxide filler showed no change.

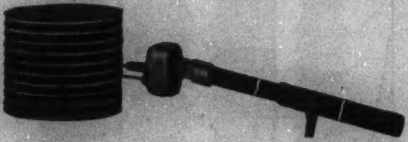

Fig. 6—Oxide Film Cell Testing Device in Position for Testing

To obtain information on the life of *O F* arresters several years ahead of outside reports, however, an intensive test has been running during the past few years. Fig. 8 gives the general scheme of circuit used.

In Fig. 8, the surge circuit is shown to the right and consists of the usual inductance, capacitance and air gap, used to obtain oscillations. The 50,000-volt transformer charges the condensers, which, upon breaking down the air gap set for a little under 50,000 volts, cause the surge through the arresters. The transformer to the left supplies the dynamic 60-cycle voltage to all the arresters running on this particular voltage. Ordinarily all the lever switches are down (Fig. 8 shows only one particular voltage, one arrester and one set of switches. With this arrangement the arresters are separate from the surge circuit. When it is desired to surge any one particular arrester, the upper lever switch corresponding to this arrester is thrown, thus paralleling the two transformers supplying dynamic voltage. The lower lever switch is then opened and this particular arrester is still on dynamic voltage, but also on the surge circuit which can now be thrown on. After surging, this arrester is thrown back on the regular dynamic transformer, and the next arrester put through a similar operation. This

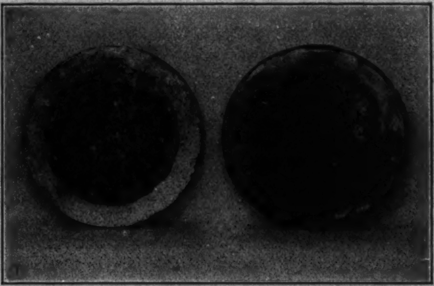

FIG. 7—INSIDE OF ELECTRODES OF *O F* LIGHTNING ARRESTER CELL RETURNED FROM 13,000-VOLT INSTALLATION AFTER FOUR YEARS OF SERVICE

arrangement of transformers and switches permits the regular dynamic voltage to all the arrester to be uninterrupted during surging operations.

O F arresters were placed on 330, 2300 and 12,700 volts respectively at 60 cycles with no series gap, and all arranged as shown in Fig. 8. These arresters have been surged daily, the surge current through the arresters having a maximum peak value of about 50 amperes and dying down to about 20 amperes at the end. This surge, having a surge impedance of 370 ohms is representative of an actual surge on a line, except that the average actual surge has a higher frequency and may at times be more powerful. It has been found, however, that the lower the frequency of a surge, the more difficult it is for an arrester to seal.

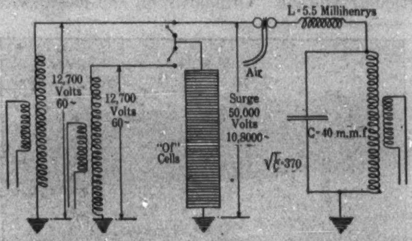

FIG. 8—CIRCUIT CONNECTION USED IN INTENSIVE LIFE RUN TESTS

330-Volt Circuit—Single O F Cells.

These cells take from 5 to 75 milliamperes leakage current and run at a temperature of about 50 deg. cent. It took about four years to record a failure with these cells. A failure then occurred by enough of the PbO_2 being reduced to cause high internal resistance and hence loss of protection. The voltage across the cells being, of course, always the same, causes this group of cells to be more permanent than the 2300- and 12,700-volt arresters. With these latter arresters the voltage distribution across the various cells may change. This adds one more variable to the action of arresters consisting of more than one cell.

2300-Volt Circuit—Eight O F Cells in Series.

These arresters so far have acted about like the single cells; that is, voltage distribution has remained normal. Voltage distribution is obtained by means of shunting vacuum tubes, which break down or glow at various voltages, across each cell in turn. It is the same idea used to test the cells in service as shown in Fig. 6. For service conditions a vacuum tube which will glow at about 1000 volts a-c. is used. As the internal condition of a cell changes, and more particularly the film, the voltage drop when in series with a number of cells may change. Although this is not an infallible method of picking out poor cells in service, it does give a reliable indication in most cases. For voltage distribution tests, tubes breaking down between 100 and 2300 volts a-c. respectively are used. For convenience in interpolating these data, a cell having a voltage drop of less than 200 is designated low, from 200 to 400 normal, from 400 to 600 high, and above 600 very high. The results can then be plotted against the respective cells by using a different color for each of the above four groups. The units on 2300 volts have shown with one or two exceptions only normal cells on voltage distribution, and the few low or high cells which have appeared from time to time, have returned again to normal. The leakage current of this group of arresters varies between 1 and 10 milliamperes, and the cells run at a temperature of about 40 deg. cent. A few units have failed or lost their protection after four years of continuous service.

12,700-Volt Circuit—Forty-seven O F Cells in Series.

These arresters have been running almost two years with no appreciable deterioration. To obtain the relative effect of dynamic and surge, similar arresters were run with different service characteristics, as follows: (a) dynamic only, (b) dynamic and surge, (c) surge only and (d) idle. The leakage current is from 5 to 10 milliamperes and about the same through all the arresters. The temperature is about 35 deg. cent. at the top of the stack, 45 deg. cent. in the middle and 30 deg. cent. at the bottom of the stack. Results to date show that

(a) and (b) types of arresters give about the same characteristics; that is, the daily surge has no ill effect on the arresters. Both (a) and (b) show a gradual tendency for low-voltage cells to appear at the bottom of the stack and higher voltage cells at the top. Here again no change has been found to be absolute; that is, unless a cell is extremely high, it may go from low to high and back again. The low cells at the bottom of a stack may be due to either capacity or temperature, but probably the latter, as all the cells are about normal when first put on the circuit. The (c) and (d) types of cells show a general scattering of high and low cells throughout the stack.

This sort of intensive test has been found extremely valuable in trying to determine ahead of time what might occur in service and also for determining the effect of changes. So far as applying to standard arresters in service, it seems fair to assume that if an arrester will stand, say four years, under such an intensive test, it will stand up several times four years in actual service. Of course, it is always possible that a more or less direct lightning stroke or a long arcing ground will destroy an arrester, so this conclusion should apply to normal average service. As yet the factor to use between test and actual service is not known, but should be when longer service results are available.

II – PERFORMANCE OF *O F* ARRESTERS

The efficiency of a lightning arrester is governed by four factors; namely, sensitiveness, current discharge capacity, reseal and life.

Sensitiveness. As most electrical apparatus is tested at twice normal voltage, an arrester should be able to begin discharging at about this voltage. This means a horn or sphere gap should not be set for over double voltage for best results.

To care for steep wave impulses the time lag of the arrester should be a minimum.

Current Discharge Capacity. To discharge the energy from a surge, the discharge path must be of a sufficiently low resistance to prevent the voltage drop being above the insulation strength of the apparatus connected to the line. Again since a double voltage test is given apparatus, the discharge capacity of an arrester is usually given at double rated voltage.

Reseal. Reseal is the act of cutting off the discharge path through the arrester when the voltage across the arrester has returned to normal. The quicker this can be accomplished the better it will be, for if an arrester has sufficient discharge capacity, dynamic or line frequency current following, not only will be apt to destroy the arrester but may also cause bad disturbances on the line.

Reseal should also permit an arrester to be ready immediately for another discharge, for with a lightning storm over a large area of transmission lines, it is fair to assume that impulses and surges can occur extremely close together; that is, at least a second apart, and sometimes several per second.

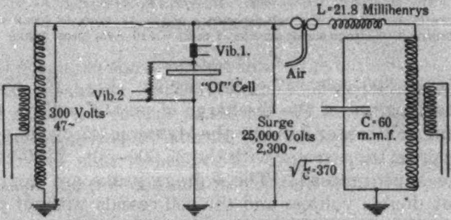

FIG. 9—CIRCUIT CONNECTION USED FOR SURGE TESTS

Life. It is difficult to define exactly whatthe life of a satisfactory arrester should be, but a good arrester should easily withstand the average surge or impulse. Arcing grounds are the most dangerous type of discharges and as they vary greatly in severity, depending upon the system and just where they occur, it is difficult to state how long an arrester should care for one.

Tests. The following results are given to show how the *O F* arrester acts in regard to the above points. A single cell was used in all the following tests in order to obtain as powerful discharges through the cell as possible with the power available.

The first set of tests was made with a circuit as shown in Fig. 9. The usual surge circuit is used, which superimposes the 25,000-volt, 2300-cycle surge on the dynamic 300-volt, 47-cycle circuit. Fig. 10 shows an oscillogram of the discharge of an *O F* cell on this circuit. Vibrator 2 shows the dynamic 47-cycle voltage across the arrester with the 25,000-volt, 2300-cycle surge superimposed. The voltage peaks are kept at about double voltage and the cell reseals without permitting any dynamic current to follow; that is, this test shows that *reseal* and *sensitiveness* are satisfactory. Although the discharge through the cell is about 50 amperes, since the surge is supplied by a 15-kv-a. trans-

Fig. 10—*O F* Cell on Circuit in Fig. 9
Vibrator 1—Current through arrester, 1 mm. = 12. amperes (peak value)
Vibrator 2—Voltage across arrester, 1 mm. = 210 volts (peak value)

Fig. 11—*O F* Cell on a 600-Volt, 40-Cycle Circuit
Vibrator 1—Current through arrester, 1 mm. = 250 amperes.
Vibrator 2—Voltage across arrester, 1 mm. = 55 volts.

former, this test is not enough in itself to demonstrate that the discharge capacity is satisfactory.

Fig. 11 shows an oscillogram taken with 600 volts, 40 cycles (double standard voltage) impressed across an *O F* cell, to show current discharge capacity. The current peaks are 3500, 4200 and 3300 amperes respectively, and the voltage peaks 110, 89 and 154 volts respectively, giving an internal resistance of 0.031, 0.021 and 0.047 ohm respectively. Due to the low resistance of the *O F* cell and its relative value to the impedance of the circuit, the impressed voltage of 600 was not sustained across the cell when the high current flowed. This *current discharge capacity* is extremely high and should be ample under all conditions. The internal resistance of a cell will vary between 0.01 and

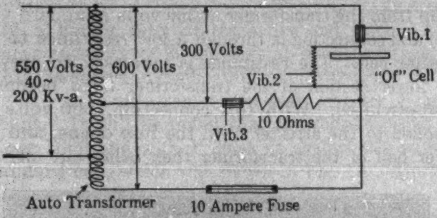

FIG. 12—CIRCUIT CONNECTION USED FOR DOUBLE-VOLTAGE SURGE TESTS

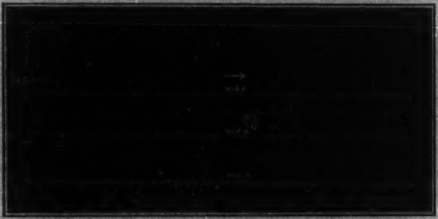

FIG. 13—*O F* CELL ON CIRCUIT IN FIG. 12

Vibrator 1—Current through arrester at 600 volts, 1 mm. = 250 amperes
Vibrator 2—Voltage across arrester, 1 mm. = 55 volts.
Vibrator 3—Current through arrester at 300 volts, 1 mm. = 12 amperes.

0.1 ohm, depending upon the particular path through the cell the discharge happens to pick out.

Fig. 12 gives the connection used for a double voltage surge test with normal voltage immediately following. This is accomplished as shown by bringing out a tap from the transformer at 300 volts (standard voltage) and connecting it through a low resistance to the arrester cell. The resistance is necessary to prevent the lower section of the transformer from becoming short-circuited. With this connection, 600 volts are supplied to the arrester until the fuse opens, and the lower half of the transformer then being cut off, 300 volts are continued across the arrester cell. This is about the most severe test that can be given a lightning arrester and only an arrester which has a *low breakdown, good current discharge capacity* and *good sealing characteristics*, will act satisfactorily. Referring to the oscillogram taken on this circuit shown in Fig. 13, the switch impressing 600 volts across the cell closed at the extreme right. The cell immediately broke down and discharged 2700 amperes. This current after one-half cycle blew the 10-ampere fuse, thereby cutting off one-half of the transformer, and causing the voltage across the cell to drop to 300 or normal. There was then a sealing current of about 2 amperes for several cycles shown by vibrator 3, which caused the small breaks in the voltage wave. After a few seconds the current through the cell had dropped to normal or a few milliamperes.

FIG. 14—*X* ARRESTER ON CIRCUIT IN FIG. 9
Vibrator 1—Current through arrester, 1 mm. = 12 amperes (peak value)
Vibrator 2—Voltage across arrester, 1 mm. = 210 volts (peak value).

To show the relation of protection and current discharge capacity, oscillograms were taken of single *O F* cells with external resistance in series on the circuit shown in Fig. 9. *X* represents an arrester with a medium internal resistance and having a discharge capacity at double voltage of 60 amperes. *Y* represents an arrester with a higher internal resistance and having a discharge capacity of 20 amperes at double voltage.

Fig. 14 shows an oscillogram taken with arrester *X* and Fig. 15 an oscillogram taken with arrester *Y* on this circuit. It will be noted that the voltage peaks with *X* are 1600 and with *Y* 3650, as against 900 with the standard cell, which was shown in Fig. 10. Moreover if the frequency were nearer what is obtained in actual service, that is, from 10,000 to 100,000 cycles instead of 2300 cycles which had to be used for oscillographic work, this difference would have been much greater due to the higher impedance of the transformer at the higher frequencies. To give, therefore, satisfactory protection, an arrester must have a good current discharge capacity on double voltage and more than these *X* and *Y* arresters show. *X* and *Y* also show the bad effect of a poor ground connection.

FIG. 15—*Y* ARRESTER ON CIRCUIT IN FIG. 9

Vibrator 1—Current through arrester, 1 mm. = 12 amperes (peak value)
Vibrator 2—Voltage across arrester, 1 mm. = 210 volts (peak value).

The *life* of an *O F* arrester was discussed in Part I, Life Run Tests on *O F* Arresters, and is believed to be satisfactory.

Sensitiveness in service is limited by the gap setting, but since no dynamic current follows a surge discharge and the leakage current is only a few milliamperes, this gap setting can be small. The gap settings used in service correspond to line voltage, so the breakdown between phases is double voltage and the breakdown to ground is 1.7 times the voltage to ground. Since the covered gap is used for outdoor installations a dry or indoor setting can be used.

The author wishes to express his appreciation to Mr. E. E. Burger for his valuable assistance in obtaining the data used in this paper.

Presented at the 365th Meeting of the American Institute of Electrical Engineers, Chicago, Ill., November 12, 1920.

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ELECTROSTATIC CONDENSERS

BY V. E. GOODWIN

General Electric Co., Pittsfield, Mass.

IN studying the progress of the electrical art one is continually confronted with the fundamental factors in Ohm's law—namely, volts, amperes, resistance, inductance and capacity. Of all these factors, the least practical use is made of the latter, and one is naturally interested to learn the reason for this apparent neglect when there are so many applications for this factor.

A review of the proceedings of the A. I. E. E. shows many references to the subject of electrostatic capacity or capacitance, but one finds only two papers on the subject of condensers, and one of these appeared over twenty years ago.

The purpose of this paper is to stimulate a more active interest in the development and application of electrostatic condensers. It is well-known that the electric condenser is one of the oldest electrical devices, and the thought naturally comes to mind as to why there has been so little progress made in its development during the past decade or two, during which there have been such rapid strides in other lines of the industry.

The author believes that a review of the subject at this time will tend to stimulate interest and promote a more rapid development in this branch of the art.

The first part of this paper will be confined to fundamental characteristics of condensers and their relations to electric circuits containing inductance and resistance, and includes a discussion of the effects of switching condensers on and off such circuits. The second part of the paper describes some of the more important applications of condensers and illustrates the great demand there is in the electrical industry for this class of apparatus.

SECTION I

An electrostatic condenser acts as a non-conductor or a very high resistance to a constant direct voltage and as a conductor while the voltage is varying. When voltage is applied and current flows, the condenser becomes a source of counter e. m. f. opposing the applied voltage, making the sum of the voltages in the circuit equal to zero. A condenser takes sufficient current so that its counter e. m. f. plus the counter e. m. f. of the series inductance and resistance is equal to the total applied voltage. Similarly, when a charged condenser discharges through a resistance and inductance, the applied voltage of the condenser is equal to the counter e. m. f. of the inductance and resistance. The characteristics of a condenser circuit are thus more or less dependent upon the characteristics of the series resistance and inductance. The above statements apply in general to all electrical insulation. Insulation is used to separate one conductor from another and therefore always forms part of a condenser.

Electrical energy can be stored in two forms, electrostatic and electromagnetic. In condensers energy is stored in the electrostatic form. In inductances energy is stored in the electromagnetic form. Electric energy in process of transfer from one point to another in an electric circuit always exists in both electrostatic and electromagnetic forms.

Both potential and current are necessary for the transmission of electrical energy. Thus, in the condenser, current must flow in order that electrostatic energy may be stored. Potential must exist across an inductance while the current and stored electromagnetic energy are increasing or decreasing. The rate (in watts or joules per sec.) at which energy is passing into or out of a condenser or inductance is equal to the product of volts times amperes.

A condenser stores energy while its voltage is increasing and gives out energy while its voltage is decreasing. An inductance stores energy while the current is increasing and gives out energy while the current is decreasing.

The fundamental equations for a condenser (using instantaneous values) are:

Quantity in coulombs $q = C\,e$ (farads × volts)

Current in amperes $i = C\dfrac{d\,e}{d\,t}$

Power in watts $p = e\,i$

Energy in joules or watt seconds $w = \dfrac{C\,e^2}{2}$

The fundamental equations for an inductance are:

Inductance in henrys $= L$

Potential in volts $e = L\,\dfrac{d\,i}{d\,t}$

Power in watts $p = e\,i$

Energy in joules or watt seconds $w = \dfrac{L\,i^2}{2}$

Oscillation Circuit. When a charged condenser discharges through an inductance the current at any instant is such that the inductance voltage is equal and opposite to the condenser voltage. The current starts at zero and can be zero only when the condenser is fully charged. The current is proportional to the rate of change of condenser voltage. The condenser voltage is equal and opposite to the inductance voltage. The inductance voltage is proportional to the rate of change of current. These conditions can only be met by a current varying according to a sine (or cosine) wave, the current lagging a quarter cycle behind the condenser voltage and leading a quarter ahead of the inductance voltage. The frequency of the sine wave oscillation is determined by the relations.

$$e = L\,\frac{d\,i}{d\,t} = \frac{1}{C}\int i\,d\,t$$

where $i = I \sin 2\,\pi\,f\,t$

$$\therefore f = \frac{1}{2\,\pi\,\sqrt{L\,C}}$$

The energy then oscillates between the condenser and inductance, being all in the condenser when the current is zero and the voltage maximum, and all in the inductance when the current is maximum and the voltage zero. With the condenser initially charged to a given maximum voltage E, the maximum value of current during discharge is then determined by the relation:

$$1/2\, C\, E^2 = 1/2\, L\, I^2$$

$$\therefore\ I = \frac{E}{\sqrt{L/C}}$$

Such a circuit is called an oscillation circuit. Since there is no loss of energy (except that due to resistance, radiation etc.), it is possible to build up a large oscillation by the addition of a large number of small amounts of energy. For example, a voltage may be induced in the inductance while it is giving energy to the condenser. The building up of an oscillation by a number of large or small increments of energy received at the proper instants is called resonance.

If the oscillation circuit contains a small resistance the oscillation is damped or decreasing in amplitude. The condenser voltage is equal to the inductance voltage plus the resistance voltage. The conditions are then satisfied by a current wave which is the product of a sine curve by a hyperbolic curve.

That is $i = I\, e - \frac{R\,C}{2\,L}\, t \sin \frac{1}{2\,\pi} \sqrt{\frac{1}{L\,C} - \frac{R^2}{4\,L^2}}\; t$

This equation does not hold and becomes imaginary if R is equal to or greater than $\sqrt{\frac{4\,L}{C}}$

If R is equal to or greater than $\sqrt{\frac{4\,L}{C}}$ the total energy is dissipated in the resistance during one rise and fall of current and there is no oscillation.

If the circuit contains resistance and negligible inductance the current starts at a maximum and decreases to zero along a hyperbolic curve. $i = I\, e^{-CRt}$

Condenser Connected to D-C. Circuit. When a condenser is switched onto a direct-current circuit containing a resistance R and a negligible amount of inductance, the current transient starts at a maximum and decreases to zero along a hyperbolic curve. If there is inductance in the circuit the current starts at zero and there must be at least one rise and fall of current. If the resistance is less than the critical value $\sqrt{\frac{4L}{C}}$, there will be a damped oscillation of current. If it were possible to have zero resistance, the oscillation would be continuous or undamped.

Condenser Connected to an A-C. Circuit. There are always more or less of current and voltage transients when the condenser is switched on. Any rise of condenser voltage above normal is due to series inductance. If the inductance is relatively small, the rise of voltage will be small as there will not be enough energy stored in the inductance to charge the condenser much above normal. If the inductance is relatively small, the current transient will be much larger (in per cent of normal) than the voltage transient. The energy of the transients is dissipated in the series resistance so that the current and voltage soon come to their normal values.

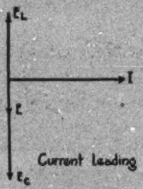


FIG. 1

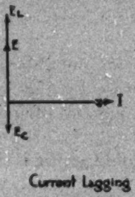


FIG. 2

The normal value of current is determined by the relation that the current is equal to the capacitance times the rate of change of voltage. The condenser current is proportional to and in phase with the rate of change of voltage. In the case of sine waves, the rate of change of voltage has the same wave shape as the voltage wave and a proportional maximum value, but passes through its maximum value a quarter cycle before the voltage wave. The rate of change of voltage at any instant is equal to $2\pi f$ times the voltage value which will exist a quarter cycle later. It is, therefore, permissible to say that the condenser current is proportional to the voltage but leads the voltage by a quarter cycle. This statement is true only for sine waves, but is generally used because commercial waves are approximately sine waves. It should be kept in mind that the fundamental fact is that condenser current at any instant is proportional to the rate of change of voltage at that instant.

When a condenser is in series with an inductance and the same sine-wave current passes through each, the inductance voltage is opposite in phase to the condenser voltage and their difference is equal to the total applied voltage.

These relations are shown in the diagrams Figs. 1 and 2.

It is thus possible to raise the voltage and current of a condenser by a series inductance or to raise the voltage and current of an inductance by a series condenser. The current is leading if the condensive reactance is greater and lagging if the inductive reactance is greater. This depends upon the frequency as well as the capacitance and inductance. If the inductive reactance is equal to the condensive reactance the current is limited only by resistance and other losses. This is the condition of resonance. With a given capacitance and inductance, resonance can be obtained by varying the frequency. At a given frequency, resonance can be obtained by varying the capacitance or inductance.

The condition for resonance is that $f = \dfrac{1}{2\pi\sqrt{LC}}$

When a condenser is connected in parallel with an inductance, the current and voltage relations are as shown in diagrams Figs. 3 and 4.

As in the series connection, the line current may be made lagging or leading by varying the inductance, capacitance, or frequency. When the inductive reactance is equal to the condensive reactance, the current is purely a circulating current in the parallel connection and no current is taken from the line. It is thus possible to reduce the line current to an inductance by connecting a condenser in multiple and vice versa.

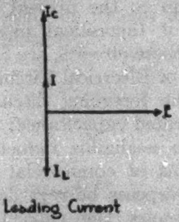


FIG. 3

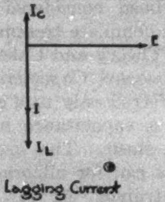


FIG. 4

In the case of complex circuits containing capacitance, inductance, and resistance there are an unlimited number of possible combinations and each must be considered individually as met with. One such circuit which is of considerable theoretical interest is that of a condenser and inductance of equal reactance in series and a resistance in multiple with either inductance or condenser as shown in Fig. 5.

The vector diagram is given in Fig. 6.

From this diagram it is evident that the voltage applied to the condenser can be divided into two vectors, one of which is equal to and opposite the voltage applied to the inductance and resistance, and the other is in phase with I_L and equal to $X_C I_R$. This

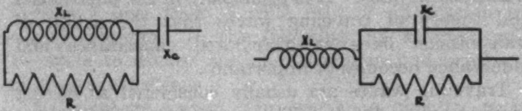


FIG. 5

latter component is the total applied voltage E for any value of R. Similar relations hold when the resistance is connected in multiple with the condenser. $\therefore I_R = E/X =$ constant for any value of R. This circuit, therefore, gives a means of obtaining constant current from constant sine-wave voltage. There are a number of other more complicated circuits giving the same results. These circuits have been known for many years but up to the present time have been considered only of theoretical interest. (For a complete treatment of these circuits, see Steinmetz' Theory and Calculation of Electrical Circuits.)

Distributed Capacitance. The foregoing discussion has referred only to a concentrated capacitance; that is, to a capacitance having a negligible inductance within itself. The usual forms of commercial condensers can for all practical purposes be regarded as concentrated. At low frequencies the capacitance of cables and transmission lines may be considered as made up of one or several concentrated capacitances. At high frequencies, transmission lines and cables must be treated as distributed capacitances and distributed inductances. High frequencies or high rates of change of voltage occur in the case of sudden changes of circuit conditions which set up traveling waves in transmission lines. It is principally in the phenomena and theory of traveling waves that the action of transmission lines as distributed capacitance and inductance becomes of importance.

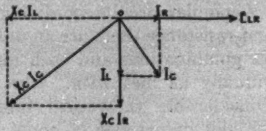


FIG. 6

Traveling waves are usually considered as having vertical wave fronts. This assumption is justified by the fact that it greatly simplifies calculations and because a vertical wave front is the limiting condition, and the effects calculated on that basis will mark the maximum limit of any effects which could occur in practise.

One of the fundamental characteristics of a traveling wave is that the total energy is equally divided between electrostatic and electromagnetic energy. This fact is expressed by the equation

$$1/2\,C\,E^2 = 1/2\,L\,I^2$$

This equation follows directly from the fundamental equations:

$$Q = C\,E = I\,t$$

$$\therefore \frac{C\,E^2}{2} = \frac{E\,I\,t}{2} = \frac{1}{2}\text{ total energy.}$$

$$\therefore \frac{L\,I^2}{2} = \text{other} \cdot \frac{1}{2} \quad \text{“} \quad \text{“}$$

It also follows directly from the above equations that

$$E = I\,\sqrt{L/C} = I\,Z$$

where Z = the "surge impedance."

Another fundamental characteristic of an electric wave is that the voltage is doubled when striking a point of total reflection. At reflection the length of the wave is reduced by one half and the energy is all electrostatic. The electrostatic energy per unit of length is thus four times as great and the voltage twice as great as in the traveling wave.

When a traveling wave strikes a point of short circuit, the current doubles and the voltage reduces to zero during reflection.

SECTION II

The characteristics of condensers as outlined in Section I give them a wide field of application in electric circuits. In the following, an attempt will be made to discuss the more common and practical of these applications.

1. *Telephone Service.* In telephone circuits, it is often necessary to confine the flow of direct current to certain sections of a network of conductors and at the same time to allow alternating and pulsating currents to flow as desired. This separation is accomplished by means of condensers connected in series with the line at the limits of the d-c. zones.

Thus it is possible to use alternating currents to call inactive stations and to separate the minute pulsating talking currents from the relatively heavy direct currents. The condenser in this service not only economizes in the amount of energy required, but also makes long distance telephony possible.

It is also interesting to mention the fact that specially constructed condensers can be used as telephone receivers. A condenser of this construction is known as the "singing" condenser as it will produce audible notes corresponding to the rates of change of the applied voltage impulses.

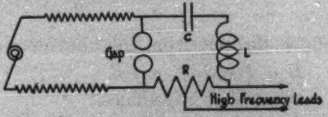


FIG. 7

2. *Condensers in Oscillating Circuits.* The condenser finds a wide application in oscillating circuits where it is desired to produce high-frequency voltages or to study the effects of impulses having steep wave fronts.

Oscillating circuits are of two general types, known as the oscillation generator and the oscillation transformer. Fig. 7 shows the diagrammatic arrangement of the oscillation generator.

The purpose of this machine is to produce an impulse or train of oscillations of definite wave form whenever the alternator voltage is raised sufficiently to spark over the gap *G*. These machines have been of great value in studying the action of exceedingly high rates of change in potential and in the study and determination of the dielectric spark lag of insulating materials.

Fig. 8 shows circuit connections of the oscillation transformer.

The application of this form of apparatus is in the production of recurrent oscillation at high frequency and voltage. By varying the adjustable inductance *L* and capacitance *C*, it is possible to obtain any desired frequency. By varying the ratio of the turns in the air transformer T, it is possible to obtain a wide variation in voltage of the high-frequency oscillation. This circuit is used extensively in wireless work, as will be described later, and in many forms of testing and laboratory work.

3. *Protection from Lightning and High-Frequency Disturbances.* The fact that a condenser stores energy as the voltage increases, and gives out energy as the voltage decreases, can be made use of in protecting electric circuits from abnormal disturbances having steep wave fronts. When a steep wave impulse reaches a point in the circuit where a suitable condenser is connected across the circuit, it delivers energy to the condenser depending upon the steepness of wave form or rate of change of potential. The amount of energy delivered depends upon the voltage of the impulse and the capacitance of the condenser. This tends to make the wave front more sloping and less dangerous and also tends to lower its maximum potential in cases where the crest of the wave is short. When the impulse passes, its potential starts to decrease in what is known as the rear, or tail of the wave. The condenser then gives its energy back into the impulse, modifying the shape of the rear of the wave by making it likewise more sloping.

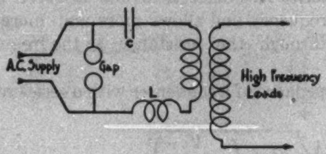


Fig. 8

Another and more common, application of condensers for protection purposes is in the absorption and consequent elimination of high frequency. This application is based upon the fact that the current flow in a condenser is proportional to the rate of change of potential and in the case of a recurrent oscillation to the frequency. The absorption is accomplished by inserting a resistance in series with the condenser. The value of resistance for maximum absorption with a given condenser is obtained when $R = \frac{1}{2\pi f C}$. The value of capacitance to be used should depend upon the nature and character of the oscillations which are to be handled by the machine. In actual practise these oscillations vary through a wide range in frequency and voltage. The actual designs of these machines employ a capacitance, for a given operating voltage, such as to allow a normal current from 0.05 to 0.10 ampere to flow at 60 cycles. Under normal conditions little energy is absorbed from the system. If, however, an abnormal recurrent disturbance is started by an arcing ground or other cause, the condenser, with its series resistance, acts like a valve operated by the frequency and allows more and more current to flow through the resistance as the frequency increases.

The impedance of a condenser with a series resistance is

$$Z = \sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}$$

Assuming $R = 100$ ohms, $C = 0.01$ microfarad
$f = 60$ cycles and $E = 13{,}200$ volts.
Then $Z = 265{,}000$ ohms.

At 13,200 volts the current at 60 cycles would be 0.05 ampere and the energy absorbed per second by the series resistance would be $0.05^2 \times 100 = 0.25$ watt-seconds.

If the frequency is 100,000 cycles,
Then $Z = 188$ ohms.

At 13,200 volts the current at 100,000 cycles would be 70.3 amperes and the energy absorbed per second $70.3^2 \times 100 = 494$ kilowatt-seconds.

From the above, it will be seen that at low frequencies the major part of the impedance is in the condenser while at 100,000 cycles the principal impedance is in the resistance R. The above calculations are based upon the resistance and capacitance remaining constant at all frequencies. This is not exactly true in practise as the capacitance of condensers and the conductance of resistance decreases as the frequency increases. This is due to the absorption factor of dielectrics and the skin effect and temperature coefficient of resistances. These are variable factors depending upon the shape and character of the materials used. With proper designs these factors should not have much effect at frequencies below 100,000 cycles.

Since many abnormal disturbances in electric circuits are of an oscillatory nature and have relatively little energy back of them, it would seem that an absorber of this character would prove an excellent protector, as its ability to absorb and consequently destroy oscillations increases so rapidly with the frequency and destructiveness of oscillations.

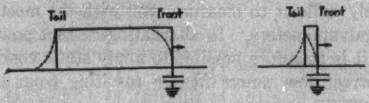


FIG. 9

Another use of condensers as a protective device is in connection with what has been termed a resonant shunt. With this apparatus the condenser is connected in series with an inductance which is tuned to resonance for a given frequency. Oftentimes in electric circuits, it is found that there is present a certain harmonic or superimposed oscillation of a definite frequency which is detrimental to the operation of other apparatus at some point in the circuit. By connecting a resonant shunt across the circuit and having it tuned to the particular frequency of the undesirable harmonic, it will provide a short-circuit path which will entirely eliminate this undesirable part of the wave form and leave the fundamental wave unchanged. This combination should have a wide field of application particularly in eliminating such harmonics as are liable to produce telephone interferences.

A further application of condensers as a protective device would be to shunt series coils on regulators, current transformers, etc. These series coils being connected in the line will subject them to high rises of potential between turns whenever a traveling oscillation passes through them. By providing a shunt path consisting of condensers of such capacity as to allow little current to pass at normal frequency, it would be possible to shunt the major part of the high frequency disturbance through the condenser and thus prevent a rise of potential between turns and between the terminals of the series coil and thus relieve the strain on the insulation.

4. *Power Factor Correction and Improvement in Line Regulation.* Static condensers have a power factor of approximately 0.003 leading and an energy of loss of less than 0.5 per cent. For practical purposes, we can consider the power factor as zero leading as it is extremely difficult to measure even with the most sensitive galvanometers. In alternating-current power service, it is therefore possible to apply static condensers to correct low power factors resulting from lagging currents.

Power factor corrections can be accomplished by either of two methods: First by retarding the voltage vector so that it becomes more nearly in phase with the current; and second, by advancing the current vector so that it becomes more nearly in phase with the voltage. The former scheme can be accomplished by the use of a suitable condenser in series with the load, and the latter by means of a condenser in shunt with the load.

The problem of power factor correction has been quite fully discussed before the Institute in connection with synchronous condensers so it will be unnecessary at this time to point out the advantages accruing therefrom. The electrostatic condenser has, however, certain advantages over the synchronous type for certain classes of work and should find a wider application when their adaptability for this service is more fully understood. The following are some of the advantages of the electrostatic condenser for power work:

1. Being a stationary apparatus, no attention is required.

2. The high efficiency of the outfit means that the annual power loss is very small.

3. With reliable condensers the maintenance costs are practically nil.

4. The corrective kv-a. can easily be increased or decreased by simply cutting in or out of sections.

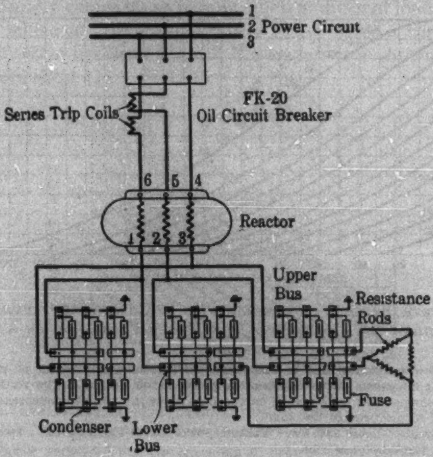

1
2 Power Circuit
3
FK-20
Oil Circuit Breaker
Series Trip Coils
6
5
4
Reactor
1
2
3
Upper
Bus
Resistance
Rods
Fuse
Condenser
Lower
Bus

Fig. 10

5. The outfit can be connected to any part of the system and thus provide correction at the points where the greatest saving can result.

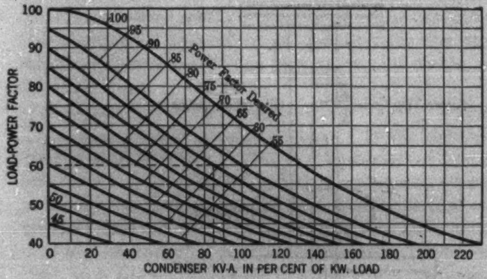
6. Noiseless and requires no special foundation.

7. Lower first cost for small installations.

8. It can also be used to improve the line regulation.

Fig 10 shows the diagrammatic connections of a static condenser used for power factor correction on a 2300-volt, three-phase circuit.

The condensers are connected in delta and each phase is divided into a number of sections each having a suitable enclosed fuse in series. A low reactance is connected in each phase between the circuit breaker and the condensers to act as a harmonic screen. This reactance can also be used to boost the voltage on the condensers so as to obtain greater corrective kv-a. with a given condenser. High resistances are connected in parallel with each phase of the condensers to discharge the condensers when they are disconnected from the line.



CURVE SHEET I—DETERMINATION OF CONDENSER CAPACITY REQUIRED FOR CORRECTING THE POWER FACTOR OF AN A-C. CIRCUIT

Follow horizontal line corresponding to present power factor of load until it intersects curve representing power factor desired. The vertical projection of this intersection on the base gives the size of condenser required in per cent of kw. load.

Example: Load 250 kw. Present power factor 60 per cent. Power factor desired 90 per cent.

Projection of intersection of 60 per cent power factor line with 90 per cent power factor curve gives desired condenser as 84.9 per cent of 250 kw. or 210 kv-a.

In studying the application of condensers for power factor work the author has found it more convenient to rate these units in terms of kv-a. at their rated voltage rather than in terms of capacitance.

If a sine wave of voltage E at a frequency f is impressed on a condenser section of capacity C farads, the current $I = 2\pi f G E$

The leading kv-a. is $E I \times 10^{-3}$.

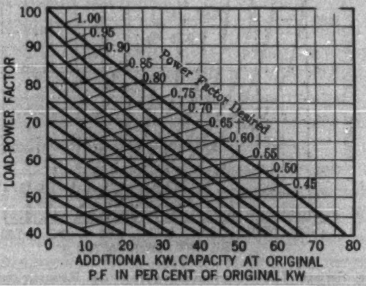
Hence, kv-a. $= 2\pi f C E^2 \times 10^{-3}$.

If C is expressed in terms of microfarads,

$$\text{kv-a.} = 2\pi f C E^2 \times 10^{-9}.$$

In the determination of the size of static condenser equipment required to correct the power factor of a given load, curve sheets I, II and III will prove of value. Curve sheet I gives the kv-a. correction required in percentage of the kw. load for any desired improvement in power factor from 40 per cent to 100 per cent.

In studying a problem of this character, the question naturally comes up as to how much additional load can be added at the original power factor after a certain correction has been added. These data are shown by curve sheet II.



Curve Sheet II—Determination of Additional Capacity Made Available by the Installation of Static Condensers

Follow horizontal line corresponding to present power factor of load until it intersects curve representing power factor desired. The vertical projection of this intersection on the base gives the per cent of the original kw. load at the original power factor made available by the condenser.

Example: Load 250 kw Present power factor 60 per cent. Power factor desired 90 per cent.

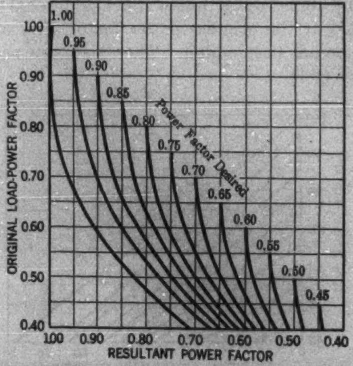
Projection of intersection of 60 per cent factor line with 90 per cent power factor curve gives kw. load at 60 per cent power factor made available as 35.8 per cent of 250 kw. or 89.5 kw.

Curve II indicates the amount of load at the original power factor that can be taken on a feeder, after correction has been added, without exceeding the initial heating in the lines and apparatus.

After the additional load indicated by Curve II has been taken on, it is desired to know the resulting power factor. These values are shown on curve sheet III. By the use of these three curve sheets, practically any information relative to loads and power factors can be quickly determined.

The static condenser is not necessarily a competitor of the synchronous condenser, since its principal application is in a field which has never been open to the latter machine.

During the past few years, there has been a gradual awakening throughout this country to the inefficiency of distribution networks resulting from low power factors. Operating companies, with a view to fairness to their customers, have added power factor clauses to their power contracts penalizing customers for low power factors. In other words, they aim to charge each cus-



Curve Sheet III—Determination of Resultant Power Factor After Original Load at Corrected Power Factor has Been Increased by Additional Load at Original Power Factor so that the Total Kv-a. of the Circuit is Equal to the Original Kv-a.

Follow horizontal line corresponding to original power factor until it intersects curve corresponding to the corrected power factor. The projection of this intersection on the base gives the resultant power factor.

Example: A load of 250 kw. having a power factor of 60 per cent is raised to 90 per cent power factor by the installation of a static condenser. A load of 89.5 kw. at 60 per cent power factor corresponding to the additional capacity made available by the correction in power factor is added so that the total kv-a. is equal to the original kv-a. The resultant power factor is obtained by projecting the intersection of the 60 per cent power factor line with the 90 per cent power factor curve upon the base and is found to be 81.5 per cent.

tomer for the delivery of his wattless or reactive load in addition to his energy load. This policy has opened up a new field for power factor correction which includes a large number of relatively small loads for which the stationary, or static condenser is particularly suited.

In the design of condenser sections the question of reliablity is of greatest importance. On the other hand, it is necessary to employ a dielectric of high permitivity. This dielectric must be very uniform throughout its entire volume. These conditions are very difficult to meet with very thin sheets even with modern insulations. It is therefore necessary to build up the dielectric of a condenser from several very thin sheets in order to obtain reliability. It has also been noted that as the number of sheets increased it becomes more difficult to radiate the heat. By studying these opposing factors for a given dielectric, we can arrive at a construction giving the most economic design. This gives a condenser which can be operated at any voltage up to its maximum reliable rating.

Since the kv-a. rating of a condenser increases as the square of the impressed voltage, operation at voltages less than normal results in reduced capacity. Hence, when considering a static condenser outfit for operating at voltages less than normal, it is often more economical to use a transformer to step up the voltage rather than to employ a greater number of condenser sections.

Another method of raising the voltage on a condenser is to install a suitable series reactance, but in doing so. care must be taken to avoid resonance.

5. *Electric Welding.* It is worthy of note in passing that reliable electric welding can be done with a circuit containing considerable capacitance. Preliminary work along this line indicates that equally good work can be done with circuits containing either inductance or capacitance, provided the power factor is of the correct low value. This low power factor is objectionable from the power supply standpoint. If, however, we divide our welding circuits into two groups, one using the condensive circuit and the other using the induc-

tive circuit, it is possible to obtain unity power factor in the supply circuit.

Another application of the condenser to electric welding is based upon the fact that it is possible to charge a condenser slowly and to discharge it at a high ampere rate. This scheme is used quite extensively where small wires are to be welded together. By this means a definite amount of energy can be put into a welding and very uniform results obtained.

6. *Motor Application and Contact Sparking.* As previously shown, the phase relationship of current to applied voltage depends upon the capacitance inductance and resistance in the receiving circuit. It is, therefore, possible with two or more receiving circuits to obtain two or more currents of different phase relationships from one supply circuit. This process is called phase splitting. These out-of-phase currents may be used to obtain starting torques or to change the running characteristics of single-phase motors. This method of starting gives better running efficiency than other methods for small single-phase motors such as fan motors, and there seem to be considerable possibilities for future developments in such uses for condensers.

A condenser may be used across the interrupter of an induction coil to eliminate sparking and consequent pitting of contacts and to increase the secondary voltages. The smaller the condenser, the greater the secondary voltage, provided there is no sparking at contacts. The smallest condenser which will eliminate sparking should be used. The formula for secondary voltage is

$$E_s = \frac{N_s}{N_p}\sqrt{\frac{L}{C}}\, I_b$$

where I_b = secondary current at break. In other cases condensers may be used merely to eliminate sparking.

7. *Laboratory Testing.* In the electrical testing laboratory, the uses for condensers may be divided into two general classes. The first class involves the condenser as a standard of capacity which may be either absolute, in which the capacity is capable of calculation from measurements of length, or compari-

son standard, which for convenience is used in the ordinary determination of the value of an unknown capacity. The usual comparison standard condensers employ either air or a very high grade of mica as dielectric. A well designed air condenser has the advantages of practically no absorption and the nearest approach to zero phase difference that can conveniently be obtained. It is, however, very bulky when a capacity of considerable magnitude is desired. The mica standard on the other hand, has an appreciable absorption and phase difference but can be constructed for capacities of several microfarads without becoming exceedingly bulky.

The second class of uses involves the condenser as an auxiliary piece of apparatus by the aid of which it is possible to obtain alternating currents of high frequency, to modify wave forms, and to duplicate and study in the laboratory many of the effects produced by atmospheric electric disturbances.

DISCUSSION ON "ELECTROSTATIC CONDENSERS" (GOODWIN), CHICAGO, ILL., NOVEMBER 12, 1920.

E. E. F. Creighton: I would like to discuss one phase of Mr. Goodwin's paper, namely, the use of condensers as high-frequency absorbers, just giving the earliest illustration of the application of the condensers in that form.

The high-frequency absorber is merely a condenser with a low resistance in series. Mr. Swan of the Narragansett Co. has a submarine cable that crosses an estuary. It is in a bad situation in that it is buried in the mud at each end, and is very difficult to repair. On that circuit at each end was installed every known form at that time of lightning arrester and choke coils, but in spite of that the cable broke down successively seven different times, and in desperation I told Mr. Swan we might try the high-frequency absorber. That was some considerable number of years ago. He put the absorber in at each end, and I regulated it for a low frequency. I had calculated from Dr. Steinmetz's formula the natural frequency of the circuit, and I judged there couldn't be over forty-thousand cycles per second. After that the cable broke down again. Then I decided that the mathematics might be wrong, and we might try the assumption that the frequencies were very high, so I readjusted it for a high-frequency discharge. That was many years ago, and since then there has not been a single break-down of the cable from any lightning troubles. They did have a break-down due to an anchor being dragged across it.

C. T. Allcutt: I would like to ask Mr. Goodwin what test is applied to the electrostatic condensers utilized for power factor correction? What is the nature of the over-voltage tests that are found necessary before putting them in service? Is it the same as other forms of electrical apparatus, or is it necessary to subject them to more severe tests in order to be sure they will operate without failure?

V. E. Goodwin: We give our condensers the standard A. I. E. E. voltage test between terminals and ground. This should not be applied between terminals since the kv-a. of the condenser increases as the square of the voltage and a double voltage would, therefore, produce four times or even more heating than normal.

Furthermore in the design of condensers, it is most economical to operate the dielectric at a high voltage stress per mil at the edges of the foil. A double voltage test is liable to produce corona at these edges and a one-minute test might injure the dielectric.

Hence, for the dielectric test we put the condensers on a heat run for several hours at normal voltage, reading the watt loss and temperature rise.

The other tests we apply are mainly tests of materials which go into the condensers. These, I believe, are the most important, as well as difficult, factors of making condensers. It also seems absolutely essential to have thorough laboratory supervision over any department that manufactures condensers.

C. T. Allcutt: A point in connection with the testing of condensers that might be of interest, is the analogy between the over-voltage tests of condensers and the over-potential tests on transformers. I don't mean the insulation tests to ground, but the over-voltage tests applied to the terminals of a transformer in order to test the insulation between turns. It is customary in transformer practise to make this test at a relatively high frequency in order not to have a very large magnetizing current flowing through the transformer. In this way it is possible to get a high voltage without a great over-load on the transformer copper.

A similar situation appears in over-voltage tests on a condenser with alternating current. The current taken by the condenser becomes very high and the heating from the internal losses becomes quite serious so it might be desirable in some cases to make the short time over-voltage tests on condensers, as distinguished from life tests, at either very low frequency or with direct current, and in this way get the high-voltage tests that may be necessary without danger of over heating the condenser.

V. E. Goodwin: We also make dielectric tests on certain parts of condensers. After they are made we take them apart and take certain sections of them and test those for dielectric strength at different points. We also give them high-frequency or energy loss test at different frequencies and different potentials.

C. T. Allcutt: About what power factor do they run?

V. E. Goodwin: The power factor of these condensers varies between 0.3 and 0.5 of a per cent for different designs. The energy losses in a good condenser are so small that they are quite difficult to measure. The operating temperature rise is very small also. We allow a maximum of 15 deg. cent. and prefer to keep this value down to between 5 deg. cent. and 10 deg. cent.

E. E. F. Creighton: Since there is an interest in that side of the condenser, the heating of it, a test that we made at one time, I think is apropos. We used slightly above a normal potential, 2600 volts being its normal potential and we applied a little over 3000 volts, but increased the frequency up to an extremely high value. I have forgotten exactly what it was, but I think something like probably half a million cycles per second. The current increased so greatly that it burned off the leads, but it didn't overheat the condenser.

H. R. Summerhayes: In my experience with a manufacturing company we have received at times some very unusual letters, and I remember quite well one letter from a man who had been operating some d-c. motors and then obtained induction motors, and he found that there was more current in the induction motors than in the d-c. motors. He wrote us that he had inquired of his electrician and was told there was idle current there. Said he thought no doubt by this time we had devised something that would set that idle current to work, and he wanted us to quote him on some devices that would make use of that idle current.

Now it occurs to me that in that static condenser we have something of that sort. We don't exactly set it to work, although we make it earn dividends,—we do as the bees do with it, if it is idle we kill it, we neutralize it, and we actually increase the output of a plant by adding a load to it. That is, we add a kv-a. load to increase its output in horse power, and to improve the voltage regulation, so it can frequently be shown that the static condenser is a good financial proposition, will frequently pay for itself in the course of a very few years.

There is something very fascinating about the idea of selling with each motor a little box that will kill the idle current. You can put in these condensers, about 5 or 10 kv-a. in a typewriter box, something about that size, so it is very compact. At first thought it would seem desirable to put a box right with each motor and so have the idle current neutralized at the source, where it would be the best place to neutralize it, but on account of the diversity factor and the fact that all the motors are not operating at one time, it generally works out best to put the condenser in somewhat larger units, at a load center.

V. E. Goodwin: When I reviewed the subject originally the thought came to me that there didn't seem to be any definite or concise name for this class of apparatus. Some writers called them "electrostatic condensers." Others called them "electric condensers." Still others mentioned them as "static condensers," while others were satisfied with common plain "condensers," and it seems to me that it might be well for the Standards Committee to take up this question of obtaining a descriptive and still a concise name by which we could differentiate between these condensers and steam condensers and a dozen other kinds of condensers which we have.

Presented at New York Section Meeting,
American Institute of Electrical Engineers,
March 26, 1920.

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GASEOUS CONDUCTION LIGHT FROM LOW-VOLTAGE CIRCUITS

BY D. MCFARLAN MOORE
Moore Light Dept., General Electric Co.

THE production of artificial light is one of the most important activities concerning the welfare of humanity. It is a very large subject, since both its practical and theoretical aspects cover vast fields. Yet, there are less than a dozen distinct methods of making light artificially, and some of them are not developed commercially, although theoretically, they possess great possibilities.

This paper is written to consider some of these methods of producing artificial light that have to do with electricity and that come under item 5 of the following list:

1. Torch (and candle)
2. Oil,
3. Gas,
4. Solid Electric Conductors,
5. Gaseous Electric Conductors.

Electricity can be used to agitate into light either solids or liquids or gases. The light of the incandescent lamp is due to electrically-heated solids, and when electricity is conducted by a gas under suitable conditions, light also results. Many varieties of lamps of this nature both in design and construction are indicated in Fig. 1, the scope of which can be enlarged almost indefinitely; for example, by the use of many other gases and vapors.

High-tension lamps require the special auxiliary transforming apparatus to generate the high potential.

The two major factors of all of these types are (1) the electrodes, (2) the gaseous conductor.

Both electrodes of alternating-current lamps can be similar, but in direct-current lamps the cathode differs from the anode. Electrode materials differ with the gas used. It is therefore seen that the construction and design of each one of the scores of lamps indicated, is a distinct, and difficult problem, the solution of many of which, has hardly been seriously attempted.

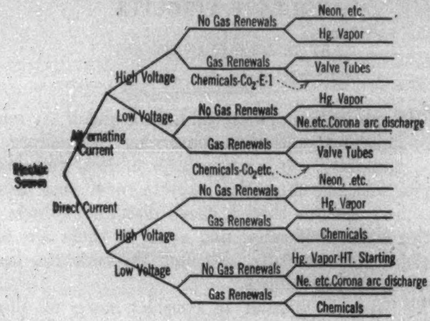
Alternating Current
Direct Current
High Voltage
Low Voltage
No Gas Renewals
Gas Renewals
Chemicals-Co_2-E-1
Neon, etc.
Hg. Vapor
Valve Tubes
High Voltage
Low Voltage
No Gas Renewals
Gas Renewals
Chemicals-Co_2etc.
Hg. Vapor
Ne.etc.Corona arc discharge
Valve Tubes
No Gas Renewals
Gas Renewals
Neon, .etc.
Hg. Vapor
Chemicals
No Gas Renewals
Gas Renewals
Hg. Vapor-HT. Starting
Ne. etc.Corona arc discharge
Chemicals

FIG. 1

As might be surmised the specific type of lamp I wish to emphasize, is the one that I have been most interested in recently, but in order to give it its proper setting, it is necessary to hastily review the past. The first natural electric light was lightning, or the aurora. It was due to gaseous conduction and so also was the first artificial electric light, which was produced

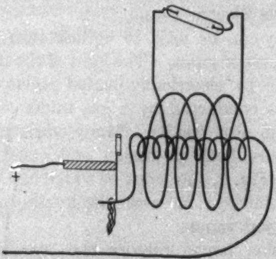

FIG. 2

with the revolving glass sphere of Hawksbee in 1750.

A hundred years later, Geissler first operated his small tubes from an induction coil. See Fig. 2. In 1879 Crookes modified them in many ways including obtaining high vacua. About 1891 Nickola Tesla delivered his famous lectures on "High Voltage and High Frequency."

Due to the rapid and very objectionable blackening that was deposited over the inside of incandescent lamp bulbs in 1893, I first began talking and thinking about the possibility of constructing a lamp without a heated filament—a filamentless lamp. In connection with the American Institute of Electrical Engineers, I explained that I meant a bulb form of lamp, the light source of which was to be, not an incandescent solid conductor but to emanate instead, solely from the enclosed gas or vapor electrically agitated by the low-tension circuits in common use.

During the twenty-six years that have intervened, this simple thought has never left me, though the tortuous road has been very dark at times, but it is now brighter than it has been before.

In order that I may not be misunderstood, I must hasten to say, perhaps sorrowfully, that it is still far too dim to even think of its competing in brilliancy with that splendid array of present day commercial illuminants—led by the incomparable tungsten lamp. Dreaming about the "Light of the Future" resulted in the effect shown in Fig. 3.

My first attempts in 1893 to obtain any light from a lamp without a heated filament on 220 volts, resulted in no light whatever. All known gases were unsuccessfully tried. Light from many of the common gases proved very interesting; for example, the bluish white light from CO_2, or the pinkish, hot and almost non-luminous light of hydrogen or the efficient orange yellow light of nitrogen, or the dull whitish light of oxygen; also many mixtures were tried together with chlorine, bromine, etc., and various vapors like those of sulphur or mercury. The prediction was made that progress would only result after the discovery of some of the gases indicated by the table of the periodic law of the elements. It was necessary therefore, in 1894, to resort to the high voltage of an induction coil, in order to obtain some light from the first gaseous conductor bulb lamp. In 1895, the vacuum vibrator displaced the induction coil and on direct-current circuits, the bulb lamps were filled with negative glow light. See Fig. 4 which shows vibrator and connections and Figs. 5 and 6 showing negative glow lamps and Fig. 7 depicting the use of negative glow for advertising purposes and means for increasing its intensity. Detailed information of the nature will be found in some of my previous papers.[1]

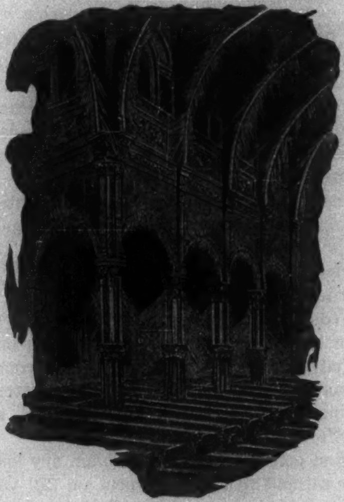

Fig. 3

After neon was discovered as hoped for, and I made nineteen years later, the first low-voltage gaseous conductor lamp, there was a certain satisfaction in proving that my original conception of utilizing the feeble light of the almost despised negative glow was correct. In 1896, seven foot vacuum tubes with external electrodes displaced the bulb lamp. See Fig. 8 for circuits. Fig. 9 shows the meeting hall of the A. I. E. E. thus lighted.

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[1] A New method for the Control of Electric Energy, Trans. A. I. E. E Sept. 20, 1893.

Recent developments in Vacuum Tube Lighting, Trans. A. I. E. E. April 22, 1898.

Light from Gaseous Conductors within Glass Tubes, Moore Light, Trans. A. I. E. E. April 26, 1907.

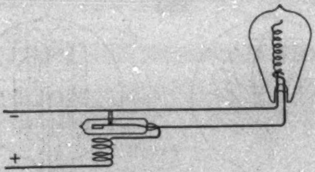

FIG. 4

The vacuum rotator succeeded the vibrator in 1897 and 98. Fig. 10 shows the Rotator Cabinet and Fig. 11 the interior of the historical "Moore Chapel." The first 220-volt direct-current tubes, started with a higher potential, from both vibrator and rotators were made and used. See Fig. 12 for the 5-in. tube which was used in taking the first instantaneous electric portrait, Chauncy M. Depew being the subject.

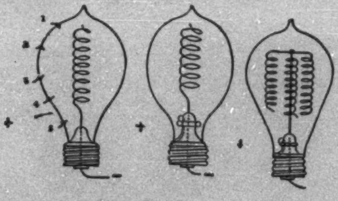

FIG. 5

The anticipated discovery of neon was announced in 1898, but even samples of it were impossible to obtain in America. Sir Wm. Ramsey, Lord Raleigh, Travers, and their brilliant contemporaries announced in rapid succession the five new monatomic elements, argon, helium, neon, xenon, and krypton, all of which will probably ultimately take important places in the world of commerce and some of which have already done so.

Vacuum-breaks were displaced in 1899, for a combination of resonance coils and a low-frequency generator, and later high-frequency generators. As shown in Fig. 13 the laboratory was thus lighted.

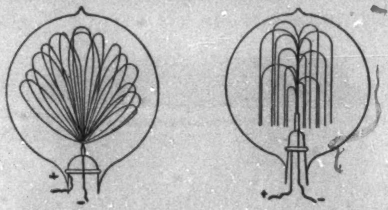

FIG. 6

In 1902, the "long tubes" (about 100 feet) appeared and they were improved in 1903 with internal electrodes. The beauty of the first long tube (Fig. 14)

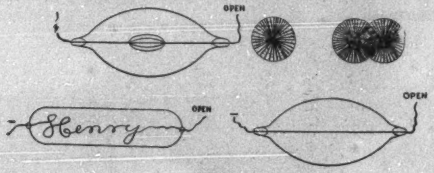

FIG. 7

was admired by thousands. The first rotary high vacuum oil pump was developed for the exhaustion of "long tubes" built in situ. Also a 24-in. CO_2 tube lamp provided with a carbon filament cathode was started with higher potential on 220-volt direct current and the resultant light was highly efficient.

Other though similar tubes and lamps, had metallic cathodes buried in lime, etc., and it was noted that when operated on alternating current, rectification took place. The combination of chronium and boron nitride with neon and also with some of the gas-feeding methods looks promising. These interesting types of lamps are shown in Fig. 15.

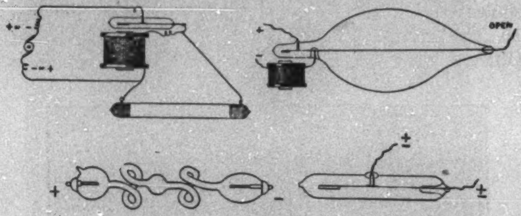

Fig. 8

It was a great advance in 1904, and 1905, to discontinue the use of a special generator with each "long tube" installation and obtain brilliant illumination from the distributed street circuits, by the use of nitrogen gas. See Fig. 16. Special electrodes were also constructed with auxiliary circuits, similar to those later used in rectifiers, pliotrons, and X-ray tubes.

Fig. 9

The life of these long tubes was extended to 10,000 hours, 1906—1909, by the invention of the electromagnetic feed valve, and over four miles of light-giving tubing was commercially installed. The lobby of Madison Square Garden is shown in Fig. 17. Fig. 18, shows the details of the magnetic feed valve and Fig. 19, the straight line tubes in the New York Post Office. No light source known today equals in

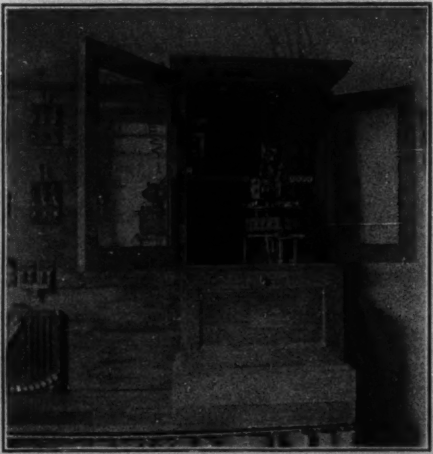

FIG. 10

efficiency a neon tube 1¾ in. in diameter and 200 ft. long. The "long tube" system is theoretically correct in so far as it provides means for generating light at the exact intensity most suitable for the eye; this in contradistinction to the generation of concentrated light at an enormous intensity and temperature that must, before it can be used by the eye, be either greatly reduced in intensity by means of some kind of semi-transparent or diffusing screen, or widely scattered by a reflector. Fundamentally the first cost of a "long tube" system is less than that of a complete

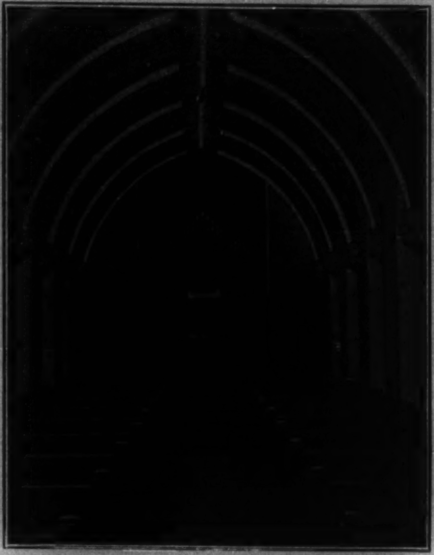

Fig. 11

incandescent lamp system and its life is longer with a resulting lower maintenance cost. It is simpler. During 1910-1911, the long tubes in the form of portable artificial daylight windows made their appearance, as shown in Fig. 20.

Between 1913-1915, several types of small tube lamps, dependent upon the new chemical gas feed

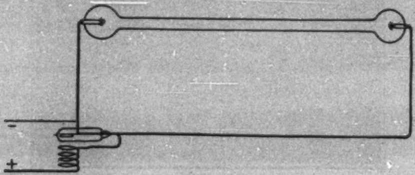

Fig. 12

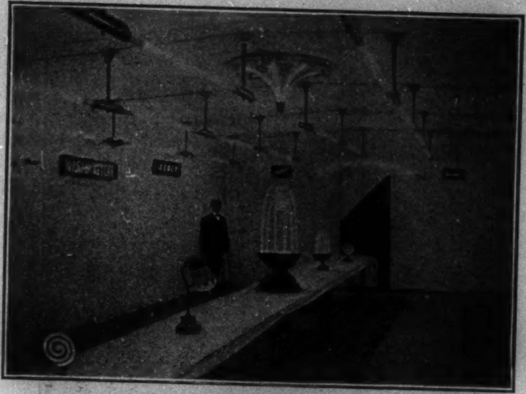

FIG. 13

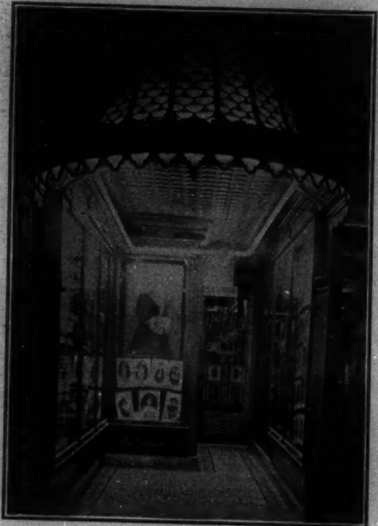

FIG. 14

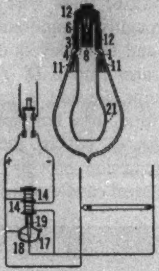

FIG. 15

principle, were invented and marketed for color matching purposes. The spectrum of this type of color matching lamp will never be surpassed as a standard light by which to judge colors. See Figs. 21, 22, and 23. Simple neon tubes operable from transformers were designed and made in many varieties. Some were equipped with screw lamp bases. These outfits consume 13 watts and are light enough to be screwed into an ordinary incandescent lamp socket. Lamps of this kind have run without change over 4000 hours.

FIG. 16

In the Fall of 1916, there was exhibited the first portable and thoroughly commercial neon tube outfit of high intensity and efficiency operated from a step-up sixty-cycle transformer. It resembles Fig. 21, except that the tube housing is twice as long. The tube was in the form of a hair-pin and had a total gas column length of 101 in. at ⅞-in. diameter. The specific consumption of this type of lamp was 0.74 watts per spherical candle power.

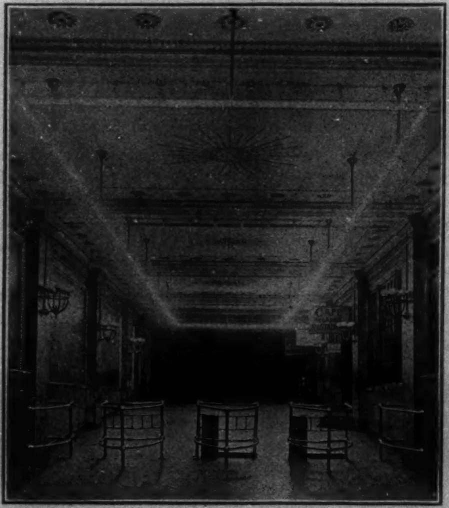

FIG. 17

Even this high efficiency can be improved considerably by using purer neon (that is, neon gas that does not contain 25 per cent helium and other impurities) together with a longer gas column of greater diameter. Also the electrode losses can be reduced. But the photometric measurements on this brilliant type of tube lamp showed a total of 180 mean spherical candle-power of 2260 lumens with 0.162 ampere passing through the gas column. Simple straight

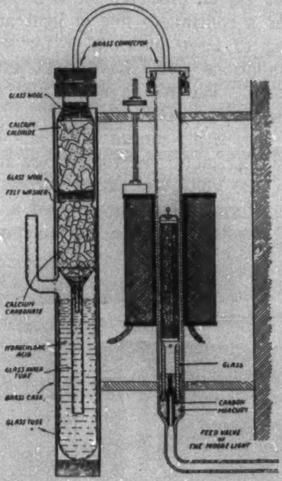
BRASS CONNECTOR
GLASS WOOL
CALCIUM CHLORIDE
GLASS WOOL
FELT WASHER
CALCIUM CARBONATE
HYDROCHLORIC ACID
GLASS INNER TUBE
BRASS CASE
GLASS TUBE
GLASS
CARBON
MERCURY
FEED VALVE OF THE MOORE LIGHT

FIG. 18

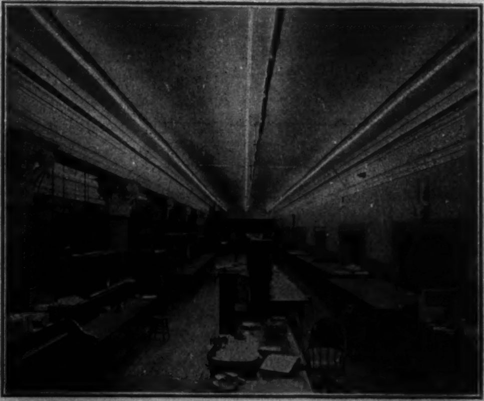

FIG. 19

tubes about 1¾ in. diameter and 8 ft. long could be arranged as a continuous line of light and used for the lighting of large interiors or for streets. The initial installations would have great advertising or display worth. The red rays will also be valuable for signaling purposes, etc.

Various alternating-current tube lamps, provided with two similar metal electrodes, were also made to operate on 220 volts alternating current without a step-up transformer, but they needed a momentary higher voltage to start the gas column discharge which is most simply obtained by short-circuiting a series inductance. The length of the gas column of this type of lamp is too long (about 3 in.) to permit 220 volts to pass any current but it will maintain the discharge, which is positive for an indefinite length of time, after once started. The necessity for starting apparatus is an objectionable feature of this particular type of lamp. When the gap or gas column between the electrodes of a tube lamp on 220 volts alternating current is less than about 1¾ in. the light is negative glow.

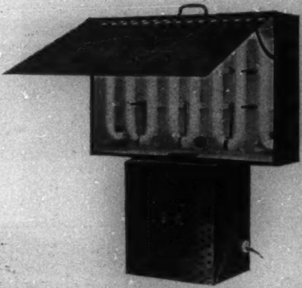

Fig. 20

Fig. 21

The direct-current lamp of the type requiring high potential for starting, involves, even when filled with neon gas, a special cathode of mercury of a K-Na amalgam. Still another type of 220-volt direct-current neon tube was started by using an auxiliary current to raise to a high temperature a portion of the cathode. Space will not permit the listing of many other varieties of gaseous lamps.

However it is to the type of lamp that has cold electrodes and is designed to start and operate without using high potential, that your attention is particularly called. As previously stated it is by no means completely developed but it seems to fulfill the original conception of a gaseous conductor lamp without any auxiliaries, for low-potential circuits.

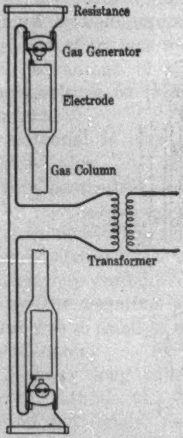

Fig. 23

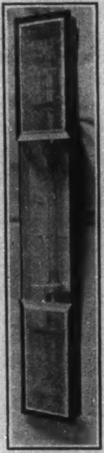

Fig. 22

The current of a 220-volt circuit passes through the neon gas and causes it to give light in a manner analogous to the way current passes through a tungsten wire and causes it to give light. No potential raising transformer is required or used. When the particular problem was the production of small units of light, its satisfactory solution by the use of the transformer was commercially impracticable, but it seemed for many years impossible to obtain any light without its use.

Fig. 24, shows a form of this type of lamp, for alternating circuits. Scores of modified designs have been made. It is a novel type of lamp. I hope that many will see in it, with me, the possibilities of a lamp of this kind. In fact diligent inquiry among scientific men has failed to find anyone who did not agree that all theory seemed to indicate the great probability that artificial light of high efficiency will result from the further development of lamps of this kind. The handwriting on the wall seems to unmistakably indicate that to further increase the luminous efficiency of light sources in general, we will need to resort to gaseous radiation, by which means it may be possible to reduce to about one-tenth the energy now required.

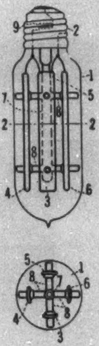

FIG. 24

Since the ice now seems to be broken on the problem, it is my earnest hope that many of the ablest of inventors will become actively interested, and through the combined knowledge, experience and ingeniousness of all who have studied and worked on gaseous conduction phenomena, the many problems involved will be solved. I believe that a very great deal remains to be learned and discovered.

The lamp shown in Fig. 24 resembles an incandescent lamp in outward form and perhaps is far more simple, yet it is not an incandescent lamp. Four electrodes made of aluminum, each 6 in. long, $\frac{5}{8}$ in. wide, and $^{1}/_{16}$ in. thick, are mounted in a 3-in. straight-sided bulb about a common center. A glass bulb, provided with radial arms of glass, supports the electrodes, which have holes in them, and through which the arms extend. The capacity of the solid radiators is objectionable and yet the effect of a solid radiator is approached by radiators, made of very fine meshed netting. An effort was made in designing this lamp, to take advantage of every factor that required minimum voltage so that it would operate on 220 volts or less.

The potential is least (volts per centimeter) for the negative glow. All of the light radiated from this new type of lamp is produced by the negative discharge;—not by the positive column as is the light in all of the long vacuum tubes when in operation from either a-c. or d-c. circuits. All text books and investigators have heretofore considered that the amount of light given by the negative glow in any vacuum tube discharge was so small as to make it entirely negligible as a light source.

An ordinary long tube discharge consists of (a) next to the cathode, the short first dark space (b) the short and not bright negative glow (c) the short second dark space, (d) the long brilliant positive column extending to the anode. But in the lamps herewith described, the positive column has been practically eliminated and substantially the only luminous discharge in the lamp is the negative glow which appears in the form of a velvety glow or corona of yellowish light over the entire surface of the alternating-current electrodes and also a uniform gaseous radiation throughout the entire interior of the bulb.

The lamp shown in Fig. 24, is designed for operation on 220-volt a-c. circuits. At this voltage it uses from the line about 0.11 ampere and 21 watts, but of this amount 3.6 watts at 33 volts is used by an ohmic resistance about 1 in. long placed in the skirt of the lamp base, because due to impurities, principally in the neon gas and the aluminum radiators, a slight blackening may form between them in time, which may cause the lamp to short-circuit.

The finished lamp will probably require no series resistance, but at the present time, the use of such a resistance affords a convenient method of adjusting the total watts consumed, the life or intensity. The specific efficiency of this particular type of lamp is low. When this lamp is consuming 17.4 watts, it gives approximately 1.16 s. c. p., which corresponds to 15 watts per s. c. p. Therefore, the most important problem still to be solved is how to decrease the number of heat waves and increase the number of luminous radiations. When the line voltage was reduced to 135 on this lamp, the light was suddenly extinguished.

The neon used had a helium content of about 25 per cent, but if it had had a nitrogen impurity of a fraction of 1 per cent, the neon lines would have been greatly reduced. The pressure of the gas when sealed off was 3.5 millimeters. The bulb temperature is about 40 deg. cent. but of course is increased when the watts are increased. The color of the light of this lamp is a beautiful yellow.

Some of the important factors given special attention in the designs of this new lamp are:

1. The attempt to use a gaseous conductor of maximum conductivity.
2. Electrodes that are subdivided and of as large a total area as possible.
3. A gas column (discharge gap) as short as possible.
4. The planes of the electrodes of opposite polarity placed parallel to each other.
5. The length of the radiator electrodes greater than the gas column and perpendicular to it.

Since the light is entirely due to negative glow, cathodic disintegration of the electrodes is one of the problems in connection with this type of lamp but it is practically nil when the cathode fall equals its minimum value. It is greater at lower gas pressures and increases as the square of the current, assuming a constant electrode area and gas pressure, but it is not an essential to transmission of current and seems to be largely due to the occluded gases, particularly hydrogen. The bulb blackening is far less with aluminum radiators than tungsten, nickel, copper, etc. Carbon in pure form is difficult to obtain. Iron radiators as well as various radiators combined with fluorescent coatings offer promise.

One of the troubles connected with the use of carbon was the difficulty of removing all of the occluded gases. However, this may be overcome by heating not only carbon electrodes but all other varieties of radiators. Radiators of whatever material should be as pure as possible and be cleansed in the best manner. A combination of light from the glowing gas, and from an incandescent radiator in the same bulb does not seem entirely practical.

This corona type of lamp produces a luminosity that is not due to arcing or even pure discharge phenomena but is due to the glow of light emanating from electrodes or radiators that normally have a temperature below red heat. According to the theory of ionization, the temperature within the negative glow, is higher than within the positive column and the velocity of the negative ion is greater than that of the positive, and this is one reason why the potential required to produce a luminous discharge from a negative pole is less than from a positive, together with the fact that in the negative glow the number of positive and negative ions are about equal.

The exceptional luminous efficiency of neon makes it unique among light sources. Immediately upon the announcement of its discovery in 1898, I proposed its use for lighting purposes. Its great scarcity until recently has made rapid progress impossible. Within the last few days announcement has been made that it can now be bought in almost any quantity and of a high degree of purity. Its luminous spectrum is almost ideally located to affect the eye in a maximum manner. It is a splendid example of selective emission or radiation that eliminates the long and therefore inefficient waves. It does for gaseous conductors just what the Welsbach mantle or the impregnated arc lamp electrode does for heated solids.

The maximum emission is between wave lengths 590 and 650 which is one of the remarkable properties of this gas. It produces over a hundred times as much luminosity for the same watts as does argon for example. Its dielectric cohesion is 5.6 which is extremely low when compared with air at 419. It has less than one-half the resistance of nitrogen. It is fortunate that the color of the resultant radiation of neon from a positive column is different from that of a negative glow.

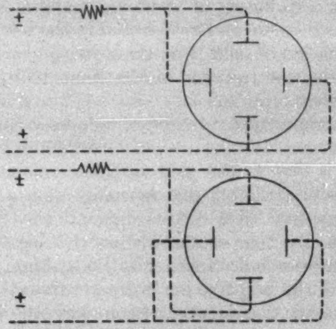

Fig. 25

Neon gas when used as a positive column of light has a color so reddish that it would be objectionable for many purposes but when the same gas is used as a negative glow, the color is yellowish. It has no blue or violet or indigo lines and very few infra red rays. It is four times better as a light producer than the yellow-white light of helium or the violet of xenor, both of which have many infra red rays. The characteristic crimson of neon has been displaced by a uniform mass of soft yellow light that somewhat resembles the color of a high class oil lamp, or that from the electric incandescent carbon lamp, the intrinsic brilliancy of which is theoretically too great.

The connections of the four radiators are shown in Fig. 25A, but the total flux of light is not very much less when the connections are as per Fig. 25B.

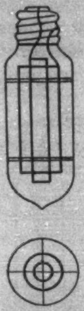

FIG. 26

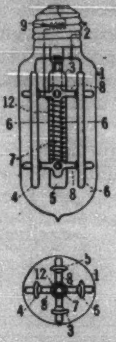

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FIG. 27

Scores of modifications and varying designs have suggested themselves. For example, such a type of

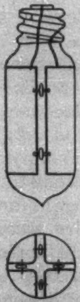

FIG. 28

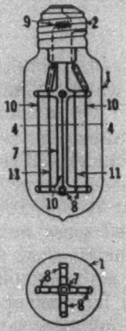

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FIG. 29

lamp that is suitable for alternating circuits will differ from a properly designed direct-current lamp. But all alternating-circuit lamps will give some light on a direct current of the same voltage. That is, only one of the radiator poles (the negative) will give any light. Therefore, in the case of the lamp shown in Fig. 24, only two of the four radiator plates will be luminous.

The positive poles will remain absolutely dark. This fact is given recognition in the design of the direct-current lamp, shown in Fig. 26. The inner cylinder is of sheet aluminum and the outer cylinder is of aluminum netting and made the negative pole.

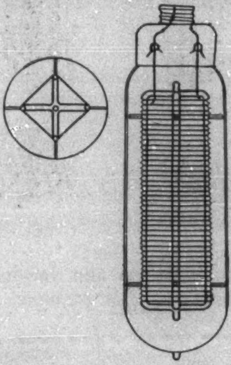

Fig. 30

Fig. 27 which is taken from the United States Patent No. 1,316,967, which was applied for November 30th, 1917, shows the positive electrode in the form of a spiral on the axis of the lamp.

Fig. 28 shows a very simple lamp for alternating circuits that is constructed by inserting into a 3-in. straight-sided glass bulb four right angles made of aluminum netting, of 0.052 wire having a mesh of eight wires per inch. Each right angle is 5 in. long. Eight glass buttons or spacers keep all portions of these four angles at a uniform distance of $^3/_{16}$ in. from each other, and they are all held in place by the walls of the bulb.

Just as a final mechanical form for the major designs of the tungsten lamp was arrived at, so also will doubtless be the case with lamps or tubes based on the corona principle. These lamps should be so designed mechanically that a maximum amount of the light that is generated has free exit or is reflected in the best manner.

Fig. 29 has a construction that closely resembles that of a standard tungsten incandescent lamp, and Fig. 30 shows wire electrodes wound parallel to each other on a glass drum. Fig. 31 shows such a lamp in the form of a tube.

The dozen most important factors involved in the design of these lamps should be examined theoretically and a definite conclusion reached concerning each one, so as to determine definitely its possibilities.

Some of the important variables are:

The gas pressure, for (a) efficiency, (b) life.

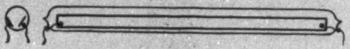

Fig. 31

Electrodes; material, form (wire) and area, best suited for a definite voltage, life, wattage, intensity and efficiency.

The exhaust programme.

The length of the gas column, that is distance between radiators of opposite polarity.

Volume and shape of the bulb.

When the gas pressure is too high (about 10 millimeters) no light appears but at 6 milliameters it consists of a velvety or luminescent glow that closely envelopes the radiators but extends further and further from them as the pressure grows less until it fills the entire bulb with a suffused glow, which, however, becomes thinner and less luminous when the pressure is still further reduced.

It seems advantageous to subdivide the radiator of each negative pole. The lamp made as shown in Fig. 32, shows that far more light is generated between such subdivisions than between areas attached to opposite poles.

It is apparent that it is very desirable to produce a brighter lamp. Photometric measurements of lamps constructed as in Fig. 30, showed 2.59 spherical candle power on 220 volts alternating current. The higher the voltage, the easier is this problem. Therefore, perhaps it would be best to first develop a lamp for the commercial 500-volt circuits. When the voltage is raised abnormally on most of these lamps they will arc destructively even though the air gap is large. Often times there seems to be less tendency to this destructive "ball discharge" arcing when the air gap is small, than when it is large, because then the ohmic series resistance can be greater. The lamp as shown in Fig. 24, has a discharge gap of ⅝ in. but in other lamps it varies from 1/32 in. to one inch.

The lamps that give the most light on 110 volts are those whose radiators were made of wire of small diameter and small total area as shown in Fig. 29.

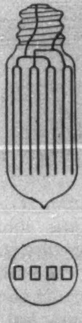

Fig. 32

Photometric data of various types of these corona lamps have been obtained by the use of an 80-in. sphere. Color corrections were made by the following procedure:

1. Hold Moore lamp at 220 volts and adjust comparison lamp to Moore lamp color.

2. Set galvanometer on zero and maintain comparison lamp at above color.

3. Adjust Mazda "B" lamp to comparison lamp color and note voltage.

4. Ascertain horizontal candle power of Mazda "B" lamp at above voltage.

TABLE I—ALTERNATING CURRENT

Lamp Nos.	Line		Lamp		Lamp s. c. p.	Lamp w.s.c.p	Line watts	Line w. per s. c. p.	Series w.
M.	V.	A.	V.	W.	A-C.	A-C.	A-C.	A-C.	
547	155 min.								
	164	0.03	149	5.	0.234	21.	4.5	24.	500 w.
	184	0.062	148	6.5	0.32	20.5	8.5	26.5	
	221	0.105	168	15.5	0.594	26.	21.	35.5	
	240	0.127	177	20.	0.780	25.5	28.	36.	
	264	0.16	184	24.5	1.04	23.5	37.5	36	
595	135 min.								300 w.
	166	0.045	153	3.9	0.258	15.	4.5	17.	
	181	0.06	162	7.	0.535	13.	8.1	14.	
	220	0.11	187	17.4	1.15	15.	21.	17.	
	239	0.14	197	23.3	1.77	13.	29.	16.	
	265	0.18	211	32.9	2.44	13.4	42.6	17	
594	127 min.								300 w.
	167	0.045	153	4.4	0.392	11.2	5.0	12.	
	182	0.075	159	9.3	0.635	14.7	11.	17.	
	220	0.135	180	21.	1.24	17.	26.4	21.2	
	240	0.17	189	27.8	1.7	16.	36.5	21 4	
	265	0.215	200	37.2	2.4	15.	51.	21.2	
430	220	0.13	155	17.	0.715	23.5	26	34.	500 w.
605	220	0.11	188	16.7	0.897	18.5	23	22.	300 w.
609	220	0.245	195	53	1.825	29.	58	32	100 w.
600	220	0.095	172	14.1	0.715	19.8	18.6	26	500 w.
674 etc.									
647	220	0.185	164.5	24.5	0.870	28.	34.5	39.	300 w.
270	220	0.205	179.	29.	1.047	27.	37.5	36.	200 w.
669	220	0.135	193.	15.9	0.645	24.	19.5	30.	200 w.
675	220	0.085	126.5	19.	0.601	31.	16.5	27.	1100w.
673	220	0.22	176.	30.3	0.847	35.	40.	47.	200 w.

5. Horizontal c. p. of Mazda "B" lamp x 0.785—s. c. p. of Mazda "B" lamp.

6. Read Mazda "B" lamp and all d-c. or a-c. Moore lamps against comparison lamp as set.

7. Multiply Moore lamp readings by ratio of Mazda "B" reading to Mazda "B" s. c. p.

The tabulations of Table I show first the performances of four lamps constructed approximately as shown in Fig. 24 on alternating current, and then follow the test data of several lamps of varying constructions.

The tabulations of Table II show most of these lamps when operating on d-c. circuits, under which circumstances of course, but one pole gives any light.

It is only safe to consider these data however as indicating very broad generalizations because no two lamps have been made alike, even as regards their mechanical construction, and they also differ as regards the purity of the gas and its pressure, the exhaust programme, etc. There were also encountered difficulties as regards the photometrical and electrical measurements, for example, when such a lamp is consuming less than two watts, yet the amount of light seems considerable to the eye in a dark room.

Table II—DIRECT CURRENT

Lamp Nos.	Line		Lamp		Lamp s. c. p.	Lamp w.s.c.p	Line watts	Line w. per s. c. p.	Series w.
M.	V.	A.	V.	W.					
547	165	0.017	156.5	2.66	0.158	16.	2.8	17.8	500 w.
	220	0.066	187	12.3	0.444	27.	14.5	32.7	
	265	0.124	203	25.1	0.880	28	32.8	37.3	
595	165	0.013	161.0	2.0	0.178	11.7	21.1	12.1	300 w.
	220	0.072	198.4	14.2	0.792	17.8	15.8	20.	
	265	0.134	224.	30.	1.88	16.	35.8	18.9	
594	165	0.014	160.8	2.2	0.178	12.3	2.3	12.9	300 w.
	220	0.061	201.7	12.3	0.663	18.7	13.4	20.2	
	265	0.13	26.1	33.9	1.53	22.	34.4	22.5	
605	165	0.015	160.	2.5	0.217	11.5	2.6	1.22	300 w.
	220	0.078	196.6	15.2	0.787	19.	17.1	21.8	
	265	0.146	221.	32.	1.48	22.3	38.7	26.	
609	220	0.16	204	32.4	1.48	21.5	35.	23.5	100 w.
430	219	0.105	166	16.2	0.796	22.5	21.9	27.4	500 w.
600	220	0.04	198	8.	0.387	20.5	8.8	22.5	500 w

Nevertheless, I believe that complete and exact specifications should be determined upon for an ideal lamp of this nature entirely independent of its comparative relations to other and seemingly far superior forms of artificial light.

Some of the conclusions that may be drawn are as follows:

1. Efficiency of these lamps is about the same whether operating on a-c. or d-c. circuits.

2. Efficiency is about the same on a-c. circuits over a wide voltage range.

3. Efficiency is about the same on a-c. circuits over a wide range of intensities.

4. S. c. p. varies approximately with the wattage on either a-c. or d-c.

5. That the lamps with a reasonably pure neon color were not as efficient as those in which gas impurities made the color whiter.

6. The general lamp performance is not very sensitive to wide variations in the length of the gas column or gap.

7. The same lamp equipped with the same resistance and operating at the same voltage takes a considerably higher line wattage on a-c. than on d-c., which doubtless is principally due to the light radiating area being double.

8. The c. p. is greater with radiators of large area.

9. The power factor of these lamps is about 85 per cent.

These lamps demonstrate that useful gaseous conductor light, that, to say the least, has advertising value, can now be produced in a simple manner from ordinary commercial circuits. Special uses will be found for such lamps for example as polarity or potential indicators. Since the internal parts are all below red heat gas explosions will not be caused by bulb breakage. Gaseous light, due to electrical agitation has, to a limited extent, been emancipated from all necessity for a heretofore ever present high potential either for starting or normal operation. The basic conception of using a gas to supplant the heated filament in an ordinary lamp seemed wholly impossible, yet, this new type of lamp makes it at least a partial reality. It constitutes an advance in that it adds to our knowledge of a very little developed subject. A new epoch in the history of gaseous conduction lighting has been reached. It is my hope that the great cause of new and better lighting methods in which my deep interest has been centered for years, may be spurred to rapid advancement in a new direction that gives promise of reward to an unlimited number of worthy investigators and inventors.

REPORT OF THE BOARD OF DIRECTORS

FOR THE FISCAL YEAR ENDING APRIL 30, 1920

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Thirty-sixth Annual Report, for the fiscal year ending April 30, 1920. A general balance sheet showing the condition of the Institute's finances on April 30, 1920, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year; more detailed information has been published from month to month in the Institute JOURNAL.

Directors' Meetings.—The Board of Directors held ten regular meetings during the year; five of these were held in New York, one at the Lake Placid Club, N. Y., in June; one in Philadelphia in October; one in Chicago in January; one in Pittsburgh in March, and one in Boston in April; one Executive Committee meeting was held on April 27, 1920.

Information regarding the more important activities of the Institute which have been under consideration of the Board of Directors, the committees, and the various officers, is published each month in the section of the JOURNAL devoted to "Institute Activities."

President.—During the year President Townley has attended many Institute and Section meetings including the Annual and Midwinter Conventions and meetings in Boston, Chicago, Detroit, Milwaukee, New York, Philadelphia, Pittsburgh, St. Louis, Schenectady and Toronto.

Annual Meeting.—The Annual Business Meeting was held at Institute headquarters, New York, on May 16, 1919. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1919, was presented as published in full in the June 1919 issue of the PROCEEDINGS. The Tellers Committee presented its report upon the election of officers for the year beginning August 1, 1919.

Directly following the business meeting came the ceremony of the presentation of the Edison Medal to Benjamin G. Lamme.

Annual Convention.—The Thirty-fifth Annual Convention was held at the Lake Placid Club, Adirondacks, N. Y., on June 24 to 27, 1919. The business and entertainment features were about equally divided, constituting a program that met with general approval. Seven technical papers were presented at two sessions. A session was devoted to the presentation of Technical Committee Reports. The usual conferences of Section delegates, attended by representatives of thirty Sections, were merged with conferences under the auspices of the Committee on Development of the Institute and they constituted one of the most important features of the Convention, as affecting future policies and activities.

Pacific Coast Convention.—The eighth annual Pacific Coast Convention was held in Los Angeles, California, September 18 to 20, 1919, under the auspices of the Los Angeles Section. Five technical papers were presented and a splendid entertainment program provided.

Joint Meeting with Radio Engineers.—A joint meeting of the A. I. E. E. and the Institute of Radio Engineers was held in New York, October 1, 1919.

Philadelphia Meeting.—A joint meeting of the Institute and the American Physical Society, to which members of the Franklin Institute were also invited, was held in Philadelphia on October 10, 1919.

New York Meeting.—The 355th Institute meeting was held in New York on November 14, 1919, under the auspices of the Power Stations Committee. Three technical papers were presented on Design and Operation of Steam Turbines.

New York Meeting.—The 356th Institute meeting was held in New York on December 12, 1919. Two technical papers were presented, one on Searchlights and one on the Automatic Telephone.

Chicago Meeting.—The 357th Institute meeting was held in Chicago, January 9, 1920, under the auspices of the Chicago Section and the Lighting and Illumination Committee.

Mid-Winter Convention.—The eighth Mid-Winter Convention was held in New York on February 18 to 20, 1920. The total attendance reached 1100. Six technical sessions were held and fourteen technical papers were presented.

Pittsburgh Meeting.—The 358th Institute meeting was held in Pittsburgh on March 12, 1920. Five technical papers were presented at two sessions under the auspices of the Pittsburgh Section and the Traction and Transportation Committee.

Boston Meeting.—The 359th Institute meeting was held in Boston on April 9, 1920. This was a joint meeting with the American Electrochemical Society. Two Institute sessions were held under the auspices of the Committee on Electrochemistry and Electrometallurgy, at which six papers were presented.

Meetings and Papers Committee.—The Meetings and Papers Committee held meetings every month of the year except during July and August.

A very large number of papers was offered during the past year, more than could be presented at regular Institute meetings. To provide an opportunity for the presentation of these papers the Committee recommended to the Board of Directors that the Section Chairmen be made ex-officio members of the Committee. Thus the Sections could be kept informed of the papers available and their representatives could ask for the assignment of appropriate papers for Section meetings. The Board approved the suggestion and each Section Chairman receives monthly a list of available papers.

On the recommendation of the Committee a number of papers of general interest were published in the JOURNAL without presentation at meetings.

The Annual Convention will be held at White Sulphur Springs, June 29 to July 2, 1920, and a Pacific Coast Convention is scheduled for Portland, Oregon, July 21 to 23, 1920.

Board's Committee on Technical Activities—Certain portions of the Development Committee's report were referred to the Committee on Technical Activities, who reported on October 10, 1919. Among the recommendations made were the following: that Chairmen of Sections become ex-officio members of the Meetings and Papers Committee, thus permitting Sections a better opportunity to obtain papers of high grade and enabling them to hold meetings of even greater interest than in the past; that papers covering progress in the art be presented from time to time at Institute meetings and that the annual reports of the Technical Committees include a review of the state of the art in the particular field covered and such developments as may be worth recording. The committee's complete report was published in the PROCEEDINGS for December 1919.

Publication Committee.—The Publication Committee was established for the first time during the present year by an amendment to the By-laws, in order to carry into effect the recommendations of the Development Committee relating to the Institute publications, as approved by the June 1919 Convention and subsequently by the Board of Directors. This committee has supervision of the monthly JOURNAL and the annual TRANSACTIONS of the Institute; the functions of the former Editing Committee have been incorporated with those of the new Publication Committee.

The chief activity of this committee has been to put into effect the recommendation of the membership of the Institute that the monthly PROCEEDINGS be enlarged both as to scope and size and that it should serve as a medium for the presentation of much general information for the membership in addition to the customary Institute papers. This change was not easy to make during the past year, due to existing conditions of the publishing and related industries, including labor troubles, shortage of supplies and the enormous increase in the cost of paper, printing and engraving.

The appropriation for publications for the present fiscal year is 50 per cent greater than for any preceding year, thus more of the benefits of the Institute are being given directly to the membership in the form of publications than ever before, and this has been accomplished without any increase in the dues.

Sections and Branches.—The year terminating May 1, 1920, marks the completion of the first year for some time past not involved with some phase of the Great War. It is therefore natural it should be marked by a return of the Sections activities to normal conditions. The accompanying figures indicate that such is the case, for they indicate that the activities of the past year have returned to a point very near those for the year ending May 1, 1917, and it may be inferred from the present trend that there will be a further increase of activities next year.

The number of Sections has been increased during the past year by the addition of those at New York, N. Y., Worcester, Mass., and Providence, R. I., while at the same time the number of Student Branches has been increased by the addition of Branches at the University of Wisconsin, Madison, Wis., and the School of Engineering of Milwaukee, Milwaukee, Wis.

The effort which the Sections Committee made in behalf of the Development Committee, culminated last June at the Lake Placid Convention, where, owing to the close conformity of interests, the Sections' delegates merged their meeting with the Committee on Development, and ratified the recommendations of the committee.

	For Fiscal Year Ending						
	May 1 1914	May 1 1915	May 1 1916	May 1 1917	May 1 1918	May 1 1919	May 1 1920
SECTIONS							
Number of Sections	30	31	32	32	34	34	36
Number of Section meetings held	233	246	251	265	245	217	262
Total Attendance	22,626	23,507	28,553	31,299	34,614	25,837	30,741
BRANCHES							
Number of Branches	47	52	54	59	59	61	62
Number of Branch meetings held	306	328	360	368	268	156	360
Attendance	11,617	12,712	15,166	16,107	10,683	6,441	16,827

Standards Committee.—The Standards Committee held during the year six regular meetings, three of which were joint meetings with the United States Committee of the International Electrotechnical Commission, as well as several meetings of a special committee with the delegates who were to attend the Brussels meeting of the Advisory Committees of the I. E. C.

Proposals were formulated and transmitted to the I. E. C. Committee which were in accordance with our present 1918 rules and which embodied also 1919 amendments which have not yet been issued. The committee felt that it would not be desirable to issue the amended rules in revised form until the return of the delegates who attended the meeting of the I. E. C. Advisory Committees which was held in Brussels in March.

It is expected that a full report of the Brussels proceedings will be placed before the Standards Committee at an early date, at which time another joint session with the I. E. C. Committee will be held and arrangements perfected to issue a new edition of the rules. This work is well along and it is hoped that the new edition can be ready for publication before the close of the present Institute year.

American Engineering Standards Committee.—The American Engineering Standards Committee upon which the Institute is represented by three members is a coordination of the various bodies engaged in the promulgation of standards, to prevent the confusion, duplication and overlapping of the efforts of those engaged in standardization, which was particularly evident in the early stages of the war. This committee makes it possible to give an international status to approved American engineering standards and to cooperate with similar foreign organizations.

During the past year the committee has prepared a revised constitution, by-laws and rules of procedure, and these revised documents have been approved by the governing bodies of the organizations represented on the committee.

The new constitution provides for the representation on the committee of other than the five original organizations. Two groups have already been admitted, namely: 1. Fire Protection Group, including, National Fire Protection Association, National Board of Fire Underwriters, and the Associated Factory Mutual Fire Insurance Companies. 2. National Safety Council, representing the Safety Group.

The following organizations have made application and have passed the committee, but final action on these applications has not yet been taken by the governing bodies. They are: 1. Society of Automotive Engineers. 2. National Electric Light Association. 3. Electrical Manufacturers Council, representing the Electric Power Club, the Association of Allied Electrical Supply Manufacturers, and the Electrical Manufacturers Club.

The American Engineering Standards Committee has already made considerable practical headway. It has approved specifications for standard pipe threads, and is represented on this subject at an international conference in Paris. Cooperation is in progress with the National

Screw Thread Commission looking forward to standard screw threads. Through this arrangement direct cooperative work with the British, which is not possible by the official Commission, is being carried out. The committee is also in active cooperation with the Canadians on bridge specifications, with the British on specifications for machine tools and also on structural steel sections and with the Swiss on specifications for ball bearings.

At a conference in which practically all national organizations interested in industrial safety were represented, it was unanimously voted that all industrial safety codes should be prepared under the auspices of the American Engineering Standards Committee.

A general sectional committee on Safety Codes has been appointed and fifteen of these codes have already been assigned to sponsors who will appoint sectional committees to prepare the codes.

Mr. A. A. Stevenson, of the Standard Steel Works Company, has been elected as the new Chairman, and the committee has been fortunate in securing the full-time services of Dr. P. G. Agnew, as Secretary.

International Electrotechnical Commission and Standardization Meetings: Paris, May 5 to 7, 1919.—The Special Committee on Rating of the I. E. C. held a meeting at Paris, May 5 to 7, 1919. This meeting was attended by Messrs. C. A. Adams, H. M. Hobart and C. O. Mailloux representing the U. S. National Committee of the I. E. C. The meeting was attended by representatives from Belgium, France, Great Britain, Italy, Sweden, Switzerland and the United States.

London, October 20 to 22, 1919.—The fourth plenary meeting of the International Electrotechnical Commission was held at the Institution of Civil Engineers, London, October 20-22, 1919. Over 50 delegates representing 19 countries were received by President Maurice Leblanc at the opening meeting. The delegates were welcomed by Sir Richard Glazebrook, C. B., President of the British National Committee, Roger T. Smith, President of the Institution of Electrical Engineers, and Mr. Everest on behalf of the British Electrical and Allied Manufacturers Association The Institute was represented on the U. S. National Committee of the I. E. C. by Messrs. Mailloux, Burke, Chubb and Hobart. At the Council Meeting on October 22, Dr. Mailloux, President of the U. S. National Committee was elected the new President of the I. E. C. A report of the meeting was published in the JOURNAL for February 1920.

Brussels, March 27, 1920—The National Committees of the I. E. C. met in Brussels March 27-29, to discuss recommendations to be made for adoption at the next plenary meeting of the I. E. C. Reports of this meeting will be published in the JOURNAL promptly after receipt.

Committee on Safety Codes.—The Chairman of the Committee on Safety Codes continued to represent the Institute on the Electrical Committee of the National Fire Protection Association, and the Institute was represented at the annual meeting for the revision of the Code, held in New York in March.

The committee was represented at a conference held in New York on the question of combining of the National Safety Code and the National Electrical Code, the decision being not to attempt to combine them at this time.

The committee was also represented at a meeting in Washington with the Bureau of Standards, in the carrying forward the project of the development of an Industrial Safety Code, and it was the sense of the conference that the matter should be handled through the American Engineering Standards Committee, as suggested at the previous conference, and a committee was appointed to carry the matter forward in this way.

Board of Examiners.—The Board of Examiners during the year held thirteen meetings, averaging about two and one-half hours each. It considered and referred to the Board of Directors a total of 3265 applications for admission or transfer to the higher grades. This is an increase of about 55% over last year.

Because of the rapidly increasing amount of work coming before the Board, particularly transfers, the personnel this past year was increased from five to nine. This increased number permitted a division of the work among subcommittees, final action in all cases being taken by the entire Board. All doubtful or borderline cases were referred for consideration to the Board as a whole.

The result of the Board's work for the year is given in the following tabulated statement:

APPLICATIONS FOR ADMISSION

Recommended for grade of Associate	1754	
Not recommended	5	1759
Recommended for grade of Member	109	
Not recommended for admission to this grade	40	149
Recommended for grade of Fellow	7	
Not recommended for admission to this grade	10	17
Recommended for enrolment as Students		1172

APPLICATIONS FOR TRANSFER

Recommended for grade of Member	120	
Not recommended for transfer to this grade	19	139
Recommended for grade of Fellow	19	
Not recommended for transfer to this grade	10	29
Total number of applications considered		3265
Applications reconsidered		9
Total		2743

Membership.—Two thousand and thirty-three applications for membership in the Institute have been received this year, which exceeds by 437 the number for the year ending May 1, 1919, the banner year in Institute history.

With the demobilization of the army and navy complete and industry returned to a peace basis, great numbers of engineers are turning their attention to the zealous pursuit of their profession which accounts largely for the decided increase in applications for admission.

During the five years ending May 1, 1917, the net increase was only 1251; the total applications received in the five years, 2440, while the year ending May 1, 1919 shows an increase of 1070 and that ending May 1, 1920 a gain of 1093, a slight increase over last year's record figures.

	Honorary Member	Fellow	Member	Associate	Total
Membership, April 30, 1919.	6	489	1467	8290	10,252
Additions:					
Transferred	....	18	106	....	....
New Members Qualified	....	6	100	1717	....
Reinstated	....	....	11	66	....
Deductions:					
Died	....	7	7	62	....
Resigned	....	2	5	127	....
Transferred	....	....	12	112	....
Dropped	....	6	43	548	....
Membership April 30, 1920	6	498	1617	9224	11345

Net increase in membership during the year....................................1093

Deaths—The following deaths have occurred during the year:

Fellows Geo. T. Hanchett, John D. Ihlder, Charles E. Knox, John Langton, Charles E. Lord, Walter J. Warder, W. D. Weaver.

Members: J. W. Bard, Leslie H. Harris, Frederick L. Neely, W. O. Oschmann, Henry A. Reed, Albert Schmid, W. I. Thomson.

Associates: Hugo Altmayar, George B. Ames, R. M. Berry, Robert M. Biggs, R. L. Cadwell, Willard E. Case, A. P. Chapman, Richard E. Cheeny, Stuart H. Clemmen, James E. Cole, David C. Collier, Henry S. Dawe, Harry J. Dixon, John W. Downie, Charles L. Easton, Charles Elstrom, L. H. Feldhake, Walter S. Garvey, F. S. Gassaway, R. M. Glaspey, L. G. Gomez, Harold W. Haldeman, L. J. Heissler, Yoshio Hiraga, Raymond S. Horth, Philip B. Hummel, C. J. R. Humphreys, Stanleigh O. Kelley, H. D. Kemp, N. C. Kingsbury, A. C. Knight, Harold H. La Fever, Frank S. Leisenring, Hugo J. Lundgren, Paul Lupke, Wallace J. Mayo, Charles J. Moore, Eldin S. Moulden, Robert W. Nolte, Cyril Orpin, Carl Ort, Nathaniel J. Owen, Raymond W. Parker, N. E.

Puhakka, George A. Roesler, S. R. Shaper, Herbert L. Sheen, Robert K. Sheppard, Oliver T. Smith, Geoffrey R. Thayer, A. C. Thompson, Guion Thompson, F. W. Throop, J. T. Tomlinson, Julian Tyng, T. N. Vail, W. O. Vickery, O. A. Walthall, R. P. Walton, K. L. Wang, Hugh M. Wilson, L. E. Wright.

Total deaths, 76.

Employment Service—The employment service which has been maintained for many years at Institute headquarters for the purpose of assisting members in obtaining positions has been coordinated with similar service of other societies. This service, now supported and directed jointly by the national societies of Civil, Mining, Mechanical and Electrical Engineers as the Engineering Societies Employment Bureau, consists principally in acting as a medium for bringing together the employer and the employee.

An engineering service bulletin is published each month in the Institute JOURNAL and has served to place many men in positions of responsibility, both in this country and abroad. The bulletin is subdivided into two parts: one containing announcements of vacancies, and the other containing lists of men available, with condensed records of their experience. All announcements are published without charge either to the employers or to the men seeking positions.

American Committee on Electrolysis.—This committee is composed of representatives of the Institute and several other organizations interested in electrolysis by stray currents. The American Electrolysis Committee has held two meetings during the year, and resumed work after a considerable period of interruption due to the war. A subcommittee, known as the Research Subcommittee, has been appointed to cooperate with the Bureau of Standards in the investigation of electrolysis questions, and this committee has secured a large amount of information. Other subcommittees are gathering material, and a report is in preparation.

Development Committee.—In order to meet the rapidly changing social, economic and industrial conditions and the ever increasing demand for widening the field of Institute activities, the Development Committee was appointed in October, 1918. This committee solicited and considered suggestions from the membership at large, and reported to the Board of Directors in August, 1919. The report, which was approved by the Board and published in full in the September 1919 PROCEEDINGS, recommended such changes in the monthly JOURNAL as would serve to greatly broaden its scope; changes in organization were also recommended to bring the membership in closer touch with Institute affairs; a closer bond of cooperation with other engineering organizations was suggested, together with the holding of an annual Engineering Congress.

The various recommendations offered were referred to properly qualified committees and reports of the actions resulting from the deliberations of these committees will be found elsewhere in this report.

Joint Conference Committee of Development Committees.—When the Committee on Development made its report on August 12, 1919, it stated that it had entered into conference with representatives of the other three national societies. A Joint Conference Committee was formed of three members of each society. A preliminary meeting was held on July 2, 1919, with several subsequent meetings. The final report of this Joint Conference Committee was made October 10, 1919, and was printed in full in the PROCEEDINGS of November 1919. This report came before the Board of Directors of the Institute for final action December 12, 1919. It recommended in brief the formation of local affiliations and a national organization consisting of representatives of local affiliated bodies and national societies.

The Board endorsed the report and referred it back to the Institute's representatives on the Joint Conference Committee for such action, in cooperation with other societies as endorse it, as may be necessary to put the recommendations into effect. A call has been issued by the Joint Conference Committee for a meeting of delegates of various national and local engineering organizations in Washington, D. C., June 3 and 4, 1920.

Franco-American Engineering Committee.—At the invitation of French Engineers issued during the latter part of 1918, a delegation of American engineers was appointed by the four national societies, the A. S. C. E., the A. S. M. E., the A. I. M. & M. E., and the A. I. E. E., to attend a Congress of Engineers in France and to study with French engineers industrial habilitation problems. This delegation sailed for France on December 5, 1918. They returned to New York and reported at a joint meeting of the societies on February 10, 1919, as outlined in the March PROCEEDINGS. During the Paris sessions of the Congress a permanent Franco-American Commission was suggested.

This permanent Franco-American Engineering Committee, composed of two representatives of each of the four Founder Societies, has met and organized by the appointment of Nelson P. Lewis as chairman. The Institute is represented by Lewis B. Stillwell and Andrew M. Hunt.

United Engineering Society.—This Society performs for the national societies of Civil, Mining, Mechanical and Electrical Engineers, certain specific acts which are governed by contracts; the primary function of the United Society being to hold in trust and to administer for these societies the Engineering Societies Building, in which the headquarters of the National societies are located.

Extracts from the annual financial report of the United Engineering Society were published in the March 1920 JOURNAL.

Engineering Societies Library.—The library of the Institute is combined with the libraries of the national societies of Civil, Mining and Mechanical Engineers, administered as the "Engineering Societies Library" under the direction of the Library Board of the United Engineering Society; this board is composed of representatives of each of the four societies referred to above.

In order to place the facilities of the library at the disposal of persons residing at a distance from New York, a Library Service Bureau has been established, and a staff of expert searchers and translators is employed to cover almost any engineering topic, in the following manner: abstracting, translating, bibliographing, statistical searches and reports, searches for patent purposes, copying, preparing reference cards, etc.

An abstract of the annual report of the Engineering Societies Library covering the calendar year 1919, was published in the March 1920 JOURNAL.

Engineering Council.—The Engineering Council represents the result of an organized effort inaugurated in the latter part of 1916 by the national societies of Civil, Mining, Mechanical and Electrical Engineers, to establish an instrument through which united action could be brought about upon matters of common interest to engineers and which would serve as a connecting medium between the engineer and the public welfare in matters of interest to the engineering profession. The first meeting of the Council was held on June 27, 1917.

For details regarding the field, aims, and activities of the Council, members are referred to the numerous statements published from time to time in the monthly Institute JOURNAL. A resume of the annual report covering the activities of the Council for the year ending February 1920, was published in the March 1920 JOURNAL.

Engineering Foundation.—The Engineering Foundation is a fund established by the United Engineering Society on January 27, 1915 through the generosity of Mr. Ambrose Swasey, of Cleveland.

The purpose of the Engineering Foundation is the advancement of the engineering arts and sciences in all their branches to the greatest good of the engineering profession and the good of mankind, which it is proposed to accomplish largely through the promotion of engineering research.

The Engineering Foundation is administered by a board upon which the Institute and other national engineering societies are represented. The annual report of the Engineering Foundation was published in the March 1920 JOURNAL.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and in addition has appointed representatives upon a number of new organizations, some of the more recent being the Franco-American Engineering Committee, the Joint Committee for Standard Thread for Insulators and Insulator Pins, and on Special Joint Committee on Determination of Power Factor in Polyphase Circuits. A complete list of representatives is published monthly in the JOURNAL.

Edison Medal.—The Edison Medal for 1918, which had been awarded to Benjamin G. Lamme "For Invention and Development of Electrical Machinery" was presented to Mr. Lamme with appropriate ceremonies at the Annual Meeting of the Institute held in New York on May 16, 1919. The Edison Medal for 1919 has been awarded to W. L. R. Emmet "For Inventions and Developments of Electrical Apparatus and Prime Movers" and the presentation will take place at the Annual Meeting of the Institute which will be held in New York on May 21, 1920.

John Fritz Medal.—The John Fritz Medal Board of Award which is composed of representatives of the national societies of Civil, Mining, Mechanical and Electrical Engineers, awarded the 1920 medal to Orville Wright. The medal was presented to Mr. Wright with appropriate ceremonies in the auditorium of the Engineering Societies Building, New York, on the evening of May 7, 1920.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report is included herein.

NEW YORK
CHICAGO
PHILADELPHIA
DETROIT
CLEVELAND
BOSTON
SAINT LOUIS
BALTIMORE
PITTSBURGH
SAN FRANCISCO
LOS ANGELES
NEW ORLEANS
SEATTLE
DENVER
ATLANTA
WATERTOWN
LONDON

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
CABLE ADDRESS "HASKSELLS"
30 BROAD STREET
NEW YORK

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

CERTIFICATE OF AUDIT

We have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1920, and

WE HEREBY CERTIFY that the accompanying General Balance Sheet properly sets forth the financial condition of the Institute on April 30, 1920, that the Summary of Income and Profit & Loss for the year ended that date is correct, and that the books of the Institute are in agreement therewith.

HASKINS & SELLS,
Certified Public Accountants.

NEW YORK,
May 17, 1920.

AMERICAN INSTITUTE OF

EXHIBIT A.

GENERAL BALANCE SHEET,

ASSETS

REAL ESTATE:			
One-fourth Interest in United Engineering Society's Real Estate, 25 to 33 West 39th Street:			
Land and Building		$472,500.00	
Real Estate Equipment		14,292.79	
Total Real Estate			$486,792.79
EQUIPMENT:			
Library—Volumes and Fixtures		$ 40,154.55	
Works of Art, Paintings, etc.		3,001.35	
Office Furniture and Fixtures		12,927.38	
Total		$ 56,083.28	
Less Reserve for Depreciation		9,470.07	
Remainder—Equipment			46,613.21
INVESTMENTS:			
Bonds—City of Wilmington, Delaware, 4¼%, 1934, Par Value $15,000.00		$ 15,729.91	
United States Third Liberty Loan 4¼% Bonds		10,000.00	
Total Investments			25,729.91
WORKING ASSETS:			
Publications Entitled "Transactions," etc.		$ 13,657.25	
Paper and Cover Paper		3,562.25	
Paper for Volume 38 and Advertising		1,093.65	
Badges		2,416.54	
Total Working Assets			20,729.69
CURRENT ASSETS:			
Cash		$ 8,545.68	
Accounts Receivable:			
Members—for Past Dues		8,634.58	
Advertisers		3,520.33	
Miscellaneous		598.87	
Miscellaneous Printing and Subscriptions		227.42	
Accrued Interest on Investments		109.38	
Accrued Interest on Bank Balance		260.23	
Total Current Assets			21,896.49
FUNDS:			
Life Membership Fund:			
Cash	$1,438.67		
Chicago, Burlington & Quincy Railroad Company Bonds, 4%, 1958, Par Value $5,000.00	4,868.75		
Accrued Interest	33.33	$ 6,340.75	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 281.01		
New York City Bonds, 4¼%, 1957, Par Value, $2,000.00	2,235.95		
New York Telephone Company Bond, 4¼%, 1939, Par Value $1.000.00	878.75		
Accrued Interest	67.50	3,463.21	
MAILLOUX FUND:			
Cash	$ 186.60		
New York Telephone Company Bond, 4¼%, 1939, Par Value	1,000.00		
Accrued Interest	22.50	1,209.10	
Midwinder Convention Fund—Cash		123.10	
Total Funds			11,136.16
Total			$612,898.25

ELECTRICAL ENGINEERS.

APRIL 30, 1920.

LIABILITIES

CURRENT LIABILITIES:		
Accounts Payable—Subject to Approval by the Finance Committee	$ 11,960.71	
Due United Engineering Society, Account Building Addition, Including Accrued Interest	5,084.11	
Dues Received in Advance	4,073.03	
Entrance Fees and Dues Advanced by Applicants for Membership	561.00	
Total Current Liabilities		$ 21,678.85
FUND RESERVES:		
Life Membership Fund	$ 6,340.75	
International Electrical Congress of St. Louis—Library Fund	3,463.21	
Mailloux Fund	1,209.10	
Midwinter Convention Fund	123.10	
Total Fund Reserves		11,136.14
SURPLUS: Per Exhibit "B"		580,083.26
Total		$612,898.25

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

SUMMARY OF INCOME AND PROFIT AND LOSS

FOR THE YEAR ENDED APRIL 30, 1920.

EXHIBIT B.

REVENUE:			
Entrance Fees		$ 9,940.00	
Dues, Current and Past		*119,072.24	
Student's Dues		5,736.00	
Transfer Fees		1,195.00	
Sales of Publications, Badges and Advertising		29,297.81	
Interest on Investments		1,100.00	
Interest on Bank Balance		985.72	
Gain on Exchange		94.02	
Total			$167,420.79
EXPENSES:			
Publications:			
Journal	$54,217.38		
"Transactions"	18,310.70		
Year Book	5,171.60	$ 77,699.68	
Meetings		6,626.26	
Administrative Expenses		35,821.60	
Sections Committee		15,152.28	
Membership Committee		2,633.84	
Standards Committee		1,082.86	
Finance Committee		150.00	
Code Committee		30.00	
Commitees on Institute Development		442.67	
Dues—International Electrotechnical Commission		250.00	
Interest on United Engineering Society Building Loan		277.06	
President's Special Appropriation		234.89	
Honorary Secretary		4,000.00	
American Engineering Standards Committee		1,300.00	
John Fritz Medal Award		41.30	
Engineering Societies Employment Bureau		2,500.00	
Engineering Council		4,333.36	
Engineering Societies Library, Maintenance		4,000.00	
Engineering Societies Library, Recataloging		2,500.04	
United Engineering Society Assessment		4,875.00	
Amortization of Premium on City of Wilmington Bonds		52.14	
Accounts Payable—Subject to Approval by the Finance Committee—for Expenses Undistributed		2,799.37	
Total			166,802.35
NET REVENUE—(Forward)			$ 618.44

51,260.00, applicable to subscriptions to the JOURNAL.

NET REVENUE—(Forward)		$ 618.44
PROFIT AND LOSS CREDIT—ADJUSTMENT OF INVENTORY OF LIBRARY VOLUMES AND FIXTURES, APRIL 30, 1919		307.00
GROSS SURPLUS FOR THE YEAR		$ 925.44
PROFIT AND LOSS CHARGES:		
Uncollectible Dues Written Off	$ 2,910.00	
Dues of Members in Military Service Written Off	1,348.75	
Total		4,258.75
DEFICIT FOR THE YEAR		$ 3,333.31
SURPLUS, MAY 1, 1919		583,416.55
SURPLUS, APRIL 30, 1920		$580,083.24

NOTE: No provision has been made for depreciation of Furniture and Fixtures as the present reserve appears to be in excess of requirements.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

STATEMENT OF CASH RECEIPTS AND DISBURSEMENTS FOR DESIGNATED PURPOSES, FOR THE YEAR ENDED APRIL 30, 1920.

EXHIBIT C.

RECEIPTS:	
Life Membership Fund	$1,220.66
International Electrical Congress of St. Louis Library Fund—Interest and Royalties	143.40
Mailloux Fund—Interest	45.00
Midwinter Convention Fund—Interest	5.46
Total	$1,414.52
DISBURSEMENTS:	
Life Membership Fund	220.66
Midwinter Convention Fund	18.25
Mailloux Fund	.75
Total	$239.66

RECEIPTS AND DISBURSEMENTS PER MEMBER.

During each fiscal year for the past eight years.

Year ending April 30	1913	1914	1915	1916	1917	1918	1919	1920
Membership, April 30, each year	7654	7876	8054	8212	8710	9282	10252	11345
Receipts per Member	$13.45	$14.08	$14.06	$13.62	$13.30	$13.17	$13.18	$15.01
Disbursements per Member	15.57	12.86	13.54	13.74	12.75	11.99	12.92	15.62
Credit Balance per Member	*$2.12	$1.22	$ .52	*$ .12	$ .55	$1.18	$ .26	*$ .61

*Deficit,

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON, *Secretary*.

New York, May 21, 1920.